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# The Embodiment of Intangible Investment Goods: a Q-Theory Approach

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# The Embodiment of Intangible Investment Goods: a Q-Theory Approach*

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**Abstract**

Recent empirical findings on firms' expenditure towards the creation and acquisition of knowledge goods, otherwise known as intangibles, suggest that their share in overall investment has grown considerably. Still, intangible investment is rarely present in investment models. In this paper, I extend the q-theory of investment to model explicitly the decision of firms to invest in intangibles. I then use the model to measure the contribution of intangible goods to the overall capital stock in the U.S. The model highlights the embodiment of intangible goods in tangibles and the role of relative price movements in the measurement of the contribution of each type of investment to the overall capital stock. In particular, given that the relative cost of the main input to intangible production, skilled labor, rose substantially in the 80s and 90s, the price of intangibles inherits this rise. As a result, the downward trend in the aggregate investment deflator series reported by national accounts, which accounts only for the presence of tangible investment goods, is found to have a significant downward bias in the 90s. The model also shows that the growth in the overall capital stock from the late-80s until 2000 was driven mainly by an increase in the contribution of intangibles. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the movements in the price of intangibles rather than focusing only on their rising share in overall investment.

*Keywords*: Intangible investment, Q-Theory, Skill Premium, Investment Deflator.
*JEL classifications*: E22, E44, O47, G31, J31, J44.

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# 1 Introduction

Recent empirical findings point to a considerable increase in the share of intangible expenditures in overall investment since the early 80s. Still, intangible investment is rarely accounted for in investment models. This paper extends the q-theory of investment to account for both the existence of intangible investment goods and the movement of their price over time. The model highlights the embodiment of intangible goods in tangibles and the role of relative price movements in the measurement of the contribution of each type of investment good to the overall capital stock. The model is used to: 1) derive the quantitative implications on the behavior of the capital stock and 2) to measure the contribution of embodied intangible investment goods to the overall capital stock in the U.S.

The increasing share of intangibles since 1950 is documented in a comprehensive study by Corrado *et al.* (2006). The authors construct a data set on U.S. firms' spending on an identified list of intangible inputs[1]. This list consists of three main categories: computerized information (software and database expenses), innovative property (scientific and non-scientific R&D) and economic competencies (training expenses, advertising expenditures and spending on organizational change and design)[2]. They find that the share of intangible investment in overall investment since 1950 has steadily increased over time and that by 2000, investment in intangible capital was as large as investment in physical capital[3].

Other authors have attempted to measure the size of intangibles by taking indirect approaches. For example, Hall (2001) uses a q-theory model to infer the amount of intangible investment such that his model matches the movements in the stock market. McGrattan and Prescott (2007a) use a general equilibrium model to generate the intangible series which makes movements in hours worked, in producing both intangibles and tangibles, match some aggregate moments.

A key limitation to all these approaches is that none of them separately identify indices for the price and quantity of intangible investment goods. Corrado *et al.* (2006) and Hall (2001)

[1]A consensus emerged overtime among national accountants on what those items should be. See Vosselman (1998).

[2]See Belhocine (2008a) for a list and a description of specific items included in each group.

[3]Similar work conducted for Canada, the U.K. and Japan respectively by Belhocine (2008a), Marrano and Haskel (2006) and Fukao *et al.* (2007) reached similar findings.

simply assume that the price of intangibles is the same as the price of tangible investment goods. McGrattan and Prescott (2007a) guess a price series in order to infer an intangible investment series.

The distinction between the price and the quantity of intangibles is an important issue because it is not clear how much of the increase in the overall expenditure on intangibles is attributable to the *real* contribution of intangibles. Moreover, Hall's lack of distinction between the price of tangibles and intangibles effectively implies some unlikely movements in the stock of intangibles, including that they were negative for a decade after 1974 and extremely volatile in the last decade.

What is more, the substantial increase in the relative cost of the main input to intangibles production, skilled labor, suggests that the relative price of intangibles also rose (Katz and Autor (1999) and Lemieux (2007) review the literature on the rise in the skill-premium). A closer look at the aforementioned list of intangible items reveals that they are mainly produced by a class of skilled workers made of university graduates and executives. The real average compensation of these two types of workers has been increasing since the early 1980s until it peaked in 2000 and declined somewhat. This compensation pattern should translate into a rise in the unit cost of intangibles up to the year 2000 and a subsequent fall.

In order to account for both the existence of intangible investment goods and the movement of their price over time, I develop and implement a generalized q-theory of investment. Previous authors have mainly assumed that intangible goods are disembodied and that the stock of intangibles can be conceptualized as separately evolving from the physical capital stock of firms. However, many intangible investments made by firms appear to be "embodied" in a composite stock made of tangible and intangible capital. One might think of this composite stock as combinations of "hardware" (e.g. office buildings, computers, machines) and "software" (organizational design, operating systems, blueprints) which have been brought together by past investments.

This is precisely how the investment process is modeled, with units of intangible and tangible goods being optimally combined as intermediates in the production of a final aggregate investment good. It is this final investment good that is accumulated overtime and used in output production. Changes in the share of expenditures on the two types of investment goods over time reflect exogenous technological change in the final investment good sector.

The price of intangible goods reflects the costs of producing them which, in turn, reflects the wages of skilled labor. I construct a price series for intangibles based upon the compensation received by university graduates and executives and estimates of their share in the production of intangibles (similar to those used by Corrado *et al.* (2006)).

Both the rise in the price of intangibles and their increasing share in overall investment results in an aggregate investment deflator whose behavior contrasts markedly with the (physical) investment deflator reported by national income accounts; while the investment deflator of national accounts exhibits a downward trend starting in the mid-1950s, the investment deflator constructed from the model to include intangible investment has a downward trend up to the mid-1980s and then rises up to 2000 and declines afterwards[4]. In other words, the behavior of the acquisition cost of capital is dramatically different when intangibles are accounted for.

The model successfully generates a smoothly-behaving series of capital stock with a market value that predominantly remains above and close to its acquisition cost; this reflects a Tobin's q that fluctuates closely around its equilibrium value, a desirable feature which is not observed in empirical measures of Tobin's q when capital exclusively consists of tangible investment. This result is a direct implication of: 1) the new trend observed in the acquisition cost of capital once intangibles are accounted for and 2) the increased size of capital stock once intangible investment goods are accounted for. These successful quantitative implications suggest that the mixed econometric success of the q-theory might be a consequence of the omission of intangible investment[5].

Finally, the model is used to measure the extent to which the composition of the stock of capital in the economy has shifted over time towards the inclusion of more intangible capital at the expense of tangible capital. The decomposition of capital stock into its investment constituents shows that the rise in overall capital stock from the mid-80s until the late 90s was mainly driven by an increase in intangible investment despite an increase in tangible

[4]The secular fall in the price of physical investment goods is a well documented fact. See in particular Greenwood *et al.* (1997) and Krusell (1998).

[5]These potential specification problems are mentioned in a related discussion by Hall (2004) pp. 914-915. See the conclusion section for an elaboration on this point.

investment. In particular, I find that once the price movements of the two investment goods are accounted for, the relative contribution of accumulated intangible investments to overall capital stock was higher prior the 1990s and substantially lower during the 1990s than other authors have found.

Taken together, the findings in this paper underscore the importance of intangible investment as a source of value for the firm and as a key component of any investment theory. Moreover, this paper provides a consistent account of the compositional changes that have occurred in the last 25 years in the U.S. economy by bringing together the evidence on the rise of the skill premium, the evidence on the increasing importance of intangible investment and the evidence on the behavior of aggregate securities.

The paper proceeds as follows. In section 2, the approach followed is contrasted with other approaches that try to account for the presence of intangibles. The most notable feature that distinguishes this work are: 1) the linkage of the unit cost of intangibles to the wage behavior of skilled workers and 2) the embodiment of intangibles in tangible investment goods. Section 3 outlines how the extension of the q-theory is constructed. It will also describe the approach used to disentangle the contribution of each type of investment good to the capital stock. Section 4 discusses how the parameters are calibrated and what data sources are used to document some of these choices. This section also details the technology by which skilled workers, identified as university graduates and executives, produce intangibles. Section 5 describes the findings with regards to the inferred capital stock, the behavior of Tobins' q and the changing composition of the capital stock over time. Section 6 compares the findings with the work and results of the two closest approaches in the literature, namely Hall (2001) and McGrattan and Prescott (2007a). Section 7 outlines the sensitivity analysis conducted to assess the robustness of the results. Section 8 concludes with a discussion of future research.

# 2 Difference with Other Approaches

The baseline neoclassical model of investment, also known as the q-theory of investment, predicts that the decision of a firm to invest is a function of a trade-off between the benefit of increasing capital by one unit and the cost of acquiring and installing the extra unit.

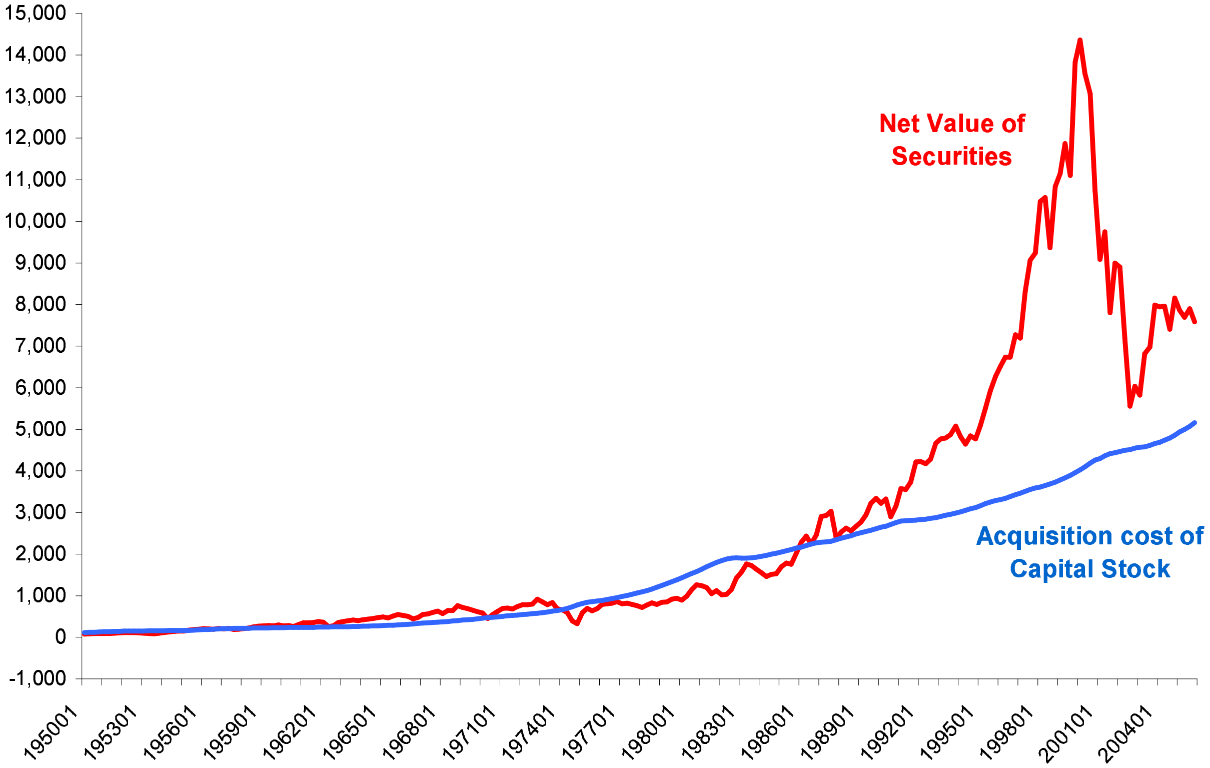


Figure 1: Net Value of Securities and Value of Capital Stock (in billions). Source: Hall (2001), NIPA and author's calculations

Hayashi (1982) showed the conditions under which the marginal benefit of increasing capital is identified with the net value of outstanding securities. Figure 1 shows the evolution of the aggregate value of net securities together with the acquisition cost of capital or its replacement cost. The q-theory states that, from this graph, the firm can deduce the benefits and costs it faces in its decision to adjust capital by one unit: if the capital in place has a higher value than the capital that is not yet installed then the firm should take advantage of the arbitrage opportunity. Any discrepancy between the cost and benefit is due to capital adjustment costs which slow down the arbitrage process.

Another way of presenting this tradeoff is through Tobin's q, which is the ratio of the net value of securities to the acquisition cost of capital. Tobin's q reflects the incentive of the firm to adjust its capital stock: the firm should invest if the value is bigger than one, otherwise the firm should disinvest. Figure 2 illustrates the behavior of Tobin's q.

Two aspects of this behavior are difficult to rationalize from the perspective of the q-theory. First, we observe a large discrepancy between the benefit and the cost of adjusting

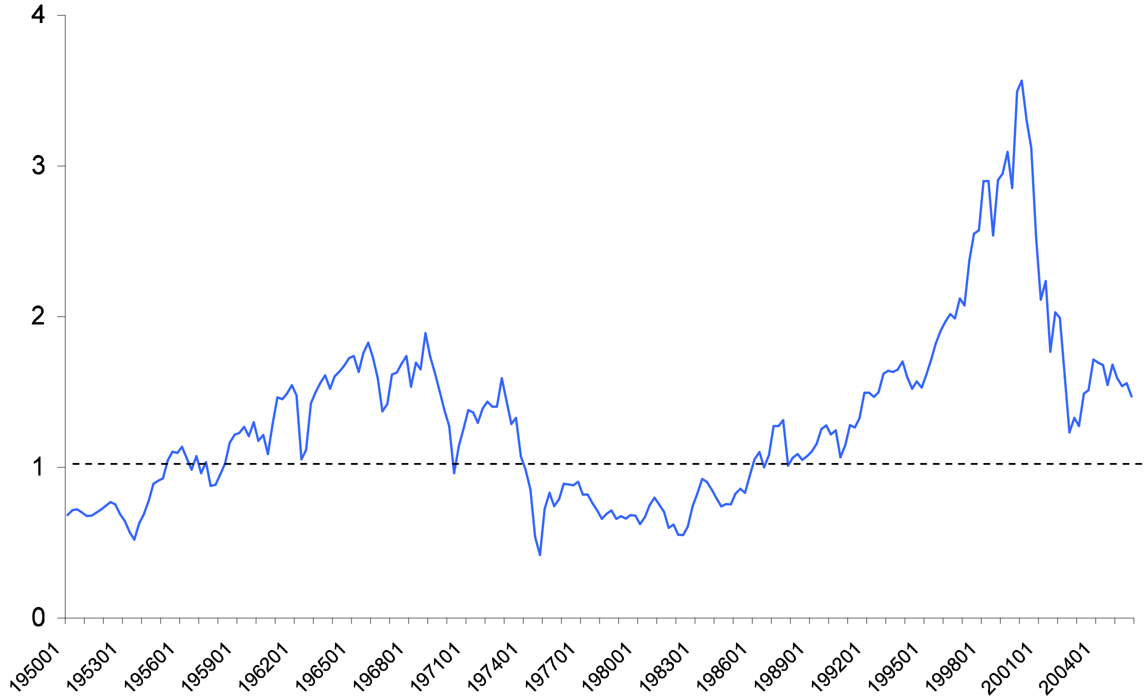


Figure 2: Behavior of Tobin's q. Source: Hall (2001), NIPA and author's calculations

capital by one unit suggesting a very long adjustment period. In fact, the econometric implementation of the q-theory by Summers (1981) implied an adjustment period of 8 years[6]. This number is viewed as unrealistic. Second, the period from the mid-1970s until the mid-1980s features an aggregate value of firms below the acquisition cost of capital. This is viewed as a puzzle because the firm's capital can be sold at a higher value than when kept inside the firm (see Baily, 1981). In other words, there are arbitrage opportunities that agents are not taking advantage of.

Four broad research directions have been pursued to explain these anomalies in Tobin's $q$. The first line of attack was to study how financial markets do not accurately reflect the fundamental value of the firm. This literature developed different models but none of them seem to be immune from problems despite the strong theoretical and empirical evidence for the existence of bubbles (see LeRoy (2004) for a review of this literature). The second line of attack was to relax the restrictive assumptions that allowed the value of the installed capital to reflect the marginal value of increasing the capital stock; the

[6]Other studies confirmed the long implied adjustment period. See Chirinko (1993) for a review of the literature. However, more recent work which uses natural experiments, exemplified by the paper of Cummins *et al.* (1994) and (1996) have found shorter adjustment periods.

reliance on perfect competition, constant returns to scale and quadratic adjustment costs might not be accurate. This literature had some success in improving on the q-theory but the improvements are sometimes viewed as marginal (see Caballero (1999) for a review of this literature). A third approach was to question the use of a representative firm at the aggregate level and to focus instead on modeling the heterogeneity observed at the plant level. This literature abandoned the macroeconomic approach to modeling investment (see the review of this literature in Bond and Van Reenen (2007)). Finally, one explanation that has received attention in the past few years is the existence of another category of investment that has not yet been accounted for in theoretical and empirical investigations. This category of investment goods consists of intangible goods. The argument is that if the net value of securities reflects, under rational valuation, the value of installed capital stock, then periods of marked departure of the net value of securities from the acquisition cost of tangible capital stock is evidence for the accumulation of intangible capital by firms.

This last possibility was proposed by Hall (2001). His paper uses the q-theory of investment to generate the stock of capital in the economy. From this inferred capital, the component which is recorded in national income accounts as physical capital is subtracted and the residual is assimilated to intangible capital. One crucial assumption in Hall's work is the assumption that the intangible and tangible investment goods are perfect substitutes and that their price is equal. These two assumptions are violated, under the framework of this paper, as discussed in the introduction: the share of intangibles in the overall capital expenditures has been increasing overtime and the unit cost of intangibles behaves the opposite of the price of physical capital. Finally, one anomaly in Hall's findings is that the quantity of intangibles falls below zero for a decade, starting in the mid-70s.

Eliades and Weeken (2004) apply Hall's methodology to the U.K. These authors also find negative intangibles throughout the 70s, the 80s and early 90s. However, they reach the same qualitative results as Hall (2001) for the late 90s. Belhocine (2008b) applies this approach to Canada for the period after 1990 and reaches the same qualitative conclusions as those found for the U.S. and the U.K.

I depart from Hall's approach by: 1) relaxing the assumption of equality between the price of intangibles and tangibles and 2) allowing some degree of substitutability between the two investment goods.

McGrattan and Prescott (2005a) make another attempt to use the unmeasured levels of intangible capital to rationalize the rise in the U.S. and U.K. stock markets in late 90s. The authors depart from Hall's work by taking a general equilibrium approach with no frictions aside from the existence of taxes. The authors rationalize the size of intangible investment found in Corrado *et al.* (2005) in the 1990s while using the change in tax regulations to account for the differing performance between the U.K. and the U.S. stock markets. McGrattan and Prescott (2005b) show that by explicitly accounting for intangible investment in an otherwise standard real business cycle model, one can explain the low productivity levels in the early 90s. In particular, they argue that GDP in national income accounts is undervalued because intangible investment is expensed, which ultimately created a downward bias in the productivity estimates for the early 90s. Taking this work further, McGrattan and Prescott (2007a) extend the baseline real business cycle model to allow for the production of intangibles by the representative agent. The goal of their work is to reconcile the real business cycle model's prediction of a fall in hours worked after the 1990s with actual evidence of their increase. In particular, their model is calibrated to match some aggregate macroeconomic series and features two stocks of capital: a tangible capital stock and an intangible capital stock.

In this paper, I calibrate the extended q-theory model to the aggregate value of securities and the extended model features only one capital stock index that embodies both tangible and intangible investment goods. Another difference with the work of McGrattan and Prescott is the way the price of intangibles is calculated: McGrattan and Prescott (2007b) state that they guessed the price of intangibles in their solution to derive a series of intangible investment. As mentioned above, I derived the price of intangibles from the behavior of the cost of its main inputs.

Finally, the proposed model is most comparable to the work of Hall (2000). In Hall's paper, he focuses on the period from 1990 to 2000 and tries to link the behavior of university graduate wages with the formation of intangible capital. Hall does not however account for the changing structure of firms' investment as done in the extended model I propose. In addition, Hall assumes that the intangible and tangible capital stocks evolve over time separately while I develop a model with a homogeneous capital stock. Moreover, the inclusion of executives in the class of skilled labor are another conceptual and empirical difference between Hall's work and the model developed below. Finally, the long-term approach I adopt illuminates the pre-1990 events as well as the post-1990 events.

Although most studies find that the size of intangible capital is substantial, Hall (2003) and Bond and Cummins (2000) are exceptions. They both show, using different data, that the returns to physical capital exhaust all payments to capital and hence, nothing is left over to reward the services of intangible capital. This is held as evidence for the absence of a substantial intangible capital stock which is puzzling in light of the findings in the previously cited papers. In the conclusion section, I discuss how to reconcile the findings of Hall (2003) and Bond and Cummins (2000) with the results obtained here.

# 3 Including Intangibles in the Q-theory of Investment

The standard neoclassical model of investment as developed in Hayashi (1982) is extended to account for the production of intangibles. Once produced, intangible investment goods are combined with tangible investment goods, which are bought from the market, to produce a final investment good that accumulates into a capital stock which is used in production. Ultimately, the model relates the value of securities to the value of the capital in place within firms. This key relationship allows the generation of a series for the capital stock and the construction of a series for Tobin's $q$.

There is perfect competition in input and output markets. The firm employs two types of labor, skilled and unskilled. An amount $l^u$ of unskilled labor is used for the production of output only. It is paid $w^u$. Skilled labor is used for two tasks: the amount $l^s$ is used for the production of output and the remainder, $h^s$, is used for the production of intangibles. Skilled labor is paid $w^s$. The production of intangibles is governed by the following technology:

$$x^I = \theta h^s \tag{1}$$

where $\theta$ is a productivity parameter. The existence of this function is motivated by the need to capture the link between the rise in the wage paid to skilled labor and the increase in the price of intangible investment goods. This function will allow the ratio of the intangible to tangible price to vary over time instead of being set equal to one as Hall (2001) does. Skilled labor consists of university graduates and executives. The price of intangible capital is denoted $p^I$ and will be given, after optimization, by

$$p^I = \frac{w^s}{\theta}. \tag{2}$$

The construction of this unit cost will be discussed in the calibration section, once the wage of skilled workers is specified as a function of the wage of university graduates and executives. The production of output proceeds according to $F(k_{t-1}, l_t^u, l_t^s)$ where $F(.)$ is assumed to be homogeneous of degree one. The price of output is the numeraire.

The model departs from the baseline q-theory model by defining a composite investment good which is accumulated into a capital stock. At each period $t$, the firm combines intangible goods $x_t^I$ with tangible investment goods $x_t^T$ to produce a final investment good $x_t$ according to the following constant elasticity of substitution (CES) function

$$x_t = \left(a_t(x_t^T)^\rho + (1 - a_t)(x_t^I)^\rho\right)^{\frac{1}{\rho}}. \tag{3}$$

The exponent $\rho$ is the elasticity of substitution between the two intermediate inputs and is assumed to be less than or equal to 1 while $a_t$ represents the income share of each input. This share is allowed to vary over time and is calibrated to capture the evidence on the increasing spending on intangibles relative to tangibles. This is important because the variation in the shares will account for the changing structure of the economy in the past 50 years towards the use of relatively more intangible capital. This CES functional form makes apparent that this changing structure is a biased technological change.

There is no evidence on the empirical value of $\rho$. Hall (2001) assumed that tangible and intangible investment are perfect substitutes so implicitly $\rho$ in his model is equal to 1 and $a_t = 1 - a_t$. In the calibration section, the baseline model is specified with $\rho = 0$ and in the sensitivity analysis section, various values of $\rho$ are considered.

Tangible investment goods are bought from the market at a price $p^T$ which is taken as given by the firm. The aggregate investment good accumulates according to

$$k_t = (1 - \delta)k_{t-1} + x_t \tag{4}$$

where $\delta$ is the depreciation rate. The adjustment of the capital stock is subject to output losses modeled as a cost function assumed to be quadratic and homogeneous of degree one that is denoted by $C(x_t, k_{t-1})$.

At each period, firm's profit is given by

$$\hat{v}_t = F(k_{t-1}, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s - w_t^s h_t^s - p_t^T x_t^T - C(x_t, k_{t-1}). \tag{5}$$

The firm's problem is to choose the optimal level of labor and investment in order to maximize the net present value of future profits subject to: 1) the technology of production of intangibles and of the final investment good, 2) the capital accumulation equation, 3) the starting level of capital and 4) the transversality condition:

$$\max_{\{l_t^u, l_t^s, h_t^s, x_t^T, x_t\}} \quad v_s = \sum_{t=s}^{\infty} \left(\frac{1}{1+r}\right)^{t-s} \hat{v}_t \tag{6}$$

s.t.

$$x_t^I = \theta_t h_t^s$$
$$x_t = \left(a_t (x_t^T)^\rho + (1-a_t)(x_t^I)^\rho\right)^{\frac{1}{\rho}}$$
$$k_t = (1-\delta) k_{t-1} + x_t$$
$$k_{t-1} \tag{7}$$
$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T v_{s+T} = 0. \tag{8}$$

The value function $v_s$ is the net present value at time $s$ of future payout to securities' holders: after the firm pays inputs their due, the leftover income is paid to owners. Their ownership materializes through the possession of titles in the form of securities. Hence, $v_s$ is also the value of the firm at time $s$.

The model can be shown to be equivalent to a standard q-theory optimization problem through a two-stage optimization procedure. The only difference will lie in the interpretation of the price index of investment goods. The first stage is a static problem which consists in choosing $x^T$ and $x^I$ to minimize the expenditure on the production of $x$ within each period. The second stage recasts the above dynamic problem in such a way that it is solved at the start.

The static problem can be written as

$$\min_{x^T, x^I} \quad p^T x^T + p^I x^I \tag{9}$$

$$s.t. \quad \left(a(x^T)^\rho + (1-a)(x^I)^\rho\right)^{\frac{1}{\rho}} \leq x. \tag{10}$$

Replacing the optimal solutions $x^{T^*}$ and $x^{I^*}$ into the objective function leads to the minimum cost function:

$$\begin{aligned} p^T x^{T^*} + p^I x^{I^*} &= x \left( \left(\frac{p^T}{a}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{p^I}{1-a}\right)^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} && (11) \\ &= x p^x && (12) \end{aligned}$$

where $p^x$ reflects the unit cost of an investment good or the price index of aggregate investment, $x$. The new dynamic problem of the firm can be written as:

$$\max_{\{l_s^u, l_s^s, x_s\}} v_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \hat{v}_s \tag{13}$$

$$\hat{v}_t = F(k_{t-1}^x, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s - p_t^x x_t - C(x_t, k_{t-1}) \tag{14}$$

s.t.

$$k_t = (1-\delta) k_{t-1} + x_t$$

$$k_{t-1}$$

$$\lim_{T \to \infty} \left(\frac{1}{1+r}\right)^T v_{s+T} = 0$$

The solution to this standard problem is detailed in Appendix A. The usual first-order condition on the equality of the lifetime return to increasing capital by one unit with its marginal cost is given by

$$\lambda_t = p_t^x + C_x(x_t, k_{t-1}) \tag{15}$$

where $\lambda_t$ is the shadow price of a unit of installed capital. The right-hand side is the marginal cost given by the summation of the acquisition price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. In order to obtain sharper results with respect to the investment decision of the firm, the adjustment cost function is specified as quadratic and homogeneous of degree one, as is often done in the literature:

$$C(x_t, k_{t-1}) = \frac{\alpha}{2} \left(\frac{x_t}{k_{t-1}}\right)^2 k_{t-1}. \tag{16}$$

Substituting this function into the first-order condition results in the following equation:

$$\frac{x_t}{k_{t-1}} = \frac{1}{\alpha}(\lambda_t - p_t^x). \tag{17}$$

This is known as the investment equation since it relates the behavior of investment to the difference between the value of capital in place $\lambda_t$ and its acquisition cost $p_t^x$. Investment is positive when the lifetime return to increasing capital by one unit exceeds its marginal cost and vice versa. To get around the fact that $\lambda_t$ is by definition unobservable, the finding of Hayashi (1982) that

$$v_t = \lambda_t k_t \tag{18}$$

is used to obtain the following expression:

$$\frac{x_t}{k_{t-1}} = \frac{1}{\alpha}(\frac{v_t}{k_t} - p_t^x). \tag{19}$$

Finally, in order to implement the model quantitatively, this relationship is combined with the equation for the investment term $x_t$ to obtain the following quadratic equation:

$$\alpha k_t^2 + ([p_t^x - \alpha(1-\delta)]k_{t-1})\, k_t - v_t k_{t-1} = 0 \tag{20}$$

Hall (2001) shows that a unique solution exists for a general convex cost function with constant returns to scale. This equilibrium is stable and is therefore not sensitive to initial conditions in the long-run. $k_t$ is the endogenous variable to be solved for and generated at each point in time. The positive root expresses the law of motion of the capital stock:

$$k_t = \frac{-[p_t^x - \alpha(1-\delta)]k_{t-1} + \sqrt{([p_t^x - \alpha(1-\delta)]k_{t-1})^2 + 4\alpha v_t k_{t-1}}}{2\alpha}. \tag{21}$$

All variables are observable and the pair $(v_t, p_t^x)$ is a sufficient statistic to generate the stock of capital in the economy. This pair of variables is assumed to be taken as given by the firm.

Once a series for the capital stock is obtained, the contribution of each type of investment to the overall capital stock can be recovered in the following way: the capital accumulation equation $k_t = (1-\delta)k_{t-1} + x_t$ is substituted forward to obtain:

$$k_{t+T} = (1-\delta)^{T+1}k_{t-1} + \sum_{i=0}^{T}(1-\delta)^i \left\{x_{t+T-i}\right\}. \tag{22}$$

Since the technology of production of the final investment good is homogeneous of degree one, the Euler theorem applied to $x_t$ leads to:

$$\begin{aligned} x_t &= \frac{\partial x_t}{\partial x_t^T} x_t^T + \frac{\partial x_t}{\partial x_t^I} x_t^I \\ &= \left(\frac{p_t^T}{p_t^x}\right) x_t^T + \left(\frac{p_t^I}{p_t^x}\right) x_t^I. \end{aligned} \tag{23}$$

It is now possible to link the stock of capital to the investment over time in each type of investment good by substituting this last relationship in the capital equation:

$$k_{t+T} = (1-\delta)^{T+1} k_{t-1} + \sum_{i=0}^{T} (1-\delta)^i \left\{ \left(\frac{p_{t+T-i}^T}{p_{t+T-i}^x}\right) x_{t+T-i}^T + \left(\frac{p_{t+T-i}^I}{p_{t+T-i}^x}\right) x_{t+T-i}^I \right\}. \tag{24}$$

This equation will be used to disentangle the contribution of each type of investment into the overall capital stock. Each type of investment is weighted by its relative price which allows the capital stock to be written in efficiency terms. This is a direct consequence of the aggregation formulation that was assumed through the production function of the final investment good.

# 4 Calibration

## 4.1 Constant Parameters

The parameter values and paths of some exogenous parameters in the law of motion of capital need to be specified.

The adjustment-cost parameter $\alpha$ represents the time it takes for the capital stock to double (halve) when $\lambda$ doubles (halves). To see this, note that if $\lambda$ doubles permanently, say from one to two, it will initially cause the investment-capital ratio to increase by $\frac{1}{\alpha}$. For the investment-capital ratio to double, the increase in $\frac{1}{\alpha}$ must be repeated for $\alpha$ periods. Hall (2001) cites the work of Shapiro (1986) to justify the choice of a doubling time parameter of 8 quarters. The depreciation rate of 2.6% per quarter is used by national income accounts for physical capital. To start the iteration on the law of motion of capital, the value of the initial capital stock $k_{t-1}$ needs to be specified. We will assume that at the pre-initial quarter, the value of the firm reflects its quantity of installed capital *i.e.* $k_{t-1} = v_{t-1}$. This is

Table 1: Parameter values

| **Name** | **Parameter** | **Value** | **Rationale** |
|---|---|---|---|
| Adjustment-cost | $\alpha$ | 8 | Shapiro (1986) |
| Depreciation rate | $\delta$ | 0.026 | Hall (2001) |
| Initial capital stock | $k_{t-1}$ | $v_{t-1}$ | Assuming $\lambda_{t-1} = 1$ |
| Elasticity of substitution | $\rho$ | 0 | Embodiment assumption |

similar to assuming that $\lambda_{t-1} = 1$. Since the recursion was shown to be insensitive to initial conditions, this assumption will not affect the behavior of the system in the long-run.

Given the desire to model intangibles as embodied in tangibles, we allow a certain degree of complementarity between the two types of investment goods. It will be enough to set $\rho$ equal to zero as a baseline case. This will transform the CES function into a Cobb-Douglas function and as a result, the composite investment good will capture the embodiment of intangibles in tangible investment goods. The shares will then be represented by the exponents on each input which makes the final investment good an aggregator of two intermediate investment goods. Indeed, it will be a share-weighted function that is apparent to an index of investment. The weights represent the share of each intermediate investment good in the overall investment expenditure. In fact, it is a Divisia index approach to combining two investment goods. Following this logic, $x$ will then be viewed as an index of aggregate investment. In the sensitivity analysis section, $\rho$ will be allowed to take a value bigger and lower than 0.

Assuming $\rho = 0$ implies that $p^x$ in equation 11 takes the following form:

$$p_t^x = \left(\frac{p_t^T}{a_t}\right)^{a_t} \left(\frac{p_t^I}{1 - a_t}\right)^{1-a_t} \tag{25}$$

Table 1 summarizes the parameter values used and the rationale for the choice of each value.

## 4.2 Varying Parameters

The market value of net financial claims (financial liabilities minus financial assets) is used as the measure of $v_t$ since the value of ownership claims are a reflection of the installed capital inside the firm. Indeed, $v_t$ was defined as the present value of payouts to securities' holders. Assuming that investors are rational, it follows that the present value of payouts $v_t$ will equal the value of securities on the market. Since for all $t$, $v_t = \lambda_t k_t$, then the value of securities equals the value of the installed capital stock.

Notice that $v_t$ includes all financial claims towards firms, net of financial assets. These claims are made of equity, bonds and all other other liabilities (loans and mortgages, short-term paper, trade payables, life insurance and pensions). The definition of $v_t$ represents a departure from most of the q-theory literature's interpretation of $v_t$. Traditionally, $v_t$ covers only equity values or equity plus bonds. This departure is mainly due to the new types of data that are available for use.

Most of the data to measure $v_t$ is taken from the national balance sheet account at market value from 1950Q1 to 2005Q4. Equity is reported at market value and all the other liabilities are at book value. These were converted by Hall (2001) into market value. Data analysis focuses on the non-farm, non-financial corporate sector. This sector was chosen because it best fits the perfectly competitive framework of this paper. The removal of the farming sector aims to control for the presence of land in the overall capital stock, a capital input in fixed supply, which therefore earns rents. The choice of the corporate sector ensures that securities are continually priced to accurately reflect new information regarding the value of the capital stock. This would not be true for the installed capital of unincorporated businesses. Another reason to focus on this sector is dictated by the fact that the farming sector, the non-corporate sector and the financial sector suffer from data quality problems. The use of the non-farm, non-financial corporate sector is not restrictive given that this sector owns around 90% of the non-residential fixed capital stock in the economy.

A full description of data sources and data manipulations can be found in Appendix B.

The paper

This specification highlights the relationship between the labor market for high-skilled workers and the cost of intangibles. Since national income accounts do not collect information on the investment of firms in intangibles, and because the market for intangibles is extremely thin[7], little is known about their aggregate price. The view taken in this paper is that the

[7]Some R&D spending leads to the creation of a patent which will carry a price if commercialized. However, the market for patents is extremely thin: very few patents change hands. For example, Serrano (2006) documents that only about 20% of all U.S. patents issued to small innovators (i.e., firms that were issued

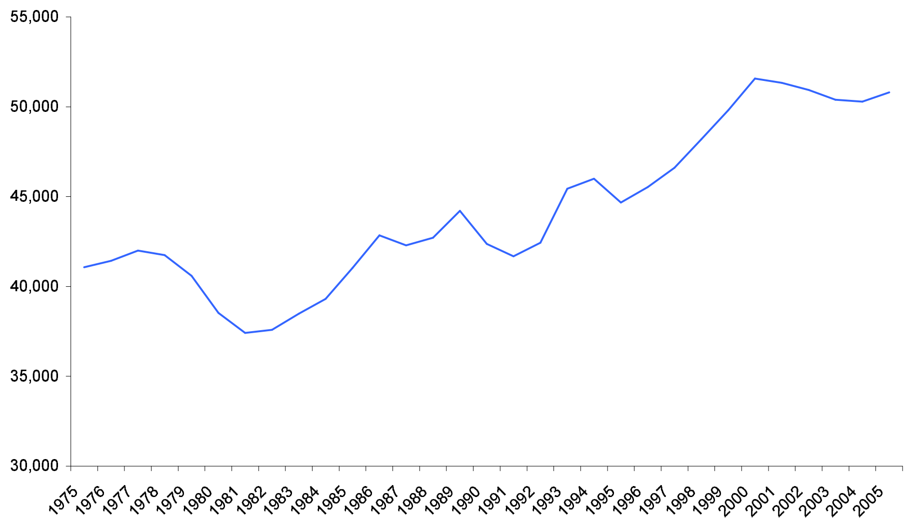


Figure 3: Real Mean Earnings of University Graduates. Source: BLS.

major manufacturers of intangible goods, as listed in Corrado et al. (2005), are university graduates and executives. Due to the rapid increase in demand of intangibles as documented by Corrado et al. (2006), this category of workers experienced a widely documented increase in their wage premium starting in the early 80s. (See Katz and Autor (1999) and Lemieux (2007) for an up-to-date review of this literature). Figure 3 shows the evolution of the real wage of university graduates from 1970 to today. The upward trend observed up to 2000 and the subsequent fall is still preserved if this wage is taken as a ratio of non-university graduates.

Another category of workers which experienced an impressive rise in their wages is executives. Frydman and Saks (2007) report data from 1935 on CEO compensation. Figure 4 reproduces their findings. There is an upward trend starting in the early 80s. The rise and fall in the stock market around 2000 seems to have had an important effect on the compensation trend. This documented evolution in the labor market for high-skilled workers is important in valuing the competitive price of produced intangible goods and thereby accurately constructing an index for the price of aggregate investment.

Using the wage of executives and college graduates together with the shares of these inputs allows the construction of the intangible unit cost[8]. The behavior of this price is

no more than five patents in a given year) are traded once or more.

[8]Note that the calibration of all starting values of the varying parameters is detailed in Appendix C.

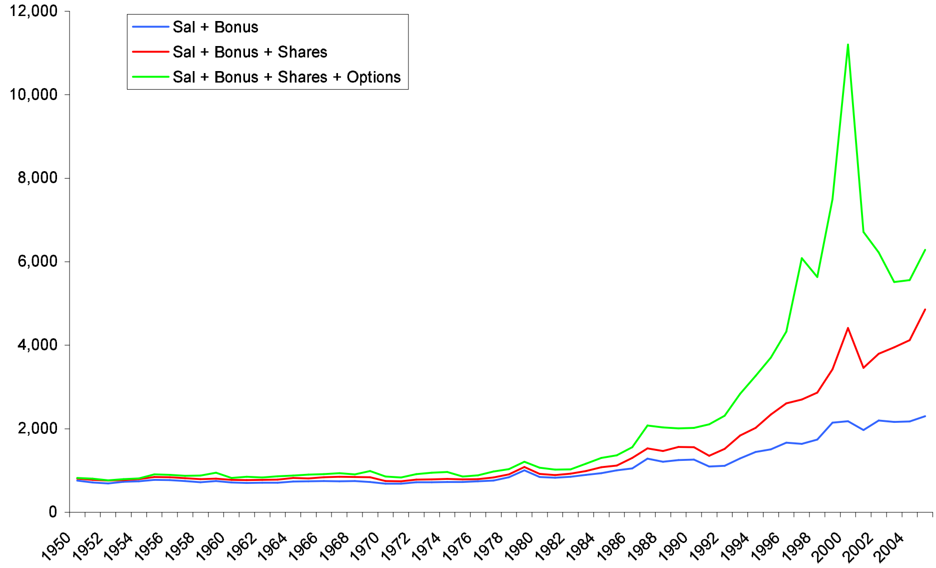


Figure 4: Real Mean Compensation of Executives (in thousands). Source: Frydman and Saks (2007)

depicted in Figure 5 in log scale. The price of intangibles $p^I$ is rising at a constant rate from 1950 until mid-80s. Afterwards, its growth rate accelerates driven by the rise in the wage of CEOs and in their increasing share in the overall wage bill up to 2000. The growth rate of the intangible price falls afterwards.

It is possible to construct an aggregate price index that combines the price of intangible and tangible investment goods once the share of tangible expenditure in overall investment expenditures given by:

$$a_t = \frac{p_t^T x_t^T}{p_t^T x_t^T + p_t^I x_t^I} \tag{26}$$

is calibrated. Corrado *et al.* (2006) report the spending of U.S. firms on an identified list of intangible inputs as mentioned before. The National Income and Product Accounts (NIPA) recorded 20% of the reported expenditures in intangibles by Corrado *et al.* (2006). $p^T x^T$ is the private fixed investment as recorded by NIPA from which the included recorded intangibles are subtracted. These recorded intangibles consisted of software, mineral exploration, and architectural and design services. The time series behavior of this ratio captures the biased technological change which resulted in the use of relatively more $x^I$. Figure 6 shows the behavior of $a$ overtime.

As mentioned in the introduction, there is a striking upward trend beginning as early as

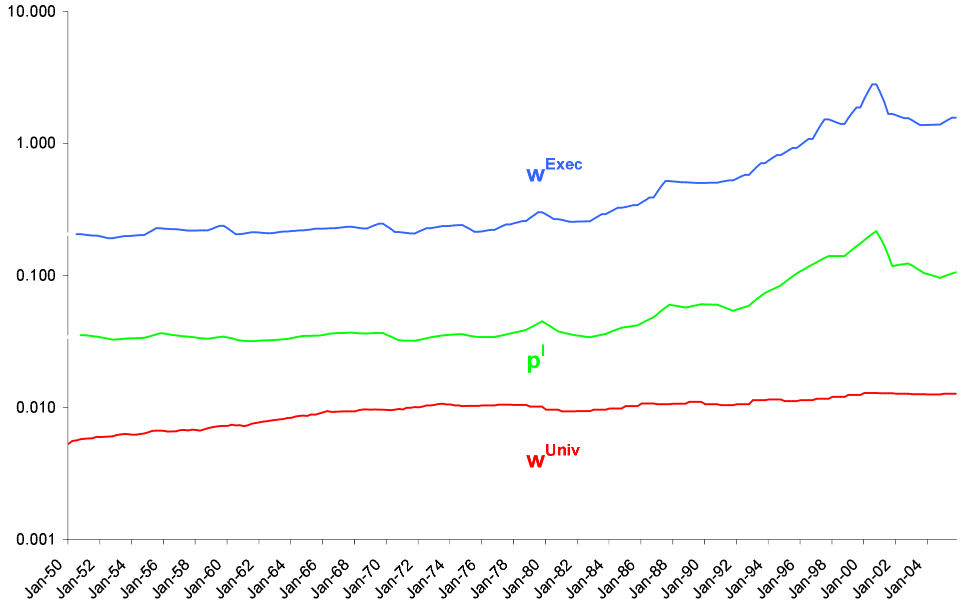


Figure 5: Real Wages and Price of Intangible Investment Goods $p^I = \frac{1}{\theta}\left(\frac{w^{Exec}}{\phi}\right)^{\phi}\left(\frac{w^{Univ}}{1-\phi}\right)^{1-\phi}$.

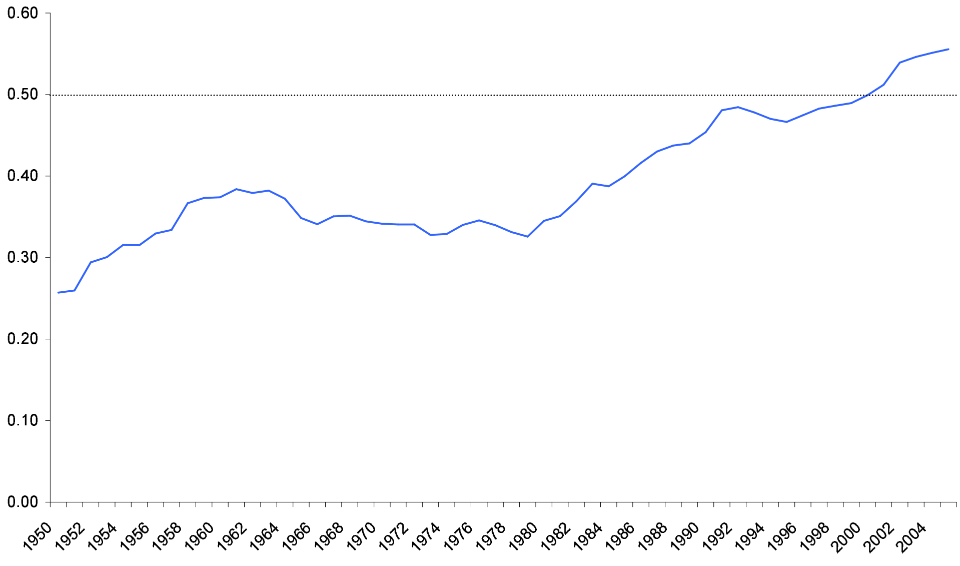


Figure 6: Share of Intangible Expenditure in Overall Investment Expenditure $a$. Source: Corrado et al. (2006) and author's calculations.

the 1950s. After 2000, firms' investment in intangibles began to surpass the investment in tangible investment goods. For the purpose of this paper, this is important in constructing an accurate measure of aggregate investment. Indeed, because the share of expenditure of firms in intangibles has been increasing overtime, the reported aggregate investment by national income accounts, which consists mainly of tangible investment, is only a partial reflection of the investment activity of firms.

It is now possible to use the behavior of the share series in the expression

$$p_t^x = \left(\frac{p_t^T}{a_t}\right)^{a_t} \left(\frac{p_t^I}{1-a_t}\right)^{1-a_t} \tag{27}$$

to derive the series on the aggregate investment price index $p^x$. This is shown in Figure 7 together with the price series of tangibles $p^T$. Both the rise in the price of intangibles and their increasing share in overall investment results in an aggregate investment deflator whose behavior contrasts markedly with the (physical) investment deflator reported by national income accounts; while the investment deflator of national accounts has a downward trend beginning in the mid-1950s, the investment deflator from the model has a downward trend until the mid-1980s and then rises until 2000 at which point it peaks and then falls[9]. In other words, the behavior of the acquisition cost of capital is dramatically different when intangibles are accounted for. In particular, note that the price of intangibles behaves opposite than that of the price of physical capital. This drives the unique shape of the aggregate price of investment since it is a shared weighted index.

# 5 Quantitative Findings and Results

## 5.1 Behavior of the Stock of Capital

Figure 8 shows how the value of the firm $v_t$ is contrasted to the generated capital stock $k_t$ and to its acquisition cost $p_t^x k_t$.

The first notable feature is that the implied stock of capital smoothly increases until the peak of 2000 and then falls and flattens out. The second notable feature is the contrast between the series of the acquisition cost of capital with the one shown in Figure 1 which uses

[9] The secular fall in the price of physical investment goods is a well documented fact. See in particular Greenwood *et al.* (1997) and Krusell (1998).

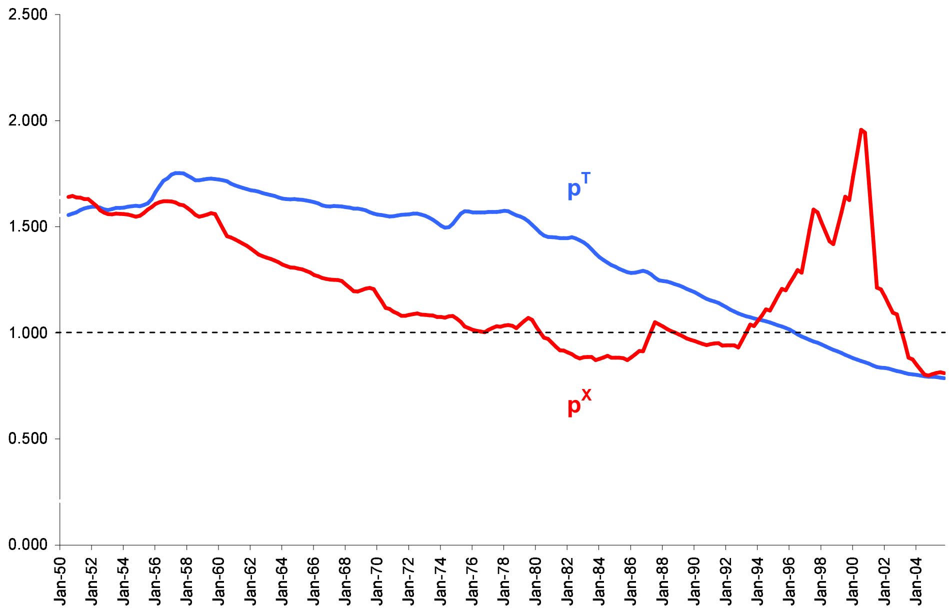


Figure 7: Real Price of Investment Goods $p^x = \left(\frac{p^T}{a}\right)^a \left(\frac{p^I}{1-a}\right)^{1-a}$. Source: NIPA and author's calculations.

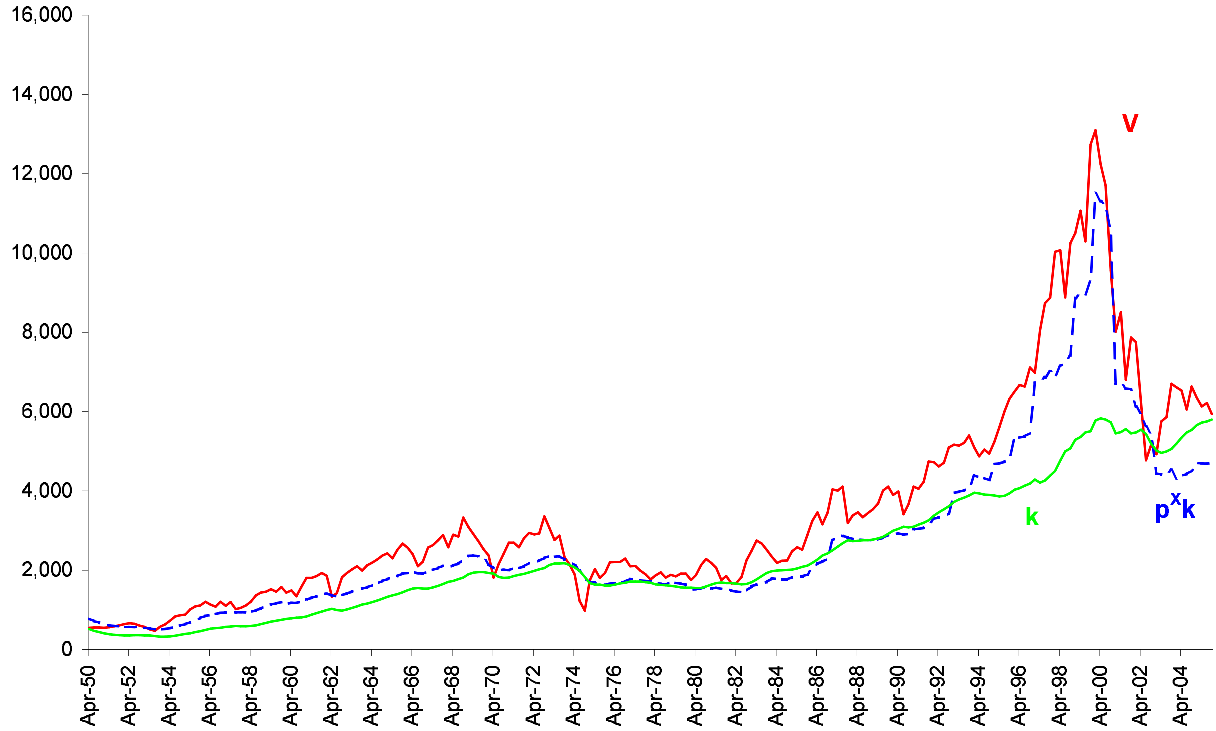


Figure 8: The Aggregate Value of Firms and the Inferred Capital Stock.

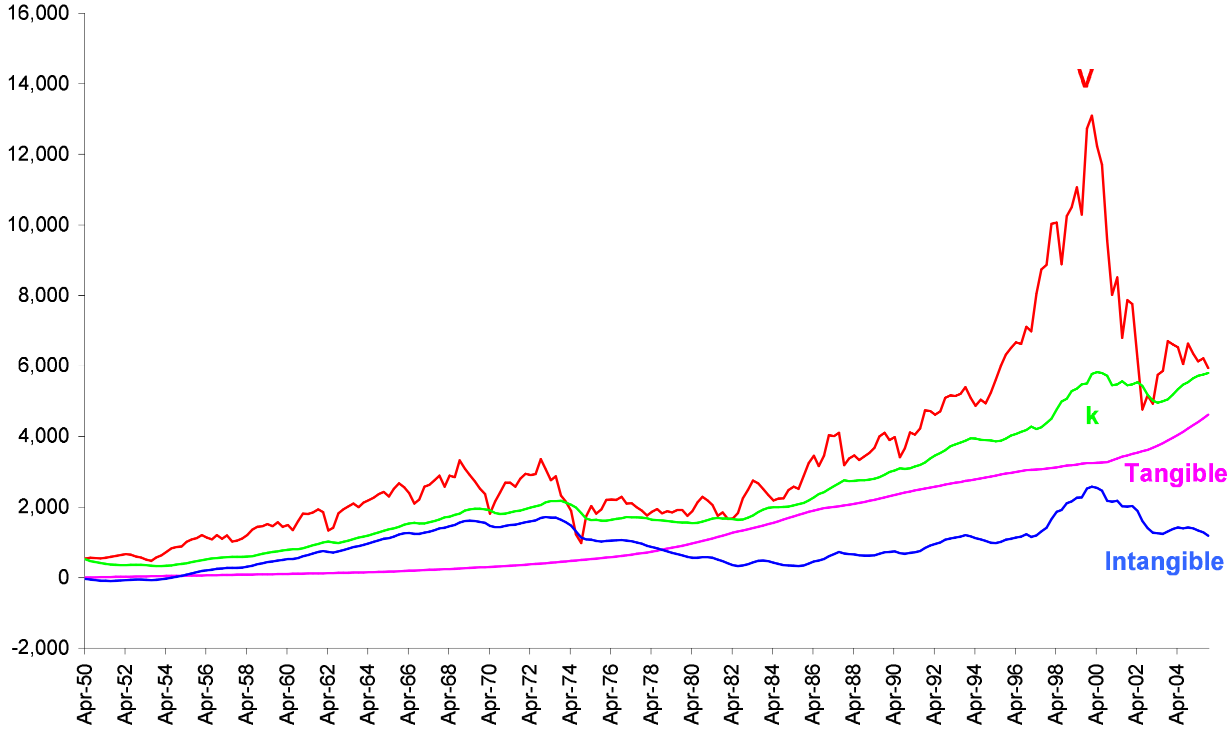


Figure 9: The Contributions to the Inferred Capital Stock

the national accounts investment deflator. The smoothness depicted from official statistics in Figure 1 is at odds with what Figure 8 depicts. The rise in the acquisition cost of capital as a result of the larger share of intangibles and their rising cost is at the source of this discrepancy. Note that allowing for irreversible investment doesn't affect the behavior of the capital stock series. Appendix D extends the model to allow for irreversible investment and discusses why this addition is innocuous.

## 5.2 Contribution of Intangibles and Tangibles to Capital Stock

Figure 9 shows the decomposition of the contribution of the series of intangibles and tangible capital. Notice that the contribution of intangible investment goods has been substantial.

The rise in the overall capital stock from the mid-80s until the late 90s was driven mainly by an increase in intangible investment. After 2000, the contribution of intangibles falls consistently to reach a level similar to the one in 1990. These movements in the contribution of intangibles underscore the importance of this type of capital as a source of variation in the value for the firm and as a key component of any investment theory. The fall in the contribution of intangibles after mid-70s draws a parallel with the explanation of Greenwood and Jovanovic (1999) on the reasons for the fall in the value of securities in the early 70s. These authors argue that it is the arrival of the information technology at a time where the stock market incumbents of the day were not ready to implement it which depressed stock prices. The arrival of this general purpose technology effectively destroyed the value of existing capital. This suggests that intangible capital experienced obsolescence and its value started to fall accordingly more rapidly than for the price of tangibles as suggested by Figure 7.

## 5.3 Implications for Tobin's q Series

Tobin's $q$ is the ratio of the value of an additional unit of capital *in place* to the price of acquiring new capital.

$$q_t = \frac{\lambda_t}{p_t^x} \tag{28}$$

Since $v_t = \lambda_t k_t$, then

$$q_t = \frac{v_t}{p_t^x k_t} \tag{29}$$

The result from the extended q-theory shows that the behavior of Tobin's $q$ is almost always positive. Figure 10 depicts the fact that the market value of the capital stock predominantly remains above and close to its acquisition cost; this reflects a Tobin's $q$ that fluctuates closely around its equilibrium value, a desirable feature which is not observed in empirical measures of Tobin's q when capital is exclusively made of physical investment goods. In particular, Tobin's $q$ does not fall below one for extended periods. Hall (2001) delivers a Tobin's $q$ which is more volatile. His result stems from an absence of a rise and then a fall in the price of intangibles which would push up $p_t^x$ before 2000 and lower it afterwards. This would offset the rise and then fall in the value of the firm and keep Tobin's $q$ around its equilibrium value. The next section explores more thoroughly the differences in the findings with Hall (2001).

Note that other authors such as Laitner and Stolyarov (2003) deliver a theoretical Tobin's $q$ that is positive and constant due to how they specify the production of intangibles in their model. In particular, their model is not one of adjustment costs but one of vintage capital

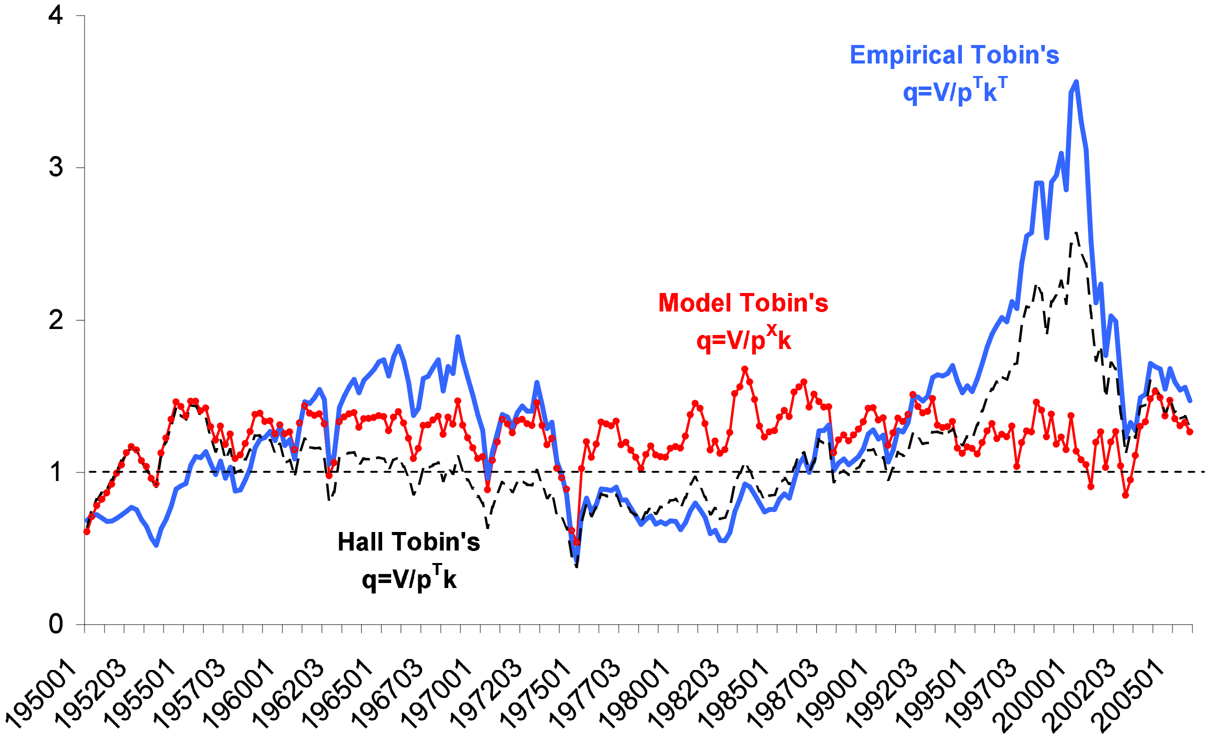


Figure 10: Behavior of Tobin's $q$

goods and technological revolutions; it is not comparable to the setup of the model in this paper.

# 6 Comparison of Findings with Existing Literature

In this section, I will compare the model developed in this paper with the models used both by Hall (2001) and by McGrattan and Prescott (2007a). The comparison will abstract from the presence of adjustment costs, taxes and difference in depreciation rates and will focus on the expression of the firm's value and the implications of assumptions made about the measurement of the size of the capital stock. The results of each paper will also be contrasted with the findings described above.

Recall that the forward substitution in the capital accumulation equation led to the general expression given by equation 24, reproduced here for convenience:

$$k_{t+N} = (1-\delta)^{N+1} k_{N-1} + \sum_{i=0}^{N} \left( (1-\delta)^i x_{t+N-i} \right).$$

The first term tends to zero as N becomes large. The equation can then be re-written as

$$k_{t+N} = \sum_{i=0}^{N}(1-\delta)^i x_{t+N-i}. \tag{30}$$

This expression will be shown to differ in important ways across approaches.

## 6.1 Reframing Hall (2001)

As mentioned in the introduction, the model developed in this paper differs from Hall (2001) by relaxing the assumption that $p^I = p^T$ and by allowing $x^T$ and $x^I$ to have some degree of substitutability. In the case of Hall (2001), the value of the firm is given by

$$V_t = p_t^T k_t = p_t^T (k_t^T + k_t^I) \tag{31}$$

where $p^T$ is the (physical) investment price deflator from national accounts. Indeed, Hall assumes that there is one price of investment goods *i.e.*, $p^T = p^I$ and implicitly assumes that there are two equations for the capital stock at each point in time, one for each type of investment good:

$$\begin{cases} k_{t+N}^T = \sum_{i=0}^{N}(1-\delta)^i x_{t+N-i}^T \\ \\ k_{t+N}^I = \sum_{i=0}^{N}(1-\delta)^i x_{t+N-i}^I \end{cases} \tag{32}$$

## 6.2 Reframing McGrattan and Prescott (2007a)

In McGrattan and Prescott (2007a), the model they use implies an expression for the value of the firm that can be written as:

$$V_t = p_t^T k_t^T + p_t^I k_t^I = k_t^T + p_t^I k_t^I \tag{33}$$

where $p_t^T$ is the price of tangible investment goods (considered to be equal to the numeraire, assumed to be the consumption good) and $p_t^I$ is the price of intangible capital goods. The price of intangibles is guessed as part of the solution to their general equilibrium model. Unfortunately, the authors do not report the series for $p_t^I$. The model of McGrattan and Prescott features two capital accumulation equations similar to Equation 32, as done by Hall as well.

## 6.3 Models' Comparison

In this paper, the value of the firm without adjustment costs can be written as

$$V_t = p_t^x k_t = \left(\frac{p_t^T}{a_t}\right)^{a_t} \left(\frac{p_t^I}{1-a_t}\right)^{1-a_t} k_t. \tag{34}$$

Note that the two investment goods in this paper are not accumulated because they are intermediate goods. However, in Hall (2001) and in McGrattan and Prescott (2007a), the two investment goods are final so they are separately accumulated over time. Moreover, the non-embodiment in Hall and in McGrattan and Prescott of investment goods results in two prices, one for each accumulated investment good while in this paper, there is only one price for the accumulated investment good. This one aggregate price deflator is attached to a single overall stock of capital, as it is commonly modeled when using an aggregate production function.

The comparison of the expressions for capital stock in each model will also reveal another consequence of the embodiment hypothesis. To see this, note that the embodiment of intangibles is reflected when equation 30 is expanded after using the Euler theorem on $x_{t+N-i}$:

$$\begin{aligned} k_{t+N} &= \sum_{i=0}^{N}(1-\delta)^i \left\{ \left(\frac{p_{t+N-i}^T}{p_{t+N-i}^x}\right) x_{t+N-i}^T + \left(\frac{p_{t+N-i}^I}{p_{t+N-i}^x}\right) x_{t+N-i}^I \right\} \\ &= \sum_{i=0}^{N}(1-\delta)^i \left(\frac{p_{t+N-i}^T}{p_{t+N-i}^x}\right) x_{t+N-i}^T + \sum_{i=0}^{N}(1-\delta)^i \left(\frac{p_{t+N-i}^I}{p_{t+N-i}^x}\right) x_{t+N-i}^I. \end{aligned} \tag{35}$$

Both parts of the right-hand-side of this last expression are similar to the corresponding equations for Hall (2001) and for McGrattan and Prescott (2007a) when we allow $p^T = p^I$. In this case,

$$k_{t+N} = k_{t+N}^T + k_{t+N}^I. \tag{36}$$

Such a capital stock exists in Hall (2001) by construction but not in McGrattan and Prescott (2007a) because this is not the capital stock which enters into the production of intangibles or in the production of final output. This is a major theoretical difference.

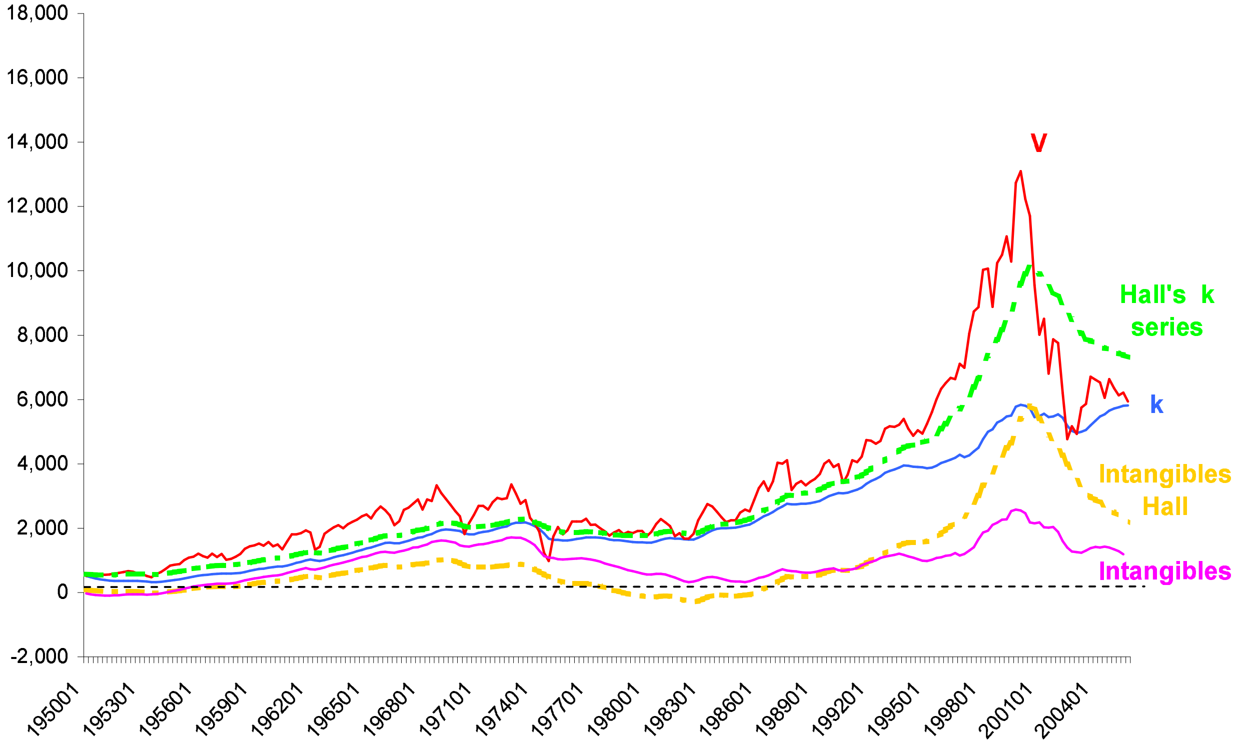


Figure 11: Comparison of findings with Hall (2001)

## 6.4 Comparison of Findings

Hall's implied capital stock series has a very pronounced bell-shape around 2000 as shown in Figure 11. Given that the acquisition cost of capital goods is identified to the tangible deflator which was falling throughout, his model's implication of a rise and then fall in the value of securities is for the firm to invest massively and then disinvest. Everything else the same, the secular fall in the price of new investment goods leads to more investment. This is not the case in the extended model presented here: the acquisition cost of capital becomes heavily skewed towards intangibles whose price begins to rise starting in the 80s. It then falls after 2000 causing the acquisition cost of capital to have a bell shape around 2000 (recall Figure 7). The implication is that the firm in this paper has less incentive to accumulate capital before 2000 and to disinvest after 2000 once the price of aggregate investment falls. This explains why the extended model features a much less pronounced bell-shape around 2000.

The assumption of the equality between the prices of intangibles and tangibles leads to Hall's finding that the quantity of intangible capital is negative from the mid-70s until the mid-80s. The model in this paper does not feature such an anomaly. To see why, observe that the constructed price series of aggregate investment in Figure 7 falls faster than the tangible price series up until the mid-80s. Firm's incentive to invest in the extended model is therefore bigger than in Hall's model. As a result, the accumulation of capital is higher during this period than in Hall, which makes the implied contribution of intangible investment higher.

Comparing the extended model's results with the findings of McGrattan and Prescott (2007a) is not as straightforward because they use a general equilibrium framework and because they emphasize different series. In particular, the price of intangibles is not reported in their paper. In their technical appendix (McGrattan and Prescott, 2007b) the authors only mention that they guessed this price series. Furthermore, McGrattan and Prescott do not report a series for intangible investment alone, which makes it hard to compare the size of intangible investment directly. However, the authors report two series that include, in one way or another, intangible investment: 1) intangible investment as a share of total GDP (their Figure 6) and 2) a graph containing the series of overall investment and tangible investment (their Figure 12).

It is not possible to use the first series to make quantitative comparisons regarding the amount of intangible investment because McGrattan and Prescott make many corrections to GDP. However, this series can be used to compare the qualitative findings. In particular, McGrattan and Prescott find that intangibles are falling in the early 90s while in this paper intangible investment was shown to rise during that period. The behavior of intangible investment around 2000 is similar.

The second series is used to approximate the difference between the overall investment and the tangible investment, which is the intangible investment found in McGrattan and Prescott. Following this method of backing out intangible investment, it seems that intangible investment is quite small between 1990 and 1994. It then rises until 1999; This rise is about 20%. Afterwards, intangible investment falls to a very low level. The drop is about 40% between 2000 and 2003. On the other hand, the extended q-theory model results in a rise of intangible investment by 150% between 1990 and 1992. Then it falls by 150% between 1992 and 1994. After 1994, intangible investment increases until 1999 by 300% and falls afterwards by 350%. The bottom line of these comparisons is that the extended model reports more intangible investment growth before 2000 and more decline after 2000 than that in McGrattan and Prescott (2007a).

To sum up, the above discussion implies that the reported results in this paper lie somehow in between the findings of Hall and of McGrattan and Prescott.

# 7 Sensitivity Analysis

In this section, two experiments are conducted to assess the robustness of the results. The first involves specifying a different value for the elasticity of substitution than was assumed in the baseline model ($\rho = 0$). In order to explore the consequence of assuming less complementarity, *i.e.* when $\rho \in (0; 1]$, and more complementarity, *i.e.* when $\rho \in (-\infty; 0)$, between the two investment goods compared to the baseline case, two values of $\rho$ are used. These values are arbitrarily chosen. To explore the consequence of less complementarity, or equivalently, more substitution, $\rho$ is set equal to 0.5. Then, to explore the consequence of more complementarity, or equivalently, less substitution, $\rho$ is set equal to $-0.5$. The impact of changing the value of $\rho$ on the model will appear through the value of $p^x$, as shown in Equation 11, and ultimately on the generated stock of capital and the contribution of each type of investment good according to Equation 24.

The second experiment involves using different wage series data when calculating the price index of intangibles. Here again, I explore the consequence of first using only the real wage of university graduates and second, different portions of the components of executive compensation, on the price of the composite investment good. The impact of the experiments on the contribution of intangibles relative to the overall capital stock is summarized in Table 2.

## 7.1 Allowing for Different Elasticity of Substitution Values

Figure 12 depicts the consequence of allowing for more and less complementarity between the two investment goods on the price of the composite investment good. By setting $\rho = 0.5$, the two investment goods are made less complementary to each other. This results in an investment price index which has an overall behavior that is similar to the price index from the baseline model but nevertheless, its rise and fall are less pronounced. On the other hand,

| Periods | Baseline model | Rho = -0.5 | Rho = 0.5 | Using only skilled labor | Using only the "salary" component | Using only the "salary+shares" component |
|---|---|---|---|---|---|---|
| 1950-1960 | 0.17 | 0.22 | 0.17 | 0.06 | 0.13 | 0.07 |
| 1960-1970 | 0.79 | 0.76 | 0.79 | 0.77 | 0.79 | 0.78 |
| 1970-1980 | 0.66 | 0.62 | 0.65 | 0.65 | 0.66 | 0.66 |
| 1980-1990 | 0.24 | 0.32 | 0.26 | 0.32 | 0.26 | 0.29 |
| 1990-1995 | 0.27 | 0.41 | 0.30 | 0.38 | 0.30 | 0.34 |
| 1995-2000 | 0.34 | 0.54 | 0.42 | 0.48 | 0.39 | 0.43 |
| 2000-2005 | 0.30 | 0.35 | 0.34 | 0.35 | 0.32 | 0.35 |

Table 2: Sensitivity of the Contribution of Intangibles Relative to the Overall Capital Stock

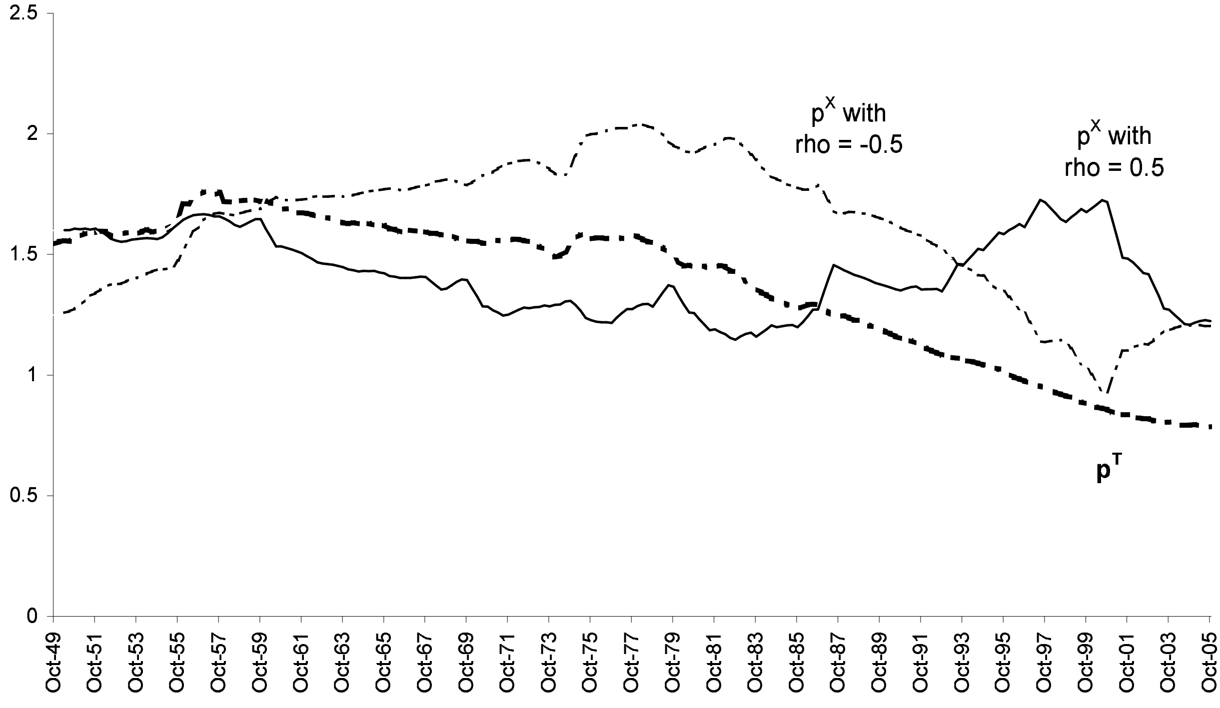


Figure 12: Impact of Different Values of the Elasticity of Substitution on the Price of Composite Investment Good

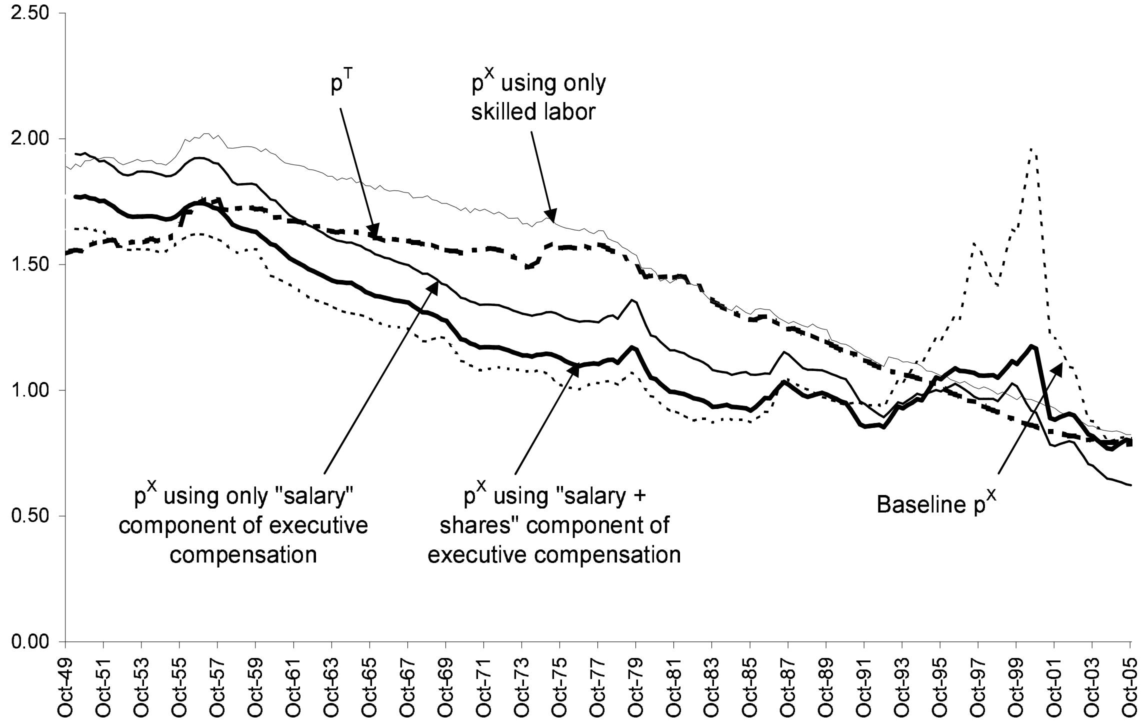


Figure 13: Impact of Different Wage Series on Price of Composite Investment Good

allowing $\rho = -0.5$ leads $p^x$ to behave almost opposite to its value with $\rho = 0.5$. By looking at Equation 11, it becomes obvious why this occurs: with more substitutability allowed, the income share of each investment good relative to its own price becomes the relevant variable. The opposite occurs when we allow for more complementarity. In any case, it is interesting to note that the behavior of the generated capital and the contribution of each input are not significantly affected in either scenario. Table 2 shows the discrepancies in the relative contribution of intangibles for each value of $\rho$.

## 7.2 Allowing for Different Wage Series

In this exercise, I consider the impact of using first only the real wage of university graduates and second, only one component of the executive compensation at a time on the price of the composite investment good. Figure 13 reproduces the findings of the baseline model shown in Figure 7 and the results of each experiment[10].

[10]Note that because the wage data series are being changed, the implied initial values had to be adjusted each time in accordance with the procedure described in Appendix C, Section 4.3 and mentioned in Footnote 27.

The first experiment involved using only the wage of skilled labor. Because labor productivity, $\theta$, grows faster than the wage of skilled workers, the price of intangibles falls monotonically over time. However, given that this fall is not as steep as the fall in the price of tangibles before the 80s and after the mid-90s, I find that the price of the composite investment good is almost always higher than the price of tangibles. Note that the rise in the composite investment good price after the mid-90s relative to the reported tangible price series is due to the increasing share of intangibles in overall investment. This causes the relatively benign fall in the price of intangibles to reduce the fall in the price index of the composite investment good. This finding suggests that official statistics still under-estimated the price of capital goods by the late 90s even though we don't observe the same pattern as in the baseline case. As for the impact on the contribution of intangibles and tangibles to the overall capital stock, the qualitative findings remain the same as the baseline specification (see Table 2). However, quantitatively, the contributions track the ones found in Hall, admittedly with a different interpretation, as explained in the previous section. This happens because the composite price index series behaves almost similarly to the price of tangibles. That the quantitative findings are similar to Hall is not surprising: the complementarity assumption works through the price index. If the constructed price index resembles the behavior of the price of tangibles, then the behavior of the generated capital and the contribution of each input is almost the same.

The second experiment involves using the different components of the executive wage compensation which are shown in Figure 4. Figure 13 shows that as I eliminate each component of executive compensation at a time, the overall behavior of the composite investment index is the same but the rise up to 2000 and the subsequent fall, are less pronounced. Note also that as I downplay the compensation of executives, and at the same time their share in wage compensation relative to skilled workers, the composite investment index price series gets closer to the price of tangibles. As a result, the implied stock of capital is closer to the one reported by Hall.

To summarize, the non-inclusion of executive compensation or the inclusion of different components of executive compensation alters the behavior of the composite investment price index but does not take away from the main message of the text: namely, that the reported index of the price of capital goods has a downward bias in the 90s due to the rise in the contribution of intangibles in overall investment. This exercise shows also that the derived capital series and the contribution of intangibles and tangibles to the overall capital stock is not qualitatively affected.

# 8 Conclusion

This paper extended the q-theory of investment to account for intangible investment and used the model to measure the contribution of embodied intangible goods to the overall capital stock in the U.S. The paper also explored the quantitative implications of the model on the behavior of the capital stock and Tobin's q.

The price trend of aggregate investment was shown to contrast markedly with the aggregate investment deflator series reported by national accounts because this deflator series does not account for the price effect of intangibles. In addition, the extended model successfully generated a smoothly-behaving series of capital stock with a market value that predominantly remains above its book value; this reflects a Tobin's q that is mostly above one, a feature which is not observed in empirical measures of Tobin's q when capital is exclusively made of tangible investment. Furthermore, the model shows that the rise in the overall capital stock from the mid-80s until the late 90s was driven mainly by an increase in intangible investment. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the movements in the price of intangibles rather than focusing only on their rising share in overall investment. These findings confirm the changing nature of the stock of capital in the economy and the quantitative importance of intangibles, underscoring the importance of intangibles as a source of value for the firm and as a key component of any investment theory.

Two main avenues are proposed for future work. First, the price of aggregate investment reflected two secular stylized facts: 1) the larger role for intangibles in production and 2) the rise in their cost of production as illustrated by the growth in compensation of skilled labor and executives. The link between the labor market for high-skilled workers and capital markets was shown to be key in obtaining these results. This feature might provide a better understanding of the mixed performance of the econometric estimation of the investment equation. These regressions tend to produce low $R^2$'s and serially correlated residuals. Moreover, additional regressors, such as output and cash flow, also appear to be important factors in the investment decision, as they typically have statistically significant coefficients (Chirinko, 1993). These problems may be due to the omission of intangibles when valuing the true cost of new investment goods. In future work, I plan to use the data series on the acquisition cost of capital corrected for the inclusion of intangibles, as constructed above, to investigate whether the econometric estimation of the investment equation did indeed suffer from specification problems.

A second future extension is to explore the insight that intangibles are produced mainly by skilled workers, in order to address the puzzle found in Hall (2003). In that paper, Hall finds that the payment to intangibles is almost nil. He reaches this conclusion after comparing the income stream accrued from physical capital as predicted by the baseline user-cost of capital theory, with the income stream observed from earnings of firms. Hall finds that the present value of the future flow of income generated by intangibles is close to zero, suggesting that their size is unimportant. I suspect that the treatment of intangibles as expenditures (as opposed to capital goods) caused reported earnings to be undervalued. This omission could deliver the finding that once physical capital is paid for its services, there is nothing left to reward intangible capital. I plan to redo Hall's (2003) exercise using earnings that are corrected for this mismeasurement and investigate whether Hall's findings hold for this correction.

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# Appendix

## A Derivation of First-Order Conditions

The firm's problem is

$$\max_{\{l_s^u, l_s^s, x_s\}} v_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \hat{v}_s$$
$$\hat{v}_t = F(k_{t-1}^x, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s - p_t^x x_t - C(x_t, k_{t-1})$$

s.t.

$$k_t = (1-\delta)k_{t-1} + x_t$$
$$k_{t-1}$$
$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T} \hat{v}_{s+T} = 0.$$

The Lagrangian $\mathcal{L}$ at time $s$ and the first order conditions are given by

$$\max_{\{l_t^u, l_t^s, x_t, k_t, \lambda_t\}} \mathcal{L}_s = \sum_{t=s}^{\infty} \left(\frac{1}{1+r}\right)^{t-s} \{F(k_{t-1}, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s$$
$$-p_t^x x_t - C(x_t, k_{t-1}) - \lambda_t[k_t - (1-\delta)k_{t-1} - x_t]\}$$

$$\frac{\partial \mathcal{L}_t}{\partial x_t} : \quad \lambda_t = p_t^x + C_x(x_t, k_{t-1})$$
$$\frac{\partial \mathcal{L}_t}{\partial l_t^u} : \quad w_t^u = F_{l^u}(k_{t-1}, l_t^u, l_t^t)$$
$$\frac{\partial \mathcal{L}_t}{\partial l_t^s} : \quad w_t^s = F_{l^s}(k_{t-1}, l_t^u, l_t^s)$$
$$\frac{\partial \mathcal{L}_t}{\partial k_t} : \quad \lambda_t(1+r) = F_k(k_{t-1}, l_t^u, l_t^s) - C_k(x_{t+1}, k_t) + (1-\delta)\lambda_{t+1}$$
$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t} : \quad k_t = (1-\delta)k_{t-1} + x_t$$

where $\lambda$ is the Lagrangian multiplier or the shadow price of an additional unit of capital. We assume that bubbles in the shadow price of capital are ruled out, *i.e.* $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T} q_{t+T} = 0$. The first equation illustrates the equality of the lifetime return to increasing capital by one unit with its marginal cost given by the price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. The second and third equation state the usual equilibrium condition for the labor market whereby the real wage is equal to the marginal product of labor. The next equation shows the dynamic equilibrium equation of $\lambda$ with its continuation value. The last equation recasts the investment technology constraint.

# B Data Sources and Definitions

The deflator used is the quarterly CPI and the base year adopted is 1996. This series is obtained from the Bureau of Labor Statistics which reports the CPI on a monthly basis for all urban consumers from 1913 until today (Series ID: CUUR0000SA0). I take the average of three consecutive months to obtain the quarterly equivalent.

The data on output per worker are published by the Bureau of Labor Statistics under the heading "major sector productivity and costs index". Because of data availability, I use the output per person of the business sector from 1950 until 1957 (Series ID: PRS84006163) and then the output per person of the non-financial corporate sector from 1957 until 1975 (Series ID: PRS88003163).

The data on wages of workers by educational attainment is collected annually by the Bureau of Labor Statistics in the Current Population Survey and reported every March for the whole economy. I use table A-3 entitled "Mean Earnings of Workers 18 Years and Over, by Educational Attainment, Race, Hispanic Origin, and Sex: 1975 to 2005" to obtain data on mean annual earnings and number of workers by educational attainment. There are 5 educational levels: not a high school graduate, high school graduate, some college/associate degree, bachelor's degree and advanced degree. Earnings refer to the total income people receive for work performed as an employee during the income year. This includes wages, salary, armed forces pay, commissions, tips, piece-rate payments, and cash bonuses earned, before deductions are made for items such as taxes, bonds, pensions, and union dues. The wage of university workers is calculated as follows. I multiply the mean annual earnings of workers with a bachelor's degree and advanced degree by their respective number of workers and divide the result by the total number of workers with a bachelor's and advanced degree. I extend the data from 1975 until 1950 using the growth of output per worker. The implicit assumption is that the mean earnings of university graduates grew at the same rate as productivity per worker from 1950 to 1975. This assumption has a strong empirical support during this period (see Lemieux (2007)).

The wage bill of skilled labor from 1975 to 2005 is obtained by multiplying the mean earnings of university graduates by the number of university graduates. I extend the data back to 1950 using the economy wide compensation of employees. This series is provided by the Bureau of Economic Analysis in their Table 1.10. entitled “gross domestic income by type of income”, line 2. This extension relies on the same assumption as the one outlined in the previous paragraph.

The data on executive compensation and its composition are taken from Frydman and Saks (2007). This series is a major improvement on previous studies which collected data on executive compensation for short samples, with different sample designs and employed different methodologies to value compensation and its components (see for example Antle and Smith (1985), Hall and Liebman (1998) and Bebchuk and Grinstein (2005).) The work of Frydman and Saks (2007) is the first comprehensive panel dataset on executive compensation that spans the period 1936 to 2003. The sample follows the compensation of individual officers in the largest 50 publicly traded corporations ranked according to the value of sales in 1940, 1960 and 1990. This amounted to a total of 102 firms. Frydman and Saks discuss the representativeness of their sample in Appendix Section 3, and conclude that it is representative of the largest 300 publicly-traded corporations. They limit their analysis to the top three officers in order to maintain a consistent group of individuals over time, but the results are robust to including the 4th and 5th highest-paid executives.

The data on the number of chief executives is taken from the Occupational Employment Statistics (OES) Survey produced by the Bureau of Labor Statistics. The occupational title is “chief executive” with the occupational code 11-1011. This data is available from 1998 to 2005. From 1983 until 1997, the OES survey used a somewhat different classification system. The closest occupational definition for our purpose is the “management, business, and financial operations occupations” (Series ID: LNU02032202). I use the growth rate of this occupation to extend the data on the number of chief executives backward to 1983. Finally, I use the growth rate of the employment in the private sector to extend this data backward to 1950. This data are produced by the Bureau of Economic Analysis under Table 6.5 A-D with the heading “full-time equivalent employees by industry”.

The wage bill of executives is calculated by multiplying the average CEO compensation reported by Frydman and Saks (2007) by the number of chief executives described in the previous paragraph.

The price of tangible investment goods is the national income and product accounts implicit deflator for fixed non-residential investment. This series is published by the Bureau of Economic Analysis under Table 7.1 "quantity and price indexes for GDP quarterly", line 32.

The data on the real capital stock and the value of securities are taken from Hall (2001) and are extended to 2005. The data on intangible expenditures are taken from Corrado *et al.* (2006).

# C Calibration of Starting Values

The initial value of $p^I$ is calibrated such that the firm's investment condition given by equation 19 holds in the long-run. In other words,

$$\delta\alpha = (\overline{\frac{v}{k}} - \overline{p^x})$$

where the variables with a bar denote their sample average. Once the value $p_0^I$ is found, $\theta_0$ is deduced and made to grow at the rate of change of output per worker. Also, $p_0^I$ is used to calculate the starting value of $p^x$.

The empirical starting value of the share of executives in overall wage bill is $\phi_0 = 0.47$. Corrado et al. (2006) calibrate this value to be $\phi_0 = 0.21$ on the basis that executives spend 20% of their time in activities that involve organizational change and design. I chose this starting value to be an average of the two values $\phi_0 = 0.33$. The results are not sensitive to departures from this initial value inside the set of values made of these two boundary values.

Finally, the starting value of $a$ is taken from the constructed series on the share of tangible expenditures in overall investment expenditures. It is found that $a_0 = 0.74$.

# D Allowing for Irreversibility in Investment

The generated stock of capital is not sensitive to the existence of irreversibility in investment. It is possible to assume that the cost function is piece-wise quadratic :

$$C(x_t, k_{t-1}) = \begin{cases} \frac{\alpha^+}{2}\left(\frac{x_t}{k_{t-1}}\right)^2 k_{t-1} & \text{if } x_t > 0 \\ \frac{\alpha^-}{2}\left(\frac{x_t}{k_{t-1}}\right)^2 k_{t-1} & \text{if } x_t < 0 \end{cases}$$

where the adjustment-cost parameter $\alpha^+$ ($\alpha^-$) has the same interpretation as in the model: it represents the time it takes for the capital stock to double (halve) when $\lambda$ doubles (halves). By allowing the downward adjustment-cost parameter to be higher than the upward adjustment-cost parameter, this asymmetry in the investment decision will reflect irreversibility of investment. Setting $\alpha^+ = 8$ as in the main text and allowing $\alpha^-$ to be arbitrarily set at up to ten times higher than the upward adjustment-cost parameter, the generated capital stock will not be affected. The result behind this finding comes from the fact that gross investment is almost always above depreciation expenditures in the data. In other words, net investment is almost always positive so there is little evidence on irreversibility at the aggregate level. In addition, when net investment is negative, its magnitude is quite small.