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Affirmative Action, Shifting Competition, and Human Capital Accumulation: A Comparative Static Analysis of Investment Contests

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Affirmative Action, Shifting Competition, and Human Capital Accumulation: A Comparative Static Analysis of Investment Contests

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Abstract. We develop a model in which many heterogeneous agents invest in human capital as they compete for better college admission slots or employment opportunities. The model provides theoretical predictions about how affirmative action or preferential treatment policies change the distribution of effort, human capital accumulation, and job/college slot allocations across different population groups. Our findings deliver two key insights. First, incentives to invest in human capital depends substantially on the strength of one's competition. Second, we find evidence of a counter-intuitive role for preferential treatment in promoting overall human capital development.

Key words and phrases. Large Contest, All-Pay Auction, Affirmative Action, College Admissions, Field Experiment, Human Capital.

JEL subject classification: J15, J24, C93, D82, D44.

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1. Introduction

Human capital (HC) investment has become a central theme in economics research. There is a vast literature studying how incentives to invest in HC stem from its ability to directly enhance a worker's productivity or quality of life. The majority of this literature concentrates on single-agent models of HC acquisition, especially through formal educational pursuits. However, many important real-world settings involve competitive investment, where agents' expected rewards depend not only on their own HC output, but also on whether they out-invest their competitors vying for the same job, college seat, or promotion. When relative investment is used as a rank-order index to allocate non-divisible rewards, the returns to one's own HC hinge on choices of one's rivals, and therefore competitive forces may also play an indirect, though important, role in educational choices. The economics literature applies contest theory to model a variety of environments involving allocation of fixed, non-divisible resources, from competition across firms for contracts, to competition within a firm for promotion, to competition across students for grades. Most of the existing literature focuses on settings where agents are homogeneous, or where there are a small number of heterogeneous agents or prizes. However, real-world settings often involve large-scale, many-to-many, rank-order matching between heterogeneous agents and heterogeneous prizes.

Economically important examples of many-to-many matching and competitive investment abound in secondary education, post-secondary education, labor markets, and matching markets. We briefly survey several prominent motivating examples in the next section below. Each one shares three salient features in common: (1) agent heterogeneity—a wide array of competitors differ in their investment costs; (2) prize heterogeneity—schools, programs, jobs, firms, or match partners differ in value, holding agent characteristics fixed; and (3) a dual role for investment—producing both direct, intrinsic returns to the investor, and indirect returns by controlling his or her access to a better reward or match partner. We combine these three features into a model of rank-order competitive HC investment. The model is adapted from a more general theoretical framework developed by Bodoh-Creed and Hickman [2018], with our focus being on a simplified version that allows for comparative static predictions. Our goal is to explore how incentives for HC investment vary with shifts in competition, and how the impact may depend on individual characteristics. In the model, a continuum of agents have a privately-known cost of HC production θ and there is a fixed continuum of vertically heterogeneous "prizes" P. There is a rank-order mechanism which allocates prizes based on relative observed HC production, h, and ex-post match utility is determined by both prize quality and HC output.

Our model delivers two novel insights concerning how behavior reacts to changes in the intensity of competition. First, holding fixed an agent's own cost type and the set of prizes, a stochastic dominance shift toward lower investment costs among the agent's competitors leads a low-cost agent to invest more aggressively, and a medium- or high-cost agent to reduce investment. In other words, the strongest agents invest more, while weaker agents invest less, as the strength of their competitors increases. We refer to this latter shift as the *discouragement effect*. Moreover, for any cost type θ in the interior of the support there exists some stochastic shift toward lower cost competitors such that θ will become discouraged and reduce investment. In that sense, the distinction between "low-cost" and "high-cost" agents is relative as well, being a function of one's quantile rank, rather than one's absolute ability level.

This comparative static is interesting because it involves no change to the direct marginal costs or benefits of improving one's HC. There are two canonical economic models which have been used to explain educational choices: the Becker model [Becker, 1973a] which interprets schooling as investment in productive HC, and the Spence model [Spence, 1973] which interprets schooling as a signaling game to separate agents by their unobservable types in an incentive-compatible way. Consensus among labor economists has evolved to predominantly favor the Becker interpretation in light of ample empirical evidence that education improves worker productivity and quality of life in a variety of ways (see Becker [1993] for a detailed discussion). Nevertheless, our first main result illustrates how Spence-like competition can play an economically important role in determining the intensity of educational activities, even when the result of education is intrinsically valuable HC. This Spence component of incentives is manifest in the form of "over-investment" above and beyond a level that can be rationalized by the direct marginal returns to holding an additional unit of productive HC.

Our second theoretical result establishes a somewhat surprising role for preferential treatment in provision of investment incentives. Suppose that there is some observable binary characteristic R that is correlated with unobservable types, so that increasing R from 0 to 1 induces a stochastic dominance shift in the distribution of human capital production costs. For ease of discussion we will refer to the group of agents for whom $R = 1$ as the *disadvantaged group*.[1] Within this context, we consider the effects of a simple policy that provides preferential treatment to the disadvantaged group: a representative quota, which reserves a proportional set of prizes for the disadvantaged group, before the competition begins. This means that disadvantaged agents compete only among themselves, effectively increasing the cost distribution of their competition compared to a situation in which all agents compete against all other agents. A representative quota leads to the majority of disadvantaged agents *increasing* their investment, and to a small share of top agents reducing their investment. Intuitively, preferential treatment can improve average investment of disadvantaged agents by shifting the distribution of competitors and thereby mitigating discouragement effects. Predictions for the $R = 0$ group are in opposite directions, but with theoretical ambiguity about how the average agent in that group will react.

These two theoretical predictions have several policy-relevant implications. First, it has been widely speculated that today's admissions process for elite colleges represents a burdensome academic arms race that imposes unnecessary costs on students and their families. Our first result shows that a shift in the distribution of competitors without any change to market-wide school quality leads elite students to further escalate the arms race. Examples of phenomena that might lead to a shift in the equilibrium distribution of HC include proliferation of new investment technologies—*e.g.*, advanced-placement courses and extra-curricular academic programs—and universities marketing their services to top students from abroad as a means of improving revenues, which would tend to select additional low-cost agents into the pool of competitors.

Second, preferential treatment schemes have become commonplace in many settings, including race-based affirmative action (widespread among many university systems worldwide), and income-based preferential admissions rules like the Texas Top 10% program. Traditional wisdom has held that these

[1] Throughout we assume that cost types exist on a common support but relative masses of high and low costs differ across groups.

programs will generally erode incentives by lowering admissions standards. Our second theoretical result largely refutes this idea: a preferential treatment policy which targets disadvantaged students may actually *increase* their effort by placing within reach outcomes which would otherwise be unattainable. This mitigates discouragement effects that arise when disadvantaged agents find themselves too far behind the competitive curve. It is even possible, within the model, to mitigate allocative inequality through a preferential admission policy while at the same time narrowing demographic achievement gaps and increasing overall HC production.

Finally, our second theoretical result tells us something about the potential sources of achievement gaps in primary and secondary education. In the US, there is a vast disparity in developmental resources (e.g., healthcare, K-12 school quality, supplementary learning aids) accessible to rich and poor. If we think of the idiosyncratic production technology θ as encapsulating factors both innate and external to the individual, then one would expect students in under-resourced schools to have systematically higher learning costs. Conventional wisdom has held that asymmetric resources creates achievement gaps in a mechanical way, simply by creating gaps in human capital production technology. However, our second theoretical result illustrates how asymmetric funding and rank-order incentives can have a combined synergistic impact. The comparative static scenario in our theory (and experimental design) holds gross payoffs and individual production technology fixed, but produces meaningful changes in learning behaviors, simply by altering the distribution of one's competitors. In short, our model suggests that academic achievement gaps are endogenous equilibrium objects, and should not be thought of as exogenous or mechanical.

Several questions remain as to the empirical relevance of these theoretical insights. In order for relative investment incentives to be a first-order consideration for social scientists and policy-makers, real-world labor-leisure decisions must be consistent with the complex, strategic, HC investment decisions that take place in the Bayes-Nash equilibrium. Moreover, this must be true at all ability levels, including for middle- and low-ability participants, and not just among the best and brightest. In a companion paper, Cotton et al. [2020], we report results from an academic field experiment in partnership with real schools that was designed to investigate these concerns empirically. The experiment involves paying large groups of middle-school students based on their relative performance on a mathematics exam. Our experiment includes students from two adjacent grades competing in math achievement for fixed monetary payoffs of varying value, where students in the lower grade (having one year less math education) serve as the disadvantaged group. Students are individually randomized into two treatments for our math competition: a control treatment in which both grade cohorts compete head-to-head, and a quota treatment, where a proportional set of prizes is reserved for the students in the disadvantaged group. The representative quota alters the set of relevant competitors, since it implies that competition occurs only within one's own group. Therefore, our treatment and control groups can be interpreted as creating a counterfactual scenario which simultaneously tests both of our comparative static results on shifts in competition and/or preferential treatment. The experimental evidence reported in Cotton et al. [2020] provides strong support for theoretical predictions on strategic forces that shape HC choices: the subjects, who range from fifth through eighth grade, respond to changes in relative incentives in a way remarkably consistent with sophisticated, Bayes-Nash behavior depicted in our game-theoretic model.

1.1. **Motivating Examples.** Before moving on, we breifly survey various prominent examples of the kinds of competitive investment contests that motivate our model and experimental study. Each of the examples discussed below shares the basic salient features depicted in our model: agent heterogeneity, many-to-many matching, and a dual role for education/effort as HC production and a rank-order index for allocation of non-divisible resources.

The academic arms race among American high-school students for university admissions is an annual competition for roughly 2 million freshman seats at a diverse set of post-secondary institutions ranging from Ivy League schools to small, regional colleges. Throughout high school, students exert effort in their studies and extra-curricular activities to acquire abilities and experiences which produce valuable human capital, but also increase their relative standing during the application process through measures such as GPA and standardized exam scores. Similarly, more than 500,000 college undergraduates take the GRE (Graduate Record Examination) each year, and approximately 100,000 take the LSAT (Law School Admission Test) as they compete for admissions to graduate programs and law schools of varying quality. These exam scores and their GPAs serve as both a reflection of real HC acquired during college, and as a rank-order index for post-graduate admissions.

European post-secondary admissions work similarly. In France, tens of thousands of secondary school graduates each year enter an intensive two-year course of study—*classes préparatoires aux grandes écoles*, or "prepa" for short—in hopes of gaining admission to an elite public university, or *grande école*. At the end of prepa, students take written and oral exams to gauge their learning, and their scores result in an individual ranking against all other test takers nationally. These rankings then determine a student's ordering for choice of limited slots at one of 250 grandes écoles; students who do not rank sufficiently high must settle for admission to non-elite universities instead. Aside from establishing a ranking, it is widely believed that these examinations are a meaningful reflection of technical skills and knowledge needed for success in their continuing studies.

Another prominent example is career track assignment in the United States armed forces. Each year thousands of people enroll for officer training in the three service academies—the US Military Academy (West Point), the US Naval Academy, and the US Air Force Academy—which are designed to teach academic knowledge, leadership, and military skills, and one's performance in acquiring and demonstrating these skills contributes to an explicit cumulative score. For example, the US Air Force Academy feeds an annual class of 1,000 graduates into dozens of career tracks, the most desirable (and best paid) of which are pilot, intelligence, JAG (law), and medical. For each cohort there are a limited number of slots open for each track, and a cadet's cumulative score rank determines the order in which he or she may choose. US Air Force Flight School trains a large fraction of the pilots who eventually end up in American commercial aviation, and is run similarly. For the roughly 1,000 junior officers who go on to flight school each year, training takes place in two stages where performance on every learning task, both in and out of the classroom, is measured and added to a cumulative score. In the first stage one's score rank determines order of choice for limited slots in different flight tracks, with fighter/bomber being the most highly paid and sought-after track. Scores are then re-set at the beginning of the second stage, and one's final score rank determines order of choice for limited slots pertaining to specific planes which pilots will go on to fly.

Such competition also extends to the post graduate job market. In 2015, approximately 39,000 new U.S. law school graduates competed for a diverse set of approximately 19,000 job openings requiring a law degree.[2] Large-scale investment competitions are not limited to academic pursuits and job markets either. Promising athletes hoping to play at the collegiate or professional levels are in competition with every other athlete having similar aspirations. Their investment of time and effort to train and/or learn proper technique differ from the other examples here in that it produces both physical and cognitive human capital. Finally, the marriage market may be seen as competition amongst many suitors for spouses, where real-effort investment occurs on traits such as physical condition, education, and/or income. While being more physically fit, more educated, and earing higher income clearly benefits one directly outside of marriage, there is evidence that these traits also help one attract a mate with more desirable characteristics as well (see [Hitsch et al., 2010]).

1.2. **Related Literature.** Olszewski and Siegel [2016] and Bodoh-Creed and Hickman [2018] develop models of large-scale contests with many heterogeneous agents and prizes, though the former has a somewhat different focus from our analysis in that it does not readily admit a human capital interpretation of effort. The latter paper, Bodoh-Creed and Hickman [2018], provides the general theoretical framework on which our model is based. The virtue of this framework is that it is rich enough to encompass a large set of possibilities, but the drawback is that with so many moving parts it is difficult to prove sharp comparative statics predictions for behavioral responses to policy change. Therefore, we present a version of the model making reasonable assumptions regarding function forms in order to explore how the distribution of effort and achievement depend on demographic group and affirmative action policies; such comparative static analyses are not possible under the more general framework from Bodoh-Creed and Hickman [2018], which instead focuses on foundational theory which establishes existence of equilibrium and explores market design questions within a broad class of mechanisms. In this paper, our focus is more narrow in one way and more broad in another.

Our paper also contributes to the literature on effort and contest design. Galton [1902] first considered how to divide a fixed pot of prize money between two prizes in a contest, and how the result depends on the number of competitors. More recently, papers have considered the optimal allocation of prizes in game theoretic models of contests [e.g. Moldovanu and Sela, 2001, 2006]. Our paper focuses on contests with an exogenous distribution of heterogenous prizes, and considers how the distribution of competitors within a contest affects effort provision and performance. The reason that the distribution of competitor ability (given a set of prizes) affects performance in our environment is similar to the reason that the distribution of prizes (given a set of competitors) affects performance in the other environment. Fullerton and McAfee [1999] and Che and Gale [2003] explore how a contest designer may be able to influence the distribution of competitor ability, but in very different ways than us. None of the earlier work involves large-scale contests, or an environment with many heterogeneous prizes.

Additionally, our paper contributes to the literatures on affirmative action and effort incentives. Earlier theoretical work includes Coate and Loury [1993] and Moro and Norman [2003]. The key differences between our model and previous work is a combination of many-to-many matching with asymmetry across demographic groups and scarcity of high value positions. In our large-contest framework, agents

[2]See https://lawschooltuitionbubble.wordpress.com/original-research-updated/law-graduate-overproduction/

compete with one another for a fixed set of heterogeneous prizes, leading to type-specific predictions. Although a preferential treatment policy will decrease incentives for the highest ability beneficiaries, the opposite effect will predominate for most of them.

There is a substantial empirical literature studying preferential treatment regimes in college admissions, especially race-based affirmative action. Two papers by Bodoh-Creed and Hickman [2019] and Akhtari et al. [2019] represent empirical investigations which are to varying degrees based on the ideas in the Bodoh-Creed and Hickman [2018] framework; that is, the idea that measured academic proficiency is partly determined by competitive forces arising from affirmative action.[3] Furthermore, as discussed in the introduction, Cotton et al. [2020] uses an experiment designed based on the theoretical framework in the current paper, showing that middle-school aged children respond to affirmative action incentives as predicted by our theory.

2. Model

Our setup is an adaptation of the model from Bodoh-Creed and Hickman [2018, henceforth, BCH].[4] By making assumptions around the functional form of agent utility functions, for example, our model gain enough analytic tractability to prove sharp comparative static predictions for behavioral responses to changes in the level of competition.

There is a continuum of agents of mass 1 who exert effort to accumulate observable human capital (HC). Their HC output also establishes a rank ordering among them as they compete for prizes. There is a mass 1 of prizes, which differ in their value, and which are awarded to agents based on relative achievement. Agents differ by their unobservable, idiosyncratic cost of achievement.

Each agent i chooses a level of HC achievement h_i in the simultaneous-moves contest. Acquiring HC requires time and effort, and is therefore thought to be costly. Agents differ by their background, abilities, access to help, and other resources that affect the rate at which time is converted into new HC. At the individual level, these factors are summarized by a privately known parameter θ_i that enters an investment cost function $c(h_i;\theta_i)$, which is strictly increasing and convex in the amount of output: $c'(h_i;\theta_i) > 0$ and $c''(h_i;\theta_i) > 0$ for all $h_i \geq \underline{h}$. We also assume $\frac{\partial c}{\partial \theta} > 0$ and $\frac{\partial^2 c}{\partial h \partial \theta} > 0$ so that smaller θ implies a more productive agent having lower costs for a given level h, and also a lower marginal cost of increasing output to $h' > h$. There is a minimum level of HC necessary to receive a prize, $\underline{h} > 0$, and the highest-cost student in the market is willing to achieve at this level (e.g., a high-school diploma).

Individual prizes (e.g., college seats) vary by quality level $P \sim F_P(p)$. Throughout the paper, we simplify discussion by assuming F_P is the uniform distribution on $[\underline{p}, \overline{p}] = [0,1]$, so that prize quality

[3] Ferman and Assuncao [2011] found evidence that test scores among black Brazilian high-school students decreased in response to an admissions quota at elite universities in Rio de Janeiro. An early study by Bowen and Bok [1998] quantified the preference given to minority students by admissions officers at elite schools. A lengthy debate in the literature focuses on the mismatch hypothesis, including papers such as Loury and Garman [1995], Sander [2004], Long [2008], Rothstein and Yoon [2008], Chambers et al. [2005], Arcidiacono et al. [2016], and Dillon and Smith [2017]. Throughout this literature SAT—"Scholastic Aptitude Test"—scores are used as a proxy for student ability, and assumed to be fixed. However, students' incentives to invest in human capital during middle and high school depend on admission policies they expect to face when applying to college. Test scores are therefore a function of student ability *and* market incentives induced by affirmative action. Our results suggest that more attention to relative incentives is needed in empirical studies of observational datasets concerning college admissions.

[4] BCH develop foundational theory—i.e., existence of equilibria and exploration of broad classes of assignment mechanisms—and as such their model does not allow for the analysis of behavioral comparative statics.

index and quantile rank are the same. An agent matching with prize p_i and having produced h_i units of human capital experiences gross benefit

$$u(p_i, h_i) = p_i^{\alpha} h_i^{\beta}, \quad \alpha \in (0,1], \quad \beta \in [0,1].$$

Our imposition of a specific functional form for match utility is the principal simplification which provides analytic tractability. Our choice the Cobb-Douglas utility form in particular is inspired by seminal work on matching theory by Becker [1973b] who showed that positive complementarity in the match utility function implies that assortative matching is socially optimal. The fact that assortative matching is a strongly established empirical fact in various contexts (e.g., marriage markets and college admissions among others) is suggestive of the empirical relevance of complementarity in the matching function. The Cobb-Douglas family of utility functions admits positive cross-partial derivatives. However, in Section 5 below we consider an alternative model where prize value p and human capital h enter match utility additively in order to demonstrate that our theoretical results are not being fundamentally driven by complementarity *per se*.

In the case of Cobb-Douglas match utility, net utility is given by

$$U(p_i, h_i; \theta_i) = p_i^{\alpha} h_i^{\beta} - c(h_i; \theta_i).$$

WWe assume $U(\underline{p}, \underline{h}; \overline{\theta}) = U(\emptyset, 0; \overline{\theta})$; i.e., all agents in the game weakly prefer producing the minimum HC to not participating in the contest. Therefore, the model is one of decisions on the intensive margin; the question of how preferential treatment may affect participation decisions on the extensive margin is left for future research.

2.1. **Demographics.** Each agent observably belongs to one of two mutually exclusive demographic subgroups, $\mathcal{A}$ and $\mathcal{D}$, with $\delta \in (0,1)$ being the mass of the latter. Costs are privately known to each individual, and agents view rivals' types in group $j = \mathcal{A}, \mathcal{D}$ as a random variable Θ_j with distribution $F_j(\theta)$ and density $f_j(\theta)$ which is strictly positive on a common support $[\underline{\theta}, \overline{\theta}]$. As convenient shorthand, we denote the unconditional random variable and distribution by Θ and $F(\theta) \equiv \delta F_{\mathcal{D}}(\theta) + (1-\delta) F_{\mathcal{A}}(\theta)$.

We assume group $\mathcal{A}$ is "advantaged" and group $\mathcal{D}$ is "disadvantaged" in that a typical agent in $\mathcal{D}$ finds HC production more costly than a typical agent in $\mathcal{A}$. Formally, the distributions of $\Theta_{\mathcal{A}}$ and $\Theta_{\mathcal{D}}$ are ordered by *likelihood ratio (LR) dominance*, where

$$\frac{\partial \left(f_{\mathcal{D}}(\theta)/f_{\mathcal{A}}(\theta) \right)}{\partial \theta} > 0 \quad \text{for all} \quad \theta \in [\underline{\theta}, \overline{\theta}].$$

LR dominance implies that for any measurable event $T \subseteq [\underline{\theta}, \overline{\theta}]$ the distributions of $\Theta_{\mathcal{A}}$ and $\Theta_{\mathcal{D}}$, conditional on T, follow first-order stochastic dominance, or $F_{\mathcal{D}}(\theta|T) \leq F_{\mathcal{A}}(\theta|T) \ \forall \ \theta \in T$. LR dominance also implies a unique point, $\widetilde{\theta} \in \left(\underline{\theta}, \overline{\theta}\right)$, at which the densities $f_{\mathcal{A}}$ and $f_{\mathcal{D}}$ cross. This fact will be useful later on. Although some "disadvantaged" agents have relatively low costs of acquiring HC, and some "advantaged" agents have relatively high costs, a student at a given cost percentile in $\mathcal{A}$ has a lower cost than his counterpart at that same percentile in $\mathcal{D}$. This reflects the idea that, on average, disadvantaged agents must expend more time and effort to overcome obstacles correlated with their demographic status.[5]

[5]E.g., in the context of education and race it is well-known that African-American and Hispanic children in the US tend to be less affluent and have less access to crucial childhood inputs like health care and high-quality public education; however, some still grow up in affluent environments which are more advantageous to childhood learning.

2.2. **Prize Allocation Rules.** Let $H \sim G(H)$ denote the overall distribution of HC output, and let $H_j \sim G_j(H_j)$ be the group-specific distribution for $j = \mathcal{A}, \mathcal{D}$. The baseline prize allocation rule is a *pure rank order* (PRO) mechanism, which ignores demographic status. It determines the quantile rank of i's achievement h_i within $G(H)$, and then matches her to a prize at the corresponding quantile rank. For example, the 75^{th} percentile agent matches with the 75^{th} percentile prize, and so on. Formally,

$$P_j^{PRO}(h_i) = P^{PRO}(h_i) \equiv G(h_i), \quad j = \mathcal{A}, \mathcal{D}. \tag{1}$$

The other prize allocation rule we consider is a *representative quota* (RQ) preferential treatment policy, which reserves a similar distribution of prizes for each group, $\mathcal{A}$ and $\mathcal{D}$, and then allocates the prizes *within* each group by rank ordering. By "similar distribution" we mean that fraction δ of all prizes at each point in the quality spectrum are earmarked *ex ante* for group $\mathcal{D}$, thus splitting prizes into two subsets having mass δ and $(1-\delta)$, but with both subsets still following the original quality distribution F_P.[6] Formally, under the RQ rule, agent i receives prize

$$P_j^{RQ}(h_i) \equiv G_j(h_i), \quad j = \mathcal{A}, \mathcal{D}. \tag{2}$$

From an agent's perspective, the distinguishing characteristic of the RQ rule is that it alters the distribution of one's competitors, while leaving all other aspects of the contest—one's own cost and the set of all prizes under competition—the same as under a PRO rule.

BCH proved existence and uniqueness of Nash equilibrium for the game we study here. They also prove general conditions (all satisfied here) under which the equilibrium of a continuum representation of the game will closely approximate the equilibria of similar games with large but finite sets of players and prizes. The advantage to working with the continuum version of the game is in analytic and computational tractability. Our main goal in this section is to illustrate qualitative model predictions which are testable through experimental methods.

3. Comparative Static Analysis

Let $h^*(\theta)$ denote the common equilibrium investment function under the benchmark PRO rule, and let $h^*_{\mathcal{A}}(\theta)$ and $h^*_{\mathcal{D}}(\theta)$ denote the group-specific investment functions under the alternative RQ rule. Moreover, let θ^*, $\theta^*_{\mathcal{A}}$, and $\theta^*_{\mathcal{D}}$ denote the relevant inverses, so that $\theta^* \equiv h^{*-1}$ and $\theta^*_j \equiv h_j^{*-1}$, $j = \mathcal{A}, \mathcal{D}$. In the next subsection we begin by developing results for a simplified version of the model where we shut down the direct marginal benefit of HC in order to isolate the implications of strategic investment incentives. In the following two subsections, we show that the strategic aspect of the model plays largely the same role in a more realistic setting where HC is intrinsically valued by agents.

3.1. **Pure Strategic Incentives.** First consider a special case where $(\alpha, \beta) = (1, 0)$. Here, net utility under PRO competition takes the form

$$U\left[P^{PRO}(h_i), h_i, \theta_i\right] = P^{PRO}(h_i) - c(h_i; \theta_i).$$

[6]A subtle but important detail is that our *RQ* policy calibrates δ to the *fraction of group $\mathcal{D}$ market participants*, which may not be the same as the mass of $\mathcal{D}$ within the population at large. E.g., South Africa mandates racial quotas for skilled professions, where quotas are pegged to the fraction of blacks in the overall population. A challenge of implementation has been insufficient South African blacks with prerequisite post-secondary degrees to fulfill the mandate. A theory of how affirmative action affects the extensive margin (i.e., labor market entry/exit) would be needed for such a scenario.

Agent i chooses HC to maximize her net payoff, given that other agents play according to h^*. Equilibrium HC investment is strictly decreasing in θ, so equation (1) can be re-written as

$$P^{PRO}(h) = 1 - F\left[\theta^*(h)\right],$$

and agent i's objective as

$$\max_{h_i \geq \underline{h}} \left\{ (1 - F\left[\theta^*(h_i)\right]) - c(h_i; \theta_i) \right\}.$$

Taking a first-order condition (FOC), we get $-f(\theta_i)\theta^{*\prime}(h_i) = c'(h_i; \theta_i)$. In equilibrium, $\theta^*(h_i) = \theta_i$ for all i, and since h^* is the inverse of θ^*, it follows that $h^{*\prime}(\theta_i) = 1/\theta^{*\prime}(h_i)$. Therefore, a change produces the following differential equation:

$$h^{*\prime}(\theta_i) = \frac{-f(\theta_i)}{c'\left[h^*(\theta_i); \theta_i\right]}, \text{ with boundary condition } h^*(\overline{\theta}) = \underline{h}. \tag{3}$$

Given the assumptions on f and c, it is easy to see that h^* is strictly decreasing in θ since its derivative is negative. Equation (3) allows us to compare investment under two alternative cost distributions.

Theorem 1. *Consider two PRO contests,* 1 *and* 2*, which differ only by their cost distributions,* F_1 *and* F_2*, respectively. Assume further that competition is more fierce under contest* 2 *in the sense that* F_1 *strictly LR dominates* F_2*. Let* $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ *denote the unique crossing point of the density functions where* $f_1(\tilde{\theta}) = f_2(\tilde{\theta})$*. There exists a unique interior crossing point* $\ddot{\theta} \in (\underline{\theta}, \tilde{\theta})$*, such that* $h_1^*(\theta) < h_2^*(\theta)$ *for all* $\theta < \ddot{\theta}$ *and* $h_1^*(\theta) > h_2^*(\theta)$ *for all* $\theta \in (\ddot{\theta}, \overline{\theta})$*.*

Proof: Recall that LR dominance implies first-order stochastic dominance. Therefore, not only do the densities have a unique crossing point, but also $f_1(\theta) > f_2(\theta)$ for $\theta > \tilde{\theta}$, and $f_1(\theta) < f_2(\theta)$ for $\theta < \tilde{\theta}$. Since the same boundary condition applies to both contests, $h_1^*(\overline{\theta}) = h_2^*(\overline{\theta}) = \underline{h}$, then equation (3) implies the initial trajectories at the boundary point are ordered in the following way:

$$h_1^{*\prime}(\overline{\theta}) = \frac{-f_1(\overline{\theta})}{c'(\underline{h}; \overline{\theta})} < \frac{-f_2(\overline{\theta})}{c'(\underline{h}; \overline{\theta})} = h_2^{*\prime}(\overline{\theta}).$$

This in turn means that $h_1^*(\theta) > h_2^*(\theta)$ within a neighborhood of $\overline{\theta}$ since the investment functions are continuous and everywhere differentiable. Note that because slopes are negative $h_1^{*\prime}(\theta) < h_2^{*\prime}(\theta)$ means h_1^*, rises in the leftward direction and is *more steep* at θ.

Now suppose there exists at least one point where h_1^* and h_2^* cross, and let $\ddot{\theta} \in (\underline{\theta}, \overline{\theta})$ denote the maximum of all such possible points, with $\ddot{h} \equiv h_1^*(\ddot{\theta}) = h_2^*(\ddot{\theta})$, if any exist. Since h_1^* crosses h_2^* from above at $\ddot{\theta}$, it must be that $h_1^{*\prime}(\ddot{\theta}) \geq h_2^{*\prime}(\ddot{\theta})$ (*i.e.*, h_1^* is *less steep* at the crossing point). However, since $f_1(\theta) > f_2(\theta)$ on $(\widetilde{\theta}, \overline{\theta}]$ by LR dominance, and since $h < h'$ implies $c'(h'; \theta) \geq c'(h; \theta)$ by convexity, the following must be true for any $\theta \in (\tilde{\theta}, \overline{\theta})$:

$$h_1^{*\prime}(\theta) = \frac{-f_1(\theta)}{c'\left[h_1^*(\theta); \theta\right]} < \frac{-f_2(\theta)}{c'\left[h_2^*(\theta); \theta\right]} = h_2^{*\prime}(\theta) \quad \Rightarrow \quad h_1^*(\theta) > h_2^*(\theta). \tag{4}$$

Therefore, $\ddot{\theta} < \tilde{\theta}$, if such a point exists. Similarly, since $f_1(\theta) < f_2(\theta)$ on $[\underline{\theta}, \widetilde{\theta})$, then any crossing point would have to obey $h_1^{*\prime}(\ddot{\theta}) = -f_1(\ddot{\theta})/c'(\ddot{h}; \ddot{\theta}) > -f_2(\ddot{\theta})/c'(\ddot{h}; \ddot{\theta}) = h_2^{*\prime}(\ddot{\theta})$. This means that h_1^* can only cross h_2^* from above (*i.e.*, at points where it is *less steep*) and so there can be at most one such crossing.

Finally, to see why a crossing point must exist, suppose for a contradiction that for all $\theta \in (\underline{\theta}, \overline{\theta})$ we have $h_1^*(\theta) > h_2^*(\theta)$. In that case, it follows that

$$\begin{aligned} h_1^*(\underline{\theta}) &= \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{c'[h_1^*(u);u]} f_1(u)du + \underline{h} < \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{c'[h_1^*(u);u]} f_2(u)du + \underline{h} \\ & \qquad\qquad\qquad \leq \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{c'[h_2^*(u);u]} f_2(u)du + \underline{h} = h_2^*(\underline{\theta}), \quad \rightarrow\leftarrow . \end{aligned} \tag{5}$$

The strict inequality follows because the first and second line depict expectations over the decreasing function $1/c'[h_1^*(u);u]$ and f_2 places more weight on strictly lower values of u (or higher values of the function). The weak inequality follows from the supposition and from c being convex and having a positive cross-partial derivative. Thus we have a contradiction, so a unique crossing $\ddot{\theta}$ exists on the open interval $(\overline{\theta}, \tilde{\theta})$ and the theorem is proved. ■

Theorem 1 provides useful insight into how competition shapes incentives. It says that, holding fixed the set of all prizes, an increase in the degree of competition will cause the most-able agents to invest more aggressively in their accumulation of HC, and will cause the less-talented agents to withdraw somewhat thereby decreasing their HC accumulation. This second shift is similar to a phenomenon in the literature on dynamic contests (with finite players and a single prize) known as the *discouragement effect*. To our knowledge, this paper is the first to characterize discouragement effects in a many-to-many, static, rank-order contest setting.

The intuition embodied in the theorem is that since investment costs must be sunk before prizes are assigned, then, holding one's own cost type θ fixed, if the distribution of competitors shifts so that one's quantile rank falls low enough, investment incentives fall. Moreover, *any* cost type but the lowest possible type can conceivably become subject to discouragement. It directly follows from the result that for any interior cost type $\theta' > \underline{\theta}$, there is some stochastic dominance shift toward lower cost competitors that is extreme enough so that the crossing point of the densities falls in the interval $\tilde{\theta} \in (\underline{\theta}, \theta')$, and type θ' will reduce investment.

Note also that the qualitative pattern predicted by the theorem depends only on the type of shift in competitor costs—being one of LR dominance—and *not* on its magnitude—i.e., how much the mean changed. In other words, a unique interior crossing point $\ddot{\theta} \in (\underline{\theta}, \overline{\theta})$ exists even when $\int_{\underline{\theta}}^{\overline{\theta}} F_1(\theta) - F_2(\theta) d\theta$ is very large or very small, provided that the shift from F_2 to F_1 conforms to LR dominance. Ultimately, the magnitude of the difference $h_1^*(\theta) - h_2^*(\theta)$ is an empirical question which depends on the magnitude of the difference $F_1(\theta) - F_2(\theta)$. Note also that the common support condition $supp(F_1) = supp(F_2) = [\underline{\theta}, \overline{\theta}]$ ensures that even if the LR dominance shift is very large, there will always exist an interior crossing point of the investment profiles with some positive fraction of low-cost agents under F_2 increasing investment when competition becomes more fierce by any margin, large or small.

There is much insight to be had from Theorem 1 when comparing alternative allocation rules as well. Under an RQ mechanism granting preferential treatment to group $\mathcal{D}$, equation (2) can be re-written as $P_j^{RQ}(h) = 1 - F_j\left[\theta_j^*(h)\right], \ j = \mathcal{A}, \mathcal{D}$, and the objective for agent i from group j is now

$$\max_{h_i \geq \underline{h}} \left\{ \left(1 - F_j\left[\theta_j^*(h_i)\right]\right) - c(h_i;\theta_i) \right\}.$$

The difference here is that the group-specific HC distribution determines allocations rather than the unconditional one. That leads to the following FOC

$$h_j^{*\prime}(\theta_i) = \frac{-f_j(\theta_i)}{c'\left[h_j^*(\theta_i);\theta_i\right]}, \text{ with boundary condition } h_j^*(\overline{\theta}) = \underline{h}, \;\; j = \mathcal{D}, \mathcal{A}. \tag{6}$$

Recall our assumption that the random variable $\Theta_{\mathcal{D}}$ LR dominates $\Theta_{\mathcal{A}}$, or in other words, the ratio $f_{\mathcal{D}}(\theta)/f_{\mathcal{A}}(\theta)$ is strictly increasing in θ. This also implies that the non-group-specific random variable Θ LR dominates $\Theta_{\mathcal{A}}$, and that $\Theta_{\mathcal{D}}$ LR dominates Θ as well. To see why, note that

$$\frac{f(\theta)}{f_{\mathcal{A}}(\theta)} = \frac{\delta f_{\mathcal{D}}(\theta) + (1-\delta) f_{\mathcal{A}}(\theta)}{f_{\mathcal{A}}(\theta)} = \delta \frac{f_{\mathcal{D}}(\theta)}{f_{\mathcal{A}}(\theta)} + (1-\delta),$$

from which it follows that $f(\theta)/f_{\mathcal{A}}(\theta)$ is strictly increasing in θ. Likewise,

$$\left(\frac{f_{\mathcal{D}}(\theta)}{f(\theta)}\right)^{-1} = \frac{\delta f_{\mathcal{D}}(\theta) + (1-\delta) f_{\mathcal{A}}(\theta)}{f_{\mathcal{D}}(\theta)} = \delta + (1-\delta)\frac{f_{\mathcal{A}}(\theta)}{f_{\mathcal{D}}(\theta)},$$

so $f_{\mathcal{D}}(\theta)/f(\theta)$ is strictly increasing in θ.

Given this, switching between allocation rule PRO—where the competition group is all agents—and the RQ rule—where competition occurs only within one's own group—entails an effective LR dominance shift in the distribution of rivals' costs while holding the distribution of prizes fixed. This fact leads to the following result:

Theorem 2. *Assume $F_{\mathcal{D}}$ strictly LR dominates $F_{\mathcal{A}}$. Let $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ denote the unique crossing point of the cost densities where $f(\tilde{\theta}) = f_{\mathcal{A}}(\tilde{\theta}) = f_{\mathcal{D}}(\tilde{\theta})$, let $h_j^*(\theta),\; j = \mathcal{A}, \mathcal{D}$ denote the equilibrium investment strategies under a RQ admissions rule, and let $h^*(\theta)$ denote the common investment strategy under PRO admissions. Then there exist crossing points $\ddot{\theta}_{\mathcal{A}}, \ddot{\theta}_{\mathcal{D}} \in (\underline{\theta}, \tilde{\theta})$, such that*

(i) $h_{\mathcal{D}}^*(\theta) < h^*(\theta)$ for all $\theta < \ddot{\theta}_{\mathcal{D}}$ and $h_{\mathcal{D}}^*(\theta) > h^*(\theta)$ for all $\theta > \ddot{\theta}_{\mathcal{D}}$, and
(ii) $h_{\mathcal{A}}^*(\theta) > h^*(\theta)$ for all $\theta < \ddot{\theta}_{\mathcal{A}}$ and $h_{\mathcal{A}}^*(\theta) < h^*(\theta)$ for all $\theta > \ddot{\theta}_{\mathcal{A}}$.

In words, under RQ admissions the academically strongest agents in group $\mathcal{D}$ *decrease* HC investment, as competition for the top prizes becomes less intense, while higher-cost individuals in group $\mathcal{D}$ exert greater effort and *increase* HC achievement, as a RQ mitigates discouragement effects by placing them in a competition group where they are less far behind the curve. Similar reasoning implies effects of the opposite sign for group $\mathcal{A}$. However, the crossing points $\ddot{\theta}_{\mathcal{A}}$, $\ddot{\theta}_{\mathcal{D}}$ need not coincide.

Since theory predicts θ-specific behavioral responses of differing magnitudes and signs, the result above begs the question of which effect will dominate. To answer this question we can use the density crossing point $\tilde{\theta}$ to partition the cost support into subsets, $T_{\mathcal{A}} \equiv (\underline{\theta}, \tilde{\theta})$ and $T_{\mathcal{D}} \equiv (\tilde{\theta}, \overline{\theta})$. We refer to these as the *typical cost sets* for each group, since T_j is the region of the support where group j is over-represented. Figure 1 provides a graphic illustration of typical cost sets, given two normally distributed type distributions. LR dominance implies some interesting properties for these sets. Since the densities have a unique crossing and since both must integrate to 1, it follows that $\int_{T_{\mathcal{A}}} [f_{\mathcal{A}}(\theta) - f_{\mathcal{D}}(\theta)] = \int_{T_{\mathcal{D}}} [f_{\mathcal{D}}(\theta) - f_{\mathcal{A}}(\theta)]$, i.e., the degree of over-representation of group $\mathcal{D}$ in the high-cost set $T_{\mathcal{D}}$ is the same as the degree of over-representation of group $\mathcal{A}$ within the low-cost set $T_{\mathcal{A}}$.

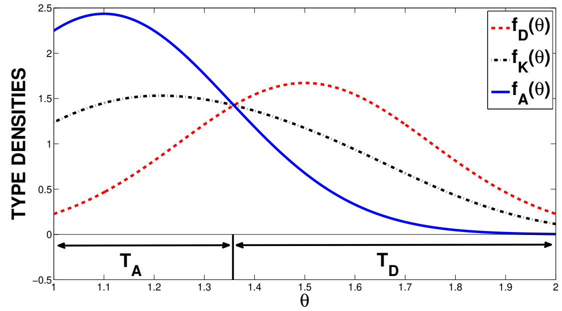


FIGURE 1. Typical Cost Sets

Theorem 3. *Assume the same conditions as in Proposition 2. Then under the RQ rule (relative to the PRO rule), typical disadvantaged agents—that is, group $\mathcal{D}$ agents with costs $\theta \in \left\{(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}] \cup T_{\mathcal{D}}\right\}$—exert higher effort and accumulate more HC. Moreover, if we define $\Delta : (\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}] \cup T_{\mathcal{D}} \to \mathbb{R}$ as the difference on this set between group $\mathcal{D}$ investment under RQ versus PRO, or $\Delta(\theta) \equiv (h^*_{\mathcal{D}}(\theta) - h^*(\theta))$, then $\Delta(\theta)$ is strictly positive and attains a maximum on the interval $(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}]$. Moreover, if investment costs are strictly convex in h, then $\Delta(\theta)$ attains its maximum on the open interval $(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta})$.*

Proof: Theorem 2 directly implies the first part of the result, and that $\Delta(\cdot)$ tends toward zero at its endpoints but is strictly positive everywhere else. Moreover, equation (4) in the proof of Theorem 1 establishes that $\Delta'(\theta) < 0$ for all $\theta \in T_{\mathcal{D}}$, meaning the difference between the two investment functions becomes steadily wider as one moves toward the density crossing $\tilde{\theta}$ from the left. If costs are strictly convex, then (4) shows that $\Delta'(\theta) < 0$ for all $\theta \in T_{\mathcal{D}} \cup \tilde{\theta}$, so the maximum cannot occur at $\tilde{\theta}$. ■

Theorem 3 provides some useful intuition on behavioral response predictions at the group level. It implies that a majority of group $\mathcal{D}$ agents actually *increase* HC investment under the preferential treatment scheme. To gain an appreciation for the strength of this result, the proposition also shows that the improved incentives extend to cost types outside the typical set $T_{\mathcal{D}}$ as well. In fact, the largest improvement of investment incentives by type (*i.e.*, where $\Delta(\cdot)$ attains its maximum) actually occurs on the interval $(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}]$. Thus, the we have an increase of investment activity for a large fraction of the disadvantaged group. On the other hand, the situation is less clear for the advantaged group. Although an analogous statement can be made—that all group $\mathcal{A}$ cost types $\theta \in \left\{(\ddot{\theta}_{\mathcal{A}}, \tilde{\theta}] \cup T_{\mathcal{D}}\right\}$ will reduce HC output under *RQ*—the statement is less informative, because investment also increases on part of the typical set $T_{\mathcal{A}}$ as well.

3.2. **Competitive Investment with Intrinsically Valued Human Capital.** Having characterized the workings of strategic forces which produce discouragement effects in our many-to-many matching contest, we now demonstrate that the above results carry over to the case where agents also derive intrinsic value from HC accumulation itself. If $\alpha = \beta = 1$ and costs are linear so that $U(p_i, h_i, \theta_i) = p_i h_i - \theta(h - \underline{h})$, and $[\underline{\theta}, \overline{\theta}] \subset (1, \infty)$, then proving statements analogous to Theorems 1–3 is relatively straightforward.[7] Under a PRO investment contest student i's objective is now

$$\max_{h_i \geq \underline{h}} \left\{ h_i \left(1 - F\left[\theta^*(h_i)\right]\right) - \theta_i (h_i - \underline{h}) \right\}.$$

Taking a FOC, we get

$$-h_i f\left[\theta_i^*(h_i)\right] \theta^{*\prime}(h_i) + \left(1 - F\left[\theta^*(h_i)\right]\right) = \theta_i.$$

In this world, student i can be thought to choose HC production in two parts. First, she raises investment to the level where she equates the direct marginal benefit (second term on right-hand side of FOC) and direct marginal cost of holding an additional unit of productive HC; BCH refer to this as the *productive channel* of incentives. Above that base level of investment, the *competitive channel* of incentives (first term on right-hand side of the FOC) arises because more HC gives her access to better match partners. This drives additional investment until higher-cost competitors no longer wish to steal her seat by out-performing her. If we define $\eta(\theta) \equiv \log\left(h^*(\theta)\right)$ then we not get the following (where η is strictly decreasing):

$$\eta'(\theta_i) = -\frac{f(\theta_i)}{\theta_i + F(\theta_i) - 1}, \text{ with boundary condition } \eta(\overline{\theta}) = \log(\underline{h}). \tag{7}$$

Theorem 4. *Assume HC production costs and gross utility are $c(h;\theta) = \theta(h - \underline{h})$, and $u(s,h) = sh$, respectively. Moreover, consider two pure rank-order contests with cost distributions, $F_1(\theta)$ and $F_2(\theta)$, where competition is more intense under F_2 in the sense that F_1 strictly LR dominates F_2. Then, letting $\tilde{\theta}$ denote the unique crossing point of f_1 and f_2, there exists a unique interior crossing point $\ddot{\theta} \in (\underline{\theta}, \widetilde{\theta})$ such that $h_1^*(\theta) < h_2^*(\theta)$ for $\theta < \ddot{\theta}$ and $h_1^*(\theta) > h_2^*(\theta)$ for $\theta > \ddot{\theta}$.*

Proof: Recall that strict LR dominance implies $f_1(\theta) \geq f_2(\theta)$ and $F_1(\theta) < F_2(\theta)$, for $\theta \in [\widetilde{\theta}, \overline{\theta})$. This with $\eta_1(\overline{\theta}) = \eta_2(\overline{\theta})$ and equation (7) together mean that $\eta_1'(\theta) < \eta_2'(\theta)$ and $\eta_1(\theta) > \eta_2(\theta)$, for each $\theta \in [\widetilde{\theta}, \overline{\theta})$. Thus, if η_1 and η_2 cross, the crossing must be on the interval $[\underline{\theta}, \widetilde{\theta})$.

Now, equation (7) can be expressed in integral form by

$$\eta_j(\theta) = \int_{\theta}^{\overline{\theta}} \frac{f_j(x)}{x + F_j(x) - 1} dx + \log(\underline{h}), \; j = 1, 2.$$

Moreover, if we impose a change of variables $y = F_j(\theta)$ within the integral, we get

$$\eta_1(\underline{\theta}) = \int_0^1 \frac{1}{F_1^{-1}(y) + y - 1} dy + \log(\underline{h}) < \int_0^1 \frac{1}{F_2^{-1}(y) + y - 1} dy + \log(\underline{h}) = \eta_2(\underline{\theta}),$$

where the inequality follows from LR dominance. Therefore, by continuity at least one crossing point exists on the open interval $(\underline{\theta}, \tilde{\theta})$. Let $\ddot{\theta}$ denote the maximum point at which $\eta_1(\ddot{\theta}) = \eta_2(\ddot{\theta})$, and note

[7]The assumption of $[\underline{\theta}, \overline{\theta}] \subset (1, \infty)$ is required because the gross utility and cost function are both linear in h. Since the direct marginal benefit of more human capital is $u_2(s,h) = s$ and the direct marginal cost is θ, we must have $\underline{\theta} > \overline{s}$, in order to rationalize all students choosing finite HC production.

that since the two functions are negatively sloped and η_1 crosses η_2 from above (moving in the leftward direction) at $\ddot{\theta}$, the following must be true when $\theta = \ddot{\theta}$:

$$\begin{aligned} \eta_1'(\theta) = -\frac{f_1(\theta)}{\theta + F_1(\theta) - 1} &> -\frac{f_2(\theta)}{\theta + F_2(\theta) - 1} = \eta_2'(\theta) \\ \Leftrightarrow \quad f_1(\theta)\left[\theta + F_2(\theta) - 1\right] &< f_2(\theta)\left[\theta + F_1(\theta) - 1\right] \end{aligned} \tag{8}$$

Now consider approaching $\underline{\theta}$ from above, beginning at $\ddot{\theta}$ and moving leftward. Since $\ddot{\theta}$ is to the left of the density crossing $\tilde{\theta}$, then by the LR dominance property, we know that if we begin at $\ddot{\theta}$ and approach $\underline{\theta}$ from above, then $f_2(\theta)$ becomes steadily larger relative to $f_1(\theta)$ as we move leftward. This also implies that $[\theta + F_1(\theta) - 1]$ becomes steadily larger relative to $[\theta + F_2(\theta) - 1]$ in the leftward direction (beginning from $\ddot{\theta}$) as well. To see why, recall that $F_1(\underline{\theta}) = F_2(\underline{\theta}) = 0$ and $f_1(\theta) < f_2(\theta)$ for each $\theta < \tilde{\theta}$. Thus, F_2 (F_1) becomes steadily larger relative to F_1 (F_2) when moving in the rightward (leftward) direction from $\underline{\theta}$ to $\ddot{\theta}$.

Therefore, the ordering between the right-hand and left-hand sides of (8) only becomes more pronounced as we move leftward from $\ddot{\theta}$. From this fact it follows that $\eta_1'(\theta) > \eta_2'(\theta)$ for each $\theta \in [\underline{\theta}, \ddot{\theta}]$, and the crossing point $\ddot{\theta}$ is therefore unique. ■

Theorems 5 and 6 below build on Theorem 4 similarly as Theorems 2 and 3 build on Theorem 1 above. In particular, Theorem 6 follows from Theorem 4 and from the properties of LR dominance: we now know that $\eta_{\mathcal{D}}'(\theta) < \eta'(\theta) < \eta_{\mathcal{A}}'(\theta)$ for each $\theta \in [\tilde{\theta}, \overline{\theta}]$, and since the log transformation preserves ordering, it follows that $h_{\mathcal{D}}^{*\prime}(\theta) < h^{*\prime}(\theta) < h_{\mathcal{A}}^{*\prime}(\theta)$ on that same interval as well. This also produces a slight strengthening of the result since the derivatives are strictly ordered at the density crossing $\tilde{\theta}$. Together, the final two results demonstrate that the model still predicts a large fraction of the disadvantaged group increasing investment under RQ with intrinsically valued HC:

Theorem 5. *Let HC production costs and gross utility be $c(h;\theta) = \theta(h - \underline{h})$, and $u(s,h) = sh$, respectively. Assume $F_{\mathcal{D}}$ strictly LR dominates $F_{\mathcal{A}}$ and let $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ denote the unique crossing of the cost densities where $f_{\mathcal{A}}(\tilde{\theta}) = f_{\mathcal{D}}(\tilde{\theta})$. Then there exist crossing points $\ddot{\theta}_{\mathcal{A}}, \ddot{\theta}_{\mathcal{D}} \in (\underline{\theta}, \tilde{\theta})$, such that*

(i) $h_{\mathcal{D}}^*(\theta) < h^*(\theta)$ for all $\theta < \ddot{\theta}_{\mathcal{D}}$ and $h_{\mathcal{D}}^*(\theta) > h^*(\theta)$ for all $\theta > \ddot{\theta}_{\mathcal{D}}$, and

(ii) $h_{\mathcal{A}}^*(\theta) > h^*(\theta)$ for all $\theta < \ddot{\theta}_{\mathcal{A}}$ and $h_{\mathcal{A}}^*(\theta) < h^*(\theta)$ for all $\theta > \ddot{\theta}_{\mathcal{A}}$.

Theorem 6. *Assume the same conditions as in Proposition 5. Then under the RQ policy (relative to the PRO policy), typical disadvantaged students—that is, group $\mathcal{D}$ students with costs $\theta \in \left\{(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}] \cup T_{\mathcal{D}}\right\}$—exert higher effort and accumulate more HC. Moreover, if we define $\Delta : (\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}] \cup T_{\mathcal{D}} \to \mathbb{R}$ as the difference on this set between group $\mathcal{D}$ investment under RQ versus PRO, or $\Delta(\theta) \equiv (h_{\mathcal{D}}^*(\theta) - h^*(\theta))$, then $\Delta(\theta)$ is strictly positive and attains a maximum on the interval $(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta})$.*

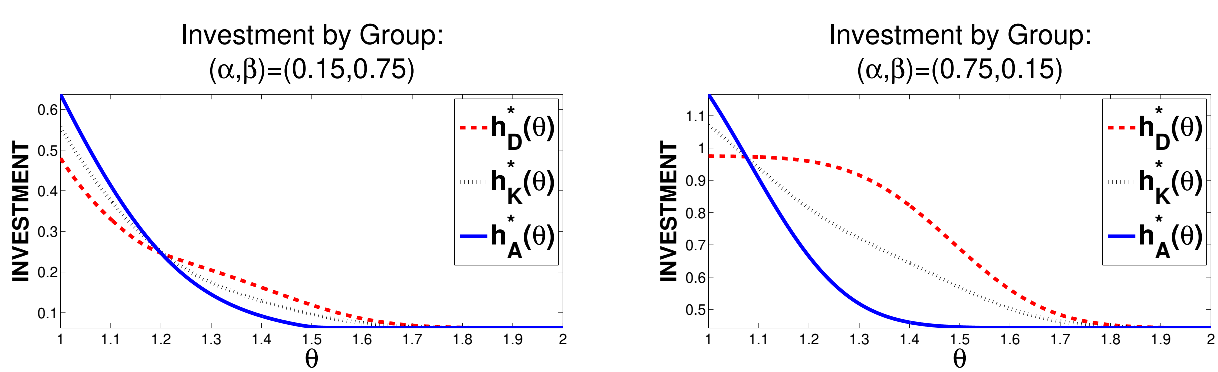
Intuitively, the policy aids the top agents from $\mathcal{D}$, but since they were already placing close to the upper bound, their outcomes cannot be commensurately improved and they rationally reduce effort. For other agents in $\mathcal{D}$, the policy alleviates discouragement effects by placing them in a competition group where their own type is not as far behind the curve, making them more competitive for higher quality outcomes and more willing to incur the costs of competition.

4. Numerical Analysis

A remaining concern is whether the patterns predicted in Theorems 1 – 6 are somehow artifacts of the special cases of the Cobb-Douglas family we consider here (i.e., $(\alpha, \beta) \in \{(1,0),\ (1,1)\}$) or other functional form choices (e.g., linear costs). In this section, we probe this idea further, first by presenting two illustrative examples in which (α, β) takes on the values $(0.15, 0.75)$ or $(0.75, 0.15)$, and then by reporting results from a comprehensive numerical analysis that computew over 1.7 million model equilibria in more than 500,000 special cases traversing the Cobb-Douglass parameter space $(\alpha, \beta) \in (0,1] \times [0,1]$ as well as various cost functions (linear, quadratic, exponential) and shapes of the type distributions. Throughout this exercise, *all* numerical examples computed follow the qualitative patterns predicted in Theorems 1–6.

4.1. **Illustrative Numerical Examples.** In this section we present numerical examples to further illustrate the model and demonstrate robustness of the qualitative patterns predicted above. In the first examples we compute, $\delta = 0.5$ (i.e., both groups have the same mass) and both $\Theta_{\mathcal{D}}$ and $\Theta_{\mathcal{A}}$ follow normal distributions truncated to a common support $[\underline{\theta}, \overline{\theta}] = [1, 2]$ with variance parameter $\sigma_{\mathcal{A}} = \sigma_{\mathcal{D}} = 0.25$. The mean parameters differ, with $\mu_{\mathcal{D}} = 1.5$ and $\mu_{\mathcal{A}} = 1.1$, which ensures that the distributions are ordered by likelihood ratio dominance, with $\Theta_{\mathcal{D}}$ being higher, on average (see Figure 1). In the first examples we specify costs as a linear function $c(h; \theta) = \theta h$, so that the maximum distance between investment under RQ and PRO occurs at the boundary between the sets $T_{\mathcal{A}}$ and $T_{\mathcal{D}}$. We numerically solve for equilibria under a PRO investment contest and a RQ contest by integrating the differential equations that arise from the first-order conditions under two cases. In the first one ("Example 1"), HC factors relatively heavily into match utility, with $\alpha = 0.15$ and $\beta = 0.75$. In the second ("Example 2"), students care less about their own HC and more about the quality of the institution they attend, with $\alpha = 0.75$ and $\beta = 0.15$. Equilibrium investment profiles under both allocation rules are summarized in figure 2, and are consistent with the analytic results proven above.

Figure 2. NUMERICAL EXAMPLES:
Investment, PRO vs RQ



(A) Example 1 (B) Example 2

Propositions 1–6 above establish several novel qualitative predictions for incentives in investment contests, which are consistent with these simple numerical examples:

(1) Holding one's own type fixed, a shift toward stiffer competition (an LR dominance shift toward lower costs among competitors) will lead low-cost types to invest more aggressively, while medium- and high-cost types invest less aggressively;
(2) in an asymmetric investment contest, a RQ allocation rule will lead to investment profiles among groups $\mathcal{A}$ and $\mathcal{D}$ which have a single interior crossing with the common investment profile under a PRO contest, where $h^*_{\mathcal{A}}(\underline{\theta}) > h^*(\underline{\theta}) > h^*_{\mathcal{D}}(\underline{\theta})$; and
(3) the typical investment change for group $\mathcal{D}$ under a RQ contest, relative to a PRO contest, is in the positive direction, with the change for group $\mathcal{A}$ being ambiguous.

4.2. **Extended Numerical Examples.** The usefulness of our comparative statics results is potentially limited by their dependence upon specific functional forms; namely, Propositions 1–3 rely on $(\alpha, \beta) = (1, 0)$, and Propositions 4–6 rely on linear costs and $(\alpha, \beta) = (1, 1)$. For general configurations of the Cobb-Douglas parameters α and β, and for arbitrary cost functions, it is difficult to prove analytic results concerning model predictions. Therefore we also execute an extensive set of numerical examples to probe for whether our theoretical results seem to generalize beyond the special cases covered by the proofs of Propositions 1–6.

Here, we compute numerical examples assuming normally distributed cost types truncated to a common support $[1, 2]$ for various values of the distributional parameters. We use values of the standard deviation parameter $\sigma \in \{0.15, 0.25, 0.35, 0.70, 1\}$, and we consider various combinations of the mean parameters $(\mu_{\mathcal{A}}, \mu_{\mathcal{D}}) \in \{(1.1, 1.3), (1.1, 1.5), (1.5, 1.9), (1.7, 1.9)\}$. For each of these the LR dominance relation is preserved, but for some of them one of the two densities is symmetric about its mean with the other being skewed, and for others, both are skewed. One may also be concerned that the results of the numerical examples hinge in some way on δ, the fraction of the population in group $\mathcal{D}$, so we consider examples where $\delta \in \{0.25, 0.5, 0.75\}$; i.e., where group $\mathcal{D}$ is either a statistical minority, a statistical majority, or neither. Another possible concern is that the results of the propositions rely crucially on linearity of costs, so we also compute numerical examples where costs are linear, display moderate curvature (quadratic), or display heavy curvature (exponential): $c(h; \theta) \in \{\theta h,\ \theta h^2,\ \theta \exp(h)\}$. Table 1 summarizes the combinations of δ, shape parameters, and cost functions that we consider in our set of extended numerical examples, with references to additional figures that display results for each case.

Finally, one may be concerned that the results of the propositions and numerical examples 1 and 2 from the body of the paper rely crucially on the values of the Cobb-Douglas match utility parameters assumed, so for each of the cells in Table 1, we compute equilibria under a PRO contest and a RQ contest for a fine grid of roughly 8,500 points $(\alpha,\beta) \in [0,1]\times[0,1]$ traversing the Cobb-Douglas parameter space.[8][9]

Across all model parameter combinations, we computed roughly 1.7 million model equilibria in more than 500,000 special cases traversing the Cobb-Douglas utility family and various combinations of cost functions and distributional parameters. For each numerical example, we compute the percent change in HC investment within group $\mathcal{A}$, group $\mathcal{D}$, and the combined population $(1-\delta)\mathcal{A}+\delta\mathcal{D}$. We also numerically check whether the qualitative predictions of Propositions 2 and 5 are satisfied: for $j \in \{\mathcal{A},\mathcal{D}\}$ we find all interior zeros of the function $h_j^*(\theta) - h^*(\theta)$ and, provided a unique interior zero is found, we check whether the difference has the appropriate sign at the lower bound of costs.

4.2.1. ***Results.*** In all computed examples we find a unique interior crossing between investment functions $h_j^*(\theta)$ and $h^*(\theta)$, $j = \mathcal{A},\ \mathcal{D}$, with an ordering of the investment functions that is consistent with the ordering predicted in Propositions 2 and 5. To investigate predictions in Propositions 3 and 6 concerning typical investment changes under an RQ contest (relative to a PRO contest), we display results on HC production percentage changes in this section. Figure 4 displays level curves of percent changes of human capital production across the Cobb-Douglas parameter space for the $(\mu_{\mathcal{A}},\mu_{\mathcal{D}},\delta) = (1.1, 1.5, 0.25)$ case with linear costs and various values of the variance parameter σ. In plots where depicted level curves are sometimes positive and sometimes negative, the zero level curve is represented with a thick line. Percent changes in HC investment range from negative to positive for group $\mathcal{A}$, whereas they are always positive for group $\mathcal{D}$ and for the population as a whole (i.e., including both groups). It is also interesting to note that the magnitudes of the changes increase considerably when σ is smaller, as this induces a greater degree of asymmetry in the PRO competition. Note that increasing the population share of group $\mathcal{D}$ has little effect, qualitatively (Figures 5 and 6), except to reduce the magnitudes of the resulting percentage changes.

Figures 7, 8, and 9 depict the same comparison for quadratic costs, and Figures 10, 11, and 12 do so for exponential costs. The main difference with cost curvature is that it leaves less room for group $\mathcal{A}$ to increase investment under a RQ contest. Only three of the panels for quadratic costs exhibit a positive region for advantaged agents, and under exponential costs they always reduce mean HC investment in all examples computed. As for group $\mathcal{D}$, it is still the case that, across all examples displayed, mean HC production strictly increases, and this increase is always large enough so that the overall population including both groups $\mathcal{A}$ and $\mathcal{D}$ exhibits a HC mean increase.

Figures 14, 15, and 16 display similar plots for linear costs and the $(\mu_{\mathcal{A}},\mu_{\mathcal{D}}) = (1.5, 1.9)$ case where costs are higher for both groups (see Figure 13). Once again, the intuition is that higher marginal costs leave less room for group $\mathcal{A}$ to respond to a RQ allocation rule with a positive mean investment

[8]Specifically, we compute all examples for the Cartesian product of a grid of values $\alpha \in \{0.05, 0.06, 0.07, \ldots, 0.99, 1\}$ in increments of 0.01 and a grid of values $\beta \in \{0.05, 0.06, 0.07, \ldots, 0.91, 0.92\}$ in increments of 0.01. We omit combinations of very small values of both α and β as these often produce numerical instability. We also omit values of β close to 1 as these occasionally produce numerical instability in combination with small values of α in some of the cells in Table 1. Where numerical instability does not result for values of β close to 1, the qualitative patterns that emerge are similar to the rest of the examples we compute and display here. MATLAB code for the numerical examples is available from the authors upon request.

[9]In each numerical example computed, we set the initial condition $\underline{h}$ of the differential equations using a "zero surplus condition" (see Bodoh-Creed and Hickman [2018]): $c(\underline{h};\overline{\theta}) = u(\underline{p},\underline{h})$. In brief, this condition implies the marginal market participant is just indifferent between being placed at the lowest-value college (which happens with probability one in a monotone equilibrium) and opting out of the college market.

change, but the universal positive response by group $\mathcal{D}$ is always enough to ensure that HC production rises on average in the overall population. Finally, Figures 17 and 18 display results for the final two rows of Table 1, where both type distributions are skewed and costs are linear. Once again, the results qualitatively agree with others described above.

5. Robustness Analysis: Alternative Utility Specifications

In focusing on the Cobb-Douglas family of match utility functions in the analysis above, several of our theoretical proofs and all numerical examples implicitly assume that the value of a college applicant's human capital choice is mediated by the value of their equilibrium college placement: when the value of the prize p is higher, the value of HC h is higher as well. Although these are reasonable modeling assumptions with strong precedent in the literature, such assumptions may not apply in all situations.

In this appendix, we explore alternate specifications of the model where match utility is either quasilinear in p or for the additive term in p to be strictly concave. In both cases we validate our previous theoretical results, with some mild caveats in the latter. The analysis shows that when match utility is additively separable in prize quality p and human capital h, the general character of Theorems 1-6, derived under complementary match utility, is largely preserved. The analysis demonstrates that the theoretical predictions do not hinge crucially on the assumption of p and h being complementary inputs in the match utility function.

6. Discussion

We have presented a model of a large contest which captures key, salient features of real-world investment contests, in which many heterogeneous agents compete for many heterogeneous prizes. The model generates predictions for how variation in the level of competition will affect incentives to expend costly effort in acquiring productive human capital. Such a framework may be used to model competition between students for post graduation opportunities, for example, allowing one to assess the impact of affirmative action or other incentive-changing policies on the distribution of effort and achievement.

The analysis identifies several behavioral patterns that are important for assessing the impact of an affirmative action policy. The key insight is that the impact of an affirmation action policy differs across population groups. It not only depends on whether one is a beneficiary of the policy (e.g., a minority student under an affirmative action policy), but also depends on one's relative ability within their group. When an affirmative action policy targets a disadvantaged population group, the best and the brightest students within this group exert less effort as they effectively face weaker competition among their peers, while others exert greater effort as better outcomes become within reach. The opposite pattern of behavior is predicted to be true among those not targeted by the affirmative action policy. The model shows how affirmative action policies can increase the average effort among minority groups, and reduce achievement gaps between advantaged and disadvantaged populations.

These insights matter only to the extent to which people actually respond to affirmative action incentives in the way that is predicted by the theory. The strategic environment is complex, and it is not certain that students, for example, will respond to affirmative action by adjust their effort in a way that is consistent with equilibrium behavior. To test whether people do respond to incentives in a manner consistent with theory, our companion paper Cotton et al. [2020] summarizes our theoretical results as three testable predictions and then designs a field experiment in which to test them.

Prediction (I): If the distribution of rivals' costs undergoes a stochastic dominance shift toward lower cost values (i.e., competition for prizes becomes more intense); then the best and brightest (low-cost) agents react by *ramping up* HC achievement, while lower-ability (higher-cost) agents *reduce* investment activity due to a *discouragement effect* as better prizes are now perceived as out of reach.

Prediction (II): Replacing a PRO rule with a RQ rule brings more valuable prizes within reach of middle- and high-cost members of $\mathcal{D}$, thus mitigating discouragement effects. At the same time, the best and brightest members of $\mathcal{D}$ invest less in HC under RQ because they now face less intense competition for the top prizes. The effects on group $\mathcal{A}$ agents with high/middle/low costs tend in opposite directions by similar logic.

Prediction (III): In the aggregate, a RQ rule will lead to gains in achievement for most group $\mathcal{D}$ agents, relative to a PRO rule. Behavioral responses among group $\mathcal{A}$ agents are generally weaker in magnitude and the theory is ambiguous about the sign of the majority response within group $\mathcal{A}$.

Cotton et al. [2020]'s field experiment changes the way that middle-school aged students are paid for performance on a competitive mathematics exam in order to approximate the RQ and PRO environments from the model. The experiment is able to track both student study effort on a tutorial website and performance on the final exam. It shows that affirmative action policies change both the distribution of study effort and final exam performance in ways that are remarkably consistent with the theoretical predictions. This suggests that children as young as grade 5 respond to affirmative action incentives in a manner that is consistent with equilibrium behavior in our theoretical analysis.

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7. Appendix: Alternative Utility Specifications

7.1. **Quasilinear Match Utility.** Suppose that the match utility function is quasilinear in university quality $u(p_i, s_i) = p_i + v(s_i)$, where $v(\cdot)$ is assumed to be a strictly positive, weakly increasing, and weakly concave function. Note also that the model of pure strategic incentives from Section 3.1 is not only a special case of the Cobb-Douglas utility formulation, but it is also a special case of the current quasilinear utility formulation as well, where $v(h) = 0$ for each h. Now, net utility in the PRO competition takes the form

$$U(p_i, h_i, \theta_i) = P^{PRO}(h_i) + v(h_i) - c(h_i; \theta_i).$$

Agent i's objective function is

$$\max_{h_i \geq \underline{h}} \left\{ (1 - F\left[\theta^*(h_i)\right]) + v(h_i) - c(h_i;\theta_i) \right\}.$$

while the FOC takes the form

$$h^{*\prime}(\theta_i) = \frac{-f(\theta_i)}{\zeta'\left[h^*(\theta_i);\theta_i\right]}, \text{ and boundary condition } h^*(\overline{\theta}) = \underline{h}, \tag{9}$$

where $\zeta(h;\theta) \equiv c(h;\theta) - v(h)$ is defined for notational simplicity. Given the similarity between equation (9) and equation (3), it is straightforward to see why results analogous to Theorems 1–3 would still follow. We state Theorem 7 (analogous to Theorem 1 above) with proof to illustrate how the logic is virtually identical to that employed above, and we state Theorems 8 and 9 (analogous to Theorems 2 and 3, respectively) without proof in the interest of brevity.

Theorem 7. *Consider two PRO contests with quasilinear match utility, call them,* 1 *and* 2*, and assume that they differ only by their cost distributions,* F_1 *and* F_2*, respectively. Assume that competition is more fierce under contest* 2 *in the sense that* F_1 *strictly LR dominates* F_2*. Let* $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ *denote the unique crossing point of the density functions where* $f_1(\tilde{\theta}) = f_2(\tilde{\theta})$*. There exists a unique interior crossing point* $\ddot{\theta} \in (\underline{\theta}, \tilde{\theta})$*, such that* $h_1^*(\theta) < h_2^*(\theta)$ *for all* $\theta < \ddot{\theta}$ *and* $h_1^*(\theta) > h_2^*(\theta)$ *for all* $\theta \in (\ddot{\theta}, \overline{\theta})$*.*

Proof: Note that since $v(h)$ is weakly concave in h it follows that $\zeta(h;\theta)$ is strictly convex in h and still has a positive cross-partial derivative for all θ. Using this fact, the proof logic of Theorem 1 follows through directly by merely replacing $c'(h;\theta)$ in the denominators of the FOC (and in particular, in equations (4) and (5)) with $\zeta'(h;\theta)$ instead. ■

Theorem 8. *Assume that match utility is quasilinear in prize quality* p *and that* $F_{\mathcal{D}}$ *LR dominates* $F_{\mathcal{A}}$*. Let* $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$ *denote the unique crossing point of the cost densities where* $f(\tilde{\theta}) = f_{\mathcal{A}}(\tilde{\theta}) = f_{\mathcal{D}}(\tilde{\theta})$*, let* $h_j^*(\theta),\ j = \mathcal{A}, \mathcal{D}$ *denote the equilibrium investment strategies under a RQ admissions rule, and let* $h^*(\theta)$ *denote the common investment strategy under PRO admissions. Then there exist crossing points* $\ddot{\theta}_{\mathcal{A}}, \ddot{\theta}_{\mathcal{D}} \in (\underline{\theta}, \tilde{\theta})$*, such that*

(i) $h_{\mathcal{D}}^*(\theta) < h^*(\theta)$ *for all* $\theta < \ddot{\theta}_{\mathcal{D}}$ *and* $h_{\mathcal{D}}^*(\theta) > h^*(\theta)$ *for all* $\theta > \ddot{\theta}_{\mathcal{D}}$*, and*
(ii) $h_{\mathcal{A}}^*(\theta) > h^*(\theta)$ *for all* $\theta < \ddot{\theta}_{\mathcal{A}}$ *and* $h_{\mathcal{A}}^*(\theta) < h^*(\theta)$ *for all* $\theta > \ddot{\theta}_{\mathcal{A}}$*.*

Theorem 9. *Assume the same conditions as in Theorem 8, including a match utility function that is quasilinear in prize quality* p*. Then under the RQ rule (relative to the PRO rule), typical disadvantaged agents—that is, group* $\mathcal{D}$ *agents with costs* $\theta \in \left\{(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}] \cup T_{\mathcal{D}}\right\}$*—exert higher effort and accumulate more HC. Moreover, if we define* $\Delta : (\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}] \cup T_{\mathcal{D}} \rightarrow \mathbb{R}$ *as the difference on this set between group* $\mathcal{D}$ *investment under RQ versus PRO, or* $\Delta(\theta) \equiv (h_{\mathcal{D}}^*(\theta) - h^*(\theta))$*, then* $\Delta(\theta)$ *is strictly positive and attains a maximum on the interval* $(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta}]$*. Moreover, if investment costs* $C(\cdot)$ *or human capital utility* $v(\cdot)$ *are strictly convex in* h*, then* $\Delta(\theta)$ *attains its maximum on the open interval* $(\ddot{\theta}_{\mathcal{D}}, \tilde{\theta})$*.*

7.2. **Additively Separable, Strictly Concave Match Utility.** As a final check on the robustness of our comparative static predictions to functional form assumptions, the appendix reports results from an analysis in which we generalize the quasilinear model by allowing for match utility to be additively separable in p and h, but strictly concave in university quality p. Match utility now takes the form

$$u(p_i, h_i) = w(p_i) + v(h_i), \tag{10}$$

where $w(\cdot)$ is strictly increasing, $v(\cdot)$ is weakly increasing, and both are strictly positive and weakly concave. Once again, this formulation nests the model of Section 3.1 since it generalizes the quasilinear model of the previous section.

In the PRO competition agent i's objective function now takes the form

$$\max_{h_i \geq \underline{h}} \left\{ w\left(1 - F\left[\theta^*(h_i)\right]\right) + v(h_i) - c(h_i;\theta_i) \right\} \tag{11}$$

with FOC

$$h^{*\prime}(\theta_i) = \frac{-f(\theta_i) w' \left[1 - F(\theta_i)\right]}{\zeta' \left[h^*(\theta_i);\theta_i\right]}, \text{ and boundary condition } h^*(\overline{\theta}) = \underline{h}, \tag{12}$$

where $\zeta(h;\theta) \equiv c(h;\theta) - v(h)$ is once again defined for notational simplicity. Given the assumptions on f, c, w, and v, it is easy to see that h^* is still strictly decreasing in θ.

Several difficulties now arise in proving results analogous to Theorems 1–3 for general forms of the function $w(\cdot)$. First, recall from Theorems 1, 4, and 7 above that under previously considered utility formulations we supposed there are two type distributions, F_1 and F_2, with the former dominating the latter according to the likelihood ratio ordering. We showed that, holding all else fixed, if the type distribution shifts from F_1 to F_2 then the resulting crossing point of the two investment functions could be bounded from above by point $\widetilde{\theta}$ where the two densities cross, or $f_1(\theta) = f_2(\theta)$. In light of the above FOC, the bound on the crossing point of the strategies would now be at a different point, call it $\hat{\theta}$, which is defined as the point where

$$f_2\left(\hat{\theta}\right) = f_2\left(\hat{\theta}\right) \frac{w'\left[1 - F_1\left(\hat{\theta}\right)\right]}{w'\left[1 - F_2\left(\hat{\theta}\right)\right]}. \tag{13}$$

By concavity of $w(\cdot)$ and the LR dominance property, it follows that any such point satisfying the definition of $\hat{\theta}$ would have to lay to the right of $\widetilde{\theta}$. Moreover, the proof logic would require that $\hat{\theta}$ must be unique, which is difficult to guarantee for general forms of w, F_1, and F_2. However, provided that $\hat{\theta}$ is unique, it is straightforward to see that an analogous result to Theorem 7 could be proven, but with $\widetilde{\theta}$ replaced by $\hat{\theta}$. This rightward shift of the bound on the strategy crossing point would therefore tend to weaken the result of Theorem 9 which follows from Theorem 7.

On the other hand, a second aspect of Theorem 7 is altered by strict concavity of w in the opposite direction. Whereas under a match utility function with positive cross-partial derivatives it was possible to show that an interior crossing point of the investment functions does in fact exist, under the current utility formulation this need not be true.[10] Since it is still true that high-cost members of group $\mathcal{D}$ increase investment when the distribution of cost types undergoes a LR dominating shift, this means

[10]To see why, recall how we prove existence of the strategy crossing point under quasilinear match utility. If one were to construct an analogous sequence of inequalities under the current utility formulation, strict concavity of w may allow for the final weak inequality to be violated.

that it is now possible that the positive sign on the behavioral response associated with a shift to RQ admissions could be true for *all* members of group $\mathcal{D}$. This possibility would strengthen the result of Theorem 9. Since the two main changes work in opposite directions, taken together they generally work against finding sharp predictions.

We prove a partial result analogous to Theorem 7 below and then briefly discuss its implications within a class of examples using a functional form of $w(p) = p^\alpha$ and the distributional assumptions from Section 4 for illustrative purposes. Within this broad class of examples, barring parameter values which lead to scenarios where competitive forces are weak or where ex-ante asymmetry is small, the general character of the results from Theorems 1–3 is preserved.

Theorem 10. *Assume match utility given by equation (10) and consider two PRO contests, call them,* 1 *and* 2*, which differ only by their cost distributions,* F_1 *and* F_2*, respectively. Assume that competition is more fierce under contest* 2 *in the sense that* F_1 *LR dominates* F_2*, and assume further that* w*,* F_1*, and* F_2 *are such that* $\widehat{\theta}$*, as defined in equation (13), is unique. Then* $h_1^*(\theta) > h_2^*(\theta)$ *for all* $\theta \in (\hat{\theta}, \overline{\theta})$*. Moreover, if the strategy functions cross, they do so at most once on the interval* $[\underline{\theta}, \widehat{\theta})$*.*

Proof: Note that the LR dominance property implies that $f_1(\theta)w'[1 - F_1(\theta)] > f_2(\theta)w'[1 - F_2(\theta)]$ for θ in a neighborhood of $\overline{\theta}$, since the ratio $\frac{w'[1-F_1(\theta)]}{w'[1-F_2(\theta)]}$ tends toward 1 as $\theta \to \overline{\theta}$, whereas the ratio $\frac{f_1(\theta)}{f_2(\theta)}$ tends toward a number strictly above 1. Since the same boundary condition applies to both contests, $h_1^*(\overline{\theta}) = h_2^*(\overline{\theta}) = \underline{h}$, then equation (12) implies the initial trajectories at the boundary point are ordered in the following way:

$$h_1^{*\prime}(\overline{\theta}) = \frac{-f_1(\overline{\theta})w'\left[1 - F_1(\overline{\theta})\right]}{\zeta'(\underline{h};\overline{\theta})} < \frac{-f_2(\overline{\theta})w'\left[1 - F_2(\overline{\theta})\right]}{\zeta'(\underline{h};\overline{\theta})} = h_2^{*\prime}(\overline{\theta}). \tag{14}$$

This in turn means that $h_1^*(\theta) > h_2^*(\theta)$ within a neighborhood of $\overline{\theta}$ since the investment functions are continuous and everywhere differentiable. Note that because slopes are negative $h_1^{*\prime}(\theta) < h_2^{*\prime}(\theta)$ means h_1^*, rises in the leftward direction and is *more steep* at θ.

Now suppose there exists at least one point where h_1^* and h_2^* cross, and let $\ddot{\theta} \in (\underline{\theta}, \overline{\theta})$ denote the maximum of all such possible points, with $\ddot{h} \equiv h_1^*(\ddot{\theta}) = h_2^*(\ddot{\theta})$, if any exist. Since h_1^* crosses h_2^* from above at $\ddot{\theta}$, it must be that $h_1^{*\prime}(\ddot{\theta}) \geq h_2^{*\prime}(\ddot{\theta})$ (*i.e.*, h_1^* is *less steep* at the crossing point). However, since $f_1(\theta)w'[1 - F_1(\theta)] > f_2(\theta)w'[1 - F_2(\theta)]$ on $(\widehat{\theta}, \overline{\theta}]$, and since $h < h'$ implies $\zeta'(h';\theta) \geq \zeta'(h;\theta)$ by convexity, the following must be true for any $\theta \in (\hat{\theta}, \overline{\theta})$:

$$h_1^{*\prime}(\theta) = \frac{-f_1(\theta)w'[1 - F_1(\theta)]}{\zeta'[h_1^*(\theta);\theta]} < \frac{-f_2(\theta)w'[1 - F_2(\theta)]}{\zeta'[h_2^*(\theta);\theta]} = h_2^{*\prime}(\theta) \quad \Rightarrow \quad h_1^*(\theta) > h_2^*(\theta). \tag{15}$$

Therefore, $\ddot{\theta} < \tilde{\theta}$, if such a point exists. Similarly, since $f_1(\theta)w'[1 - F_1(\theta)] < f_2(\theta)w'[1 - F_2(\theta)]$ on $[\underline{\theta}, \hat{\theta})$, then any crossing point would have to obey

$$h_1^{*\prime}(\ddot{\theta}) = \frac{-f_1(\ddot{\theta})w'\left[1 - F_1\left(\ddot{\theta}\right)\right]}{\zeta'(\ddot{h};\ddot{\theta})} > \frac{-f_2(\ddot{\theta})w'\left[1 - F_2\left(\ddot{\theta}\right)\right]}{\zeta'(\ddot{h};\ddot{\theta})} = h_2^{*\prime}(\ddot{\theta}).$$

This means that h_1^* can only cross h_2^* from above (*i.e.*, at points where it is *less steep*) and so there can be at most one such crossing. ■

Theorem 11. *Assume match utility given by equation (10) and that $F_{\mathcal{D}}$ LR dominates $F_{\mathcal{A}}$. Let $\widehat{\theta}$ be as defined above with $F_{\mathcal{D}} = F_1$ and $F_{\mathcal{A}} = F_2$. Finally, let $h_j^*(\theta)$, $j = \mathcal{A}, \mathcal{D}$ denote the equilibrium investment strategies under a RQ admissions rule, and let $h^*(\theta)$ denote the common investment strategy under PRO admissions. Then*

(i) $h_{\mathcal{D}}^*(\theta) > h^*(\theta)$ for all $\theta > \widehat{\theta}$, and

(ii) $h_{\mathcal{A}}^*(\theta) < h^*(\theta)$ for all $\theta > \widehat{\theta}$.

Moreover, if the strategy functions cross, they do so at most once on the interval $[\underline{\theta}, \widehat{\theta})$.

What is unclear from the two results above is whether it is still true (as it was in Theorems 3, 6, and 3) that a majority of group $\mathcal{D}$ agents increase their effort level under a representative quota, relative to a pure rank-order contest. This is complicated now by the fact that the bound on the strategy crossing point $\widehat{\theta}$ is to the right of the density crossing point $\tilde{\theta}$, which means that some agent types within the typical set $\mathcal{T}_{\mathcal{D}}$ may reduce effort under a RQ contest. However, given specific forms for the utility function and the type distributions, one can compute the quantile rank of the crossing bound $\widehat{\theta}$ within the type distribution $F_{\mathcal{D}}$. Whenever $\hat{\theta}$ is at or below the median for group $\mathcal{D}$, it follows that at least half of group $\mathcal{D}$ agents respond to RQ incentives by increasing effort (relative to a PRO contest).

7.3. **Numerical Examples of Alternative Utility Cases.** With this assumption in mind, we can re-visit the wide variety of parameter combinations covered in the numerical examples presented in Section 4. In particular, we consider a fine grid of utility parameter values $\alpha \in \{0.05, 0.06. \dots, 0.99, 1\}$ and truncated normal type distributions with common variance parameter $\sigma \in \{0.15, 0.25, 0.35, 0.70, 1\}$ and mean parameters $(\mu_{\mathcal{A}}, \mu_{\mathcal{D}}) \in \{(1.1, 1.3), (1.1, 1.5), (1.5, 1.9), (1.7, 1.9)\}$. As noted above, the LR dominance property is satisfied by all of these parameter combinations. For all combinations of these parameter values, we can compute the quantile rank of $\hat{\theta}$ (as defined in equation (13)) which provides an upper bound on the set of points where a crossing of the strategies $h_{\mathcal{D}}^*(\theta)$ and $h^*(\theta)$ could exist. Whenever $\hat{\theta}$ falls at or below the median relative to $F_{\mathcal{D}}$, we can guarantee that at least half of group $\mathcal{D}$ agents increase effort after a switch from PRO admissions to RQ admissions. Note that in order to do so, it is not necessary to assume forms or parameter values for $v(\cdot)$ or $c(\cdot)$, since they do not affect the definition of $\widehat{\theta}$.

Initially, across all parameter combinations described above, it would appear that a substantial weakening of the theoretical predictions had occurred, since $\hat{\theta}$ is at or below the median in only 40% of the 1152 parameter combinations examined. However, the vast majority of the cases resulting in $\hat{\theta}$ above the median involve either $\alpha < 0.5$—where marginal benefits to better placement are very small for most agents and therefore competitive incentives are relatively unimportant—or where $\sigma \in \{0.7, 1\}$—where ex-ante asymmetry is much weaker (e.g., see Figure 3) and therefore changes in the strategic environment for group $\mathcal{D}$ are less salient.[11] If we exclude the corresponding parameter combinations for each of these relatively uninteresting scenarios, we get a very different picture. Of the remaining 408 special cases, over 87% of them involve $\hat{\theta}$ at or below the median type of group $\mathcal{D}$, with the maximal quantile rank being just below 0.66. Note also that $\hat{\theta}$ is merely an upper bound on locations where

[11]Note that small values of α were more meaningful in the numerical examples of Section 4 because of the positive cross-partial derivative in the match utility function, which meant that small values of α still allow for prize quality p to yield indirect benefits of increasing the marginal payoff of holding an additional unit of human capital h.

strategy crossing points may exist, though they need not exist. While the theory shows that all group-$\mathcal{D}$ cost types above $\widehat{\theta}$ increase effort under RQ admissions, there is a positive mass below this point that do so as well.

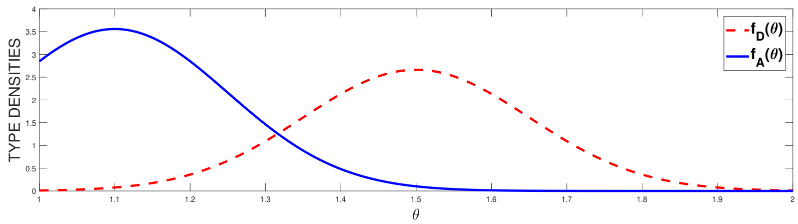
The analysis shows that when match utility is additively separable in prize quality p and human capital h, the general character of Theorems 1-6, derived under complementary match utility, is largely preserved. The analysis demonstrates that the theoretical predictions do not hinge crucially on the assumption of p and h being complementary inputs in the match utility function.

Table 1. EXTENDED NUMERICAL EXAMPLES

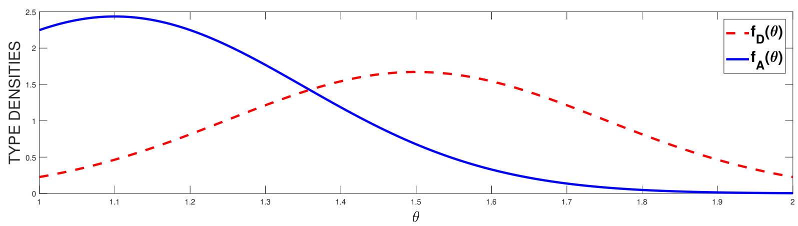
Distributional Parameters	δ: 0.25 (Figs 4, 7, 10, 14, 17, 18)	δ: 0.50 (Figs 5, 8, 11, 15, 17, 18)	δ: 0.75 (Figs 6, 9, 12, 16, 17, 18)
$\mu_{\mathcal{A}} = 1.1$, $\mu_{\mathcal{D}} = 1.5$, $\sigma = 0.15$ (Fig. 3)	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.1$, $\mu_{\mathcal{D}} = 1.5$, $\sigma = 0.25$ (Fig. 3)	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.1$, $\mu_{\mathcal{D}} = 1.5$, $\sigma = 0.35$ (Fig. 3)	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.1$, $\mu_{\mathcal{D}} = 1.5$, $\sigma = 0.70$ (Fig. 3)	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.1$, $\mu_{\mathcal{D}} = 1.5$, $\sigma = 1$ (Fig. 3)	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear, quadratic, exponential $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.5$, $\mu_{\mathcal{D}} = 1.9$, $\sigma = 0.15$ (Fig 13)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.5$, $\mu_{\mathcal{D}} = 1.9$, $\sigma = 0.25$ (Fig 13)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.5$, $\mu_{\mathcal{D}} = 1.9$, $\sigma = 0.35$ (Fig 13)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.5$, $\mu_{\mathcal{D}} = 1.9$, $\sigma = 0.70$ (Fig 13)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.5$, $\mu_{\mathcal{D}} = 1.9$, $\sigma = 1$ (Fig 13)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.7$, $\mu_{\mathcal{D}} = 1.9$, $\sigma = 0.25$ (Fig 17)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$
$\mu_{\mathcal{A}} = 1.1$, $\mu_{\mathcal{D}} = 1.3$, $\sigma = 0.25$ (Fig 18)	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$	linear $(\alpha, \beta) \in [0.05, 1] \times [0.05, 0.92]$

Notes: For the first 5 rows of the table, the group $\mathcal{D}$ cost density is symmetric about its mean and the group $\mathcal{A}$ density is right-skewed. For the next 5 rows, the group $\mathcal{A}$ density is symmetric about its mean and the group $\mathcal{D}$ density is left-skewed. In the penultimate row both densities are left-skewed, and in the final row both densities are right-skewed. All combinations of distributional parameters are chosen to ensure that group $\mathcal{D}$ costs stochastically dominate group $\mathcal{A}$ costs according to the likelihood ratio ordering.

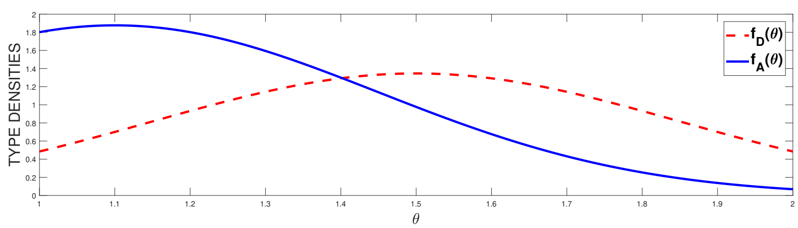
FIGURE 3. Truncated Normal Cost Densities: $\mu_D = 1.5$ and $\mu_A = 1.1$



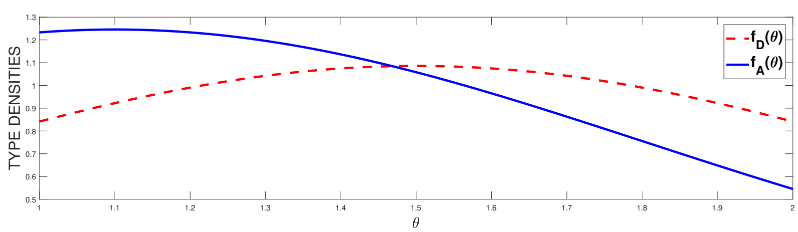
(A) $\sigma = 0.15$



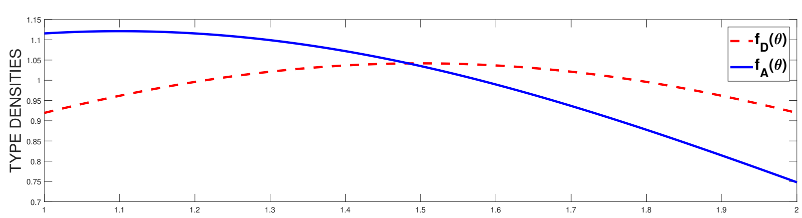
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$

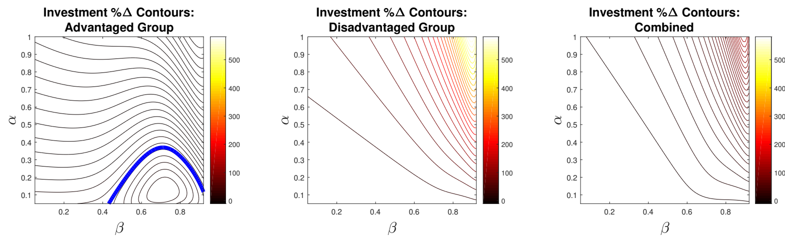


(D) $\sigma = 0.70$

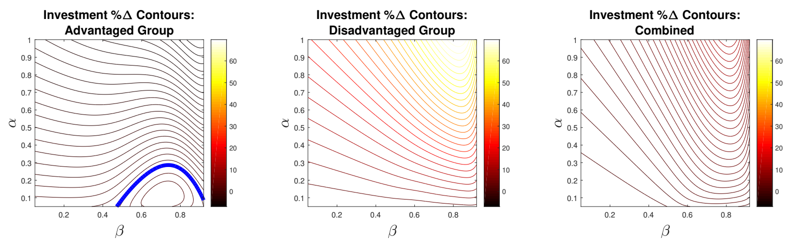


(E) $\sigma = 1$

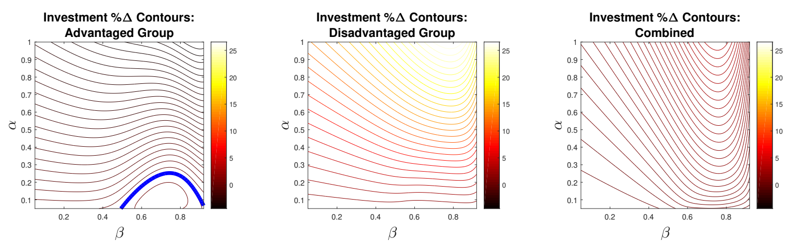
Figure 4. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.25$ (Disadvantaged Group is Statistical Minority) with Linear Costs**



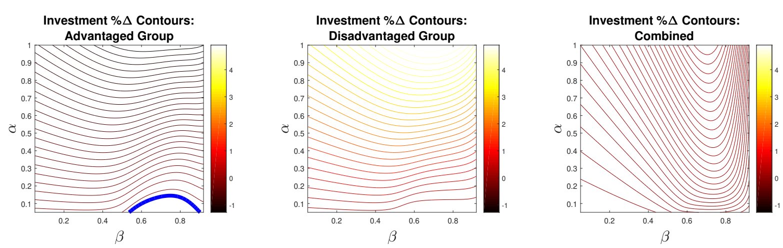
(A) $\sigma = 0.15$



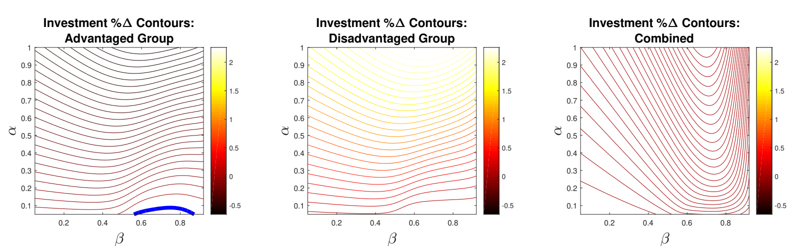
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



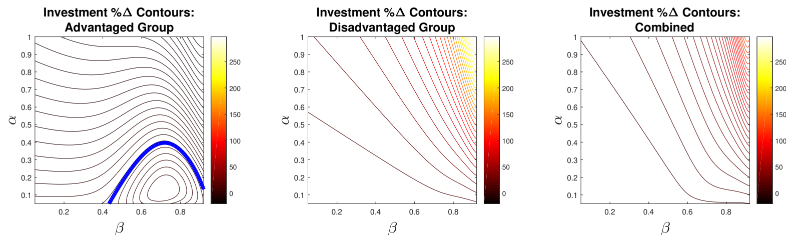
(D) $\sigma = 0.70$



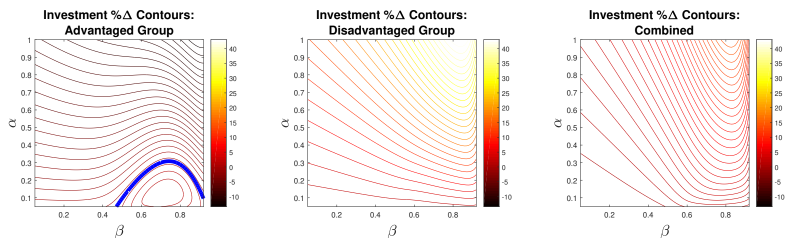
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the disadvantaged group and for the combined population are positive.

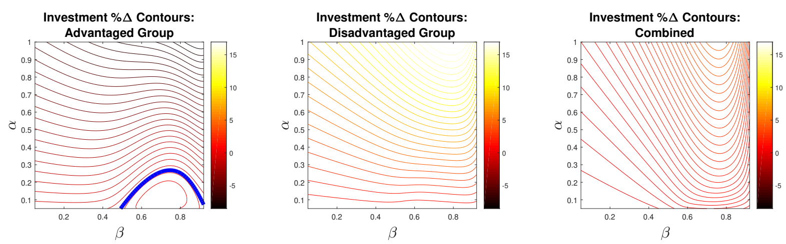
FIGURE 5. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.50$ (Both Groups Same Size) with Linear Costs**



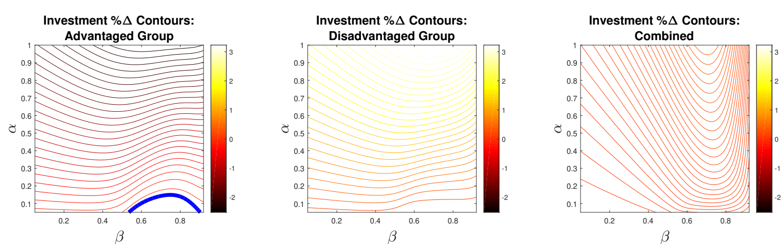
(A) $\sigma = 0.15$



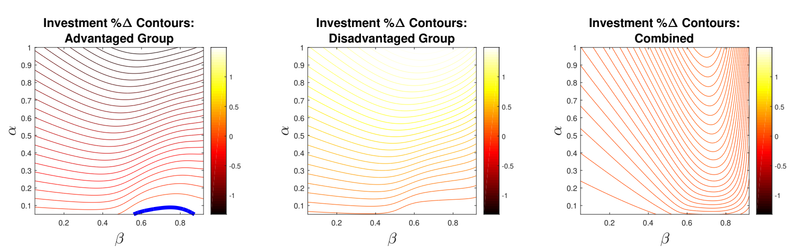
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



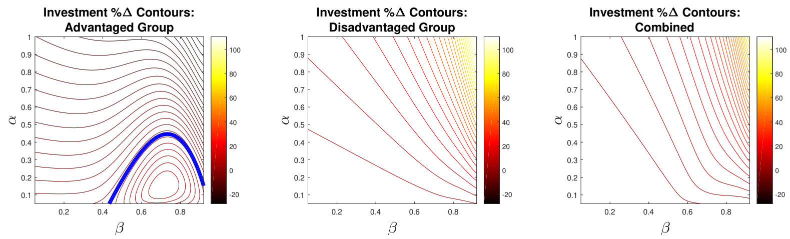
(D) $\sigma = 0.70$



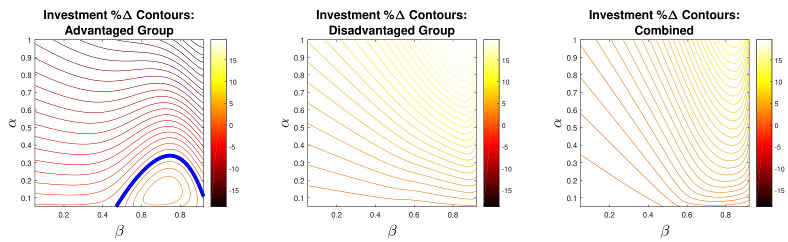
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the disadvantaged group and for the combined population are positive.

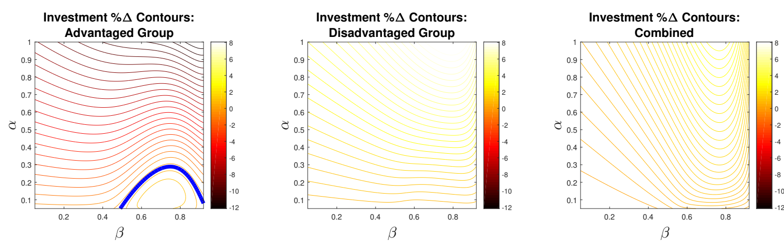
Figure 6. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.75$ (Disadvantaged Group is Statistical Majority) with Linear Costs**



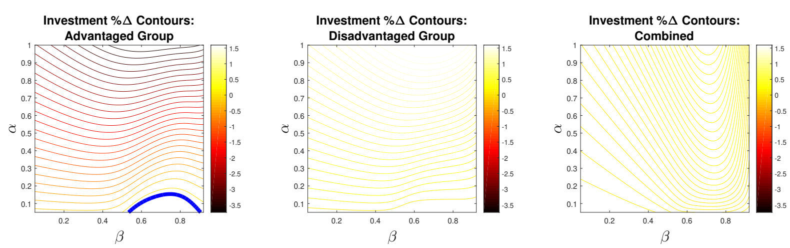
(A) $\sigma = 0.15$



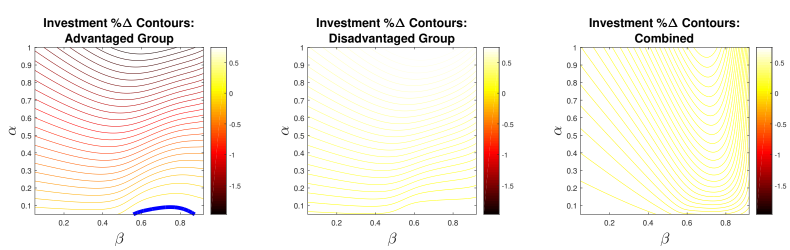
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



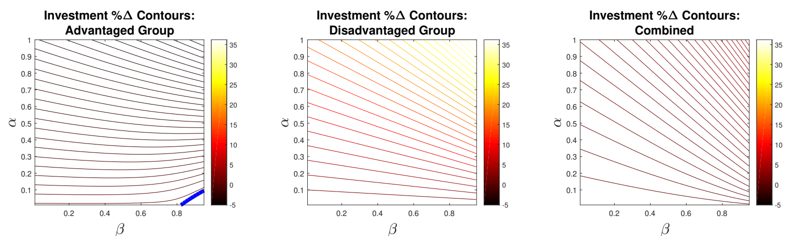
(D) $\sigma = 0.70$



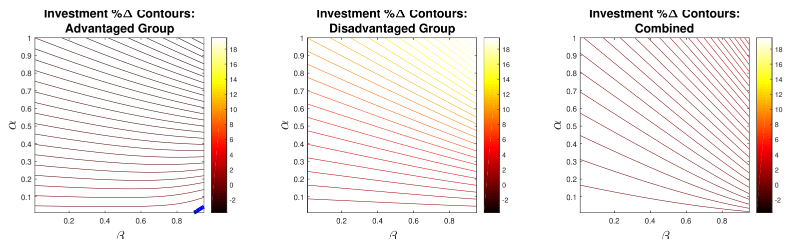
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the disadvantaged group and for the combined population are positive.

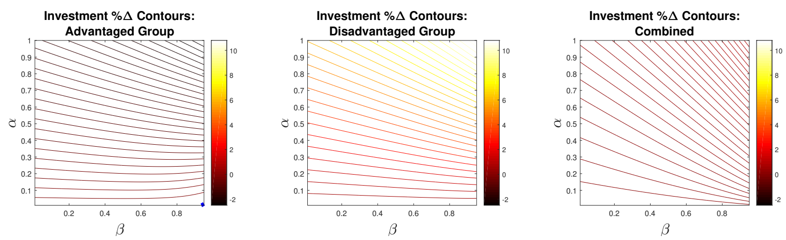
Figure 7. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.25$ (Disadvantaged Group is Statistical Minority) with Quadratic Costs**



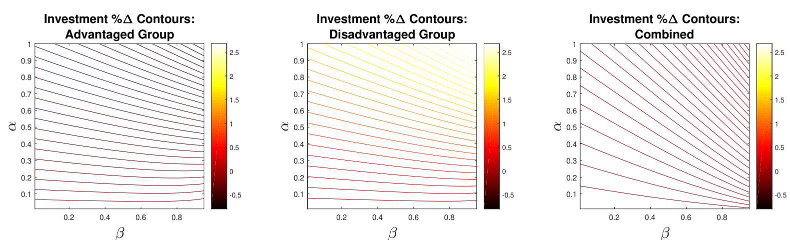
(A) $\sigma = 0.15$



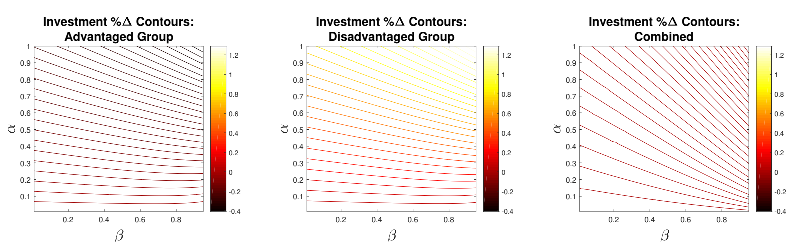
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



(D) $\sigma = 0.70$

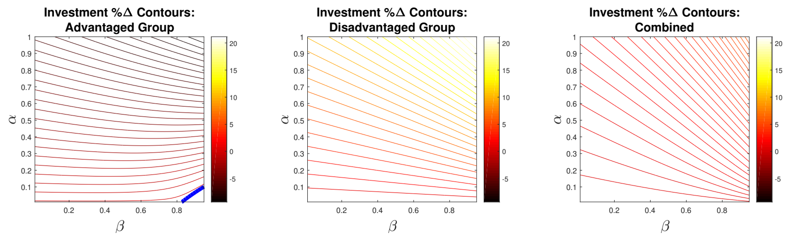


,

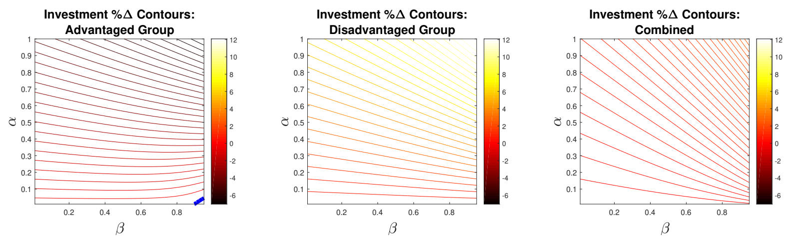
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the advantaged group in panels (D) and (E) are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

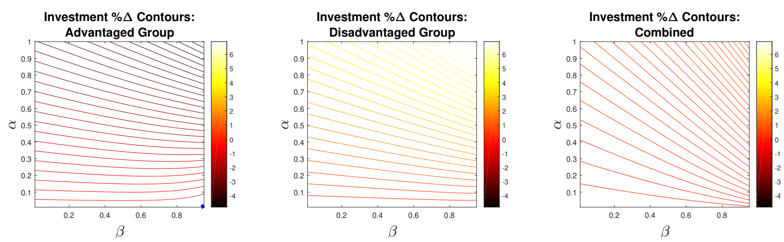
Figure 8. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.50$ (Both Groups Same Size) with Quadratic Costs**



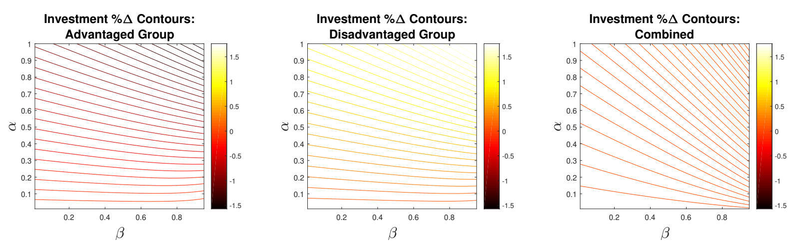
(A) $\sigma = 0.15$



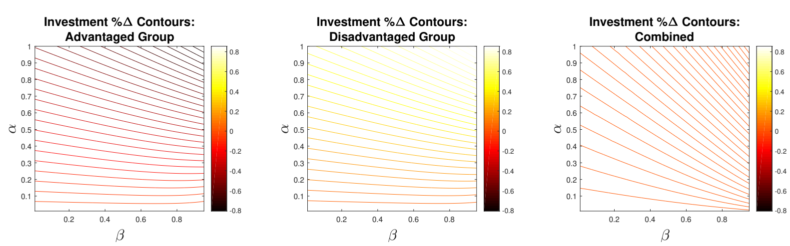
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



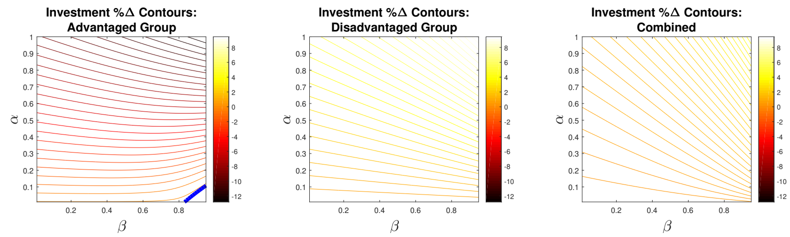
(D) $\sigma = 0.70$



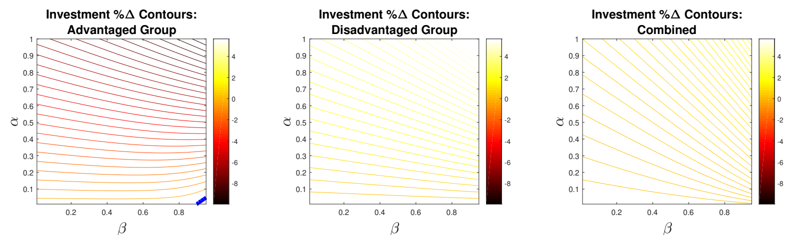
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the advantaged group in panels (D) and (E) are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

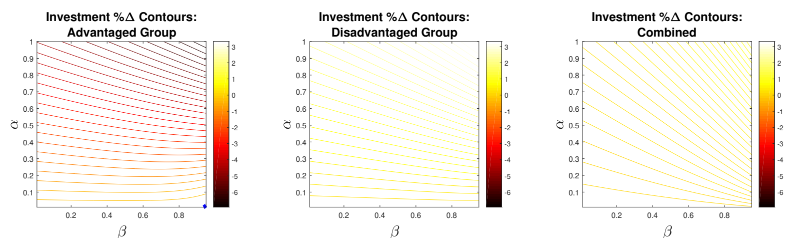
FIGURE 9. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.75$ (Disadvantaged Group is Statistical Majority) with Quadratic Costs**



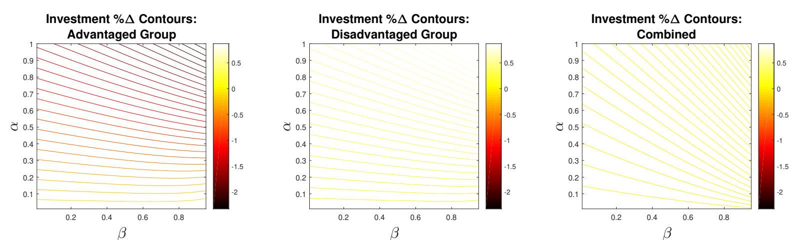
(A) $\sigma = 0.15$



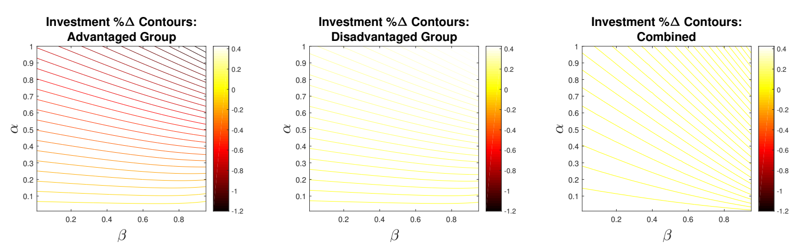
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



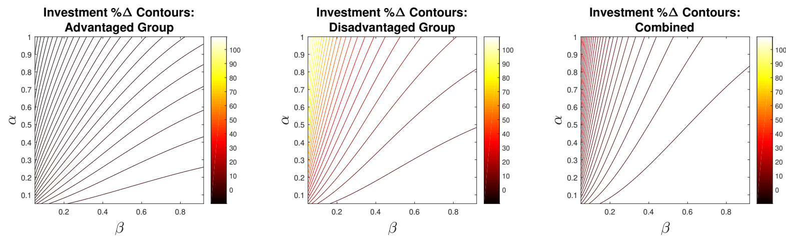
(D) $\sigma = 0.70$



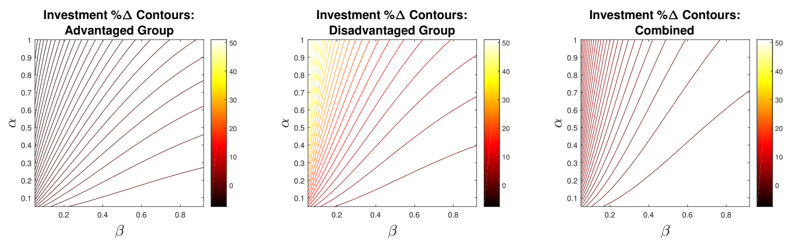
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the advantaged group in panels (D) and (E) are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

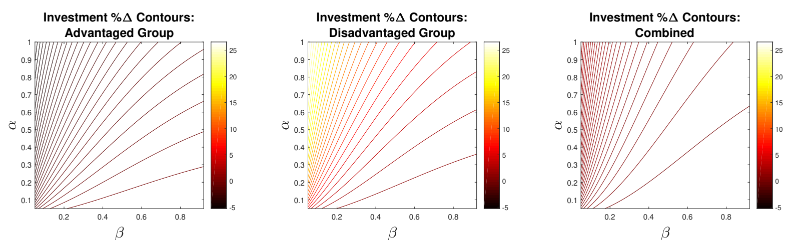
Figure 10. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.25$ (Disadvantaged Group is Statistical Minority) with Exponential Costs**



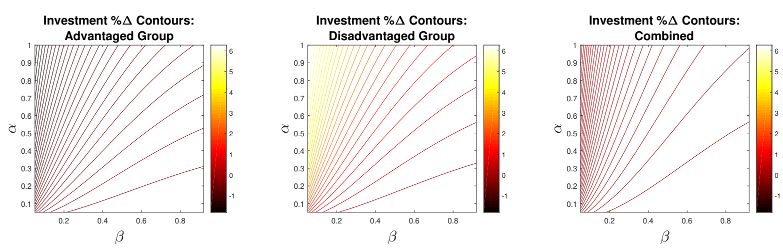
(A) $\sigma = 0.15$



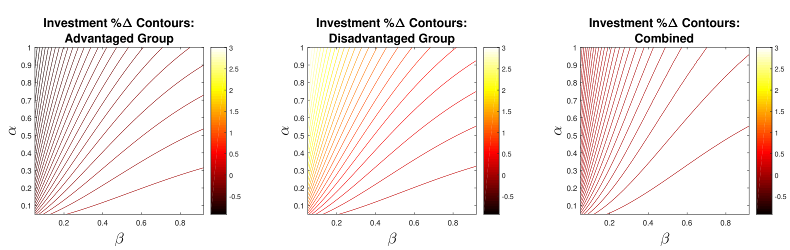
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



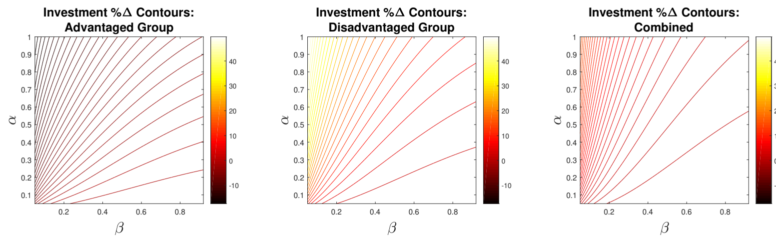
(D) $\sigma = 0.70$



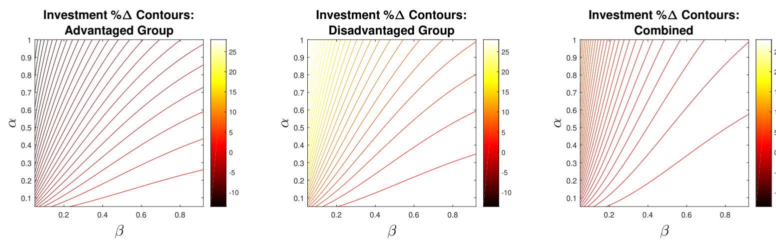
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. All depicted percent changes for the advantaged group are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

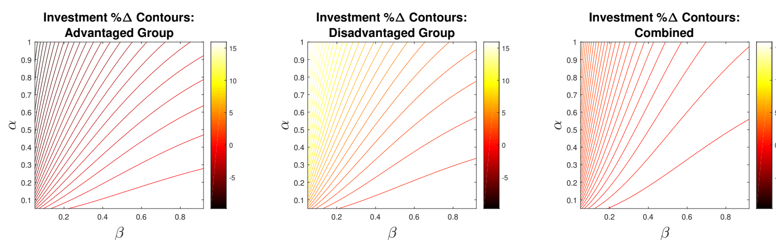
Figure 11. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.50$ (Both Groups Same Size) with Exponential Costs**



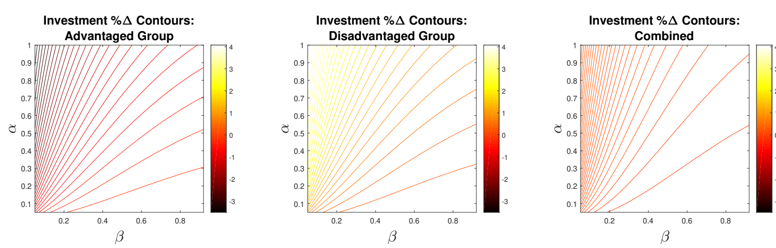
(A) $\sigma = 0.15$



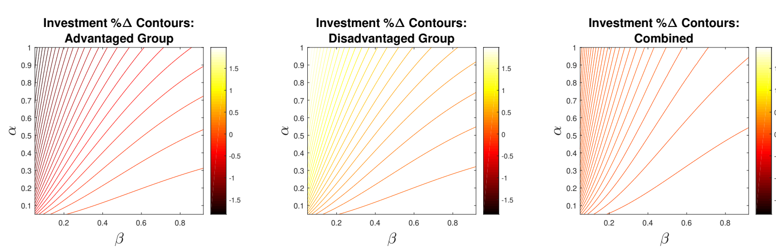
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



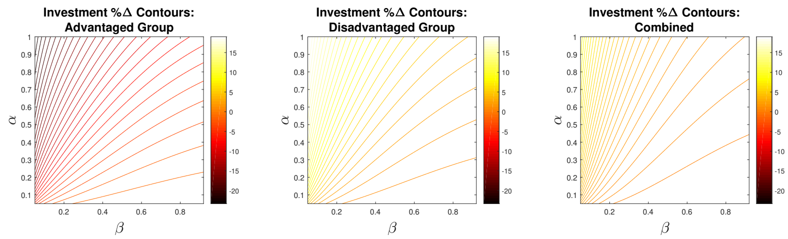
(D) $\sigma = 0.70$



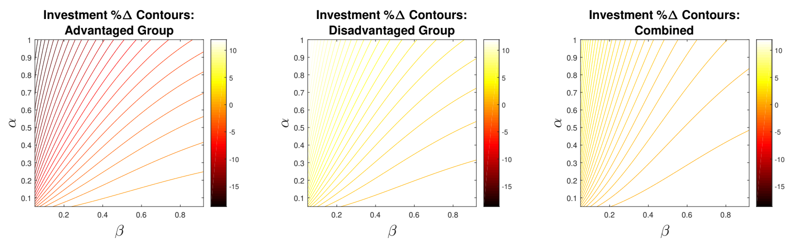
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. All depicted percent changes for the advantaged group in this figure are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

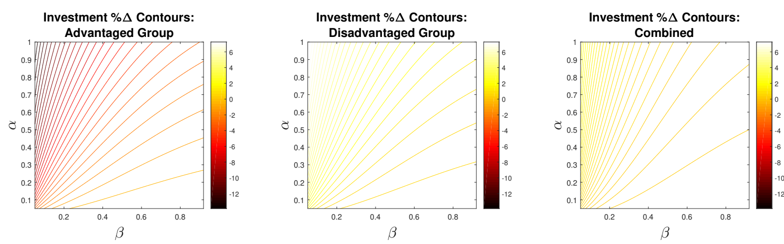
FIGURE 12. **Numerical Examples: $\mu_D = 1.5$, $\mu_A = 1.1$; and $\delta = 0.75$ (Disadvantaged Group is Statistical Majority) with Exponential Costs**



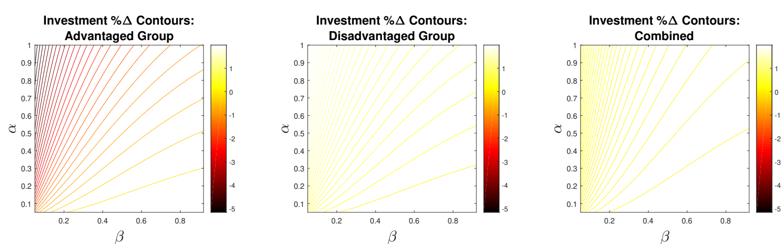
(A) $\sigma = 0.15$



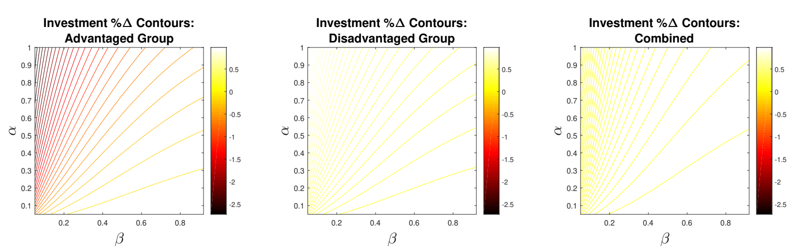
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



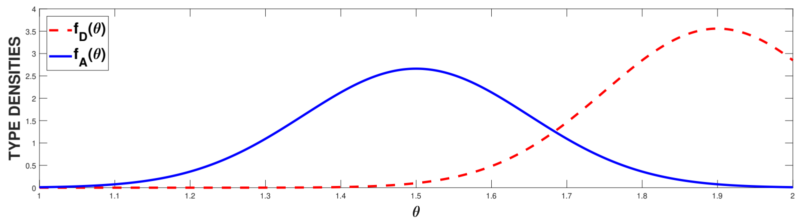
(D) $\sigma = 0.70$



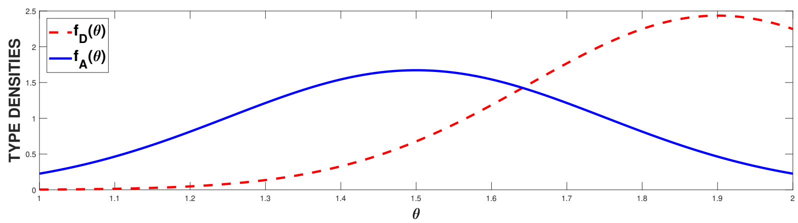
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. All depicted percent changes for the advantaged group in this figure are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

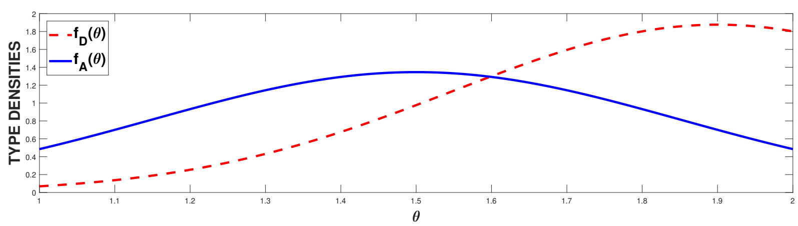
FIGURE 13. **Truncated Normal Cost Densities:** $\mu_D = 1.9$ **and** $\mu_A = 1.5$



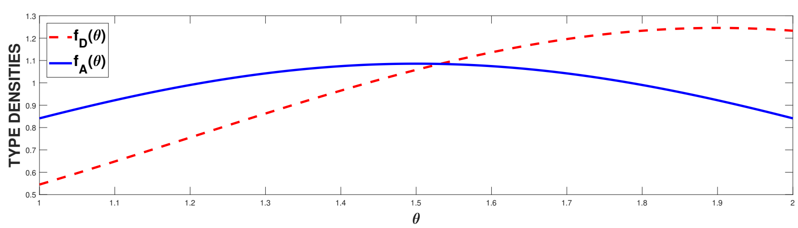
(A) $\sigma = 0.15$



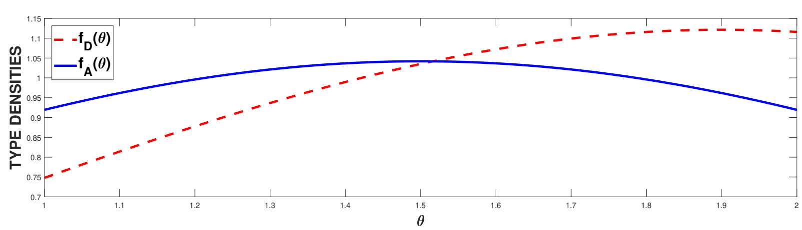
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$

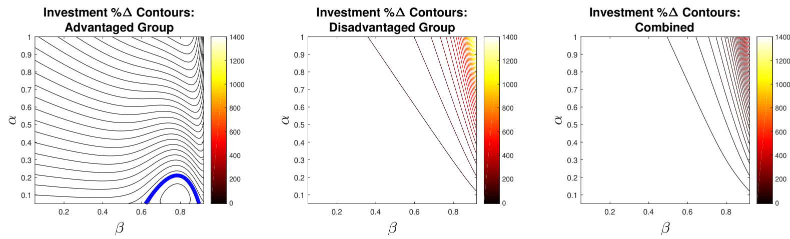


(D) $\sigma = 0.70$

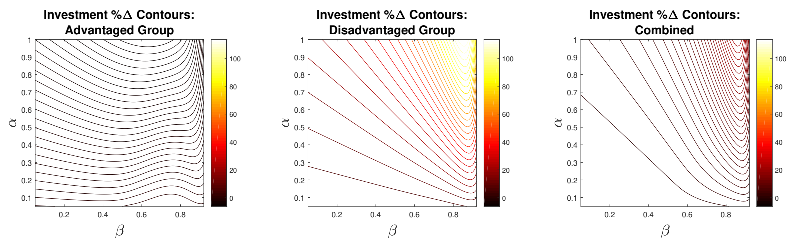


(E) $\sigma = 1$

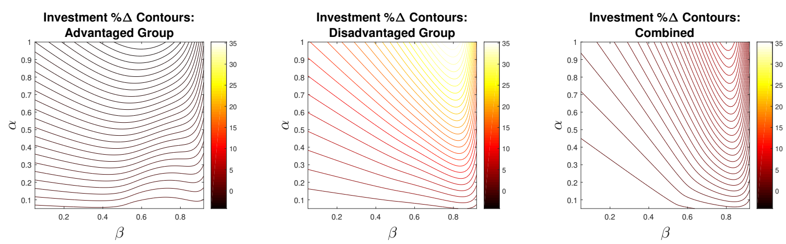
FIGURE 14. **Numerical Examples: $\mu_D = 1.9$, $\mu_A = 1.5$; and $\delta = 0.25$ (Disadvantaged Group is Statistical Minority) with Linear Costs**



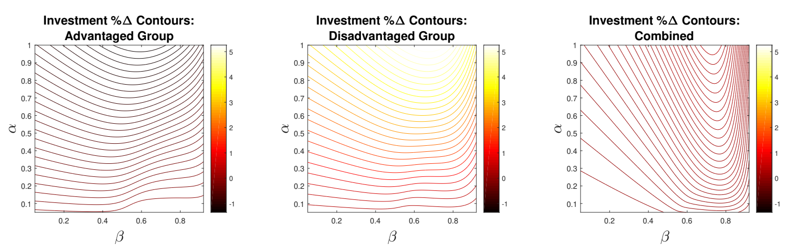
(A) $\sigma = 0.15$



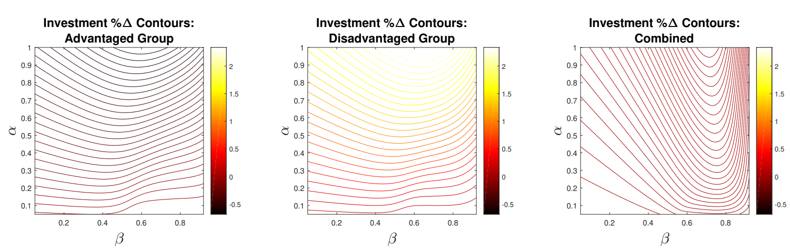
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



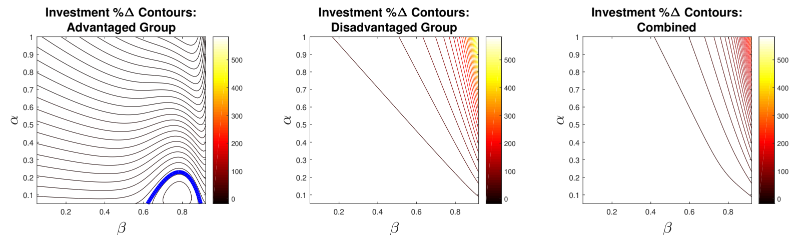
(D) $\sigma = 0.70$



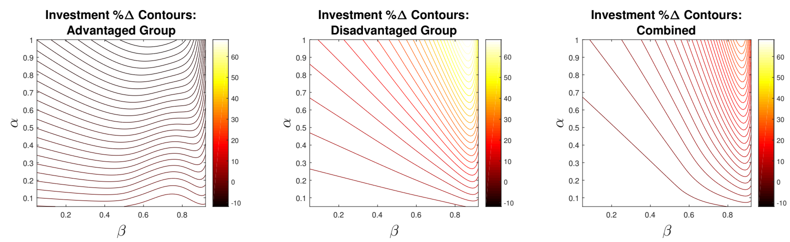
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the advantaged group in panels (B)–(E) are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

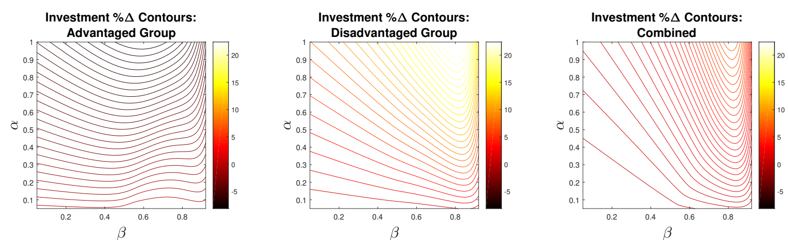
FIGURE 15. **Numerical Examples: $\mu_D = 1.9$, $\mu_A = 1.5$; and $\delta = 0.50$ (Both Groups Same Size) with Linear Costs**



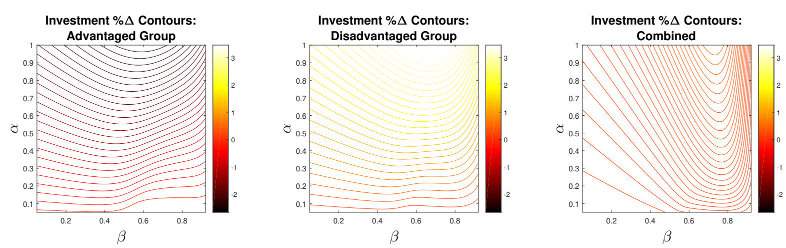
(A) $\sigma = 0.15$



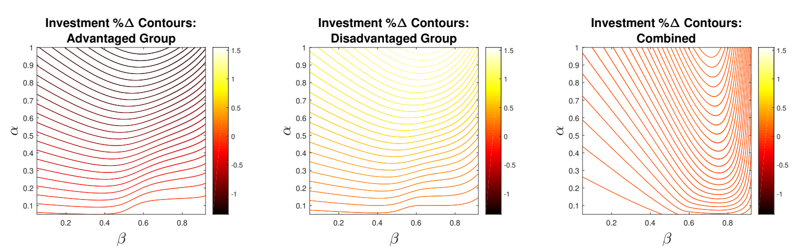
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



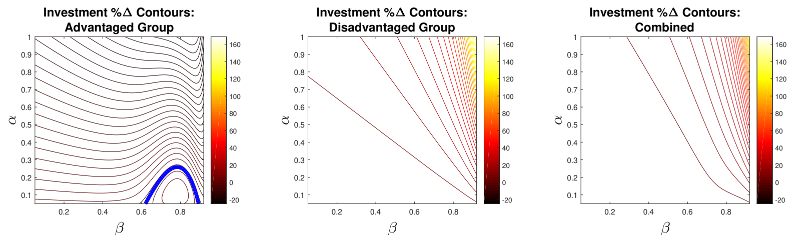
(D) $\sigma = 0.70$



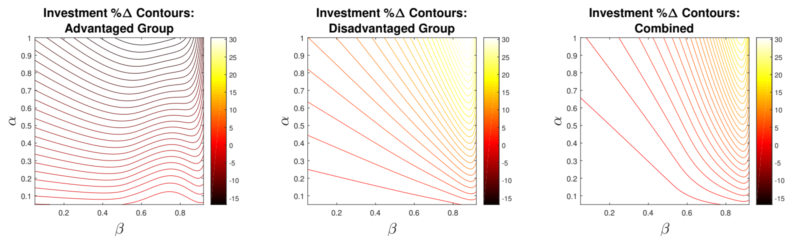
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the advantaged group in panels (B)–(E) are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

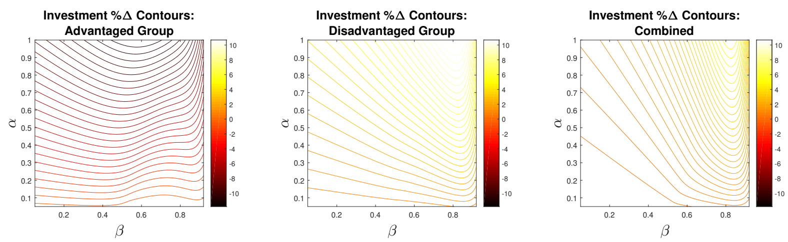
Figure 16. **Numerical Examples: $\mu_D = 1.9$, $\mu_A = 1.5$; and $\delta = 0.75$ (Disadvantaged Group is Statistical Majority) with Linear Costs**



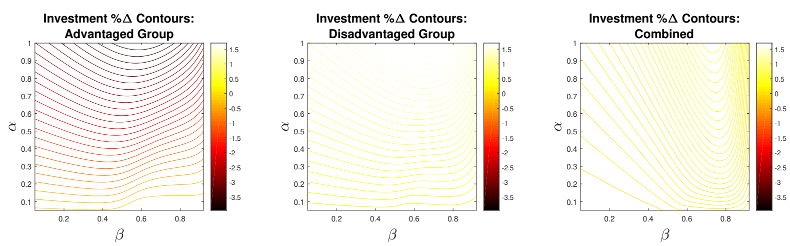
(A) $\sigma = 0.15$



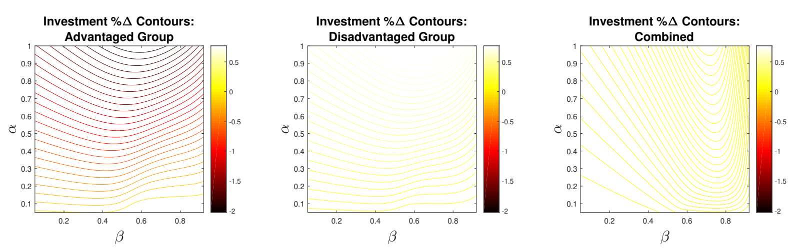
(B) $\sigma = 0.25$



(C) $\sigma = 0.35$



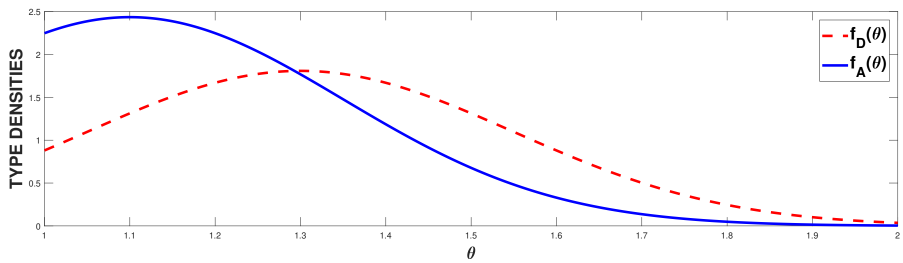
(D) $\sigma = 0.70$



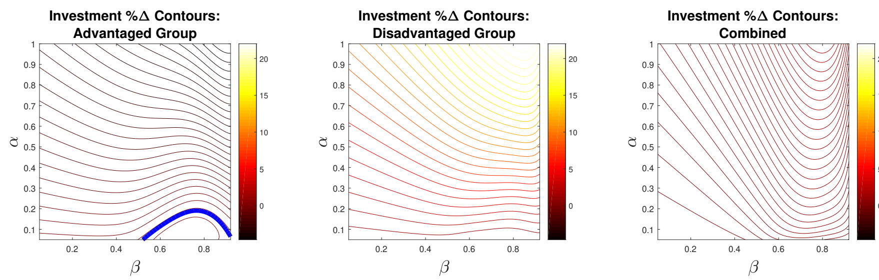
(E) $\sigma = 1$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the advantaged group in panels (B)–(E) are negative, while all percent changes for the disadvantaged group and for the combined population are positive.

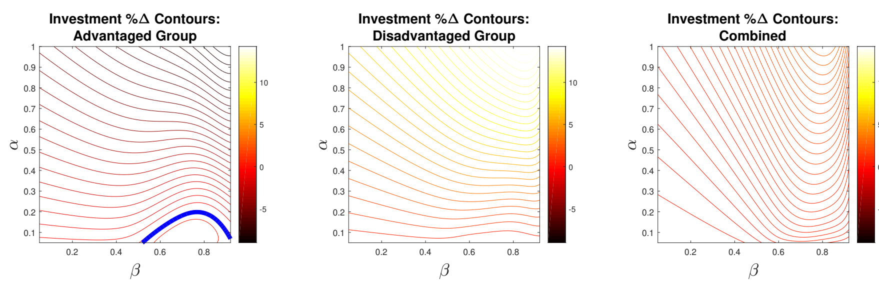
Figure 17. **Numerical Examples: $\mu_D = 1.3$, $\mu_A = 1.1$; various δ's; Linear Costs**



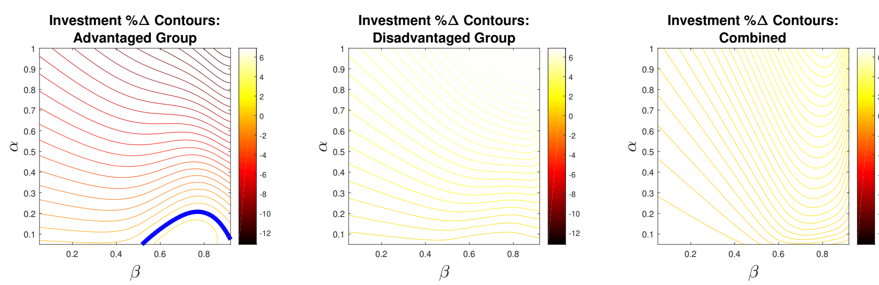
(A) Type Densities



(B) $\delta = 0.25$



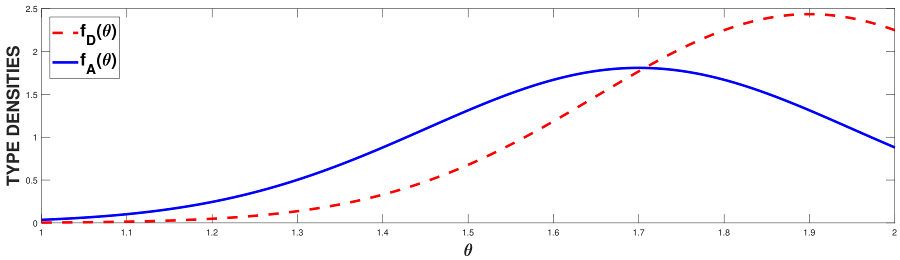
(C) $\delta = 0.50$



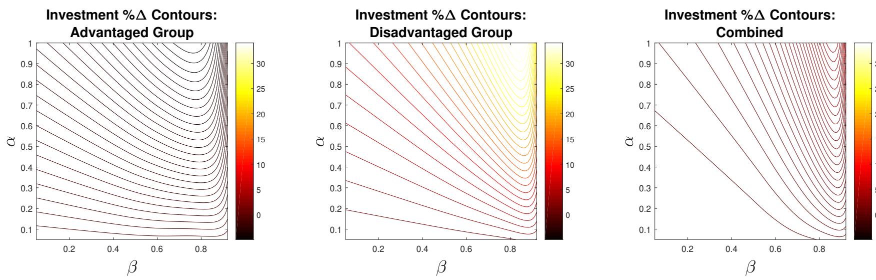
(D) $\delta = 0.75$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. Thick lines represent the zero level curve. All depicted percent changes for the disadvantaged group and for the combined population are positive.

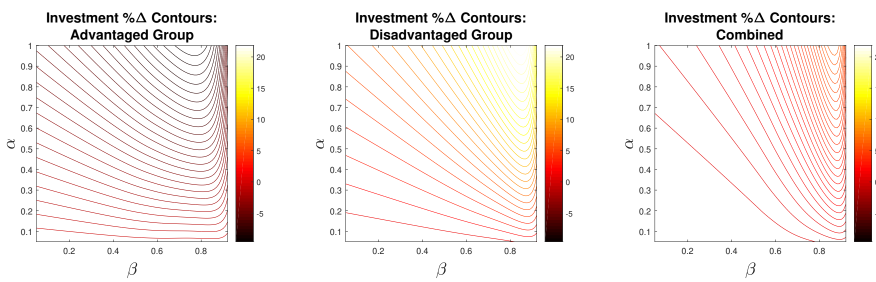
FIGURE 18. **Numerical Examples: $\mu_D = 1.9$, $\mu_A = 1.7$; various δ's; Linear Costs**



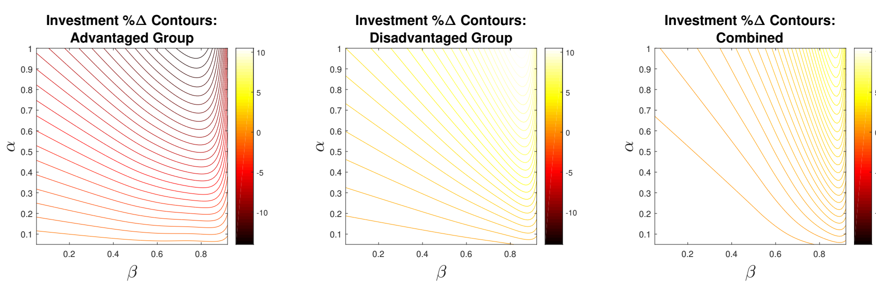
(A) Type Densities



(B) $\delta = 0.25$



(C) $\delta = 0.50$



(D) $\delta = 0.75$

NOTES: For the Cobb-Douglass match utility function, α is the weight on college quality and β is the weight on human capital. All figures depict level curves of percent changes for mean human capital investment under different configurations of the utility parameters. All depicted percent changes for the advantaged group are negative, while all percent changes for the disadvantaged group and for the combined population are positive.