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Bait Contracts

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Abstract

This paper explores contracting in the presence of ambiguity. It revisits Holmström's (1979) sufficient statistic result of when to condition a contract on an outside signal. It is shown that if the signal is ambiguous, in the sense that its probability distribution is unknown, then Holmström's result can be overturned. Specifically, uninformative ambiguous signals can be valuable.

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1 Introduction

In recent years a number of papers on contracting and mechanism design under ambiguity have appeared. The increasing interest is well motivated, as ambiguity seems especially

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relevant for such problems. However, it is important to fully understand the implications of introducing ambiguity. Specifically, there is a number of well-known results for contracting in standard settings that may or may not hold under ambiguity.

The present paper shows that when allowing for ambiguity, one such well-known standard result, namely Holmström's (1979) sufficient statistic result no longer holds. In a standard model with no ambiguity, Holmström considers the conditions under which a principal can improve upon a contract by conditioning on an outside signal. His sufficient statistic result shows that it is optimal to make the contract contingent on the outside signal if and only if the signal is not orthogonal to the directly payoff relevant variables of interest.

This paper considers the canonical principal-agent model with hidden information with the distinctive feature that there exists an ambiguous public signal which the contract may be made contingent on. Both the principal (she) and the agent (he) are assumed to have preferences that are represented by the α-maxmin expected utility model axiomatized in Ghirardato, Maccheroni, and Marinacci (2004). Here, each party has a set of priors (rather than a single prior) on the underlying state space and an attitude towards the realization of ambiguity that is captured by a parameter α, which can be interpreted as the party's optimism. The payoff from any action is computed by weighing the maximum and the minimum expected utility of the action over the set of priors with weight α on the maximum and $1 - \alpha$ on the minimum.

The first result of this paper is that for a public signal that is uninformative about the private information of the agent, that is, for a signal which is a payoff irrelevant random variable (henceforth PIRV), the principal can benefit from conditioning the contract on the public signal if and only if the signal is ambiguous. Hence, even PIRVs can be of value if they are ambiguous. Contracts that condition on the signal are denoted 'bait contracts'.[1] The second result shows that a bait contract cannot be decomposed into a standard (non conditioning) contract and a pure side bet. The optimality of a bait contract which conditions on a PIRV shows that ambiguity overturns Holmström's result.

The presence of the ambiguous public signal provides the principal with an additional instrument she can use when designing the contract, namely that of deliberately introducing ambiguity into the contract. By doing so, she can exploit optimistic ambiguity attitudes,

[1]The intuition for the use of the term 'bait contract' should become clear below.

which will be left unexploited with a contract that does not condition on the signal. By conditioning on the PIRV the principal creates endogenous heterogeneity in the parties' overall decision weights, or effective beliefs. Such heterogeneity results in the parties having a motive to bet on the resolution of the PIRV. If the agent is optimistic, the betting will be to the principal's advantage, and the optimal contract is a bait contract.

A bait contract thus fulfills two purposes. On one hand, it serves the usual purpose of ensuring participation of and providing incentives for the agent. On the other hand, the parties are betting on their differences in effective beliefs through the contract. If the problem was broken up into separate contracting and betting problems, the betting would upset the marginal trade-offs in the solution to the contracting problem. Thus, bundling the two problems dominates solving them separately, as the joint solution allows all marginal trade-offs, both those for given and those across signal realizations, to be at their optima.

The introduction of ambiguity is motivated by experimental and theoretical work. Ellsberg (1961) showed that ambiguity can affect the choice of a decision maker in a fundamental way that cannot be captured by a framework that assumes a unique prior. Many subsequent papers have underlined the importance of ambiguity and of decision makers' attitude towards it in understanding observed behavior.

As mentioned, there appears to have been increasing interest recently in contracting under ambiguity and in other non-standard choice theoretic settings. Mukerji (1998) considers a moral hazard problem with firms in a vertical relationship and a discrete choice set and shows that ambiguity aversion among the parties can rationalize incomplete contracts. Mukerji and Tallon (2004) also consider a contracting problem where agents are ambiguity averse. Lopomo, Rigotti, and Shannon (2011) consider a principal-agent model with moral hazard where the agent's beliefs are imprecise due to incomplete preferences. Vierø (2012) considers contracting between risk neutral parties when the contracting environment itself is vague (or ambiguous), and shows that the presence of vagueness or ambiguity often leads to the standard 'sell the firm to the agent' contract being suboptimal. None of these papers consider the issue of conditioning on an outside signal. Kotowski (2012) considers a principal-agent problem with moral hazard in which the agent is ambiguity averse and the principal can be ambiguous concerning the contract's evaluative criteria.

A different group of related papers considers contracting when the parties have heterogeneous beliefs. These include Adrian and Westerfield (2009) and Carlier and Renou (2005, 2006). When beliefs are heterogeneous, the parties also have a motive to bet on the resolution of uncertainty, but there is no possibility for the principal to influence the agent's weight on the different final scenarios. With precise information and heterogeneous beliefs, all differences between the contracting parties are exogenous. These papers also do not consider the issue of conditioning on an outside signal.

A third group of related papers analyzes mechanism design problems under uncertainty. Levin and Ozdenoren (2004) consider auctions when there is ambiguity about the number of bidders, while Bose, Ozdenoren, and Pape (2006) and Bose and Daripa (2009) study auctions when there is ambiguity about the bidders' valuations. De Castro and Yanellis (2011) show that when individuals have MEU preferences, then any efficient allocation is incentive compatible. Lopomo, Rigotti, and Shannon (2009) consider mechanism design when preferences are incomplete.

The paper is organized as follows: Section 2 presents the model with ambiguity. Section 3 contains the results. Section 4 concludes. Proofs can be found in the appendix.

2 Model

Consider the canonical principal-agent problem with hidden information.[2] A principal, who is risk neutral, wants to hire a risk averse agent to complete a task. It is assumed that the agent's effort can be measured by a one-dimensional variable $e \in [0, \infty)$. The principal's gross profit is a continuous function of the agent's effort, $\pi(e)$, with $\pi(0) = 0$, first-order derivative $\pi'(e) > 0 \ \forall e > 0$, and second-order derivative $\pi''(e) < 0 \ \forall e > 0$. The principal's Bernoulli utility function is given by her net profits,

$$u_P(w, e) = \pi(e) - w,$$

where w denotes the wage she pays to the agent.

The agent's utility is assumed to depend on a variable, measuring how well suited to the required task he will find himself. The value of this variable is realized after the contract is signed. For convenience, it is referred to as the agent's efficiency level, but it could be

[2]See, for example, Mas-Colell, Whinston, and Green (1995, chp. 14.C).

interpreted in a variety of ways. The agent's Bernoulli utility function depends on his wage w, how much effort he exerts e, and his efficiency x, which affects how much disutility, denoted $g(e, x)$, he experiences from effort. There are assumed to be two possible values of x: the agent will be either of high-efficiency type x_H or of low-efficiency type x_L. It is assumed that the efficiency level is unobservable to the principal. Effort, on the other hand, is observable and contractible.

Assume further that the agent's Bernoulli utility function is of the form

$$u_A(w, e, x) = v(w - g(e, x)), \text{ with } v'(\cdot) > 0 \text{ and } v''(\cdot) < 0.$$

The disutility $g(e, x)$ is assumed to satisfy the following standard conditions: the first-order derivative w.r.t. e is $g_e(e, x) > 0 \ \forall e > 0$ and the second-order derivative w.r.t. e is $g_{ee}(e, x) > 0 \ \forall e$, such that his disutility from effort is increasing at an increasing rate, $g(0, x_H) = g(0, x_L) = g_e(0, x_H) = g_e(0, x_L) = 0$, such that the agent suffers no disutility if he does not exert any effort, and $g_e(e, x_L) > g_e(e, x_H) \ \forall e > 0$, such that his marginal disutility from positive effort is higher if he is of low-efficiency type. Note that these conditions imply that $g(e, x_L) > g(e, x_H) \ \forall e > 0$, that is, the disutility of any positive effort level is also higher for the low-efficiency type. Finally, let $\overline{u}$ denote the agent's reservation utility, which for simplicity (and without effect on the results) is assumed to equal zero.

Suppose there is a publicly observable and verifiable outside signal, which can take values y_H or y_L. The state space is the Cartesian product of the two possible realizations of the agent's type and the two possible realizations of the signal. Thus, there are four possible states of the world.

I assume that the signal and the agent's type are independent, i.e. the probability of realizing the pair (x_i, y_j), $i, j = H, L$, is the product of the relevant marginal probabilities. Because the signal is orthogonal to, and thus uninformative about, the agent's type, it is referred to as a PIRV. Let $p = (p_H, p_L)$, with $p_H \in (0, 1)$, denote the marginal probability distribution over the agent's types. That is, the agent will be of type x_i with probability p_i. This probability is known to both parties to the contract. Hence, the contracting environment itself is unambiguous, i.e. there is no ambiguity about the directly payoff relevant variable x.

The PIRV, however, is potentially ambiguous. That is, the contracting parties may not know the precise marginal probability with which the PIRV will take the value y_H. Instead they only know a possible set $Q = \{q = (q_H, q_L) : \; q_H \in [a,b], q_L = 1 - q_H, 0 \le a \le b \le 1\}$ of this marginal distribution. That is, the probability of y_H is $q_H \in [a,b] \subseteq [0,1]$. The parties therefore have common but, in general, ambiguous knowledge of the marginal probability of the PIRV taking value y_H. The PIRV is only unambiguous in the special case $a = b$.[3]

Contracting is assumed to take place ex-ante, i.e. before the agent learns his type and before the PIRV is realized. Ex-ante contracting has two stages: the agent first agrees to a menu of wage-effort pairs. Then, once he learns his type, the agent selects one of the wage-effort pairs in the menu by announcing his type. It is assumed that the principal is unable to observe the agent's efficiency level at any point in time; hence there is asymmetric information at the interim.

The principal can choose whether or not to make the contract contingent on the ambiguous signal. That is, she can decide whether or not to make it a bait contract. A bait contract, denoted C, consists of different wage-effort pairs for different values of the agent's type *and* of the signal:

$$\begin{aligned} C &= \big(e(x_H, y_H), w(x_H, y_H), e(x_H, y_L), w(x_H, y_L), e(x_L, y_H), w(x_L, y_H), e(x_L, y_L), w(x_L, y_L)\big) \\ &\equiv \big(e_{HH}, w_{HH}, e_{HL}, w_{HL}, e_{LH}, w_{LH}, e_{LL}, w_{LL}\big). \end{aligned}$$

Since the PIRV is publicly observable, conditioning the wage-effort pairs on it does not lead to any further informational asymmetry between the contracting parties. For a particular realization of the PIRV, the agent must choose one of the corresponding wage-effort pairs at the interim: If $y = y_H$, then the agent's choice is between (e_{HH}, w_{HH}) and (e_{LH}, w_{LH}), while if $y = y_L$, then the agent's choice is between (e_{HL}, w_{HL}) and (e_{LL}, w_{LL}). A bait contract therefore has to be incentive compatible given each value of the signal.

For an incentive compatible contract, the agent will truthfully reveal his type. Given a high value of the PIRV, he will thus exercise effort e_{HH} and be paid wage w_{HH} when he is of high-efficiency type x_H, and exercise effort e_{LH} and be paid wage w_{LH} when he is of low-efficiency type x_L. Similarly, if the value of the PIRV is low, he will exercise effort e_{HL} and be paid wage w_{HL} when he is of high-efficiency type x_H, and exercise effort e_{LL} and be paid wage w_{LL} when he is of low-efficiency type x_L. Note that the agent's Bernoulli utility is

[3]It is assumed that we do not have $a = b = 0$ or $a = b = 1$, in which case there would be ex-ante certainty about the realization of the PIRV.

state-dependent, since his disutility depends on the realization of his type. This is captured by letting x be an argument of the utility function. For ease of notation, define $z \equiv (w, e, x)$.

The preferences of the parties are represented by the α-maxmin expected utility (henceforth α-MMEU) model axiomatized in Ghirardato, Maccheroni, and Marinacci (2004). A similar representation under objective ambiguity is axiomatized in Olszewski (2007) and Vierø (2009).[4] In the present context, the assumption of α-MMEU preferences implies that both the principal and the agent maximize utility of the following form:

$$U_k(C) = \alpha_k \sum_{\substack{j\in\{H,L\}\\ i\in\{H,L\}}} \overline{q}_{k,j} p_i u_k(z_{ij}) + (1-\alpha_k) \sum_{\substack{j\in\{H,L\}\\ i\in\{H,L\}}} \underline{q}_{k,j} p_i u_k(z_{ij}), \tag{1}$$

where j indexes the value of the PIRV, i indexes the agent's type, $k \in \{P, A\}$, u_k is k's Bernoulli utility function defined over $z_{ij} = (w_{ij}, e_{ij}, x_i)$, and $\alpha_k \in [0,1]$ is a parameter that captures k's ambiguity attitude or degree of optimism. Finally, $\overline{q}_k = (\overline{q}_{k,H}, \overline{q}_{k,L})$ and $\underline{q}_k = (\underline{q}_{k,H}, \underline{q}_{k,L})$ are, respectively, the best and worst marginal probability distributions in the set Q from k's point of view given the contract C. That is,

$$\overline{q}_k = \arg\max_{q\in Q} \sum_{\substack{j\in\{H,L\}\\ i\in\{H,L\}}} q_j p_i u_k(z_{ij}) \tag{2}$$

and

$$\underline{q}_k = \arg\min_{q\in Q} \sum_{\substack{j\in\{H,L\}\\ i\in\{H,L\}}} q_j p_i u_k(z_{ij}), \tag{3}$$

where z_{ij} is specified by the contract. Since x and y are independent, there is a one-to-one correspondence between these best and worst marginal probabilities and the best and worst overall probabilities over the four states.

It is important to note that which probabilities are best and worst depend on the contract offered. Therefore, the contract offered endogenously determines the beliefs of the agent and principal. Consequently, the agent and the principal may endogenously have heterogeneous beliefs. This is the driving force behind the optimality of bait contracts. The ambiguous PIRV provides the principal with an extra instrument when designing the contract, which enables her to exploit optimistic ambiguity attitudes.

[4] Gilboa and Schmeidler (1989) axiomatize the case of $\alpha = 0$, while Ahn (2008) provides an alternative representation of preferences under objective ambiguity.

To focus on the case in which the principal cares about expected profits, it is henceforth assumed that she is ambiguity neutral, that is, $\alpha_P = \frac{1}{2}$. Given this assumption, the principal's effective weight on (or effective belief about) the PIRV taking value y_H is $\frac{1}{2}\overline{q}_{P,H} + \frac{1}{2}\underline{q}_{P,H}$ and thus independent of whether the principal is best off when the PIRV takes value y_H or y_L. Put differently, when the principal is ambiguity neutral her effective weight on the PIRV taking value y_H is the same when $\overline{q}_{P,H} = a$, $\underline{q}_{P,H} = b$ and when $\overline{q}_{P,H} = b$, $\underline{q}_{P,H} = a$, since this weight will be equal to the midpoint of the interval $[a, b]$.[5]

Let $q^e_{P,H}$ denote the midpoint of the interval $[a, b]$, i.e. $q^e_{P,H} \equiv \frac{1}{2}a + \frac{1}{2}b$. As described in the previous paragraph, the principal's effective weight on the PIRV taking value y_H will be equal to $q^e_{P,H}$. Also, any set Q of possible marginal probability distributions over the signal y can be described by the corresponding $q^e_{P,H}$ and a parameter $\delta \geq 0$, and is given by $Q = \{(q_H, 1 - q_H) \mid q_H \in [q^e_{P,H} - \delta, q^e_{P,H} + \delta]\}$ where $\delta = b - q^e_{P,H}$. Thus, for any set Q, ambiguity is symmetric around the ambiguity neutral principal's effective weight $q^e_{P,H}$. This situation is the same as one in which the principal has standard preferences with belief $q^e_{P,H}$ that the signal takes value y_H and ambiguity being symmetric around the principal's beliefs. It is in this sense that the principal cares about expected profits.

Using the notation just introduced, the agent's effective weight on the PIRV taking high value y_H will be

$$q^e_{A,H} = q^e_{P,H} + (2\alpha_A - 1)\delta \tag{4}$$

if the contract makes the agent best off when the PIRV takes value y_H, and

$$q^e_{A,H} = q^e_{P,H} + (1 - 2\alpha_A)\delta \tag{5}$$

if the contract makes the agent worst off when the PIRV takes value y_H. If $\delta = 0$, the PIRV is unambiguous, and by (4) and (5) the parties' effective weights will be equal. If $\delta > 0$, the PIRV is ambiguous, and, moreover, by (4) and (5) the parties will have different effective weights for all $\alpha_A \neq \frac{1}{2}$.

[5]It follows immediately that the principal's weight on the PIRV taking value y_L is also independent of the value of the PIRV under which she is best off.

3 Bait contracts

It is now shown that there exists a set of principals and agents for which the principal will write deliberately ambiguous contracts. That is, the principal will introduce ambiguity into a contracting situation where there is otherwise no ambiguity, since the principal will optimally choose to condition the contract on the ambiguous PIRV. Therefore, signals can be valuable, even if they are independent of the unobserved variables that directly affect the parties' payoffs, as long as they are ambiguous.

In a standard model with no ambiguity, Holmström (1979) considers the question of when an outside signal can be used to improve upon a contract. He shows that it is optimal to make the contract contingent on the outside signal if and only if the signal is not orthogonal to the directly payoff relevant variables of interest. Holmström obtains his result in a moral hazard model, but a similar result is easily derived in a model with adverse selection and unambiguous information. I now show that Holmström's result can be overturned if the outside event adds ambiguity to the contracting situation.

The principal can choose whether or not to make the contract contingent on the ambiguous PIRV, i.e. whether to write a bait contract or a standard contract. A standard contract does not condition on the signal and therefore has $e(x_H, y_H) = e(x_H, y_L) \equiv e_H$, $e(x_L, y_H) = e(x_L, y_L) \equiv e_L$, $w(x_H, y_H) = w(x_H, y_L) \equiv w_H$, and $w(x_L, y_H) = w(x_L, y_L) \equiv w_L$. Hence, it is a special case of conditioning and will appear as the solution to the problem below if optimal.

Vierø (2012, Theorem 1) shows that the revelation principle holds in the presence of ambiguity, i.e., that any general incentive compatible contract can be implemented with a truthful revelation mechanism. Given that, the principal's problem of finding the optimal contract is given by

$$max_{\substack{w_{HH}, e_{HH} \geq 0 \\ w_{HL}, e_{HL} \geq 0 \\ w_{LH}, e_{LH} \geq 0 \\ w_{LL}, e_{LL} \geq 0}} q^e_{P,H} \Big(p_H \big(\pi(e_{HH}) - w_{HH} \big) + p_L \big(\pi(e_{LH}) - w_{LH} \big) \Big)$$
$$+ \big(1 - q^e_{P,H}\big) \Big(p_H \big(\pi(e_{HL}) - w_{HL} \big) + p_L \big(\pi(e_{LL}) - w_{LL} \big) \Big)$$

subject to

$$\begin{aligned} &q^e_{A,H}\Big(p_H v\big(w_{HH} - g(e_{HH}, x_H)\big) + p_L v\big(w_{LH} - g(e_{LH}, x_L)\big)\Big) \\ &+\big(1 - q^e_{A,H}\big)\Big(p_H v\big(w_{HL} - g(e_{HL}, x_H)\big) + p_L v\big(w_{LL} - g(e_{LL}, x_L)\big)\Big) \geq 0, \end{aligned} \tag{PC}$$

$$v\big(w_{HH} - g(e_{HH}, x_H)\big) \geq v\big(w_{LH} - g(e_{LH}, x_H)\big), \tag{IC_{HH}}$$

$$v\big(w_{LH} - g(e_{LH}, x_L)\big) \geq v\big(w_{HH} - g(e_{HH}, x_L)\big), \tag{IC_{LH}}$$

$$v\big(w_{HL} - g(e_{HL}, x_H)\big) \geq v\big(w_{LL} - g(e_{LL}, x_H)\big), \tag{IC_{HL}}$$

$$v\big(w_{LL} - g(e_{LL}, x_L)\big) \geq v\big(w_{HL} - g(e_{HL}, x_L)\big), \tag{IC_{LL}}$$

where $q^e_{P,H} = \frac{1}{2}a + \frac{1}{2}b$ and $q^e_{A,H}$ is given by (4) if the agent is best off when $y = y_H$, by (5) if he is best off when $y = y_L$, and equals $q^e_{P,H}$ if he is equally well off under the two realizations of the PIRV.[6]

Conditioning on the ambiguous PIRV results in the agent receiving different utility for each possible realization of the PIRV. His effective weight on $y = y_H$ will be given by (4) or (5) and thus be different from the principal's. Hence, conditioning generally creates an endogenous heterogeneity in the effective weights (or effective beliefs) that the parties assign to the two possible realizations of the signal. Consequently, the contract fulfills two purposes. Not only does it serve the usual purpose of ensuring participation of and providing incentives for the agent, but the parties are also placing side-bets on their differences in effective beliefs through the contract. If the agent is ambiguity loving, the principal can exploit the heterogeneity, and therefore it is worthwhile for her to generate a difference in the agent's utility. Thus, with ambiguity loving (or optimistic) agents, the principal can use the ambiguous PIRV to her advantage.

The following theorem shows that bait contracts can be optimal when there is ambiguity, i.e. that there exist principals and agents for which the principal will condition the wage-effort pairs on an ambiguous PIRV.

Theorem 1 (Optimality of bait contracts). *Let the set of possible marginal probability distributions over the PIRV y be given by $Q = \{(q_H, 1 - q_H) \mid q_H \in [q^e_{P,H} - \delta, q^e_{P,H} + \delta]\}$, where $\delta \geq 0$. Then the following two statements are equivalent:*

[6] When the agent is equally well off under the two realizations of the PIRV, all probabilities in Q are equally good from his point of view, and $\overline{q}_{A,H}$ and $\underline{q}_{A,H}$ can be chosen such that $q^e_{A,H} = q^e_{P,H}$.

i. There exists a set of agents characterized by $\{\alpha_A, v(\cdot), g(e,x)\}$ *for which the optimal contract conditions on the PIRV.*

ii. The PIRV is ambiguous, that is, $\delta > 0$.

Proof: See the appendix.

Theorem 1 states that if and only if the outside signal is ambiguous, there exists a set of principals and agents for which the principal will optimally choose to make the contract contingent on the realization of the outside signal, even when this signal is uninformative about the variable that is subject to asymmetric information at the interim and directly affects the parties' payoffs. By conditioning on the ambiguous PIRV, the principal introduces ambiguity into the otherwise unambiguous contracting situation. That way she creates an endogenous heterogeneity in the effective beliefs that the parties assign to the different realizations of the PIRV. This endogenous heterogeneity is a consequence of the ambiguity and depends on the contract offered.

The heterogeneity in effective beliefs introduces a betting motive into the contract. If the agent's effective weight on the signal realization being high is higher than the principal's, the principal can improve upon the standard contract by offering the agent a higher utility when $y = y_H$. With such a contract, the best distribution in Q from the agent's point of view has $\overline{q}_{A,H} = b$, while the worst has $\overline{q}_{A,H} = a$. In this case, the agent will therefore assign higher effective weight to the signal realization being high if and only if he is ambiguity loving, i.e. $\alpha_A > \frac{1}{2}$. If instead the agent's effective weight on the signal realization being low is higher than the principal's, the principal can improve upon the standard contract by offering the agent a higher utility when $y = y_L$. By an argument similar to that just made, the agent will in this case again assign higher effective weight to the signal realization being low if and only if he is ambiguity loving.[7]

[7] In the present paper, it is assumed that the principal is ambiguity neutral, i.e. that $\alpha_P = \frac{1}{2}$. Although not fully explored, numerical computations indicate that the result in Theorem 1 continues to hold for $\alpha_P \neq \frac{1}{2}$ as long as $\alpha_A + \alpha_P > 1$, that is, if the parties are on average ambiguity loving. When $\alpha_P = \frac{1}{2}$, the condition that $\alpha_A > \frac{1}{2}$ ensures that the parties are ambiguity loving on average. For general α_P, the endogenous heterogeneity in effective beliefs has two possible sources. The first source of heterogeneity is the potential difference in those marginal probability distributions that are the best and worst for each of the contracting parties, and the second source is the parties' ambiguity attitudes as captured by the parameters

The side bet makes it easier to get the agent to participate: with a bait contract the average (according to the ambiguity neutral principal's beliefs) compensation needed for the agent's participation is lower than with a contract that does not condition on the signal. By writing a bait contract, the principal attempts to bait the agent by offering him a high payoff given one value of the signal. If the agent is sufficiently optimistic, he will put a lot of weight on this high payoff and take the bait. This means that the agent will accept contracts with an average compensation that would otherwise be insufficient to ensure his participation.

The next theorem turns to the question of whether it is possible to mimic a bait contract by offering a contract and a side bet separately.

Theorem 2 (No decomposition)**.** *There exists a set of agents characterized by* $\{\alpha_A, v(\cdot), g(e,x)\}$ *for which the optimal contact is a bait contract and this bait contract cannot be decomposed into a standard contract and a pure side bet.*

Proof: See the appendix.

By Theorem 2, bundling dominates breaking up the problem into distinct contracting and betting problems. The intuition is as follows. Since a standard contract does not condition on the PIRV, it specifies the same effort level and wage for a particular type of agent, regardless of the value of the PIRV. As it is the case for the bait contract, the optimal non-bait contract distorts effort for the low-efficiency type in order to ensure incentive compatibility for the high-efficiency type. The exact amount of distortion is such that the principal's marginal loss in profit for the low-efficiency type x_L equals the marginal gain in profit for the high-efficiency type x_H:

$$\begin{aligned}&(1-p_H)\big[\pi'(e_L) - g_e(e_L, x_L)\big]\\ &= p_H\Big[1 - \tfrac{v'(w_L - g(e_L, x_H))}{p_H v'(w_L - g(e_L,x_H)) + (1-p_H) v'(w_L - g(e_L, x_L))}\Big]\big[g_e(e_L, x_L) - g_e(e_L, x_H)\big],\end{aligned}$$

which is the non-conditioning equivalent to conditions (24) and (25) in the proof of Theorem 1 in the appendix. Suppose now that the principal also offers a side bet, which consists of a payment of T_H that the principal pays to the agent when the PIRV takes value y_H and a payment T_L that the agent pays to the principal when the PIRV takes value y_L. The optimal payments will be such that the parties' marginal rates of substitution between the

(α_A, α_P).

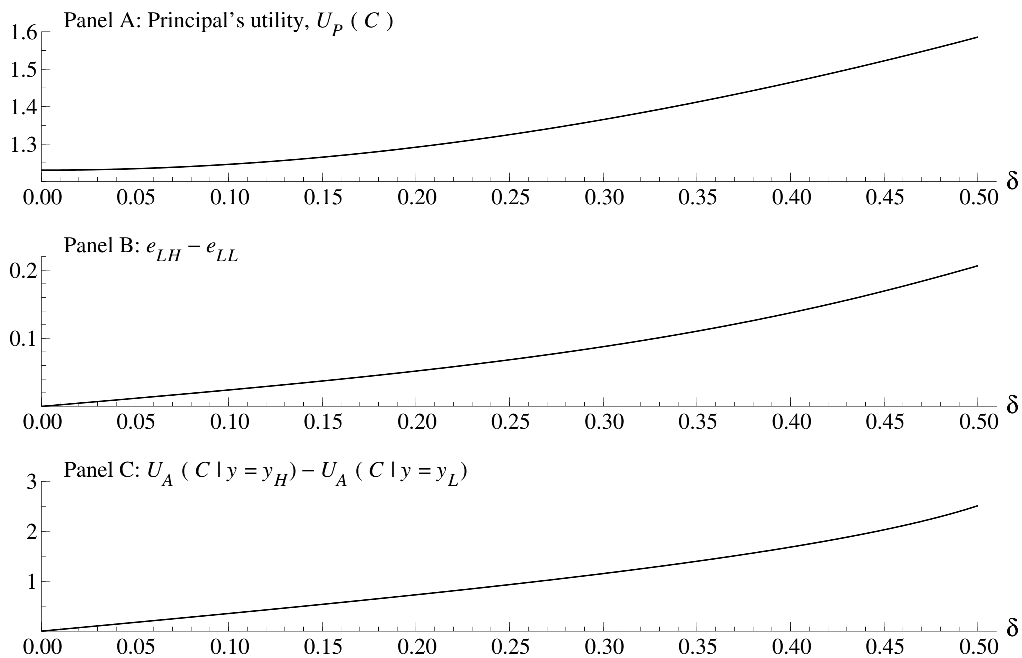
two possible realizations of the signal are equal. That is, the optimal T_H and T_L are such that (22) and (23) in the appendix hold for $e_{LH} = e_{LL} = e_L$, $w_{LH} = w_L + T_H$, and $w_{LL} = w_L - T_L$. However, since the agent is risk averse with strictly concave v, (24) and (25) will no longer be satisfied after the transfer.

When the principal instead solves the contracting and betting problems jointly, she can adjust the effort level for the low-efficiency type for the two values of the signal such that not only the parties' marginal rates of substitution given the two possible signal realizations are equal, but also the marginal loss in profit for the low-efficiency type continues to equal the marginal gain in profit for the high-efficiency type for either value of the PIRV. By bundling the problems, the principal thus ensures that all marginal trade-offs will be at their optima. This cannot be achieved by the decomposed contract.

As an example of the results in Theorems 1 and 2, consider the solution to the principal's problem when $v(\cdot) = \log(\cdot)$, $g(e, x) = \frac{e^2}{2x}$, and $\pi(e) = 2e^{1/2}$, and the parameters take values $x_L = 1$, $x_H = 8$, $p_H = \frac{1}{2}$, $\alpha_A = 0.95$, and the interval $[a, b]$ is symmetric around $\frac{1}{2}$. In this case, the solution when there is no ambiguity (i.e. when $\delta = 0$) has $(e_{HH}, w_{HH}, e_{LH}, w_{LH}) = (e_{HL}, w_{HL}, e_{LL}, w_{LL}) = (4.000, 2.197, 0.908, 1.248)$. This optimal contract consists of the same wage-effort pairs for either realization of the PIRV and has effort for the high-efficiency type at the first-best level, while effort for the low-efficiency type is distorted downwards to ensure incentive compatibility for the high-efficiency type. With this contract, the utility for either type of agent is zero (the reservation utility).

If instead $\delta = 0.1$, the PIRV is ambiguous. In this case, $(e_{HH}, w_{HH}, e_{LH}, w_{LH}) = (4.000, 2.355, 0.918, 1.408)$ and $(e_{HL}, w_{HL}, e_{LL}, w_{LL}) = (4.000, 2.005, 0.893, 1.055)$. The optimal contract thus consists of different wage-effort pairs for the two realization of the PIRV. Effort for the high-efficiency type remains at the first-best level for either realization, but the effort levels for the low-efficiency type and the wages differ. When the signal takes value y_H, either type of agent is paid a higher wage and the effort for the low-efficiency type is less distorted than when there is no ambiguity. On the contrary, when the PIRV takes value y_L, either type of agent is paid a lower wage and the effort for the low-efficiency type is more distorted than when there is no ambiguity. The agent gets positive utility when the PIRV-realization is high and negative utility when the PIRV-realization is low. The principal gets higher utility with this bait contract than with a standard contract. Figure 1 illustrates that,

Figure 1: Numerical example



for the example under consideration, the principal's utility (Panel A) as well as the difference $e_{LH} - e_{LL}$ (Panel B) and the difference in the agent's utility given the two realizations of the PIRV (Panel C) are all increasing in the level of ambiguity δ.

The presence of ambiguity and the assumption of α-MMEU preferences together drive the results. If the outside signal is unambiguous, the set of probability distributions is a singleton and the best and worst distributions always coincide. In that case, we are in the standard model. When the signal is ambiguous on the other hand, the best and worst probability distributions differ and the assumption of α-MMEU preferences makes a difference.

4 Concluding remarks

This paper has revisited Holmström's (1979) sufficient statistic result and shown that for a signal that is uninformative about the agent's superior information, the optimal contract conditions on this signal if and only if the signal is ambiguous. Contracts that condition on the realization of the signal are referred to as "bait contracts." The basic intuition behind the optimality of bait contracts is that the ambiguous signal gives the principal an extra instrument she can use when designing contracts. By conditioning on the ambiguous signal, the principal creates a motive for betting and can thereby take advantage of optimistic ambiguity attitudes. The analysis above showed that this extra instrument can indeed be valuable. It was also shown that the bait contract cannot be decomposed into a standard (non-conditioning) contract and a pure side bet. The reason is that when solving the contracting and betting problems jointly, the principal can adjust the agent's effort such that the amount of distortion that occurs to ensure incentive compatibility for each realization of the signal is optimal given the wealth transfer across states.

Appendix

Proof of Theorem 1: It is first shown that *i.* implies *ii.* by showing the contrapositive. This follows simply by noting that if *ii.* is not true (i.e., if $a = b$) we are in the standard model with precise information, which implies that *i.* is not true.

It is now shown that *ii.* implies *i.*. The Lagrangian for the principal's problem is

$$
\begin{aligned}
\mathscr{L} \;=\;& q^e_{P,H}\Big(p_H\big(\pi(e_{HH}) - w_{HH}\big) + (1-p_H)\big(\pi(e_{LH}) - w_{LH}\big)\Big) \\
& +\big(1-q^e_{P,H}\big)\Big(p_H\big(\pi(e_{HL}) - w_{HL}\big) + (1-p_H)\big(\pi(e_{LL}) - w_{LL}\big)\Big) \\
+\;& \gamma\Big[q^e_{A,H}\Big(p_H\, v\big(w_{HH} - g(e_{HH}, x_H)\big) + (1-p_H)v\big(w_{LH} - g(e_{LH}, x_L)\big)\Big) \\
& +\big(1-q^e_{A,H}\big)\Big(p_H\, v\big(w_{HL} - g(e_{HL}, x_H)\big) + (1-p_H)v\big(w_{LL} - g(e_{LL}, x_L)\big)\Big)\Big] \\
+\;& \lambda_{HH}\big[v\big(w_{HH} - g(e_{HH}, x_H)\big) - v\big(w_{LH} - g(e_{LH}, x_H)\big)\big] \\
+\;& \lambda_{HL}\big[v\big(w_{HL} - g(e_{HL}, x_H)\big) - v\big(w_{LL} - g(e_{LL}, x_H)\big)\big] \\
+\;& \lambda_{LH}\big[v\big(w_{LH} - g(e_{LH}, x_L)\big) - v\big(w_{HH} - g(e_{HH}, x_L)\big)\big] \\
+\;& \lambda_{LL}\big[v\big(w_{LL} - g(e_{LL}, x_L)\big) - v\big(w_{HL} - g(e_{HL}, x_L)\big)\big],
\end{aligned}
$$

where γ, λ_{HH}, λ_{HL}, λ_{LH}, and λ_{LL} are the Lagrange multipliers. The first-order conditions for the problem are

$$p_H q^e_{P,H} \pi'(e_{HH}) - \gamma p_H q^e_{A,H} v'(w_{HH} - g(e_{HH}, x_H)) g_e(e_{HH}, x_H)$$
$$-\lambda_{HH} v'(w_{HH} - g(e_{HH}, x_H)) g_e(e_{HH}, x_H) + \lambda_{LH} v'(w_{HH} - g(e_{HH}, x_L)) g_e(e_{HH}, x_L) \leq 0, \tag{6}$$

$$\begin{aligned} &-p_H q^e_{P,H} + \gamma p_H q^e_{A,H} v'(w_{HH} - g(e_{HH}, x_H)) \\ &+\lambda_{HH} v'(w_{HH} - g(e_{HH}, x_H)) - \lambda_{LH} v'(w_{HH} - g(e_{HH}, x_L)) = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} &(1-p_H) q^e_{P,H} \pi'(e_{LH}) - \gamma (1-p_H) q^e_{A,H} v'(w_{LH} - g(e_{LH}, x_L)) g_e(e_{LH}, x_L) \\ &+\lambda_{HH} v'(w_{LH} - g(e_{LH}, x_H)) g_e(e_{LH}, x_H) - \lambda_{LH} v'(w_{LH} - g(e_{LH}, x_L)) g_e(e_{LH}, x_L) \leq 0, \end{aligned} \tag{8}$$

$$\begin{aligned} &-(1-p_H) q^e_{P,H} + \gamma (1-p_H) q^e_{A,H} v'(w_{LH} - g(e_{LH}, x_L)) \\ &-\lambda_{HH} v'(w_{LH} - g(e_{LH}, x_H)) - \lambda_{LH} v'(w_{LH} - g(e_{LH}, x_L)) = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} &p_H \big(1 - q^e_{P,H}\big) \pi'(e_{HL}) - \gamma p_H \big(1 - q^e_{A,H}\big) v'(w_{HL} - g(e_{HL}, x_H)) g_e(e_{HL}, x_H) \\ &-\lambda_{HL} v'(w_{HL} - g(e_{HL}, x_H)) g_e(e_{HL}, x_H) + \lambda_{LL} v'(w_{HL} - g(e_{HL}, x_L)) g_e(e_{HL}, x_L) \leq 0, \end{aligned} \tag{10}$$

$$\begin{aligned} &-p_H \big(1 - q^e_{P,H}\big) + \gamma p_H \big(1 - q^e_{A,H}\big) v'(w_{HL} - g(e_{HL}, x_H)) \\ &+\lambda_{HL} v'(w_{HL} - g(e_{HL}, x_H)) - \lambda_{LL} v'(w_{HL} - g(e_{HL}, x_L)) = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &(1-p_H)\big(1 - q^e_{P,H}\big) \pi'(e_{LL}) - \gamma (1-p_H)\big(1 - q^e_{A,H}\big) v'(w_{LL} - g(e_{LL}, x_L)) g_e(e_{LL}, x_L) \\ &+\lambda_{HL} v'(w_{LL} - g(e_{LL}, x_H)) g_e(e_{LL}, x_H) - \lambda_{LL} v'(w_{LL} - g(e_{LL}, x_L)) g_e(e_{LL}, x_L) \leq 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &-(1-p_H)\big(1 - q^e_{P,H}\big) + \gamma (1-p_H)\big(1 - q^e_{A,H}\big) v'(w_{LL} - g(e_{LL}, x_L)) \\ &-\lambda_{HL} v'(w_{LL} - g(e_{LL}, x_H)) - \lambda_{LL} v'(w_{LL} - g(e_{LL}, x_L)) = 0, \end{aligned} \tag{13}$$

$$\begin{aligned} &q^e_{A,H} \left(p_H \, v\big(w_{HH} - g(e_{HH}, x_H)\big) + (1-p_H) v\big(w_{LH} - g(e_{LH}, x_L)\big) \right) \\ &+\big(1 - q^e_{A,H}\big) \left(p_H \, v\big(w_{HL} - g(e_{HL}, x_H)\big) + (1-p_H) v\big(w_{LL} - g(e_{LL}, x_L)\big) \right) \geq \overline{u}, \end{aligned} \tag{PC}$$

$$v\big(w_{HH} - g(e_{HH}, x_H)\big) \geq v\big(w_{LH} - g(e_{LH}, x_H)\big), \tag{IC_{HH}}$$

$$v\big(w_{LH} - g(e_{LH}, x_L)\big) \geq v\big(w_{HH} - g(e_{HH}, x_L)\big), \tag{IC_{LH}}$$

$$v\big(w_{HL} - g(e_{HL}, x_H)\big) \geq v\big(w_{LL} - g(e_{LL}, x_H)\big), \tag{IC_{HL}}$$

$$v\big(w_{LL} - g(e_{LL}, x_L)\big) \geq v\big(w_{HL} - g(e_{HL}, x_L)\big), \tag{IC_{LL}}$$

where (6), (8), (10), (12), (PC), (IC_{HH}), (IC_{LH}), (IC_{HL}), and (IC_{LL}) hold with equality if, respectively, e_{HH}, e_{LH}, e_{HL}, e_{LL}, γ, λ_{HH}, λ_{LH}, λ_{HL}, and λ_{LL} are strictly greater than zero.

It follows from (7) and (9) that $\gamma > 0$. Furthermore, since $\pi'(0) > 0$ and $g_e(0, x_L) = g_e(0, x_H) = 0$, it follows from (6), (8), (10), and (12), respectively, that $e_{HH} > 0$, $e_{LH} > 0$, $e_{HL} > 0$, and $e_{LL} > 0$. Thus, we have equality in (PC), (6), (8), (10), and (12).

The following constellations of the Lagrange multipliers,

$$\lambda_{HL} = 0 \text{ and } \lambda_{LL} = 0,$$

$$\lambda_{HL} > 0 \text{ and } \lambda_{LL} > 0,$$

$$\lambda_{HL} = 0 \text{ and } \lambda_{LL} > 0,$$

$$\lambda_{HH} = 0 \text{ and } \lambda_{LH} = 0,$$

$$\lambda_{HH} > 0 \text{ and } \lambda_{LH} > 0,$$

and

$$\lambda_{HH} = 0 \text{ and } \lambda_{LH} > 0,$$

all lead to contradictions, as will now be shown. When $\lambda_{HL} = \lambda_{LL} = 0$, (11) and (13) imply that $w_{LL} - g(e_{LL}, x_L) = w_{HL} - g(e_{HL}, x_H)$, which violates ($IC_{HL}$) and ($IC_{LL}$). A similar argument gives a contradiction when $\lambda_{HH} = \lambda_{LH} = 0$.

When $\lambda_{HH} > 0$ and $\lambda_{LH} > 0$, (IC_{HH}) and (IC_{LH}) give that $e_{HH} = e_{LH}$ and $w_{HH} = w_{LH}$. By (6), (7), (8), and (9), it then follows that $\pi'(e_{HH}) - g_e(e_{HH}, x_H) < \pi'(e_{HH}) - g_e(e_{HH}, x_L)$, which contradicts the assumptions on the function $g(e, x)$. A similar argument can be used to show that $\lambda_{HL} > 0$ and $\lambda_{LL} > 0$ leads to a contradiction.

Finally, when $\lambda_{HL} = 0$ and $\lambda_{LL} > 0$, (11), (13), and (IC_{LL}) can be combined to solve for λ_{LL} as a function of e_{HL} and w_{HL}. The condition that $\lambda_{LL} > 0$ then gives that $w_{HL} - g(e_{HL}, x_H) < w_{LL} - g(e_{LL}, x_L)$, which violates ($IC_{HL}$) and ($IC_{LL}$). By a similar argument, $\lambda_{HH} = 0$ and $\lambda_{LH} > 0$ leads to a contradiction.

This leaves one case to investigate, namely

$$\lambda_{HH} > 0, \ \lambda_{LH} = 0, \lambda_{HL} > 0, \text{ and } \lambda_{LL} = 0.$$

In this case, (6) and (7) imply that

$$\pi'(e_{HH}) = g_e(e_{HH}, x_H), \tag{14}$$

while (10) and (11) imply that

$$\pi'(e_{HL}) = g_e(e_{HL}, x_H). \tag{15}$$

Thus, $e_{HH} = e_{HL} = e^o_H$, i.e. at their undistorted levels. Also, by (8) and (9),

$$\begin{aligned}(1-p_H)q^e_{P,H}\big(\pi'(e_{LH} - g_e(e_{LH}, x_L)\big)\\ = \lambda_{HH}v'(w_{LH} - g(e_{LH}, x_H)\big(g_e(e_{LH}, x_L) - g_e(e_{LH}, x_H)\big),\end{aligned} \tag{16}$$

and by (12) and (13),

$$\begin{aligned}(1-p_H)\big(1-q^e_{P,H}\big)\big(\pi'(e_{LL} - g_e(e_{LL}, x_L)\big)\\ = \lambda_{HL}v'(w_{LL} - g(e_{LL}, x_H)\big(g_e(e_{LL}, x_L) - g_e(e_{LL}, x_H)\big).\end{aligned} \tag{17}$$

It follows from (IC_{HH}) and (IC_{HL}) that

$$w_{HH} - g(e_{HH}, x_H) = w_{LH} - g(e_{LH}, x_H)$$

and

$$w_{HL} - g(e_{HL}, x_H) = w_{LL} - g(e_{LL}, x_H).$$

Then (7) and (IC_{HH}) imply that

$$\lambda_{HH} = \frac{p_H q^e_{P,H}}{v'(w_{LH} - g(e_{LH}, x_H)} - \gamma p_H q^e_{A,H}, \tag{18}$$

which together with (9) implies that

$$\gamma = \frac{q^e_{P,H}}{p_H q^e_{A,H} v'(w_{LH} - g(e_{LH}, x_H) + (1-p_H) q^e_{A,H} v'(w_{LH} - g(e_{LH}, x_L))}. \tag{19}$$

Similarly, (11) and (IC_{HL}) give that

$$\lambda_{HL} = \frac{p_H\big(1-q^e_{P,H}\big)}{v'(w_{LL} - g(e_{LL}, x_H)} - \gamma p_H\big(1-q^e_{A,H}\big), \tag{20}$$

which together with (13) results in

$$\gamma = \frac{1-q^e_{P,H}}{p_H(1-q^e_{A,H})v'(w_{LL} - g(e_{LL}, x_H)) + (1-p_H)(1-q^e_{A,H})v'(w_{LL} - g(e_{LL}, x_L))}. \tag{21}$$

Using (19) and (21), I now have that

$$\frac{q^e_{P,H}}{1-q^e_{P,H}} = \frac{q^e_{A,H}}{1-q^e_{A,H}} \frac{p_H v'(w_{LH} - g(e_{LH}, x_H) + (1-p_H)v'(w_{LH} - g(e_{LH}, x_L)}{p_H v'(w_{LL} - g(e_{LL}, x_H) + (1-p_H)v'(w_{LL} - g(e_{LL}, x_L)}. \tag{22}$$

Also, (PC), (IC_{HH}), and (IC_{HL}) imply that

$$\begin{aligned} & q^e_{A,H}\Bigg(p_H v\big(w_{LH} - g(e_{LH}, x_H)\big) + (1-p_H) v\big(w_{LH} - g(e_{LH}, x_L)\big)\Bigg) \\ & + \big(1 - q^e_{A,H}\big)\Bigg(p_H v\big(w_{LL} - g(e_{LL}, x_H)\big) + (1-p_H) v\big(w_{LL} - g(e_{LL}, x_L)\big)\Bigg) = 0. \end{aligned} \tag{23}$$

From equations (16), (18), and (19) it follows that

$$\begin{aligned} & p_H v'(w_{LH} - g(e_{LH}, x_H))\big[\pi'(e_{LH}) - g_e(e_{LH}, x_H)\big] \\ & +(1-p_H) v'(w_{LH} - g(e_{LH}, x_L))\big[\pi'(e_{LH}) - g_e(e_{LH}, x_L)\big] \\ & = p_H v'(w_{LH} - g(e_{LH}, x_L)\big[g_e(e_{LH}, x_L) - g_e(e_{LH}, x_H)\big]. \end{aligned} \tag{24}$$

Furthermore, it follows from equations (17), (20), and (21) that

$$\begin{aligned} & p_H v'(w_{LL} - g(e_{LL}, x_H))\big[\pi'(e_{LL}) - g_e(e_{LL}, x_H)\big] \\ & +(1-p_H) v'(w_{LL} - g(e_{LL}, x_L))\big[\pi'(e_{LL}) - g_e(e_{LL}, x_L)\big] \\ & = p_H v'(w_{LL} - g(e_{LL}, x_L)\big[g_e(e_{LL}, x_L) - g_e(e_{LL}, x_H)\big]. \end{aligned} \tag{25}$$

Equations (24) and (25) reflect the well-known requirement that the exact amount of distortion of the low-efficiency type's effort is such that the principal's marginal loss in profit for the low-efficiency type equals the marginal gain in profit for the high-efficiency type. Equation (22) is the, also well-known, condition that the parties' marginal rates of substitution with respect to consumption in different states are equal. The latter equation implies that if the parties' decision weights differ, then there must be conditioning on the signal. However, since the decision weights depend on the contract offered, further investigation is required.

With a contract that does not condition on the signal, all marginal distributions in Q are equally good for each of the parties. With such a contract, the distributions $\overline{q}_k$ and $\underline{q}_k$, $k \in \{A, P\}$, can therefore be any of the distributions in Q and thus chosen such that the parties have equal marginal rates of substitution. Hence, there exists a contract that does not condition on the signal that satisfies the first-order conditions.

However, there may also be bait contracts that satisfy the first-order conditions with different wage-effort pairs given the two realizations of the signal. Hence, the first-order conditions must be solved for these, and the value of the principal's objective function must be compared with the value when there is no conditioning.

Equations (14) and (15) pin down e_{HH} and e_{HL}. The other variables are, however, determined by the set of nonlinear equations (22)-(25), which can not be solved for general functional forms. Therefore, to show existence, consider the specific functional forms

$$v(\cdot) = \log(\cdot),\ g(e, x) = \frac{e^2}{2x},\ \text{and } \pi(e) = 2e^{1/2}.$$

The nonlinear system of equations (22)-(25), and hence the principal's optimization problem, is solved numerically using Ox for the case where $x_L = 1$, $x_H = 8$, and $\alpha_A = 0.95$ on a grid of $(p_H, q^e_{P,H})$ where both probabilities range from 0.001 to 0.999, with increments of 0.001. In the computations, $q^e_{A,H}$ is assumed to be defined by (4) and below it is confirmed that this is indeed correct.

For positive levels of δ, on a grid with 0.001 increments from 0 to $\min\{q^e_{P,H}, 1-q^e_{P,H}\}$, the difference in the principal's utility is computed when the contract conditions on the PIRV and when the contract does not condition on the PIRV. This difference is positive for all values on the grid. This implies that the principal will condition on the PIRV when δ is strictly positive. The difference in the agent's utility given $y = y_H$ and his utility given $y = y_L$ is also computed under the optimal contract, and this difference is positive when $\delta > 0$, showing that it is indeed equation (4) that is relevant for computing the agent's weight on y_H.[8]

The numerical computations finally show that the derivative of the principal's utility is positive when evaluated at $\delta = 0$ for all values of $(p_H, q^e_{P,H})$ on the grid. Thus even an infinitesimal amount of ambiguity will make the principal condition on the PIRV. ■

Proof of Theorem 2: Equations (22)-(25) in the proof of Theorem 1 together with equation (4) implicitly define e_{LH}, w_{LH}, e_{LL}, and w_{LL} as functions of the ambiguity parameter δ. This can be used to find the derivatives of e_{LH} and e_{LL} with respect to δ. For the same functional forms, parameter values, and grid for $(p_H, q^e_{P,H})$ as used in the proof of Theorem 1, numerical computations show that when evaluated at $\delta = 0$ the values of these derivatives

[8]There are actually two optimal contracts for each level of ambiguity. The two contracts are symmetric in the sense that (e_{LH}, w_{LH}) for one contract will equal (e_{LL}, w_{LL}) for the other contract, and vice versa. With the symmetric contract, $e_{LH} < e_{LL}$ and the agent will be best off when the PIRV takes value y_L, thus it will be (5) that is relevant for computing the agent's weight on y_H.

are non-zero and that of e_{LH} is positive while that of e_{LL} is negative.[9]

For positive levels of δ, on the same grid as in the proof of Theorem 1, the difference $e_{LH} - e_{LL}$ is computed for the optimal bait contract, and this difference is positive as illustrated in Panel B of Figure 1 for $p_H = q^e_{P,H} = \frac{1}{2}$.

Since the optimal contract always has $e_{LH} \neq e_{LL}$, and a contract that does not condition on the PIRV necessarily has $e_{LH} = e_{LL}$, the bait contract cannot be mimicked by breaking up the problem into separate contracting and betting problems for any $\delta > 0$. ■

References

[1] Adrian, Tobias and Mark M. Westerfield (2009): "Disagreement and learning in a dynamic contracting model," *Review of Financial Studies* 22, 3873-3906.

[2] Ahn, David S. (2008): "Ambiguity without a state space," *Review of Economic Studies* 75, 3-28.

[3] Bose, Subir and Arup Daripa (2009): "A dynamic mechanism and surplus extraction under ambiguity," *Journal of Economic Theory* 144, 2084-2114.

[4] Bose, Subir, Emre Ozdenoren, and Andreas Pape (2006): "Optimal auctions with ambiguity," *Theoretical Economics* 1, 411-438.

[5] Carlier, Guillaume and Ludovic Renou (2005): "A costly state verification model with diversity of opinions," *Economic Theory* 25, 497-504.

[6] Carlier, Guillaume and Ludovic Renou (2006): "Debt contracts with ex-ante and ex-post asymmetric information: an example," *Economic Theory* 28, 461-473.

[7] De Castro, Luciano and Nicholas C. Yanellis (2010): "Ambiguity aversion solves the conflict between efficiency and incentive compatibility," http://kellogg.northwestern.edu/faculty/decastro/htm/personal/maximin.pdf

[9]As noted in footnote 8, there are actually two optimal contracts for each level of ambiguity, which are symmetric in the values given the two possible realizations of the PIRV. For the symmetric solution, the derivative of e_{LH} would be negative while that of e_{LL} would be positive.

[8] Ellsberg, Daniel (1961): "Risk, ambiguity, and the Savage axioms," *Quarterly Journal of Economics* 75, 643-669.

[9] Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci (2004): "Differentiating ambiguity and ambiguity attitude," *Journal of Economic Theory* 118, 133-173.

[10] Gilboa, Itzhak and David Schmeidler (1989): "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics* 18, 141-153.

[11] Holmström, Bengt (1979): "Moral hazard and observability," *Bell Journal of Economics* 10, 74-91.

[12] Kotowski. Maciej H. (2012): "Engineered ambiguity in the principal-agent problem," working paper, Kennedy School of Government, Harvard University.

[13] Levin, Dan and Emre Ozdenoren (2004): "Auctions with uncertain number of bidders," *Journal of Economic Theory* 118, 229-251.

[14] Lopomo, Giuseppe, Luca Rigotti, and Chris Shannon (2009): "Uncertainty in mechanism design,"
http://www.pitt.edu/∼luca/Papers/mechanismdesign.pdf

[15] Lopomo, Giuseppe, Luca Rigotti, and Chris Shannon (2011): "Knightian uncertainty and moral hazard," *Journal of Economic Theory* 146, 1148-1172.

[16] Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green (1995): *Microeconomic Theory*, Oxford University Press, Oxford.

[17] Mukerji, Sujoy (1998): "Ambiguity aversion and incompleteness of contractual form," *American Economic Review* 88, 1207-1232.

[18] Mukerji, Sujoy and Jean-Marc Tallon (2004): "Ambiguity and the absence of wage indexation," *Journal of Monetary Economics* 51, 653-670.

[19] Olszewski, Wojciech (2007): "Preferences over sets of lotteries," *Review of Economic Studies* 74, 567-595.

[20] Vierø, Marie-Louise (2009): "Exactly what happens after the Anscombe-Aumann race? Representing preferences in vague environments," *Economic Theory* 41, 175-212.

[21] Vierø, Marie-Louise (2012): "Contracting in vague environments," *American Economic Journal: Microeconomics* 4, 104-130.