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Heterogeneity, Frictional Assignment And Home-ownership

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Heterogeneity, Frictional Assignment and Home-ownership*

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Abstract

We study the long-run composition of the housing stock across city-level ownership and rental markets in a dynamic equilibrium model of frictional assignment. Newly constructed and existing houses may be rented or sold to heterogeneous households that sort over housing quality. Due to the endogenous interaction between matching frictions and an ownership surplus that rises with quality, more housing is offered for sale per buyer in higher quality segments. Consequently, even in the absence of financial frictions and constraints on which houses can be rented, higher income households are more likely to own and lower quality housing is more likely to be rented. When calibrated to match key features of housing markets in the average U.S. city, the model is qualitatively consistent with observed cross-city relationships between underlying determinants and key market outcomes. We study the implications of the model for affordability and ownership across cities and for the impact of progressive property taxes and other affordable housing policies.

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1 Introduction

Average rates of home-ownership and the relative average costs of owning and renting vary dramatically and persistently across U.S. cities. Amongst metropolitan statistical areas (MSA's) in the 2010 American Community Survey (ACS five-year estimates), for example, home-ownership rates vary from 51% to 81% and ratios of average prices to average rents from 8.7 to 54.2. In the 2017 ACS, the variation is even greater.[1] To a large extent, the ratio of average prices to average rents in a given city reflects a *composition effect*: high value housing units are disproportionately supplied to owner-occupied markets while lower value units tend to be supplied to rental markets. But why is this, and what determines the long-run, cross-city variation in these composition effects? Moreover, how do these effects determine the affordability of home-ownership in different cities?

To study these issues, we develop a dynamic equilibrium model of *frictional assignment* in which the quality distribution of the housing stock across city-level owner-occupied and rental markets and the relative affordability of home-ownership are determined by the optimal decisions of households, landlords and developers. In our model, whether a housing unit of a given quality is offered for sale or rent depends on (a) households' relative willingness to pay to own rather than rent the unit and (b) the liquidity of the owner-occupied market relative to the rental market for that unit. The former depends on multiple factors including preferences, relative maintenance costs and the income levels of the households that constitute the buyers in the relevant market segment.[2] The relative liquidity of a given segment of the housing market depends on matching frictions, the number of potential buyers/renters and the quantity of housing that is available for purchase.

We assume that housing units are indivisible and that their observable characteristics (*e.g.* size, number of rooms, location, proximity to amenities, weather, etc.) can be summarized by a single index which we refer to as their "quality".[3] A city is comprised of heterogeneous housing units and inhabited by a growing population of households with stochastic lifetimes who are differentiated by their permanent income. New houses may be of any quality and are built by a development industry comprised of a large number of firms under conditions of free entry. Construction/land costs

[1] For this calculation we use the ratio of the mean price-asked for each MSA to mean annual rent (see Appendix B.1). Note that this is not the relative price and rent of an equivalent unit.

[2] We use "segment" throughout to refer to the market for all housing of a given quality level.

[3] This is a common approach in the literature and is consistent with the hedonic methodology to real estate valuation. Housing is similarly modelled recently by Landvoigt et al. (2015), Epple et al. (2020) and Nathanson (2020).

increase with quality and, once built, a house's observable quality is permanently fixed. All households require housing, but could either rent or own. We assume that asset markets are complete and focus on a balanced growth path where new housing is built every period.[4]

A fundamental distinction between owning and renting in our model derives from the fact that ownership confers the right to customize housing units to suit the idiosyncratic preferences of the occupants. We assume that only certain housing units in a given segment are suitable in this respect and that different houses in the same segment appeal to different households. Households therefore derive greater utility from owning certain specific houses than they do from owning others. While we allow for trading frictions that are common to both rental and owner-occupied markets, the lower likelihood with which households find housing units which can be customized to match their specific requirements or tastes creates a "matching friction" which is specific to owner-occupied markets.[5]

A second distinction, in our baseline specification, is that the maintenance costs incurred by landlords increase more rapidly with quality than those for owner-occupiers. This assumption is intended to reflect the idea that the costs of moral hazard associated with renting increase with house quality.[6] The main consequence of this assumption is that the surplus associated with ownership increases with quality. An increasing surplus also arises if maintenance costs are constant but there is a utility premium for ownership that rises with quality (see Section 4.5).

In equilibrium, ownership patterns solve a frictional assignment problem in the sense of Shi (2001, 2005). Vacant houses of any quality may either be rented or offered for sale through directed search. Unmatched households either rent or purchase an affordable home of their preferred quality. Since searching for a house to buy is costless, whether or not households own depends on their willingness to pay and the incentives facing *supply-side* participants. Free entry of sellers and the directed search

[4] The assumption of complete markets here functions mainly to avoid complications of *ex post* heterogeneity arising from idiosyncratic shocks at the household level.

[5] Available evidence, though disparate, suggests rental units are leased more rapidly on average than houses for sale are sold. Gabriel and Nothaft (2001) estimate the average duration of rental vacancies to be 1.5-2 months for 29 MSAs during 1986-97. Allen et al. (2009) document an average of 71 days on the market for single family residential rental (leasehold) property interests in Dallas-Fort Worth during 2003-4. Adjusting for censoring bias, Carrillo and Pope (2012) estimate the average days on the market for housing for sale in Fairfax County (VA) to be 138 days in 2007. Similarly, during 2010, the simple average across MSAs of Zillow "days on the market" for housing for sale was 126 days.

[6] See Sweeney (1974); Henderson and Ioannides (1984). A similar assumption also plays a role in determining home-ownership in Chambers et al. (2009a,b).

strategies of buyers in each segment of the market cause matching hazards to adjust until the willingness/capacity to pay a price premium equals the liquidity-adjusted present value of the ownership surplus. Because the ownership surplus rises with housing quality, a high price premium induces relatively more housing to be offered for sale rather than for rent in higher quality segments. While owning a well-matched unit is more desirable than renting for any given house quality, proportionately more poor households end up renting because they cannot afford housing in market segments where housing for sale is more plentiful.

The equilibrium of the model is consistent with two broad empirical observations: First, the likelihood of home-ownership is strongly increasing in household permanent income after controlling for other household characteristics (including age and family composition), neighbourhood characteristics and cyclical factors.[7] Second, the likelihood that a given housing unit is sold rather than rented out rises with the value of the unit.[8] Notwithstanding these overall tendencies, some low value housing units are owned while many high value ones are rented.

We calibrate the model's balanced growth path to match several median features of U.S. MSA housing markets, including vacancy rates, average ownership duration, the ratio of average prices to average rents and the distribution of ownership across income quintiles. We use the calibrated model to characterize the steady state effects of variation in several key fundamentals. In particular, the model predicts that the ownership rate increases with median income and age and decreases with inequality and construction costs. The ratios of average prices to average rents increase with median income, inequality and construction costs.[9] The vacancy ratio — *i.e.* the vacancy rate of owner-occupied housing relative to the overall vacancy rate — increases with income and inequality and decreases with median age and construction costs.

Using the ACS, we characterize the empirical relationships between ownership rates, average price-rent ratios, vacancy ratios and city-level characteristics. Guided by our model, we estimate the association between these outcomes and median income, inequality, age and land costs using cross-city regressions.[10] Controlling for numerous factors potentially affecting the desirability of living in a given city and the desire to own, we find that, qualitatively, the patterns observed in cross-city data are remarkably consistent with the predictions of our model. Moreover, these patterns

[7]See, for example, Rosen (1979), Goodman (1988), Kan (2000) and Carter (2011).

[8]See Glaeser and Gyourko (2007) and Halket et al. (2020).

[9]The relationship between median age and the price-rent ratio is ambiguous.

[10]For our sample of 366 MSA's, median incomes range from \$31,264 to \$86,286 and income Gini coefficients from .388 to .537. See also Glaeser et al. (2009).

appear robust to alternative specifications and inclusion of other explanatory factors.

To further study the theory's quantitative predictions, we use the calibrated model to generate predicted cross-city variation in outcomes resulting from observed and inferred variation in MSA-level characteristics. We find that, while the distribution of income and age are important, differences in construction/land costs and average amenities across cities play a key role in accounting for observed cross-city variation in ownership, ratios of average prices to average rents and vacancy ratios.

Given the observed variation in fundamentals, the model generates substantial variation in the *affordability* of housing across cities. Having less affordable housing reduces housing quality for all households and, importantly, makes ownership less attainable for those with relatively low permanent income. We use the model to study the long-run effects of increasing the progressivity of property taxes. We show that such a policy change improves the well-being of lower permanent income households relative to that of higher income ones, mainly by increasing housing quality. Moreover, it significantly increases home-ownership despite not targeting it directly. Households with high permanent income (who effectively bear the cost of the policy) continue to own at roughly the same rate, but live in lower quality houses. They can compensate for this to a large extent by increasing their non-housing consumption.[11]

Households in our model face binding financial limitations imposed by their intertemporal budget constraints. Much of the macroeconomic housing literature focuses on the role of additional financial frictions in the form of exogenous borrowing constraints and minimum downpayment requirements.[12] The critical assumption in the determination of home-ownership in these models is that small or low quality housing units cannot be purchased in the owner-occupied market. Without such a restriction, these types of constraints will normally affect house size or quality rather than ownership *per se*.[13] In our theory, ownership patterns are driven by the optimal decisions of buyers and sellers faced with a choice between rental markets and relatively less liquid owner-occupied markets. Rental housing is more likely to be of low quality because the price incentive to supply such housing to the owner-occupied market is weak.

[11] A more progressive tax policy can replicate the effects of subsidizing the construction of low quality housing and taxing that of high quality housing. In this respect the exercise is comparable to one considered by Nathanson (2020).

[12] See, for example, Stein (1995), Gervais (2002), Chambers et al. (2009a,b), Iacoviello and Pavan (2013), Sommer et al. (2013), Floetotto et al. (2016) and Sommer and Sullivan (2018).

[13] In the partial equilibrium model of Bajari et al. (2012), younger households rent to avoid non-convex adjustment costs incurred when upgrading.

While financial frictions can have important effects on the short-run dynamics of ownership, their role in determining long run cross-city variation is less clear. In their detailed structural modelling of the determinants of home-ownership in U.S. housing markets, Garriga et al. (2020) find no significant long-run relationship with changes in credit constraints, even with stark assumptions about the life cycle. Relatedly, using a stochastic life cycle Aiyagari–Bewley–Huggett economy with incomplete markets, uninsurable idiosyncratic earnings risk, down-payment requirements and endogenous house prices and rents, Sommer et al. (2013) find no steady-state impact of a change in average income on either the home-ownership rate or the average price-rent ratio. That said, in Section 7.2 we study the implications of an extended version of our model that incorporates a payment-income ratio constraint.

Search and matching models have been used extensively to understand housing market microstructure (see Wheaton, 1990; Krainer, 2001; Albrecht et al., 2007; Halket and di Custoza, 2015; Lu and Strange, 2015; Piazzesi et al., 2020) and to study various aspects of the dynamics of housing markets (*e.g.* Diaz and Jerez, 2013; Head et al., 2014; Ngai and Sheedy, 2020; Hedlund, 2016; Garriga and Hedlund, 2020; Garriga et al., 2020; Anenberg and Bayer, 2020). Relatively little work, however, has focused on the implications of trading frictions for the composition of the housing stock across ownership and rental markets and the relative cost of owning versus renting. Rather, search-theoretic models typically focus either on one of these markets in isolation or, where both are incorporated, assume that they are largely disconnected on the supply side.[14] This contrasts with the standard user-cost framework in which the incentives for heterogeneous housing units to be sold or rented are a primary concern.

The remainder of the paper is organized as follows. Section 2 describes the theoretical environment which will be used to study housing tenure in equilibrium and for comparisons across cities, both qualitative and quantitative. Section 3 defines a stationary balanced growth path in a directed search equilibrium. The calibration is detailed in Section 4 along with the implied characteristics of housing markets within the city. Section 5 describes variation across cities with regard to housing tenure and the ratio of average prices to average rents in both the data and the model. The affordability of housing and the effects of policies designed to improve it are considered in Section 6 and additional implications are discussed in Section 7. Section 8 concludes and outlines future work.

[14] Head et al. (2014) develop a search model with a rent *vs.* sell decision but where housing is homogeneous.

2 A Model of Construction and Housing Tenure

2.1 The environment

Consider a dynamic economy in discrete time, consisting of a single city populated by a growing number of households with stochastic life-times. Each period, new households enter the city either through migration from elsewhere or by its members attaining an age at which they live independently. The rate of entry/household formation is constant, and denoted by ν. Households die with probability δ each period.[15] The population of the city, L_t, thus evolves according to:

$$L_{t+1} = (1 + \nu - \delta)L_t. \tag{1}$$

Households differ *ex ante* with regard only to their permanent income, y. As we discuss in Section 4, their actual income at a given point in time may also depend on their age.[16] Household permanent incomes are distributed according to a cumulative distribution function, F, with positive and continuous support.

Households consume both goods and housing services. In particular, each period they must live in a single, indivisible *house*, which they may either rent or own. Houses differ with regard to their characteristics, and we represent these by a single index of *quality*, $q \in \mathbb{R}_+$. Households maximize expected utility over their stochastic lifetimes. Preferences are represented by

$$U = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + h(z_t, q_t)\right], \tag{2}$$

where $u(c_t)$ denotes utility from consuming goods c_t and $h(z_t, q_t)$ represents the current period utility flow from living in a house of quality q_t.[17] Here $z_t \in \{0, 1\}$ is an indicator: if $z_t = 1$, the household owns the house *and* it matches with household's idiosyncratic preferences in period t. If $z_t = 0$, the house is rented. We assume that $u_c > 0$, $u_{cc} < 0$, $h_q > 0$, $h_{qq} < 0$, and that $h(1, q) \geq h(0, q)$ for all q (with equality

[15] Rather than dying, households could leave the city for elsewhere randomly. This would make little difference for our results, although it would require a re-interpretation of certain parameters.

[16] Given our assumption below of complete markets, households could also face idiosyncratic shocks to their income flow with no changes to our analysis or results.

[17] Separability between c and h greatly simplifies the household's portfolio problem since it implies that the marginal utility of consumption does not depend on housing services (see below).

for $q = 0$).[18] In our baseline calibration below, we also assume that the utility a household would derive from buying and living in a house that does not match their idiosyncratic preferences is the same as if the house provided no housing services at all, $h(1,0)$. In (2), β is the household's discount factor adjusted for the probability of survival and $\beta/(1-\delta)$ reflects the pure rate of time preference. The discount factor satisfies $\beta = (1-\delta)/(1+\rho)$ where ρ is the exogenous world interest rate.[19]

Each period, with probability π, owner-occupiers receive an idiosyncratic preference shock which results in them no longer valuing the housing services from living in *that particular house*. The utility flow of remaining in this house would fall to $h(1,0)$. As a result, the household will move to their optimal rental unit and start the search for a new house to own that is a good match with their current preferences. This *mobility shock* is intended to capture a household's evolving taste for the idiosyncratic features of a house, and generates turnover in housing markets.[20] This implies that, in our baseline specification, only renters search and all housing for sale is vacant. As we demonstrate in Section 7.1, a more complex set of assumptions that results in mis-matched owners remaining in their houses while they search generates very similar results once appropriately calibrated.

In addition renters are also subject to an idiosyncratic moving shock: with probability π_R they relocate within the city (e.g. due to eviction, change of employment, etc.). Because there are no explicit costs to moving, this shock does not affect households' utility but allows us to match certain flows in the data. There is no restriction on how many houses a household can own. Each household must, however, live in (and thereby receive housing services from) one, and only one, house at a time.

Houses of different qualities are built using a construction technology through which the cost of land and construction required to build a house of quality q is $T(q)$, where $T'(q) > 0$.[21] Construction is undertaken by an industry comprised of a large number of identical, risk neutral *developers* under conditions of free entry.

[18]In our baseline calibration, we set the ownership utility premium to zero, so that $h(1,q) = h(0,q)$ for all q. See Section 4.5 for discussion of an alternative formulation with an increasing ownership utility premium.

[19]This assumption is necessary for there to be a stationary balanced growth path. One justification is that ρ is set in a stationary "rest of the world", and taken as given in the city.

[20]In general, mobility risk may depend on house quality and/or age. For example, a case in which $\pi'(q) < 0$ is consistent with the finding of Piazzesi et al. (2020) that in the San Francisco Bay area, less expensive market segments tend to be less "stable" (*i.e.* moving shocks occur more frequently). For simplicity, however, here we hold π constant across house types.

[21]With complete markets, other housing costs that depend only on q, and not on tenure, occupational status or income, could be included in $T(q)$. In particular, as we discuss in Section 6, it can include the appropriately discounted value of future property tax obligations.

Once produced, the quality of a given house is fixed, permanently.[22] Construction of a house takes one period, and development firms are owned by households, remitting their profits (if any) lump-sum. Because of free entry, firms build houses of each type as long as the discounted future value of a house is not less than $T(q)$. This parsimonious representation of housing supply is suitable given our focus on the balanced growth path. The study of transitional dynamics would likely require a more detailed specification of the production process (*e.g.* Head et al., 2014).

An owner of a vacant house, whether a developer or household, may either rent it to a prospective tenant or offer it for sale. Rental markets are Walrasian and $x_t(q)$ represents the current rent for a house of quality q. In contrast, we represent the process by which potential buyers match with specific houses that can be customized appropriately as a directed search process. Vacant houses of a given quality are offered for sale in sub-markets characterized by a posted price and a pair of matching probabilities, one each for both buyers and sellers. We assume a constant returns to scale (CRS) matching function and refer to the ratio of buyers to sellers as market tightness, denoted θ_t. The matching rates for buyers and sellers (λ and γ, respectively) are functions of tightness and satisfy:[23]

Assumption 1. *The matching probabilities have the following properties:*

(i) $\lambda(\theta) \in [0,1]$ *and* $\gamma(\theta) \in [0,1]$ *for all* $\theta \in [0,\infty]$*;*

(ii) $\lambda'(\theta) < 0$ *and* $\gamma'(\theta) > 0$ *for all* $\theta \in (0,\infty)$*; and*

(iii) $\lim_{\theta\to 0}\gamma(\theta) = 0$ *and* $\lim_{\theta\to\infty}\gamma(\theta) = \bar{\gamma} \leq 1$.

All available houses, whether they are to be sold or offered for rent, are also subject to common frictions that determine how quickly they become occupied. We assume that it may take time for newly built or vacated houses to be assigned to either the rental or owned market in the relevant segment. Specifically, an available house, that has not yet been assigned, becomes so with a common probability ϖ each period. This is a simple way of capturing various market frictions that are unrelated to households' search for the right house to own. For example, it could include the time taken to undertake maintenance or renovations, to advertise appropriately or find the right real estate agent and to identify the correct market segment. Houses may be optimally assigned to either the rental or owner-occupied market.

[22] As described below, any physical depreciation is assumed to be offset via ongoing maintenance.

[23] The likelihood of a match could, in principle, depend on the quality of the house. This could reflect, for instance, higher quality houses being more diverse and thus specifically appealing to a smaller fraction of buyers who visit them. Here we assume that the matching probabilities depend only on market tightness.

Occupancy of houses results in depreciation which we assume to be completely offset by maintenance, the cost of which depends on quality and whether or not the house is occupied. Specifically, $Z_R(q)$ represents the per period cost of maintaining a house of quality q when rented and $Z_N(q)$ denotes that cost when owner-occupied. We assume that $Z_R(q) \geq Z_N(q)$ and that $Z'_R(q) \geq Z'_N(q)$. Houses depreciate when rented or owner-occupied, but not while vacant.

At each point in time, the total stock of housing in the city is given by

$$M_t = \int_0^\infty M_t(q)dq \tag{3}$$

Each of these houses may be either owned by or rented to its occupant, remain unassigned, or held vacant for sale. Let N_t denote the measure of owner-occupied houses (or, equivalently, the measure of homeowners), R_t denote the measure of houses currently rented (or of renting households), S_t the measure of houses vacant for sale, and S_t^U the measure of currently unassigned vacant housing. We then have

$$M_t = N_t + R_t + S_t + S_t^U \tag{4}$$

$$L_t = N_t + R_t. \tag{5}$$

Thus, in each period, the measure of houses in the city exceeds that of resident households by total vacancies, $S_t + S_t^U$. Note that for each house quality, q, equations analogous to (4) and (5) hold also.

Finally, there exist competitive markets in a complete set of one-period-ahead state-contingent claims paying off in units of the non-storable consumption good. These enable households to fully insure their idiosyncratic risks in the housing market (associated with λ and γ) and of losing the housing services from their current house (associated with π).[24] Households are also required to purchase/issue contingent claims to settle their estate in the event of death (associated with δ). Households face no financial constraints beyond that implied by their life-time budgets.

In addition to the assumptions made above, further joint restrictions on functional forms are required to guarantee a monotonic mapping between household income and house quality:

Assumption 2. *The joint properties of the functions u, h, T, λ, γ, Z_R and Z_N are such that the optimal choices of households and developers in equilibrium generate a bijective mapping between household income, y, and house quality, q, in the owner-occupied market.*

[24] The mobility risk faced by renters does not affect them in any real way.

In Section 4 we specify functional forms and use them to compute a steady state equilibrium with this feature. In Appendix E we characterize sufficient conditions and verify that they hold for the functional forms and parameters we use. Eeckhout and Kircher (2010) discuss general properties required for positive assortative matching in a related but static environment. The sufficient conditions we derive in Appendix E have a similar interpretation.

3 Equilibrium

Throughout, we focus on balanced growth paths in which rents, house prices and values all remain constant. Along such a path, the distributions of both households and houses across rental and owner-occupied markets are time-invariant.

3.1 The supply-side decision problem

An important choice in the economy is the rent *vs.* sell decision, made by owners of available houses, be they households or developers. Let $V_t(q)$ and $V_t^u(q)$, respectively, denote the value of assigned and unassigned houses of quality q. Unassigned houses may subsequently become assigned and so

$$V_t^u(q) = \frac{1}{1+\rho}\left[\varpi V_{t+1}(q) + (1-\varpi)V_{t+1}^u(q)\right]. \tag{6}$$

Once assigned, a house is either rented immediately or held vacant for sale in the current period. Its value is

$$V_t(q) = \max\left\{ \begin{array}{c} x_t(q) - Z_R(q) + \frac{1}{1+\rho}\left[(1-\pi_R(1-\varpi))\, V_{t+1}(q) + \pi_R(1-\varpi)V_{t+1}^u(q)\right], \\ \frac{1}{1+\rho}\max_p\left[\gamma(\theta_t\,(q,p))p + (1-\gamma(\theta_t\,(q,p)))V_{t+1}(q)\right] \end{array} \right\} \tag{7}$$

The first expression in curly brackets in (7) is the value of renting the house and the second is that of holding it vacant for sale. The maximization operator in the second expression reflects the optimal choice of sub-market by the seller. The seller, taking as given the search behavior of buyers, anticipates how market tightness, and consequently the matching probability, responds to the price.

All sellers value houses of a particular quality identically.[25] Thus, their indifference across *active* sub-markets gives rise to an equilibrium relationship between price and

[25]This implication stems from two assumptions. First, the mobility shock induces mismatched

tightness for houses in a given market segment:

$$\gamma(\theta_t(q,p)) = \frac{(1+\rho)V_t(q) - V_{t+1}(q)}{p - V_{t+1}(q)}. \tag{8}$$

Moreover, as sellers may freely decide whether to rent or hold a house vacant-for-sale, we have

$$V_t(q) = x_t(q) - Z_R(q) + \frac{1}{1+\rho}\left[(1-\pi_R(1-\varpi))\,V_{t+1}(q) + \pi_R(1-\varpi)V^u_{t+1}(q)\right]. \tag{9}$$

Conditions (8) and (9) equate house values across rental and sales markets. Free entry then implies that, along the balanced growth path, house values and rental costs can therefore be represented by time-invariant functions of house quality:

$$V(q) = \Big(\frac{\varpi+\rho}{\varpi}\Big)T(q) \tag{10}$$

$$x(q) = Z_R(q) + \Big(\frac{\varpi+\rho+(1-\varpi)\pi_R}{\varpi(1+\rho)}\Big)\rho T(q). \tag{11}$$

Moreover, tightness for active sub-markets is a time-invariant function of p and q:

$$\gamma(\theta(q,p)) = \frac{\rho V(q)}{p - V(q)}. \tag{12}$$

Note that this indifference condition implies the same equilibrium relationship between the ratio $p/V(q)$ and market tightness, θ, in every segment. As we discuss below, this supply-side condition plays a key role in determining the distribution of home-ownership across segments.

3.2 The household decision problem

As households are risk-averse, have separable utility and markets are complete, they carry out financial transactions to smooth consumption completely. Specifically, at the beginning of each period, through the purchase and sale of contingent claims, households insure themselves against risks associated with preference shocks (which determine whether the household remains happy with their house), matching outcomes and death, all of which are random. From this point on, where possible, time subscripts will be suppressed.

households to move out so that their vacant home becomes a purely financial asset. Second, the complete markets assumption implies that households with different consumption levels value such financial assets equally. Section 7.1 relaxes the former assumption without changing the main results.

Consider first a homeowner. This household may purchase/issue w_S units of a security, each of which pays one unit of consumption good in the next period contingent on receiving a preference shock and becoming a renter, and w_N units of a security that pay contingent on remaining a homeowner. The homeowner may also sell up to $\varpi V(q) + (1-\varpi)V^u(q) = (1+\rho)T(q)$ units of contingent claims which pay-off in the event that the household dies at the end of the period. The payment of these claims is financed by the sale of the household's then-vacant house.

Similarly, a household renting while searching to buy a house may purchase w_B units of insurance that pay contingent on buying a house, and w_R units of a security that pay contingent on having failed to buy and continuing to rent. Finally, we impose that a household searching to buy in sub-market p of segment q must purchase $p-(1+\rho)T(q)$ units of a claim that pays off contingent on the household committing to buy a house but then dying at the end of the period.[26]

The prices of the contingent securities, in the order defined, are denoted ϕ_S, ϕ_N, ϕ_D, $\phi_B(q,p)$, $\phi_R(q,p)$, and $\phi_D(q,p)$. The last three prices depend on the house quality and the price as they insure against outcomes in a particular sub-market of a particular housing market segment.

At the beginning of each period, renters, depending on their *total wealth*,[27] choose: (i) a type/quality of house to rent, q_R; (ii) whether or not to search for a house to buy; and, if searching, (iii) a particular sub-market, p, (associated with a particular house type, q_S) in which to search; and (iv) a consumption level, c, and savings in the form of a portfolio of claims, $\{w_B, w_R\}$. For simplicity, the decision not to search for a house to purchase will be represented by the choice of $q_S = 0$ and $p = 0$. Accordingly, $\theta(0,0) = \infty$ and $\lambda(\theta(0,0)) = 0$. A renter with wealth w then has value:

$$W^R(w) = \max_{\substack{c,w_R,w_B,\\ q_R,q_S,p}} \left\{ \begin{array}{l} u(c) + h(0,q_R) \\ +\beta\left[\lambda(\theta(q_S,p))W^N(q_S,w_B-p) + (1-\lambda(\theta(q_S,p)))\,W^R(w_R)\right] \end{array} \right\} \quad (13)$$

subject to:

$$c + \phi_B(q_S,p)w_B + \phi_R(q_S,p)w_R + \phi_D(q_S,p)(p-(1+\rho)T(q_S)) + x(q_R) = w, \quad (14)$$

where $W^N(q_S, w_B - p)$ is the value of entering next period as an owner of a house of

[26]Although renters are subject to a moving shock, this does not affect their wealth or utility since they are able to immediately move into a unit in the same segment.

[27]Total wealth, w, is defined to include all available financial resources. In particular, the present discounted value of future permanent income, $y/(1-\beta)$, is contained in w. See Appendix A.1 for details.

quality q_S with wealth $w_B - p$. This value is given by:

$$W^N(q,w) = \max_{c,w_N,w_S,w_D} \left\{ \begin{array}{l} u(c) + h(1,q) \\ +\beta \left[\pi W^R(w_S + (1+\rho)T(q)) + (1-\pi)W^N(q,w_N)\right] \end{array} \right\} \tag{15}$$

subject to:

$$c + \phi_S w_S + \phi_N w_N + \phi_D w_D + Z_N(q) = w \tag{16}$$

$$w_D + (1+\rho)T(q) \geq 0. \tag{17}$$

3.2.1 Consumption and portfolio choice

Appendix A.1 contains the solution to households' portfolio allocation problem and the derivation of the optimal consumption by income. Let q_R, q_S, and p denote the optimal rental and search choices for a household with permanent income y.[28] This household's (constant) per period consumption is given by

$$\begin{aligned} c = y &- \frac{\lambda(\theta(q_S,p))\left[\rho+\delta+\pi(1-\delta)\right]\left[p-(1+\rho)T(q_S)\right]}{(1+\rho)\left[\rho+\delta+(\pi+\lambda(\theta(q_S,p)))(1-\delta)\right]} \\ &- \frac{\left[\rho+\delta+\pi(1-\delta)\right]x(q_R) + \lambda(\theta(q_S,p))(1-\delta)\left[Z_N(q_S)+\rho T(q_S)\right]}{\rho+\delta+(\pi+\lambda(\theta(q_S,p)))(1-\delta)} \end{aligned} \tag{18}$$

This is the highest attainable constant consumption sequence satisfying the present value budget constraint given the cost of insuring against both preference shocks and the matching risks associated with housing-related transactions.

3.2.2 Renting

The rent decision is determined by the *intratemporal* Euler equation associated with the choice of q_R in (13):

$$u'(c)x'(q_R) = \frac{\partial h(0,q_R)}{\partial q}. \tag{19}$$

Here $x'(q_R)$ is the derivative of the time-invariant rental cost function from equation (11). First order condition (19) combined with (18) pins down the quality of housing rented by a household with permanent income y.

[28]We suppress the dependence of q_R, q_S, p and c on y for brevity.

3.2.3 Home-ownership

Maximizing with respect to q_S and p in (13) reflects optimal search decisions with regard to house type and sub-market, where tightness $\theta(q,p)$ is determined by (12). A pair of *intertemporal* Euler equations (one for segment choice, q_S, the other for choice of sub-market, p) characterize the solution to the household's search problem, derived in Appendix A.2. In steady state, the optimal choice of segment for a given sub-market is characterized by

$$\frac{\partial h(1,q_S)}{\partial q} = \frac{\frac{[\rho+\delta+\pi(1-\delta)]\left[p-(1+\rho)T(q_S)\right]}{(1+\rho)(1-\delta)}\frac{T'(q_S)}{T(q_S)} + Z_N'(q_S) + \rho T'(q_S)}{u'(c)}. \tag{20}$$

The optimal sub-market within a given segment reflects the household's understanding that higher priced sub-markets will attract more units for sale relative to buyers (higher $1/\theta$). The indifference condition (12) for potential sellers is illustrated in Figure 1.[29] For a given unit, this condition governs how the ratio of price to its present discounted value in the rental market must vary with the ratio of units for sale to searchers. For a given quality of house, households prefer lower prices and a higher ratio of vacant houses for sale to buyers, as illustrated by the potential buyers' indifference curves. Given that all households in the sub-market are identical, they will therefore optimally select the same unique sub-market (p,θ).[30]

The implications of this directed search equilibrium are summarized by the price premium that the household pays to acquire owner-occupied housing.

$$p - V(q_S) = \mathcal{L}(\theta(q_S,p))\left\{\frac{h(1,q_S)-h(0,q_R)}{u'(c)} - \big(\hat{x}(q_S)-x(q_R)\big)\right\} \tag{21}$$

where

$$\hat{x}(q) = Z_N(q) + \rho T(q) + \frac{\rho(1-\varpi)\left(\rho+\delta+\pi(1-\delta)\right)}{\varpi(1+\rho)(1-\delta)}T(q) \tag{22}$$

$$\mathcal{L}(\theta) = \frac{(1+\rho)(1-\delta)(1-\eta(\theta))}{\rho+\delta+(\pi+\eta(\theta)\lambda(\theta))(1-\delta)}, \tag{23}$$

and $\eta(\theta) = \theta\gamma'(\theta)/\gamma(\theta)$. The function $\hat{x}$ imputes the user cost of home ownership. Analogous to (11) for rental properties, it includes maintenance expenses, debt service

[29] Its linearity stems from the particular matching function we assume in our calibration below.

[30] A more formal treatment of the household's search problem is provided in Appendix E.

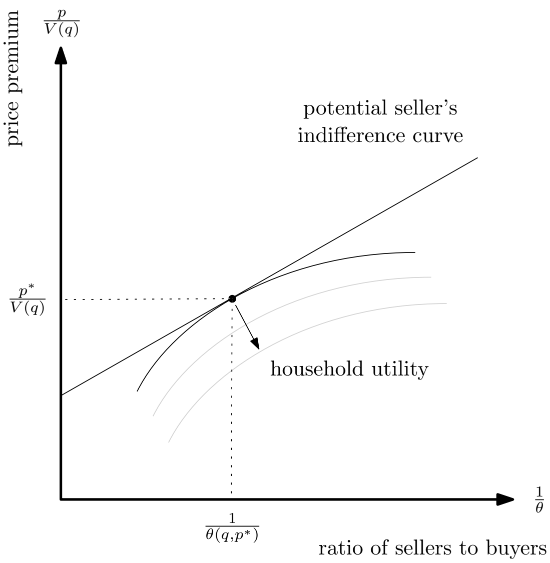


Figure 1: Equilibrium submarket.

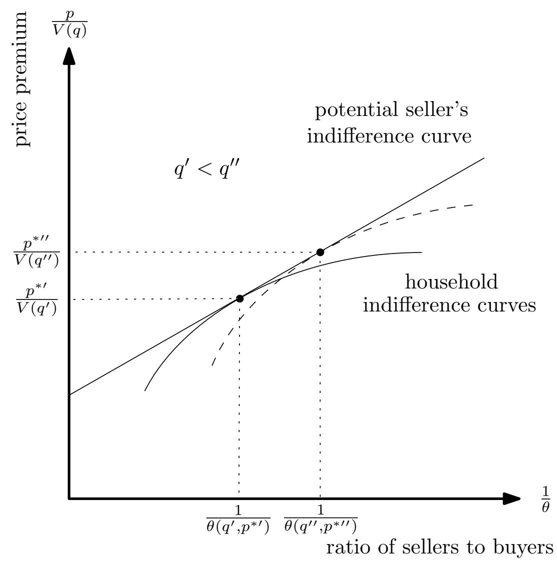


Figure 2: Equilibrium segments.

costs, as well as the anticipated cost of maintaining an unavailable vacancy upon the arrival of a mobility shock. The term in curly brackets on the right-hand side of (21) is therefore the overall flow surplus associated with the change in the household's living arrangements from renting a unit of quality q_R to owning a unit of quality q_S. The term $\mathcal{L}(\theta)$ is the present value factor for this stream of ownership surplus and reflects the frictions in sub-market p of segment q_S.

The potential sellers' indifference condition (12) implies the same positive equilibrium relationship between the price premium, p/V, and the ratio of units for sale relative to potential buyers, $1/\theta$, in every segment. As we move to higher segments (and households with higher income), the rising ownership surplus results in a greater willingness to pay the price premium.[31] This implication is illustrated by the shift in the buyers' indifference curves in Figure 2. In response, more housing units are offered for sale rather than rented causing $1/\theta$ to rise and $\gamma(\theta)$ to fall until the potential sellers' infifference condition is satisfied again. An increase in q is therefore associated with an increase in both the equilibrium price premium and the likelihood of finding a house to own, $\lambda(\theta)$.[32] Even though owning is more desirable, lower income households cannot afford houses in market segments where many are offered for sale and are therefore more likely to rent.

[31]This can be due to either a rising maintenance cost difference or ownership utility premium.
[32]This implication is consistent with the findings of Piazzesi et al. (2020). See below.

Household optimization is summarized by the decision rules, $c(y)$, $q_R(y)$, $q_S(y)$ and $p(y)$. These satisfy (18), (19), (20) and (21), with market tightness, $\theta(y) \equiv \theta(q_S(y), p(y))$, determined by (12).

3.3 An Equilibrium with Balanced Growth

The measure of renters with income level y evolves according to:

$$R_{t+1}(y) = (1 - \lambda(\theta(y)))(1-\delta)R_t(y) + \pi(1-\delta)N_t(y) + \nu f(y) L_t. \tag{24}$$

Here the first term is the measure of unsuccessful, surviving searchers who remain as renters, the second term is that of mismatched surviving owners who enter the renter pool and the last is that of new entrants into the housing market.

Similarly, the measure of owners with income level y evolves according to:

$$N_{t+1}(y) = (1-\pi)(1-\delta)N_t(y) + \lambda(\theta(y))(1-\delta)R_t(y). \tag{25}$$

This consists of surviving owners who remain well-matched and surviving renters who successfully match and buy a home. Dividing all quantities by the population, L_t, and using lower case letters for *per capita* values, the relative measures along a balanced growth path can be expressed as

$$r(y) = \frac{\nu + \pi(1-\delta)}{\nu + (1-\delta)[\pi + \lambda(\theta(y))]} f(y) \tag{26}$$

$$n(y) = \frac{(1-\delta)\lambda(\theta(y))}{\nu + (1-\delta)[\pi + \lambda(\theta(y))]} f(y). \tag{27}$$

Given (26) and (27), the (normalized) stocks of owner-occupied and rental housing of type q are $n(q_S^{-1}(q))$ and $r(q_R^{-1}(q))$, where $q_S^{-1}(q)$ and $q_R^{-1}(q)$ denote the income levels for households that buy and rent in segment q, respectively. Similarly, the (normalized) stock of vacant houses for sale in segment q is $r(q_S^{-1}(q))/\theta(q, p(q_S^{-1}(q)))$. The per capita total stock of housing of each type therefore satisfies:

$$m(q) = n\left(q_S^{-1}(q)\right) + r\left(q_R^{-1}(q)\right) + \frac{r\left(q_S^{-1}(q)\right)}{\theta(q, p(q_S^{-1}(q)))} + s^U(q), \tag{28}$$

Here $s^U(q)$ is the per capita stock of unassigned vacant housing which is given by

$$s^U(q) = \frac{1-\varpi}{\varpi\,(1+\nu-\delta)} \left\{ \begin{array}{l} [\nu + (1-\delta)\pi]\, n(q_S^{-1}(q)) \\ +\, [\nu - \delta(1-\gamma(\theta_S(q)))]\, r(q_S^{-1}(q))/\theta_S(q) \\ +\, [\nu + (1-\delta)\,(\pi_R + (1-\pi_R)\lambda(\theta_R(q)))]\, r(q_R^{-1}(q)) \end{array} \right\}. \tag{29}$$

where

$$\theta_R(q) = \theta(q_S(q_R^{-1}(q)), p(q_R^{-1}(q))) \quad \text{and} \quad \theta_S(q) = \theta(q, p(q_S^{-1}(q))).$$

We then have the following:

Definition 1. *A Directed Search Equilibrium Balanced Growth Path is a list of time-invariant functions of income $y \in \mathbb{R}_+$ and house quality, $q \in \mathbb{R}_+$:*

i. *household values, $W^R(w)$ and $W^N(q,w)$, and decision rules: $w_R(y)$, $w_B(y)$, $w_N(y)$, $w_S(y)$, $w_D(y)$, $c(y)$, $q_R(y)$, $q_S(y)$, and $p(y)$;*

ii. *house values, $V(q)$, and rents, $x(q)$;*

iii. *a function for market tightness, $\theta(q,p)$;*

iv. *shares of households renting and living in owner-occupied housing $r(y)$ and $n(y)$;*

v. *per capita stocks of housing, $m(q)$ and $s^U(q)$;*

such that

1. *W^R and W^S satisfy the household Bellman equations,* (13) *and* (15)*, with the associated policies $q_R(y)$, $q_S(y)$, $p(y)$, $c(y)$, $w_R(y)$, $w_N(y)$, $w_B(y)$, $w_S(y)$ and $w_D(y)$ satisfying* (18)—(21) *and* (A.16)—(A.20)*;*
2. *free entry into new housing construction and rental markets: that is, $V(q)$ and $x(q)$ satisfy* (10) *and* (11)*;*
3. *optimal price posting strategies by sellers of houses in the owner-occupied market: that is, $\theta(q,p)$ satisfies* (12)*;*
4. *the per capita measures of households, $r(y)$ and $n(y)$, satisfy* (26) *and* (27)*;*
5. *the per capita measures of housing, $m(q)$ and $s^U(q)$, satisfy* (28) *and* (29).

4 A Calibrated Economy

We parameterize the model to match several characteristics of the median MSA in a sample of U.S. cities. We then assess the extent to which the predictions of the model are consistent with observations of the within-city distributions of houses and households across rental and owner-occupied markets. In Section 5, we compare the cross-city predictions of the model with the data.

4.1 Baseline Calibration

4.1.1 Functional Forms

We use the following form for preferences:

$$u(c) = (1 - \alpha) \ln c \qquad \text{and} \qquad h(z, q) = \alpha \ln q. \tag{30}$$

This form is consistent with the observation of Davis and Ortalo-Magné (2011) that the share of income allocated to rent is roughly constant across renting households. Note that in (30), the housing services from renting and owning a well-matched house are assumed equal. In this baseline calibration, the benefit to owning a housing unit of a given quality derives from the maintenance cost advantages to doing so. Specifically, we assume that the maintenance costs incurred by owner-occupiers are proportional to q and that those incurred by landlords are greater and rise more than proportionately with q:

$$Z_N(q) = \zeta_0 q \quad \text{and} \quad Z_R(q) = \zeta_0 q + \zeta_1 \left(e^q - 1\right). \tag{31}$$

The exact functional forms assumed allow us to fit ownership rates by income quintile, given the other functional forms assumed (see below). In Section 4.5, we discuss the equivalence of an alternative calibration where maintenance costs are equal across owned and rented units, but the ownership premium varies with q.

The sum of land and construction costs (and the discounted value of property taxes) are assumed to be proportional to house quality:

$$T(q) = \tau q. \tag{32}$$

Note that since q is itself an index, the simple form of (32) is not very restrictive. For example, any monotonically increasing function of q will generate identical results with an appropriate adjustment of the preference specification and the maintenance cost functions. The advantage of this simple specification is that it allows us to interpret τ as the average cost of supplying housing per unit of quality for a given city.

Our matching probabilities correspond to the so-called "telephone line" function derived from a congestion and coordination problem (e.g. Cox and Miller, 1965; Stevens, 2007):

$$\lambda(\theta) = \frac{\chi}{1+\chi\theta} \quad \text{and} \quad \gamma(\theta) = \frac{\chi\theta}{1+\chi\theta} \tag{33}$$

Here $\chi \in (0,1)$ is a matching efficiency parameter reflecting buyers' search intensity. Note that these matching probabilities satisfy Assumption 1.

Households enter the city at age 25 which, given constant entry and exit rates ν and δ, implies a stationary age distribution. We model the log of household income as a deterministic function of a permanent component, β_0, and the age of the householder:

$$\ln Y = \beta_0 + \beta_1 \min\{75, \text{age}\} - \beta_2 \left(\min\{75, \text{age}\}\right)^2, \quad \beta_0 \sim \mathcal{N}(\mu_0, \sigma_0^2). \tag{34}$$

This specification is a simplified version of the type of life-cycle income models commonly estimated (Feiveson and Sabelhaus, 2019). It allows for a hump-shaped age profile which we truncate beyond age 75.[33] For a given age, log income is therefore normally distributed with mean $\mu_0 + \beta_1 \min\{75, \text{age}\} - \beta_2 \left(\min\{75, \text{age}\}\right)^2$ and variance σ_0^2. The unconditional income distribution is therefore a mixture of normal distributions, weighted by the probability mass of household age. For our calibrated economy, this mixture distribution of log income for households is well-approximated by a normal distribution, and hence the annual income distribution is approximately log-normal:

$$Y \mathrel{\dot\sim} \log\mathcal{N}(\mu, \sigma^2). \tag{35}$$

This implies that incomes are bounded below by zero and that their approximate distribution can be summarized by its first two moments. It also implies a one-to-one relationship between the standard deviation and the implied Gini coefficient.[34]

A household's optimal decisions depend on permanent rather than annual income. We therefore compute, for each β_0, the quarterly cash flow of a perpetuity with a

[33] Since the death rate is constant, the age distribution is not very realistic, with the possibility that some householders live forever. However, with ongoing entry, only a very small fraction of the population lives beyond 90.

[34] Below we use observed Gini coefficients computed for MSA's by the U.S. Census Bureau.

present discounted value equal to that of their quarterly income stream from age 25 onward:

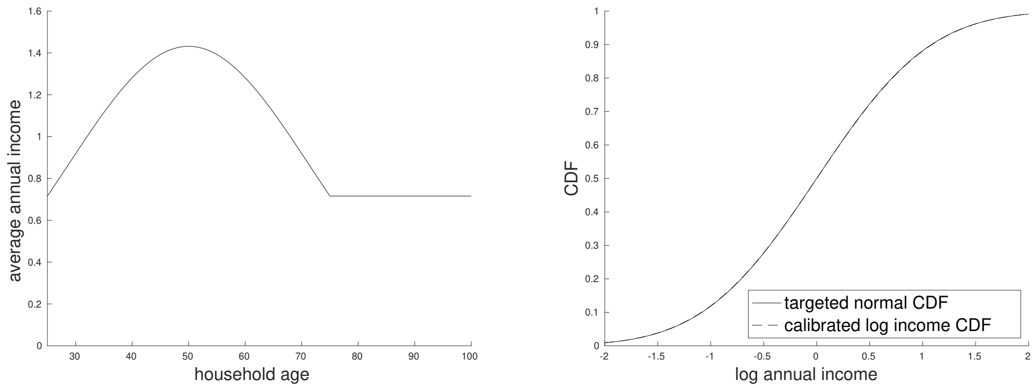
$$\begin{aligned}\frac{y}{1-\beta} = &\sum_{\tau=0}^{199} \frac{\beta^{\tau}}{4} \exp\left(\beta_0 + \beta_1 \left(25 + \frac{\tau}{4}\right) - \beta_2 \left(25 + \frac{\tau}{4}\right)^2\right) \\ &+ \frac{\beta^{200}}{4} \frac{\exp\left(\beta_0 + \beta_1 75 - \beta_2 75^2\right)}{1-\beta}.\end{aligned} \tag{36}$$

The relationship between permanent income y and actual income Y arising from the age-income profile in equation (34) allows us to target moments in the data that are calculated from actual household income despite the formalization of the model in terms of permanent income.

4.1.2 Parameterization

Table 1 contains calibrated parameter values, together with the economy statistics with which each is most closely associated. The entry and exit rates, ν and δ, are jointly chosen to achieve a steady state population growth rate of 1 percent annually and a median household age equal to the average across MSA's. In line with typical estimates of the age-earnings profile (e.g., Feiveson and Sabelhaus, 2019), we set β_1 and β_2 so that the income process in equation (34) implies that maximum income is earned at age 50, and that income doubles from age 25 to age 50. The age-income profile of the calibrated economy is plotted in Figure 3a. The log-normal distribution of the permanent component of log quarterly income in equation (34) is calibrated based on the implied distribution of annual income. Specifically, the mean and standard deviation of the permanent component of log income, μ_0 and σ_0, are chosen so that the distribution of annual income closely approximates a log-normal distribution with a median normalized to unity and a Gini coefficient that corresponds to the average across MSA's. As shown in Figure 3b, the calibrated distribution of annual household income closely approximates the targeted log-normal distribution.

The parameter ϖ, which captures the frictions governing the availability of vacant housing, is set to achieve an overall vacancy rate equal to the average across MSA's. The matching parameter, χ, is set to generate an average time to sell of four months (in addition to the stochastic period of unavailability), which corresponds to the simple average across MSA's of "days on Zillow" in 2010. The moving probabilities for owners and renters, π and π_R, are chosen so that the annual household turnover



(a) Household age-income profile.

(b) Distribution of household income.

Figure 3: Household income.

rate is 15% (Bachmann and Cooper, 2014) and the *vacancy ratio*, defined as

$$\text{vacancy ratio} = \frac{\text{(vacancies for sale or sold) / (owner-occupied homes)}}{\text{(total vacancies) / (rental and owner-occupied homes)}},$$

corresponds to the average across MSA's.

Given these parameters, the preference parameter, α, is set so that the model generates a mean rent to median income ratio equal to the corresponding average across MSA's. The parameter ζ_0 in the maintenance cost functions was chosen to match the average city-level price-rent ratio. For these price-rent ratio calculations, we divide the mean price-asked for each MSA by the mean annual rent. The remaining maintenance parameter, ζ_1, was chosen to minimize the sum of the squared differences between the equilibrium ownership rates by income quintile and those reported in Table 1 which are computed using census summary tables for all U.S. households in 2010.[35] Figure 4 depicts the implied maintenance cost by market segment for both rentals and owner-occupied houses.

[35]See https://www.census.gov/data/tables/time-series/demo/income-poverty/cps-hinc/hinc-05.2010.html. For the U.S. economy as a whole, ownership rates for the bottom quintiles are likely higher than for households residing in MSA's. A disproportionate fraction of lower-income households live outside MSA's and own at a higher rate than those in MSA's.

Table 1: Calibrated Parameters

statistic	target value	model value	calibrated parameter	parameter value
median household age	49.75	49.750	ν	0.0069
population growth rate (%)	1	1.0000	δ	0.0044
peak income age	50	50.000	β_1	0.1109
income at age 50 / income at age 25	2	2.0000	β_2	0.0011
median annual income	1	1.0000	μ_0	-2.4148
Gini coefficient for income	0.45	0.4500	σ_0	0.8093
annual interest rate (%)	4	4.0000	ρ	0.0099
overall vacancy rate (%)	11	11.461	ϖ	0.3125
vacancy ratio ($\times$ 100)	26	26.169	π	0.0191
annual turnover rate (%)	15	14.667	π_R	0.0374
average time to sell (quarters)	1.33	1.3003	χ	0.4254
ratio of mean rent to median income	0.19	0.1902	α	0.2067
average price-rent ratio	21	21.004	ζ_0	0.0125
normalization			τ	1.0000
average ownership rate, Q1 of income dist. (%)	44	43.99		
average ownership rate, Q2 of income dist. (%)	56	56.38		
average ownership rate, Q3 of income dist. (%)	67	66.67	ζ_1	0.00006
average ownership rate, Q4 of income dist. (%)	77	77.04		
average ownership rate, Q5 of income dist. (%)	87	87.08		

4.2 The City in Equilibrium

Figure 5 illustrates the housing decisions of households by permanent income (i.e., income y, calculated according to equation (36)). Clearly, quality is strictly increasing in permanent income for both rental (q_R) and owner-occupied (q_S) housing. At low permanent incomes, households search to own houses of comparable quality to those they rent. At permanent income levels above the median, however, households search for houses to buy of notably higher quality than they rent. This reflects the fact that higher quality houses are relatively expensive to rent, given their high relative maintenance costs.

Figure 6 plots home-ownership by log annual income. Annual income varies with age as per equation (34), and ownership rates vary by age because households enter

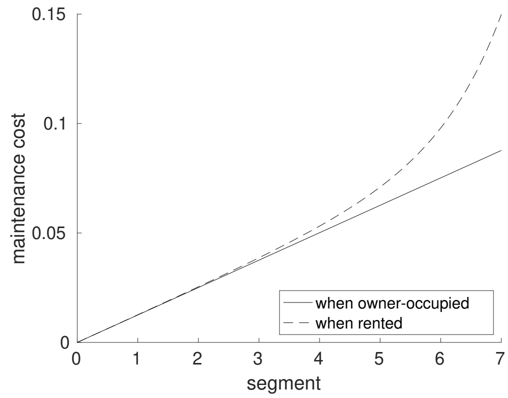


Figure 4: Maintenance costs by housing segment.

the city as renters at age 25 and it takes time to find a suitable house to buy in the owner-occupied market. The dashed and dotted lines plot the relationship between income and ownership for households aged 25.5 and 75, whereas the solid line is for the household of median age. The supply of owner-occupied housing to the very poorest households is low: these households choose to search in sub-markets with low prices and hence low matching rates. Beyond a point, however, home-ownership rises rapidly and then flattens out at high incomes. Intuitively, an increasing ownership rate manifests because the relatively high cost of maintaining higher quality rental houses translates into higher equilibrium rents. High income households seek high quality homes and thus target high price sub-markets where buying probabilities are higher.

Figure 7 plots the relative cost of owning *vs.* renting by household permanent annual income. For a household with a given permanent income, the figure compares the price of the house for which they search to buy to their annual rental cost while searching. The relationship in the figure reflects the differences in households' choice of house quality when renting versus owning (see Figure 5). Due to the scarcity of high quality rentals that results from their high rental costs, higher income households search to buy houses of much higher quality than they rent while searching. At levels of income below the median, the effect is small, even negligible, reflecting the relatively small gap in maintenance costs between rented and owner-occupied homes of a given quality.

The rapid rise in relative maintenance costs as quality increases results in the relationship depicted in Figure 8, which plots the price-rent ratio by market segment. These maintenance costs (which depend on an interaction between occupant household's tenure and the physical characteristics of the house) may be seen as an example of "unobservable costs of renting" in the language of Halket et al. (2020) who, as noted above, observe a price-rent ratio declining in house quality. Figure 8 also plots the ratio of the price to rent net of maintenance cost by market segment, which is rising slightly with quality.

The price-rent ratio for a house of a given quality is low, yet the calibrated model delivers an aggregate price-rent ratio of 21. This reflects households' heterogeneous search strategies on the demand side of the owner-occupied market, as well as the price incentives facing owners and developers to supply owner-occupied and rental housing. Figure 9 displays the histogram of housing market transactions by income for both sales and rentals. High income households are responsible for a large share of transactions in the owner-occupied market, whereas the opposite is true for the rental market. Both of these contribute to a high average price to average rent ratio.[36] The distributions of transactions by income play a crucial role in the calculation of aggregate statistics and are important determinants of the cross-city implications derived below.

4.3 Non-targeted implications

Table 2 compares the distribution of buyer matching rates implied by the model baseline with that estimated by Piazzesi et al. (2020) in their detailed study of segmented search behaviour in the San Francisco Bay Area during 2008-11. Their estimates are based on online search behaviour and can only pin down the relative buyer matching rates by segment.[37] Moreover, the buyer matching rates by segment in our baseline calibration are for the median U.S. city, so their levels are not directly comparable with those for San Francisco. Table 2 therefore reports the implied buyer matching rates for each quartile of the distribution *relative* to the mean in each case.[38]

Although these were not targets of the calibration, the overall dispersion in the matching rates are comparable. The rates for the bottom two quartiles relative to the means are very similar, while that for the top quartile is somewhat higher in the model. The dispersion in buyer matching rates is determined to a large extent by the

[36] The ratio of two weighted averages can be a lot higher than the average of the underlying ratios if the weights are inversely related.

[37] Piazzesi et al. (2020) compute an estimate of the mean separately from this estimation.

[38] We converted the monthly estimates documented by Piazzesi et al. (2020) into quarterly ones using $\lambda_q = 1 - (1 - \lambda_m)^3$.

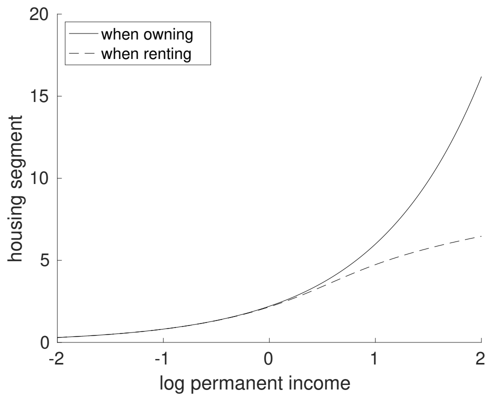


Figure 5: Market segment *vs.* income by housing tenure.

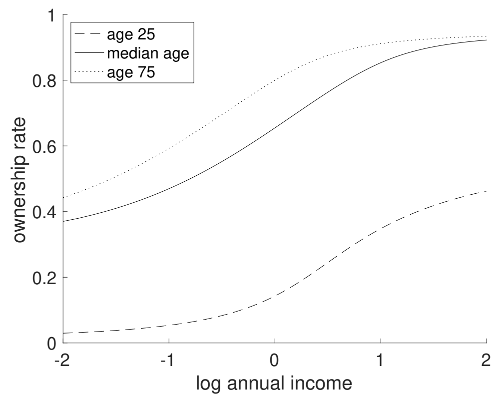


Figure 6: Ownership rate *vs.* household income.

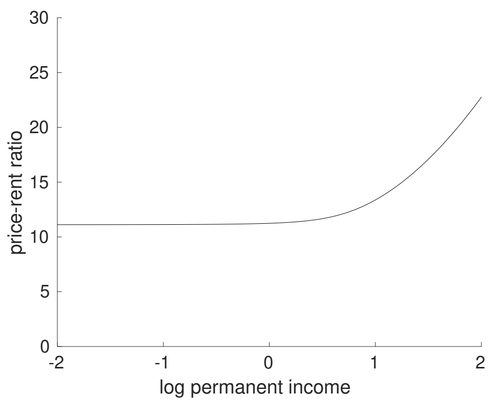


Figure 7: Price-rent ratio by household income.

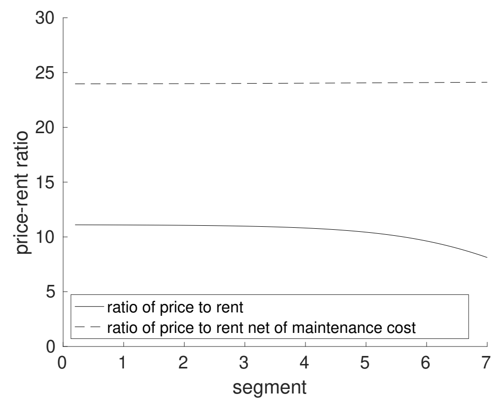


Figure 8: Price-rent ratio by housing segment.

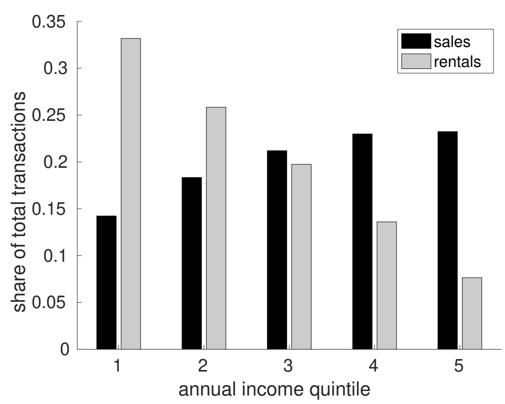


Figure 9: Histograms of house purchases and rentals by income.

Table 2: Buyer matching rates relative to mean

Quartile	Model Baseline	Piazzesi et al. (2020)
Q25	0.40	0.42
Q50	0.77	0.73
Q75	1.41	1.16

calibrated dispersion in ownership rates.

In Table 3 we compare ownership rates by age range and income quintile implied by the model with corresponding estimates from the data. Again, while these were not targets of the calibration, the patterns of ownership are similar. Notably, the young in our model own at a higher rate than in the data and ownership among the very old declines a little in the data but not in the model.

Table 3: Home-Ownership by Age and Income

	Model					Data*				
Income quintile:	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
age 25 to 34	23	37	51	67	82	18	30	44	57	67
age 35 to 44	43	55	66	77	88	31	50	67	80	88
age 45 to 54	46	56	66	76	87	42	62	76	87	94
age 55 to 64	48	59	68	78	88	55	72	84	89	95
age 65 to 74	53	66	75	83	90	59	79	87	90	95
age $\geq$ 75	56	71	80	87	91	65	71	81	89	92

* *Authors' estimates based on the 2011 American Household Survey.*

Our calibration implies an average tenure spell for owners of almost 11 years. Conditional on remaining in the city until the next move, the average tenure spell for owners is 13 years, which is considerably shorter than that implied by the transition probabilities estimated by Bachmann and Cooper (2014). The conditional average rental spell in the calibrated model is almost 7 years, which is again less than the 9 and a half year average implied by Bachmann and Cooper's (2014) estimates. In Section 7.1, we consider a more complicated set of assumptions that result in mis-matched owners remaining in their homes while searching. When re-calibrated to match the targets in Table 1, these implied tenure spells and the cross-city predictions of the model described below remain very similar.

4.4 Comparisons across equilibrium balanced-growth paths

We now consider the effects of changes in median income, income inequality (*i.e.* the Gini coefficient), median age and land/construction costs on home-ownership, the relative cost of owner-occupied *vs.* rental housing (measured by the ratio of average prices to average rents) and the vacancy ratio along the balanced growth path. In Figures 10, 11 and 12 the ranges depicted on the horizontal axes reflect the relevant ones in the data across MSA's. Note that if relative maintenance costs (and therefore the ownership surplus) were independent of quality, home-ownership, the ratio of average prices to average rents and the vacancy ratio would not vary at all.

4.4.1 Median income

We vary incomes holding the distribution of ages constant, so this corresponds to variation in the permanent component. As city median income increases, holding inequality constant, the distribution of housing quality desired by households shifts up. Since the ownership surplus rises with quality, the distribution of price premia that households are willing to pay to own rather than rent and the corresponding equilibrium distribution of the ratio of houses for sale to potential buyers shift up. It follows that buyer matching probabilities increase and, consequently, so does the rate of home-ownership (see Figure 10a).

In general, the implication of rising city median income for the ratio of average prices to average rents is ambiguous. As households demand higher quality housing both the average purchase price and average rents rise. Which increase is larger depends on the distribution of the home-ownership rate by income. This depends on the relationship between matching probabilities and income. Under our calibration, the ratio of average prices to average rents rises with median income over the empirically relevant range.

While the effect of rising median income for the aggregate vacancy ratio is also ambiguous in general, it rises under our calibration. As households demand higher quality housing, the surplus associated with ownership rises. Consequently, more houses are offered for sale, driving up the vacancy rate for owned housing relative to the overall vacancy rate.

4.4.2 Inequality

As inequality increases, holding median income constant, the quality of housing desired by relatively high income households increases while that desired by relatively low income households declines. Since the relative costs of renting rise sharply for increases in quality at the upper end, but fall only minimally as quality declines at the lower end, the fraction of high quality houses supplied to the owner-occupied market rises while that of low quality houses falls.

The impact on the aggregate home-ownership rate is in general ambiguous and again depends on the relationship between ownership rates and household income. In our calibration, the relative costs of owning and renting imply a concave relationship over most of the income distribution (see Figure 6). Consequently (see Figure 10b), home-ownership falls with inequality as measured by the income Gini coefficient.

At the same time, increased income inequality *raises* the ratio of average prices to average rents through a composition effect. Having more low-income households results in the construction of more low quality houses. While this lowers *both* prices and rents, the effect on the former is mitigated by the fact that low income households buy houses at a low rate. Similarly, having more high-income households results in more high quality houses being built and drives *up* both prices and rents. In this case, however, the effect on the *latter* is minor as high-income households rarely rent. Overall, as shown in Figure 11b the increase in the purchase prices of high quality homes and the reduction in rents together result in an increase in the ratio of average prices to average rents.

The impact on the aggregate vacancy ratio reflects similar forces. Due to the housing supply response, the measure of houses that are vacant-for-sale rises at the top end and falls at the bottom end. Since the ratio of houses for sale to potential buyers increases with quality in our calibration, the increase in the measure of high-quality houses that are vacant-for-sale outweighs the decrease in the measure of low-quality houses that are vacant-for-sale. The vacancy rate in the owner-occupied market therefore increases, and so too does the overall ratio of vacancy rates.

4.4.3 Median age

We focus on the impact of variation in the entry rate, ν as the main source of variation in median age (and steady state population growth). Variation in the exit rate, δ, within a range that ensures non-negative population growth does not affect median age much in our model. Figures 10c, 11c and 12c depict the relationships between median age and home-ownership, the ratio of average prices to average rents and the vacancy ratio, respectively, resulting from variation in ν.

Ownership increases monotonically with median age. An older city has a lower rate of entry, and as such a smaller fraction of the population renting while searching for an initial house to own. This accounts directly for the effect of age on ownership. The impact of median age on the price-rent ratio is small and, while for this calibration it is non-monotonic, for others it can be positive or negative. The vacancy ratio falls with median age simply because with relatively fewer households searching for owned housing, a lower share of vacant housing is assigned to that market.

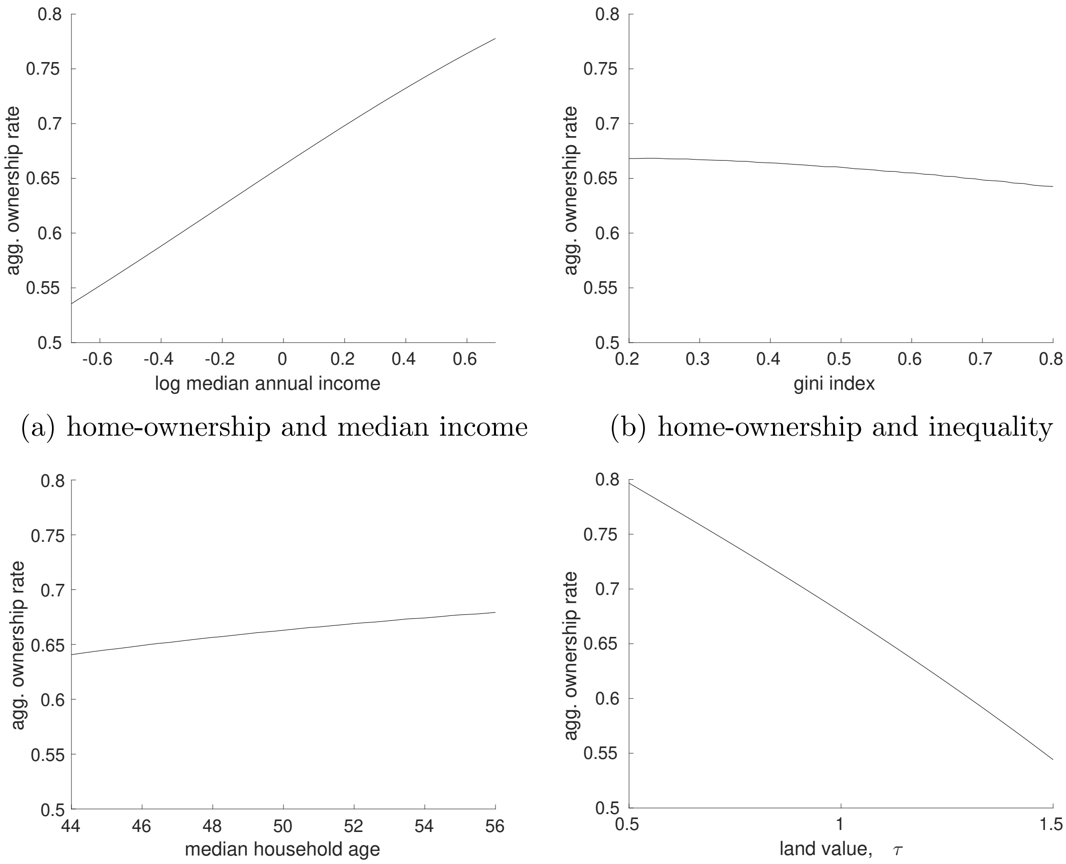
4.4.4 Construction costs and city-wide amenities

The parameter τ represents the cost of building per unit of housing quality. As such, it reflects city-wide amenities (*e.g.* climate) and costs (*e.g.* regulatory hurdles) as well as the choices of developers (*e.g.* land, size, construction materials, etc.). Variations in τ capture all the costs of providing housing that are *independent* of whether the occupying household owns or rents.

An increase in τ translates into a proportional increase in the value of vacant housing required to induce competitive developers to supply new housing of any quality. The resulting rise in both rents and purchase prices causes households at every given income level to choose lower quality housing which, in turn, reduces the relative costs of renting. Consequently, as shown in Figure 10d, the aggregate ownership rate declines as τ increases.

In general, the impact of an increase in τ on the ratio of average prices to average rents is more ambiguous. While the purchase prices and rents paid for houses of a given quality rise, this is largely offset at the household level by the reduced quality of such houses. A more important factor determining the relative impact on average prices and average rents is the implied distribution of changes in the ownership rate across income levels. This, in turn, depends on the shift in the steady state mappings between income and matching probabilities.

As may be seen in Figure 11d, our calibration implies the ratio of average prices to average rents increases with τ. This results from the combination of two effects. First, overall the quality distribution shifts to the left as houses become less affordable. This



(a) home-ownership and median income

(b) home-ownership and inequality

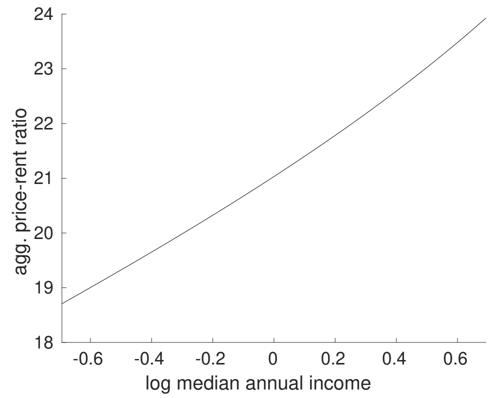
(c) home-ownership and median age

(d) home-ownership and construction costs

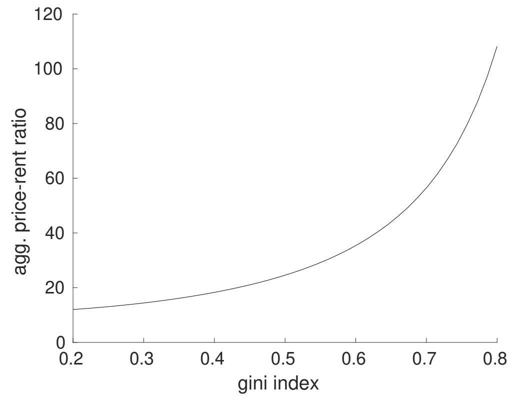
Figure 10: Cross-city relationships between home-ownership and fundamentals

results in a general reduction of relative rental costs, and hence an increase in the price-rent ratios, segment by segment (see Figure 8). The second, and more significant effect, comes from the fact that any reduction in home-ownership is overwhelmingly concentrated amongst low-income households. High-income ones continue to own, paying relatively high prices simply because they live in high quality houses.

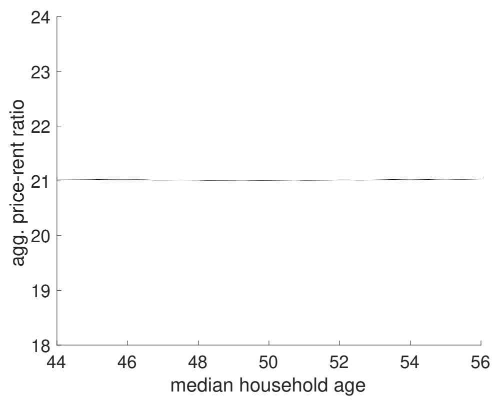
In general, variation in housing supply costs per unit of quality has an ambiguous impact on the vacancy ratio. For the range in which most cities fall (according to our estimates below), however, the impact is negative.



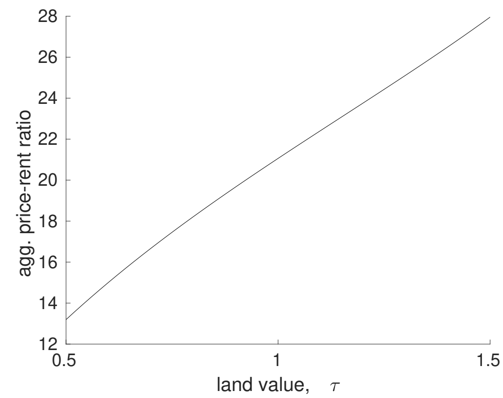
(a) price-rent ratio and median income



(b) price-rent ratio and inequality



(c) price-rent ratio and median age



(d) price-rent ratio and construction costs

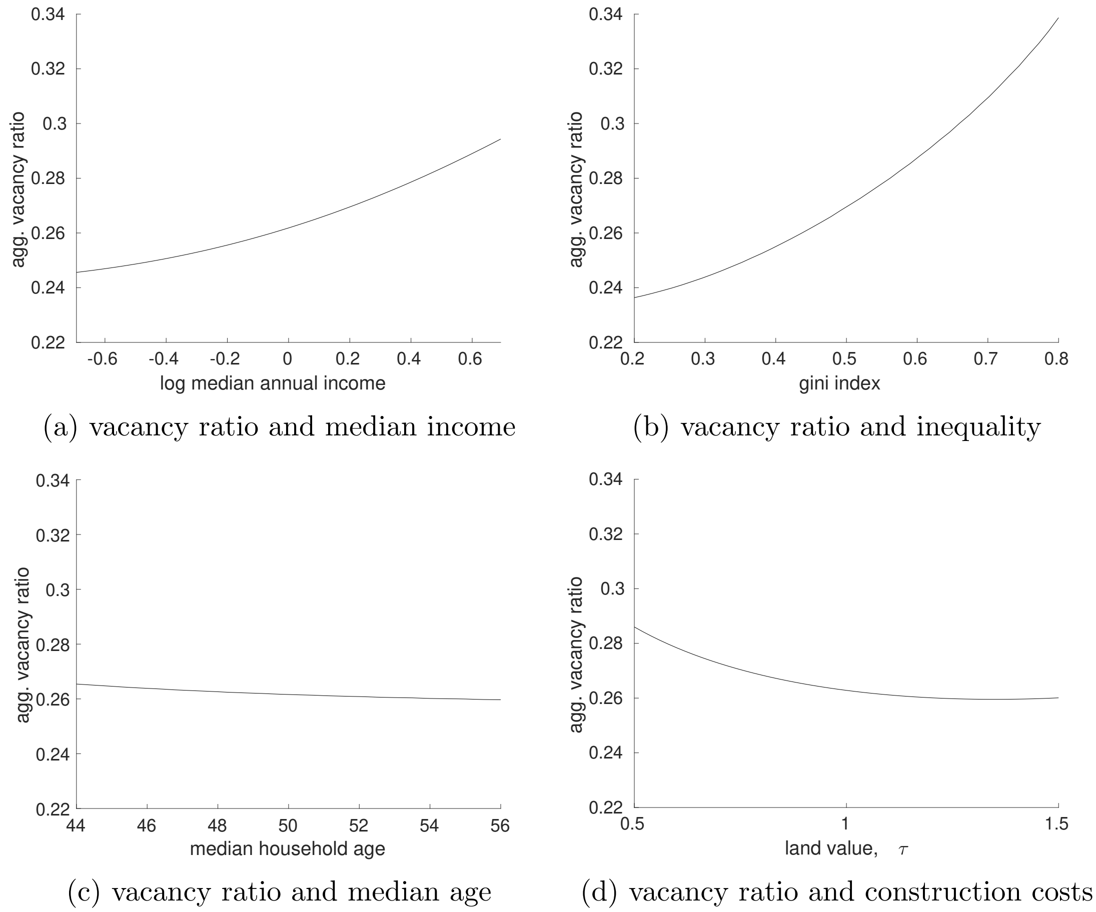
Figure 11: Cross-city relationships between the price-rent ratio and fundamentals

4.5 An Alternative Calibration

In our baseline calibration, the net surplus associated with home-ownership increases with quality due to the specification of maintenance costs. Here we describe briefly an alternative calibration of the baseline model with an explicit utility benefit to home ownership that rises with quality.[39] Specifically, we assume that the utility from housing services is given by:

$$h(z,q) = \alpha(\ln q + zg(q)), \tag{37}$$

[39]That owners derive greater utility than renters from a given dwelling is a common assumption (Rosen, 1985; Poterba, 1992; Kiyotaki et al., 2011; Iacoviello and Pavan, 2013; Floetotto et al., 2016).



(a) vacancy ratio and median income

(b) vacancy ratio and inequality

(c) vacancy ratio and median age

(d) vacancy ratio and construction costs

Figure 12: Cross-city relationships between the vacancy ratio and fundamentals

where the ownership premium is

$$g(q) = \psi_0 + \frac{\psi_1 q}{1 + e^{\psi_2 - \psi_3 q}} \qquad q > 0. \tag{38}$$

Maintenance costs are now assumed to be equal across owned and rented units of a given quality:

$$Z_N(q) = Z_R(q) = \zeta q. \tag{39}$$

When this version of the model is calibrated to match the same targets as above, we find that the implications are qualitatively unchanged and quantitatively very similar.[40] The only substantive difference is that the price-rent ratio for equivalent

[40]These results are available upon request.

housing units now rises with quality, in contrast to the negative relationship depicted in Figure 8.[41] Empirical studies of the relationship between price-rent ratios and housing quality find mixed results. A number of studies, using various approaches and in different locations, have suggested an increasing price-rent ratio by quality.[42] Using data from London, Halket et al. (2020), however, attribute much of this to the effects of selection on unobservable quality. Correcting for this bias, they find that the price-rent ratio falls with value.

The equivalence between these alternative calibrations for our main results and the additional implications reported below, demonstrate a more general observation. There are many other possible differences in the benefits and costs of owning versus renting. These include transactions costs of buying that are unrelated to liquidity, differential income tax treatment (*e.g.* mortgage interest deductibility), differences in property tax implications, etc. If we were to incorporate all of these into the model and then re-calibrate preference and/or maintenance cost parameters to target the distribution of ownership by income as above, the main results would not change.

5 Cross-City Variation in the Data and the Theory

We now document observed variation across U.S. cities with regard to home-ownership, the relative costs of owning and renting (price-rent ratios) and the relative liquidity of owned vs. rental markets (the vacancy ratio). We consider the extent to which these are associated with variation in median incomes, inequality, median age and land costs, controlling for other factors affecting the desirability of living in different cities. We then compare the corresponding variation generated by the model to these characteristics of the data. In principle, we could consider relationships between these variables across various "collections of households". We focus on MSAs (rather than, say, states or counties) largely because of the availability of the necessary combination of observables.

[41] Note that a positive relationship also arises in the baseline calibration when we replace the rent with the rent net of maintenance costs (see Figure 8).

[42] See, for example, Verbrugge (2008), Heston and Nakamura (2009), Garner and Verbrugge (2019),Verbrugge and Poole (2010), Epple et al. (2020), Landvoigt et al. (2015), Bracke (2015) and Hill and Syed (2016).

5.1 Variation Among a Sample of U.S. Cities

Our base sample consists of the 366 primary MSA's from the 2010 American Community Survey (ACS), 5-year estimates.[43] For all of these MSA's ownership rates, average price-rent ratios, vacancy rates, and Gini coefficients are computed directly from the ACS. Median nominal incomes were also obtained from the ACS. To put these in real terms, we then divided by a local cost of living index (COLI) for non-housing expenditures.[44]

For MSAs the ACS reports the median age of the population. While it might be more appropriate to use the median age of the "householder" (i.e. the member who completed the survey), this does not appear to be reported. Instead, only a coarse age distribution for householders by MSA can be obtained. A problem could potentially arise if variation in the median age of the population were largely driven by variation in the number of children rather than the age of householders (these are, of course, likely to be correlated). In our regressions, we also include the share of the adult population with their own children under 18 ("Kids"). Thus, the coefficient on "Age" should reflect the impact of the variation in median age that is orthogonal to the fraction of children in the population.

We take the view that land values are the main source of variation in overall construction costs across cities. Land values are taken from Albouy et al. (2018) who computed them at the MSA level using land transactions data, adjusted for geographic selection in location and limited sample sizes. Their calculations are based on the 1999 OMB definitions of MSA's. There is not an exact match between MSA's in the two samples for several reasons. For example, there were several new primary MSA's in the 2010 census resulting from population growth, some MSA's were subdivided and others experienced name changes. We matched the MSA's as closely as possible and, where they were subdivided, applied the same land values to each.

To isolate the role of costs per unit of housing quality we control for "natural" amenities taken from Albouy (2016). This data is also based on the 1999 OMB definitions of MSA's. Moreover, in this case some of the observations are for consolidated MSA's, which combine congruent primary MSA's. In order to maximize our sample size, we use the amenity controls computed for the consolidated MSA's to approximate those for each constituent primary MSA.

[43]An MSA is an urban agglomeration containing at least 50,000 households. These 366 MSA's contain over 83% of U.S. households.

[44]See Appendix B.1 for details.

The COLIs, land values and amenity controls are not available for every MSA in the ACS and, as a result, the sample size used in our empirical analysis is reduced to 200 MSA's. As documented in Appendix C, this sub-sample of cities is representative of the full sample.

Table 4: U.S. MSAs (2006-10): OLS Estimates

	Ownership Rate	**Price-Rent ratio**		**Vacancy ratio**
		Est. value	Price-asked	
Income	1.163	1.461	1.389	2.546
Robust	(0.400***)	(0.397***)	(0.636**)	(0.807***)
Clustered	(0.541**)	(0.547***)	(0.647**)	(0.786***)
Gini	-0.439	0.474	0.548	0.013
Robust	(0.118***)	(0.149***)	(0.243**)	(0.240)
Clustered	(0.111***)	(0.182**)	(0.259**)	(0.278)
Age	1.055	0.080	-0.009	-0.805
Robust	(0.067***)	(0.064)	(0.128)	(0.259***)
Clustered	(0.075***)	(0.066)	(0.110)	(0.276***)
Kids	0.383	-0.042	-0.044	-0.292
Robust	(0.063***)	(0.052)	(0.098)	(0.212)
Clustered	(0.059***)	(0.043)	(0.096)	(0.227)
Land Price	-0.953	1.049	0.479	1.102
Robust	(0.178***)	(0.192***)	(0.290*)	(0.305***)
Clustered	(0.163***)	(0.326***)	(0.433)	(0.303***)
Controls	Yes	Yes	Yes	Yes
$\mathbf{R}^2$	0.70	0.75	0.47	0.34
# obs	200	200	200	200

Notes:
(1) Both heteroskedasticity robust standard errors and those clustered at the state level are shown in parenthesis.
(2) ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

In the first column of Table 4, the dependent variable is the ownership rate by MSA, computed as the ratio of owner-occupied units to total (owner and renter) occupied units. This regression, which also controls for amenities, accounts for 70% of the variation in the ownership rate across cities.[45] As predicted by the model, the ownership rate is positively associated with median income and age and negatively associated with inequality and land price per acre. The share of the adult population

[45]Estimated coefficients for the additional controls are provided in Appendix C.

with children ("Kids") is also significantly associated with average ownership rates across cities. Whether or not this variable is included does not change the basic associations with the other core variables but does substantially add to the explanatory power of the regression. This likely reflects its independent importance in determining cross-city variation in home-ownership rates, over and above its correlation with median age (see Table 15 in Appendix C).

For the second and third columns of Table 4, the dependent variable is the ratio of average prices to average rents by MSA. In all cases, *rent* refers to the mean gross rent of renter-occupied housing units. In the second column *price* refers to the mean estimated value of all owner-occupied housing units whereas in the third, it refers to the mean price asked for vacant units for sale. The regressions, which also control for city-wide amenities, account for 75% and 47%, respectively, of the variation in these measures of the relative average costs of owning and renting across cities. Again, as predicted by the model, the estimates imply that average price-rent ratios are positively and significantly associated with median income, inequality and land values. The effect of median age in each case is small and less robust.[46]

For the fourth column, the dependent variable is the vacancy ratio: the vacancy rate for housing which is for sale or sold relative to the overall vacancy rate. This regression accounts for substantially less of the variation in the dependent variable, reflecting a much noisier relationship. As predicted by the model, median income exhibits a statistically significant, positive association with the vacancy ratio, whereas median age exhibits a negative one. However, the Gini index appears unrelated to the vacancy ratio and the land price is positively associated. As noted above, in the model this latter relationship is ambiguous in general but negative over the relevant range for our baseline calibration.

Overall, these results conform well qualitatively with most of the cross-city predictions of our model. While these OLS estimates cannot be interpreted as causal, they do not immediately reject the qualitative predictions of the model. Although we do not place great emphasis on the exact values of the estimated coefficients from these regressions, it is conceivable that some sources of simultaneity could be sufficiently severe that they would reverse the implied signs.

Of particular concern is the role of land prices, which we have treated as being exogenous in our model, but are undoubtedly endogenously determined in the land market. From the perspective of the OLS regressions above, the question arises as

[46]Similarly, the share of households with children is not statistically significant in accounting for the price-rent ratio or the vacancy ratio.

to whether or not accounting for this endogeneity results in biased and inconsistent estimates. For example, if higher ownership rates were to "cause" lower land prices, this might lead the negative coefficient on land prices in the ownership OLS regression to be biased upwards in absolute terms.[47] Moreover, if cross-city variation in income, say, also significantly affects land prices this reverse causation could induce the coefficient on income to be biased upwards too.

We explore this possibility further using an instrumental variables approach. We consider two potential instruments for average MSA land prices per acre. The first is population-weighted density, which is a weighted average of population density by census tract where the weights are the share of the MSA population in each census tract. The second instrument we consider is an index of the average slope of the land in the metropolitan area. This is an important factor determining the costs of converting land to residential use (Saiz, 2010).

These variables are statistically significant drivers of real land prices, after controlling for the other explanatory variables. Both factors reduce the elasticity of new land supply and so, other things equal, imply a higher equilibrium land price. The key exclusion assumption we make is that their correlation with the dependent variables is entirely due to this impact. While one could argue that density and slope might directly affect the relative demand for and supply of owned housing in other ways, we conjecture that, in comparison, these impacts would be negligible on average.

Table 5 documents IV results for home-ownership. The first and second stage estimates are provided, first using weighted density, then average slope and finally both variables as instruments. As may be see, the general pattern in each case is the same: in the main (second-stage) regression, the coefficients on both the land price and real income *increase* substantially in absolute value while other coefficients do not change much.[48] The statistical significance of the robust score test (Wooldridge, 1995) performed in each case suggest that we do need to account for the endogeneity of land prices in determining home-ownership.

While we cannot test the exclusion assumption directly, the fact that we have two potential instruments allows us to perform an over-identification test, which is shown at the bottom of the last column. The statistical insignificance of this test implies that we cannot reject the validity of these instruments. We conclude that this 2SLS regression provides consistent coefficient estimates of the impact of the core variables

[47] In fact, as we show below, the opposite is true.

[48] Note that dropping real income from these regressions does not change the implications for the impact of land price.

Table 5: U.S. MSAs: IV Results for Home-ownership

Instruments:	**Weighted Density**		**Average Slope**		**Density & slope**	
Stage:	**Main (2SLS)**	**First stage**	**Main (2SLS)**	**First stage**	**Main (2SLS)**	**First stage**
Income	2.000	0.625	2.310	0.886	2.034	0.701
Robust	(0.376***)	(0.167***)	(0.817***)	(0.182)	(0.384***)	(0.161***)
Clustered	(0.526***)	(0.182***)	(0.813***)	(0.192***	(0.522***)	(0.178***)
Gini	-0.312	-0.028	-0.265	0.116	-0.307	-0.037
Robust	(0.110***)	(0.056)	(0.162*)	(0.084)	(0.111***)	(0.054)
Clustered	(0.099***)	(0.052)	(0.151*)	(0.085)	(0.100***)	(0.051)
Age	1.103	0.049	1.121	0.038	1.105	0.039
Robust	(0.074***)	(0.027*)	(0.089***)	(0.030)	(0.075***)	(0.028)
Clustered	(0.087***)	(0.024**)	(0.089***)	(0.027)	(0.087***)	(0.024)
Kids	0.352	-0.052	0.341	-0.028	0.351	-0.051
Robust	(0.076***)	(0.028*)	(0.094***)	(0.031)	(0.077***)	(0.026**)
Clustered	(0.069***)	(0.027*)	(0.077***)	(0.026)	(0.069***)	(0.024**)
Land Price	-1.980		-2.361		-2.022	
Robust	(0.220***)		(0.904***)		(0.236***)	
Clustered	(0.193***)		(0.866***)		(0.222***)	
Density		0.031				0.031
Robust		(0.007***)				(0.007***)
Clustered		(0.008***)				(0.008***)
Slope				0.129		0.142
Robust				(0.065**)		(0.056**)
Clustered				(0.065**)		(0.051***)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
$\mathbf{R^2}$	0.63	0.65	0.58	0.53	0.63	0.67
# obs	200	200	200	200	200	200
Endog. test	9.588 [P=0.002]		3.372 [P=0.066]		12.432 [P=0.002]	
Over ID test					0.200 [P=0.655]	

on home-ownership, allowing for the endogeneity of land prices.

We also estimated the other three regressions using weighted density as an instrumental variable for land price (see Tables 12 and 13 in Appendix B.2). Doing so did not significantly change the coefficient estimates relative to the OLS estimates. Moreover, the statistical insignificance of the endogeneity tests does not support the need to allow for the endogeneity of land prices when estimating these relationships. Thus, while the share of homes owned has a causal impact on land prices, this does not appear to be the case for the relative composition or the liquidity of owned versus rented housing.

In Appendix C we consider the robustness of these relationships to the inclusion of a number of additional variables that could also be associated with home-ownership and the relative costs of owning and renting across cities. These include the share of the adult population with a college degree, employment rates, debt-to-income ratios, land-use regulation and property taxes. Note finally, that the period on which our baseline estimates are based (2006-10), was chosen to be consistent with the land value estimates of Albouy et al. (2018). We also provide similar regressions using ACS data for a later period (2013-17) in Appendix C. While the estimated relationships are qualitatively similar, they are harder to interpret as the land price estimates reflect an earlier period.

5.2 Cross-city Variation in the Theory

Here we quantify the relative importance of variation in MSA-level characteristics in driving cross-city variation in ownership rates and price-rent ratios that is implied by our theory. While we can set median income, the Gini coefficient and median age in the model to match those observed for the corresponding city in the 2010 ACS, there is no single variable in the data that corresponds to τ: construction costs per unit of quality.

Variation in the model parameter τ ultimately depends, in the data, on variation in multiple variables related to construction/land costs and city-wide amenities. We therefore essentially impose our theory on the data in order to infer the distribution of τ's across cities. Specifically, we compute city-specific τ's to minimize the weighted distance between actual and predicted log values of home-ownership and the ratio of average prices to average rents, where the weights are inversely proportional to their variance.[49] As in Table 4, we consider house prices based on both estimated values and price-asked from the ACS.

Table 6: Relationship between inferred τ's and Land Prices and Amenities

	Inferred τ							
	Prices based on est. value				**Prices based on price-asked**			
Land Price	0.031	(0.007***)	0.019	(0.007**)	0.021	(0.008**)	0.024	(0.008***)
DCDD	0.088	(0.021***)			0.074	(0.026***)		
DHDD	0.043	(0.011***)			0.049	(0.013***)		
Sunshine	0.352	(0.141**)			0.290	(0.170*)		
Coast	0.327	(0.228)			0.544	(0.308*)		
Latitude	0.013	(0.004***)			0.009	(0.005*)		
Ave. Slope	0.017	(0.004***)			0.023	(0.005***)		
Density	0.001	(0.000***)			0.001	(0.000***)		
Amenity			1.035	(0.163***)			0.877	(0.166***)
Constant	0.480	(0.173***)	0.992	(0.024***)	0.599	(0.195***)	0.922	(0.025***)
R-squared	0.60		0.54		0.45		0.40	
# obs	200		202		200		202	

To assess the validity of this procedure, Table 6 reports estimates of a least-squares regression across MSA's of our inferred τ's on average land values (from Albouy et al., 2018), average slope, population-weighted density and various natural amenity measures (from Albouy, 2016). As seen in the first column of Table 6 these explanatory variables are statistically significant and account for 60% of the variation in τ when house prices are based on estimated values. The second column replaces the direct amenity measures with a city-level amenity index constructed by Albouy (2016). This more parsimonious representation accounts for almost as much of the overall variation as before. While there are obviously numerous unobserved factors which determine the variation in these inferred τ's, it seems clear that it does indeed reflect variation in actual land costs and amenities.

Table 7 documents the changes implied by the model in the ownership rate, price-rent ratio, and vacancy ratio as a result of a one standard deviation change in each of the MSA-level characteristics. Overall, these results demonstrate that while variation in income, inequality and population growth play a role in determining variation in the ownership rates and ratios of average prices to average rents across cities, variation in land costs and average amenities is the most important factor, quantitatively, for understanding these relationships.

[49] For details on the procedure used, see Appendix B.

Table 7: Relative Importance of MSA-level Characteristics

	One Standard Deviation (SD) Increase			
	Median Income	Gini Index	Median Age	Construction Cost
Ownership Rate (in SD's)	0.389	−0.014	0.194	−1.462
Price-Rent Ratio (in SD's)	0.142	0.421	0.004	1.245
Vacancy Ratio (in SD's)	0.857	0.583	−0.220	−0.817

Note: This table displays the effects on the ownership rate, the price-rent ratio, and the vacancy ratio of a one SD increase for each of the four factors indicated in the column headers. These effects are expressed in SD's of the dependent variable of interest.

6 The Affordability of Housing

In the model, all households would prefer to own since the overall costs of owning a house of any given quality level are lower than renting.[50] However, lower income households can only afford housing in market segments where, in equilibrium, there are relatively few housing units for sale. Consequently, they are more likely to rent. Thus, the overall affordability of housing is inextricably linked to home-ownership. To study these issues further, we consider the implications for housing affordability using a standard definition taken from the National Association of Realtors (NAR).

6.1 The Housing Affordability Index (HAI)

The Housing Affordability index (HAI) is defined as:

$$\text{HAI} = \frac{Y_m}{Y_q} \times 100; \tag{40}$$

where Y_m denotes median (annual) household income,[51] and Y_q denotes *qualifying* income. The latter is taken to represent the annual income flow required to afford, "reasonably", the mortgage payment on the median-priced home.[52] Consequently, the HAI measures, more specifically, the affordability of *home-ownership*.

[50]In the alternative formulation of Section 4.5 there is a utility premium derived from ownership.

[51]As we target a log-normal distribution of annual income, Y, with mean μ, $Y_m = e^{\mu}$.

[52]The method used by the NAR in constructing HAI's for various housing markets is described online at: https://www.nar.realtor/topics/housing-affordability-index/methodology.

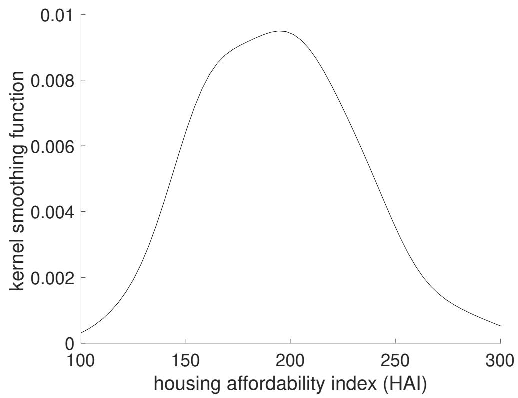


Figure 13: Housing affordability across cities (kernel density estimate).

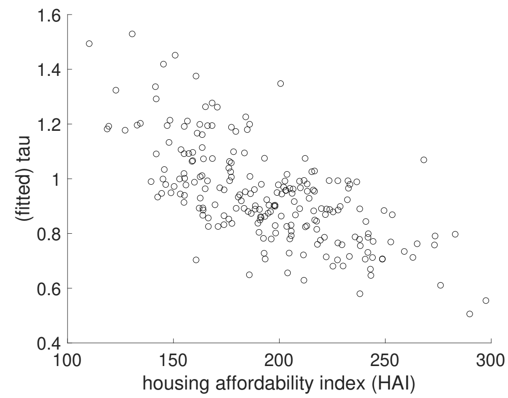


Figure 14: Fitted measure of construction costs and housing affordability.

Our notion of "reasonableness", with regard to affording a mortgage payment, is based on a household that makes a 20% downpayment, faces a mortgage rate, i_m, equivalent to 4% annually over an amortization period of 30 years and spends no more than 25% of income on the payment.

If p_m denotes the median price of a house, then:

$$Y_q = 4 \times \underbrace{\frac{(1-0.2)p_m i_m}{1-\left(\frac{1}{1+i_m}\right)^{360}}}_{\text{monthly payment}} \times 12. \tag{41}$$

A lower HAI indicates that housing is *less* affordable.

In our baseline calibration the HAI = 183. That is, a household with median income has a quarterly income flow 1.83 times that of the qualifying income. Using the values of τ fitted to match the variation in ownership rates and price-rent ratios across U.S. MSA's (see Figures 19 and 20 in Appendix B), the HAI ranges from 110 to over 300. Figure 13 depicts a kernel density estimate for the distribution of HAI's across the cities in our sample. Figure 14 plots the HAI for each of these cities against the fitted value of τ. Clearly, housing affordability in the model is closely related negatively to the cost of producing a unit of housing quality.

In the model, city-level housing affordability affects *home-ownership* significantly for lower income households only. Figure 15 plots average home-ownership rates for households in the bottom, middle and top income quintiles against the HAI for each city in the sample. In the figure it is clear that affordability is effectively uncorrelated

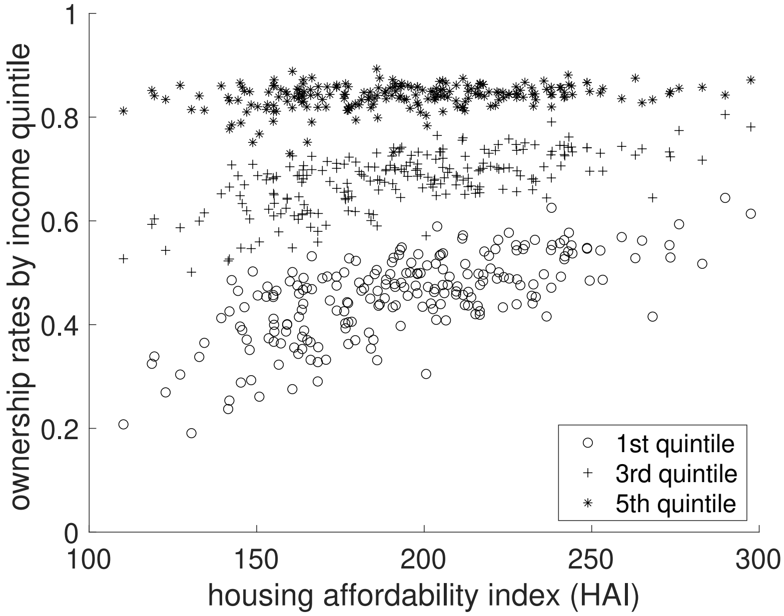


Figure 15: Ownership across cities by income quintile.

with home-ownership for households above the 80th percentile, and shows only a slight positive correlation for those in the middle quintile. For households below the 20th percentile, however, the correlation of affordability with ownership is striking.

6.2 Implications of progressive property taxation

Most jurisdictions impose property taxes of various kinds, usually to pay for local amenities. Several observers have argued recently that these taxes should be more progressive. For example, Leachman and Waxman (2019) argue that flat rate property taxes are regressive at the state level and propose state-wide surcharges on luxury homes. Others propose such a tax surcharge as a way of raising significant revenue (Choi et al., 2018).[53] In this section, we consider the broader housing market implications of adopting more progressive property taxes.

Suppose property taxes are initially proportional to q.[54] With complete markets,

[53]With rising house values and expanding public debt, such proposals are likely to become increasingly common.

[54]Property taxes are typically based on assessed value rather than transactions prices. In principle,

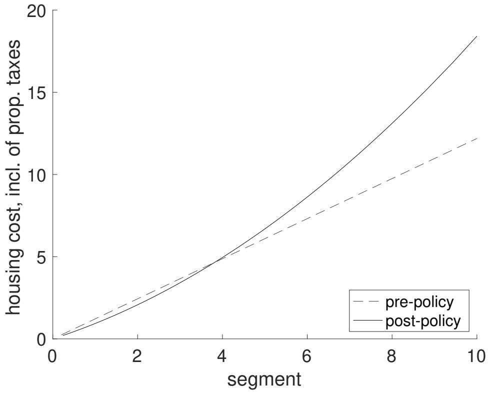


Figure 16: Housing costs.

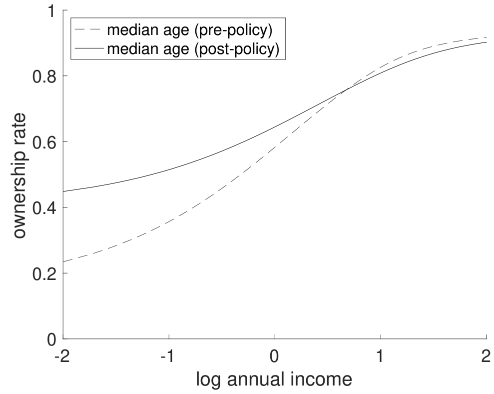


Figure 17: Ownership rate *vs.* income.

period-by-period payments are equivalent to a single initial payment upon construction equal to the discounted future liability. We can therefore think of the property tax liability as being a portion of $T(q)$, say ωq, where $\omega < \tau$. We then consider replacing this with a more progressive but revenue-neutral tax function given by

$$\hat{\Omega}(q) = [\hat{\omega}_0 + \hat{\omega}_1(q - \hat{q})]\,\omega q \tag{42}$$

where $\hat{\omega}_0 \leq 1$, $\hat{\omega}_1 > 0$, and $\hat{q} > 0$. Under the progressive property tax, owners (landlords or owner-occupiers) of houses of quality $q < \hat{q} + (1 - \hat{\omega}_0)/\hat{\omega}_1$ pay proportionately less, while those of houses of quality $q > \hat{q} + (1 - \hat{\omega}_0)/\hat{\omega}_1$ pay proportionately more.

Now, consider a relatively "unaffordable" city with an HAI one standard deviation below the average for the simulated MSA's in Section 5.2. We then consider a revenue-neutral shift to a progressive property tax of the form (42) that raises the HAI for the city to the *average* affordability. As illustrated in Figure 16, property taxes on relatively high quality housing must be increased substantially in order to offset the somewhat lower taxes on relatively low quality housing. This is because the income distribution is such that the majority of households experience relatively lower taxes, and because relatively high income households substitute towards lower quality housing units (see below).

The progressive property tax affects the levels and distributions of home-ownership, housing quality, consumption, and welfare. It generally redistributes from upper to lower income households. It has, however, effects throughout the distribution.

Home-ownership: While the policy change is revenue neutral, the overall home-

these should be based on the observable characteristics of each house which are summarized by q.

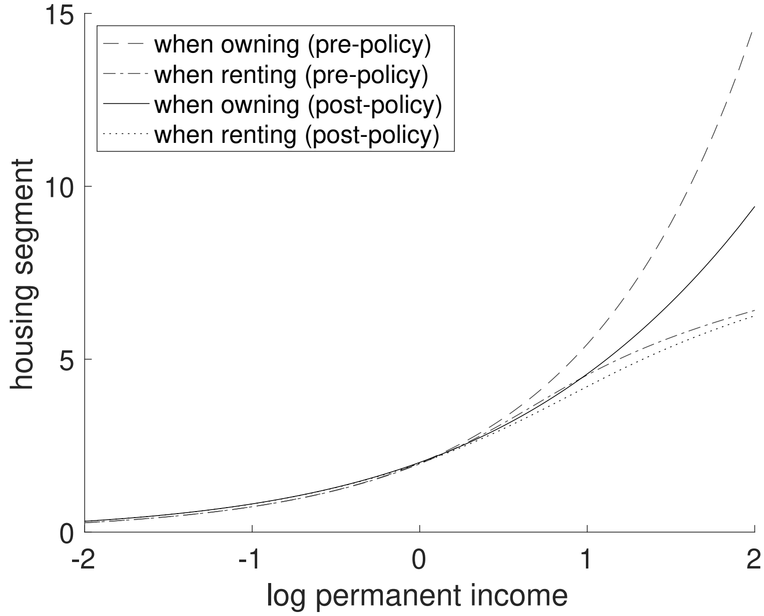


Figure 18: Market segment *vs.* income by housing tenure.

ownership rises significantly, from 60.42% to 64.19%. This occurs in spite of the fact that the policy does not favor home-ownership *per se*. The increase in home-ownership is concentrated overwhelmingly in the bottom half of the income distribution, as can be seen in Figure 17. For low quality housing, the relative cost of renting does not change much with q (see Figure 4). A small reduction in the property tax lowers the cost of housing services, induces households to demand higher quality houses, and thus increases the relative cost of renting. Since a significant share of the population lives in relatively low quality rental housing, this leads leads to a large increase in home-ownership for this segment of the population. In contrast, for higher income households the progressive property tax increases market tightness, sending prospective buyers into lower quality sub-markets but has only very minor effects on their rates of ownership.

Housing quality: Table 8 documents the average quality of housing consumed by a representative household at the midpoint of each quintile of the permanent income distribution both before and after the tax change. As can be seen, the average quality of housing for the representative household rises for the bottom two quintiles, remains relatively unchanged for the middle quintile, and falls for the top two. The decline in housing quality is large for households in the top quintile. Overall, the city-wide average quality of rented housing rises while that of owner-occupied housing declines.

Consumption and welfare: Table 9 decomposes the impacts on welfare for the median household in each permanent income quintile using the percentage change in goods consumption that would have the effect on household utility equivalent to that of the policy change. Overall, the policy change increases welfare for the median household in each of the bottom four quintiles while reducing welfare for that of the top quintile. The gain is largest for the bottom quintile and falls as income rises. The first five rows of Table 9 measure the contribution of changes in the various determinants of utility. For households in the bottom quintile, welfare gains come mainly from increases in housing quality. For those in the top two quintiles, the effects of reduced housing quality are paramount, but are mitigated significantly by a relatively large increase in consumption. Households in the third quintile experience only small changes in housing quality and negligible effects on home-ownership but gain mainly due to increased consumption.

Table 8: Housing Quality under the Progressive Property Tax

Household	q When Renting		q When Owning		Average q	
Permanent Income	*Pre-*	*Post-*	*Pre-*	*Post-*	*Pre-*	*Post-*
$F^{-1}(0.1)$	0.7589	0.8400	0.7606	0.8424	0.7595	0.8411
$F^{-1}(0.3)$	1.3975	1.4691	1.4059	1.4779	1.4016	1.4740
$F^{-1}(0.5)$	2.1157	2.1174	2.1466	2.1426	2.1345	2.1334
$F^{-1}(0.7)$	3.1328	2.9737	3.2748	3.0557	3.2355	3.0324
$F^{-1}(0.9)$	4.7926	4.4562	6.0061	4.9382	5.8153	4.8490
Agg. Average q	1.8216	1.9456	3.6089	2.9356	2.9015	2.5811

7 Alternative specifications and other implications

7.1 Mis-matched owners remain in their home while selling and searching

In our baseline model, mismatched owners immediately move out of their houses and rent in their preferred segment until they find a new house to own. This is optimal because the value of housing services from a mismatched house drops to zero. This simplifying assumption ensures that their mismatched house becomes a

Table 9: Percentage Welfare Effects (Expressed in Consumption Equivalents)

	Household Permanent Income				
Post-Policy Outcome(s)	$F^{-1}(0.1)$	$F^{-1}(0.3)$	$F^{-1}(0.5)$	$F^{-1}(0.7)$	$F^{-1}(0.9)$
Consumption	0.8136	1.3502	1.8443	2.4066	3.2470
Housing Services	2.6853	1.3101	−0.0176	−1.6441	−4.3838
Quality While Renting	1.8616	0.7423	0.0093	−0.4463	−0.3659
Quality While Owning	0.8087	0.5636	−0.0269	−1.2032	−4.0327
Home Ownership	0.0066	0.0106	0.0112	−0.0101	−0.1906
All Post-Policy Outcomes	3.5293	2.6889	1.8357	0.7167	−1.3648

purely financial asset that (with complete markets) is valued equally by all sellers. In reality, however, many owners do not pass through a period of renting but rather remain in their original house while selling and searching for a new one.

Suppose instead that the value of housing services from a mis-matched house falls only to the extent that homeowners are exactly indifferent between remaining in their current home and moving to rent in their optimal segment. Suppose further that an arbitrary fraction remains and the rest moves out. In this case, the only change to the model would be that potential buyers in each sub-market would now include both renters and "mismatched, remaining owners", who are otherwise equivalent. Because it would require a re-calibration of the model to match the within-city facts, this would change the quantitative implications somewhat, but not the qualitative conclusions. In Appendix D we flesh out this alternative specification and present the results for a particular case where all mismatched households remain in their houses while selling.

7.2 A Version with Financial Constraints

Households in our model face the financial limitations imposed by their intemporal budget constaints. This is why poorer households cannot afford to search for housing to own in segments where houses for sale are relatively abundant and why cheap housing is more likely to be rented. As we have noted in the introduction, models that emphasize the role of additional tighter borrowing constraints rest on the assumption that no one is allowed to sell cheap housing units. Although households in our model have access to complete asset markets, we can explore the implications of financial frictions by imposing a simple type of payment-to-income (PTI) constraint for home buyers:

$$Z_N(q_S) + \rho T(q_S) \leq 0.3y \tag{43}$$

While this is not, strictly-speaking a borrowing constraint, the PTI constraint does have a similar effect by restricting searchers to market segments in which the imputed flow cost of home-ownership is no more than 30 percent of their income.

We introduce this constraint into a version of our model with a utility premium for ownership that rises with quality (see Section 4.5) and calibrate it to match the same targets as before.[55] When we do so, our main qualitative results remain largely unchanged.[56] In this equilibrium, the PTI constraint is binding for relatively high income households. Consequently, and more realistically, the share of income spent on consumption does not rise dramatically with income as it does in the unconstrained version of the model.

7.3 The price-rent ratio of the median quality house

The main focus of this paper is on the role of trading frictions in driving cross-city variation in housing composition across owned and rental markets. We have used the ratio of average prices to average rents aross cities because a large part of the variation in this measure is driven by housing composition. Our theory, however, also has implications for cross-city variation in the price-rent ratio of the average quality house. This concept is often viewed by housing economists as a potential gauge of the role of financial or speculative factors in driving relative activity in the owned housing market (*e.g.* Gilbukh et al., 2017). Empirically, however, it is challenging to estimate the price to rent ratio of equivalent housing units, precisely because the composition of owned and rented housing is so different. A substantial literature has used various approaches to adjust for quality and the implications are somewhat mixed. For example, Zillow Research provides MSA-level estimates of the price-rent ratio of the median quality house based on a hedonic approach.

As seen in Figure 21 in Appendix C.4, there is a significant positive empirical correlation between the price-rent ratio of the median quality house provided by Zillow and the ratio of average prices to average rents estimated from the ACS data.[57] When

[55] Using the baseline version is not very interesting because the share of income spent on housing is roughly constant. Consequently, the contraint binds either for all households or for none.

[56] Details are available upon request.

[57] There are fewer observations in Figure 21 than in Figure 22 because there are missing Zillow

we incorporate cross-city variation in the fundamentals, all versions of our model also predict a positive correlation between the two price-rent ratio concepts. This is true even for the baseline calibration that exhibits a declining price-rent ratio by quality (see Figure 22) because a large part of the cross-city variation comes from supply-side variation driven by housing production costs per unit of quality, τ. Both price-rent ratio concepts are strongly increasing with τ.

8 Conclusion

Across U.S. MSAs, there is a large variation in the aggregate share of housing which is owner-occupied and average price-rent ratios. To account for the long run determinants of these aggregates across cities, it is necessary to understand what determines differences in the *composition* of the housing that is supplied to ownership and rental markets. To do so we have developed a dynamic model of frictional assignment in the housing market in which heterogeneous households search for endogenously-supplied, heterogeneous housing units in owned and rental markets.

In the theory, the likelihood that a housing unit of a given quality is owned or rented is determined by the incentives faced by developers, landlords and moving home-owners when supplying their vacant housing units to the ownership or rental markets. These incentives, in turn, reflect a trade off between the relative illiquidity of ownership markets (due to search/matching frictions) and maintenance cost differentials between rental and owned housing (and/or a preference for owning). As we move up the housing quality ladder, the maintenance cost differential rises and households are willing to pay a higher price premium to own. This induces developers in high quality segments to assign proportionally more housing starts to ownership markets rather than rental markets, which causes an increase in the expected time to to sell. Equilibrium occurs when the share of housing assigned to ownership markets in each segment rises to the point where the implied increase in liquidity costs matches the increased willingness of buyers to pay to own.

In equilibrium, proportionately more housing units are put up for sale in high quality market segments where buyers are willing and able to pay a price premium to compensate sellers for the liquidity cost. Proportionately more poor households end up renting because they cannot afford housing in sub-markets where housing for sale is relatively more plentiful. The cross-city variation in the aggregate share of housing

values for some of the MSA's.

which is owned, average price-rent ratios and vacancy ratios reflects the composition of the housing stock across ownership and rental markets which is determined by the distribution of households across housing market segments. This distribution, in turn, depends on the distribution of household permanent income, the age of the population and construction costs per unit of quality.

The model is calibrated to replicate relevant features of the median U.S. city's housing market. We use the calibrated model to generate steady-state predictions of how the ownership rate, the average price-rent ratio and the vacancy ratio vary in response to changes in the distribution of permanent income, median age and land prices. When we compare these predictions to those observed across U.S. MSAs, we find that they are qualitatively consistent.

We also use the calibrated model to study the relationship between ownership and affordability across US cities and the long-run effects of adopting a more progressive property tax. Our model demonstrates that cities in which construction costs are lower will tend to have higher ownership rates, especially amongst lower income households, and that progressive property tax policies can induce greater ownership.

In order to focus on the role of trading frictions, our model does not explicitly incorporate numerous other factors that are potentially important in understanding the relative incentives to own and rent. These include supply constraints, differential tax treatment, risk factors, life cycle issues and exogenous transactions costs. Many of their effects on incentives could be replicated by adjusting our assumptions regarding housing supply costs, maintenance costs and/or preferences. Of course, in order to consider explicitly the effects of changes in these factors, the model can be extended to incorporate them.

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Online Appendices

A Additional Model Details

A.1 The consumption-saving decision

Owners issue claims to achieve zero wealth in the event of death: $w_D = -(1+\rho)T(q_S)$. These claims work essentially as a reverse mortgage. The intertemporal Euler equations are

$$\phi_R(q_S,p) = \beta(1-\lambda(\theta(q_S,p)))\frac{u'(c_R^{+1})}{u'(c_R)} = \frac{(1-\delta)(1-\lambda(\theta(q_S,p)))}{1+\rho}\frac{u'(c_R^{+1})}{u'(c_R)} \tag{A.1}$$

$$\phi_B(q_S,p) = \beta\lambda(\theta(q_S,p))\frac{u'(c_B^{+1})}{u'(c_R)} = \frac{(1-\delta)\lambda(\theta(q_S,p))}{1+\rho}\frac{u'(c_B^{+1})}{u'(c_R)} \tag{A.2}$$

$$\phi_N = \beta(1-\pi)\frac{u'(c_N^{+1})}{u'(c_N)} = \frac{(1-\delta)(1-\pi)}{1+\rho}\frac{u'(c_N^{+1})}{u'(c_N)} \tag{A.3}$$

$$\phi_S = \beta\pi\frac{u'(c_S^{+1})}{u'(c_N)} = \frac{(1-\delta)\pi}{1+\rho}\frac{u'(c_S^{+1})}{u'(c_N)} \tag{A.4}$$

The no-arbitrage conditions are:

$$\phi_N = \beta(1-\pi) = \frac{(1-\delta)(1-\pi)}{1+\rho} \tag{A.5}$$

$$\phi_S = \beta\pi = \frac{(1-\delta)\pi}{1+\rho} \tag{A.6}$$

$$\phi_D = \frac{\beta\delta}{1-\delta} = \frac{\delta}{1+\rho} \tag{A.7}$$

$$\phi_R(q,p) = \beta(1-\lambda(\theta(q,p))) = \frac{(1-\delta)(1-\lambda(\theta(q,p)))}{1+\rho} \tag{A.8}$$

$$\phi_B(q,p) = \beta\lambda(\theta(q,p)) = \frac{(1-\delta)\lambda(\theta(q,p))}{1+\rho} \tag{A.9}$$

$$\phi_D(q,p) = \frac{\beta\delta\lambda(\theta(p;q))}{1-\delta} = \frac{\delta\lambda(\theta(q,p))}{1+\rho} \tag{A.10}$$

The Euler equations then imply a constant consumption stream. To achieve this, wealth positions, $\{w_R, w_B, w_N, w_S\}$, must satisfy $w_B = w_N + p$, $w_S = w_R - (1+\rho)T(q_S)$

and the following budget constraints:

$$c + \frac{1-\delta}{1+\rho}\Big\{\lambda(\theta)\left[w_N + p\right] + (1-\lambda(\theta))w_R\Big\} + \frac{\delta\lambda(\theta)}{1+\rho}\left[p - (1+\rho)T_S\right] + x_R = w_R \tag{A.11}$$

$$c + \frac{1-\delta}{1+\rho}\Big\{\pi\left[w_R - (1+\rho)T_S\right] + (1-\pi)w_N\Big\} - \frac{\delta}{1+\rho}(1+\rho)T_S + Z_S = w_N \tag{A.12}$$

For convenience, the notation here has been simplified by replacing, for example, $\theta(q_S, p)$ with θ, $T(q_S)$ with T_S, $x(q_R)$ with x_R, and $Z_N(q_S)$ with Z_S. Combining both budget constraints to eliminate c yields

$$w_R = w_N + (1+\rho)T_S + \frac{\lambda(\theta)\left[p - (1+\rho)T_S\right] + (1+\rho)\left[x_R - Z_S - \rho T_S\right]}{\rho + \delta + (\pi + \lambda(\theta))(1-\delta)} \tag{A.13}$$

This relationship and a budget constraint determine the level of consumption:

$$\begin{aligned} c = \frac{\rho+\delta}{1+\rho}w_R &- \frac{\lambda(\theta)\left[\rho + \delta + \pi(1-\delta)\right]\left[p - (1+\rho)T_S\right]}{(1+\rho)\left[\rho + \delta + (\pi + \lambda(\theta))(1-\delta)\right]} \\ &- \frac{\left[\rho + \lambda + \pi(1-\delta)\right]x_R - \lambda(\theta)(1-\delta)\left[Z_S + \rho T_S\right]}{\rho + \delta + (\pi + \lambda(\theta))(1-\delta)} \end{aligned} \tag{A.14}$$

A household with permanent income y initially enters the city as a renter with wealth equal to the present discounted value of lifetime income:

$$w_R = \frac{y}{1-\beta} = \frac{1+\rho}{\rho+\delta}y$$

The constant consumption level for this household is therefore

$$\begin{aligned} c = y &- \frac{\lambda(\theta)\left[\rho + \delta + \pi(1-\delta)\right]\left[p - (1+\rho)T_S\right]}{(1+\rho)\left[\rho + \delta + (\pi + \lambda(\theta))(1-\delta)\right]} \\ &- \frac{\left[\rho + \delta + \pi(1-\delta)\right]x_R + \lambda(\theta)(1-\delta)\left[Z_S + \rho T_S\right]}{\rho + \delta + (\pi + \lambda(\theta))(1-\delta)} \end{aligned} \tag{A.15}$$

Note that (A.15) is (18) in the text. The financial wealth of a household changes with every transition to a different ownership status and in the event of death according to

$$w_R = \frac{1+\rho}{\rho+\delta} y \tag{A.16}$$

$$\begin{aligned} w_B = {} & \frac{1+\rho}{\rho+\delta} y + p - (1+\rho) T_S \\ & - \frac{\lambda(\theta)\left[p - (1+\rho)T_S\right] + (1+\rho)\left[x_R - Z_S - \rho T_S\right]}{\rho + \delta + (\pi + \lambda(\theta))(1-\delta)} \end{aligned} \tag{A.17}$$

$$\begin{aligned} w_N = {} & \frac{1+\rho}{\rho+\delta} y - (1+\rho) T_S \\ & - \frac{\lambda(\theta)\left[p - (1+\rho)T_S\right] + (1+\rho)\left[x_R - Z_S - \rho T_S\right]}{\rho + \delta + (\pi + \lambda(\theta))(1-\delta)} \end{aligned} \tag{A.18}$$

$$w_S = \frac{\rho+\delta}{1+\rho} y - (1+\rho) T_S \tag{A.19}$$

$$w_D = -(1+\rho) T_S \tag{A.20}$$

A.2 The search decision

Households direct their search by choosing a particular sub-market (price) and market segment (quality). The search decision appears in the Bellman equation for a household that is currently renting:

$$W^R(w) = \max_{q_S, p} \left\{ \begin{aligned} & u(c) + h(0, q_R) + \frac{(1-\delta)\left(1 - \lambda(\theta(q_S, p))\right)}{1+\rho} W^R(w_R) \\ & \qquad + \frac{(1-\delta)\lambda(\theta(q_S, p))}{1+\rho} W^N(q_S, w_B - p) \end{aligned} \right\} \tag{A.21}$$

subject to

$$\begin{aligned} c = w - x(q_R) - \frac{(1-\delta)\lambda(\theta(q_S,p))}{1+\rho} w_B - \frac{(1-\delta)\left(1 - \lambda(\theta(q_S,p))\right)}{1+\rho} w_R \\ - \frac{\delta\lambda(\theta(q_S,p))}{1+\rho}\left[p - (1+\rho)T(q_S)\right] \end{aligned}$$

and

$$\gamma(\theta(q_s, p))\left[p - V(q_S)\right] = \rho V(q_S). \tag{A.22}$$

where the latter constraint imposes that the searching household correctly anticipates that market tightness, θ, is determined by the free entry of sellers according to (12).

By substituting this constraint into the household's objective function, the optimal choice of segment and sub-market solve the following:

$$W^R(w) = \max_{q_S,\theta} \left\{ \begin{array}{r} u(c) + h(0,q_R) + \dfrac{(1-\delta)\,(1-\lambda(\theta))}{1+\rho} W^R(w_R) \\ + \dfrac{(1-\delta)\lambda(\theta)}{1+\rho} W^N \left(q_S, w_B - \left[\dfrac{\varpi+\rho}{\varpi} + \dfrac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)} \right] T(q_S) \right) \end{array} \right\} \tag{A.23}$$

subject to

$$c = w - x(q_R) - \frac{1-\delta}{1+\rho}\left[\lambda(\theta)w_B + (1-\lambda(\theta))w_R\right] - \frac{\delta\lambda(\theta)}{1+\rho}\left[\frac{\rho(1-\varpi)}{\varpi} + \frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)}\right] T(q_S)$$

The first order condition with respect to q_S is

$$\begin{aligned} & u'(c)\delta \left[\frac{\rho(1-\varpi)}{\varpi} + \frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)} \right] T'(q_S) \\ & \qquad = (1-\delta) \left\{ \frac{\partial W^N}{\partial q} - \frac{\partial W^N}{\partial w} \left[\frac{\varpi+\rho}{\varpi} + \frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)} \right] T'(q_S) \right\} \end{aligned} \tag{A.24}$$

and the first order condition with respect to θ is

$$\begin{aligned} & \left\{ u'(c) \left[w_R - w_B - \frac{\delta}{1-\delta} \frac{\rho(1-\varpi)}{\varpi} T(q_S) \right] + W^N - W^R \right\} \frac{\theta\lambda'(\theta)}{\lambda(\theta)} \\ & \qquad + u'(c) \frac{\delta}{1-\delta} \frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)} T(q_S) + \frac{\partial W^N}{\partial w} \frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)} \frac{\theta\gamma'(\theta)}{\gamma(\theta)} T(q_S) = 0. \end{aligned} \tag{A.25}$$

The value associated with home-ownership satisfies

$$W^N(q_S,w) = u(c) + h(1,q_S) + \frac{1-\delta}{1+\rho}\left[\pi W^R(w_S + (1+\rho)T(q_S)) + (1-\pi)W^N(q_S,w_N)\right] \tag{A.26}$$

subject to

$$c = w - \frac{1-\delta}{1+\rho}\left[\pi w_S + (1-\pi)w_N\right] + \delta T(q_S) - Z_N(q_S)$$

The Benveniste-Scheinkman conditions are therefore

$$\frac{\partial W^N(q_S,w)}{\partial w} = u'(c) \tag{A.27}$$

$$\frac{\partial W^N(q_S,w)}{\partial q} = \frac{1+\rho}{\rho+\delta+\pi(1-\delta)} \left\{ \begin{array}{l} \dfrac{\partial h(1,q_S)}{\partial q} - u'(c) Z_N'(q_S) \\ + u'(c)\left(\delta + \pi(1-\delta)\right) T'(q_S) \end{array} \right\} \tag{A.28}$$

Substituting (A.22), (A.27) and (A.28) into first order conditions (A.24) and (A.25) yields

$$\begin{aligned}
&u'(c)\delta\big[p-(1+\rho)T(q_S)\big]\frac{T'(q_S)}{T(q_S)}+u'(c)(1-\delta)\frac{pT'(q_S)}{T(q_S)}= \\
&\quad\frac{(1+\rho)(1-\delta)}{\rho+\delta+\pi(1-\delta)}\left\{\frac{\partial h(1,q_S)}{\partial q}+u'(c)\big[\left(\delta+\pi(1-\delta)\right)T'(q_S)-Z_N'(q_S)\big]\right\}
\end{aligned}\tag{A.29}$$

and

$$\begin{aligned}
&\left\{u'(c)\left[w_R-w_B-\frac{\delta}{1-\delta}\frac{\rho(1-\varpi)}{\varpi}T(q_S)\right]+W^N-W^R\right]\left(1-\eta(\theta)\right) \\
&\quad=u'(c)\frac{\delta}{1-\delta}\frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)}T(q_S)+u'(c)\eta(\theta)\frac{\rho(\varpi+\rho)}{\varpi\gamma(\theta)}T(q_S)
\end{aligned}\tag{A.30}$$

where $\eta(\theta)=\theta\gamma'(\theta)/\gamma(\theta)=\theta\lambda'(\theta)/\lambda(\theta)+1$. Condition (A.29) simplifies to

$$u'(c)\left\{\frac{\rho+\delta+\pi(1-\delta)}{(1+\rho)(1-\delta)}\big[p-(1+\rho)T(q_S)\big]\frac{T'(q_S)}{T(q_S)}+Z_N'(q_S)+\rho T'(q_S)\right\}=\frac{\partial h(1,q_S)}{\partial q}.$$

With substitutions from (A.16), (A.17), (A.23), (A.26), condition (A.30) becomes

$$\begin{aligned}
p=V(q_S)&+\frac{(1+\rho)(1-\delta)(1-\eta(\theta))}{\rho+\delta+(\pi+\eta(\theta)\lambda(\theta))(1-\delta)} \\
&\times\left[\frac{h(1,q_S)-h(0,q_R)}{u'(c)}-\left(Z_N(q_S)+\rho T(q_S)+\frac{\rho(1-\varpi)\left(\rho+\delta+\pi(1-\delta)\right)}{\varpi(1+\rho)(1-\delta)}T(q_S)-x(q_R)\right)\right].
\end{aligned}$$

These correspond to conditions (20) and (21) in the text.

B Data Definitions, Sources and Calculations

B.1 Data definitions and sources

All data on housing and households is from the American Community Survey (2010, 5-year estimates) accessed via the Census Bureau at data.census.gov/cedsci.

House price: To compute mean house prices by urban area, we use the data on housing value obtained from Housing Question 16 in the ACS. The question was asked at housing units that were owned, being bought, vacant for sale, or sold and not occupied at the time of the survey. The estimated value is the respondent's estimate of how much the property (house and lot, mobile home and lot, or condominium unit) would sell for if it were for sale. We also used the average price-asked on vacant housing for sale only and on housing sold but unoccupied. This was calculated by dividing the aggregate price-asked by the number for sale and sold but unoccupied.

Rent: To compute mean rent for each urban area, we use the data on gross rent obtained from answers to Housing Questions 11a-d and 15a in the ACS. Gross rent is the contract rent plus the estimated average monthly cost of utilities if these are paid by the renter.

Housing tenure: To compute the number of owning and renting households we use data obtained from Housing Question 14 in the ACS. The question was asked at occupied housing units. Occupied housing units are classified as either owner occupied or renter occupied:

Owner occupied – A housing unit is owner occupied if the owner or co-owner lives in the unit even if it is mortgaged or not fully paid for.

Renter occupied – All occupied housing units which are not owner occupied, whether they are rented or occupied without payment of rent, are classified as renter occupied.

Households – the number of households was computed as the sum of owner and renter occupied units.

Vacant housing units – A housing unit is vacant if no one is living in it at the time of interview. Units occupied at the time of interview entirely by persons who are staying two months or less and who have a more permanent residence elsewhere are considered to be temporarily occupied, and are classified as "vacant." New units not yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place.

Median household income: The data on income during the last 12 months were derived from answers to Questions 47 and 48, which were asked of the population 15 years old and over. Household income includes the income of the householder and all other individuals 15 years old and over in the household. "Total income" is the sum of the amounts reported separately for wage or salary income; net self-employment income; interest, dividends, or net rental or royalty income or income from estates and trusts; Social Security or Railroad Retirement income; Supplemental Security Income (SSI); public assistance or welfare payments; retirement, survivor, or disability pensions; and all other income

Gini coefficient: The Gini index of income inequality for each urban area comes from the ACS and measures the dispersion of the household income distribution.

Median age refers to the median age of the population taken from the 2010 ACS. (It is not the median age of the householder, which is not available by MSA.)

Kids is the share of adult population with own children 17 and under by MSA from the 2010 ACS.

Land values are from column 5 of Albouy et al. (2018) appendix Table A2. The MSA's for which they provide these estimates are based on the 1999 OMB definitions. The MSA's used in the current paper are from the 2010 census and are slightly different and greater in number. This is in part because some MSA's have been subdivided and also because there are some new MSA's.[58] Where MSA's have been subdivided, we use the same average land values for both. Where we could find no match, we dropped the MSA from the data set.

COLI: The local cost of living index is the ACCRA Cost of Living Index provided by Council for Community and Economic Research (https://www.c2er.org/). It measures relative price levels for consumer goods and services in participating areas for a mid-management standard of living. Weights are based on the Bureau of Labor Statistics' 2004 Consumer Expenditure Survey. We use these weights to extract a COLI for non-housing expenditures only. Data are for selected urban areas within

[58]The MSA's in 2010 are based on the concept of Core Based Statistical Areas.

the larger MSA. In the few cases where there are multiple areas within a given MSA, we used a simple average.

Amenity controls: "Natural" amenities are from Albouy (2016).:

Heating and cooling degree days (Annual) – Degree day data are used to estimate amounts of energy required to maintain comfortable indoor temperature levels. Daily values are computed from each day's mean temperature, (max+min)/2. Daily heating degree days (**DHDD**) are equal to $\max\{0; 65 - \text{mean temp}\}$ and daily cooling degree days (**DCDD**) are $\max\{0; \text{mean temp} - 65\}$. Annual degree days are the sum of daily degree days over the year. The data here refer to averages from 1970 to 1999.

Sunshine – Average percentage of possible sunshine. The total time that sunshine reaches the surface of the earth is expressed as the percentage of the maximum amount possible from sunrise to sunset with clear sky conditions.

Coastal proximity – Equal to the logarithm of the inverse distance in miles to the nearest coastline from PUMA centroid.

Latitude – Measured in degrees from the equator.

Instruments:

Population-weighted density for metropolitan areas is from the Census Bureau:

http://www.census.gov/population/metro/data/pop_pro.html.

It is calculated as a weighted-average of population densities for each census tract where the weights are the share of the total area population in each census tract. It is intended to reflect the population density experienced by the average person in the urban area.

Average slope – The slope of the land in the metropolitan area (percent), using an average maximum slope technique based on a 30 arcsec x 30 arcsec grid. From Albouy (2016).

B.2 Additional empirical estimates

Tables 10, 11 and 13 contains the parameter estimates for the amenity controls associated with Table 4, 5 and 12, respectively.

Table 10: Coefficient estimates for amenity controls in Table 4

Dependent Variable	Ownership Rate	Price-Rent ratio		Vacancy ratio
		Est. value	Price-asked	
DCDD	-1.540	3.303	3.416	0.148
Robust	(0.598**)	(0.562***)	(0.948***)	(1.354)
Clustered	(0.736**)	(0.529***)	(1.403**)	(1.310)
DHDD	-1.221	0.865	0.847	1.798
Robust	(0.321***)	(0.276***)	(0.416**)	(0.621***)
Clustered	(0.382***)	(0.353**)	(0.478*)	(0.623***)
Sunshine	-7.032	12.177	11.867	5.277
Robust	(3.764*)	(4.350***)	(5.931**)	(9.468)
Clustered	(4.496)	(7.386)	(9.907)	(8.037)
Coastal Prox.	-10.654	27.772	48.599	-50.134
Robust	(7.763)	(8.146***)	(11.418***)	(14.469***)
Clustered	(6.000*)	(8.261***)	(12.037***)	(13.804***)
Latitude	-0.443	0.192	-0.039	1.127
Robust	(0.132***)	(0.136)	(0.217)	(0.331***)
Clustered	(0.140***)	(0.152)	(0.189)	(0.247***)
Ave. Slope		0.731	0.891	-0.896
Robust		(0.104***)	(0.260***)	(0.235***)
Clustered		(0.126***)	(0.282***)	(0.237***)
Constant	47.552	-18.073	-10.105	14.336
Robust	(10.434***)	(12.793)	(21.658)	(24.876)
Clustered	(10.720)	(15.512)	(22.867)	(21.592)

Table 11: Coefficient estimates for amenity controls in Table 5

Instruments:	**Weighted Density**		**Average Slope**		**Density & slope**	
Stage:	**Main (2SLS)**	**First stage**	**Main (2SLS)**	**First stage**	**Main (2SLS)**	**First stage**
DCDD	-1.069	0.274	-0.894	0.283	-1.050	0.079
Robust	(0.722)	(0.362)	(0.924)	(0.434)	(0.733)	(0.375)
Clustered	(0.888)	(0.432)	(1.091)	(0.540)	(0.902)	(0.427)
DHDD	-0.422	0.666	-0.126	0.713	-0.390	0.593
Robust	(0.387)	(0.136***)	(0.848)	(0.156***)	(0.405)	(0.133***)
Clustered	(0.414)	(0.116***)	(0.869)	(0.142***)	(0.438)	(0.112***)
Sunshine	2.282	5.810	5.738	8.055	2.662	4.665
Robust	(4.678)	(1.998***)	(9.296)	(2.127***)	(4.739)	(2.007**)
Clustered	(4.201)	(1.976***)	(8.330)	(2.250***)	(4.190)	(1.762***)
Coastal Prox.	3.712	4.990	9.042	14.712	4.298	5.680
Robust	(9.152)	(2.976*)	(14.253)	(4.078***)	(9.066)	(2.991*)
Clustered	(6.511)	(2.630*)	(14.380)	(3.845***)	(6.840)	(2.424**)
Latitude	-0.154	0.187	-0.046	0.256	-0.142	0.158
Robust	(0.158)	(0.067***)	(0.325)	(0.069***)	(0.165)	(0.066**)
Clustered	(0.167)	(0.040***)	(0.342)	(0.050***)	(0.180)	(0.039***)
Constant	26.604	-7.921	18.832	-19.357	25.750	-6.642
Robust	(10.991**)	(5.174)	(21.480)	(6.721***)	(11.194**)	(5.041)
Clustered	(12.054**)	(4.413*)	(21.849)	(6.384***)	(12.375**)	(4.317)

Table 12: IV estimates for all variables using pop.-weighted density as an instrument

Dependent Variable	**Ownership rate**	**Price-rent ratio** Est. value	Price-asked	**Vacancy Ratio**	**Land Price**
Stage	**Main**	**Main**	**Main**	**Main**	**First-stage**
Income	1.976	1.535	1.586	2.398	0.701
Robust	(0.383***)	(0.452***)	(0.802**)	(0.836***)	(0.161***)
Clustered	(0.541***)	(0.504***)	(0.743**)	(0.887***)	(0.178***)
Gini	-0.309	0.484	0.574	-0.006	-0.037
Robust	(0.110***)	(0.146***)	(0.236**)	(0.244)	(0.054)
Clustered	(0.099***)	(0.170***)	(0.239**)	(0.288)	(0.051)
Age	1.107	0.083	0.000	-0.811	0.039
Robust	(0.074***)	(0.063)	(0.124)	(0.248***)	(0.028)
Clustered	(0.086***)	(0.065)	(0.106)	(0.261***)	(0.024*)
Kids	0.352	-0.045	-0.050	-0.287	-0.051
Robust	(0.077***)	(0.051)	(0.097)	(0.207)	(0.026**)
Clustered	(0.070***)	(0.041)	(0.093)	(0.221)	(0.024***)
Land Price	-1.984	0.965	0.256	1.269	
Robust	(0.218***)	(0.297***)	(0.585)	(0.423***)	
Clustered	(0.194***)	(0.305***)	(0.584)	(0.491***)	
Density					0.031
Robust					(0.007***)
Clustered					(0.008***)
Controls	Yes	Yes	Yes	Yes	Yes
$\mathbf{R}^2$	0.63	0.75	0.46	0.34	0.67
# obs	200	200	200	200	200
Endog. Test	9.55 [0.002]	0.12 [0.735]	0.19 [0.660]	0.15 [0.696]	

Table 13: Coefficient estimates for amenity controls in Table 12

Dependent Variable	**Ownership rate**	**Price-rent ratio**		**Vacancy Ratio**	**Land Price**
		Est. value	Price-asked		
Stage	**Main**	**Main**	**Main**	**Main**	**First**
DCDD	-1.001	3.326	3.479	0.101	0.079
Robust	(0.736)	(0.536***)	(0.932***)	(1.335)	(0.375)
Clustered	(0.904)	(0.494***)	(1.319***)	(1.207)	(0.427)
DHDD	-0.394	0.925	1.005	1.679	0.593
Robust	(0.399)	(0.318***)	(0.545*)	(0.618***)	(0.133***)
Clustered	(0.435)	(0.368**)	(0.562*)	(0.698**)	(0.112***)
Sunshine	2.700	12.851	13.658	3.933	4.665
Robust	(4.674)	(4.593***)	(7.155*)	(9.291)	(2.007**)
Clustered	(4.217)	(6.146**)	(9.395)	(9.210)	(1.762***)
Coastal Proximity	3.495	29.003	51.871	-52.589	5.680
Robust	(9.231)	(8.629***)	(13.337***)	(15.811***)	(2.991*)
Clustered	(6.543)	(8.323***)	(13.354***)	(14.702***)	(2.424**)
Latitude	-0.143	0.214	0.018	1.084	0.158
Robust	(0.164)	(0.149)	(0.241)	(0.316***)	(0.066**)
Clustered	(0.180)	(0.164)	(0.202)	(0.280***)	(0.039***)
Average Slope	-0.049	0.742	0.920	-0.918	0.142
Robust	(0.101)	(0.108***)	(0.269***)	(0.237***)	(0.056**)
Clustered	(0.097)	(0.126***)	(0.288***)	(0.230***)	(0.051***)
Constant	26.131	-19.693	-14.410	17.567	-6.642
Robust	(10.991**)	(13.506)	(22.641)	(23.936)	(5.041)
Clustered	(12.143**)	(14.378)	(21.306)	(23.504)	(4.317)

B.3 Fitting construction costs (τ) to cross-city data

We compute MSA-level construction cost parameter values that best match the observed ownership rates and price-rent ratios. We allow median income, the income Gini and median age to match the characteristics of each MSA.[59] For a given construction cost parameter value τ, the resulting simulated economies yield predicted ownership rates and aggregate price-rent ratios. For each MSA, we choose the value for τ that best matches the observed MSA-level statistics in the following sense: we minimize a weighted average of the squared differences between the simulated statistics and their empirical counterparts (in logs). For the weights, we use the inverse of the variances of the logged MSA ownership rates and price-rent ratios.

Figures 19 and 20 depict the results where prices are based on the price-asked. The ratio of the explained sum of squares to the total sum of squares implied by this procedure is 0.38.

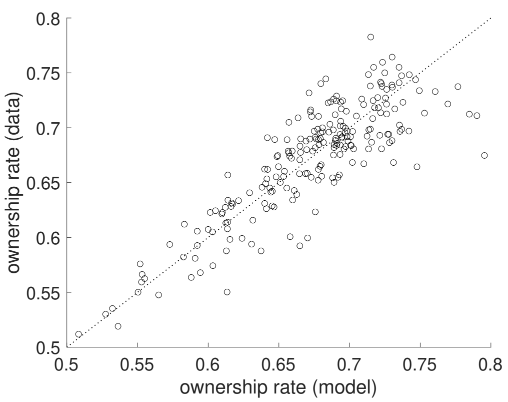


Figure 19: Ownership in the data and the model with variation in construction/land costs.

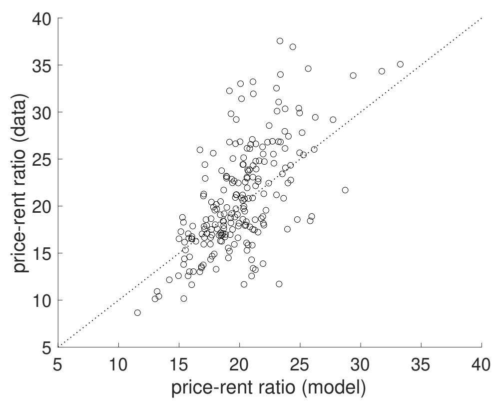


Figure 20: Price-rent ratio in the data and the model with variation in construction/land costs.

[59]Since we do not observe median householder ages in the data, we adjust median population ages upward by a constant equal to the difference in the average of each across MSAs. In the model, median age is controlled by population growth, which is subject to the requirement that it be between 0% and 5% annually.

C Additional Empirical Results

C.1 Additional explanatory variables

Employment – Employment rate by MSA. Source: American Community Survey 2010, 5-year estimates, US Census Bureau.

College – Share of adult population with a college degree. Source: Albouy (2016).

Debt-to-income – Lower bound of Debt-to-Income ratio category by MSA in 2010. Source: (www.federalreserve.gov/releases/z1/dataviz/household debt/) Enhanced Financial Accounts of the Federal Reserve Board Household debt is calculated from FRBNY Consumer Credit Panel/Equifax Data, and household income is reported by the Bureau of Labor Statistics.

WRLURI – Wharton Residential Land Use Regulation Index (2008) by MSA. Source: Gyourko et al. (2008) (real-faculty.wharton.upenn.edu/gyourko/land-use-survey/) and Albouy (2016).

Tax rate – Estimated average property tax rate index by MSA. The Lincoln Institute of Land Policy provides data on the property taxes paid per person within 150 or so "Fiscally Standardized Cities" (FSC) (www.lincolninst.edu/subcenters/fiscally-standardized-cities/.) While these do not correspond to MSAs, it is possible to match them with the main cities within many MSA's. We then estimated an index of the average property tax as a percentage of land value in 2010 using available data as follows:

$$\textit{Estimated MSA average property tax rate index} =$$

$$\frac{\text{Property Tax rate per person for FSC} \times \text{population density of MSA}}{\text{Estimated Land value per acre in MSA containing FSC}}.$$

C.2 Summary Statistics

Table 14 compares summary statistics from the full sample of MSAs and the sub-samples used in the various regressions in the main text and below. Note, in particular, that the sub-sample of 200 US cities used in Table 4 appears to be representative of the full sample of 366 MSAs. Table 15 contains unconditional correlations of the variables in the sub-samples.

Table 14: Summary Statistics

	Full Sample					Sub-Samples				
Variable	#	Mean	s.d.	Min	Max	#	Mean	s.d.	Min	Max
Ownership Rate (%)	366	67.74	5.53	51.19	81.09	200	67.42	5.05	51.19	78.26
Price/rent (est. val.)	366	23.18	5.73	13.12	44.14	200	22.61	5.20	13.12	42.91
Price/rent (p-asked)	366	21.38	6.76	8.67	54.20	200	20.84	5.88	8.67	43.77
Vacancy ratio	366	26.31	9.51	5.27	71.97	200	26.54	8.15	8.43	63.28
Real Med. Income	220	4.89	0.64	3.37	7.44	200	4.91	0.65	3.37	7.44
Gini index (%)	366	44.53	2.36	38.80	53.70	200	44.95	2.29	38.80	53.70
Median Age	366	36.68	4.18	24.30	54.80	200	36.39	3.54	25.50	48.60
Share with kids (%)	366	28.95	4.76	16.15	49.54	200	29.22	3.97	20.22	48.44
Real land price	202	2.31	1.93	0.49	13.81	200	2.25	1.76	0.49	11.20
Employment rate	366	58.60	5.19	41.20	72.50	200	59.75	4.96	42.70	72.50
Share with college	332	23.31	6.94	10.20	47.20	200	23.85	6.55	11.20	47.20
Debt/income	355	1.66	0.52	0.51	2.57	198	1.60	0.47	0.51	2.57
WRLURI	294	-0.10	0.80	-1.76	4.31	175	-0.22	0.75	-1.68	1.94
Tax Rate index	119	0.79	0.90	0.07	4.96	95	0.80	0.93	0.07	4.96

C.3 Alternative Empirical Specifications

In Tables 16 to 19, we consider a number of alternative controls that may be considered important in determining equilibrium housing market outcomes across cities. In each case, we present results for the ownership rate regression based on both OLS and 2SLS estimators, where the latter incudes both weighted density and average slope as instruments. As in the main text, we find no evidence supporting of the need to treat land price as endogenous in the other three regressions.

Table 16 documents the impact of adding two income-related MSA-level variables: the employment rate and the share of adults with a college degree. Not surprisingly, as may be seen from Table 15, these variables are highly correlated with median income. Controlling for the other variables, they do not add much explanatory power to the regression. That said, it appears that there is multi-colinearity arising in the regressions for the price-rent ratio due to the strong correlation between the employment rate and median income.

Table 17 documents the impact of adding the average household debt-to-income ratio. Unconditionally, this variable is strongly positively correlated with land prices and price-rent ratios. Controlling for other variables, the debt-to-income has no

Table 15: Unconditional Correlations

	Own. Rate	P/r est	P/r asked	Vac. Ratio	Med. Inc.	Gini index	Med. Age	Land Price	Share Kids	Emp. Rate	Coll. Deg.	Debt /inc.	WRL URI	Tax Rate
Ownership Rate	1.00													
P/r (est.value)	-0.24	1.00												
P/r (P-asked)	-0.22	0.82	1.00											
Vacancy ratio	-0.27	0.09	-0.03	1.00										
Median income	0.07	0.39	0.23	0.32	1.00									
Gini index	-0.37	0.06	0.17	-0.06	-0.25	1.00								
Median Age	0.56	0.22	0.13	-0.29	0.05	-0.18	1.00							
Share w/children	0.04	-0.20	-0.15	0.07	0.11	-0.04	-0.46	1.00						
Land price	-0.36	0.66	0.48	0.21	0.34	0.13	0.13	-0.05	1.00					
Employment rate	-0.02	0.21	0.08	0.32	0.67	-0.15	-0.12	0.09	0.05	1.00				
Share w/college	-0.31	0.44	0.30	0.29	0.54	0.14	-0.16	0.35	-0.26	0.52	1.00			
Debt/income	-0.19	0.57	0.54	0.10	0.09	-0.06	0.05	0.53	-0.09	-0.19	0.20	1.00		
WRLURI	-0.20	0.48	0.37	0.17	0.34	-0.04	0.07	0.42	0.01	0.10	0.33	0.43	1.00	
Tax Rate	-0.13	0.34	0.37	-0.20	0.23	0.23	0.33	0.14	-0.20	0.16	0.31	-0.11	0.38	1.00

Table 16: Impact of adding income-related variables

	Ownership Rate			**Price-Rent ratio**		**Vacancy ratio**
				Est. value	Price-asked	
Method	**OLS**	**2SLS**		**OLS**	**OLS**	**OLS**
		Main	**1st-stage**			
Real Income	1.336	1.464	0.123	0.248	0.131	2.900
Robust	(0.600**)	(0.646**)	(0.273)	(0.640)	(1.059)	(1.279**)
Clustered	(0.598**)	(0.648**)	(0.270)	(0.722)	(1.077)	(1.207**)
Gini	-0.439	-0.391***	-0.098	0.355	0.432	0.069
Robust	(0.128***)	(0.124)	(0.054*)	(0.177**)	(0.289)	(0.268)
Clustered	(0.135***)	(0.116***)	(0.050*)	(0.208*)	(0.322)	(0.297)
Median Age	1.069	1.194	0.096	0.192	0.092	-0.877
Robust	(0.082***)	(0.097***)	(0.036***)	(0.067***)	(0.147)	(0.262***)
Clustered	(0.081***)	(0.103***)	(0.034***)	(0.066***)	(0.124)	(0.255***)
Kids	0.408	0.445	0.004	0.048	0.031	-0.366
Robust	(0.075***)	(0.090***)	(0.031)	(0.058)	(0.107)	(0.220*)
Clustered	(0.062***)	(0.090***)	(0.036)	(0.066)	(0.133)	(0.216*)
College	0.043	0.132	0.069	0.111	0.085	-0.109
Robust	(0.054)	(0.065**)	(0.026***)	(0.052**)	(0.080)	(0.143)
Clustered	(0.048)	(0.067**)	(0.029**)	(0.072)	(0.108)	(0.140)
Employment	-0.083	-0.045	0.025	0.117	0.157	0.055
Robust	(0.077)	(0.078)	(0.026)	(0.072)	(0.111)	(0.197)
Clustered	(0.087)	(0.084)	(0.026)	(0.101)	(0.117)	(0.193)
Land Price	-0.969	-2.071		0.911	0.351	1.182
Robust	(0.174***)	(0.249***)		(0.192***)	(0.287)	(0.302)
Clustered	(0.160***)	(0.249***)		(0.300***)	(0.386)	(0.283***)
Density			0.030			
Robust			(0.007***)			
Clustered			(0.008***)			
Slope			0.129			
Robust			(0.053**)			
Clustered			(0.048***)			
Controls	Yes	Yes	Yes	Yes	Yes	Yes
$\mathbf{R^2}$	0.70	0.63	0.69	0.77	0.48	0.35
# obs	200	200	200	200	200	200

statistically significant association with the ownership rate across cities. It is, however, significantly and positively associated with the price-rent ratio and the vacancy ratio. Its addition does not, however, significantly affect the association of the dependent variables with the core explanatory variables, nor does it add much in the way of explanatory power.

The costs of supplying of housing is likely to reflect regulations of various kinds that affect housing markets (e.g. zoning, size limitations). A commonly referenced index of regulation across locations is the Wharton Residential Land Use Regulation Index (WRLURI) proposed by Gyourko et al. (2008). In Table 18, we assess the impact of adding this index to out baseline regressions (this reduces our sample of cities to 175). Controlling for other variables, we find no statistically significant association of the dependent variables with the WRLURI. From Table 15 we can see that there is a high positive unconditional correlation of land prices with the WRLURI: the role of regulations is already incorporated through the impact of high land prices.

A potentially important factor that varies considerably across U.S. cities are property taxes. Property taxes are incurred by both owner-occupiers and landlords and therefore can be thought of as part of the effective cost of supplying housing of a given quality, τ. While we have not been able to obtain detailed data on average property taxes at the MSA level, we compute a proxy for some cities (see Section C.1). Table 19 documents the results adding this estimated tax rate index to the baseline regression. The resulting restricted sample contains only 95 MSA's. Nevertheless, as may be seen, the qualitative nature of the results regarding the other factors are unchanged from Table 3. As is consistent with our model, both the ownership rate and the vacancy rate exhibit statistically significant negative partial correlations with tax rates while the price-rent ratio exhibits a positive relationship.

Table 20 provides estimates of the same regressions using data from the 2017 ACS (five-year average). The land price data, however, still corresponds to the 2006-10 period for which it was calculated by Albouy et al. (2018).

C.4 The price-rent ratio of equivalent units

Figures 21 and 22 plot two different approaches to calculating price-rent ratios using empirical and simulated data. The price-rent ratio of equivalent units is plotted relative to the ratio of average prices to average rents. A positive correlation between the two price-rent ratio concepts is evident both empirically (Figure 21), and in our

Table 17: Impact of adding average debt-to-income ratio

	Ownership Rate			**Price-Rent ratio**		**Vacancy ratio**
				Est. value	Price-asked	
Method	**OLS**	**2SLS**		**OLS**	**OLS**	**OLS**
		Main	**1st-stage**			
Real Income	1.245	1.999	0.649	1.521	1.487	2.549
Robust	(0.394***)	(0.390***)	(0.171***)	(0.362***)	(0.598**)	(0.797***)
Clustered	(0.501***)	(0.494***)	(0.179***)	(0.459***)	(0.544***)	(0.770***)
Gini	-0.367***	-0.222	-0.004	0.613	0.748	0.105
Robust	(0.114)	(0.114*)	(0.054)	(0.134***)	(0.224***)	(0.244)
Clustered	(0.123***)	(0.129*)	(0.054)	(0.152***)	(0.233***)	(0.295)
Median Age	1.062	1.113	0.051	0.121	0.053	-0.790
Robust	(0.066***)	(0.073***)	(0.028*)	(0.063*)	(0.120)	(0.258***)
Clustered	(0.074***)	(0.084***)	(0.021**)	(0.070*)	(0.115)	(0.276***)
Kids	0.399	0.389	-0.033	0.004	0.027	-0.276
Robust	(0.063***)	(0.072)	(0.023)	(0.049)	(0.090)	(0.211)
Clustered	(0.060***)	(0.072***)	(0.025)	(0.044)	(0.095)	(0.227)
DTI	1.191	2.103	0.987	2.873	4.310	1.283
Robust	(0.551**)	(0.610***)	(0.278***)	(0.544***)	(0.875***)	(1.367)
Clustered	(0.499**)	(0.737***)	(0.329***)	(0.878***)	(1.251***)	(1.396)
Land Price	-1.044	-1.981		0.834	0.149	1.035
Robust	(0.180***)	(0.226***)		(0.181***)	(0.286)	(0.297***)
Clustered	(0.134***)	(0.213***)		(0.251***)	(0.345)	(0.257***)
Density			0.032			
Robust			(0.007***)			
Clustered			(0.008***)			
Slope			0.071			
Robust			(0.057)			
Clustered			(0.048)			
Controls	Yes	Yes	Yes	Yes	Yes	Yes
$\mathbf{R}^2$	0.70	0.65	0.70	0.79	0.53	0.35
# obs	198	198	198	198	198	198

Table 18: Impacts of adding a regulation index

	Ownership Rate			Price-Rent ratio Est. value	Price-Rent ratio Price-asked	Vacancy ratio
Method	**OLS**	**2SLS Main**	**2SLS 1st-stage**	**OLS**	**OLS**	**OLS**
Real Income	1.011	1.839	0.738	1.558	1.792	2.540
Robust	(0.482**)	(0.438***)	(0.183***)	(0.431***)	(0.607***)	(0.887***)
Clustered	(0.695)	(0.547***)	(0.195***)	(0.497***)	(0.518***)	(0.878***)
Gini	-0.617	-0.476	-0.025	0.550	0.872	-0.099
Robust	(0.170***)	(0.129***)	(0.072)	(0.173***)	(0.236***)	(0.272)
Clustered	(0.153***)	(0.119***)	(0.075)	(0.192***)	(0.245***)	(0.323)
Median Age	1.002	1.055	0.029	0.071	-0.025	-0.927
Robust	(0.083***)	(0.084***)	(0.033)	(0.075)	(0.130)	(0.313***)
Clustered	(0.083***)	(0.087***)	(0.029)	(0.074)	(0.136)	(0.352**)
Kids	0.358	0.331	-0.055	-0.034	-0.027	-0.349
Robust	(0.069***)	(0.081***)	(0.028**)	(0.053)	(0.098)	(0.232)
Clustered	(0.062***)	(0.074***)	(0.030*)	(0.047)	(0.100)	(0.253)
WRLURI	-0.418	-0.164	0.062	0.468	0.358	0.839
Robust	(0.323)	(0.369)	(0.159)	(0.370)	(0.454)	(0.827)
Clustered	(0.337)	(0.402)	(0.218)	(0.391)	(0.442)	(0.808)
Land Price	-0.869	-1.879		0.899	0.176	0.924
Robust	(0.196***)	(0.247***)		(0.189***)	(0.250)	(0.299***)
Clustered	(0.192***)	(0.224***)		(0.289***)	(0.328)	(0.254***)
Density			0.030			
Robust			(0.007***)			
Clustered			(0.008***)			
Slope			0.242			
Robust			(0.066***)			
Clustered			(0.046***)			
Controls	Yes	Yes	Yes	Yes	Yes	Yes
$\mathbf{R}^2$	0.71	0.65	0.68	0.76	0.54	0.37
# obs	175	175	175	175	175	175

Table 19: Impact of adding estimated property tax rates

	Ownership Rate			Price-Rent ratio		Vacancy ratio
				Est. value	Price-asked	
Method	**OLS**	**2SLS**		**OLS**	**OLS**	**OLS**
		Main	**1st-stage**			
Real Income	1.849	2.204	1.443	1.991	2.731	1.596
Robust	(0.694***)	(0.602***)	(0.200***)	(0.581***)	(0.865***)	(0.947*)
Clustered	(0.722**)	(0.554***)	(0.177***)	(0.653***)	(0.819***)	(0.865*)
Gini	-0.398	-0.343	0.002	0.628	0.904	0.253
Robust	(0.223**)	(0.182*)	(0.075)	(0.225***)	(0.340***)	(0.276)
Clustered	(0.212**)	(0.168**)	(0.079)	(0.233***)	(0.359**)	(0.286)
Median Age	1.108	1.107	0.054	-0.115	-0.342	0.223
Robust	(0.207***)	(0.188***)	(0.052)	(0.199)	(0.295)	(0.348)
Clustered	(0.206***)	(0.189***)	(0.051)	(0.198)	(0.269)	(0.345)
Kids	0.442	0.414	-0.110	-0.155	-0.048	0.502
Robust	(0.163***)	(0.150***)	(0.039***)	(0.156)	(0.231)	(0.264*)
Clustered	(0.154***)	(0.144***)	(0.040***)	(0.123)	(0.211)	(0.247**)
Tax rate	-1.045	-1.102	-0.771	0.567	0.856	-1.567
Robust	(0.288***)	(0.308***)	(0.161***)	(0.376)	(0.503*)	(0.520***)
Clustered	(0.285***)	(0.298***)	(0.155***)	(0.405)	(0.520)	(0.527***)
Land Price	-1.136	-1.387		0.824	0.324	1.160
Robust	(0.236***)	(0.235***)		(0.209***)	(0.274)	(0.298***)
Clustered	(0.246***)	(0.240***)		(0.223***)	(0.268)	(0.270***)
Density			0.040			
Robust			(0.006***)			
Clustered			(0.006***)			
Slope			0.226			
Robust			(0.055***)			
Clustered			(0.057***)			
Controls	Yes	Yes	Yes	Yes	Yes	Yes
$\mathbf{R}^2$	0.68	0.67	0.86	0.84	0.69	0.44
# obs	95	95	95	95	95	95

Table 20: Results for 2017 ACS (5 year estimates)

	Ownership Rate			Price-Rent ratio		Vacancy ratio
				Est. value	Price-asked	
Method	**OLS**	**2SLS**		**OLS**	**OLS**	**OLS**
		Main	**1st-stage**			
Real Income	1.354	1.772	0.537	1.327	1.104	0.393
Robust	(0.313***)	(0.301***)	(0.150***)	(0.339***)	(0.634*)	(0.707)
Clustered	(0.454***)	(0.450***)	(0.189***)	(0.355***)	(0.581*)	(0.735)
Gini	-0.413	-0.306	0.017	0.587	0.682	-0.003
Robust	(0.110***)	(0.113***)	(0.063)	(0.146***)	(0.322**)	(0.340)
Clustered	(0.091***)	(0.088***)	(0.063)	(0.160***)	(0.346*)	(0.322)
Median Age	1.084	1.136	0.079	0.124	-0.016	-0.367
Robust	(0.060***)	(0.059***)	(0.028***)	(0.058*)	(0.132)	(0.181**)
Clustered	(0.059***)	(0.057***)	(0.029***)	(0.051**)	(0.131)	(0.169**)
Kids	0.107	0.109	-0.110	0.012	0.042	-0.021
Robust	(0.017***)	(0.018***)	(0.006)	(0.010)	(0.024*)	(0.035)
Clustered	(0.019***)	(0.019***)	(0.005)	(0.008)	(0.220*)	(0.030)
Land Price	-1.118	-1.174		0.864	1.664	0.413
Robust	(0.163***)	(0.217***)		(0.209***)	(0.402***)	(0.327)
Clustered	(0.128***)	(0.200***)		(0.315***)	(0.501***)	(0.351)
Density			0.029			
Robust			(0.007***)			
Clustered			(0.008***)			
Slope			0.129			
Robust			(0.051**)			
Clustered			(0.047***)			
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.73	0.71	0.66	0.73	0.49	0.16
# obs	198	198	198	198	198	198

model simulations with variation in construction/land costs (Figure 22). Moreover, while the relationship in the data is noisier than implied by the model, its slope is quite similar.

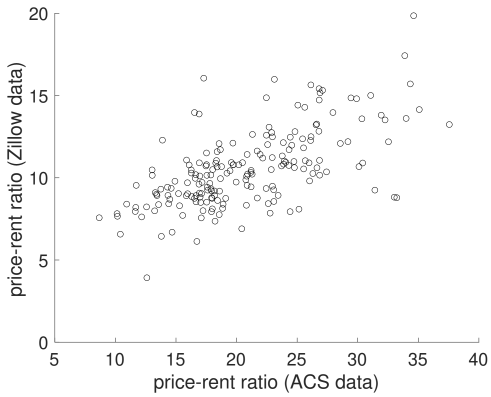


Figure 21: The two price-rent ratio concepts in the data: the price-rent ratio of the median quality house provided by Zillow and the ratio of average price-asked to average rents estimated from ACS data.

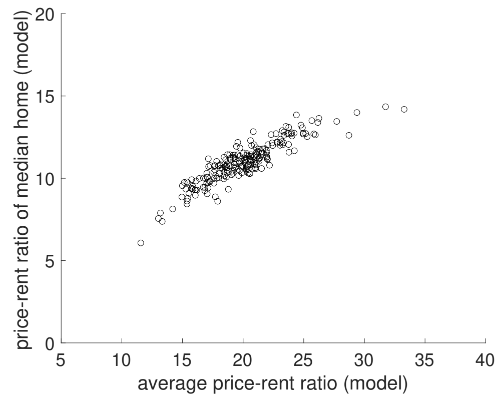


Figure 22: The two price-rent ratio concepts in the model: the price-rent ratio of the median quality house and the ratio of average prices to average rents.

D Mismatched Ownership While Searching/Selling

This section sketches out a version of the model in which owners remain in their houses while simultaneously searching to buy and listing for sale. Specifically, owners that experience the mobility shock choose to remain in their houses with probability $\xi \in [0, 1]$. With probability $1-\xi$, they rent in their preferred segment until they find a new house to purchase. Either choice is optimal provided the flow utility they receive as *mismatched owners* is such that they are exactly indifferent between remaining in and moving out.

The transition probabilities for households with permanent income level y are depicted in Figure 23. For homes bought and sold by households of income level y (i.e., of quality $q_s(y)$), Figure 24 depicts the transition probabilities. As in Section 3, these transition probabilities can be used to derive the relative measures of households and houses in an equilibrium with balanced growth, given exogenous parameter ξ. In this version of the model, prospective buyers include both renters and mismatched owners. Market tightness in the owner-occupied market, and market clearing in the rental market determine the allocation of newly constructed and available homes into the rental and sales markets.

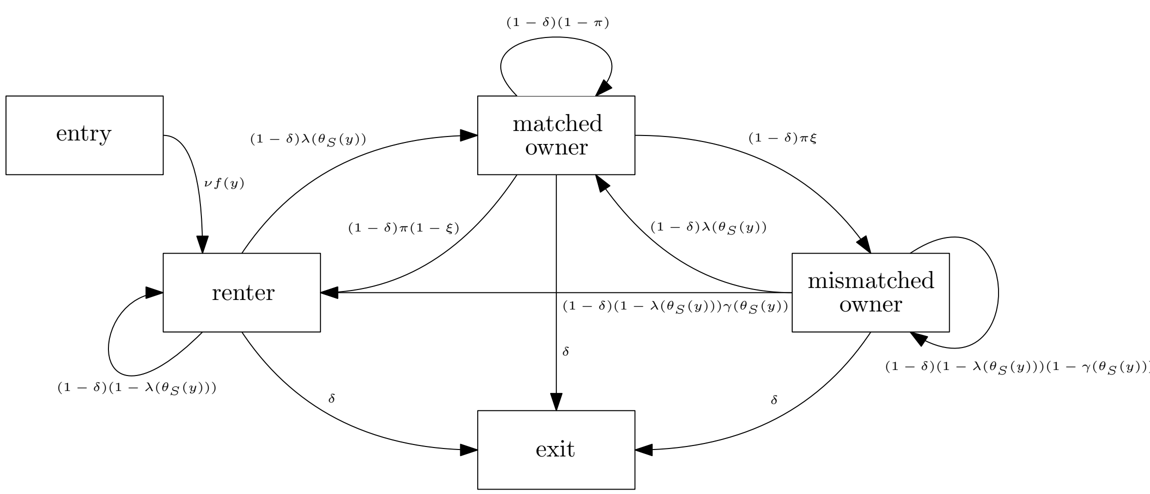


Figure 23: Transition probabilities for households with income level y.

We set $\xi = 1$ so that all households choose to remain in their houses after experiencing the mobility shock, at least until a new home is purchased or their current house sells. We then re-calibrate this version of the model to match the same targets as our baseline calibration in Section 4. Table 21 contains the calibrated parameter

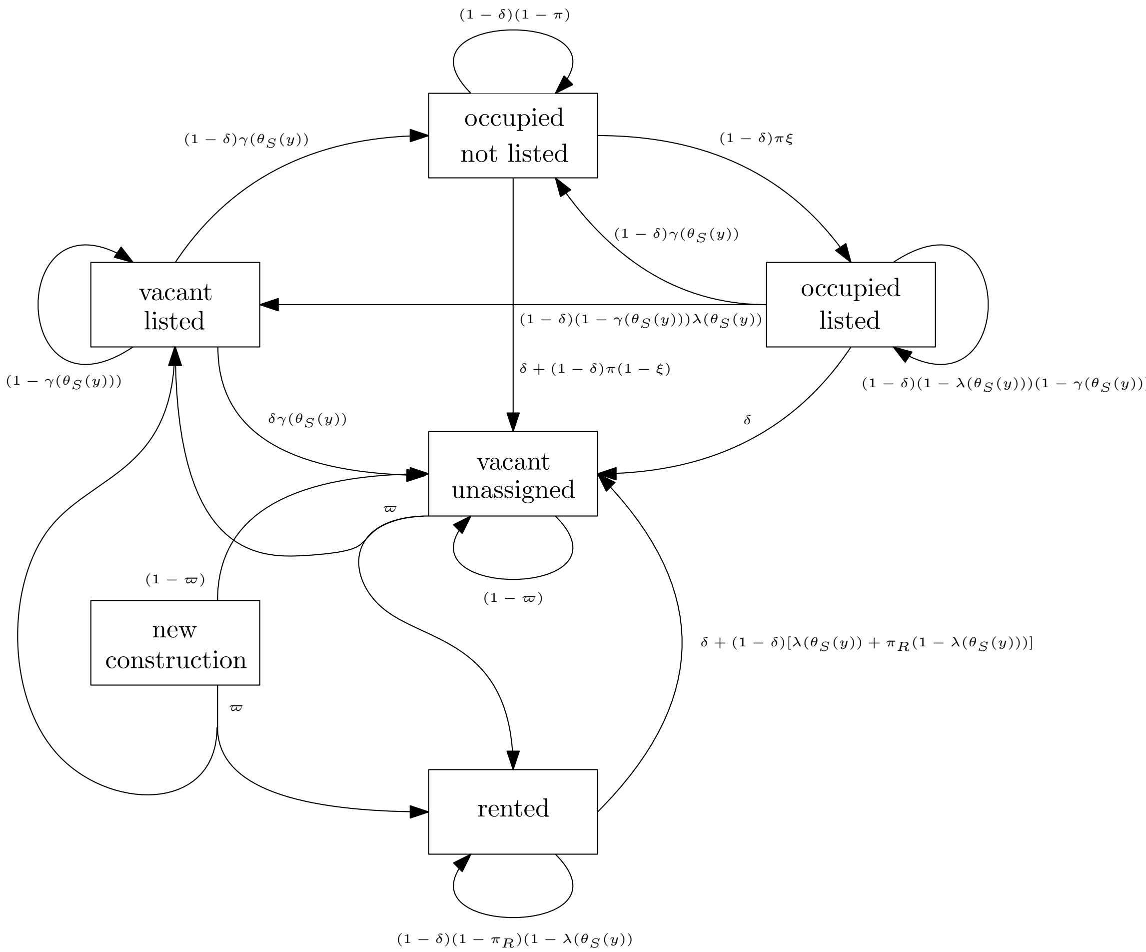


Figure 24: Transition probabilities for houses of quality $q_S(y)$.

values and economic statistics that are different from the baseline calibration. Figures 25, 26, 27, and 28 display the housing decisions, ownership rates, and price-rent ratios within the city. Figure 29 displays the histogram of housing market transactions by income for both sales and rentals. These within-city patterns are nearly indistinguishable from those in our baseline calibration in Section 4.

We use this re-calibrated version of the model to generate predicted cross-city variation in outcomes resulting from observed and inferred variation in MSA-level characteristics. As with the benchmark model, we consider the effects of changes in median income, income inequality (*i.e.* the Gini coefficient), median age (or population growth) and land/construction costs on home-ownership, the relative cost of owner-occupied *vs.* rental housing (measured by the ratio of average prices to average

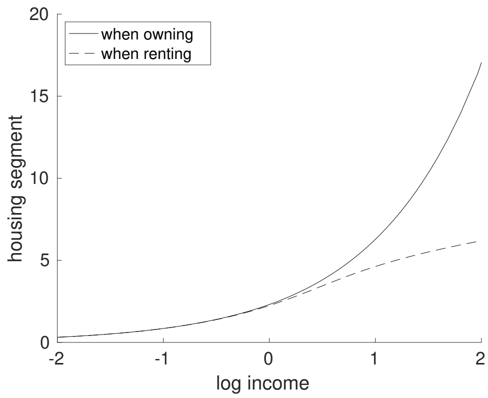


Figure 25: Market segment *vs.* income by housing tenure.

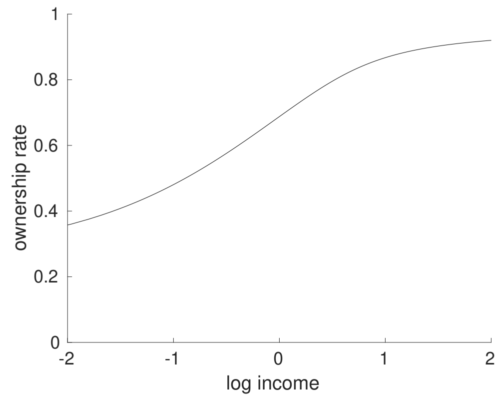


Figure 26: Ownership rate *vs.* household income.

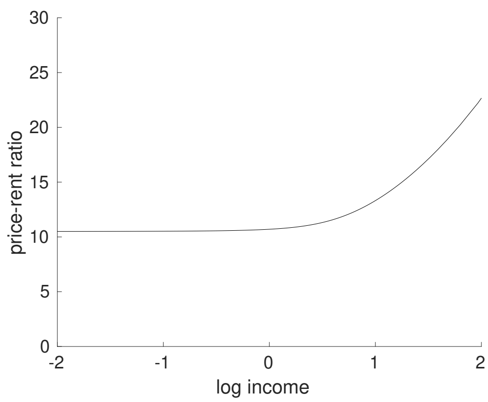


Figure 27: Price-rent ratio by household income.

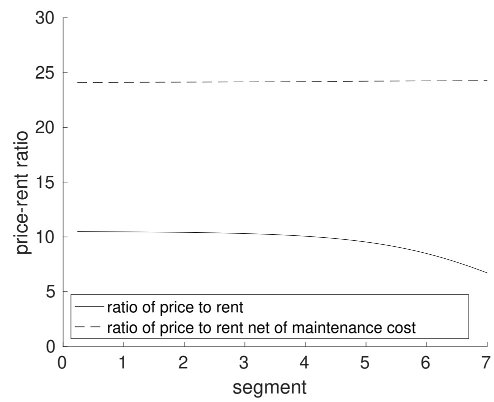


Figure 28: Price-rent ratio by housing segment.

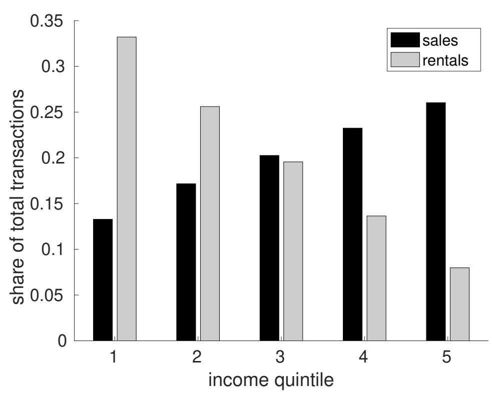
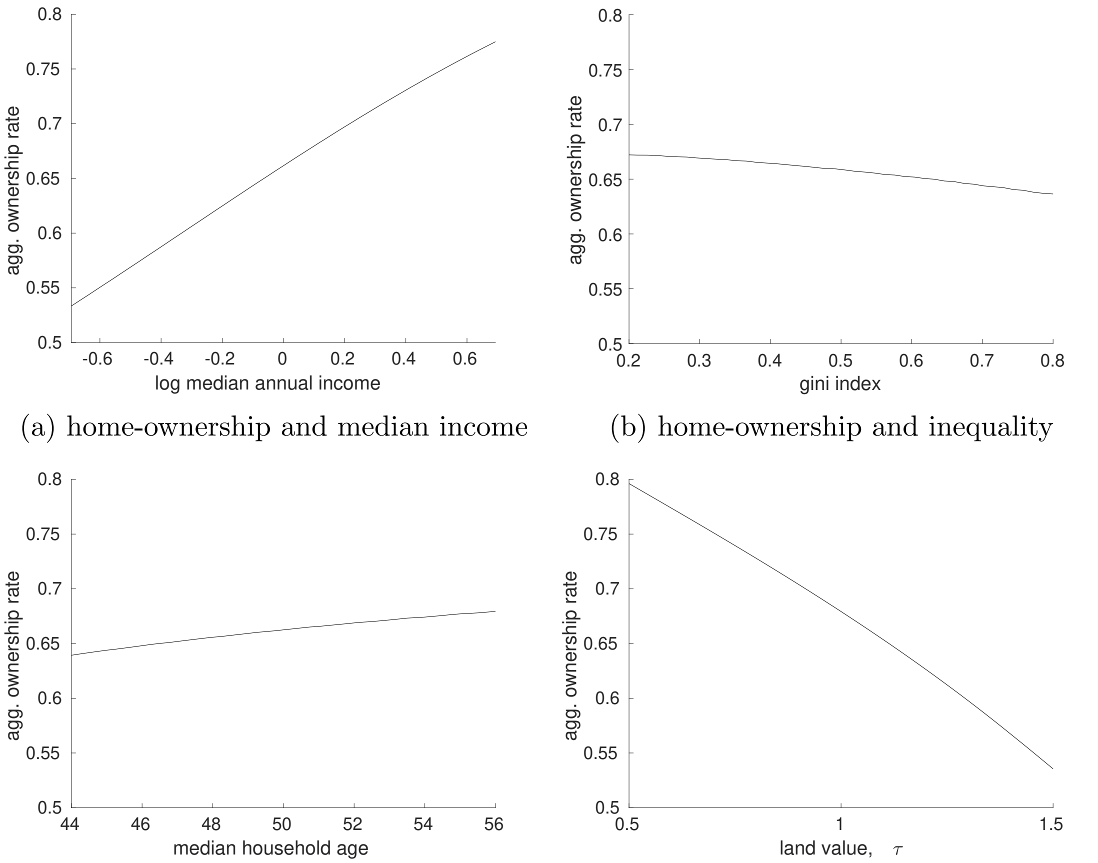


Figure 29: Histograms of house purchases and rentals by income.

Table 21: Calibrated Parameters

statistic	target value	model value	calibrated parameter	parameter value
overall vacancy rate (%)	11	7.8456	ϖ	0.4089
vacancy ratio (× 100)	26	25.077	π	0.0164
annual turnover rate (%)	15	15.185	π_R	0.0509
average time to sell (quarters)	1.33	1.3738	χ	0.3828
ratio of mean rent to median income	0.19	0.1915	α	0.2281
average price-rent ratio	21	20.899	ζ_0	0.0139
average ownership rate, Q1 (%)	44	43.75		
average ownership rate, Q2 (%)	56	56.65		
average ownership rate, Q3 (%)	67	66.89	ζ_1	0.0001
average ownership rate, Q4 (%)	77	76.88		
average ownership rate, Q5 (%)	87	86.46		

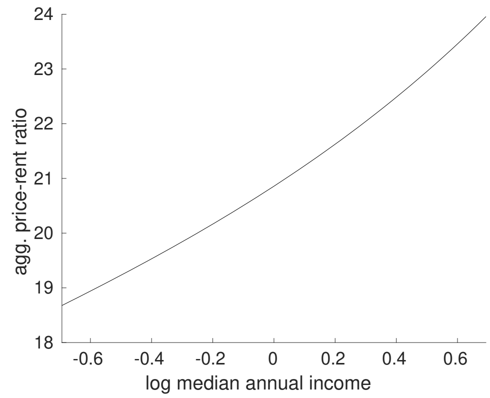
rents), and the vacancy ratio along the balanced growth path. These predictions are plotted in Figures 30, 31 and 32. Once again, the results are very similar to those obtained with the benchmark model. The only notable exception is the relationship between median household age and the aggregate price-rent ratio, plotted in Figure 31c. The impact of median age on the price-rent ratio is small in magnitude and its direction is not robust.



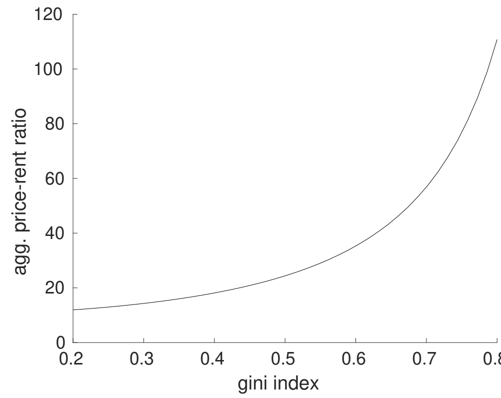
(a) home-ownership and median income (b) home-ownership and inequality

(c) home-ownership and median age (d) home-ownership and construction costs

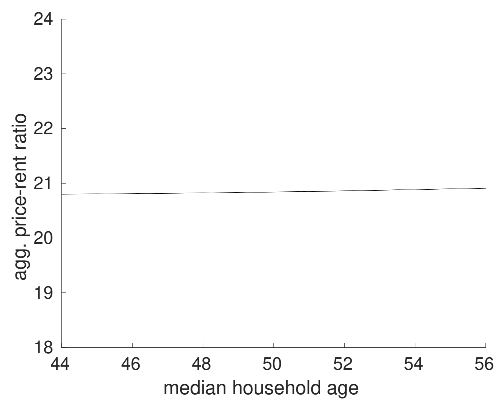
Figure 30: Cross-city relationships between home-ownership and fundamentals



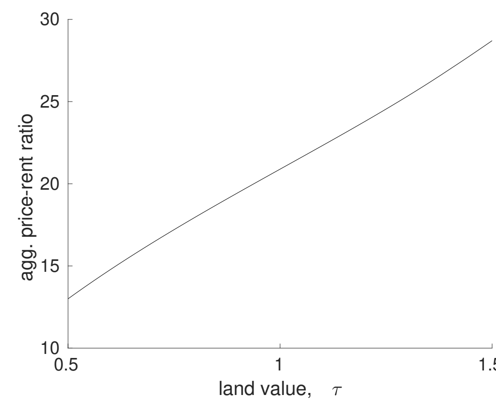
(a) price-rent ratio and median income



(b) price-rent ratio and inequality

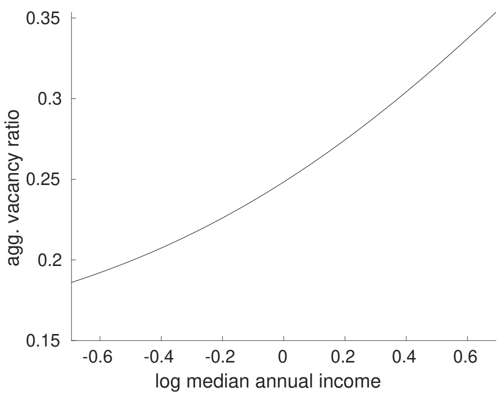


(c) price-rent ratio and median age

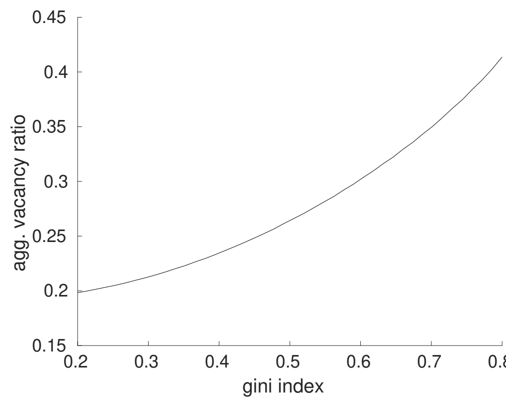


(d) price-rent ratio and construction costs

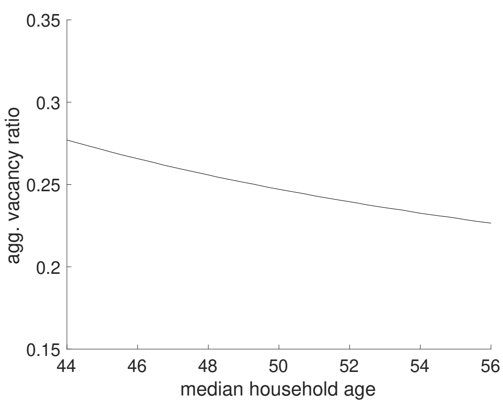
Figure 31: Cross-city relationships between the price-rent ratio and fundamentals



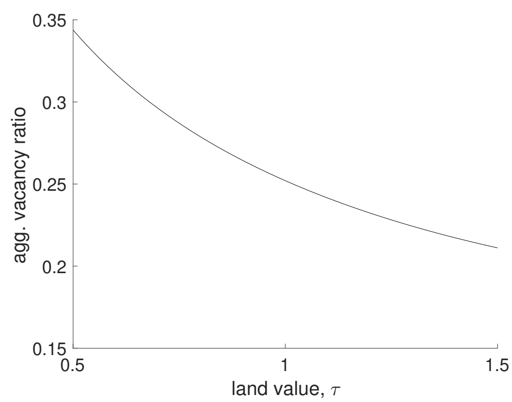
(a) vacancy ratio and median income



(b) vacancy ratio and inequality



(c) vacancy ratio and median age



(d) vacancy ratio and construction costs

Figure 32: Cross-city relationships between the vacancy ratio and fundamentals

E Optimality Conditions

E.1 Stationary formulation of the Renter's problem

Demonstrating uniqueness of the steady state equilibrium boils down to showing that the optimization problem of a renting household yields a unique vector of choices over the quality of rental and owned units and the submarket in which it searches, subject to the free entry conditions for home production and market entry. Conditional on these optimal choices, all households (whether owning or renting) with a given income choose the same consumption level under complete markets. In steady state, the renter's choice problem can be expressed as a stationary constrained optimization problem.

Proposition E1: *Given the conditional consumption function with complete markets, the optimization problem of a currently renting household (or new entrant) in steady state can be represented as a constrained optimization problem with Lagrangian given by:*

$$\begin{aligned}\hat{L}\left(q_R, q_S, \theta, p, \mu\right) = {} & \left[\Psi + (1-\delta)\,\lambda(\theta)\right] u(\hat{C}\left(y, q_R, q_S, p, \theta\right)) + \left[\rho + \delta + (1-\delta)\,\pi\right] h(0, q_R) \\ & + (1-\delta)\,\lambda(\theta)\left(h(1, q_S) - \left(\frac{\rho+\delta}{1+\rho}\right) W^R(w_R\left(y\right))\right) \\ & + \mu\left[\gamma(\theta)p - (\rho + \gamma(\theta))\left(\frac{\varpi + \rho}{\varpi}\right) T(q_S)\right] \end{aligned} \tag{E.1}$$

where μ denotes the Lagrange multiplier associated with the sellers' free entry condition and

$$\hat{C}\left(y, q_R, q_S, p, \theta\right) = y - \frac{\lambda(\theta)\left(\frac{\Psi}{1+\rho}p - A\left(q_S\right)\right) + \Psi x\left(q_R\right)}{\Psi + (1-\delta)\,\lambda(\theta)} \tag{E.2}$$

with $\Psi = \rho + \delta + (1-\delta)\,\pi$ and $A\left(q_S\right) = \left[\Psi - (1-\delta)\,\rho\right] T(q_S) - (1-\delta) Z_N(q_S)$.

Because of the dimensionality of this problem, it is convenient to first derive the optimality conditions for the choice of q_R and then substitute these into the remaining optimization problem of the renter over (q_S, p, θ). This is possible because the choice of q_R depends only on consumption.[60] Specifically, as in the main text, the optimal

[60] While this is equivalent to solving the problem in one step, it avoids having to derive and organize the determinant of a 5x5 bordered Hessian matrix when analyzing the second order conditions.

choice of quality in the rental market satisfies

$$-x'(\hat{q}_R)u_C + h_q(0, q_R) = 0. \tag{E.3}$$

The second order condition for this problem is

$$-x^{''}(\hat{q}_R)u_C + x'(\hat{q}_R)u_{CC} + h_{qq}(0, q_R) < 0 \tag{E.4}$$

which holds globally since $u_{CC} < 0$, $h_{qq}(0, q_R) < 0$ and $x''(\hat{q}_R) \geq 0$. Moreover, these conditions imply a positive monotonic, differentiable relationship between consumption and the choice of rental quality which we denote $\hat{q}_R(C)$ with

$$\hat{q}'_R(C) = -\frac{x'(\hat{q}_R(C))u_{CC}(C)}{x''(\hat{q}_R(C))u_C(C) - h_{qq}(0, \hat{q}_R(C))} > 0 \tag{E.5}$$

The Lagranian for the remainder of the renter's problem can then be expressed as

$$\begin{aligned} L\left(q_S, \theta, p, \mu\right) = {} & \left[\Psi + (1-\delta)\,\lambda(\theta)\right] u(C\left(y, q_S, p, \theta\right)) \\ & + (1-\delta)\,\lambda(\theta)\left(h(1, q_S) - \left(\frac{\rho+\delta}{1+\rho}\right) W^R(w_R\left(y\right))\right) \\ & + \mu\left[\gamma(\theta)p - (\rho + \gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right) T(q_S)\right] \end{aligned} \tag{E.6}$$

where $C\left(y, q_S, p, \theta\right)$ implicitly satisfies

$$C\left(y, q_S, p, \theta\right) = y - \frac{\lambda(\theta)\left(\frac{\Psi}{1+\rho}p - A\left(q_S\right)\right) + \Psi x\left(\hat{q}_R(C\left(y, q_S, p, \theta\right))\right)}{\Psi + (1-\delta)\,\lambda(\theta)}. \tag{E.7}$$

The first order necessary conditions for an optimum are

$$L_\mu = \gamma(\theta)p - (\rho + \gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau q_S = 0 \tag{E.8}$$

$$L_q = \left[\Psi + (1-\delta)\,\lambda(\theta)\right] u_C C_q + (1-\delta)\,\lambda(\theta) h_q(1, q_S) - \mu\left(\rho + \gamma(\theta)\right)\left(\frac{\varpi+\rho}{\varpi}\right)\tau = 0 \tag{E.9}$$

$$L_p = \left[\Psi + (1-\delta)\,\lambda(\theta)\right] u_C C_p + \mu\gamma(\theta) = 0 \tag{E.10}$$

$$L_\theta = \begin{pmatrix} (1-\delta)\left[u(C)+h(1,q_S)-\left(\frac{\rho+\delta}{1+\rho}\right)W^R(w_R(y))\right]\lambda'(\theta) \\ +\left[\Psi+(1-\delta)\lambda(\theta)\right]u_C C_\theta+\mu\left[p-\left(\frac{\varpi+\rho}{\varpi}\right)\tau q_S\right]\gamma'(\theta) \end{pmatrix} = 0 \tag{E.11}$$

where

$$C_q = \frac{\Omega(C,\theta)\lambda(\theta)A'(q_S)}{\Psi+(1-\delta)\lambda(\theta)} \tag{E.12}$$

$$C_p = -\frac{\Omega(C,\theta)\lambda(\theta)\Psi}{(1+\rho)\left[\Psi+(1-\delta)\lambda(\theta)\right]} < 0 \tag{E.13}$$

$$C_\theta = -\Psi\Omega(C,\theta)\left(\frac{\frac{\Psi}{1+\rho}p - A(q_S) - (1-\delta)x(\hat{q}_R(C))}{\left[\Psi+(1-\delta)\lambda(\theta)\right]^2}\right)\lambda'(\theta) > 0 \tag{E.14}$$

with

$$\Omega(C,\theta) = \frac{\Psi+(1-\delta)\lambda(\theta)}{\Psi(1+\varepsilon(C))+(1-\delta)\lambda(\theta)} \in (0,1) \tag{E.15}$$

and

$$\varepsilon(C) = x'(\hat{q}_R)\hat{q}'_R(C) \tag{E.16}$$

It is straightforward to verify that these conditions yield the same first order condition as in the main text.

The sufficient conditions for this problem to have a unique maximum are that the principle minors of the following bordered Hessian alternate in sign starting with a positive value:

$$H = \begin{bmatrix} 0 & L_{\mu q} & L_{\mu p} & L_{\mu\theta} \\ L_{\mu q} & L_{qq} & L_{qp} & L_{q\theta} \\ L_{\mu p} & L_{qp} & L_{pp} & L_{p\theta} \\ L_{\mu\theta} & L_{q\theta} & L_{p\theta} & L_{\theta\theta} \end{bmatrix}. \tag{E.17}$$

Here

$$L_{\mu q} = -(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau < 0 \tag{E.18}$$

$$L_{\mu p} = \gamma(\theta) > 0 \tag{E.19}$$

$$L_{\mu\theta} = \left(p - \left(\frac{\varpi+\rho}{\varpi}\right)T(q_S)\right)\gamma'(\theta) > 0 \tag{E.20}$$

$$
\begin{aligned}
L_{qq} &= [\Psi + (1-\delta)\,\lambda(\theta)]\left[u_{CC}C_q^2 + u_C C_{qq}\right] + (1-\delta)\,\lambda(\theta) h_{qq}\,(1, q_S) \\
&\quad - \mu\,(\rho + \gamma(\theta))\left(\frac{\varpi + \rho}{\varpi}\right) T''(q_S)
\end{aligned} \tag{E.21}
$$

$$
L_{qp} = [\Psi + (1-\delta)\,\lambda(\theta)]\,[u_{CC}C_q C_p + u_C C_{qp}] > 0 \tag{E.22}
$$

$$
\begin{aligned}
L_{q\theta} &= [\Psi + (1-\delta)\,\lambda(\theta)]\,[u_{CC}C_q C_\theta + u_C C_{q\theta}] \\
&\quad + (1-\delta)\,[u_C C_q + h_q\,(1, q_S)]\,\lambda'(\theta) - \mu\left(\frac{\varpi + \rho}{\varpi}\right) T'(q_S)\gamma'(\theta) < 0
\end{aligned} \tag{E.23}
$$

$$
L_{pp} = [\Psi + (1-\delta)\,\lambda(\theta)]\left[u_{CC}C_p^2 + u_C C_{pp}\right] < 0 \tag{E.24}
$$

$$
\begin{aligned}
L_{p\theta} &= [\Psi + (1-\delta)\,\lambda(\theta)]\,[u_{CC}C_p C_\theta + u_C C_{p\theta}] \\
&\quad + (1-\delta) u_C C_p \lambda'(\theta) + \mu\gamma'(\theta) > 0
\end{aligned} \tag{E.25}
$$

$$
\begin{aligned}
L_{\theta\theta} &= [\Psi + (1-\delta)\,\lambda(\theta)]\left[u_{CC}C_\theta^2 + u_C C_{\theta\theta}\right] + 2\,(1-\delta)\,\lambda'(\theta) u_C C_\theta \\
&\quad + (1-\delta)\left[u(C) + h(1, q_S) - \left(\frac{\rho + \delta}{1 + \rho}\right) W^R(w_R\,(y))\right]\lambda''(\theta) \\
&\quad + \mu\left[p - \left(\frac{\varpi + \rho}{\varpi}\right) T\,(q_S)\right]\gamma''(\theta)
\end{aligned} \tag{E.26}
$$

where

$$
C_{qq} = \frac{\Omega_C}{\Omega} C_q^2 + C_q \frac{A_{qq}}{A_q} < 0 \tag{E.27}
$$

$$
C_{qp} = \frac{\Omega_C}{\Omega} C_q C_p > 0 \tag{E.28}
$$

$$
C_{q\theta} = \left(\frac{\Psi}{\Psi + (1-\delta)\,\lambda(\theta)}\right) C_q \frac{\lambda'(\theta)}{\lambda(\theta)} + \frac{\Omega_C}{\Omega} C_q C_\theta < 0 \tag{E.29}
$$

$$
C_{pp} = \frac{\Omega_C}{\Omega} C_p^2 < 0 \tag{E.30}
$$

$$
C_{p\theta} = \left(\frac{\Psi}{\Psi + (1-\delta)\,\lambda(\theta)}\right) C_p \frac{\lambda'(\theta)}{\lambda(\theta)} + \frac{\Omega_C}{\Omega} C_p C_\theta > 0 \tag{E.31}
$$

$$
C_{\theta\theta} = C_\theta\left[\frac{\lambda''(\theta)}{\lambda'(\theta)} - \frac{2\,(1-\delta)\,\lambda'(\theta)}{\Psi(1 + \varepsilon(C)) + (1-\delta)\,\lambda(\theta)}\right] + \frac{\Omega_C}{\Omega} C_\theta^2 \tag{E.32}
$$

with

$$
\Omega_C = -\Omega \frac{\Psi \varepsilon'(C)}{\Psi(1 + \varepsilon(C)) + (1-\delta)\,\lambda(\theta)} \tag{E.33}
$$

and

$$\varepsilon'(C) = x'(\hat{q}_R)\hat{q}''_R(C) + x''(\hat{q}_R)\left[\hat{q}'_R(C)\right]^2 \tag{E.34}$$

E.2 Verifying that the conditions hold

We now check whether the second order conditions hold for the functional forms we have assumed. First note that with linearity of the production and owner maintance cost functions we have $T(q_S) = \tau q_S$, $Z_N = \xi_0 q_S$ and

$$A\left(q_S\right) = B q_S \tag{E.35}$$

where $B = \left[\Psi - (1-\delta)\,\rho\right]\tau - (1-\delta)\xi_0 > 0$. It follows that we can write the first-order derivatives of the consumption function as

$$C_q = \frac{\Omega\left(C,\theta\right)\lambda(\theta)B}{\Psi + (1-\delta)\,\lambda(\theta)} \tag{E.36}$$

$$C_p = -\frac{\Psi}{(1+\rho)B}C_q \tag{E.37}$$

$$C_\theta = -\frac{\Psi}{B}\left(\frac{\Psi\frac{p}{1+\rho} - A\left(q_S\right) - (1-\delta)x\left(\hat{q}_R(C)\right)}{\Psi + (1-\delta)\,\lambda(\theta)}\right)\frac{\lambda'(\theta)}{\lambda(\theta)}C_q \tag{E.38}$$

Given that the utility function is logarithmic w.r.t. consumption, we can express the second-order derivatives of the Lagrangian as

$$L_{\mu q} = -\left(\rho + \gamma(\theta)\right)\left(\frac{\varpi + \rho}{\varpi}\right)\tau \tag{E.39}$$

$$L_{\mu p} = \gamma(\theta) > 0 \tag{E.40}$$

$$L_{\mu\theta} = \left(\frac{\varpi + \rho}{\varpi}\right)\rho\tau q_S\frac{\gamma'(\theta)}{\gamma\left(\theta\right)} > 0 \tag{E.41}$$

$$
\begin{aligned}
L_{qq} &= \Phi\frac{C_q}{C}\left[-\frac{C_q}{C}+\frac{C_{qq}}{C_q}\right]+(1-\delta)\,\lambda(\theta)h_{qq}\left(1,q_S\right)<0\\
L_{qp} &= \Phi\frac{C_q}{C}\left[-\frac{C_p}{C}+\frac{C_{qp}}{C_q}\right]>0\\
L_{q\theta} &= \Phi\frac{C_q}{C}\left[-\frac{C_\theta}{C}+\frac{C_{q\theta}}{C_q}\right]+(1-\delta)\left[(1-\alpha)\frac{C_q}{C}+h_q\left(1,q_S\right)\right]\lambda'(\theta)\\
&\quad-\mu\left(\frac{\varpi+\rho}{\varpi}\right)\tau\gamma'(\theta)
\end{aligned}
$$

$$
\begin{aligned}
L_{pp} &= \Phi\frac{C_p}{C}\left[-\frac{C_p}{C}+\frac{C_{pp}}{C_p}\right]<0\\
L_{p\theta} &= \Phi\frac{C_p}{C}\left[-\frac{C_\theta}{C}+\frac{C_{p\theta}}{C_p}\right]+(1-\delta)(1-\alpha)\frac{C_p}{C}\lambda'(\theta)+\mu\gamma'(\theta)>0
\end{aligned}
$$

$$
\begin{aligned}
L_{\theta\theta} &= \Phi\frac{C_\theta}{C}\left[-\frac{C_\theta}{C}+\frac{C_{\theta\theta}}{C_\theta}\right]+2\left(1-\delta\right)\lambda'(\theta)(1-\alpha)\frac{C_\theta}{C}\\
&\quad+(1-\delta)\left[u(C)+h(1,q_S)-\left(\frac{\rho+\delta}{1+\rho}\right)W^R(w_R\left(y\right))\right]\lambda''(\theta)\\
&\quad+\mu\left[p-\left(\frac{\varpi+\rho}{\varpi}\right)\tau q_S\right]\gamma''(\theta)
\end{aligned}
$$

where $\Phi=(1-\alpha)\left[\Psi+(1-\delta)\,\lambda(\theta)\right]$.

The second order derivatives of the consumption function satisfy

$$
\frac{C_{qq}}{C_q}=\frac{\Omega_C}{\Omega}C_q \tag{E.42}
$$

$$
\frac{C_{qp}}{C_q}=\frac{\Omega_C}{\Omega}C_p \tag{E.43}
$$

$$
\frac{C_{q\theta}}{C_q}=\left(\frac{\Psi}{\Psi+(1-\delta)\,\lambda(\theta)}\right)\frac{\lambda'(\theta)}{\lambda(\theta)}+\frac{\Omega_C}{\Omega}C_\theta \tag{E.44}
$$

$$
\frac{C_{pq}}{C_p}=\frac{\Omega_C C_q}{\Omega} \tag{E.45}
$$

$$
\frac{C_{pp}}{C_p}=\frac{\Omega_C C_p}{\Omega} \tag{E.46}
$$

$$
\frac{C_{p\theta}}{C_p}=\left(\frac{\Psi}{\Psi+(1-\delta)\,\lambda(\theta)}\right)\frac{\lambda'(\theta)}{\lambda(\theta)}+\frac{\Omega_C}{\Omega}C_\theta \tag{E.47}
$$

$$\frac{C_{\theta\theta}}{C_\theta} = \left[\frac{\lambda''(\theta)}{\lambda'(\theta)} - \frac{2\,(1-\delta)\,\lambda'(\theta)}{\Psi(1+\varepsilon(C)) + (1-\delta)\,\lambda(\theta)}\right] + \frac{\Omega_C}{\Omega} C_\theta \tag{E.48}$$

Letting $\Theta(C) = \Phi\left[1 - \frac{\Omega_C C}{\Omega}\right]$, we can therefore further simplify the second-order derivatives of the Lagrangian as

$$L_{qq} = -\Theta(C)\left(\frac{C_q}{C}\right)^2 + (1-\delta)\,\lambda(\theta) h_{qq}\,(1, q_S) < 0 \tag{E.49}$$

$$L_{qp} = -\Theta(C)\frac{C_q}{C}\frac{C_p}{C} > 0 \tag{E.50}$$

$$L_{q\theta} = -\Theta(C)\frac{C_q}{C}\frac{C_\theta}{C} + \Phi\frac{C_q}{C}\frac{\lambda'(\theta)}{\lambda(\theta)} + (1-\delta) h_q\,(1, q_S)\,\lambda'(\theta) - \mu\left(\frac{\varpi+\rho}{\varpi}\right)\tau\gamma'(\theta) < 0 \tag{E.51}$$

$$L_{pp} = -\Theta(C)\left(\frac{C_p}{C}\right)^2 < 0 \tag{E.52}$$

$$L_{p\theta} = -\Theta(C)\frac{C_p}{C}\frac{C_\theta}{C} + \Phi\frac{C_p}{C}\frac{\lambda'(\theta)}{\lambda(\theta)} + \mu\gamma'(\theta) > 0 \tag{E.53}$$

$$\begin{aligned} L_{\theta\theta} = &-\Theta(C)\left(\frac{C_\theta}{C}\right)^2 + \frac{C_\theta}{C}\left[\Phi\frac{\lambda''(\theta)}{\lambda'(\theta)} + 2\,(1-\delta)\,(1-\alpha)\,(1-\Omega)\,\lambda'(\theta)\right] \\ &+ (1-\delta)\left[u(C) + h(1, q_S) - \left(\frac{\rho+\delta}{1+\rho}\right) W^R(w_R\,(y))\right]\lambda''(\theta) \\ &+ \mu\left[p - \left(\frac{\varpi+\rho}{\varpi}\right)\tau q_S\right]\gamma''(\theta) \end{aligned} \tag{E.54}$$

The sufficient conditions for a maximum are

$$|H_1| = \begin{vmatrix} 0 & L_{\mu q} & L_{\mu p} \\ L_{\mu q} & L_{qq} & L_{qp} \\ L_{\mu p} & L_{qp} & L_{pp} \end{vmatrix} > 0 \tag{E.55}$$

and

$$|H_2| = \begin{vmatrix} 0 & L_{\mu q} & L_{\mu p} & L_{\mu\theta} \\ L_{\mu q} & L_{qq} & L_{qp} & L_{q\theta} \\ L_{\mu p} & L_{qp} & L_{pp} & L_{p\theta} \\ L_{\mu\theta} & L_{q\theta} & L_{p\theta} & L_{\theta\theta} \end{vmatrix} < 0 \tag{E.56}$$

Evaluating (E.55) yields

$$|H_1| = -L_{\mu q}^2 L_{pp} - L_{\mu p}^2 L_{qq} + 2L_{\mu q}L_{\mu p}L_{qp}$$

Substituting using (E.39), (E.40), (E.49), (E.50) and (E.52) yields

$$\begin{aligned}
|H_1| = &\left((\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\right)^2 \Theta\left(\frac{C_p}{C}\right)^2 + \gamma(\theta)^2\left(\Theta\left(\frac{C_q}{C}\right)^2 - (1-\delta)\,\lambda(\theta)h_{qq}\,(1,q_S)\right)\\
&- 2\left((\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\right)\gamma(\theta)\Theta\frac{C_q}{C}\frac{(-C_p)}{C}\\
&= \Theta\left((\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\left(\frac{-C_p}{C}\right) - \gamma(\theta)\left(\frac{C_q}{C}\right)\right)^2\\
&- \gamma^2(\theta)\,(1-\delta)\,\lambda(\theta)h_{qq}\,(1,q_S)\,.
\end{aligned}$$

Since $h_{qq}\,(1,q_S) < 0$, it follows that $|H_1| > 0$.

Evaluating (E.56) yields

$$\begin{aligned}
|H_2| = &-L_{\mu q}^2\left(L_{pp}L_{\theta\theta} - L_{p\theta}^2\right) + L_{\mu q}L_{qp}\left(L_{\mu p}L_{\theta\theta} - L_{\mu\theta}L_{p\theta}\right) - L_{\mu q}L_{q\theta}\left(L_{\mu p}L_{p\theta} - L_{\mu\theta}L_{pp}\right)\\
&+ L_{\mu p}L_{\mu q}\left(L_{qp}L_{\theta\theta} - L_{q\theta}L_{p\theta}\right) - L_{\mu p}L_{qq}\left(L_{\mu p}L_{\theta\theta} - L_{\mu\theta}L_{p\theta}\right) + L_{\mu p}L_{q\theta}\left(L_{\mu p}L_{q\theta} - L_{\mu\theta}L_{qp}\right)\\
&- L_{\mu\theta}L_{\mu q}\left(L_{qp}L_{p\theta} - L_{q\theta}L_{pp}\right) + L_{\mu\theta}L_{qq}\left(L_{\mu p}L_{p\theta} - L_{\mu\theta}L_{pp}\right) - L_{\mu\theta}L_{qp}\left(L_{\mu p}L_{q\theta} - L_{\mu\theta}L_{qp}\right)\\
|H_2| = &-L_{\mu q}^2\left(L_{pp}L_{\theta\theta} - L_{p\theta}^2\right) + 2L_{\mu q}L_{qp}L_{\mu p}L_{\theta\theta} - 2L_{\mu q}L_{qp}L_{\mu\theta}L_{p\theta} - 2L_{\mu q}L_{q\theta}L_{\mu p}L_{p\theta} + 2L_{\mu q}L_{q\theta}L_{\mu\theta}L_{pp}\\
&- L_{\mu p}^2\left(L_{qq}L_{\theta\theta} - L_{q\theta}^2\right) + 2L_{\mu p}L_{qq}L_{\mu\theta}L_{p\theta} - 2L_{\mu p}L_{q\theta}L_{\mu\theta}L_{qp} - L_{\mu\theta}^2\left(L_{qq}L_{pp} - L_{qp}^2\right)
\end{aligned}$$

It is convenient to organize $|H_2|$ into four components which are written below on separate lines:

$$\begin{aligned}
|H_2| = &\,|H_1|\,L_{\theta\theta} &\text{(C1)}\\
&+ 2L_{\mu\theta}\left[L_{p\theta}\left(L_{\mu p}L_{qq} - L_{\mu q}L_{qp}\right) + L_{q\theta}\left(L_{\mu q}L_{pp} - L_{\mu p}L_{qp}\right)\right] &\text{(C2)}\\
&- L_{\mu\theta}^2\left(L_{qq}L_{pp} - L_{qp}^2\right) &\text{(C3)}\\
&+ \left(L_{p\theta}L_{\mu q} - L_{q\theta}L_{\mu p}\right)^2 &\text{(C4)}
\end{aligned}$$

We show that components C1, C2 and C3 must each be negative.

Component C1: The following result proves useful:

Lemma E1: If the matching function satisfies

$$\frac{\lambda''(\theta)}{-\lambda'(\theta)} + \frac{\gamma''(\theta)}{\gamma'(\theta)} \leq 0$$

then $L_{\theta\theta} < 0$.

For the assumed matching function, it is straightforward to show that this condition holds with equality. It follows that component C1 is negative since $|H_1| > 0$.

Component C2: Using (E.39), (E.40), (E.49), (E.50) and (E.52) the following products of second-order derivatives can be expressed as follows:

$$\begin{aligned}
L_{\mu p}L_{qq} &= \gamma(\theta)\left(-\Theta\left(\frac{C_q}{C}\right)^2 + (1-\delta)\,\lambda(\theta)h_{qq}\,(1,q_S)\right) < 0 \\
L_{\mu q}L_{qp} &= -\,(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\left(-\Theta\frac{C_q}{C}\frac{C_p}{C}\right) \\
&= -\frac{\Psi}{(1+\rho)B}\,(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\Theta\left(\frac{C_q}{C}\right)^2 < 0 \\
L_{\mu q}L_{pp} &= (\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\Theta\left(\frac{C_p}{C}\right)^2 \\
&= (\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\left(\frac{\Psi}{(1+\rho)B}\right)^2\Theta\left(\frac{C_q}{C}\right)^2 > 0 \\
L_{\mu p}L_{qp} &= \gamma(\theta)\left(-\Theta\frac{C_q}{C}\frac{C_p}{C}\right) = \gamma(\theta)\frac{\Psi}{(1+\rho)B}\Theta\left(\frac{C_q}{C}\right)^2 > 0
\end{aligned}$$

It follows that

$$\begin{aligned}
L_{\mu p}L_{qq} - L_{\mu q}L_{qp} &= \gamma(\theta)\left(-\Theta\left(\frac{C_q}{C}\right)^2 + (1-\delta)\,\lambda(\theta)h_{qq}\,(1,q_S)\right) \\
&\quad - \frac{\Psi}{(1+\rho)B}\,(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\Theta\left(\frac{C_q}{C}\right)^2 \\
&= \left[\frac{\Psi}{(1+\rho)B}\,(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau - \gamma(\theta)\right]\Theta\left(\frac{C_q}{C}\right)^2 + (1-\delta)\,\lambda(\theta)h_{qq}\,(1,q_S)
\end{aligned}$$

and

$$\begin{aligned}
L_{\mu q}L_{pp} - L_{\mu p}L_{qp} &= (\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\Theta\left(\frac{\Psi}{(1+\rho)B}\right)^2\left(\frac{C_q}{C}\right)^2 \\
&\quad - \gamma(\theta)\frac{\Psi}{(1+\rho)B}\Theta\left(\frac{C_q}{C}\right)^2 \\
&= \frac{\Psi}{(1+\rho)B}\left[\frac{\Psi}{(1+\rho)B}\,(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau - \gamma(\theta)\right]\Theta\left(\frac{C_q}{C}\right)^2
\end{aligned}$$

The component C2 is equal to $2L_{\mu\theta}$ mutiplied by

$$L_{p\theta}\left(L_{\mu p}L_{qq}-L_{\mu q}L_{qp}\right)+L_{q\theta}\left(L_{\mu q}L_{pp}-L_{\mu p}L_{qp}\right)$$

Using the above, this can be expressed as

$$\left(L_{p\theta}+\frac{\Psi}{(1+\rho)B}L_{q\theta}\right)\left[(\rho+\gamma(\theta))\left(\frac{\varpi+\rho}{\varpi}\right)\tau\frac{\Psi}{(1+\rho)B}-\gamma(\theta)\right]\Theta(C)\left(\frac{C_q}{C}\right)^2$$
$$+L_{p\theta}\left(1-\delta\right)\lambda(\theta)\gamma(\theta)h_{qq}\left(1,q_S\right)$$

The first term in brackets is

$$\begin{aligned}
L_{p\theta}+\frac{\Psi}{(1+\rho)B}L_{q\theta} &= -\Theta\frac{C_p}{C}\frac{C_\theta}{C}+\Phi\frac{C_p}{C}\frac{\lambda'(\theta)}{\lambda(\theta)}+\mu\gamma'(\theta)\\
&\quad+\frac{\Psi}{(1+\rho)B}\left(\begin{array}{c}-\Theta\frac{C_q}{C}\frac{C_\theta}{C}+\Phi\frac{C_q}{C}\frac{\lambda'(\theta)}{\lambda(\theta)}+(1-\delta)h_q\left(1,q_S\right)\lambda'(\theta)\\ -\mu\left(\frac{\varpi+\rho}{\varpi}\right)\tau\gamma'(\theta)\end{array}\right)\\
&=\Theta\frac{\Psi}{(1+\rho)B}\frac{C_q}{C}\frac{C_\theta}{C}-\Phi\frac{\Psi}{(1+\rho)B}\frac{C_q}{C}\frac{\lambda'(\theta)}{\lambda(\theta)}+\mu\gamma'(\theta)\\
&\quad+\frac{\Psi}{(1+\rho)B}\left(\begin{array}{c}-\Theta\frac{C_q}{C}\frac{C_\theta}{C}+\Phi\frac{C_q}{C}\frac{\lambda'(\theta)}{\lambda(\theta)}+(1-\delta)h_q\lambda'(\theta)\\ -\mu\left(\frac{\varpi+\rho}{\varpi}\right)\tau\gamma'(\theta)\end{array}\right)\\
&=\frac{\Psi}{(1+\rho)B}(1-\delta)h_q\left(1,q_S\right)\lambda'(\theta)-\left[\frac{\Psi}{(1+\rho)B}\left(\frac{\varpi+\rho}{\varpi}\right)\tau-1\right]\mu\gamma'(\theta)<0
\end{aligned}$$

This expression is negative since $\frac{\varpi+\rho}{(1+\rho)\varpi}>1$ and $\Psi\tau>B=\Psi\tau-(1-\delta)\,\rho\tau-(1-\delta)\xi_0$. Since $L_{\mu\theta}>0$ and $h_{qq}\left(1,q_S\right)<0$, it follows that C2 must be negative.

<u>Component C3</u>: Using (E.49), (E.50) and (E.52) we have

$$\begin{aligned}
L_{qq}L_{pp}-L_{qp}^2 &= \left(-\Theta\left(\frac{C_q}{C}\right)^2+(1-\delta)\,\lambda(\theta)h_{qq}\left(1,q_S\right)\right)\left(-\Theta\left(\frac{C_p}{C}\right)^2\right)-\left(-\Theta\frac{C_q}{C}\frac{C_p}{C}\right)^2\\
&=-\Theta\left(\frac{C_p}{C}\right)^2(1-\delta)\,\lambda(\theta)h_{qq}\left(1,q_S\right)>0.
\end{aligned}$$

Clearly, component C4 is positive. However, for the parameters we consider it turns out to be very close to zero for all y. For each set of parameter values we consider, we verify numerically that the sum of C1, C2 and C3 outweighs C4 so that $|H_2|<0\,\forall\,y$.

E.3 Proofs and derivations for Appendix E

Proof of Proposition E1: Given the optimal consumption choices under complete markets, from (A.26) with $w = w_N$, we have that the utility of owners conditional on (y, q_R, q_S, p, θ) is given by

$$W^N(q_S, w_N(y, q_S, p, \theta)) = u(\hat{C}) + h(1, q_S) + \frac{1-\delta}{1+\rho}\left[\pi W^R(w_R) + (1-\pi)W^N(q_S, w_N)\right] \tag{E.57}$$

where $\hat{C}$ is the optimal consumption level of households conditional on (y, q_R, q_S, p, θ) given by (E.2). It follows that

$$W^N(q_S, w_N) = \frac{u(\hat{C}) + h(1, q_S) + \frac{1-\delta}{1+\rho}\pi W^R(w_R)}{1 - \frac{1-\delta}{1+\rho}(1-\pi)} \tag{E.58}$$

Since, given the optimal conditional consumption/savings choices, $w_N = w_B - p$, the optimization problem of a household who is currently renting and searching for a house to own can then be expressed as

$$W^R(w_R(y)) = \max_{q_R, q_S, p, \theta}\left\{\begin{array}{l} u(\hat{C}) + h(0, q_R) + \frac{1-\delta}{1+\rho}\left[1-\lambda(\theta)\right]W^R(w_R(y)) \\ +\frac{\frac{1-\delta}{1+\rho}\lambda(\theta)}{1-\frac{1-\delta}{1+\rho}(1-\pi)}\left[u(\hat{C}) + h(1, q_S) + \frac{1-\delta}{1+\rho}\pi W^R(w_R(y))\right] \end{array}\right\} \tag{E.59}$$

subject to

$$\gamma(\theta)p = (\gamma(\theta) + \rho)\left(\frac{\varpi + \rho}{\varpi}\right)T(q_S) \tag{E.60}$$

The maximand can be reformulated as

$$\max_{q_R, q_S, p, \theta}\left\{\begin{array}{l} \left(1 - \frac{1-\delta}{1+\rho}(1-\pi) + \frac{1-\delta}{1+\rho}\lambda(\theta)\right)u(\hat{C}) + \left(1 - \frac{1-\delta}{1+\rho}(1-\pi)\right)h(0, q_R) \\ +\frac{1-\delta}{1+\rho}\lambda(\theta)h(1, q_S) + \left(\left(1 - \frac{1-\delta}{1+\rho}(1-\pi)\right)\left[1-\lambda(\theta)\right] + \frac{1-\delta}{1+\rho}\pi\lambda(\theta)\right)\frac{1-\delta}{1+\rho}W^R(w_R(y)) \end{array}\right\} \tag{E.61}$$

$$= \max_{q_R, q_S, p, \theta}\left\{\begin{array}{l} \left(1 - \frac{1-\delta}{1+\rho}(1-\pi) + \frac{1-\delta}{1+\rho}\lambda(\theta)\right)u(\hat{C}) + \left(1 - \frac{1-\delta}{1+\rho}(1-\pi)\right)h(0, q_R) \\ +\frac{1-\delta}{1+\rho}\lambda(\theta)h(1, q_S) + \left(1 - \frac{1-\delta}{1+\rho}(1-\pi) - \lambda(\theta)\left(1 - \frac{1-\delta}{1+\rho}\right)\right)\frac{1-\delta}{1+\rho}W^R(w_R(y)) \end{array}\right\} \tag{E.62}$$

$$= \left(\frac{1}{1+\rho}\right)\max_{q_R, q_S, p, \theta}\left\{\begin{array}{l} \left[\rho + \delta + (1-\delta)(\pi + \lambda(\theta))\right]u(\hat{C}) + \left[\rho + \delta + (1-\delta)\pi\right]h(0, q_R) \\ +(1-\delta)\lambda(\theta)h(1, q_S) + \left[\rho + \delta + (1-\delta)\pi - \lambda(\theta)(\rho + \delta)\right]\frac{1-\delta}{1+\rho}W^R(w_R(y)) \end{array}\right\} \tag{E.63}$$

and, since $W^R(w_R(y))$ depends only on y, the optimization problem can be expressed more concisely as

$$\max_{q_R,q_S,p,\theta}\left\{\begin{array}{c}[\rho+\delta+(1-\delta)(\pi+\lambda(\theta))]\,u(\hat{C})+[\rho+\delta+(1-\delta)\,\pi]\,h(0,q_R)\\+(1-\delta)\,\lambda(\theta)\left(h(1,q_S)-\frac{\rho+\delta}{1+\rho}W^R(w_R(y))\right)\end{array}\right\} \tag{E.64}$$

subject to (E.2) and (E.60).

Derivation of $C_{\theta\theta}$: We can express $C_{\theta\theta}$ as

$$C_\theta=-\Psi\left(\Psi\frac{p}{1+\rho}-A(q_S)-(1-\delta)x(\hat{q}_R(C))\right)\frac{\lambda'(\theta)}{[\Psi(1+\varepsilon(C))+(1-\delta)\,\lambda(\theta)]\,[\Psi+(1-\delta)\,\lambda(\theta)]}.$$

Differentiating with respect to θ yields

$$\begin{aligned}C_{\theta\theta}=&-\Psi\left(\Psi\frac{p}{1+\rho}-A(q_S)-(1-\delta)x(\hat{q}_R(C))\right)\\&\times\frac{\left(\begin{array}{c}[\Psi(1+\varepsilon(C))+(1-\delta)\,\lambda(\theta)]\,[\Psi+(1-\delta)\,\lambda(\theta)]\,\lambda''(\theta)\\-\lambda'(\theta)\left([\Psi+(1-\delta)\,\lambda(\theta)]+[\Psi(1+\varepsilon(C))+(1-\delta)\,\lambda(\theta)]\right)(1-\delta)\,\lambda'(\theta)\end{array}\right)}{[\Psi(1+\varepsilon(C))+(1-\delta)\,\lambda(\theta)]^2\,[\Psi+(1-\delta)\,\lambda(\theta)]^2}\\&+\frac{\Psi(1-\delta)x'(\hat{q}_R(C))\,\hat{q}_R'(C)C_\theta\lambda'(\theta)}{[\Psi(1+\varepsilon(C))+(1-\delta)\,\lambda(\theta)]\,[\Psi+(1-\delta)\,\lambda(\theta)]}+\frac{C_\theta}{\Omega}\Omega_C C_\theta.\end{aligned}$$

Reorganizing and simplifying yields

$$C_{\theta\theta}=C_\theta\left[\frac{\lambda''(\theta)}{\lambda'(\theta)}-\frac{2\,(1-\delta)\,\lambda'(\theta)}{\Psi(1+\varepsilon(C))+(1-\delta)\,\lambda(\theta)}\right]+\frac{\Omega_C}{\Omega}C_\theta^2$$

Proof of Lemma A1: Substituting using (E.11) and (E.48) we can express $L_{\theta\theta}$ as

$$\begin{aligned}L_{\theta\theta}=&-\Theta(C)\left(\frac{C_\theta}{C}\right)^2+\frac{C_\theta}{C}\left[\Phi\frac{\lambda''(\theta)}{\lambda'(\theta)}+2\,(1-\delta)\,(1-\alpha)\,(1-\Omega)\,\lambda'(\theta)\right]\\&+(1-\delta)\left[u(C)+h(1,q_S)-\left(\frac{\rho+\delta}{1+\rho}\right)W^R(w_R(y))\right]\lambda''(\theta)\\&+\left(-(1-\delta)\left[u(C)+h(1,q_S)-\left(\frac{\rho+\delta}{1+\rho}\right)W^R(w_R(y))\right]\lambda'(\theta)-\Phi\frac{C_\theta}{C}\right)\frac{\gamma''(\theta)}{\gamma'(\theta)}\end{aligned}$$

Reorganizing yields:

$$L_{\theta\theta} = -\Theta(C)\left(\frac{C_\theta}{C}\right)^2 + \frac{C_\theta}{C}\left[\Phi\frac{\lambda''(\theta)}{\lambda'(\theta)} + 2\left(1-\delta\right)\left(1-\alpha\right)\left(1-\Omega\right)\lambda'(\theta)\right] - \Phi\frac{C_\theta}{C}\frac{\gamma''(\theta)}{\gamma'(\theta)}$$
$$+ \left(1-\delta\right)\left[u(C) + h(1, q_S) - \left(\frac{\rho+\delta}{1+\rho}\right)W^R(w_R\left(y\right))\right]\left[\lambda''(\theta) - \lambda'(\theta)\frac{\gamma''(\theta)}{\gamma'(\theta)}\right]$$

$$L_{\theta\theta} = -\Theta(C)\left(\frac{C_\theta}{C}\right)^2 + \frac{C_\theta}{C}\left[\Phi\left(\frac{\lambda''(\theta)}{\lambda'(\theta)} - \frac{\gamma''(\theta)}{\gamma'(\theta)}\right) + 2\left(1-\delta\right)\left(1-\alpha\right)\left(1-\Omega\right)\lambda'(\theta)\right]$$
$$+ \left(1-\delta\right)\left[u(C) + h(1, q_S) - \left(\frac{\rho+\delta}{1+\rho}\right)W^R(w_R\left(y\right))\right]\left[\lambda''(\theta) - \lambda'(\theta)\frac{\gamma''(\theta)}{\gamma'(\theta)}\right]$$

This expression is negative if

$$\frac{\lambda''(\theta)}{-\lambda'(\theta)} + \frac{\gamma''(\theta)}{\gamma'(\theta)} \leq 0$$

With the assumed matching function this expression equals zero and so:

$$L_{\theta\theta} = -\Theta(C)\left(\frac{C_\theta}{C}\right)^2 + 2\frac{C_\theta}{C}\left(1-\delta\right)\left(1-\alpha\right)\left(1-\Omega\right)\lambda'(\theta) < 0.$$