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# Exchange Rate Pass-Through: A Competitive Search Approach

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# Exchange Rate Pass-Through: A Competitive Search Approach*

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**Abstract**

We develop an open economy monetary model with heterogeneous households which is characterized by incomplete pass-through of exchange rate movements to import prices. Partial pass-through arises in our environment due to the presence of competitive search in international goods' markets. Under competitive search, agents choose a submarket in which to exchange goods, where different submarkets are characterized by different price and trading probability combinations. Preference and policy shocks which induce exchange rate movements cause households to choose a different submarket for their purchases of traded goods–an extensive margin response. These responses mitigate the direct effect of nominal exchange rate changes on equilibrium traded goods' prices, thereby generating incomplete exchange rate pass-through to goods' prices. In the calibrated model, exchange rate pass-through due to foreign shocks ranges between 19% and 62%, which is in the range of import price pass-through estimates for developed economies. Due to risk aversion by households, the magnitude of pass-through depends on the size and direction of the initial shock, making the model consistent with the observed phenomenon of asymmetric pass-through. Importantly, by incorporating household heterogeneity, we are able to examine the role of precautionary savings in affecting pass-through, characterize how pass-through varies across different types of households, and examine the distributional effects of exchange rate movements.

Keywords: Exchange Rate Pass-Through, Competitive Search, Monetary Policy.

JEL: F31,O24, E58

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# 1 Introduction

In this paper, we develop an open economy monetary model with heterogeneous households which is characterized by incomplete pass-through of exchange rate movements to prices. Importantly, partial pass-through arises in our environment due to the presence of competitive search in international goods' markets. Under competitive search, agents choose a submarket in which to exchange goods, where different submarkets are characterized by different price and trading probability combinations. Preference and policy shocks which induce exchange rate movements cause households to choose a different submarket for their purchases of traded goods–an extensive margin response. These endogenous changes in submarkets mitigate the direct effect of nominal exchange rate changes on equilibrium traded goods' prices, thereby generating incomplete pass-through. By emphasizing the role of heterogeneous buyers' behavior in generating incomplete exchange rate pass-through, a novel feature of our analysis is variation in pass-through rates across households and distributional effects of exchange rate movements. Furthermore, by incorporating household heterogeneity and uncertainty, we are able to examine the role of precautionary savings in affecting pass-through rates.

In the calibrated model, exchange rate pass-through due to small foreign shocks varies between 19% and 62%, which is in the range of import price pass-through estimates for developed economies, (see Burstein and Gopinath (2014), for example). More generally, because the degree of pass-through is dependent on household responses and households are risk averse in our model, the magnitude of exchange rate pass-through depends on the type, size, and direction of the initial shock. For example, pass-through in response to foreign shocks is lower than in response to analogous domestic shocks. In addition, pass-through rates are highest in response to domestic monetary policy shocks, which is consistent with Taylor's (2000) hypothesis that stable monetary policy is associated with relatively low exchange rate pass-through. Furthermore, because pass-through varies with the size and direction of shocks, the model is consistent with the observed phenomenon of asymmetric pass-through.

It is well known that empirically, nominal exchange rates exhibit more volatility than goods' prices and this implies a high degree of correlation between real and nominal exchange rates. This economic phenomenon is often referred to as Exchange Rate Disconnect (Obstfeld and Rogoff (2000)) and is associated with incomplete pass-through of exchange rate movements to goods' prices. Burstein and Gopinath (2014) estimate exchange rate pass-through to import and consumer prices for several countries. Their estimates for short-run pass-through range from 13% to 75% for import prices and from -4% to 11% for consumer prices. Their findings are consistent with the general consensus in the literature that exchange rate pass-through to goods' prices is incomplete and is lower for consumer prices than for import prices.

The theoretical literature has explored many explanations for the observed low degrees of pass-through including firms engaged in pricing to market, imported and domestic inputs, nominal rigidities, local distribution networks, etc. (see Burstein and Gopinath (2014) for a survey). Most of those models incorporate non-competitive market structures where firms are strategically choosing their price adjustments in response to shocks and this feature is crucial for generating incomplete pass-through. In this paper, we focus on exchange rate pass-through to import prices in an environment with search frictions but where markets are "competitive" in the sense that sellers (firms) and buyers (households) take the terms of trade and the tightness of all markets as given when choosing which market to enter.

While most of the theoretical literature on exchange rate pass-through has focused primarily on the role of sellers' behavior in generating incomplete pass-through, our approach places more emphasis on the role of buyers' behavior.[1] In particular, we explore a novel factor which may contribute to incomplete exchange rate pass-through to prices: buyers shift their demand toward less-expensive imported goods when their currency depreciates and toward more expensive ones under an appreciation. Consider a domestic currency depreciation. If the price of a traded good denominated in the foreign currency does not change, then domestic buyers will face a higher domestic currency price for that good. However, if those buyers are able to switch their expenditure to a seller which posts a lower price denominated in the foreign currency (at the cost of a lower probability of obtaining the good), then this will mitigate the impact of the depreciation on the domestic currency price that buyers pay. This response by households implies that the domestic currency price in trades will increase by less than the nominal exchange rate and pass-through will be incomplete. Thus, expenditure switching across submarkets for imported goods by buyers in response to nominal exchange rate movements could be a contributing factor to incomplete exchange rate pass-through. We show that a competitive search model in which buyers are facing a tradeoff between price and the probability of acquiring the good when choosing in which market to shop is consistent with this story.

The magnitude of exchange rate pass-through has important implications for at least two aspects of monetary policy. First, it is an important determinant for predicting how much a depreciation of domestic currency will stimulate an economy as domestic consumers substitute home goods for foreign goods. Thus, the extent of pass-through of exchange rate movements to prices is relevant for the impact of those movements on quantities and consumer welfare. Secondly, the relationship between prices and exchange rates is relevant for predicting inflation rates. In perfectly competitive markets, according to traditional monetary theory, price inflation and exchange rate depreciations should be highly correlated since exchange rate movements will be directly transmitted into import and consumer prices. Thus, the degree of pass-through will influence policy makers' forecasts of the future path of inflation. In fact, some researchers have noted that recent shifts toward low inflation and more credible monetary policy environments in developed countries has coincided with documented declines in exchange rate pass-through. Taylor (2000) suggests that there may, in fact, be a "virtuous cycle" between stable monetary

[1] Bergin and Feenstra (2001), Froot and Klemperer (1989), Knetter (1994), Marston (1990), and Ravn et al. (2007) are among the papers which analyze models where firms engaged in pricing to market are concerned with maintaining market share. Atkeson and Burstein (2008), Burstein and Gopinath (2014), and Baggs et al. (2018) are among the papers which examine environments in which exchange rate movements affect the costs of selling in markets characterized by imperfect competition. Bachetta and van Wincoop (2005), Devereux et al. (2004), Devereux and Yetman (2010), Engel (2006), and Gopinath et al. (2010) are among the papers which analyze the behavior of price-setting firms in the presence of nominal rigidities. Alessandria (2009) and Baggs et al. (2018) are among the papers which incorporate random consumer search to study pass-through. See Burstein and Gopinath (2014) for a more complete survey of theoretical papers exploring incomplete exchange rate pass-through.

policy and low pass-through.

Our environment has two countries, each with their own currency and endogenous determination of real and nominal exchange rates. Heterogeneous households consume a non-traded good in their country and traded goods produced in the other country. The traded goods are our focus and are sold in frictional markets with producer currency pricing. These frictional markets consist of a variety of submarkets and are characterized by competitive search, as in Menzio, Shi, and Sun (2013) and Sun and Zhou (2018). Submarkets differ according to their posted price (terms of trade) and the probability that a firm and household will match for a trade in the submarket. Households choose in which submarket to participate, taking into account the tradeoff between price and matching probability. Firms choose the measure of shops to open in each submarket, also taking into account this tradeoff. Households and firms take the price and matching probability as given when making their optimal choices. Within a submarket, firms and households are randomly matched pair-wise according to the matching probability in that submarket. Households are heterogeneous in that they receive idiosyncratic preference shocks each period and hence, face, uninsurable income risk.

As noted in Menzio, Shi, and Sun (2013) and Sun and Zhou (2018), the competitive search environment with household heterogeneity is a particularly useful monetary model which generates a non-degenerate wealth distribution yet is tractable as household problems can be solved independently from the endogenous wealth distribution. Adopting this framework for our purposes allows us to examine the distributional effects of policy and non-policy shocks which change real and nominal exchange rates. Furthermore, it is precisely the competitive search environment which generates incomplete exchange rate pass-through, even in the absence of household heterogeneity. That is, the ability of households to choose a submarket for the same good with a different price and matching probability when the nominal exchange rate changes mitigates the effect of the exchange rate on transaction prices and leads to incomplete pass-through.

Our framework allows us to examine both the relationship between nominal exchange rates and nominal prices (nominal exchange rate pass-through) and the relationship between real exchange rates and real prices (real exchange rate pass-through). We are also able to examine pass-through in a domestic economy in response to both foreign and domestic shocks. With shocks to the foreign economy, the only impact on household choices are through movements in exchange rates. In contrast, under domestic economy shocks there is both a direct effect of the shock and an indirect effect via exchange rates on household choices.

We calibrate our economy and demonstrate, using numerical experiments, that both types of exchange rate pass-through in the domestic economy are less than 100% (i.e. incomplete) in our baseline calibration for all types of shocks except domestic money growth shocks. We also observe that in our economy, pass-through of foreign shocks is significantly lower than for analogous domestic shocks. We show that pass-through rates are significantly different from the baseline economy when we eliminate income uncertainty or household heterogeneity. These results are explained by the important role of a precautionary motive for generating empirically plausible pass-through levels. Furthermore, the baseline economy exhibits pass-through levels which vary with the type, direction, and magnitude of shocks, implying that the model generates asymmetric pass-through rates, consistent with observed pass-through phenomenon. In addition, by incorporating heterogenous buyers, our framework allows us to study the distributional effects of monetary and fiscal policies through their impact on exchange rates.

The two most closely related papers to ours are Sun and Zhou (2018) and Deviatov and Dodonov (2006). Sun and Zhou (2018) develop a closed economy model with competitive search and heterogeneous households to study the effects of fiscal and monetary policies on the intensive and extensive margins of exchange in frictional markets. The paper by Deviatov and Dodonov (2006) studies real and nominal exchange rate dynamics in a model with search frictions. In this paper, we focus on the role of household heterogeneity in affecting pass-through patterns while those authors are more interested in exchange rate dynamics and our theoretical environment differs from theirs in several important ways. First, in contrast with their work where a household's desire for the consumption of a foreign good is driven by an idiosyncratic preferences shock, the choice of consuming traded goods in our model is endogenous and depends on a household's income. Second, Deviatov and Dodonov (2006) assume a country-specific cash-in-advance constraint, while in our environment, money has an essential role. Third, search is competitive in our model whereas search is random in their environment. Finally, we focus on the role of household heterogeneity in affecting the distributional effects of and the level of exchange rate pass-through, which is not the focus of their work.

The remainder of the paper is organized as follows. In Section 2, we present and analyze the theoretical model. We calibrate our economy in Section 3 and present the results of a series of numerical experiments designed to characterize the nature of exchange rate pass-through in response to various types of shocks. In that section, we highlight the role of household heterogeneity and precautionary savings in generating incomplete exchange rate pass-through and demonstrate that the model generates asymmetric pass-through. We end that section by examining how pass-through rates vary across heterogeneous households and the distributional effects of shocks through their impact on exchange rates. Section 4 concludes.

# 2 Model

## 2.1 The Environment

There are two countries, Home (H) and Foreign (F). Foreign variables are denoted with a *hat* ( $\hat{}$ ). Time is discrete and each period consists of two sub-periods. Each country is populated by a continuum of infinitely lived households of measure one with discount factor $\beta \in [0, 1]$. Each household consists of a worker and a buyer. Households in each country supply labor to produce a non-traded good and they consume a non-traded good in the first sub-period. Non-traded goods are traded in competitive and frictionless markets. In the second sub-period, households in H (F) supply labor to produce traded goods and a household may choose to try to purchase one variety of a traded good produced in Country F (H).

Each period, each H (F) household receives a preference shock which determines which variety of the F (H) traded goods they will try to purchase if they seek to purchase a traded good. Traded goods are exchanged in frictional markets where trading frictions arise due to households' random preferences for purchasing a particular variety of the traded good produced in the other country. The two non-traded goods and the traded differentiated goods are perfectly divisible and non-storable.

Members of a household share income, consumption of non-traded and traded goods, and labor costs. Household preferences take the same form in both countries and are given by:

$$\mathcal{U}(y, q, \ell) = U(y) + u(q) - \theta\ell, \tag{1}$$

where $y$ is consumption of the non-traded good, $q$ is consumption of an imported good, and $l$ is labor input. The parameter $\theta$ is an idiosyncratic shock, where $\theta \in [\theta_L, \theta_H]$ and $0 < \theta_L < \theta_H < \infty$. The preference parameter is *i.i.d.* across households and over time, is realized at the beginning of each period, and is drawn from the distribution function $\mathcal{F}(\theta)$ with $E(\theta) = \bar{\theta}$ in $H$ and $\hat{\mathcal{F}}(\theta)$ with $E(\theta) = \hat{\bar{\theta}}$ in $F$. $U(.)$ and $u(.)$ are twice continuously differentiable with $U' > 0$, $u' > 0$, $U^{''} < 0$, $u^{''} < 0$, $U'(0)$ and $u'(0)$ are large but finite, and $U(0) = u(0) = U'(\infty) = u'(\infty) = 0$. Households own equal shares of all firms in their country. Households cannot borrow or lend and there is no insurance for fluctuations in household income.

Firms last for one period and new firms are formed at the beginning of each period. Firms hire labour at the beginning of each period to produce non-traded and traded goods and the labor market is competitive and frictionless. The non-traded good's market in each country is a perfectly competitive, frictionless market with a measure one of firms. One unit of labor is required to produce one unit of the non-traded good in each country.

Each firm produces one variety of traded good. Firms are divided into equally sized groups and firms belonging to the same group produce the same variety of the traded good for sale to buyers from the other country. Traded goods are exchanged in frictional markets characterized by competitive search with various submarkets. These markets are established in the sellers' country where only buyers from the other country are allowed to trade. Firms choose how many shops to set up in each submarket and must hire $\kappa > 0$ units of labor per shop in order to sell their good. In addition, the labor required to produce $q$ units of a variety of a traded good for sale in any submarket equals $\psi(q)$ where $\psi(.)$ is twice continuously differentiable with $\psi(0) = 0$, $\psi' > 0$, and $\psi^{''} < 0$.

Each submarket is characterized by its terms of trade and matching probabilities for a buyer, $b$, and a shop, $s$. A submarket consists of a variety of shops selling differentiated goods all at the same terms of trade. Firms and consumers take the characteristics of submarkets as given and choose a submarket to enter. Buyers and sellers in a submarket are randomly matched according to a constant returns to scale technology characterized by a matching function, $s = \mu(b)$. We assume that for all $b \in [0, 1]$, $\mu(b) \in [0, 1]$, $\mu(0) = 1$, $\mu(1) = 0$, $\mu'(b) < 0$ and $[1/\mu(b)]$ is strictly convex. In the frictional markets, buyers and sellers are anonymous and there is no record-keeping technology for trades. Hence, there is a need for a medium of exchange to facilitate transactions in these markets.

There is a government in each country which issues distinct fiat objects called H money and F money. These objects are perfectly divisible and storable. The money stock per capita in each country evolves according to:

$$M_{t+1} = \gamma M_t \qquad \hat{M}_{t+1} = \hat{\gamma}\hat{M}_t \qquad \forall t, \tag{2}$$

where $\gamma > \beta$ and $\hat{\gamma} > \beta$ are the money growth rates. Although only one type of money is sufficient to overcome exchange issues in frictional traded goods' markets, we assume that goods in these markets must be purchased using the seller's currency. Hence, both types of money (currencies) will exist in equilibrium. Currencies can be exchanged in a competitive currency exchange market at nominal exchange rate $S$, where $S$ is the number of units of F money needed to buy one unit of H money. Each currency is also used for transactions in the relevant non-traded good's market.

The government in Country $H$ $(F)$ also imposes proportional taxes on households' wage income at rate $\tau \in [0, 1]$ $(\hat{\tau} \in [0, 1])$. Every period, each government makes lump-sum transfers to domestic households, which are financed by additional printed money and income taxes collected from households. All tax payments and lump-sum transfers are made with domestic currency.

Home labor is the numeraire and we define Home real variables as nominal variables (denoted in $H$ currency) divided by the Home nominal wage, $W$. Letting $m$ denote a household's *real* money balances, then the amount of Home currency associated with this real balance equals $mW$. We also let $w$ denote the *normalized* Home wage: $w = \frac{W}{M}$ and set its value to 1. For consistency, we define Foreign real variables as nominal variables (denoted in $F$ currency) divided by the Foreign nominal wage, $\hat{W}$. Finally, we let $\hat{w}$ denote the Foreign normalized wage relative to the Home normalized wage.

## 2.2 Firms' Decisions

We focus on a representative Home firm who seeks to maximize the sum of its profits from sales of the non-traded good and sales of its traded good variety. A firm takes the price of the non-traded good, $p$, (measured in Home labor units) as given and chooses output of the non-traded good, $Y$. We index traded goods' submarkets in $H$ by their terms of trade $(\hat{x}, \hat{q})$, where $\hat{x}$ is the amount of $H$ currency received from a Foreign household in exchange for $\hat{q}$ units of a traded good.[2] An $H$ firm chooses the measure of shops to create in each submarket and we let $N(\hat{x}, \hat{q})$ denote the distribution of shops across submarkets. Because there is endogenous entry by households and firms into submarkets, the equilibrium matching probabilities in each submarket are functions of its terms of trade: $s(\hat{x}, \hat{q})$ is the probability that a Home shop makes a sale and $b(\hat{x}, \hat{q})$ is the probability that a Foreign buyer makes a purchase.

The expected benefit of creating a shop in submarket $(\hat{x}, \hat{q})$ is $s(\hat{x}, \hat{q})\,(\hat{x} - \psi(\hat{q}))$ . In submarkets where

[2]The terms of trade are denoted with a *hat* because these are the terms of trade facing a Foreign consumer. The terms of trade facing a Home consumer are denoted $(x, q)$ below.

$s(\hat{x},\hat{q})\,(\hat{x}-\psi(\hat{q}))>\kappa$, it is optimal for a firm to create infinitely many shops but this cannot occur as it implies $s(\hat{x},\hat{q})=0$, which is inconsistent with this case. It is optimal for a firm to create no shops in submarkets where $s(\hat{x},\hat{q})\,(\hat{x}-\psi(\hat{q}))<\kappa$ and without loss of generality, we set $s(\hat{x},\hat{q})=1$ and $b(\hat{x},\hat{q})=0$ in those markets. If $s(\hat{x},\hat{q})\,(\hat{x}-\psi(\hat{q}))=\kappa$, a firm is indifferent among different measures of shops in those submarkets. In equilibrium, entry into each submarket occurs until expected profits equal zero. Hence, the equilibrium matching probability for a shop in submarket $(\hat{x},\hat{q})$ is given by:

$$s(\hat{x},\hat{q})=\mu(b(\hat{x},\hat{q}))=\begin{cases}\frac{\kappa}{\hat{x}-\psi(\hat{q})}, & \kappa\leq\hat{x}-\psi(\hat{q})\\ 1, & \kappa>\hat{x}-\psi(\hat{q})\end{cases} \tag{3}$$

This expression clarifies our claim above that equilibrium matching probabilities in a submarket are determined by its terms of trade. We have a similar expression for matching probabilities in submarkets with Home buyers and Foreign sellers, $s(x,q)$ and $b(x,q)$.

## 2.3 Households' Decisions

Before turning to the households' decision problems, we clarify how they interact with firms in the labor markets. At the beginning of a period, workers from households receive an IOU from firms based on their expected labor supply. A worker's actual labor input depends on the submarket they are assigned to and whether or not the shop where they are working makes a sale. Since households are risk neutral in labor supply, this arrangement is acceptable to them. Firm IOUs are a firm's promise to make wage payments at the end of a period in money. These IOUs are feasible because firm's revenues and costs are deterministic given that they own a continuum of shops. Recalling that firms last for only one period, we note that IOUs circulate for only one period.

We focus on a representative household in Country $H$. At the beginning of each period, a household has real money balances from the end of last period, $m$, they observe their preference shock, $\theta$, and they receive a lump-sum transfer, $T$, from the Home government. At the beginning of the first sub-period, taking the price of the non-traded good, $p$, and the terms of trade of all submarkets for traded goods as given, a household chooses consumption of the non-traded good, $y\geq 0$, expected labor supply, $\ell\geq 0$, the asset balance to be used for purchases of a traded good, $z\geq 0$, and a precautionary balance to carry into the next period, $h\geq 0$. We assume there is a maximum real money balance, $\overline{m}$, that a household is able to carry across periods, such that $0<\overline{m}<U'^{-1}(\overline{\theta})$.[3] In addition, households pay income taxes based on their expected labour income.[4]

If the household chooses a strictly positive $z$, then at the beginning of the second sub-period, the household buyer chooses a submarket to enter to try and match with a shop selling a traded good. If the

[3]We follow Sun and Zhou (2018) in assuming the existence of an exogenous upper bound on money holdings to maintain block recursivity in the model, as explained in footnote 5 of that paper.

[4]Households are also entitled to receive dividends but in equilibrium no dividends are distributed because firms earn zero profits.

buyer chooses a submarket with terms of trade $(x, q)$, he is matched with a shop with probability $b(x, q)$. If he is matched with a shop, he must spend $\hat{W}x$ units of $F$ currency and so must convert $\frac{\hat{W}x}{S}$ units of $H$ currency to $F$ currency. This equals $\frac{\hat{W}x}{SW}$ in real terms or $\frac{x}{\delta}$, where $\delta$ is the (normalized) real exchange rate defined as:

$$\delta \equiv S\left(\frac{W}{\hat{W}}\right). \tag{4}$$

This is appropriately termed a real exchange rate as it expresses the value of F labor per one unit of H labor. If the buyer is not matched with a shop, $z$ is carried into the next period along with the precautionary balance $h$. If the household chooses $z = 0$, then they do not participate in any traded good submarket and they carry $h$ into the next period.

We turn to analyzing a household's decisions in each sub-period. Letting $\mathcal{W}(m, \theta)$ be the value function of a household with individual state $(m, \theta)$ at the beginning of a period, we have the following Bellman equation:

$$\mathcal{W}(m, \theta) = \max_{(y,\ell,h,z)} \{U(y) - \theta\ell + V(z, h)\} \tag{5}$$

$$s.t. \quad py + z + h \leq m + (1-\tau)\ell + T,$$

where $V(z, h)$ is the household's value function at the beginning of the second sub-period. The characteristics of $V(z, h)$ are analyzed below when we examine the second sub-period decisions. Noting that the budget constraint will bind and that $p = 1$ in equilibrium, we can eliminate $\ell$ in the objective function and rewrite the value function at the beginning of the period as:

$$\mathcal{W}(m, \theta) = \frac{\theta(m+T)}{1-\tau} + \max_{y \geq 0}\left\{U(y) - \frac{\theta y}{1-\tau}\right\} + \max_{z,h}\left\{V(z, h) - \frac{\theta(z+h)}{1-\tau}\right\}. \tag{6}$$

We see from this expression that a household's real money balances, $m$, does not affect their choices of $y$, $z$, or $h$. Thus this model has the well-known property that the equilibria will be block recursive, a property of directed search models first introduced by Shi (2009).

It is convenient to write the value function as

$$\mathcal{W}(m, \theta) = \frac{\theta m}{1-\tau} + \tilde{\mathcal{W}}(\theta), \tag{7}$$

where the definition of $\tilde{\mathcal{W}}(\theta)$ is clear. We now analyze a household's decision in the second sub-period and work backwards so we will return to a household's optimal choices of $y$, $z$, and $h$.

### 2.3.1 Second Sub-Period Decisions

A household with $h \geq 0$, and $z > 0$ at the beginning of the second-subperiod chooses a traded goods' submarket to solve the following maximization problem:

$$\max_{x \leq \delta z, q \geq 0}\left\{b(x, q)\left[u(q) + \beta E\mathcal{W}\left(\frac{z - \frac{x}{\delta} + h}{\gamma}, \theta\right)\right] + (1 - b(x, q))\,\beta E\left[\mathcal{W}\left(\frac{z+h}{\gamma}, \theta\right)\right]\right\}, \tag{8}$$

where $b(x, q)$ is given by the Home analog of (3). Using that equation, we can write $q$ as a function of $x$ and $b$ in active submarkets:

$$q(x, b) = \psi^{-1}\left(x - \frac{\kappa}{\mu(b)}\right). \tag{9}$$

Using this and exploiting the linearity of $\mathcal{W}(m, \theta)$, we can rewrite a household's objective at the beginning of the second sub-period as:

$$\max_{x \leq \delta z,\ b \in [0,1]} b\left[u\left(q(x, b)\right) - \frac{\beta E(\theta)\ x}{\delta\gamma(1-\tau)}\right] + \beta E \mathcal{W}\left(\frac{z+h}{\gamma}, \theta\right). \tag{10}$$

The first order condition for $x$ is given by:

$$\frac{u'\left(q(x, b)\right)}{\psi'\left(q(x, b)\right)} - \frac{\beta E(\theta)}{\delta\gamma(1-\tau)} \quad \geq \quad 0 \qquad\qquad x \leq \delta z, \tag{11}$$

where the two inequalities hold with complementary slackness. The first order condition for $b$ is given by:

$$u\left(q(x, b)\right) - \frac{\beta E(\theta)\ x}{\delta\gamma(1-\tau)} \quad + \quad \left[\frac{u'\left(q(x, b)\right)}{\psi'\left(q(x, b)\right)}\right] \frac{kb\mu'(b)}{\mu(b)^2} \quad \leq \quad 0, \qquad b \geq 0, \tag{12}$$

where the two inequalities hold with complementary slackness.[5] Let $\tilde{x}(z)$ and $\tilde{b}(z)$ denote the household's optimal policy functions and use equation (9) to define $\tilde{q}(z) \equiv q\left(\tilde{x}(z), \tilde{b}(z)\right)$. Define $z^*$ as the maximum value such that the second inequality in equation (11) is binding and we derive $z^*$ in Appendix B. Because households are able to engage in precautionary savings and because $U'(.) > 0$, a household will not choose a $z$ above the amount they are planning to spend in the traded goods market. Thus, without loss of generality, we need only consider $z \in [0, z^*]$ in the analysis and we know that $\tilde{x}(z) = \delta z$ for $z$ in that range. We now turn to characterization of $\tilde{b}(z)$.

For $z \in [0, z^*]$, a household's objective given by equation (10) can be written as

$$\mathcal{B}(z) + \beta E \mathcal{W}\left(\frac{z+h}{\gamma}, \theta\right), \tag{13}$$

where

$$\mathcal{B}(z) = \max_{b \in [0,1]} b\left[u\left(q(\delta z, b)\right) - \frac{\beta E(\theta)\ z}{\gamma(1-\tau)}\right]. \tag{14}$$

For future reference, let $z_0$ be defined by $\mathcal{B}'(z_0)z_0 = \mathcal{B}(z_0)$. Using equation (9) and recalling that $\mu(0) = 1$, we see that $\mathcal{B}(z) = 0$ for $0 \leq z \leq z_1$ and $\mathcal{B}(z) > 0$ for $z_1 < z \leq z^*$, where $z_1$ satisfies

$$u\left(\psi^{-1}\left(\delta z_1 - k\right)\right) = \frac{\beta E(\theta) z_1}{\gamma(1-\tau)}. \tag{15}$$

From this we can determine that for $0 \leq z \leq z_1$, $\tilde{b}(z) = 0$ and for $z_1 < z \leq z^*$, $\tilde{b}(z) > 0$ satisfies the first

[5] As noted in Sun and Zhou (2018), $b = 1$ cannot be an equilibrium outcome as it would imply $s = 0$. In that case, firms would earn positive profits which violates the free entry condition.

equation in (12) with equality which we can now write as

$$u\left(\psi^{-1}\left(\delta z-\frac{k}{\mu(\tilde{b}(z))}\right)\right)-\frac{\beta E(\theta)\ z}{(1-\tau)\gamma}\quad+\quad\left[\frac{u'\left(\psi^{-1}\left(\delta z-\frac{k}{\mu(\tilde{b}(z))}\right)\right)}{\psi'\left(\psi^{-1}\left(\delta z-\frac{k}{\mu(\tilde{b}(z))}\right)\right)}\right]\frac{k\tilde{b}(z)\mu'(\tilde{b}(z))}{[\mu(\tilde{b}(z))]^2}\ =\ 0. \tag{16}$$

To summarize, if a household has $z\in[0,z_1]$, the household does not participate in the traded goods' market and without loss of generality, we set $\tilde{b}(z)=\tilde{q}(z)=0$ and $\tilde{x}(z)=\delta z$. If a household has $z\in(z_1,z^*]$, the household participates in a traded goods' submarket with terms of trade $(\delta z,\tilde{q}(z))$ and buyer match probability equal to $\tilde{b}(z)$ where $0<\tilde{b}(z)<1$ satisfies (16) and $\tilde{q}(z)$ satisfies equation (9) evaluated at $(\tilde{x}(z),\tilde{b}(z))$.

It is relevant for the first-period optimization problem of households considered in the next section to note here that $\mathcal{B}(z)$ is not strictly concave for $0\le z\le z^*$. Furthermore, we cannot guarantee that $\mathcal{B}(z)$ is strictly concave for $z_1<z\le z^*$ because it is a function of $\tilde{b}(z)$ and the properties of that function are as yet unknown. This has been pointed out by Menzio, Shi and Sun (2013) and Sun and Zhou (2018) and we follow their methodology for using lotteries to make households' value functions in the second sub-period concave. These lotteries are characterized by $(L_1,L_2,\pi)$ such that $\pi L_1+(1-\pi)L_2=z$. Using such lotteries allows us to replace $\mathcal{B}(z)$ with a concave function denoted by $\tilde{V}(z)$. Details of this approach and the definition of $\tilde{V}(z)$ are contained in Appendix A.

Hence, replacing $\mathcal{B}(z)$ with $\tilde{V}(z)$ and using equations (7) and (10), we are now able to write the value function at the beginning of the second sub-period as follows:

$$\mathcal{V}(z,h)=\ \tilde{\mathcal{V}}(z)\ +\ \frac{\beta E(\theta)(z+h)}{\gamma(1-\tau)}\ +\ \beta E\tilde{\mathcal{W}}(\theta). \tag{17}$$

We further characterize the properties of the functions associated with household optimization in the second sub-period ($\tilde{x}(z)$, $\tilde{b}(s)$, $\tilde{q}(z)$, $\mathcal{B}(z)$, $\tilde{\mathcal{V}}(z)$, $\mathcal{V}(z,h)$) in Appendix B.

### 2.3.2 First Sub-Period Decisions

We are now in a position to examine a households choices of $y$, $\ell$, $z$, and $h$ at the beginning of the first sub-period. The household's value function at the beginning of the period given by equations (6)-(7) can now be written as:

$$\begin{aligned}\mathcal{W}(m,\theta)=\frac{\theta(m+T)}{1-\tau}+\beta E\tilde{\mathcal{W}}(\theta)\ +\ \max_{y\ge0}\left\{U(y)-\frac{\theta y}{1-\tau}\right\}\\ +\max_{0\le z\le z^*,\ \ 0\le h\le\bar{m}-z^*}\left\{\tilde{\mathcal{V}}(z)+\frac{\beta E(\theta)(z+h)}{\gamma(1-\tau)}-\frac{\theta(z+h)}{1-\tau}\right\}.\end{aligned} \tag{18}$$

Now optimal consumption of the non-traded good, $y(\theta)$, satisfies:

$$U'(y(\theta))=\frac{\theta}{1-\tau}. \tag{19}$$

Noting that $\tilde{\mathcal{V}}'(z) = \frac{\mathcal{B}(z_0)}{z_0}$ for $z \in [0, z_0]$ and defining $\tilde{\theta} \equiv (1-\tau)\frac{B(z_0)}{z_0} + \frac{\beta E(\theta)}{\gamma}$, then the optimal level of real balances that a household sets aside for purchases of a traded good, $z(\theta)$, satisfies:

$$z(\theta) = \begin{cases} z^* & if \quad \theta \leq \frac{\beta E(\theta)}{\gamma} \\ \check{z}(\theta) \in [z_0, z^*] & if \quad \frac{\beta E(\theta)}{\gamma} < \theta \leq \tilde{\theta} \\ 0 & if \quad \theta > \tilde{\theta}, \end{cases} \tag{20}$$

where $\check{z}(\theta)$ satisfies

$$\tilde{\mathcal{V}}'(\check{z}(\theta)) = \frac{\theta}{1-\tau} - \frac{\beta E(\theta)}{\gamma(1-\tau)}. \tag{21}$$

The optimal level of precautionary savings, $h(\theta)$, satisfies:

$$h(\theta) = \begin{cases} \overline{m} - z^* & if \quad \theta \leq \frac{\beta E(\theta)}{\gamma} \\ 0 & if \quad \theta > \frac{\beta E(\theta)}{\gamma} \end{cases} \tag{22}$$

Finally, a household's labor supply, $\ell(\theta, m)$, satisfies:

$$\ell(\theta, m) = \left(\frac{1}{1-\tau}\right)(y(\theta) + z(\theta) + h(\theta) - m - T). \tag{23}$$

We note that, as expected, all of these functions are decreasing or weakly decreasing in $\theta$ and labour supply is decreasing in a household's real balances, $m$. In addition, we point out that all households participate in the non-traded goods' market, while only households with sufficiently low $\theta$ will participate in the traded goods' market, as in Sun and Zhou (2018). This is because the traded goods' market is risky due to trading frictions whereas the non-traded goods' market is riskless so households use the latter market for consumption smoothing purposes. We note that $h(\theta) > 0$ only when $z(\theta) = z^*$, i.e. when $\mathcal{V}_z(z, h) = \mathcal{V}_h(z, h) = \frac{\beta E(\theta)}{\gamma(1-\tau)}$, which is consistent with the principles of optimal asset investment.

Given these household policy functions from the first sub-period, we can now write second sub-period choices as a function of $\theta$: $x(\theta) \equiv \tilde{x}(z(\theta))$, $b(\theta) \equiv \tilde{b}(z(\theta))$, $q(\theta) \equiv \tilde{q}(z(\theta))$, $L_1(\theta) \equiv \tilde{L}_1(z(\theta))$, $L_2(\theta) \equiv \tilde{L}_2(z(\theta))$, and $\pi(\theta) \equiv \tilde{\pi}(z(\theta))$, Finally, as discussed above, we again note that a household's choice of $y$, $z$, $h$, $x$, $b$, $q$, $L_1$, $L_2$, and $\pi$ are independent of their level of real balances, $m$, while their labor supply does depend on $m$. Optimal functions for foreign households are derived using similar methods, have similar properties, and are denoted with a *hat*.

## 2.4 Stationary Equilibrium

### 2.4.1 Definition

A stationary symmetric equilibrium consists of households' optimal policy functions in each country: $\Big(\ell(\theta, m), y(\theta), z(\theta), h(\theta), x(\theta), b(\theta), q(\theta), L_1(\theta), L_2(\theta), \pi(\theta)\Big)$,

$\left(\hat{\ell}(\theta,m),\hat{y}(\theta),\hat{z}(\theta),\hat{h}(\theta),\hat{x}(\theta),\hat{b}(\theta),\hat{q}(\theta),\hat{L}_1(\theta),\hat{L}_2(\theta),\hat{\pi}(\theta)\right)$;
households' optimal value functions in each country:
$\left(\mathcal{W}(m,\theta),\mathcal{V}(z,h),\mathcal{B}(z),\tilde{\mathcal{V}}(z)\right)$, $\left(\hat{\mathcal{W}}(m,\theta),\hat{\mathcal{V}}(z,h),\hat{\mathcal{B}}(z),\hat{\tilde{V}}(z)\right)$ ;
a distribution of real balances across households in both countries: $G(m)$ and $\hat{G}(m)$; representative firm's choices in both countries: $(Y,N(\hat{x},\hat{q}))$ and $(\hat{Y},\hat{N}(x,q))$; domestic good prices and normalized wages in both countries: $(p,w)$ and $(\hat{p},\hat{w})$; and nominal and real exchange rates: $(S,\delta)$, that satisfy the following:
(i.) households' policy functions maximize the objective functions given by (6), (10), and (17) (and the foreign counterparts) and give rise to the relevant value functions;
(ii.) firms' choices maximize expected profits;
(iii.) the expected profits of a shop in each submarket equals zero;
(iv.) non-traded goods markets, traded goods' markets, labor markets, and currency markets clear (market clearing conditions are given in Appendix C);
(v.) prices, quantities, and distributions of real balances are time invariant; and
(vi.) households with the same $\theta$ within each country make the same choices.

### 2.4.2 Properties

In the remainder of the paper, we impose additional restrictions on the distributions for households' idiosyncratic preference parameters, $\mathcal{F}(\theta)$ and $\hat{\mathcal{F}}(\theta)$ as follows. (i.) $E(\theta)$ is low enough so that $z(\theta)>0$ for some $\theta$ and (ii). $\theta \leq \frac{\beta E(\theta)}{\gamma}$ for some $\theta \in [\theta_L,\theta_H]$ so that, $h(\theta)>0$ for some $\theta$. These additional restrictions ensure that some households will participate in the traded goods market and some households will hold precautionary savings in equilibrium.

Let $\Omega \equiv (\tau,\gamma,\bar{\theta},\hat{\tau},\hat{\gamma},\hat{\bar{\theta}})$ denote the vector of relevant exogenous variables. Henceforth, we rewrite household policy functions with explicit reference to these parameters; for example, we write $z(\theta,\Omega)$. The open economy analogs with taxation and money growth of Lemmas 1-3 and Theorem 2 from Sun and Zhou (2018) hold in this economy.[6] In particular, a stationary equilibrium exists. That equilibrium is unique if and only if lottery policy functions are unique $\forall\ \theta \in [\theta_L,\theta_H]$. The policy functions $y(\theta,\Omega)$, $h(\theta,\Omega)$, $z(\theta,\Omega)$, $b(\theta,\Omega)$, $x(\theta,\Omega)$, and $q(\theta,\Omega)$ and their foreign counterparts are all decreasing functions of $\theta$ while $\ell(\theta,\Omega,m)$ and its foreign counterpart are decreasing in both $\theta$ and $m$.

Turning to the effects of changes in the elements of $\Omega$ on these variables, the results described in Lemma 2 and Proposition 2 of Sun and Zhou (2018) regarding the impact of changes in domestic parameters on domestic household choices do not necessarily hold in our open economy. This occurs because real and nominal exchange rates are affected by changes in the parameters contained in $\Omega$ and this has additional effects on household choices, making the overall effects generally ambiguous. We demonstrate this in Appendix E. Given these theoretical ambiguities, we use numerical simulations in

[6] The open economy versions with taxation and money growth of those propositions and their proofs are presented in Appendix D.

Sections 3 and 4 below to analyze the effects of changes in domestic and foreign policies on exchange rates and households' choices in our open economy.

## 2.5 Exchange Rate Pass-Through

As just discussed, changes in the economy parameters in $\Omega$ affect nominal and real exchange rates, which affect household choices and equilibrium prices of traded goods. We examine those effects using two measures of exchange rate pass-through.

First, we define the import price index in Country $H$ as the average price paid (expressed in units of $H$ labor) across traded goods' submarkets in that country:

$$IPI(\Omega) \equiv \frac{\int_{\theta_L}^{\theta_H} \left(\frac{z(\theta,\Omega)}{q(\theta,\Omega)}\right) \; b(\theta,\Omega)d\mathcal{F}(\theta)}{\int_{\theta_L}^{\theta_H} b(\theta,\Omega)d\mathcal{F}(\theta)}. \tag{24}$$

Recall that the nominal exchange rate, $S$, is units of $F$ currency per one unit of $H$ currency and the real exchange rate, $\delta$, is units of $F$ labour per one unit of $H$ labor.

Using $1/\delta$ as the relevant measure of the real exchange rate from the viewpoint of Country $H$, we define the following measure of real exchange rate pass-through due to a change in the vector of exogenous variables from $\Omega$ to $\Omega'$:

$$ERPT = \frac{\left(\frac{IPI(\Omega')}{IPI(\Omega)}\right) \; - \; 1}{\left(\frac{\delta(\Omega)}{\delta(\Omega')}\right) \; - \; 1} \tag{25}$$

Turning to the appropriate measure for nominal exchange rate pass-through, we note that we must correct for inflation when the economy changes from one period to the next when an element of the vector of economy parameters changes. Suppose that we have $\Omega_{t-1} = \Omega$ and $\Omega_t = \Omega'$, then nominal exchange rate pass-through from $t-1$ to $t$, correcting for inflation is as follows:

$$NERPT_t = \frac{\left(\frac{1}{\gamma_t}\right)\left(\frac{W_t IPI_t}{W_{t-1}IPI_{t-1}}\right) \; - \; 1}{\frac{\left(\frac{1}{S_t}\right)}{\left(\frac{1}{S_{t-1}}\right)} \; - \; 1} = \frac{\left(\frac{1}{\gamma_t}\right)\left(\frac{w_t M_{t-1}\gamma_t IPI_t}{w_{t-1}M_{t-1}IPI_{t-1}}\right) \; - \; 1}{\frac{S_{t-1}}{S_t} \; - \; 1}. \tag{26}$$

Hence, because the money supply terms and the money growth rate terms cancel in the above expression, our general measure of nominal exchange rate pass-through when the vector of exogenous variables changes from $\Omega$ to $\Omega'$ is given by

$$NERPT = \frac{\left(\frac{w(\Omega')IPI(\Omega')}{w(\Omega)IPI(\Omega)}\right) \; - \; 1}{\left(\frac{S(\Omega)}{S(\Omega')}\right) \; - \; 1}. \tag{27}$$

Recalling that all values are expressed in terms of H labor, then ERPT is a normalized exchange rate pass-through measure that can be interpreted as real ERPT. NERPT is a standard measure of nominal exchange rate pass-through in an economy with inflation due to money growth.

Before examining the quantitative predictions of the model, we first explain why we expect this economy to exhibit incomplete real exchange rate pass-through and deviations from purchasing power parity. To facilitate the explanation and the economic intuition, we consider an economy in which all households have the same disutility of labor–the homogeneous household case. In such an economy, there will be a single submarket for traded goods in each country.

Consider Country $H$ where the price of a foreign traded good expressed in $H$ labor is $\frac{z}{q} = \left(\frac{1}{\delta}\right)\left(\frac{x}{q}\right)$. If the economy experiences an exogenous shock (such as a tax or preference shock) which causes an endogenous real depreciation for Country $H$, i.e. an increase in $\frac{1}{\delta}$, then if the terms of trade for traded goods, $(x, q)$, does not change, the rise in the normalized real exchange rate would be fully transmitted into the price of the traded good expressed in units of $H$ labor. Thus, the real price in $H$ of a traded good will rise in proportion to the real exchange rate change and exchange rate pass-through will be complete. However, the real exchange rate depreciation causes $H$ buyers and $F$ sellers to optimally respond to the price increase by choosing a less expensive submarket. This implies that in the new equilibrium, $\frac{x}{q}$ will be lower. Therefore, the degree of increase in the $H$ price of a traded good, $\frac{z}{q}$, will be less than the degree of increase in the real exchange rate $\frac{1}{\delta}$, and real exchange rate pass-through will be incomplete. The key feature that generates this incomplete pass-through is the competitive search that allows buyers and sellers to rationally react to movements in the exchange rate by choosing a different submarket with a different terms of trade (and different matching probabilities). This important role of extensive margin responses by agents to shocks in this model with competitive search is consistent with the emphasis on the role of this margin of response in affecting the impacts of policy shocks as discussed in Sun and Zhou (2018).

We also note that the model will exhibit deviations from purchasing power parity (PPP) due to differences in local market conditions. That is, the terms of trade (when expressed in the same labor units) will differ across the submarkets in the two countries according to country characteristics such as the mean of preference parameters, money growth rates, and/or income tax rates. However, the direction of the price differences is not obvious. For example, suppose that the two countries are initially identical so there are no deviations from PPP and the real exchange rate equals 1. Now suppose that Country $H$ lowers its tax rate so $\tau < \hat{\tau}$. We demonstrate in Appendix B that, holding the real exchange rate fixed, the domestic tax cut increases the after tax income of $H$ households and this induces them to choose a higher level of real balances to carry into the traded goods market. They also choose a submarket with a higher probability of matching and a less favorable terms of trade ($z$, $b$, $x$, $z/q$ and $x/q$ increase). There is no direct effect of the tax cut in $H$ on $F$ household choices so if this were the end of the story then the low tax country would face higher prices for traded goods when expressed in the same labor units ($z/q > \hat{x}/\hat{q}$ and $x/q > \hat{z}/\hat{q}$). However, this is not the end of the story as the increased demand for $F$ currency by $H$ households as they increase $z$ will cause a nominal and real appreciation of $F$ currency and $\frac{1}{\delta}$ will rise. As described in the previous paragraph, this will induce $H$ households to choose a market with a lower $x/q$ but $z/q$ will increase (i.e. pass-through will be positive but incomplete). The households in Country $F$ will do the opposite. Hence the tax cut in $H$ will cause both $z/q$ and $\hat{x}/\hat{q}$ to rise relative to the symmetric equilibrium and their relative change will determine the direction of price differences across countries.

In the remainder of the paper, we use numerical experiments to demonstrate that both types of pass-through in this economy will typically be incomplete and conduct a series of experiments to clarify the various forces that affect the levels of pass-through. We also characterize the effects of various economy parameters on real and nominal exchange rate pass-through levels, document the presence of asymmetric pass-through, and examine how pass-through differs across heterogeneous households.

# 3 Numerical Experiments

## 3.1 Calibration

Because of the complexity and nonlinearity of model equations, it is not possible to derive closed form solutions for equilibrium variables (and equilibria may include the use of lotteries). Hence, we simulate the model numerically and illustrate its properties through a series of numerical experiments. To do so, we choose functional forms for preferences and technologies as follows: $U(y) = U_0\left[\frac{(y+a)^{1-\lambda}-a^{1-\lambda}}{1-\lambda}\right]$, $u(q) = u_0\left[\frac{(q+a)^{1-\sigma}-a^{1-\sigma}}{1-\sigma}\right]$, $\psi(q) = \psi_0 q^{\varphi}$, and $\mu(b) = (1 - b^{\rho})^{\frac{1}{\rho}}$. We assume that $F(\theta)$ is uniform on $[\theta_L, \theta_H]$ and that the Foreign economy has functions of the same form.

To select baseline economy parameters for our numerical experiments, we take a subset of parameters from Sun and Zhou (2018) and calibrate remaining parameters to match moments from U.S. data. In the baseline calibration, we assume that both countries have the same economy parameters prior to shocks. The parameters that we take directly from Sun and Zhou (2018) are listed in the top portion of Table 1 and are as follows. A period in the model is one year so $\beta = 0.96$ is chosen to match a 4% annual interest rate. The utility constant, $a = 0.001$, is a small number and the curvature parameter for the non-traded good, $\lambda = 2.00$, is a typical value in the macroeconomics literature. The lower bound for the disutility parameter, $\theta_L = 0.50$, is chosen so that $\bar{m}$ does not restrict households' real balance holdings. The upper bound on the disutility parameter, $\theta_H = 3.00$, and on money holdings, $\bar{m} = 2.50$, are taken from the calibration by Sun and Zhou (2018). Turning to production of traded goods, Sun and Zhou (2018) set the cost of setting up a shop to a small number, $\kappa = 0.01$, and they normalize the constant in the production function to one, $\psi_0 = 1.00$. The curvature parameter in the production function, $\varphi = 1.50$, is taken from the calibration of Sun and Zhou (2018). Finally, the policy parameters, $\gamma = 1.02$ and $\tau = 0.00$, are the same as in the baseline experiments of Sun and Zhou (2018) except that we set $\tau = 0.01$ when we calculate elasticities in response to changes in tax rates.

We choose the remaining parameters, $(U_0, \sigma, \rho)$, to match three targeted moments from the data given the baseline policy variables. Given an annual growth rate of money of 2%, we use a money velocity target equal to 5.59, which is the value for M1 in the U.S. in 2016. The degree of heterogeneity across households ($\theta_L$ and $\theta_H$) and the degree of curvature in the utility function for traded goods ($\sigma$) affects the degree of price dispersion in the economy. To capture an empirical measure of price dispersion, we choose a target of relative price variability equal to 0.035 following Wang (2016) and Sun and Zhou (2018). Our final target is a measure of dispersion in money holdings for transaction purposes. We use median transaction accounts holdings data from the Survey of Consumer Finances for the U.S. from 2016 and calculate the Gini coefficient of those accounts (excluding the top decile). The empirical value we calculate equals 0.52 and this is our target for dispersion of money holdings in the simulated model. Jointly calibrating these targets in our baseline simulations gives the economy parameters listed in the middle columns of Table 1.

Table 1: Baseline Economy Parameters

| Chosen Parameter | Value from Sun and Zhou (2018) | Calibrated Parameter | Calibrated Value | Targeted Moment | Data Moment | Model Moment |
|---|---|---|---|---|---|---|
| $\beta$ | 0.96 | $U_o$ | 16.65 | Money | 5.59 | 5.59 |
| $a$ | 0.001 | $\sigma$ | 1.18 | Velocity | | |
| $\lambda$ | 2.00 | $\rho$ | 2.75 | Relative Price | 0.035 | 0.043 |
| $u_o$ | 1.00 | | | Variability | | |
| $\theta_L$ | 0.50 | | | Transactions | 0.52 | 0.55 |
| $\theta_M$ | 3.00 | | | Gini Coefficient | | |
| $\bar{m}$ | 2.50 | | | | | |
| $\kappa$ | 0.01 | | | | | |
| $\psi_o$ | 1.00 | | | | | |
| $\varphi$ | 1.50 | | | | | |
| $\gamma$ | 1.02 | | | | | |
| $\tau$ | $\in \{0.00, 0.01\}$ | | | | | |

## 3.2 Baseline Results

We begin by considering cases where the two countries initially have the same utility parameters ($\bar{\theta} = \hat{\bar{\theta}}$) and policy variables ($\gamma = \hat{\gamma}$; $\tau = \hat{\tau}$) – a symmetric equilibrium with $S = \delta = 1$. We then consider the magnitude of responses in each country when the economy is shocked away from this initial symmetric equilibrium. In Table 2, we report real and nominal exchange rate pass-through levels averaged across submarkets for $F$ goods sold in $H$ in response to a 1% increase in each of the elements of $\Omega$. We also present corresponding price elasticities averaged across submarkets and the associated real and nominal exchange rate elasticities.

We first focus on shocks that occur in Country $F$. In the first two rows of the table, we see that a rise in the mean disutility of leisure or a rise in the tax rate in Country F leads to less demand by households in $F$ for $H$ goods, less demand for $H$ currency, and thus, both a real and a nominal depreciation for Country $H$. In row 3, we see that, as expected, a *rise* in the growth rate of money in $F$ leads to a real and nominal appreciation in $H$. In the next paragraph, we consider an increase in the mean disutility

Table 2: Baseline Model: Heterogeneous Agents Under Uncertainty

Country H Responses to a 1% Increase in an Exogenous Variable

| Row | Exogenous Variable | ERPT (%) | NERPT (%) | Elasticity of $\left(\frac{1}{\delta}\right)$ | Elasticity of $\left(\frac{1}{S}\right)$ | Elasticity of $\left(\frac{x}{q}\right)$ | Elasticity of $\left(\frac{z}{q}\right)$ |
|---|---|---|---|---|---|---|---|
| 1 | $E(\hat{\theta})$ | 79.62 | 62.47 | 1.154 | 1.441 | -0.233 | 0.919 |
| 2 | $\hat{\tau}$ | 80.06 | 22.07 | 0.131 | 0.465 | -0.026 | 0.105 |
| 3 | $\hat{\gamma}$ | 79.47 | 19.35 | -0.459 | -1.844 | 0.095 | -0.365 |
| 4 | $E(\theta)$ | 97.21 | 99.04 | -1.141 | -1.421 | 0.032 | -1.109 |
| 5 | $\tau$ | 95.84 | 99.36 | -0.131 | -0.462 | 0.006 | -0.126 |
| 6 | $\gamma$ | 101.23 | 47.03 | 0.461 | 1.879 | 0.006 | 0.467 |

of leisure, an increase in the tax rates, or a decrease in money growth in $F$ so that we may consistently discuss the effects of an $H$ depreciation caused by a shock in $F$.

Beginning with real prices and exchange rates, we note that as described above, in each of these cases the $H$ real depreciation induces $H$ households to choose submarkets for traded foreign goods with more favorable terms of trade expressed in F labor but with a lower probability of making a trade ($\frac{x}{q}$ and $b$ fall). This offsets some of the real depreciation for $H$ and although $H$ consumers pay a higher price in units of $H$ labor for the traded good ($\frac{z}{q}$ rises), it does not increase as much as the real exchange rate and so real exchange rate pass-through levels are less than one hundred percent; i.e. pass-through of a real exchange rate movement to real import prices is incomplete.

We now turn to nominal variables, the nominal exchange rate, $(1/S) = (1/\delta)(W/\hat{W})$, the nominal price paid by $H$ consumers, $W(z/q)$, and nominal exchange rate pass-through. The qualitative effects on nominal exchange rates are similar to those for real for all of the $F$ shocks considered. In each case, however, there is a relative nominal wage response that compounds the effect of the shock and so nominal exchange rate elasticities are larger than those for real exchange rates. In addition, the fall in the nominal wage in $H$ due to a fall in $H$ labor demand implies that nominal prices do not rise as much as real prices. These two forces combine to generate lower NERPT than ERPT for Country $H$ in response to $F$ shocks. We also note that the values for NERPT in response to small foreign shocks in our baseline calibration are in line with the estimates of short-run nominal pass-through to import prices presented in Burstein and Gopinath (2014), which range between 13% and 75%.

Rows 4 and 5 of Table 2 demonstrate that an increase in the expected value of the disutility parameter or the tax rate in Country **H** causes a real and nominal *appreciation* in $H$ and this induces opposite movements in $\frac{x}{q}$ and $\frac{z}{q}$ as just described for a Country $F$ shock. However, there are also direct effects of a Country $H$ shock which affect movements in these prices and the degree of pass-through. An increase in the expected value of the disutility parameter or the tax rate in $H$ directly (i.e. not via exchange rate movements) induces $H$ households to choose submarkets with lower $\frac{x}{q}$. This offsets some of its rise due to a real appreciation of $H$ currency and so the magnitude of the response of $\frac{z}{q}$ is larger here than when the shock occurs in Country $F$, causing both real and nominal pass-through to be higher under $H$ shocks.

Row 6 demonstrates that an increase in money growth in $H$ leads to a $H$ real depreciation but the increase in inflation directly induces $H$ households to choose submarkets with a *higher* $\frac{x}{q}$. This reinforces the exchange rate effect on $z/q$ and the baseline economy exhibits more than complete ERPT. This result is explained by a "hot potato" effect present in the competitive search environment as discussed by Sun and Zhou (2018). When inflation increases in $H$, money carried into the next period loses some of its value so due to that effect, $H$ households tend to choose more expensive submarkets with higher matching probabilities. This offsets, to some degree, their desire to choose less expensive submarkets when they face a real depreciation resulting from the increase in $H$ money growth. Thus, in the baseline economy, an increase in money growth in $H$ actually causes households to choose a higher $x/q$ due to the direct inflation effect, thereby making ERPT more than complete.[7] The fall in the nominal wage in $H$, however, leads to incomplete NERPT.

We also note that across all the types of shocks we consider, ERPT is highest in the presence of $H$ money growth shocks. Our finding that ERPT is lower under non-monetary shocks implies that our model predicts that real pass-through will be lower in economies with more stable monetary policies – i.e. in economies which are not subject to money growth shocks. This finding is consistent with Taylor's (2000) hypothesis that stable monetary policy is associated with lower exchange rate pass-through.

It is worthwhile to compare and contrast the mechanism in our model for generating incomplete pass-through to the more "firm-based" models discussed in the introduction by examining the response of markups to exchange rate movements. In the simulations presented in Table 2, average markups of real prices over marginal costs either does not change or moves in the same direction as the exchange rate (i.e. an $H$ currency depreciation is associated with an increase in average markups). Furthermore, markup elasticities are relatively low; between 0.00 and 0.11, depending on the shock. This stands in contrast to several firm-based models of incomplete pass-through in which firms keep prices relatively constant in the face of exchange rate movements precisely by adjusting their markups. However, our finding of muted markup responses in our model is consistent with empirical evidence suggesting little adjustment in retail markups in response to exchange rate movements (see Goldberg and Hellerstein (2006) and Gopinath et al. (2011)).

Turning to the model's implications for deviations from purchasing power parity, we note that because the elasticities reported in Table 2 are calculated from the symmetric equilibrium, this table also provides information regarding the direction of price differences due to disparities in country characteristics. Due to symmetry, the $x/q$ and $z/q$ entries in each row associated with a shock in $F$, $\hat{\omega} \in \{E(\hat{\theta}), \hat{\gamma}, \hat{\tau}\}$, are equal to their counterpart elasticities for $F$ households in response to a shock in $H$, $\omega \in \{E(\theta), \gamma, \tau\}$. Hence, for example, the table indicates that a 1% increase in $E(\hat{\theta})$ from the symmetric equilibrium decreases $\frac{x}{q}$ (the price in $H$ denoted in $F$ labour units) by 0.233% but it decreases $\frac{\hat{z}}{\hat{q}}$ (the price in $F$ denoted in $F$

[7] Other simulations not reported here show that ERPT may be more or less than complete for a wide range of magnitudes of $H$ money growth shocks but its value remains very close to 100%.

labour units) by 1.109% so the ratio of the price in $H$ to the price in $F$ expressed in $F$ labour units rises above one. Similarly the increase in $E(\hat{\theta})$ increases $\frac{z}{q}$ (the price in $H$ denoted in $H$ labour units) more that it increases $\frac{\hat{x}}{\hat{q}}$ (the price in $F$ denoted in $H$ labour units) so the ratio of the price in $H$ to the price in $F$ expressed in $H$ labour units also rises above one. Thus, we can conclude that for our calibrated parameters, the prices paid by households in a country with a high $\theta$ relative to its trading partner will be lower than in the other country. Similarly, we see that households in a relatively high tax country or a relatively low money growth country will pay relatively lower prices.

In the intuition described in the previous paragraphs, we did not discuss the role of precautionary savings in the presence of uncertainty across heterogenous households. To better understand this, we present the results of numerical experiments in which households are heterogeneous with respect to their disutility parameters but those parameters are fixed over time for each household. In comparing the results in Table 2 to those in Table 3, we see that the levels of ERPT and NERPT are affected by the presence of uncertainty, particularly in the case of a monetary growth shock in either country.

Table 3: Heterogeneous Agents With Constant Disutility Parameters

Country H Responses to a 1% Increase in an Exogenous Variable

| Row | Exogenous Variable | ERPT (%) | NERPT (%) | Elasticity of $\left(\frac{1}{\delta}\right)$ | Elasticity of $\left(\frac{1}{S}\right)$ | Elasticity of $\left(\frac{x}{q}\right)$ | Elasticity of $\left(\frac{z}{q}\right)$ |
|---|---|---|---|---|---|---|---|
| 1 | $E(\hat{\theta})$ | 85.92 | 79.62 | 1.061 | 0.616 | -0.148 | 0.912 |
| 2 | $\hat{\tau}$ | 93.30 | 22.14 | 0.121 | 0.646 | -0.008 | 0.112 |
| 3 | $\hat{\gamma}$ | 430.79 | -3550.84 | 0.007 | -0.003 | 0.023 | 0.030 |
| 4 | $E(\theta)$ | 98.40 | 3.33 | -1.050 | -0.612 | 0.017 | -1.033 |
| 5 | $\tau$ | 85.16 | 92.44 | -0.120 | -0.642 | 0.018 | -0.102 |
| 6 | $\gamma$ | -346.40 | 3683.67 | -0.007 | 0.003 | 0.031 | 0.024 |

The intuition behind these results is as follows. Consider a decrease in $\hat{\gamma}$. In the baseline case with uncertainty, due to the decrease in inflation in their country, low $\hat{\theta}$ households in Country $F$ will increase their precautionary savings and decrease expenditure on traded goods, i.e. they will choose a higher $\hat{h}$ and a lower $\hat{z}$. This decreases the demand for $H$ currency and leads to a significantly larger nominal depreciation for $H$ than in the case with no uncertainty, where there is almost no effect on the nominal exchange rate. Hence, in the absence of precautionary motives, the change in the relative nominal wage dominates and $H$ experiences a real *appreciation*. This causes $\frac{x}{q}$ and $\frac{z}{q}$ to move in the same direction (they both fall) which results in a much higher level of ERPT than in the baseline case with uncertainty. Because the nominal price and the nominal exchange rate move in opposite directions in the absence of uncertainty, NERPT is negative in this case. We see similar contrasting results between the two cases in response to a change in the money growth rate in $H$. These results illustrate that precautionary savings play a very important role in generating incomplete ERPT and NERPT in the presence of money growth shocks.

It is also useful to explore the role of heterogeneity across households in generating levels of ERPT and NERPT that are empirically plausible. Here we examine the special case in which households within a country are homogeneous and all households have constant preference parameter $E(\theta) = E(\hat{\theta}) = 1.75$. In this case, both the real money balances and income distributions will be degenerate and there is no need for precautionary savings as there is no income uncertainty. In Table 4, we report pass-through levels and exchange rate and price elasticities for this case. Examining the last column in this table, we see that in the absence of both precautionary savings and multiple submarkets for traded goods, our economy in not able to generate empirically plausible levels of NERPT. These last two sets of experiments demonstrate that there is a role for both heterogeneity and precautionary savings in generating empirically plausible levels of exchange rate pass-through in our model of competitive search.

Table 4: Homogeneous Agents (Without Uncertainty)

Country H Responses to a 1% Increase in an Exogenous Variable

| Row | Exogenous Variable | ERPT (%) | NERPT (%) | Elasticity of $\left(\frac{1}{\delta}\right)$ | Elasticity of $\left(\frac{1}{S}\right)$ | Elasticity of $\left(\frac{x}{q}\right)$ | Elasticity of $\left(\frac{z}{q}\right)$ |
|---|---|---|---|---|---|---|---|
| 1 | $E(\hat{\theta})$ | 77.46 | 19.32 | 1.276 | -1.197 | -0.284 | 0.989 |
| 2 | $\hat{\tau}$ | 77.60 | -0.246 | 0.013 | 0.956 | -0.003 | 0.100 |
| 3 | $\hat{\gamma}$ | 77.55 | 0.003 | 0.0002 | -1.034 | -0.0004 | 0.0001 |
| 4 | $E(\theta)$ | 95.19 | 4.12 | -1.260 | 1.211 | 0.061 | -1.200 |
| 5 | $\tau$ | 95.15 | 101.23 | -0.013 | -0.947 | 0.001 | -0.012 |
| 6 | $\gamma$ | -760.78 | 4.40 | -0.0002 | 1.045 | 0.001 | 0.001 |

### 3.3 Sensitivity of Pass-Through Rates to Economy Parameters

In this section we seek to determine how the levels of real and nominal exchange rate pass-through vary with select economy parameters and with the size of shocks. We first undertake a series of numerical exercises in which we maintain symmetry in the baseline and vary tax rates ($\tau$ and $\hat{\tau}$), the mean of labour disutility ($E(\theta)$ and $E(\hat{\theta})$), money growth rates ($\gamma$ and $\hat{\gamma}$), the coefficient of risk aversion ($\sigma$), and the parameter in the matching function ($\rho$) to examine the effects of these parameters on our measures of exchange rate pass-through in Table 2.

In Figure 3.1, we observe that only $H$ NERPT in response to a shock to the tax rate in $F$ is increasing in the tax rate; our other three measures of pass-through are relatively insensitive to changes in the size of tax rates. For the range of tax rates we considered, both types of pass-through are incomplete. Figure 3.2 demonstrates that changes in the mean level of labour disutility have only very small effects on the degree of ERPT and of NERPT in $H$ due to tax shocks. For most levels of this parameter, pass-through is incomplete although for the relatively high levels of mean $\theta$ depicted here, NERPT becomes slightly more than complete in the presence of domestic tax shocks. Figure 3.3 shows that ERPT and NERPT in response to tax shocks are nearly invariant to the level of the common money growth rate (and, therefore, to the level of inflation).

Turning to the coefficient of risk aversion, our simulations indicate that pass-through levels in response to tax rate shocks are sensitive to changes in $\sigma$, although the effects are small. Figure 3.4 indicates that ERPT rates are generally increasing in $\sigma$ while NERPT rates are generally decreasing but the effects become non-monotonic for relatively high levels of risk aversion. Interestingly, NERPT can be more than complete at low levels of $\sigma$. This is intuitive given our earlier description of the role of uncertainty and precautionary savings in affecting households' responses to shocks. At lower levels of risk aversion, households do not substitute money balances to carry into the next period, $h$, for expenditures in the traded goods' market, $z$, to the same degree as for higher levels of $\sigma$ and this generates the relatively high levels of pass-through in response to domestic tax shocks.

Figure 3.5 depicts how exchange rate pass-through rates vary with changes in the parameter that determines the curvature in the matching function, $\mu(b)$. The figure indicates that pass-through rates in response to domestic tax shocks are nearly invariant to changes in this parameter. In contrast, pass-through in response to foreign tax shocks are increasing for $\rho < 1$, peak when the matching function is linear and then fall as $\rho$ increases above one, although the amount of variation is quite low.

Overall, our results in this section indicate that our results from our baseline economy are generally robust to other economy parameter settings. Pass-through levels tend to be incomplete and pass-through rates in response to foreign tax shocks are lower than in response to domestic ones. Furthermore, as in the baseline economy, real exchange rate pass-through is higher than nominal exchange rate pass-through in response to foreign shocks, but these two types of pass-through are of similar magnitudes in response to domestic tax shocks.

## 3.4 Asymmetric Exchange Rate Pass-Through

In this section we characterize how levels of ERPT and NERPT depend on the magnitude and direction of changes in exchange rates. Figures 4.1-4.4 depict the levels of ERPT and NERPT in $H$ for movements in exchange rates in response to shocks to tax rates or preferences in each country. In those figures, the horizontal axis depicts changes in $1/\delta$ or $1/S$ so a negative change reflects an $H$ appreciation while a positive change is an $H$ depreciation. Figures 4.1 and 4.3 show that in all cases, real exchange rate pass-through rates are higher for $H$ real appreciations than for depreciations, larger real appreciations are associated with higher pass-through, and larger real depreciations are associated with lower pass-through.

These results occur because the curvature in the households' utility functions implies that households respond differently to price increases than they do to price decreases. The intuition can be understood by considering the standard problem of choosing consumption over time by a household with constant relative risk aversion preferences. We know in such an environment that consumption elasticities with respect to price increases are smaller in absolute value than for price decreases due to curvature in the

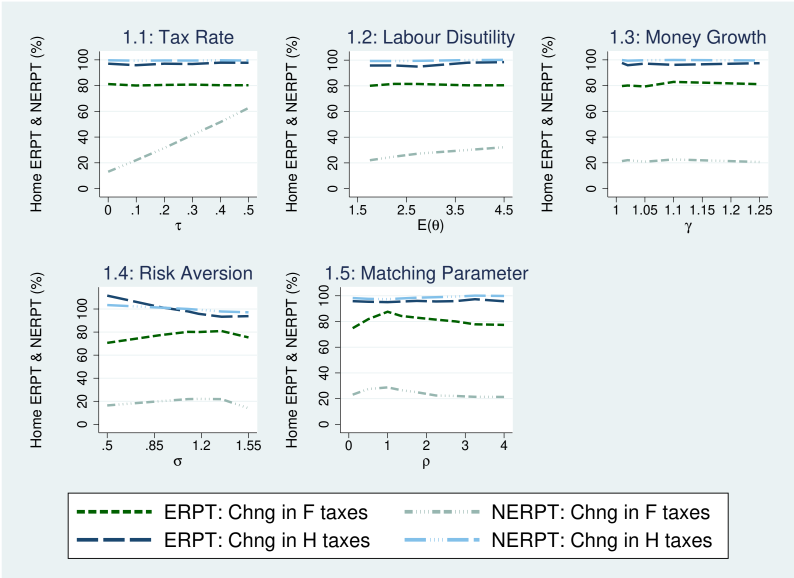


Figure 1: Effect of Economy Parameters on Pass-Through Due to Tax Shocks

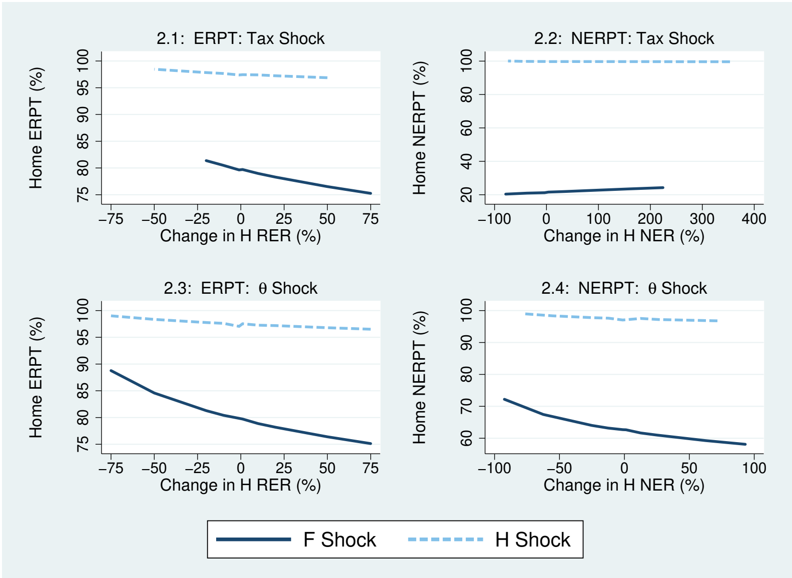


Figure 2: Asymmetric Pass-Through

utility function. We have a parallel result in our economy: households optimally choose a smaller change in their expected consumption of traded goods after an $H$ real depreciation (which is analogous to a real price increase) than after an $H$ real appreciation. Choosing a smaller expected fall in consumption in our model means they must choose to switch to a submarket that generates a larger decrease in the terms of trade denominated in $F$ labour $(x/q)$ when they switch in response to the initial rise in $z/q$. This larger change in $x/q$ results in a smaller final change in $z/q$ when there is a real $H$ depreciation and so generates lower pass-through than after a real $H$ appreciation. To summarize, due to concavity in preferences, an $H$ real depreciation, through optimal household responses, generates a smaller fall in consumption, a larger fall in $x/q$, and a smaller rise in $z/q$ than the rise in consumption and in $x/q$ and the fall in $z/q$ that results from an $H$ real appreciation of equal magnitude.

Figures 4.2 and 4.4 demonstrate a similar relationship between nominal exchange rate movements and pass-through levels except in the case of an $F$ tax shock where we observe slightly higher nominal pass-through for larger nominal depreciations. The intuition we described above continues to be relevant here but recall from equation (27) that nominal pass-through levels are also affected by the magnitude of real wage changes. When there is an increase in the tax rate in $F$ this leads to an $H$ nominal depreciation but it also leads to a decrease in the real wage in $H$ due to the fall in labor demand in $H$. Again, due to curvature in the utility function, larger depreciations will lead to a smaller labor demand response and so a smaller decrease in the real wage in $H$. Combining this with the smaller decrease in $z/q$ associated with larger depreciations as discussed above, we see that the fall in $W(z/q)$ could increase or decrease with the size of the $H$ depreciation. In our calibrated economy, the smaller rise in $W$ dominates and so NERPT rises with the size of the depreciation for $F$ tax shocks. For the other four shocks, the consumption effect dominates and NERPT pass-through rises with the size of an $H$ nominal appreciation and falls with the size of an $H$ nominal depreciation.

In comparing our theoretical asymmetry results for exchange rate pass-through with other theories of incomplete pass-through, we note that some firm-based theories of incomplete pass-through predict a relationship consistent with our primary finding (pass-through is greater under an $H$ appreciation than under a depreciation) while others predict the opposite. For example, models where incomplete pass-through arises as firms seek to maintain market share (for example, Froot and Klemperer (1989), Knetter (1994), and Marston (1990)) and in models where firms with market power use both domestic and imported inputs (for example, Burstein and Gopinath (2014) and Webber (2000)), the direction of the relationship between the size and direction of exchange rate movements is in line with our predictions. In contrast, in models where capacity constraints lead to incomplete pass-through (for example, Gil-Pareja (2000) and Knetter (1994)), the relationship is reversed. Finally, in models with nominal price rigidities (for example, Devereux et al. (2004) and Gopinath et al. (2010)), the relationship between the size of exchange rate movements and the degree of exchange rate pass-through depends on the currency in which exporters are setting their prices.

The empirical literature which examines asymmetric exchange rate pass-through has also produced mixed results. Pollard and Coughlin (2003) survey several papers and conclude that previous studies do not provide clear evidence on the direction of the asymmetry. Furthermore, in that paper, they analyze exchange rate pass-through into U.S. import prices for thirty industries and find that over one-half of the industries exhibit asymmetries in pass-through but the direction of asymmetry varies across industries. More recent papers such as Delatte and Lopez-Villavicencio (2012) and Brun-Aguerre et al. (2017) find that nominal depreciations are passed through to prices at a higher rate than are appreciations while Campa et al. (2008) find the opposite.

Our emphasis in this paper on the important role of households' responses to price movements in affecting exchange rate pass-through may shed light on the mixed empirical results. Models of incomplete pass-through which emphasize firm responses to exchange rate movements can potentially address these mixed empirical findings by arguing that different industries are characterized by different market structures, by differing degrees of nominal price rigidities, by differences in invoicing currencies, etc. Our model suggests that the degree of search frictions, curvature in households' utility, and the nature of the shock that leads to a movement in exchange rates also affect the behavior of exchange rate pass-through. Combining differences across time, industries, and countries in the market conditions from the firm side along with conditions from the household side allows us to better understand why exchange rate pass-through asymmetry results vary considerably across empirical studies. Understanding the full nature of the asymmetry and the contributions from the supply side relative to the demand side of goods market is important for the policy implications surrounding exchange rate pass-through under asymmetries.

### 3.5 Pass-Through Heterogeneity

In this section we explore how exchange rate pass-through varies across heterogeneous households and the distributional effects of exchange rate movements. To investigate this, we examine ERPT in Country $H$ in response to a tax shock in $F$ for two types of households: $\theta \in \{0.50, 3.00\}$, which we label respectively as high- and low-income households. Figure 3 depicts the degree of ERPT for each household type in $H$ for a range of real exchange rate movements due to variation in the size of $F$ tax shocks. We see from that figure that low-income households experience higher rates of real ERPT in response to tax shocks in the foreign country. The difference in pass-through between the two types of households results as low-income households are less responsive in changing the submarkets in which they search in response to an exchange rate change than are high-income households. As with other features of this model, this result is explained by the important role of precautionary savings. High-income households use their precautionary savings to allow them to make relatively large changes in submarkets in order to offset price changes due to exchange rate movements. This implies that they face relatively small domestic currency price shocks and relatively stable consumption paths. This is in comparison with low-income households who do not hold precautionary savings and so are more constrained in their ability to change submarkets and smooth consumption.

To further understand the details of the different households' responses, consider an $H$ depreciation

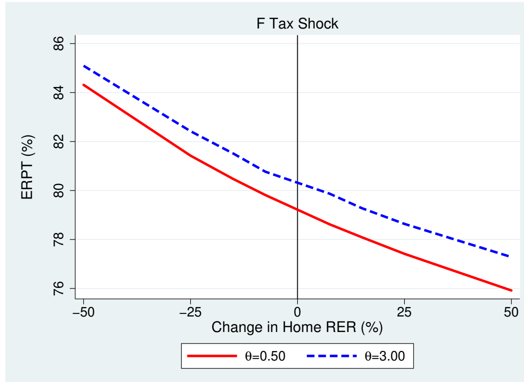


Figure 3: Pass-Through Heterogeneity

due to a rise in the tax rate in $F$. In response to the depreciation, both types of households choose submarkets with lower terms of trade and lower probabilities of trade. However, because high-income households have precautionary savings, they are less constrained than low-income households and so are able to choose a larger decrease in $\frac{x}{q}$ than the low-income households. It is optimal for them to do so because it results in a smaller increase in the $\frac{z}{q}$ that they pay for traded goods and so a smaller decrease in their consumption, $q$. In contrast, low-income households must adjust to the depreciation by increasing their labor supply as they have no precautionary savings. Because they are more constrained, they do not choose as large of a decrease in $\frac{x}{q}$ and so face a larger increase in $\frac{z}{q}$ and a larger decrease in consumption than do the high-income households.[8]

Similar consumption smoothing motives and more flexibility for high-income households leads to analogous results in an $H$ appreciation: high-income households are able to choose a larger increase in $\frac{x}{q}$ and so a smaller decrease in $\frac{z}{q}$ than are low-income households. In summary, high-income households are able to choose a smoother price and consumption path in response to exchange rate movements because they have access to precautionary savings; hence they exhibit lower levels of exchange rate pass-through.

These results imply that shocks which cause real exchange rate movements will have distributional effects across heterogeneous households. In particular, because high-income households face lower pass-through rates, a real depreciation of the home currency results in a lower percentage increase in the real price of traded goods expressed in units of $H$ labor (i.e. $z/q$) for high-income than for low-income households. As discussed above, this means that the percentage fall in consumption by high-income households is lower than for low-income households, leading to an increase in consumption inequality across these two types of households. Thus, real depreciations redistribute real purchasing power from poorer to richer households, increasing real consumption inequality across households. In contrast, real

[8] In fact, high-income households decrease their precautionary savings in an amount exactly equal to the increase in their holdings of real balances as can be seen from the fact that $h + z = \bar{m}$ for these households before and after the tax shock. We note that this also implies that high-income households do not adjust their labor supply at all in response to the depreciation as can be seen from equation (23).

appreciations increase price differences across households and because high-income households increase their consumption by less than low-income households in percentage terms, consumption inequality falls. These are particularly interesting results that arise in our model with heterogeneous agents: the effect on inequality across heterogeneous households of foreign shocks which generate real exchange rate movements depends on the direction of the shocks.

# 4 Conclusions

We have developed a tractable framework for analyzing relationships between exchange rates and prices in the presence of households facing idiosyncratic preference shocks, search frictions, and monetary and fiscal policy shocks. The model generates international price differences and incomplete exchange rate pass-through due to frictional markets for imported goods and competitive search. Incomplete pass-through arises in our environment as households respond to exchange rate movements by changing the submarket where they choose to search for purchases of traded goods. This extensive margin response mitigates the direct effect of exchange rate changes, thereby generating incomplete pass-through. We have demonstrated that precautionary savings plays an important role in generating reasonable values of both real and nominal exchange rate pass-through.

Furthermore, we have shown that our simulated economy is consistent with several empirical regularities regarding exchange rate pass-through. Firstly, pass-through in our economy is highest when monetary policy is unstable which is consistent with Taylor's (2000) findings. Secondly, because the degree of pass-through is determined by optimal household responses to shocks, consumption levels and the direction of change in those levels matter for the quantitative impact of exchange rate movements on prices. This feature implies that exchange rate pass-through is asymmetric in our model where the role of household responses to exchange rate changes is emphasized. These findings are consistent with empirical evidence suggesting the presence of asymmetries in pass-through rates. Thirdly, our model is consistent with the empirical evidence which suggests that markups are not highly correlated with exchange rate movements.

Finally, our environment also has the feature that households face different levels of pass-through due to differences in households' holdings of precautionary savings. High-income households are able to use their precautionary savings to smooth shocks and so, through their submarket choices, face lower rates of exchange rate pass-through than low-income households. This implies that shocks which induce exchange rate movements have distributional effects across households with depreciations increasing inequality and appreciations decreasing inequality. This is an interesting and novel finding in the pass-through literature.

# A Lotteries

In this appendix, we explain the application of lotteries pioneered by Menzio, Shi, and Sun (2013) to render households' second sub-period value functions concave. First note that Figure 4 depicts the approximate shape of $\mathcal{B}(z)$.

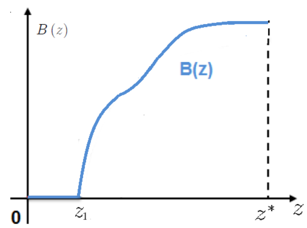


Figure 4: B(z)

We assume that lotteries are available to households at the beginning of the second sub-period prior to entering a submarket. Lotteries are characterized by $(L_1, L_2, \pi)$ such that $\pi L_1 + (1-\pi)L_2 = z$, where the household receives $L_1$ with probability $\pi$ and receives $L_2$ with probability $1-\pi$. A household with $z$ faces the following problem in its lottery choice:

$$\tilde{\mathcal{V}}(z) = \max_{(L_1 \geq 0, L_2 \geq 0, \pi \in [0,1])} \pi \mathcal{B}(L_1) + (1-\pi)\mathcal{B}(L_2), \tag{28}$$

$$\text{s.t. } \pi L_1 + (1-\pi)L_2 = z.$$

Let a household's optimal lottery choices be denoted by $\tilde{L}_1(z)$, $\tilde{L}_2(z)$, and $\tilde{\pi}(z) = \frac{\tilde{L}_2(z)-z}{\tilde{L}_2(z)-\tilde{L}_1(z)}$. A household will choose to play a lottery when it has a $z$ in the area where $\mathcal{B}(z)$ is convex and will choose $0 \leq \tilde{L}_1 \leq z$, $z < \tilde{L}_2 \leq z^*$, and $0 < \tilde{\pi}(z) < 1$. Otherwise, a household will not play a lottery and will choose $\tilde{L}_1(z) = \tilde{L}_2(z) = z$ and $\tilde{\pi}(z) = 1$, without loss of generality. Figure 5 depicts the shape of $\tilde{V}(z)$ when households have access to lotteries.

# B Derivation of $\tilde{x}(z)$

If $\tilde{b}(z) = 0$, $q$ and $x$ are irrelevant since the household will not participate in the foreign market. Without loss of generality, let $\tilde{x}(z) = z\delta$ in that case.

Now consider $z$ such that $\tilde{b}(z) > 0$. Define $\Phi(q) = \frac{u'(q)}{\psi'(q)}$. If $\tilde{x}(z) < \delta z$, then the first equation in (11) holds with equality. Combining this with equation (9) allows us to derive a household's choice of $q$ when $x < \delta z$:

$$q^* = \Phi^{-1}\left(\frac{\beta E(\theta)}{\delta\gamma(1-\tau)}\right). \tag{29}$$

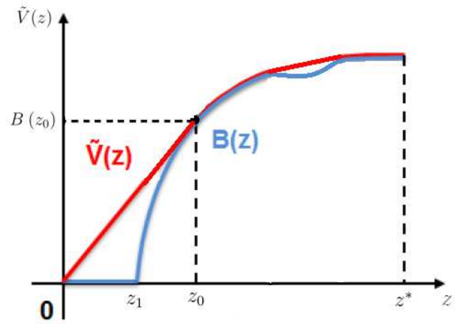


Figure 5: $\tilde{\mathcal{V}}(z)$

We can also derive their choice of $b$ in this case from the first equation in (12):

$$u(q^*) - \frac{\beta E(\theta)}{\gamma(1-\tau)\delta}\left[\psi(q^*) + \frac{k}{\mu(b^*)}\right] + \left[\frac{u'(q^*)}{\psi'(q^*)}\right]\frac{kb^*\mu'(b^*)}{[\mu(b^*)]^2} \quad = \quad 0 \tag{30}$$

Note that $q^*$ and $b^*$ are independent of $z$.

Because the left hand side of equation (30) is strictly increasing in $b^*$, then $b^* > 0$ exists and is unique if the first three terms in (30) are positive when $b$ is at its maximum value of 1; i.e. if $E(\theta)$ satisfies

$$u\left(\Phi^{-1}\left[\frac{\beta E(\theta)}{\delta\gamma(1-\tau)}\right]\right) - \frac{\beta E(\theta)}{\delta\gamma(1-\tau)}\left[\psi\left(\Phi^{-1}\left[\frac{\beta E(\theta)}{\delta\gamma(1-\tau)}\right]\right) + k\right] \quad > \quad 0. \tag{31}$$

If (31) does not hold, then $\tilde{b}(z) = 0$ and there will be no activity in the traded goods market. Hence, we only consider values for $E(\theta)$ in which (31) holds. In this case, the household's unique choice of $x$ can be determined from (9):

$$x^* = \frac{k}{\mu(b^*)} + \psi(q^*). \tag{32}$$

Now, if $\delta z < x^*$, then the constraint will bind and $\tilde{x}(z) = \delta z$ and if $\delta z \geq x$, then the constraint will not bind and $\tilde{x}(z) = x^*$. Thus, $z^* = \frac{x^*}{\delta}$ and $\tilde{x}(z) = \delta z$ for $z \in [0, z^*]$.

# C Other Equilibrium Equations

In this section, we present additional equilibrium equations for Country $H$. Analogous equations hold for Country $F$.

## C.1 Government

The government must balance its budget each period: aggregate government transfers must equal the sum of tax revenues and the increase in the money supply. Letting $LS$ denote aggregate labor supply, the government budget constraint is given by

$$T = \frac{wM\tau LS}{wM'} + \frac{(\gamma-1)M}{wM'} = \frac{\tau LS}{\gamma} + \frac{(\gamma-1)}{w\gamma}. \tag{33}$$

## C.2 Non-Traded Goods

The market-clearing condition for the $H$ non-traded good is

$$Y = \int_{\theta_L}^{\theta^H} y(\theta)\ d\mathcal{F}(\theta). \tag{34}$$

## C.3 Traded Goods

For traded goods markets to clear, it is sufficient to have the measure of buyers and the measure of shops participating in each submarket consistent with the corresponding terms of trade in each submarket.

## C.4 Currency Exchange Markets

Currency exchange market clearing requires that the demand for $H$ currency by $F$ households for transactions in the home traded goods markets equals the amount of $H$ currency that $H$ households want to trade for $F$ currency for transactions in the foreign traded goods markets. Because each buyer enters the currency exchange market only if they are matched with a shop, currency exchange market clearing is thus written as

$$\sum_{i=1}^{2}\left(\int_{\theta_L}^{\theta_H} \hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))\hat{\pi}_i(\hat{z}(\hat{\theta}))\hat{L}_i(\hat{z}(\hat{\theta}))d\hat{\mathcal{F}}(\hat{\theta})\right) = \delta\sum_{i=1}^{2}\left(\int_{\theta_L}^{\theta_H} b(L_i(z(\theta)))\pi_i(z(\theta))L_i(z(\theta))d\mathcal{F}(\theta)\right). \tag{35}$$

## C.5 Labor Markets

For the labor market to clear, aggregate labor demand (LD) must equal aggregate labor supply (LS) within each country. We begin with labor demand. Given the linear production technology, the demand for labor in production of non-traded goods equals $LD_{NT} = Y$.

Recall that sellers in Country $H$ who employ $H$ labor are active in submarkets with buyers from $F$ with relevant submarket variables denoted by a *hat*. For each shop in submarket $(\hat{q}, \hat{b})$, the expected demand for Country $H$ labor is $k + \psi(\hat{q})\mu(\hat{b})$. A Country $F$ household's realization of $\hat{\theta}$ determines its real money balances and thus, its lottery choices: $\left(\hat{L}_i(\hat{z}(\hat{\theta})), \hat{\pi}_i(\hat{z}(\hat{\theta}))\right)$ for $i \in \{1,2\}$ and where $\hat{\pi}_1(\cdot) = \hat{\pi}(\cdot)$ and $\hat{\pi}_2(\cdot) = 1 - \hat{\pi}(\cdot)$. The measure of foreign buyers specialized in consumption of any one variety of the traded good equals $\frac{1}{N}$, where $N$ is the number of varieties of traded goods. The measure of these buyers holding $\hat{L}_i(\hat{z}(\hat{\theta}))$ after the lotteries equals $\hat{N}_b(i,\hat{\theta}) = \frac{\hat{\pi}_i(\hat{z}(\hat{\theta}))d\hat{\mathcal{F}}(\hat{\theta})}{N}$. Now, given the constant-returns-to-scale matching technology, the measure of shops selling to these buyers equals

$$N_s(i,\hat{\theta}) = N_b(i,\hat{\theta}) \left( \frac{\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))}{\mu(\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta}))))} \right) = \left( \frac{\hat{\pi}_i(\hat{z}(\hat{\theta}))d\hat{\mathcal{F}}(\hat{\theta})}{N} \right) \left( \frac{\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))}{\mu(\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta}))))} \right). \tag{36}$$

Hence, multiplying this by expected labour demand per shop and aggregating that across the two lottery outcomes, the heterogeneous buyers, and the $N$ traded goods varieties gives the total expected labor demand for production of traded goods:

$$LD_T = \sum_{i=1}^{2} \left( \int_{\theta_L}^{\theta^H} \left( \frac{\hat{\pi}_i(\hat{z}(\hat{\theta}))\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))}{\mu(\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta}))))} \right) \left( k + \psi(\hat{q}(\hat{L}_i(\hat{z}(\hat{\theta})))) \ \mu(\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))) \right) d\hat{\mathcal{F}}(\hat{\theta}) \right) \tag{37}$$

The zero profit condition for an $H$ firm implies $\forall i \in \{1,2\}$ and $\forall \hat{\theta} \in [\theta_L, \theta_H]$

$$k + \psi(\hat{q}(\hat{L}_i(\hat{z}(\hat{\theta})))) \ \mu(\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))) = \frac{\hat{L}_i(\hat{z}(\theta))}{\delta} \ \mu(\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))). \tag{38}$$

Substituting this into (39) gives

$$LD_T = \left(\frac{1}{\delta}\right) \sum_{i=1}^{2} \left( \int_{\theta_L}^{\theta^H} \left( \hat{\pi}_i(\hat{z}(\hat{\theta}))\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))\hat{L}_i(\hat{z}(\hat{\theta})) \right) d\hat{\mathcal{F}}(\hat{\theta}) \right) \tag{39}$$

Thus, total labor demand is given by

$$LD = LD_{NT} + LD_T = \int_{\theta_L}^{\theta^H} y(\theta)d\mathcal{F}(\theta) \ + \ \left(\frac{1}{\delta}\right) \sum_{i=1}^{2} \left( \int_{\theta_L}^{\theta^H} \left( \hat{\pi}_i(\hat{z}(\hat{\theta}))\hat{b}(\hat{L}_i(\hat{z}(\hat{\theta})))\hat{L}_i(\hat{z}(\hat{\theta})) \right) \ d\hat{\mathcal{F}}(\hat{\theta}) \right) \tag{40}$$

Using the currency exchange market clearing condition, (35), we can rewrite aggregate labor demand as

$$LD = \int_{\theta_L}^{\theta^H} y(\theta)d\mathcal{F}(\theta) \ + \ \sum_{i=1}^{2} \left( \int_{\theta_L}^{\theta^H} \left( \pi_i(z(\theta))b(L_i(z(\theta)))L_i(z(\theta)) \right) d\mathcal{F}(\theta) \right) \tag{41}$$

We now turn to the derivation of aggregate labor supply. Labor supply is given by

$$LS = \int_{\theta_L}^{\theta^H} \int \ell(m,\theta) \ dG_a(m) \ d\mathcal{F}(\theta) \tag{42}$$

where $G_a(m)$ is the money distribution across households at the beginning of a period. Substituting in the households' policy functions for labour supply from equation (23) gives

$$LS = \int_{\theta_L}^{\theta^H} \int \left( \frac{1}{1-\tau} \right) (y(\theta) + z(\theta) + h(\theta) - m - T) \ dG_a(m) \ d\mathcal{F}(\theta). \tag{43}$$

Substituting in for $T$ from the government's budget constraint given by (33) and isolating $LS$ gives

$$LS = \left(\frac{\gamma}{\gamma(1-\tau)+\tau}\right)\left(\int_{\theta_L}^{\theta^H}\Big(y(\theta)+z(\theta)+h(\theta)\Big)d\mathcal{F}(\theta) \;-\; \int mdG_a(m) \;-\; \frac{\gamma-1}{w\gamma}\right). \qquad (44)$$

Now, aggregate real money balances at the beginning of a period consists of the precautionary savings and the balance unspent in the traded market in the previous period:

$$\int mdG_a(m) = \left[\frac{1}{\gamma}\right]\left[\int_{\theta_L}^{\theta_H} h(\theta)\; d\mathcal{F}(\theta) + \sum_{i=1}^{2}\left(\int_{\theta_L}^{\theta_H} \pi_i(z(\theta))\left(1 - b\left(L_i(z(\theta))\right)\right)L_i(z(\theta))d\mathcal{F}(\theta)\right)\right]. \quad (45)$$

Substituting this into (44), noting that $\sum_{i=1}^{2}\pi_i(z(\theta))L_i(z(\theta)) = z(\theta)$, and imposing labor market clearing, $LD = LS$, allows us to solve for the equilibrium normalized wage in Country $H$:

$$w^{-1} = \int_{\theta_L}^{\theta_H}\Big(\tau y(\theta)+z(\theta)+h(\theta)\Big)d\mathcal{F}(\theta) - (1-\tau)\sum_{i=1}^{2}\left(\int_{\theta_L}^{\theta_H}\pi_i(z(\theta))b(L_i(z(\theta)))L_i(z(\theta))d\mathcal{F}(\theta)\right). \quad (46)$$

# D Properties of Value Functions and Existence and Uniqueness of a Stationary Equilibrium

Lemmas 1-4 presented below are the open economy analogs with taxation and money growth of Lemmas 1-3 and Theorem 1 from Sun and Zhou (2018). All of these results hold for the foreign country counterparts.

*Lemma 1:*

$\mathcal{W}(m,\theta)$ given by equation (5) is continuous, differentiable, and affine in $m$.

*Proof:*

The form of the value function is the same as in Sun and Zhou (2018) except for the tax and transfer terms in the budget constraint. The proof of their Lemma 1 can be applied directly.

*Lemma 2:*

(i.) The value function $\mathcal{B}(z)$ given by equation (14) is continuous, positive, differentiable, and increasing for $z \in (z_1, z^*]$.

(ii.) The value function $\tilde{\mathcal{V}}(z)$ given by equation (28) is continuous, differentiable, increasing, and concave for $z \in [0, z^*]$.

(iii.) There exists a $z > 0$ such that $\tilde{b}(z) > 0$ if and only if there exists a $q > 0$ that satisfies $u(q)\delta\gamma(1-\tau) - \beta E(\theta)\left(\psi(q)+\kappa\right) > 0$.

(iv.) The policy functions $\tilde{b}(z)$ and $\tilde{q}(z)$ are unique and increasing in $z$.

(vi.) There exists a $z_o > z_1$ such that a household with $z < z_o$ will play the lottery with prize $z_o$, $b(z_o) > 0$, $\mathcal{B}(z_o) = \tilde{\mathcal{V}}(z_o)$, and $\mathcal{B}'(z_o) = \tilde{\mathcal{V}}'(z_o) > 0$.

*Proof:*

(i.) For $z \in (z_1, z^*]$, $\tilde{b}(z) > 0$ and the terms in brackets in equation (14) is positive. Upon inspection of (14), we see that $\mathcal{B}(z)$ is continuous and differentiable. Using the Envelope Theorem, we have

$$\mathcal{B}'(z) = \tilde{b}(z) \left[ \delta \left( \frac{u'(\tilde{q}(z))}{\psi'(\tilde{q}(z))} \right) - \frac{\beta E(\theta)}{\gamma(1-\tau)} \right] > 0.$$

Thus, equation (11) implies that $\mathcal{B}'(z) > 0$.

(ii.) Because $\mathcal{B}(z) = 0$ for $z \in [0, z_1]$, then (i.) implies that $\mathcal{B}(z)$ is continuous on $[0, z^*]$. Hence the proof from Menzio and Shi (2010) that lotteries make $\tilde{V}(z)$ continuous and concave applies here. The proof that $\tilde{V}(z)$ is differentiable and increasing from Sun and Zhou (2018) can be directly applied here.

(iii.) Define the left-hand side of (12) with $x = \delta z$ as

$$LHS(b, z) \equiv u(q) - \frac{\beta E(\theta)\ z}{\gamma(1-\tau)} \quad + \quad \left[ \frac{u'(q)}{\psi'(q)} \right] \frac{kb\mu'(b)}{\mu(b)^2}, \tag{47}$$

where $q$ is given by (9) with $x = \delta z$. Hence

$$LHS(0, z) = u(q) - \frac{\beta E(\theta)\ (\psi(q) + k)}{\delta\gamma(1-\tau)}. \tag{48}$$

This implies that $LHS(0, z) > 0$ if and only if there exists a $q > 0$ such that $u(q)\delta\gamma(1-\tau) - \beta E(\theta)\,(\psi(q) + \kappa) > 0$. Now $LHS(1, z) = -\infty$ and $\frac{\partial LHS(b,z)}{\partial b}$ is the same expression as in the proof of Lemma 2 in Sun and Zhou (2018) where they show that this is strictly negative. As in their proof, these results imply that there exists a $z > 0$ such that $\tilde{b}(z) > 0$.

(iv.) The derivations in (iii.) imply that $\tilde{b}(z)$ is unique and so it follows from (9) that $\tilde{q}(z)$ is unique. Equation (11) implies that $\frac{u'(q)}{\phi'(q)} - \frac{\beta E(\theta)}{\delta\gamma(1-\tau)} > 0$ because $x = \delta z$. Thus, as shown in the proof of Lemma 2 in Sun and Zhou (2018), we have $\frac{\partial LHS(b,z)}{\partial z} > 0$. Combining this with $\frac{\partial LHS(b,z)}{\partial b} < 0$, we have $\tilde{b}'(z) > 0$. Our expression for $\tilde{q}'(z)$ is the same as in Sun and Zhou (2018) except that $\beta E(\theta)$ is replaced by $\frac{\beta E(\theta)}{\delta\gamma(1-\tau)}$ so their logic holds here and we have $\tilde{q}'(z) > 0$.

(vi.) The proof from Lemma 2, part (iii.) of Sun and Zhou (2018) applies directly here.

*Lemma 3:*

(i.) The value function $\mathcal{V}(m, h)$ given by equation (17) is continuous and differentiable in $(z, h)$, increasing and concave in $z$ for $z \in [0, z^*]$, and affine in $h$.

(ii.) The policy functions, $y(\theta, \Omega)$, $z(\theta, \Omega)$, and $h(\theta, \Omega)$ are decreasing in $\theta$. The policy function $\ell(\theta, m, \Omega)$ is decreasing in both $m$ and $\theta$.

(iii.) The policy function $y(\theta, \Omega) > 0$ for all $\theta$.

(iv.) If there does not exist a $q > 0$ that satisfies $u(q) - \beta E(\theta)\,[\psi(q) + \kappa] > 0$, then $z(\theta, \Omega) = 0$ for all $\theta$. Otherwise $z(\theta, \Omega) > 0$ for some $\theta$.

*Proof:*

(i.) This follows from inspection of the definition of $\mathcal{V}(m, h)$ in (17) and part (ii.) of Lemma 2.

(ii.) Because $U(y)$ and $\tilde{V}(z)$ are concave, these results follow from inspection of equations (19)-(23).
(iii.) Follows from equation (19).
(iv.) The proof of Theorem 1, part (ii.) from Sun and Zhou (2018) applies directly here.

*Lemma 4:*
(i.) A stationary equilibrium exists.
(ii.) The stationary equilibrium is unique if and only if $\{\tilde{L}_1(z), \tilde{L}_2(z), \tilde{\pi}(z)\}$ are unique for all $z$.
*Proof:*

The proof of the first part of Theorem 1 from Sun and Zhou (2018) applies directly here.

# E Properties of Policy Functions

In this section, we examine the properties of $z(\theta, \Omega)$, $b(\theta, \Omega)$, $q(\theta, \Omega)$, and $x(\theta, \Omega)$.

Define the following functions:

$$F(b, z, \Omega; \delta) \equiv u\left(\Psi^{-1}\left(\delta z - \frac{k}{\mu(b)}\right)\right) - \frac{\beta E(\theta) z}{\gamma(1-\tau)}; \tag{49}$$

$$G(b, z, \Omega; \delta) \equiv b F_z(b, z, \Omega; \delta); \tag{50}$$

$$J(\theta, \Omega) \equiv \frac{\beta E(\theta)}{\gamma(1-\tau)} - \frac{\theta}{1-\tau}. \tag{51}$$

With this notation, the first-order conditions for an H household's choice of $b$ and $z$ are as follows:

$$F(b, z, \Omega; \delta) + b F_b(b, z, \Omega; \delta) = 0 \tag{52}$$

$$G(b, z, \Omega; \delta) + J(\theta, \Omega) = 0. \tag{53}$$

These two equations determine a household's optimal choices of real balances for the traded goods market, $z(\theta, \Omega; \delta)$ and the probability of matching in the submarket they choose, $b(\theta, \Omega; \delta)$. We write these as a function of the endogenous real exchange rate, $\delta$, because in this section, we initially hold $\delta$ fixed so that we may separate the effects on household choices of changes in an element in $\Omega$ between the direct effects and the effects that work through the real exchange rate.

We also have

$$q(\theta, \Omega; \delta) = \psi^{-1}\left(\delta z\left(\theta, \Omega; \delta\right) - \frac{k}{\mu\left(b\left(\theta, \Omega; \delta\right)\right)}\right) \tag{54}$$

$$x(\theta, \Omega; \delta) = \delta z(\theta, \Omega; \delta). \tag{55}$$

From these equations and part (iv.) of Lemma 2, we know that the derivatives of these two variables with respect to $\theta$ or any element of $\Omega$ will have the same sign as the derivative of $z(\theta, \Omega; \delta)$ with respect to those variables.

Holding $\delta$ fixed and differentiating equations (52) and (53) with respect to $\nu$ where $\nu \in \{\theta, \Omega\}$, rearranging, and dropping the arguments of functions for parsimony gives the following system:

$$\begin{bmatrix} 2F_b + bF_{bb} & G_b \\ G_b & G_z \end{bmatrix} \begin{bmatrix} \frac{\partial b}{\partial \nu} \\ \frac{\partial z}{\partial \nu} \end{bmatrix} = - \begin{bmatrix} F_\nu \\ G_\nu + J_\nu \end{bmatrix} \tag{56}$$

Solving this for the derivatives of interest gives

$$\frac{\partial z}{\partial \nu} = \frac{G_b F_\nu - (G_\nu + J_\nu)(2F_b + bF_{bb})}{\Delta}, \tag{57}$$

$$\frac{\partial b}{\partial \nu} = \frac{G_b (G_\nu + J_\nu) - G_z F_\nu}{\Delta} \tag{58}$$

where $\Delta > 0$ is the determinant of the left-hand-side matrix.

Now, given that $\tilde{V}(z)$ is strictly concave, that equation (11) holds with $x = \delta z$, and that the second-order sufficient condition for the choice of $b$ is satisfied, we have:

$$G_z < 0 \qquad G_b > 0 \qquad 2F_b + bF_{bb} < 0. \tag{59}$$

We first consider how a household's choices vary with its preference parameter $\theta$. We have

$$F_\theta = 0 \qquad G_\theta = 0 \qquad J_\theta = \frac{-1}{1-\tau} < 0. \tag{60}$$

It follows then that

$$\frac{\partial z(\theta, \Omega; \delta)}{\partial \theta} < 0, \qquad \frac{\partial b(\theta, \Omega; \delta)}{\partial \theta} < 0, \qquad \frac{\partial q(\theta, \Omega; \delta)}{\partial \theta} < 0, \qquad \frac{\partial x(\theta, \Omega; \delta)}{\partial \theta} < 0. \tag{61}$$

This is an expected result and consistent with Sun and Zhou (2018): households with higher disutility of working have lower labour supply and less income. Thus, they choose a lower level of real balances for the traded goods market and choose submarkets with a lower probability of matching.

We now consider each element of $\Omega$. For $\nu = \tau$,

$$F_\tau = \frac{-\beta E(\theta) z}{\gamma(1-\tau)^2} < 0 \qquad G_\tau + J_\tau = \frac{(1-b)\beta E(\theta) - \gamma\theta}{\gamma(1-\tau)^2} < 0. \tag{62}$$

Hence, from equations (57) and (58), we have that when $\delta$ is held fixed, $\forall \theta$,

$$\frac{\partial z(\theta, \Omega; \delta)}{\partial \tau} < 0, \qquad \frac{\partial b(\theta, \Omega; \delta)}{\partial \tau} < 0, \qquad \frac{\partial q(\theta, \Omega; \delta)}{\partial \tau} < 0, \qquad \frac{\partial x(\theta, \Omega; \delta)}{\partial \tau} < 0. \tag{63}$$

For $\tau = \gamma$, and $\tau = E(\theta)$, we have

$$F_\gamma = \frac{\beta E(\theta) z}{\gamma^2(1-\tau)} > 0 \qquad G_\gamma + J_\gamma = \frac{(b-1)\beta E(\theta)}{\gamma^2(1-\tau)} < 0. \tag{64}$$

$$F_{E(\theta)} = \frac{-\beta z}{\gamma(1-\tau)} < 0 \qquad G_{E(\theta)} + J_{E(\theta)} = \frac{(1-b)\beta}{\gamma(1-\tau)} > 0. \tag{65}$$

From equations (57) and (58), we see then that the effect of each of these variables on $z$, $b$, $q$, and $x$ are ambiguous (holding $\delta$ fixed). Finally, for $\omega \in \{\hat{\tau}, \hat{\gamma}, E(\hat{\theta})\}$, each of these derivatives are zero, holding $\delta$ fixed, as there are no direct effects in Country $H$ when a parameter changes in Country $F$.

We now examine the indirect effects on household choices of a change in an element of $\Omega$ that works through the real exchange rate, $\delta$. We can evaluate this by setting $\nu = \delta$ in (57) and (58) where $J_\delta = 0$ and

$$F_\delta = z\left(\frac{u'(q)}{\psi'(q)}\right) > 0 \qquad G_\delta = b\left[\left(\frac{u'(q)}{\psi'(q)}\right) + \left(\frac{\delta z}{(\psi'(q))^2}\right)(u''(q)\psi'(q) - u'(q)\psi''(q))\right], \tag{66}$$

where $q = \psi^{-1}\left(\delta z - \frac{k}{\mu(b)}\right)$. Without a specific utility function and production function for traded goods, the sign of $G_\delta$ is ambiguous. Suppose that we restrict attention to preferences and technologies for traded goods so that $\frac{\partial z}{\partial \delta} < 0$ – i.e. a depreciation of $H$ currency, ceteris paribus, induces the household to hold higher real balances in $H$ currency for exchanges in the traded goods' market. A necessary but not sufficient condition for this is $G_\delta < 0$. However, from equations (54), (55), and (58), we see that the signs of $\frac{\partial b}{\partial \delta}$, $\frac{\partial q}{\partial \delta}$, and $\frac{\partial x}{\partial \delta}$ remain ambiguous even under this condition. Thus, we conclude that without further restrictions on the economy, the qualitative effects of policy changes in either country on household choices cannot be determined.

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