Canadian Mortgage Contract Duration and Interest-Rate Risk

by

Stephanie Renaud

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1 Introduction

With historically low levels of interest rates and financial deregulation, borrowing conditions have been eased for Canadian households, which has contributed to the accumulation of debt, and particularly mortgage debt. With the constant evolution of financial and economic vulnerabilities, the Bank of Canada, among other institutions, closely monitors the resilience of the Canadian financial system.

Since 2018, higher interest rates and other policy measures targeted at the housing and mortgage markets reduced credit growth and improved the quality of Canadian mortgage debt [2]. In particular, Canadian regulators tightened mortgage underwriting standards and incorporated mortgage stress tests to which borrowers must now qualify to be eligible for mortgage loans. While the implementation of such measures improved the quality of mortgage lending, highly indebted households remain particularly sensitive to interest-rate fluctuations, which can have significant consequences for their financial well-being. [4].

Since mid-2017, the Bank of Canada has been progressively increasing its policy rate in reaction to a stronger economy, thus pushing up borrowing costs for Canadian households. Households opting for adjustable-rate mortgage loans are immediately affected by higher interest rates, in contrast to those choosing fixed-rate mortgages, who feel the impact of higher rates only at renewal. In Canada, 45 percent of outstanding mortgages are 5-year, fixed-rate mortgages, and according to the Bank of Canada, approximately 20 percent of fixed-mortgages are renewed every year.

As interest rates have been on an upward trend since mid-2017, federal oversight bodies and other regulators have started to express some concerns with respect to the ability of mortgage holders to adjust to higher interest rates. According to the Bank of Canada, the majority of mortgage holders have not taken the maximum amount they were allowed to borrow to finance their home, and thus are likely to have the financial capacity to manage higher monthly mortgage payments [2]. In addition, since 2018, mortgage interest rate stress tests are mandatory for borrowers contracting mortgage loans from federally regulated financial institutions. Mortgage holders who have demonstrated their ability to manage higher interest rates should therefore be able to accommodate an increase in payments when they renew their mortgage [2].

While numerous regulatory policy changes have been implemented to increase the resilience of households to changes in interest rates, the Bank of Canada recently raised the possibility for borrowers to use fixed-rate mortgage contracts as a tool to mitigate their exposure to interest-rate risk. In his speech at the Canadian Credit Union Association and Winnipeg Chamber of Commerce on May 6, 2019, Governor Poloz promoted longer-term mortgages to reduce the frequency at which borrowers face the risk of higher interest rates. To quote Governor Poloz:

> From the consumer point of view, a longer term means they face the risk of having to renew at higher interest rates less often. (Poloz, 2019, p. 4)

According to the Bank of Canada, longer-term fixed-rate mortgage contracts could be a solution to reduce the interest-rate risk exposure of Canadian households, and as a result, contribute to a safer financial system and more stable economy [3]. The objective of this essay is to assess the validity of this view, and determine whether Canadian households would be better off following such a proposal.

While the argument that a more resilient household sector would contribute to financial and economic stability follows economic logic, it is unclear that longer-term mortgages would indeed improve the resilience of borrowers to economic conditions, or at the very least, reduce their interest-rate risk exposure.

For borrowers, renewing at a higher interest rate could have several consequences. For example, borrowers could face higher monthly payments and higher interest payments throughout the duration of the contract. As the interest-rate risk is a key concept in this statement and it is unclear whether it refers to the variability in monthly payments or total interest payments, the definition of this risk should be carefully considered.

As a result of the key features of the Canadian mortgage market, longer-term mortgages have also been a less popular alternative over the past decades compared to shorter-term contracts. Furthermore, longer-term mortgages are usually associated with higher interest rates, which increase the cost of borrowing. To quote Governor Poloz:

> Of course, a longer-term mortgage will carry a higher interest rate, but some homebuyers may be willing to pay more to lower their risk. And a longer-term mortgage might not be much more expensive in the long run depending on the details of the loan and the prepayment penalties that apply. (Poloz, 2019, p. 4)

While extending the duration of mortgage loans implies higher costs of borrowing for households, perhaps adjustments to the mortgage contract design, as suggested by Governor Poloz, could allow longer-term mortgages to compete with shorter-term mortgages in terms of costs. To verify the validity of this statement, the cost of longer-term mortgages should be analyzed under extended periods of amortization. Prepayment features should also be considered.

Using both historical simulations and out-of-sample forecasts, this essay assesses the validity of the statement made by the Bank of Canada that longer-term fixed-rate mortgages could mitigate the interest-rate risk faced by Canadian households.

Associating lower interest-rate risk exposure with lower total interest payments over a 10-year period, I will evaluate whether 10-year fixed-rate mortgages provide a better protection against interest-rate risk than a combination of two 5-year fixed-rate mortgages.

The essay is organized as follows. Section 2 presents the stylized facts, including the definition of interest-rate risk in the context of fixed-rate mortgage contracts, key features of the Canadian residential housing market, and the historical evolution of mortgage underwriting standards. Section 3 presents the data used for this analysis. Section 4 formulates the borrower's choice problem with respect to mortgage contract selection. Using various approaches, Section 5 presents historical simulations based on the borrower's choice model to test whether 10-year mortgage loans reduce interest-rate risk exposure. Section 6 discusses the importance of prepayment privileges in this analysis, with particular attention paid to the possibility of refinancing before the end of the term. Section 7 presents the vector autoregressive (VAR-X) model with exogenous variables that will be used to obtain out-of-sample 5-year and 10-year mortgage rate forecasts. Model adequacy, forecasting performance, and the borrower's choice over the forecast horizon are also discussed. Finally, Section 8 concludes.

The main finding of this essay is that 10-year mortgages systematically lead to higher interest payments than two 5-year mortgages, regardless of the method used to predict future 5-year mortgage rates. Similar results are obtained when I extend the amortization period from 25 to 30 years and include prepayment privileges. Allowing borrowers who made incorrect predictions regarding the future path of 5-year mortgage rates to refinance their 10-year mortgage at a lower 5-year rate also leads to higher interest payments. Finally, using a VAR-X to forecast the future path of mortgage rates, I find that the same conclusion holds in out-of-sample forecasts. Therefore, when the interest-rate risk is associated with total interest payments, I conclude that 10-year mortgage contracts are not a solution to mitigate borrowers' exposure to interest-rate risk.

2 Stylized facts

2.1 Defining the interest-rate risk

With a highly indebted household sector, the normalization of interest rates may lead to severe economic consequences, and thus exacerbate current vulnerabilities in the system. While the interest-rate risk is clearly defined in the context of bonds and banks' balance sheets, there is no clear definition of this risk in mortgage contracts. This subsection provides a review of the definition of interest-rate risk proposed by domestic and international institutions in the context of the mortgage market.

The Bank of Canada broadly defines the interest-rate risk in mortgage contracts as the unexpected increase in interest rates. At renewal, borrowers holding fixed-rate mortgages may not have the financial capacity to qualify for their renewed contract if mortgage interest rates have increased [1]. Borrowers with a precarious financial position are more sensitive to fluctuations in interest rates, and thus are more exposed to interest-rate risk. The Office of the Superintendent of Financial Institutions (OSFI) also shares this view on the definition of interest-rate risk. Following the Financial Stability Board (FSB) guidelines, OSFI promotes sound mortgage underwriting practices through the requirement of an accurate evaluation of the borrower's ability to withstand potential shocks to financial and economic conditions, including higher interest rates [14, 26].

Global institutions such as the Bank for International Settlements (BIS) and the International Monetary Fund (IMF) also discuss the implications of interest-rate risk. The BIS defines the interest-rate risk as the exposure of borrowers' and lenders' financial conditions to adverse movements in interest rates [5]. The interest-rate sensitivity of a household's debt service burden increases with its stock of debt, which affects its ability to cope with unanticipated, but plausible, shocks such as lower income, lower asset prices, or higher interest rates. Given that households' wealth in housing is relatively illiquid and represents a little over 80 percent of total household debt, highly indebted borrowers may be forced to reduce their savings, and even consumption, to support their financial obligations in the event of unanticipated shocks such as higher interest rates [6, 31]. Focusing on housing market corrections, the IMF associates the interest-rate risk to a faster-than-expected rise in mortgage rates, which could increase the exposure of the household sector to economic and financial shocks [16].

Supported by these definitions, I will consider the interest-rate risk as the impact of unexpected movements in interest rates on the ability of households to meet their financial obligations with respect to their mortgage contract. Given that the focus of this essay is on fixed-rate mortgage contracts, I assume that borrowers face the interest-rate risk only at the time of renewal, which occurs every five years for the majority of Canadian borrowers. A 10-year mortgage contract will be considered to mitigate the interest-rate risk exposure if it offers lower total interest payments than a combination of two 5-year mortgages. For this statement to be true, 5-year mortgage rates would have to sharply increase over time such that borrowers renewing their 5-year mortgage at the new, higher, rate would end up paying a greater amount in interest over a 10-year period than they would have under a 10-year mortgage. Allusions to interest-rate risk throughout the essay will refer to this definition.

2.2 Key features of the Canadian mortgage market

In Canada, federally regulated financial institutions are supervised and regulated by OSFI. To ensure the soundness of the market, OSFI sets out guidelines for prudent residential mortgage underwriting standards for lenders and mortgage insurers. The mortgage insurance, also known as the default insurance, corresponds to a financial product offered by a mortgage insurer that protects the lender against losses in the event a borrower defaults on a mortgage loan [27]. OSFI regulates Canada Mortgage and Housing Corporation (CMHC), a federal Crown corporation, and two private insurers, Glenworth Financial Mortgage Insurance Company Canada and Canada Guaranty Mortgage Insurance Company. CMHC is the largest mortgage insurer and CMHC-insured mortgages are 100 percent guaranteed by the federal government in the event the insurer becomes insolvent [8]. The two private insurers account for about 25 percent of outstanding mortgage insurance and are also covered by a government guarantee, subject to a deductible of 10 percent of the insured principal, to allow them to compete with CMHC.

According to Section 418 of Canada's Bank Act, mortgage insurance is required on all high-ratio mortgages — mortgages with a loan-to-value (LTV) exceeding 80 percent of the purchased value of the property — issued by federally regulated financial institutions [18]. By design, mortgage insurance can be seen as a risk-sharing tool that reduces the risk faced by lenders and allows borrowers with only 5 percent of equity to have access to homeownership. For individuals contracting high-ratio mortgage loans covered by the mortgage insurance, a mandatory mortgage insurance premium between 2.40 and 4 percent is imposed, which is passed to the borrower by the lender [9]. The maximum amortization period for CMHC-insured mortgage loans is 25 years. Maximum thresholds for the gross debt service (GDS) and total debt service (TDS) ratios are 39 and 44 percent, respectively. A minimum credit score of

600 is also required to be considered a creditworthy borrower [10]. While mortgage insurance is often negatively perceived as it increases the cost for borrowers due to the mortgage insurance premium, its existence and effectiveness, backed by the federal government, supports access to homeownership and promotes stable and sound conditions in the housing and financial market [12].

Fixed-rate mortgage contracts with a 5-year term and 25-year amortization period are predominant in the Canadian residential mortgage market. Over 95 percent of mortgages have a term between six months and five years, and approximately two-third of mortgage contracts have a fixed interest rate [8]. Given the popularity of this mortgage structure, the majority of Canadian borrowers face interest-rate risk at renewal. Although longer contracts could potentially mitigate this risk, mortgages with terms beyond five years are highly uncommon.

The scarcity of such contracts may be explained by several factors, such as the prepayment risk and the qualification requirements for deposit insurance. As lenders tend to match the maturity between their assets and liabilities, the possibility of prepayment arising from longer-term mortgage contracts imposes a risk on their balance sheets. To limit this risk, prepayment penalties have been introduced. They depend on several factors such as the outstanding balance of the loan, the number of months left in the contract, the level of interest rates, and are calculated using the highest of three months' interest on the remaining balance or the interest rate differential [15]. Under Section 10 of Canada's Interest Act, homeowners are allowed to prepay in full their mortgage with a term to maturity greater than five years after five years of payments for a fixed prepayment penalty equal to three months' interest. Required to hedge prepayment risk over a longer period of time, lenders charge a risk premium on mortgages with longer terms, making such contracts more expensive, and thus less appealing to borrowers. Also, given that the Canadian Insurance Deposit Corporation (CDIC) only insures deposits for up to five years, deposit-taking financial institutions likely favour 5-year mortgages to longer-term mortgages [13]. Therefore, the prepayment penalty and the cap on government guaranteed deposit insurance make mortgages beyond five years more expensive for borrowers and riskier for lenders, which may explain the scarcity and the price of such contracts in the mortgage market [22]. In section 6, I will model borrowers' prepayment behaviour to assess whether prepayment privileges affect the choice of mortgage contract.

2.3 Mortgage underwriting standards and stress tests

Over the past decade, significant changes have been made to mortgage eligibility rules and macroprudential policy tools. During the 2006–2007 period of financial deregulation, several changes regarding the maximum allowable LTV ratio and amortization length were implemented. Three rounds of expansion of the maximum amortization period were undertaken by CMHC in 2006, extending the amortization period from 25 to 40 years. The maximum LTV ratio was also expanded from 90 to 100 percent during that period, allowing zero-down payment mortgages to be insured [7].

The Great Financial Crisis marked the beginning of the "tightening period" of mortgage underwriting standards. With the objective to improve the quality of new mortgage lending, the maximum amortization period and LTV ratio on new high-ratio mortgages were reduced to 25 years and 95 percent, respectively. A maximum of 80 percent LTV was also imposed for mortgage refinancing and purchase of investment properties [8]. These regulatory policies have not changed since 2012.

In addition to the maximum amortization period and LTV ratio, prudent measures for debt serviceability are set by federally regulated financial institutions to assess borrowers' affordability, especially in the event of financial or economic stress. Each borrower's debt serviceability is determined based on the gross debt service (GDS) and total debt service (TDS) ratios [28]. The GDS ratio is defined as the carrying costs of the home relative to the borrower's income, while the TDS ratio is more broadly defined and corresponds to the ratio of household income required to cover housing expenses, including other debt payments. To qualify for mortgage insurance, maximum GDS ratio and TDS ratio requirements were also reduced during the tightening period and are now set to 39 percent and 44 percent, respectively [12]. Minimum credit score requirements were also tightened during this period. In 2016, the government also extended the application of the mortgage insurance eligibility rules for high-ratio mortgages to low-ratio mortgages. As a result, all insured mortgages are subject to the same regulations since 2016 [1].

An important change in mortgage underwriting was the implementation of mortgage interest rate stress tests for federally regulated mortgages. Such stress tests are designed to evaluate the ability of highly indebted borrowers to adjust to higher future interest rates on their mortgage. To be eligible for a mortgage, borrowers have to prove that their debt-service ratios are not higher than the maximum thresholds imposed. In order to verify the ability of borrowers to accommodate higher interest rates, stress test calculations use the higher of the mortgage contract rate or the Government of Canada 5-year benchmark rate.[1] Before 2012, federally mandated stress tests were only required for high-ratio mortgages with variable rates or fixed terms of less than five years. Starting in 2012, OSFI periodically extended the application of mortgage rate stress tests, also known as OSFI Guidelines B-20. As of today, mortgage rate stress tests are mandatory for all Canadian mortgages with variable and fixed interest rate, regardless of the contract term, with the exception of mort-

[1] Note that the stress test only affects the size of the maximum loan, but not mortgage payments.

gages provided by financial institutions that are not federally regulated, such as credit unions and private lenders [2].

With the majority of mortgages being issued by federally regulated institutions, changes to mortgage underwriting practices have a strong impact on the Canadian residential mortgage market [11]. As a result, tightening measures implemented over the past decade effectively dampened credit growth and improved the quality of mortgage lending, particularly in heated housing markets such as Vancouver and Toronto. The introduction of new eligibility requirements for high-ratio mortgages in 2016 reduced by half, from 20.1 to 9.9 percent, the share of high-ratio borrowers with mortgage debt representing more than 450 percent of their gross income. The revised Guideline B-20 in 2018 also reduced the share of low-ratio mortgages with a loan-to-income over 450 percent, from 20 percent in 2017 to 14 percent in 2018 [1]. With the introduction of mandatory stress tests as an eligibility requirement for federally regulated mortgage loans, lenders also contribute to limiting the exposure of borrowers to interest-rate risk.

As noted by the Bank of Canada, vulnerabilities arising from the residential mortgage market and the highly indebted household sector can worsen the impact of economic stress, thus putting the stability of the Canadian financial system at risk. Although Canadian households remain vulnerable to adverse shocks to income and interest rates, tighter mortgage standards have slowed down the accumulation of mortgage debt, in particular among highly indebted households, and thus increased the resilience of the household sector. Promoting longer-term mortgages can be seen as a demand-oriented policy with the objective to increase the resilience of households to increasing mortgage rates. While mortgage rate stress tests discussed above ensure that borrowers have the financial capacity to manage higher interest rates, the Bank of Canada argues that longer-term mortgage contracts, by reducing the frequency of renewal, could reduce the likelihood to face higher interest rates at time of renewal. In this essay, I will analyze whether 10-year fixed-rate mortgage contracts, as an addition to current regulatory policies, reduce borrowers' exposure to interest-rate risk, as defined in Section 2.1. I will also test whether the concept of prepayment could explain the scarcity of longer-term mortgage contracts in the Canadian residential mortgage market.

3 Data

This section presents the data that will be used to estimate the VAR-X model and conduct the various simulations presented in Sections 5 to 7. Appendix A presents detailed information on exact data sources. I employ monthly data from May 2006 to February 2019 on interest rates and additional exogenous variables to improve the forecasting performance of the model. Based on data availability, this sample captures the impact of the 2008 financial crisis and the loosening and tightening cycles of mortgage underwriting standards on mortgage rates. For historical and prepayment simulations presented in Sections 5 and 6, discounted mortgage rates will be used to assess the impact of borrowers' expectations of future 5-year mortgage rates on total interest payments — our selected measure of interest-rate risk exposure. Later, in Section 7, a VAR-X model including various interest rates will be used to obtain out-of-sample forecasts of mortgage rates. The model will capture historical regulatory changes in the maximum allowed amortization period and LTV ratio over the 2006–2019 period.

3.1 Selection of variables

To analyze interest rate dynamics in the residential mortgage market, I use 3-year and 5-year posted and discounted interest rates on fixed-rate mortgages.[2] Given that the Bank of Canada does not track posted rates on 10-year fixed mortgage contracts, I only use discounted interest rates on 10-year mortgage loans. Discounted mortgage rates correspond to the actual rates paid by borrowers, which are lower than the posted rates for the selected mortgage terms. Posted mortgage rates are obtained from Statistics Canada and discounted mortgage rates are obtained from Ratehub.ca, which collects proprietary and discount brokerage data to build a comprehensive database of discounted mortgage rates. To capture how mortgage rates fluctuate with other key interest rates, I also use the Bank of Canada's policy rate, and the Government of Canada 3-year, 5-year, and 10-year benchmark bond yields. Mortgage spreads on 3-year, 5-year and 10-year mortgages are also considered and are calculated as the difference between the discounted mortgage rate and Government of Canada benchmark bond yield of corresponding maturity. Additionally, given that this essay focuses on fixed-rate mortgages, the prime rate is not included in this analysis as it only matters for variable-rate mortgages.

Furthermore, inflation and the unemployment rate are good indicators of economic performance. The Bank of Canada uses the policy rate as a monetary policy tool to maintain low, stable, and predictable inflation [30]. The assessment of the current macroeconomic situation also constitutes an important factor in the monetary policy decision-making process. The labour market, and particularly the unemployment rate, is a key indicator of the level of economic activity, and is thus taken into consideration by the Bank of Canada when setting the policy rate [30]. Therefore, given that price and unemployment dynamics reflect economic conditions and are

[2] Posted rates are only used in Section 5.3.

key drivers of interest rates, inflation and unemployment data will be considered as additional endogenous variables in the VAR-X model. Finally, I will also consider the inclusion of GDP growth. In the selected sample, the inflation, unemployment rate, and GDP growth are on average 1.69, 6.90, and 1.70 percent, respectively.

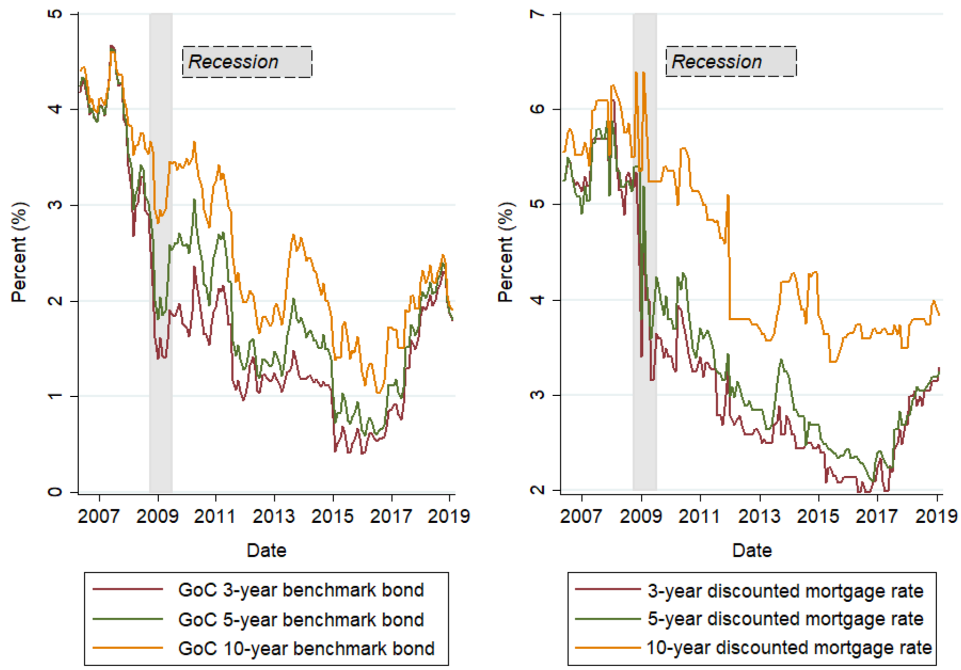
3.2 Historical evolution of mortgage spreads

In economics, the yield curve is a key concept reflecting the positive relationship between the yield of a financial product and its maturity. In our sample, bond yields and mortgage rates reflect this relationship between interest rates and maturity. As illustrated in Figure 1, 3-year, 5-year, and 10-year government benchmark bonds offer an average return of 1.80, 2.10 and 2.60 percent, respectively. The positive relationship is also supported in the mortgage market, where mortgage rates increase with the term to maturity. Between 2006 and 2019, borrowers paid on average an interest rate of 3.31 percent on a 3-year mortgage contract, 3.51 percent on a 5-year contract, and 4.52 on a 10-year contract.

Figure 1 presents the historical evolution of the Government of Canada benchmark bond yields and discounted mortgage rates with 3-year, 5-year and 10-year maturity. Over the selected sample, the average discounts negotiated on 3-year and 5-year posted mortgage rates are 1.20 and 1.42 percent, respectively. Due to data availability, we cannot compute the average discount obtained on 10-year mortgages. As shown in Figure 1, mortgage rates are also higher than benchmark bond yields.

Furthermore, the historical evolution of mortgage spreads, defined as the difference between the Government of Canada benchmark bond yield and discounted mortgage rate of similar maturity, is illustrated in Figure 2. On average, the 3-year and 5-year spreads are 1.51 and 1.42 percent, respectively. Even though the two spreads follow

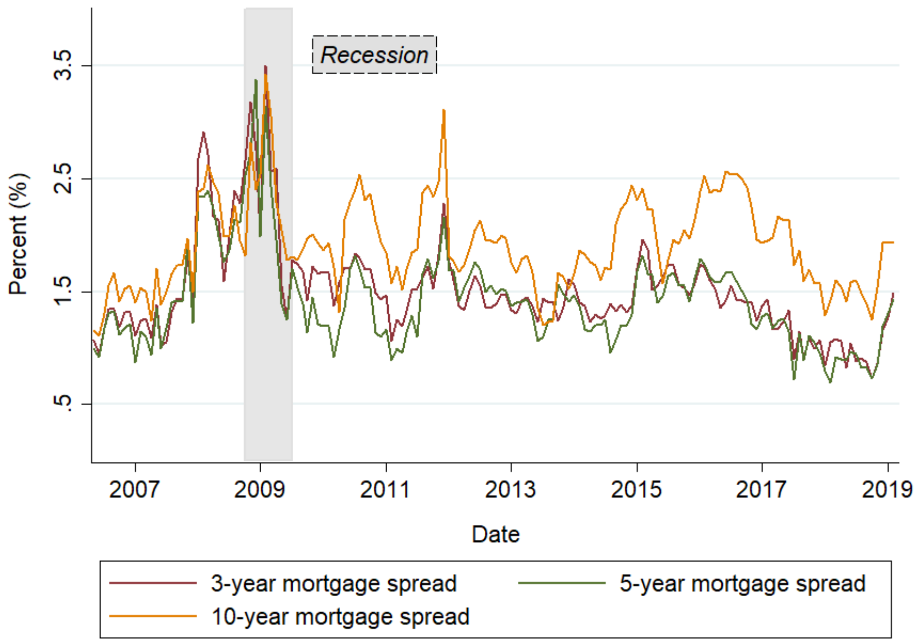
Figure 1: Benchmark bond yields and discounted mortgage rates, by maturity



a very similar path, the spread on 5-year mortgages is on average 0.09 percent lower than the one on 3-year mortgage contracts. One possible explanation is that financial institutions exhibit a competitive behaviour, and thus tend to offer greater discounts on 5-year rates to attract borrowers. Institutional factors also provide incentives to financial institutions to offer 5-year mortgages since the Government of Canada 5-year benchmark bond is more liquid than the 3-year benchmark bond.

Additionally, the 10-year mortgage spread is on average 1.92 percent, which is 0.5 percent higher than the 5-year mortgage spread. While every mortgage contract includes risks associated with prepayment and default, longer-term contracts require financial institutions to hedge such risks over a longer period of time. As discussed in Section 2, the prepayment penalty and the cap on government guaranteed deposit in-

Figure 2: Mortgage spreads, by maturity



surance make mortgages beyond five years significantly riskier for lenders. The larger spread on 10-year mortgages reflects the premium imposed by financial institutions to mitigate those risks. The financial crisis also had a significant impact on mortgage spreads. Due to fragile economic conditions, Government of Canada benchmark bond yields adjusted right away as monetary policy was loosened. Given that it requires some time for financial institutions and other lenders to observe and react to financial and economic conditions, discounted mortgage rates were revised downward with a lag. As a result of higher discounted mortgage rates and lower benchmark bond yields, mortgage spreads spiked at the end of 2008.

Notwithstanding the variance of mortgage spreads in Figure 2, comparing the two panels of Figure 1 shows that fixed mortgage rate dynamics are mainly driven by fluctuations in Government of Canada benchmark bond yields of corresponding maturities, which reflect the cost of funding faced by financial institutions to finance mortgage loans provided to households. Being guaranteed by the Government of Canada, bank deposits are a risk-free source of funding for financial institutions, and thus require a remuneration similar to that of government bonds for individuals to have an incentive to make deposits at the bank. Hence, Canadian bond yields are a representative benchmark for the cost of funds faced by financial institutions. While bank deposits reduce the cost of funding faced by Canadian banks as chequing accounts do not bear any interest, they do not correspond to the optimal source of funding to support mortgages as they can be withdrawn at any time. Instead, banks mainly rely on mortgage bonds and other financial products that cannot be withdrawn upon demand. As a result, yield fluctuations in the mortgage bond market are highly correlated to those in the government bond market, which makes the government bond market a key driver for mortgage interest rates.

3.3 Additional endogenous and exogenous variables

The estimation of the VAR-X model in Section 7 is also conditioned on some exogenous variables capturing regulatory policies over the sample period. Exogenous variables are determined outside of the model and are included to improve the ability of the model to forecast endogenous variables. In addition to the maximum amortization period and LTV ratio, I will consider the inclusion of the consumer confidence index, the housing price index, and the Bank of Canada energy price index as exogenous variables. Similar to the inflation rate, unemployment rate, and GDP growth, these variables capture economic performance and may have an impact on bond yield dynamics, and, as a result, mortgage rates. In Section 7.2, I test which additional endogenous and exogenous variables should be included to improve the forecasting performance of the model.

4 The borrower's choice problem

As discussed in Section 2.1, I compare total interest payments over ten years to determine whether a borrower would be better off by choosing one 10-year mortgage instead of a combination of two 5-year mortgages. To reduce his interest-rate risk exposure, a borrower should opt for the contract that involves paying the lowest amount of interest over the 10-year term. For comparison purposes, I use a representative mortgage loan L of \$300,000 dollars with an amortization period A of 25 years, which is the standard amortization period in Canada. For each mortgage loan generated at month t, I follow the following steps to calculate total interest payments under both types of mortgage contract to assess which one would make borrowers better off.

4.1 Preliminary concepts

First, following the methodology proposed by Kau et al. (1995) and Souissi (2007), I calculate the monthly payment M_t associated with (1) one 10-year mortgage contracted at month t, and (2) a combination of a 5-year mortgage contracted at time t and another 5-year mortgage contracted at time $t + 60$. Superscripts $10y$, $5y_1$, and $5y_2$ refer to the type of mortgage contract, and the subscript t indicates the month at which it was originated.[3] The monthly payments M_t^{10y} and $M_t^{5y_1}$ are a function of the mortgage interest rate — i_t^{10y} and $i_t^{5y_1}$ — at origination. Similarly, the expected monthly payment $E_t\left[M_{t+60}^{5y_2}\right]$ associated with the 5-year mortgage contracted at time $t + 60$ is a function of the 5-year rate $i_{t+60}^{5y_2}$ that will prevail at renewal [29, 20]. These monthly payments are calculated as follows:

$$M_t^{10y} = \frac{\left(\frac{i_t^{10y}}{12}\right)\left(1 + \frac{i_t^{10y}}{12}\right)^{(A\times 12)}}{\left(1 + \frac{i_t^{10y}}{12}\right)^{(A\times 12)} - 1} \times L, \tag{1}$$

[3] Note that superscripts $5y_1$ and $5y_2$ refer to the first 5-year mortgage and the second 5-year mortgage, respectively, contracted over a total period of ten years.

$$M_t^{5y_1} = \frac{\left(\frac{i_t^{5y_1}}{12}\right)(1+\frac{i_t^{5y_1}}{12})^{(A\times 12)}}{(1+\frac{i_t^{5y_1}}{12})^{(A\times 12)}-1} \times L, \tag{2}$$

$$E_t\big[M_{t+60}^{5y_2}\big] = E_t\left[\frac{\left(\frac{i_{t+60}^{5y_2}}{12}\right)(1+\frac{i_{t+60}^{5y_2}}{12})^{((A-5)\times 12)}}{(1+\frac{i_{t+60}^{5y_2}}{12})^{((A-5)\times 12)}-1}\right] \times OMB_{t+60}^{5y_1}. \tag{3}$$

Based on the present value formula of an annuity, the first two formulas indicate the monthly payment associated with a principal L amortized over 300 months — the number of monthly payments over 25 years — at the monthly compounded interest rate i_t^{10y} and $i_t^{5y_1}$. Equation (3) takes into account that at $t+60$, the outstanding balance $OMB_{t+60}^{5y_1}$ of the first 5-year contract will be renewed at a new 5-year mortgage rate $i_{t+60}^{5y_2}$ and amortized over 20 years.[4] These monthly payments ensure that the loan will be paid off in full with interest at the end of the amortization period. As the interest-rate risk refers to unexpected changes in interest rates at renewal, the focus will be on modeling how borrowers make their expectations.

Second, I define OMB_{t+120}^{10y} as the outstanding balance of the 10-year contract at the end of the 10-year period. It refers to the remaining balance to be refinanced, and corresponds to the value of the initial loan L and accrued interest minus the total amount applied on the principal over of the term. As mortgage payments are made, a larger percentage of monthly payments is applied to reduce the principal. It is calculated as follows:

$$OMB_{t+120}^{10y} = \frac{(1+\frac{i_t^{10y}}{12})^{(A\times 12)} - (1+\frac{i_t^{10y}}{12})^{((10\times 12)}}{(1+\frac{i_t^{10y}}{12})^{(A\times 12)}-1} \times L. \tag{4}$$

Similarly, the expected outstanding balance of the combination of two 5-year mortgage contracts at the end of the 10-year period denoted by $E_t\big[OMB_{t+120}^{5y}\big]$ is

[4] At each renewal, the amortization period is reduced by the duration of the previous term.

defined according to the following equation:

$$E_t\big[OMB^{5y}_{t+120}\big] = E_t\Bigg[\frac{(1+\frac{i^{5y_2}_{t+60}}{12})^{((A-5)\times 12)} - (1+\frac{i^{5y_2}_{t+60}}{12})^{(5\times 12)}}{(1+\frac{i^{5y_2}_{t+60}}{12})^{((A-5)\times 12)} - 1}\Bigg] \times OMB^{5y_1}_{t+60}, \tag{5}$$

where the outstanding mortgage balance at the end of the first 5-year mortgage contract $OMB^{5y_1}_{t+60}$ to be refinanced is defined as

$$OMB^{5y_1}_{t+60} = \Bigg(\frac{(1+\frac{i^{5y_1}_{t}}{12})^{(A\times 12)} - (1+\frac{i^{5y_1}_{t}}{12})^{(5\times 12)}}{(1+\frac{i^{5y_1}_{t}}{12})^{(A\times 12)} - 1}\Bigg) \times L. \tag{6}$$

4.2 Using total interest payments to assess borrowers' outcomes

For each contract, the total amount paid in interest at the end of the 10-year period defined as $TINTPMT_{t+120}$ is calculated according to the following expressions:

$$TINTPMT^{10y}_{t+120} = \big(M^{10y}_t \times 120 \text{ months}\big) - \big(L - OMB^{10y}_{t+120}\big), \tag{7}$$

$$E_t\big[TINTPMT^{5y}_{t+120}\big] = E_t\Bigg[\big((M^{5y_1}_t + M^{5y_2}_{t+60}) \times 60 \text{ months}\big) - \big(L - OMB^{5y}_{t+120}\big)\Bigg]. \tag{8}$$

The first term on the right-hand side of these equations corresponds to the sum of mortgage payments made by a borrower at time $t+120$, and the second term reflects the total amount paid to reduce the principal owed. The difference between those two reflects the total amount paid in interest by the borrower at time $t+120$.

A borrower choosing a 10-year mortgage contract to reduce his exposure to interest-rate risk is better off if and only if the total amount paid in interest is lower under such a contract than under the combination of two 5-year contracts, i.e.

$$TINTPMT^{10y}_{t+120} < E_t\big[TINTPMT^{5y}_{t+120}\big] \tag{9}$$

When selecting the optimal mortgage contract duration, borrowers face uncertainty regarding future interest rates. Given that the borrower's choice problem rests on the individual' expectations of what 5-year mortgage rates will be at renewal, it is essential to consider the implications of Jensen's inequality in the context of the choice of mortgage contract. Jensen's inequality is a mathematical statement that $E(f(x))$ is below (above) $f(E(x))$ for any concave (convex) function $f(x)$ [17].

In general, a risk averse borrower would agree to pay a higher rate on a 10-year mortgage to avoid having to face the uncertainty associated with renewing after five years at a potentially much higher interest rate. The willingness to pay for such product depends in part on the agent's expectation of the rate at which the contract will be renewed. In a world with uncertainty, it is therefore important to assess if the approach used by borrowers to form their expectations impacts the outcomes.

Given that borrowers are aware of the 5-year and 10-year rates at the moment of contracting a mortgage, they only need to form an expectation with respect to the 5-year mortgage rate that will prevail five years later. Hence, there is no uncertainty with respect to a 10-year contract, but there is in the case of two 5-year contracts, which arises in the last five years. Using a $300,000 mortgage loan amortized over 25 years, I use two hypothetical values of 5-year mortgage rates — $i_L^{5y_2}$ and $i_H^{5y_2}$ — with each probability 1/2 to test the implications of Jensen's inequality. Setting the two mortgage rates to 3 and 4 percent respectively, I calculate expected total interest payments using (1) the expected value of the $E[i_{L,H}^{5y_2}]$, which is the mean of i_L and i_H each with probability 1/2, and (2) each interest rate $i_L^{5y_2}$ and $i_H^{5y_2}$ to compute a *low* and *high* state of total interest payments, and then take the expected value of the two amounts to obtain a final result. Mathematically, I want to determine the sign and the difference arising from the following expression

$$E_t\left[\frac{1}{2} \times TINTPMT^{5y}_{t+120,i_L} + \frac{1}{2} \times TINTPMT^{5y}_{t+120,i_H}\right] \gtreqless TINTPMT^{5y}_{t+120}\big[E_t(i_{L,H})\big], \tag{10}$$

where the first and second approach reflects the calculation on the left-hand side and right-hand side of equation (10), respectively. I find that the difference in total interest payments between the two sides of equation (10) is 0.05 percent. Changing the interest rate values by one standard deviation, from 3 and 4 percent to 1.9 and 5.1 percent respectively, increases the difference to 0.82 percent. In both cases, $E(f(x)) > f(E(x))$, which suggests that the function describing total interest payments is slightly convex. Given that the difference from Jensen's inequality is less than 1 percent, I choose to use the expected value of 5-year mortgage rates at renewal — $E_t(i^{5y_2}_{t+60})$ — to compute total interest payments. Historical simulations presented in Section 5 will show that the way borrowers make their expectations with respect to future 5-year mortgage rates has a significant impact on the interest payment outcomes.

4.3 Optimal mortgage contract selection

Based on the expression of total interest payments (9), a borrower is better off taking one 10-year mortgage instead of a combination of two 5-year mortgages if and only if the total amount paid in interest is lower under the 10-year contract than under two 5-year contracts. Mathematically,

$$TINTPMT^{10y}_{t+120} < TINTPMT^{5y}_{t+120}\big[E_t(i^{5y_2}_{t+60})\big] \tag{11}$$

Recall from Section 2 that interest-rate risk refers to the impact of unexpected movements in interest rates on the borrower's ability to meet his financial obligations, and particularly with respect to his mortgage contract. While lower interest rate volatility reduces interest-rate risk, the level of interest rates is also important. Hence, if total interest payments made by borrowers under a 10-year mortgage are such that the condition represented by equation (11) is met, choosing a 10-year mortgage contract instead of two 5-year contracts could be a solution to mitigate the exposure of borrowers to interest-rate risk.

5 Historical simulations

Based on the choice model presented in Section 4, this section presents historical simulations of a borrower's decision between 5-year and 10-year mortgage contracts of $300,000 amortized over 25 years. Using equation (11), I simulate total interest payments under both types of contract to determine the optimal choice of mortgage to mitigate interest-rate risk. Historical simulations cover hypothetical mortgage loans originated monthly between May 2006 and February 2014, for a total of 94 months. These simulations assume that the borrower's decision with respect to mortgage contract duration relies on the forecast of the discounted rate $i^{5y_2}_{t+60}$ based on one of (1) perfect foresight, (2) a naive approach, (3) historical average, and (4) recursive forecasts using other interest rates. The first approach assumes perfect information, while the last three take into account the uncertainty around the 5-year mortgage rate in $t+60$. In addition, I will conduct a simulation under an extended amortization period for 10-year mortgages to determine whether adjustments to the mortgage contract design have an impact on the borrower's choice. Finally, I will compute 5-year mortgage rates that would make borrowers indifferent between the two contracts.

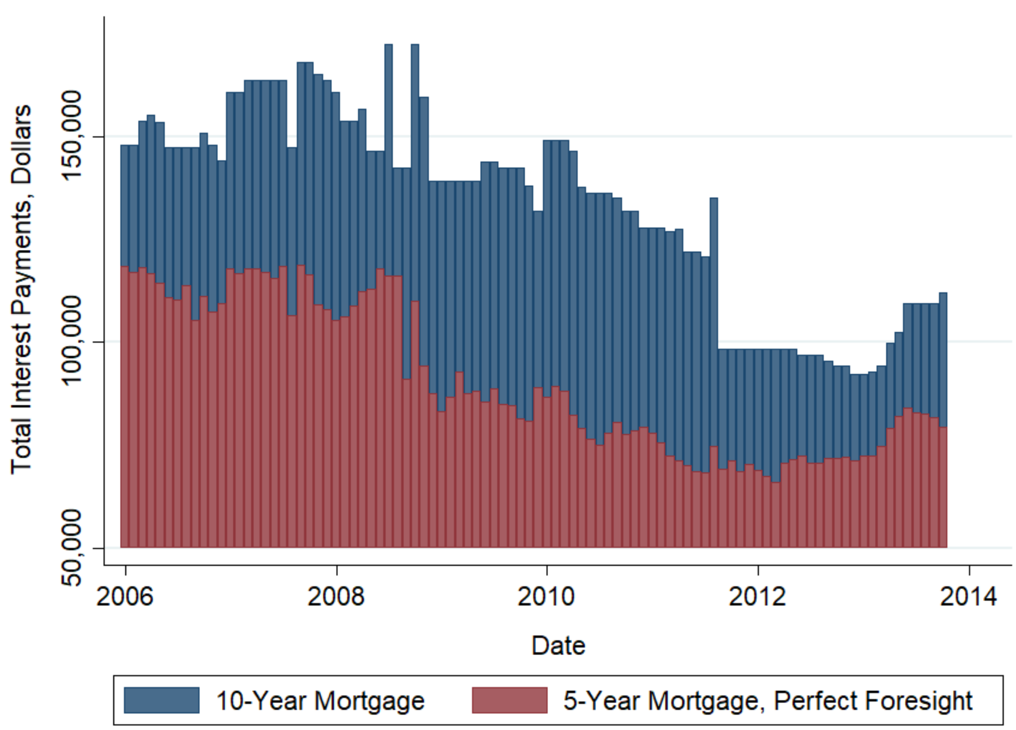
5.1 Perfect foresight

The first scenario I consider is perfect foresight. Perfect foresight refers to the correct prediction of future events, where borrowers have all the relevant information to accurately forecast future values of $i^{5y_2}_{t+60}$. Although this seems unrealistic, this approach allows us to evaluate if borrowers would experience ex-post regrets regarding their choice of mortgage contract. Mathematically, perfect foresight can be represented by the following equation:

$$E_t[i^{5y_2}_{t+60}] = i^{5y_2}_{t+60}, \tag{12}$$

where the expected value of 5-year mortgage rates in five years is equal to the actual value. Using the borrower's choice model summarized by equation (11) and the value of $E_t[i^{5y_2}_{t+60}]$ as given by equation (12), I simulate actual total interest payments generated by a 10-year and a combination of two 5-year mortgages over ten years.

Figure 3: Total interest payments over 10 years in dollars, perfect foresight



In Figure 3, each bar indicates total interest payments made over ten years under both types of contracts, and corresponds to an individual mortgage contracted at a specific time during the sample period. For mortgages contracted between May 2006 and February 2014, this figure clearly indicates that interest payments are much lower under two 5-year mortgage contracts.

As a result, a borrower opting for a 10-year mortgage contract would therefore regret his decision and have paid on average $42,683 more in interest. This significant difference arises from the fact that 10-year contracts are offered at systematically higher rates. Over the past decades, mortgage rates have also been falling, and thus borrowers choosing two 5-year contracts have had the opportunity to renew at a lower rate, which is on average 1.3 percent lower than the rate at which they contracted their previous 5-year mortgage. Therefore, under perfect foresight, borrowers are better off choosing a combination of two 5-year mortgages instead of a 10-year mortgage as the former provides lower interest payments.

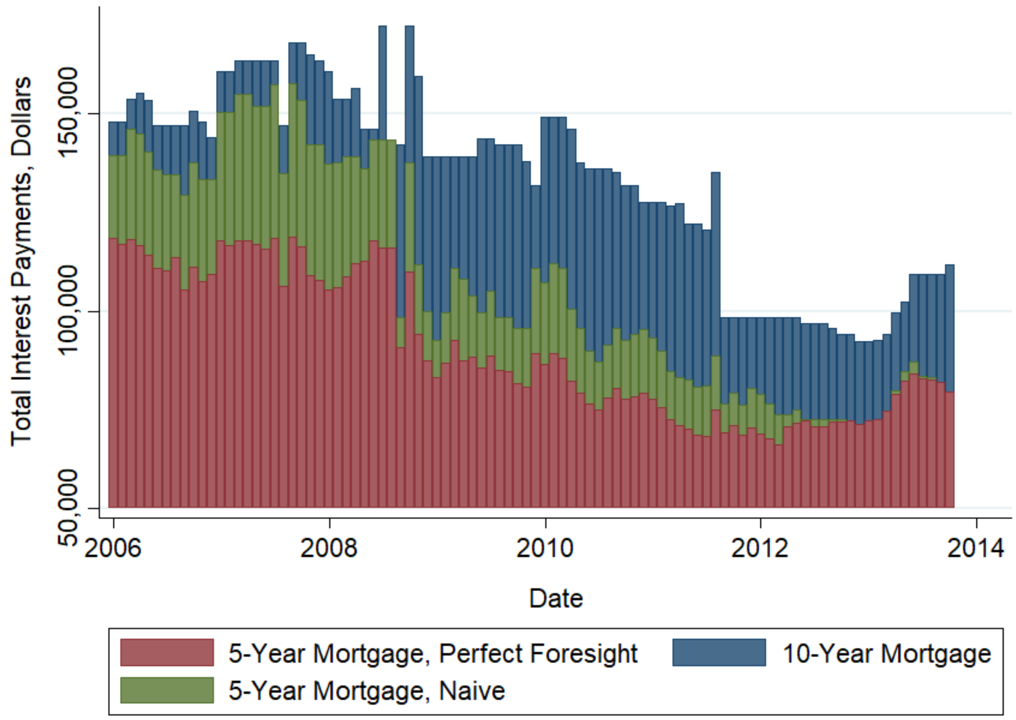
5.2 Naive approach

Reintroducing uncertainty, suppose next that borrowers predict $i^{5y_2}_{t+60}$ using a naive approach. Specifically, borrowers assume that mortgage rates will remain at the same level in five years from the day they contract their mortgage loan. Mathematically, this naive approach can be represented as

$$E_t[i^{5y_2}_{t+60}] = i^{5y_2}_t \tag{13}$$

Using the previously defined choice model and the value of $E_t[i^{5y_2}_{t+60}]$ as given by equation (13), I simulate total interest payments under a 10-year and two 5-year contracts to determine the borrower's choice. Figure 4 illustrates total interest payments under the naive approach. The outcome under perfect foresight is also included to illustrate the difference between the two approaches.

Figure 4: Total interest payments over 10 years in dollars, naive approach



The naive approach also leads to the conclusion that two 5-year mortgage contracts generate lower interest payments than one 10-year contract. Essentially, an individual who would have used a naive approach to forecast the renewal rate in 5 years from origination would have opted for two 5-year contracts and would have made the right decision in all cases, except in December 2008. In this specific case, the total amount paid in interest under a 10-year loan is $1,148 lower than the amount paid under two 5-year loans, and thus the individual would have preferred to choose a 10-year contract. However, he would have made the wrong decision since the true amount paid in interest is ultimately represented by the perfect foresight case, which is much lower than total interest payments calculated under the naive approach. Furthermore, the difference in interest payments between the two types of contracts is much smaller for contracts originated before 2009. Hence, comparing this naive approach where $E_t[i^{5y_2}_{t+60}] = i^{5y_2}_t$ to the outcome under perfect foresight, individuals who contracted mortgages before 2009 would have overestimated, by almost $30,000, total interest payments under two 5-year mortgage loans.[5] The difference in the borrower's expectations between perfect foresight and the naive approach however becomes insignificant after mid-2012, with an average difference of less than $700. Although there is a significant difference between predictions obtained under the perfect foresight and the naive approach, both lead to the conclusion that two 5-year mortgage contracts provide lower interest payments than one 10-year contract.

5.3 Historical average

A more sophisticated, though still naive, approach would be to assume that mortgage rates at $t+60$ will be equal to the historical average of *posted* rates, minus the discount received on the 5-year posted rate at time t. Given the range of historical data on posted and discounted mortgage rates available, the historical average is calculated using a fixed start date, that is May 2006, and the window is expanded to include additional information as t increases. I believe this is a reasonable assumption given that borrowers can easily consult historical mortgage rate data on online platforms, and then calculate the historical average based on the range of data available at that time. Being aware of their bargaining power, it is also reasonable to assume that borrowers deduct the discount received on their current mortgage quote to this historical average as they expect to be able to negotiate a similar discount at the time of renewal.

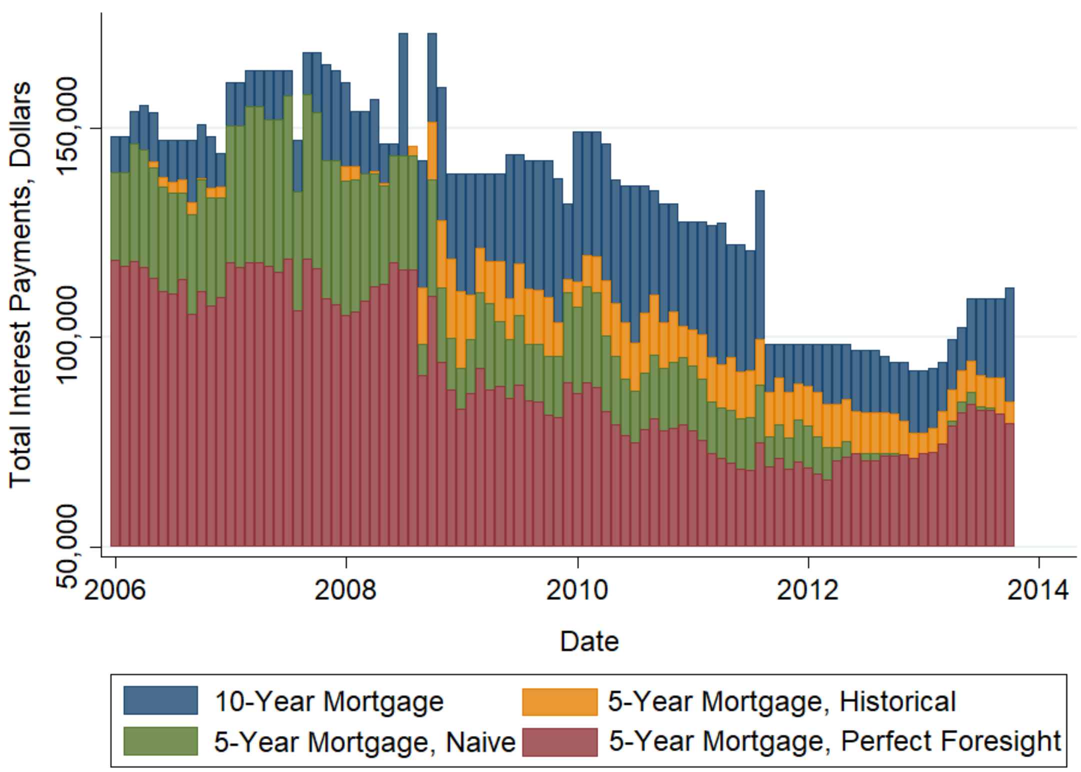
For example, a hypothetical borrower contracting a 5-year mortgage at the rate prevailing in July 2006, that is 5.49 percent, will make the assumption that the rate at which he will have to renew his 5-year mortgage in July 2011 will be the average of 5-year mortgage rates he observes between May 2006 and July 2006, minus the

[5] Total interest payments calculated under perfect foresight corresponds to the actual amount of interest paid under two 5-year contracts.

1.46 percent discount he received on the 6.95 percent 5-year posted rate in July 2006. Similarly, for a mortgage contracted at 3.79 percent in January 2010, the predicted 5-year mortgage rate that will prevail in January 2015 will be the average of 5-year mortgage rates observed between May 2006 and January 2010, minus the discount of 1.7 percent obtained in January 2010, and so on.

Using the previously defined choice model and the value of $E_t[i^{5y_2}_{t+60}]$ as given by this average, I simulate total interest payments under a 10-year and two 5-year contracts to determine the choice of mortgage contract borrowers would have made in the past under this historical approach.

Figure 5: Total interest payments over 10 years in dollars, naive approach



For mortgages originated before 2009, the historical approach provides a similar, overestimated, forecast of total interest payments under two 5-year contracts as the naive approach. Post 2009, the historical approach overestimates by a larger amount the total payment in interest to be made on two 5-year mortgages than the naive and perfect foresight approaches. By conception, borrowers using the historical approach to forecast future mortgage rates will take the average of 5-year mortgage rates available between May 2006 and the date at which they contract their first 5-year mortgage, minus the discount they received. Given that interest rates were following a downward sloping path, which was further amplified by the 2008 financial crisis, borrowers using this approach to predict 5-year mortgage rates five years later would have mechanically overestimated their interest payments. This outcome is also further magnified by an underestimation of the discount rate to be received at renewal, which increased from an average of 1.56 percent before 2009 to 2.12 percent after 2009.

Similar to the results obtained with the naive approach, 5-year mortgage loans offer lower interest payments for each of the hypothetical loans analyzed, with the exception of loans originated in December 2008, where a 10-year contract would have reduced total interest payments by $3,251. Therefore, based on Figure 5, I conclude that borrowers using the historical average approach would have been better off choosing two 5-year mortgages than one 10-year mortgage as the former implies lower interest payments over ten years.

5.4 Recursive forecasts using other interest rates

I next assume that borrowers form their expectations on the 5-year mortgage rate that will prevail in 5 years using information on other interest rates available today. Given that I only consider fixed-rate mortgages in this analysis, fluctuations in 5-year and 10-year rates between renewal dates do not matter. This exercise considers that borrowers use two distinct regression models to forecast the 5-year mortgage rate at renewal. As the objective is to maximize the forecasting performance, I will compare the performance of these models using the root mean square error (RMSE).

First, I consider the following simple regression model, where borrowers construct their prediction on 5-year mortgage rates that will prevail in five years, i^{5y}_{t+60}, using publicly available information on today's 5-year mortgage rates:

$$i^{5y}_{t} = \beta_0 + \beta_1 \ i^{5y}_{t-60} + \epsilon_t \tag{14}$$

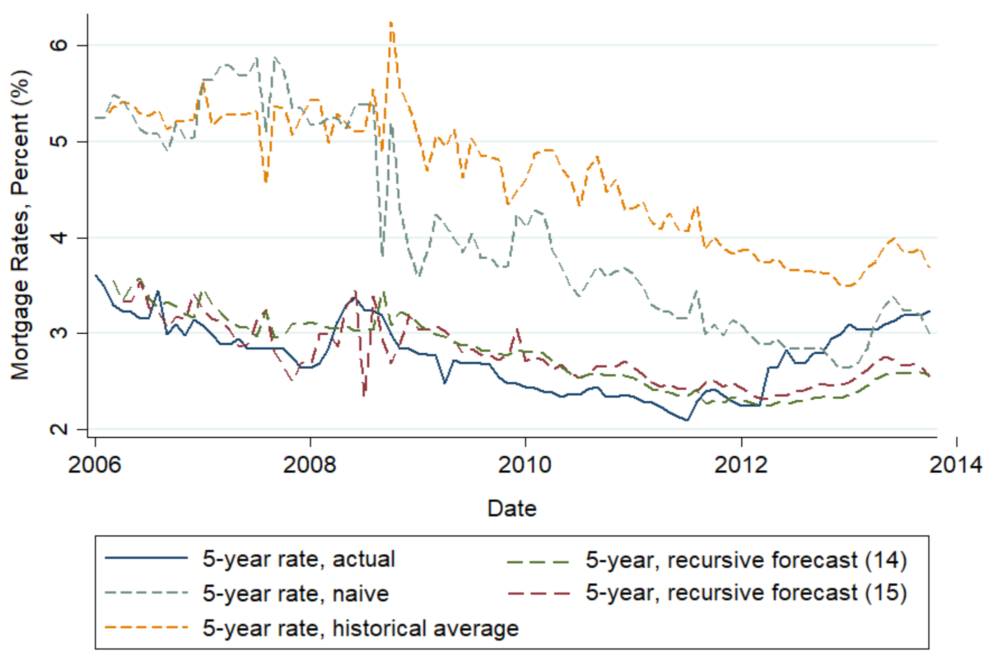
Second, assuming that borrowers are aware that 5-year and 10-year mortgage rates follow a similar path, I consider the following regression model where borrowers include information on both 5-year and 10-year rates to forecast i^{5y}_{t+60}:

$$i^{5y}_{t} = \beta_0 + \beta_1 \ i^{5y}_{t-60} + \beta_2 \ i^{10y}_{t-60} + \epsilon_t \tag{15}$$

Assuming that borrowers are aware of the term structure of interest rates, which corresponds to the relationship between interest rates and different maturities, then these two regression models may provide a reasonable estimation of how individuals form their expectations. Particularly, as implied in equation (15), borrowers would take into account that mortgage contracts taken over a longer period of time have higher interest rates. Additionally, the expectations hypothesis of the term structure of interest rates implies that an upward sloping yield curve forecasts an increase in the short-maturity yield, which supports the inclusion of multiple maturities in the second model. Thus, incorporating term structure dynamics into their expectations may increase the accuracy of borrowers' 5-year mortgage rate forecasts.

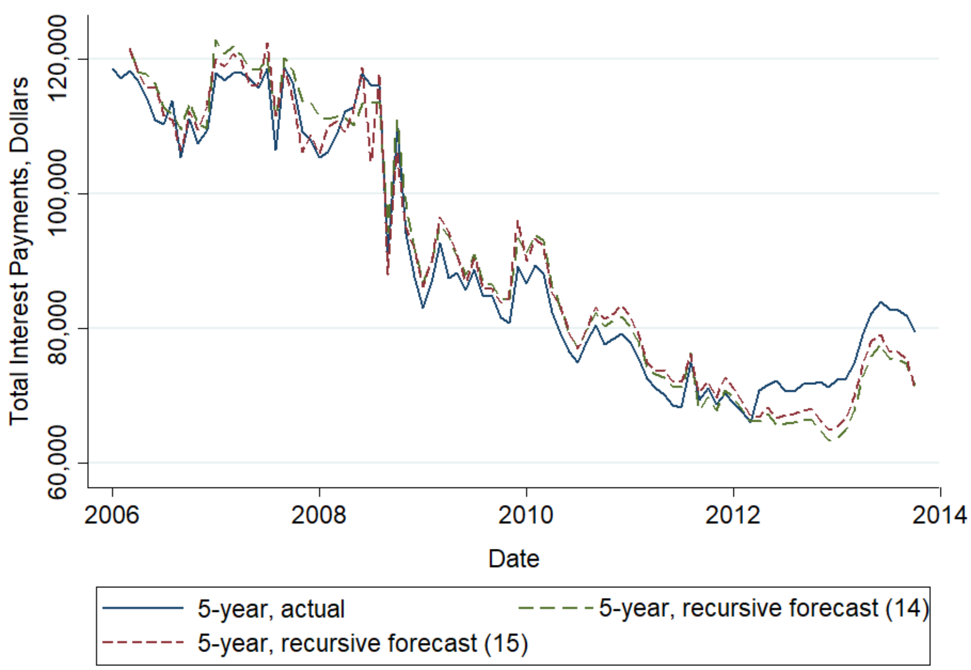
In both models, the regression coefficients are significant. Regression results are presented in detail in Table B.1 in Appendix B. Given that $\hat{\beta}_1$ is significantly less than one in the first model, we can now rule out the possibility that a random walk is the most accurate forecast of 5-year mortgage rates, as assumed in the naive approach in Section 5.2. Hence, 5-year mortgage rates do not follow a random walk and more complex methods can provide a better forecast than the naive approach. For comparison purposes, Figure 6 presents the recursive forecasts of i^{5y}_{t+60} based on these two regression models, as well as those based on perfect foresight, historical average, and the naive approach. Each of these rates corresponds to the predicted 5-year mortgage rate that borrowers expect to face in five years from the day they contracted their 5-year mortgage, conditional on the approach used to form their expectations.

Figure 6: Forecasted 5-year mortgage rates, by approach



As observed in Figure 6, the recursive forecast obtained from the second regression model defined by equation (15) provides a better fit to historical data. By including the latest information available in their forecasts, informed borrowers are more likely to make predictions today that are closer to the true value of 5-year mortgage rates in $t + 60$. To assess how the predictions obtained from the two regression models affect the borrower's choice, Figure 7 presents total interest payments expected to be made under two 5-year mortgage contracts relative to actual total interest payments, as calculated under perfect foresight.

Figure 7: Total interest payments over 10 years in dollars, by regression model



Total interest payments under the two models are very similar, as shown on Figure 7. However, Model (15) appears to provide a slightly better fit to actual total interest payments to be made by borrowers under two 5-year contracts. Model (15) also provides the lowest RMSE for both forecasted 5-year mortgage rates and total interest payments, as shown in Table B.2. Based on recursive forecasts obtained from the second regression model, the total interest payment over ten years under two 5-year mortgages is on average \$90,316, which is much lower than the total payment of \$133,228 to be made under a 10-year mortgage contract. As a result, borrowers are still better off opting for 5-year mortgages. This conclusion also holds for mortgages originated in December 2009. Therefore, based on historical data, borrowers never expected to be better off with a 10-year mortgage contract than with two 5-year contracts as the former leads to significantly higher interest payments.

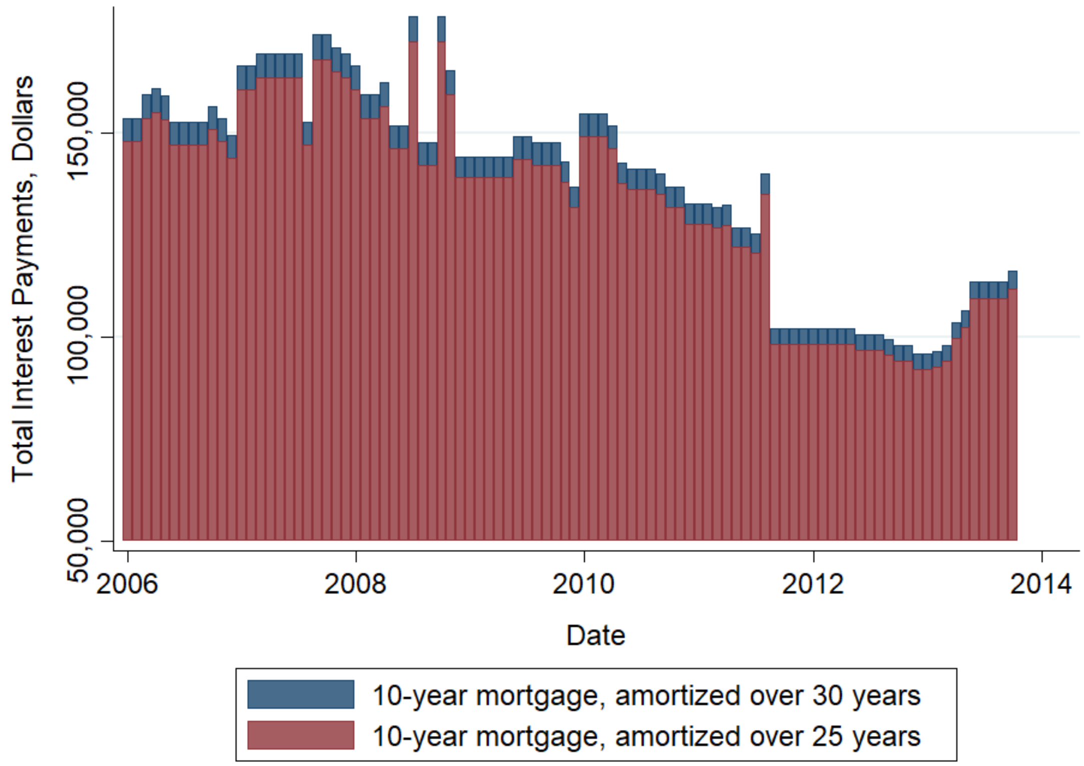
5.5 What if 10-year mortgages were amortized over 30 years?

Based on the argument that longer-term mortgages could reduce interest-rate risk exposure, the Bank of Canada believes that the Government should consider allowing a 30-year amortization period *conditional* on taking a 10-year mortgage contract. Historical simulations clearly indicate that borrowers would *not* be better off if they were choosing 10-year mortgages if such contracts are amortized over 25 years, which is the standard amortization period in Canada. In this subsection, I analyze the impact of increasing the amortization period from 25 to 30 years on the decision process. As borrowers only form their expectations on what 5-year mortgage rates will be in five years, a longer amortization period will only affect borrowers' decision with respect to the duration of their mortgage contract. Figure 8 presents total interest payments for a 10-year mortgage contract amortized over 25 years and 30 years, respectively.

As shown in Figure 8, extending the amortization period increases the average total interest payments to be made over ten years by \$5,045, thus making 10-year mortgages even less appealing. This reinforces the conclusion that borrowers are always better off choosing two 5-year mortgages instead of one 10-year one.

In short, every historical simulation leads to the conclusion that borrowers are much better off opting for two 5-year mortgage contracts instead of one 10-year. The

Figure 8: Total interest payments over 10 years in dollars, longer amortization



size of the loan also has no impact on the optimal choice of contract. This result is also reinforced if the amortization period of 10-year mortgages is increased from 25 to 30 years. Hence, the argument that longer-term mortgages would reduce the interest-rate risk faced by borrowers seems to be unfounded.

5.6 Indifference threshold

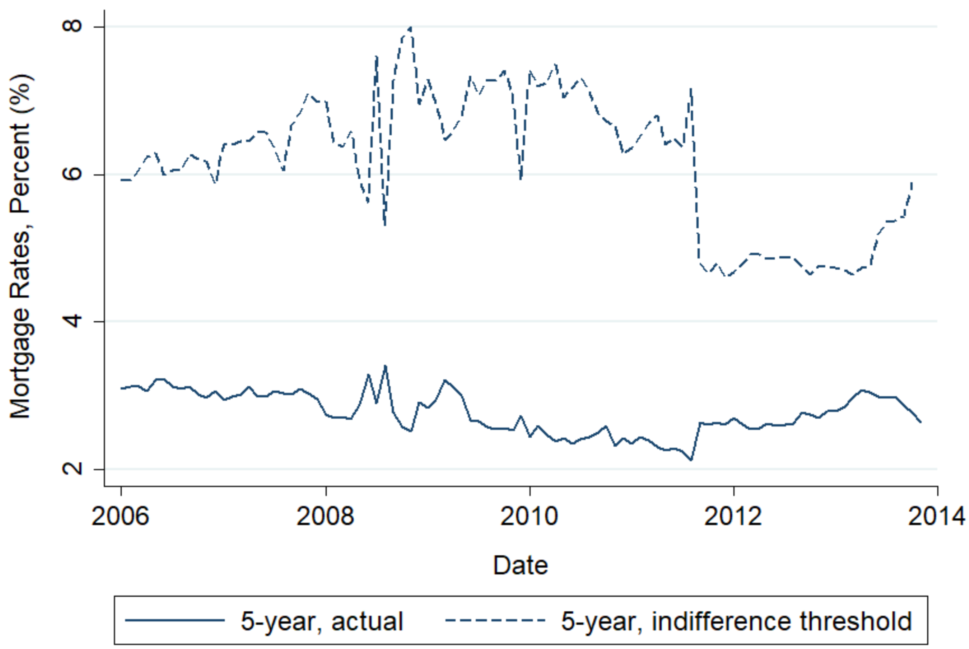
As shown by the historical simulations, borrowers do not mitigate their exposure to interest-rate risk by opting for a 10-year mortgage contract. To support this conclusion, I calculate the expected 5-year mortgage rate threshold for borrowers to be indifferent between the two products. The threshold is obtained by solving for

$E_t[i^{5y_2}_{t+60}]$ such that

$$TINTPMT^{10y}_{t+120} = TINTPMT^{5y}_{t+120}\left[E_t(i^{5y_2}_{t+60})\right] \tag{16}$$

Figure 9 plots 5-year mortgage rates required to make an individual indifferent between the two types of contract with the actual 5-year mortgage rates that prevailed at the time of renewal.

Figure 9: 5-year mortgage rates, indifference threshold



As shown in Figure 9, the expected 5-year mortgage rate must be much higher than the actual 5-year rate at renewal for borrowers to be indifferent between the two types of contract. Specifically, borrowers would have to overestimate the 5-year rate that would prevail at renewal by 3.42 percent on average to be indifferent between a 10-year contract and two 5-year contracts. This indifference threshold is also above the interest rate at which they could contract a 10-year mortgage, which is on average 5.03 percent. This further supports the conclusion that 10-year mortgages imply larger interest payments, and thus do not mitigate borrowers' exposure to interest-rate risk.

6 The importance of mortgage prepayments

This section discusses an alternative version of the borrower's choice model introduced in Section 4 to capture the possibility of prepayment. In Canada, borrowers are allowed as part of their mortgage contract to make partial prepayments every year without being subject to a penalty as long as they respect the conditions imposed by their lender. Borrowers can also prepay their mortgage in full by paying the penalty associated with the termination of the contract. As per Section 10 of the Interest Act, prepayment penalties after the first five years of a mortgage contract are relatively low and consist of three months worth of interest [19]. As a result, it is natural to wonder whether borrowers who choose 10-year mortgages do so since they have the possibility to pay a relatively small fee to refinance their remaining balance with a 5-year mortgage in the event that future 5-year mortgage rates are significantly lower. The impact of partial and full prepayment behaviours on the choice of mortgage contract are assessed in the following subsections.

6.1 Theoretical choice model of mortgage prepayment

As the objective is to understand how prepayment affects the ability of 10-year mortgage loans to reduce the interest-rate risk exposure of borrowers, I only focus on prepayment and thus ignore the possibility of default. To reflect the significant penalties associated with prepayments in the first five years of a mortgage contract, it is assumed that individuals opting for 5-year mortgages will not make any prepayment in full during their contract, while individuals opting for 10-year contracts will consider the option of prepayment only starting in the sixth year. To present the theoretical choice model with prepayment, I assume for now that prepayment privileges, which correspond to the possibility to make additional payments, are not considered by borrowers.

Among other factors, households may prepay their mortgage as a consequence of moving, prepay to refinance at a lower interest rate, or refinance for a higher amount in order to include renovations. For this analysis, I will consider the decision of prepayment as a maneuver to save by locking in a lower mortgage rate. Specifically, prepayment is an interesting option if 5-year mortgage rates fall below the 10-year rate contracted by a borrower such that the expected present value of remaining mortgage payments $PVRP_t$ is greater than the outstanding mortgage balance OMB_t.

I will use some preliminary concepts defined in Section 4.1, such as monthly payments M_t and the outstanding mortgage balance OMB_t, to compute the present value of remaining mortgage payments $PVRP_t$ and prepayment penalties. Following Souissi (2007), the present value of remaining mortgage payments $PVRP_t$ at the end of each period is defined as

$$PVRP_t = \frac{PVRP_{t+1}}{1 + \frac{E_t[i_{t+1}]}{12}}, \tag{17}$$

where $PVRP_{t+1}$ simply reflects the sum of all future monthly payments to be made until the contract ends, and $E_t[i_{t+1}]$ is the mortgage rate expected to prevail next period [29].

As I am considering the option of prepayment before the end of the contract term, a prepayment penalty must be included. In general, the penalty corresponds to the highest of 3 months' interest $Penalty^{3M}$ or the interest rate differential $Penalty^{IRD}$, which are defined as

$$Penalty_t^{3M} = \left(\frac{OMB_t \times i^m}{12}\right) \times 3 \text{ months} \tag{18}$$

$$Penalty_t^{IRD} = \left(\frac{OMB_t \times (i^m - i_t^c)}{12}\right) \times \text{nb. of months left on term}, \tag{19}$$

where OMB_t is the outstanding mortgage balance, i^m is the annual interest rate at which the mortgage is contracted, and i_t^c is the closest rate at time t on mortgages with a term equal to the remaining term on the mortgage [15]. Given that I will only consider the possibility of prepayment after five years into the contract for borrowers taking on mortgage loans for a duration of 10-years, Section 10 of the Interest Act applies, and therefore only three months worth of interest can be charged as a penalty for prepayment.

Using these concepts, a borrower will prepay the mortgage in full if and only if the present value of all future payments is higher than the outstanding mortgage balance plus the penalty equal to three months worth of interest. Hence, for a borrower engaging in a 10-year mortgage contract, the prepayment decision takes the following form:

$$P_t = \begin{cases} 0, & \text{for } t \leq 60 \\ 1, & \text{iff } PVRP_t^{10y} > OMB_t^{10y} + Penalty_t^{3M}, \text{ for } 60 < t \leq 120. \end{cases} \tag{20}$$

For prepayment in full to occur, this condition states that the sum of outstanding balance and penalty must be lower than the present value of remaining payments under the 10-year contract. If the condition is met, P_t is equal to 1 and a rational and forward-looking borrower should prepay his 10-year mortgage at time t.

Given that the penalty is a function of the outstanding mortgage balance and prepayment in full would require a significant amount of capital, I assume that borrowers who consider this option only do so to refinance their 10-year mortgage with a 5-year mortgage at the beginning of the sixth year to benefit from a lower mortgage rate. This prepayment condition will be used to analyze the desirability to switch from a 10-year to a 5-year mortgage in Section 6.3.

6.2 Including prepayment privileges in historical simulations

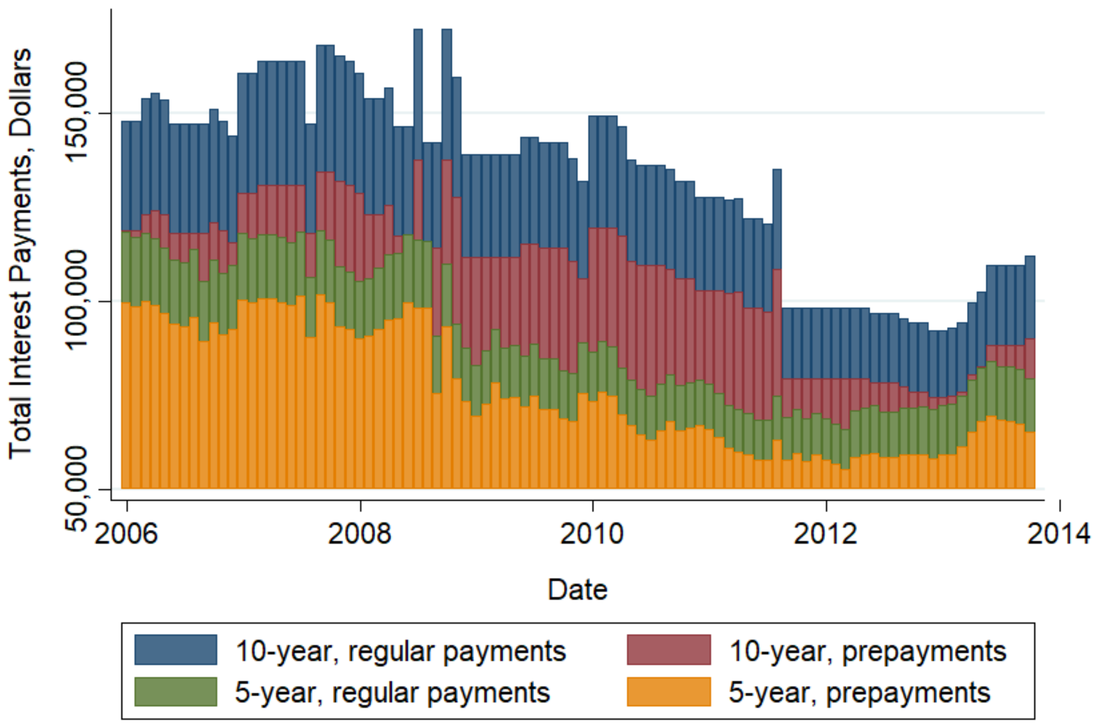
In this subsection, I relax the assumption that borrowers do not take advantage of their prepayment privilege. In Canada, financial institutions allow mortgage borrowers to make additional payments *once a year*, either by increasing their monthly payments by up to a certain amount or making a lump-sum payment of up to 15 percent of the original principal. According to the Canadian Association of Accredited Mortgage Professionals 2011 Survey, on average 17 percent of borrowers make lump-sum payments to accelerate the repayment of their mortgage. With the same objective, 16 percent increase their periodic payments, and 5 percent increase the frequency of their payments. Finally, among these borrowers, on average 36 percent take one or more actions to speed up the repayment process, while 64 percent do not [25]. The objective is to determine whether prepayment behaviours significantly affect total interest payments, and by extension, the choice of mortgage contract.

For this analysis, I make the following assumptions. First, I assume that individuals using their prepayment privilege do so through lump-sum payments of $10,000, which is less than the maximum of 15 percent of the principal borrowed authorized. As the objective of such payments is to reduce the principal owed and interest paid, I also assume that such payments are made at the beginning of the year, starting in the second year. The ability of borrowers to make such payments depends on a variety of characteristics, including income, propensity to consume, preferences, and so on. However, a simplifying and credible assumption would be to assume that borrowers who make such prepayments are not liquidity-constrained. Other factors such as the utility for liquidity relative to the importance given to debt repayment may also play a role at explaining this phenomenon.

To capture the impact of prepayment privileges on the mortgage choice faced by borrowers, I consider two extreme cases: (1) a liquidity-constrained borrower who never uses this privilege, and (2) an unconstrained borrower who always takes advantage of this privilege. This design provides a lower and upper bound of the impact of prepayment privileges on the choice of mortgage contract. This exercise is repeated to evaluate the sample of 94 mortgages, from May 2006 to February 2014, where both types of borrowers consider contracting a 10-year mortgage, or two 5-year mortgages. I assume both contracts are amortized over 25 years to facilitate the comparison. This choice is also supported by the existing rule stating that the maximum amortization period for mortgage with less than a 20 percent down payment is 25 years, which applies to the majority of Canadian mortgages. this exercise is conducted under perfect foresight to evaluate the possibility that borrowers experience ex-post regrets. Every year, borrower (1) will simply make his mortgage payments as scheduled, while borrower (2) will make an annual lump-sum payment of $10,000 in addition to his scheduled monthly payments starting in the second year of his contract.

Figure 10 illustrates the impact of prepayment privileges on total interest payments. As concluded in Section 5.1, borrowers not taking advantage of their prepayment privilege, and thus only making regular payments, are better off opting

Figure 10: Total interest payments over 10 years in dollars, prepayment privilege



for two 5-year contracts. Comparing total interest payment outcomes with prepayment under perfect foresight, I find that annual prepayments of $10,000 starting in the second year of the contract do not affect the borrower's choice with regards to the optimal mortgage contract duration. Given that a borrower who always takes advantage of prepayment privileges will always do so regardless of the choice of mortgage, prepayment only reduces total interest payments under both a 10-year and two 5-year contracts. Therefore, based on this simulation, borrowers who select a 10-year mortgage would experience ex-post regrets, even if they take advantage of their prepayment privilege. On average, periodic prepayments significantly reduce total interest payments made under a 10-year contract and two 5-year contracts by $26,280 and $14,234, respectively. In short, borrowers remain better off with a combination of two 5-year mortgages. Prepayments also significantly reduce the cost of borrowing.

6.3 The desirability to switch from a 10-year to a 5-year mortgage

Given that prepayment penalties are relatively low under the Interest Act, borrowers contracting a 10-year mortgage may decide to pay a relatively small fee to have the possibility to refinance with a 5-year mortgage and benefit from a lower rate. This subsection presents historical simulations including the possibility of prepayment in full to assess the desirability to refinance at the beginning of the last five years of 10-year mortgage contracts.

As in Section 5, the prepayment decision model is applied to the sample of 94 hypothetical mortgages between May 2006 and February 2014. I also use a 25-year amortization period. This choice is based on the results obtained in Section 5.5, where I showed that increasing the amortization period from 25 to 30 years simply increases total interest payments. In addition, prepayment privileges are ignored as they only reduce the level of total interest payments under both types of contract without affecting the optimal choice of mortgage contract duration, as concluded in Section 6.2. Focusing on the penalty associated with prepayment in full, the objective is to determine whether borrowers contracting 10-year mortgages would be tempted to switch from a 10-year to a 5-year mortgage in the last five years of their contract in order to obtain a lower mortgage rate. Using the remaining balance after five years of a 10-year mortgage contract originated at time t, the prepayment penalty and total interest payments are calculated to assess whether refinancing with a 5-year mortgage would allow borrowers to reduce their interest-rate risk exposure.

Based on the prepayment decision defined in (20), equation (7) can be adjusted as follows to include the possibility to switch from a 10-year to a 5-year contract after five years through refinancing. Total interest payments made under a 10-year mortgage contract refinanced after five years $TINTPMT_{t+120}^{10y_{ref}}\left[E_t(i_{t+60}^{5y_2})\right]$ is a function of the expected 5-year mortgage rate that will prevail in five years. Total interest payments reflect the difference between total monthly mortgage payments made over ten years and total payments applied on the principal, which are calculated by deducting the outstanding balance at the end of the term from the principal.

$$\begin{aligned} TINTPMT_{t+120}^{10y_{ref}}\left[E_t(i_{t+60}^{5y_2})\right] = &\left(M_t^{10y} + M_{t+60}^{5y_2}\left[E_t(i_{t+60}^{5y_2})\right]\right) \times 60 \text{ months} \\ &- \left(L - OMB_{t+120}^{10y_{ref}}\left[E_t(i_{t+60}^{5y_2})\right]\right) \end{aligned} \tag{21}$$

The outstanding balance of a 10-year mortgage refinanced after five years $OMB_{t+120}^{10y_{ref}}$ is a function of the expected 5-year mortgage rate at which the mortgage will be refinanced. It is given by

$$OMB_{t+120}^{10y_{ref}}\left[E_t(i_{t+60}^{5y_2})\right] = OMB_{t+60}^{10y} \times \left[\frac{(1+\frac{E_t(i_{t+60}^{5y_2})}{12})^{((A-5)\times 12)} - (1+\frac{E_t(i_{t+60}^{5y_2})}{12})^{(5\times 12)}}{(1+\frac{E_t(i_{t+60}^{5y_2})}{12})^{((A-5)\times 12)} - 1}\right], \tag{22}$$

where the outstanding balance at time $t+60$ to be refinanced with a 5-year mortgage OMB_{t+60}^{10y} is defined as

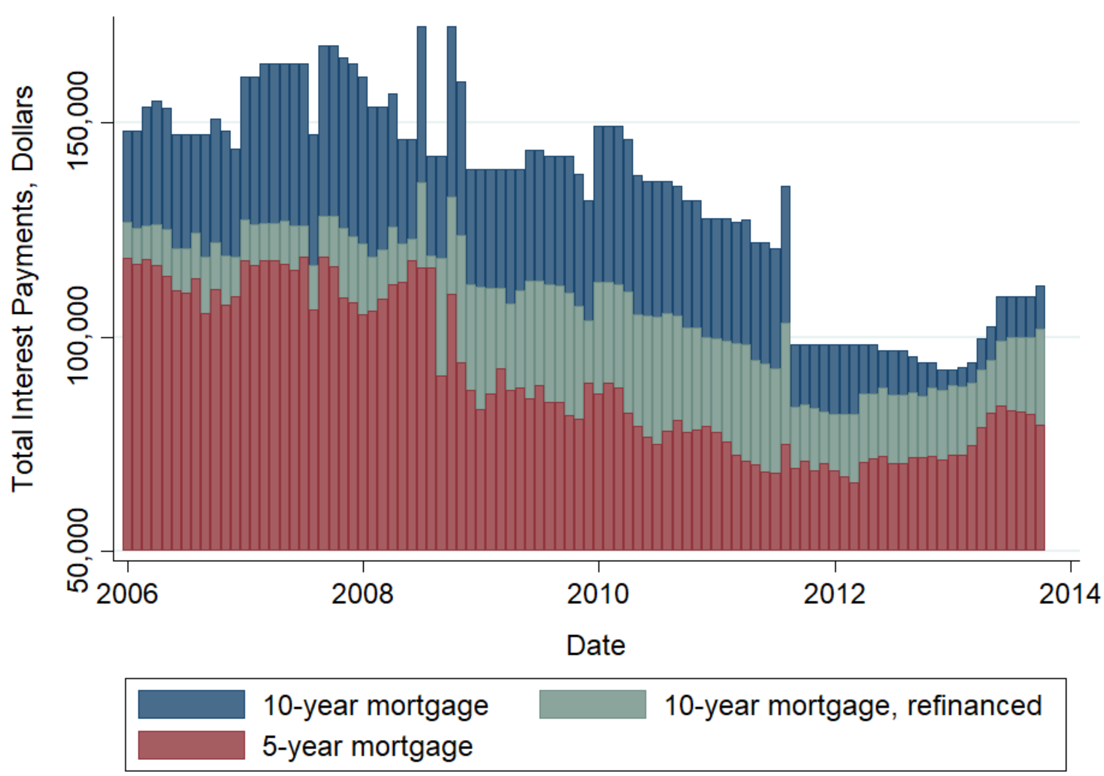
$$OMB_{t+60}^{10y} = \frac{(1+\frac{i_t^{10y}}{12})^{(A\times 12)} - (1+\frac{i_t^{10y}}{12})^{(5\times 12)}}{(1+\frac{i_t^{10y}}{12})^{(A\times 12)} - 1} \times L. \tag{23}$$

Taking into consideration that refinancing at $t = 60$ includes a prepayment penalty as defined by equation (18), expression (9) can be adjusted to compare the outcome under both contracts. It is optimal for a borrower who selected a 10-year mortgage to prepay at $t = 60$ to refinance at a lower 5-year mortgage rate if and only if

$$TINTPMT^{10y_{ref}}_{t+120}\left[E_t(i^{5y_2}_{t+60})\right] + Penalty^{3M}_{t+60} < TINTPMT^{10y}_{t+120} \tag{24}$$

Using historical data, I simulate total interest payments under both a 10-year mortgage and a 10-year mortgage refinanced at the beginning of the last five years with a 5-year mortgage contract. Figure 11 presents total interest payments made by borrowers under a 10-year mortgage, with and without refinancing, and two 5-year mortgages. To accurately reflect the costs of refinancing in equation (24), the penalty is included in total interest payments made under refinanced 10-year mortgages.

Figure 11: Total interest payments over 10 years in dollars, refinancing



As shown in Figure 11, there is a significant advantage for every hypothetical borrower who contracted a 10-year mortgage between May 2006 and February 2014 to switch to a 5-year mortgage after five years. On average, borrowers opting for a 10-year mortgage pay a total of \$133,228 in interest over ten years. On average, this payment can be reduced by \$25,215 if they decide to refinance at a lower 5-year mortgage rate at the beginning of the last five years. With refinancing, borrowers pay on average \$108,013 in interest, which includes a prepayment penalty of \$3,341 on average. Therefore, borrowers who made incorrect predictions regarding the future path of 5-year mortgages rates can use mortgage refinancing as a solution to reduce their total interest payments. However, as noted in Section 5.1, borrowers pay on average \$42,683 more interest under 10-year mortgages compared to two 5-year mortgages. Therefore, with an average of \$25,215 in savings with refinancing, this option remains more expensive than opting for a combination of two 5-year mortgages. In conclusion, even with the inclusion of prepayment privileges and the possibility of refinancing, Section 6 indicates that there is no clear rationale for borrowers to opt for 10-year mortgages in order to reduce their exposure to interest-rate risk.

7 Forecasting based on a VAR-X model

To complement the historical simulations presented in Section 5 and 6, I now build a vector autoregressive (VAR-X) model with exogenous variables with the objective to generate accurate and realistic out-of-sample forecasts for fixed mortgage rates. Specifically, the VAR-X will be used to forecast the path of 5-year and 10-year mortgage rates over the next ten years. I discuss the model specification as well as its forecasting performance. I also test whether additional variables would improve the accuracy of the forecasts. Based on the borrower's choice problem presented in Section 4, I also compute total interest payments using the future path of 5-year and

10-year mortgage rates to determine whether the conclusion obtained from historical simulations holds in out-of-sample forecasts.

7.1 Determining model adequacy

For the out-of-sample analysis, the estimated reduced form model takes the following general VAR-X(p, s) representation

$$Y_t = a_0 + \sum_{i=1}^{p} A_j Y_{t-j} + \sum_{j=1}^{s} B_j X_{t-j} + \epsilon_t, \tag{25}$$

where $Y_t = \{bench_{5yr,t},\ spread_{5y,t},\ spread_{10y,t},\ spread_{bench5-10,t},\ real_t\}$ is the vector of endogenous variables. The variables are the Government of Canada 5-year benchmark bond, the 5-year and 10-year mortgage spreads, and the spread between the 5-year and 10-year Government of Canada benchmark bonds. I also add the real interest rate, calculated by subtracting inflation from the Bank of Canada policy rate, to improve the forecast accuracy of the various spreads. This is the benchmark model.

The vector of exogenous variables X_t contains the regulatory policies such as the maximum LTV ratio, and maximum amortization period. Changes to these policies implemented over time will be captured in the recursive forecasts from February 2009 to February 2019. Given that Y_t has no impact on X_t, the exogenous variables are determined outside of the model for out-of-sample forecasts. Over the out-of-sample forecasting horizon, I assume no regulatory change in the maximum amortization period and LTV ratio, and that each additional exogenous variable included in the model follows an AR(1) process. This VAR-X model is based on monthly data from May 2006 to February 2019, as described in Section 3. Forecasts of 5-year and 10-year mortgage rates are constructed using forecasted values of benchmark bond yields as well as benchmark and mortgage spreads. In this VAR-X representation, a_0 is a 5×1

vector of intercepts and A_j is a 5×5 endogenous coefficient matrix. The exogenous coefficient matrix is given by B_j. The vector of innovations ϵ_t is a serially uncorrelated process with mean zero and constant variance Σ_ϵ.

7.1.1 Unit root testing

A first step in time series analysis is to verify the stationarity of the data. As indicated in Tables B.3 and B.4, both the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests suggest that the null hypothesis of unit root cannot be rejected at any significance level for all the variables of interest, except for 5-year and 10-year mortgage spreads. However, the ADF and PP tests strongly reject non-stationarity once applied to the data in first difference, suggesting that the variables are integrated of order 1. Figures B.1 and B.2 in Appendix B support this conclusion.

Standard hypothesis testing does not treat the null and the alternative hypothesis symmetrically. In particular, we tend to not reject the null unless we have strong evidence against it. The ADF and PP tests take non-stationarity as the null hypothesis: however there is no particular reason to do so. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test reverses these roles [23]. Results of the KPSS test presented in Table B.5 indicate that the null hypothesis of trend stationarity cannot be rejected at the 5 percent significance level for 5-year and 10-year mortgage spreads in levels. However, the null is rejected for the 5-year benchmark rate, the benchmark spread and the real interest rate. Once the first-difference transformation is applied, Table B.6 shows that trend stationarity cannot be rejected for all variables, which reinforces the conclusion that the data is integrated of order 1.

Given that Government of Canada benchmark bond yields and other interest rates move together, a VAR representation in first-difference is not appropriate as it would not capture the potential cointegration relationship that must exist, at least theoretically, between the variables. As discussed by Kilian and Lutkepohl (2017), a VAR in levels is recommended in the presence of cointegrated variables as the Least Squares/ ML estimated coefficient matrices are still consistent and normally distributed [24]. Thus, for the rest of this exercise, I focus on a VAR-X in levels of the $I(1)$ variables.

7.1.2 Lag order selection

The order p and s of the VAR-X(p, s) represented in equation (25) are unknown and must be estimated. While the order p must be large enough to ensure that the innovations of the VAR-X are uncorrelated, it must not be too large as higher orders reduce the precision of the forecasts generated by the model. Given that forecasting is the main objective, the optimal lag length p is selected to minimize the forecast mean squared error (MSE). To determine the optimal lag order p, I rely on two common lag selection criteria, namely the final predictor error criterion (FPE) and Akaike's information criterion (AIC). Other information criteria such as Hannan-Quinn and Schwarz information criteria are also useful if the main objective is to determine the 'true' lag order as they are both consistent estimators of the lag order, as opposed to FPE and AIC. Since the objective of this exercise is to maximize the forecasting performance of the VAR-X, I focus on the FPE and AIC. As presented in Table B.7, both FPE and AIC tests recommend 3 lags.

Additionally, the lag order s imposed on the exogenous variables must be selected. Given that mortgage loans originated by federally-regulated institutions must respect OSFI's prudent guidelines once they come into effect, regulatory policies implemented throughout the selected sample are assumed to have a direct impact on mortgage rates. For this reason, I set the order s to zero. Given the estimation in levels in the presence of $I(1)$ variables, I follow Kilian and Lutkepohl (2017) and use a VAR-X $(4, 0)$ specification as the benchmark model [24].

7.1.3 Model diagnostic

To determine whether the benchmark VAR-X$(4, 0)$ model appropriately captures the dynamics of the variables, I verify that the estimated innovations $\hat{\epsilon}_t$ are uncorrelated. I use the modified Ljung-Box portmanteau test, a standard approach to verify residual autocorrelation in VAR models with small samples. The Portmanteau test for innovation tests the null that $\mathbb{E}(\epsilon_t \epsilon'_{t-j}) = 0$ for all j against the alternative that at least one of the autocovariances is nonzero.

Based on the test statistic indicated in Table B.8, the null hypothesis that there is no serial autocorrelation in residuals cannot be rejected at the 5% significance level. This suggests that residuals obtained from the VAR-X$(4, 0)$ are white noise and that this specification captures well the dynamics in the data.

7.2 Model performance over the forecasting horizon

To complete this analysis, I look at the forecasting performance of the VAR-X over a 5-year and 10-year forecasting horizon. To maximize the forecasting performance of the model, the prediction that minimizes the root mean square error (RMSE) generated by the forecasts is selected. Being a measure of how spread out the residuals are, the RMSE is frequently used to compare forecasts to observed data. Given that the model does not include 5-year and 10-year discounted mortgage rates, they can be constructed using the forecasted values of the spreads and Government of Canada benchmark bonds. Appendix A presents the details on how discounted mortgage rates can be constructed from mortgage spreads. I will focus on the ability of the model to provide an accurate forecast of the 5-year and 10-year mortgage rates, as these rates enter directly into the calculations of total interest payments. As discussed in the previous sections, I assume the total amount paid in interest is the key driver of borrowers' choice of mortgage contract.

I analyze the forecasting performance of the model starting in February 2009. This starting point is selected to compare the accuracy of the forecasts over a 5-year and 10-year period, and to capture the significant reduction in interest rates in the aftermath of the 2008 financial crisis. For this exercise, I compare the RMSE of 5-year and 10-year discounted mortgage rate forecasts from the benchmark model to two sets of alternative models to determine which specification is more appropriate.

The first set of alternative models consists of the benchmark model to which I add additional endogenous variables, namely the 3-year mortgage spread, inflation, GDP growth, the policy rate, and the unemployment rate. The second set of alternative models consists of adding additional exogenous variables to the benchmark model, namely the house price index, the consumer confidence index, and the Bank of Canada's commodity price index for energy. As discussed in Section 3, changes in interest rates, and particularly the policy rate, are closely linked the the economic activity. As lenders pass changes in the policy rate to borrowers through revised mortgage rates, I believe that adding variables capturing the dynamics of the economy would increase one's ability to capture and forecast the dynamics of Canadian 5-year and 10-year mortgage rates. I will choose the model with the best forecasting performance. Once I have specified the model, I then use the same approach to assess whether a forecasting horizon of five years is better than ten years. The results are summarized in Table B.9.

Comparing the RMSE values across model specifications and forecasting horizons, the analysis indicates that alternative models including additional endogenous variables do not provide a better forecasting performance than the benchmark model. In contrast, I find that including additional exogenous variables slightly improves the forecasting performance, especially over the 10-year forecasting horizon. As a result, I decide to add both the consumer confidence index and commodity price index for energy as exogenous variables to the benchmark model as this specification provides the lowest RMSE.[6] In choosing the optimal mortgage contract, borrowers' perception of the economy is an important factor. Hence, including the consumer confidence index allows us to obtain forecasts that are more in line with borrowers' expectations. Moreover, given that the Canadian economy highly relies on natural resources, and particularly on energy, the commodity price index for energy is included to capture the key dynamics of the economy driving changes in interest rates. I also choose to focus on a 120-month forecasting exercise as this horizon improves the forecasting performance. This choice extends the out-of-sample analysis by an additional five years. Assuming this forecasting tool would be used by policymakers to predict the evolution of mortgage rates, I also compute forecasts using a recursive-VAR experiment to see how the new model performs in real time.

The 120-month forecasting exercise and the recursive VAR experiment both start in February 2009 and are illustrated in Figure B.3 of Appendix B. For completeness, I also include Figures B.4 and B.5, which present the results for the variables included in the model that were used to compute 5-year and 10-year mortgage rates, namely the Government of Canada 5-year benchmark bond, the 5-year and 10-year mortgage spreads, and the spread between 5-year and 10-year Government of Canada benchmark bonds. I also include the results for the Government of Canada 10-year benchmark bond given that it is used jointly with the 10-year mortgage spread to

[6] Based on the alternative models considered in Table B.9, the selected model corresponds to Model (2.6).

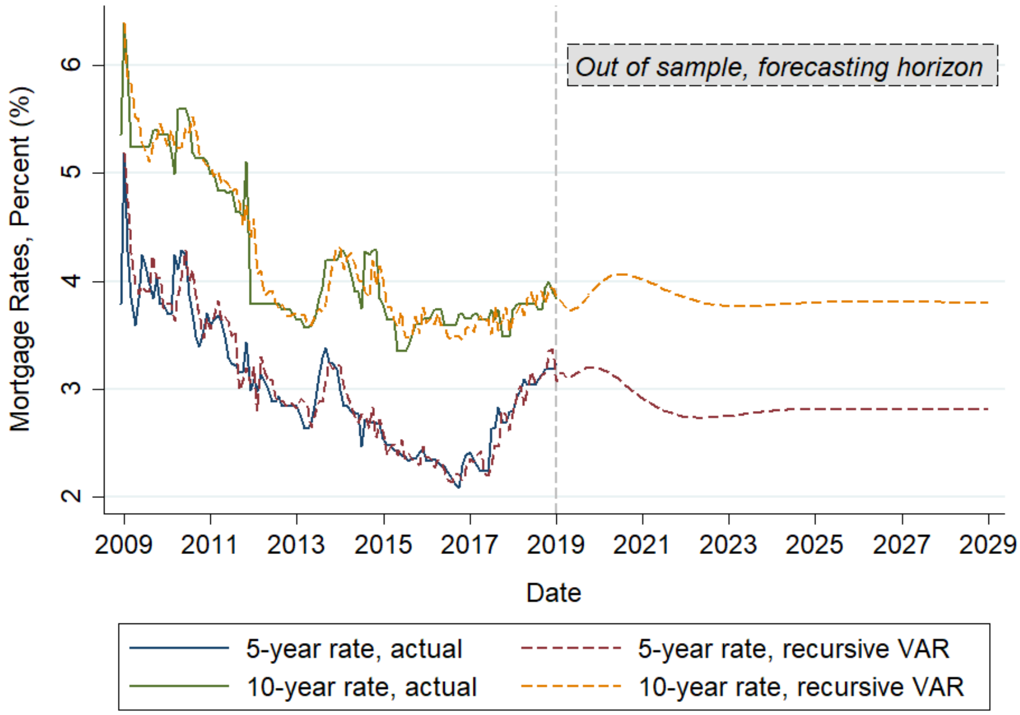
obtain the 10-year discounted mortgage rate. Based on in-sample forecasting performance tests, I conclude that the model provides forecasts that follow quite well the historical data. In addition, the recursive VAR experiment highlights the ability of the model to adjust to new information as the VAR is updated periodically, thus enhancing the accuracy of the forecasts.

Finally, based on the full sample, the recursive VAR experiment is extended to provide out-of-sample 120-month forecasts of 5-year and 10-year mortgage rates from February 2019 to February 2029 . Given that a VAR-X does not generate forecasts of exogenous variables, some assumptions with respect to the future path of the selected exogenous variables are made. I first assume that the regulations remain unchanged, and so I set the LTV ratio and the maximum amortization period to 95 percent and 25 years, respectively. I believe this is a reasonable assumption as these regulations have not changed since October 2008 and July 2012, respectively. For the consumer confidence index (CCI) and commodity price index for energy (CPIE), I assume for simplicity that they each follow a AR(1) process, which I use to forecast the future values of these two indices.[7] As a result, both variables therefore tend towards their sample average. Figure 12 shows in-sample recursive forecasts of both 5-year and 10-year mortgage rates from February 2009 to February 2019, and out-of-sample forecasts from March 2019 to February 2029.

As illustrated in Figure 12, the model indicates that 5-year mortgage rates are expected to follow a slightly downward trend until May 2019, reaching 3.10 percent. Similarly, 10-year mortgage rates are expected to follow a downward trend until June 2019, reaching 3.73 percent. In the second half of 2019, 5-year mortgage rates are expected to rise until March 2020, while 10-year rates will keep increasing until August

[7] Forecasts of the consumer confidence index (CCI) and commodity price index for energy (CPIE) are obtained from $CCI_t = \rho_0 + \rho_1 CCI_{t-1} + \epsilon_t$ and $CPIE_t = \rho_0 + \rho_1 CPIE_{t-1} + \epsilon_t$, respectively.

Figure 12: **5-year and 10-year mortgage rates forecast over 10 years**



2020. As observed in Figure 12, 10-year mortgage rates appear to be more persistent than 5-year ones, which would explain why they require more time to adjust to economic conditions. Over the next ten years, 5-year and 10-year mortgage rates stabilize to 2.81 and 3.80 percent, respectively. While the level to which mortgage rates converge is approximately 0.70 percent lower than the historical average, the spread between the forecasted 10-year and 5-year discounted mortgage rates remains on average 1 percent, which matches the average spread observed in historical data.

7.3 Borrower's choice over forecast horizon

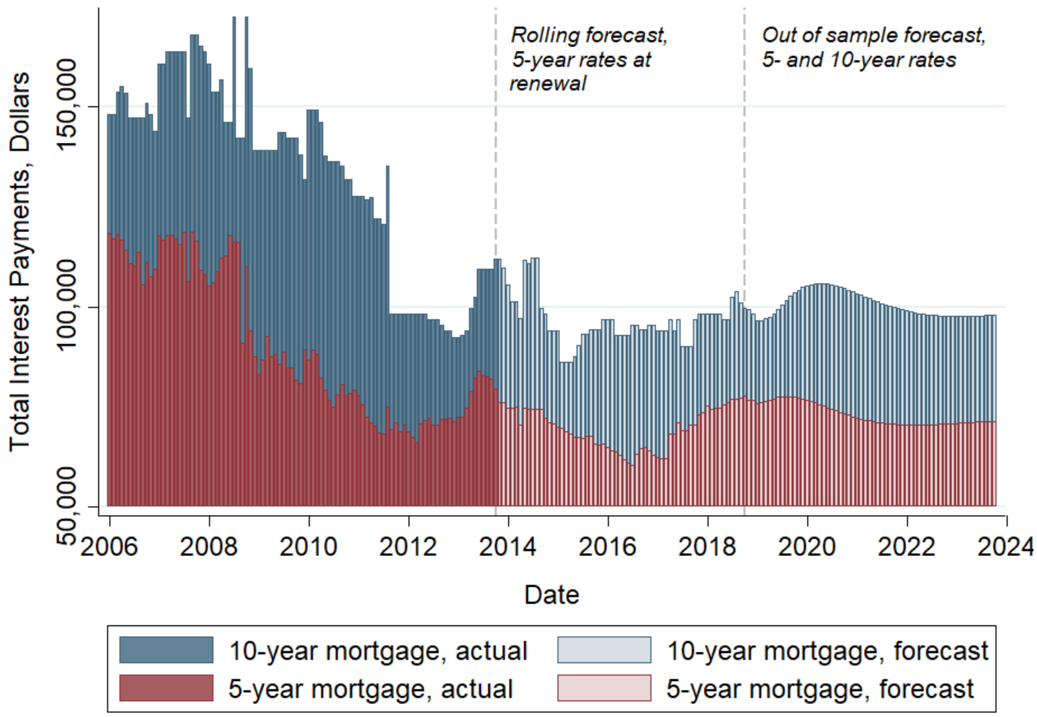
Based on the various historical scenarios, I conclude that borrowers do not mitigate their interest-rate risk exposure by opting for 10-year mortgage contracts instead of a combination of two 5-year contracts, regardless of the approach used to compute

5-year mortgage rates at renewal. The objective of this section is to assess whether this conclusion holds in out-of-sample forecasts.

To do so, I follow the borrower's choice problem under perfect foresight as presented in Section 5.1 and calculate total interest payments under both contracts using equations (7) and (8). Using the forecasted values of mortgage rates, we now have 214 hypothetical mortgages originated between May 2006 to February 2024. I use historical data on mortgage rates, but include 5-year mortgage rate forecasts, from March 2019 to February 2024, to compute 5-year mortgage rates at renewal. As we evolve through time, borrowers acquire additional information, and thus recursive forecasts are used to obtain 5-year mortgage rates at renewal. Using out-of-sample forecasts of 5-year and 10-year mortgage rates, I compute total interest payments for future mortgages to be originated between March 2019 and February 2024. These calculations rely completely on mortgage rate forecasts generated by the VAR-X model, as illustrated in Figure 12. The following figure presents the distribution of total interest payments made under 10-year and 5-year mortgages originated between May 2006 and February 2024.

As highlighted in Figure 13, total interest payments made under 10-year mortgages are much higher than those under two 5-year mortgages. This simulation based on forecasted mortgage rates shows that the conclusion obtained from historical simulations also holds in the future. Borrowers opting for a 10-year mortgage contract between March 2014 and February 2019 pay on average $97,012 in interest, which is 39 percent higher than what borrowers pay under two 5-year contracts. Looking at out-of-sample forecasts of total interest payments, a 10-year mortgage contract is associated with an interest payment of $100,525, which is much higher than the average of $73,111 paid under the combination of two 5-year contracts.

Figure 13: Total interest payments over 10 years in dollars, forecasts



To understand the dynamics of interest payments over time, we can take a closer look at the 5-year and 10-year mortgage rates across the three periods analyzed, namely May 2006 to February 2014, March 2015 to February 2019, and March 2019 to February 2024. During the first period, 5-year and 10-year mortgage rates were on average 3.47 and 4.66 percent, respectively. During the second period, the average 5-year and 10-year mortgage rates fell to 2.60 and 3.74 percent, respectively, and then increased to an average of 2.91 and 3.87 percent. Based on mortgage rates dynamics presented in Figure 12, total interest payments significantly fell from the first period to the second, and are now on an upward trend again. Although total interest payments are increasing due to rising mortgage rates, both 5-year and 10-year mortgage rates follow the same trend, which indicates that the conclusion is not an artifact of a declining trend in mortgage rates.

Based on equation (11) summarizing the choice problem, borrowers remain better off with two 5-year mortgages than one 10-year mortgage. I have shown that under no circumstance borrowers should consider choosing a 10-year mortgage to mitigate their interest-rate risk exposure as they would face higher interest payments. This reinforces the conclusion that the Bank of Canada's statement appears unfounded.

8 Conclusion and further research

This essay analyzes the statement made by the Bank of Canada that borrowers could use longer-term mortgage contracts to mitigate their exposure to interest-rate risk. Given the central role played by the interest-rate risk in this analysis, I carefully define this concept as the impact of unexpected movements in interest rates on the ability of borrowers to meet their financial obligations with respect to their mortgage contract. In Canada, 10-year mortgages are a less popular choice than 5-year mortgages due to the design of the residential mortgage market. On the demand side, borrowers are required meet various regulatory requirements, such as maximum loan-to-value and debt service ratios. They must also prove their ability to face higher interest rates by passing the mortgage stress test. On the supply side, lenders impose a premium on mortgages with maturity of more than five years in order to mitigate their exposure to prepayment risk. Combined with the cap on Government guaranteed deposit insurance, longer-term mortgages therefore impose a risk on lenders' balance sheets. As a result, 10-year mortgage loans are more expensive for borrowers, which may explain the scarcity of such contracts in the Canadian mortgage market.

Using data on Canadian mortgage rates and other interest rates, I discuss the evolution of 5-year and 10-year mortgage spreads, as well as the overall interest rate dynamics. I develop a borrower's choice model to assess whether borrowers would be better off with a 10-year mortgage instead of a combination of two 5-year mortgages, as suggested by the Bank of Canada. To determine which contract would limit borrowers' exposure to interest-rate risk, I rely on a measure of total interest payments over a 10-year period. Based on the various historical simulations presented in Section 5, I conclude that borrowers are better off choosing two 5-year mortgages instead of one 10-year one, regardless of the approach used to compute 5-year mortgage rates at renewal. I also show that extending the amortization period of 10-year mortgages from 25 to 30 years increases total interest payments under a 10-year contract, and thus leads to no change in the optimal choice of mortgage contract. Adding prepayment privileges leads to the same conclusion. Furthermore, I show that there is always a desire to refinance the outstanding mortgage balance of a 10-year mortgage with a 5-year contract starting in the sixth year to benefit from a lower rate and save on interest payments. While refinancing can be a solution for borrowers who made incorrect predictions regarding future 5-year mortgage rates, this option remains more expensive than initially opting for a combination of two 5-year contracts.

Following the borrower's choice problem, I conduct a similar analysis using out-of-sample forecasts of 5-year and 10-year mortgage rates obtained from a VAR-X. Given that the objective is to generate accurate and realistic forecasts of mortgage rates, I discuss the model adequacy and test its in-sample forecasting performance over a 5-year and 10-year forecasting horizon. I choose to forecast 5-year and 10-year mortgage rates over a 10-year horizon. I compute total interest payments based on these out-of-sample forecasts to assess whether borrowers should opt for a 10-year mortgage or a combination of two 5-year mortgages in the future in order to mitigate their exposure to interest-rate risk. Based on both historical and out-of-sample simulations, I conclude that there is a strong evidence against the statement made by the

Bank of Canada as 10-year mortgages impose significantly larger interest payments on borrowers.

While this essay is a first step towards understanding the complexity of interest-rate risk in the context of residential mortgage finance, further analyses including alternative methods to evaluate interest-rate risk exposure should be considered. A similar exercise with a focus on the variability of monthly payments could be performed. Borrowers opting for 10-year fixed-rate mortgages would secure a given rate for a longer period of time, which could potentially reduce their *future* exposure to higher interest rates at renewal. Additionally, further analyses including micro data would be required to identify the key characteristics of borrowers and mortgages expected to be more sensitive to changes in interest rates. Finally, to obtain a more accurate assessment of borrowers' exposure to interest-rate risk, this analysis could be modified to include variable-rate mortgages. Such analyses would provide essential information to policy makers and regulators that could be used in the design and implementation of policies aimed at mitigating the exposure of the household sector to interest-rate risk.

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[31] Statistics Canada (2019). Indebtedness and Wealth Among Canadian Households. Economic Insights. Catalogue no. 11-626-X — 2019003 - No. 089. 44 pages.

A Appendix A: Data sources

This appendix presents the sources of data used in this essay. The sample period covered is May 2006 to February 2019.

- Posted 3-year and 5-year mortgage rates: Statistics Canada, Table 10-10-0122-01 (formerly CANSIM 176-0043), Financial market statistics, last Wednesday unless otherwise stated, Bank of Canada
- Government of Canada 5-year and 10-year Benchmark Bonds: Statistics Canada, Table 10-10-0139-01 (formerly CANSIM 176-0048), Bank of Canada, money market and other interest rates
- Discounted (paid) 3-year, 5-year, and 10-year mortgage rates: Ratehub.ca, online: https://www.ratehub.ca/5-year-fixed-mortgage-rate-history
- Bank of Canada's policy rate: Statistics Canada, Table 10-10-0139-01 (formerly CANSIM 176-0048), Bank of Canada, money market and other interest rates, target rate
- Inflation rate: Canada Consumer Price Index, Statistics Canada, obtained from YChart
- Maximum loan-to-value (LTV) ratio and amortization period: Obtained from the BIS's discussion on the impact of housing finance macroprudential tools in Canada. See reference [7].
- Housing Price Index: Statistics Canada, Table 18-10-0205-01 (Formerly CANSIM 327-0056), New housing price index, monthly, house only
- Energy Price Index: Monthly Bank of Canada commodity price index - Energy, online: https://www.bankofcanada.ca/rates/price-indexes/bcpi/
- GDP growth rate: Statistics Canada. Table 36-10-0434-01 (Formerly CANSIM 379-0031), Gross domestic product (GDP) at basic prices, annual growth rate, by industry, monthly.
- Unemployment rate: Statistics Canada, Table 14-10-0287-01 (formerly CANSIM 282-0087), Labour Force Characteristics, monthly, Canada, both sexes, 15 years and over, seasonally adjusted

I also calculate the following variables:

- Mortgage spreads of corresponding maturity: $Spread_t = Discounted_t - Benchmark_t$
- Out-of-sample forecasts of mortgage rates: $Discounted_t = Spread_t + Benchmark_t$
- Real interest rate: $Real_t = Policy_t - Inflation_t$

B Appendix B: Additional statistical evidence

Recall from 5.4 the two regression models used to predict 5-year mortgage rates at $t + 60$:

$$i_t^{5y} = \beta_0 + \beta_1 \; i_{t-60}^{5y} + \epsilon_t \tag{14}$$

$$i_t^{5y} = \beta_0 + \beta_1 \; i_{t-60}^{5y} + \beta_2 \; i_{t-60}^{10y} + \epsilon_t \tag{15}$$

Table B.1 and B.2 present full-sample regression estimates and the RMSE comparison of recursive forecasts obtained from these two models, respectively.

Table B.1: Regression Estimates, by Model

	Model (14) Paid 5-year mortgage rate	Model (15) Paid 5-year mortgage rate
Paid 5-year mortgage rate $_{t-60}$	0.163***	0.454***
	(4.97)	(7.00)
Paid 10-year mortgage rate $_{t-60}$		-0.396***
		(-5.02)
Constant	2.101***	2.900***
	(15.21)	(14.42)
N	94	94

Note: t statistics in parentheses; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$; Full-sample estimates

Table B.2: Recursive Forecasts by Model, RMSE Comparison

	RMSE	
Variable	Model (14)	Model (15)
5-Year Mortgage Rate	0.00339	0.00308
Total Interest Payments, 5-Year Mortgage	4086.95	3722.42

Note: RMSE calculated based on recursive estimates.

Table B.3: Augmented Dickey-Fuller Tests, Level and First-Difference

	Test Statistic		Interpolated Dickey-Fuller Critical Values		
Variable	Level	First Diff.	1%	5%	10%
5-Year Spread	-3.013	-6.210	-3.494	-2.887	-2.577
10-Year Spread	-3.923	-7.324	-3.494	-2.887	-2.577
Benchmark Spread	-1.868	-5.437	-4.024	-3.443	-3.143
5-Year Benchmark Bond	-1.674	-5.769	-4.024	-3.443	-3.143
Real Interest rate	-2.471	-6.470	-4.024	-3.443	-3.143

Note: Includes 4 lags and a trend, except for 5-year and 10-year spreads

Table B.4: Phillips-Perron Tests, Level and First-Difference

	Test Statistic		Interpolated Dickey-Fuller Critical Values		
Variable	Level	First Diff.	1%	5%	10%
5-Year Spread	-4.045	-18.072	-3.492	-2.886	-2.576
10-Year Spread	-4.544	-14.454	-3.492	-2.886	-2.576
Benchmark Spread	-1.686	-10.125	-4.022	-3.443	-3.143
5-Year Benchmark Bond	-1.705	-9.441	-4.022	-3.443	-3.143
Real Interest rate	-2.230	-12.496	-4.022	-3.443	-3.143

Note: Includes 4 Newey-West lags and a trend, except for 5-year and 10-year spreads

Table B.5: KPSS Tests, Level

	Number of Lags				
Variable	0	1	2	3	4
5-Year Spread	0.417	0.237	0.168	0.134	0.113
10-Year Spread	0.491	0.276	0.201	0.162	0.140
Benchmark Spread	2.140	1.090	0.744	0.570	0.465
5-Year Benchmark Bond	2.240	1.150	0.782	0.600	0.492
Real Interest rate	1.860	0.970	0.670	0.520	0.429

Note: H_0 is trend stationary. Critical values: 10%: 0.119, 5%: 0.146, 1%: 0.216.

Table B.6: KPSS Tests, First-Difference

	Number of Lags				
Variable	0	1	2	3	4
5-Year Spread	0.0197	0.0314	0.0320	0.0342	0.0361
10-Year Spread	0.0188	0.0223	0.0252	0.0254	0.0270
Benchmark Spread	0.0457	0.0388	0.370	0.0374	0.0383
5-Year Benchmark Bond	0.0427	0.0343	0.0322	0.0312	0.0312
Real Interest rate	0.0332	0.0338	0.0348	0.0349	0.0364

Note: H_0 is trend stationary. Critical values: 10%: 0.119, 5%: 0.146, 1%: 0.216.

Table B.7: Lag Order Selection – Information Criteria

Lag	FPE	AIC	HQIC	SBIC
(1) Benchmark Model				
0	0.00002	3.51789	3.64020	3.81895
1	3.1e-08	-3.10864	-2.78247	-2.3058*
2	2.2e-08	-3.45695	-2.92693*	-2.15234
3	2.0e-08*	-3.52318*	-2.78931	-1.71680
4	2.5e-08	-3.34045	-2.40272	-1.03229
(2) Selected Model				
0	5.9e-06	2.14361	2.34746	2.64538
1	2.5e-08	-3.30652	-2.89882	-2.30298*
2	2.0e-08	-3.56018	-2.94862*	-2.05487
3	1.8e-08*	-3.64512*	-2.8297	-1.63803
4	2.1e-08	-3.48753	-2.46825	-.978663

Note: 150 observations, sample 2006:9–2019:2

Exog. variables: (1) LTV, Amort.

Exog. variables: (2) LTV, Amort, CCI, CPIE.

Table B.8: Modified Portmanteau Test for Residual Autocorrelation

Model	Multivariate Ljung-Box Statistic	$Prob > \chi^2$
(1) Benchmark Model	1045.0312	0.1570
(2) Selected Model	1043.8304	0.1633

Note: Test conducted on VAR residuals, 5 variables and 40 lags.

Exog. variables: (1) LTV, Amort., (2) LTV, Amort., CCI, CPIE.

Recall from section 7.1 the VAR-X(p, s) representation of the benchmark model:

$$Y_t = a_0 + \sum_{i=1}^{p} A_j Y_{t-j} + \sum_{j=1}^{s} B_j X_{t-j} + \epsilon_t, \tag{25}$$

where the vector of endogenous variables is $Y_t = \{bench_{\ 5yr,t},\ spread_{\ 5y,t},\ spread_{\ 10y,t},\ spread_{\ bench5-10,t},\ real_t\}$ and the vector of exogenous variables is $X_t = \{LTV_t,\ Amort_t\}$.

Table B.9 presents forecasting performance tests based on RMSE with additional endogenous and exogenous variables.

Table B.9: Forecasting Performance Tests - RMSE Comparison

Model	**Forecast Horizon (months)**	**RMSE (February 2009)**	
		5-year rate	**10-year rate**
(1) Benchmark	60	0.27830	0.32491
	120	0.32491	0.29312
1. Alternative models – additional endogenous variables			
(1.1) Benchmark + 3-year spread	60	0.28194	0.33487
	120	0.31891	0.29328
(1.2) Benchmark + inflation	60	0.31521	0.32773
	120	0.32842	0.31135
(1.3) Benchmark + GDP growth	60	0.29519	0.33474
	120	0.33488	0.30242
(1.4) Benchmark + policy rate	60	0.31521	0.32773
	120	0.32842	0.31135
(1.5) Benchmark + unemployment rate	60	0.28636	0.31629
	120	0.35240	0.30171
2. Alternative models – additional exogenous variables			
(2.1) Benchmark + exo (HPI)	60	0.27451	0.33168
	120	0.33820	0 .30607
(2.2) Benchmark + exo (CCI)	60	0.32457	0.31751
	120	0.30582	0.29396
(2.3) Benchmark + exo (CPIE)	60	0.30642	0.33085
	120	0.28624	0.29800
(2.4) Benchmark + exo (HPI + CCI)	60	0.31172	0.31363
	120	0.29318	0.28639
(2.5) Benchmark + exo (HPI + CPIE)	60	0.29484	0.32300
	120	0.31036	0.30251
(2.6) Benchmark + exo (CCI + CPIE)	60	0.31773	0.32422
	120	0.27569	0.28531

Note: The model selected for the forecasting exercise is Model (2.6). The forecasting horizon is 120 months.

Figure B.1: 5-Year, 10-Year and Benchmark Bonds Spreads, January 2009–February 2019

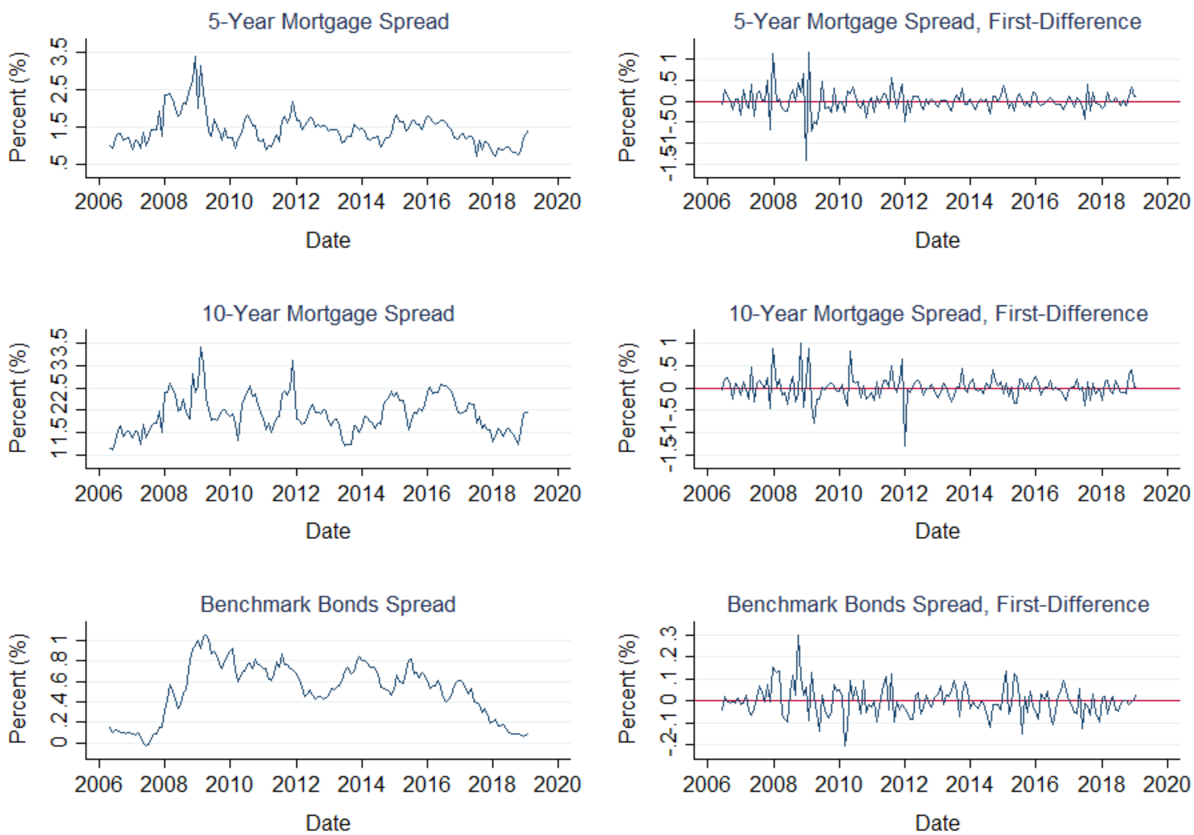

Figure B.2: Government of Canada Benchmark Bond Yields and Real Interest Rate, January 2009–February 2019

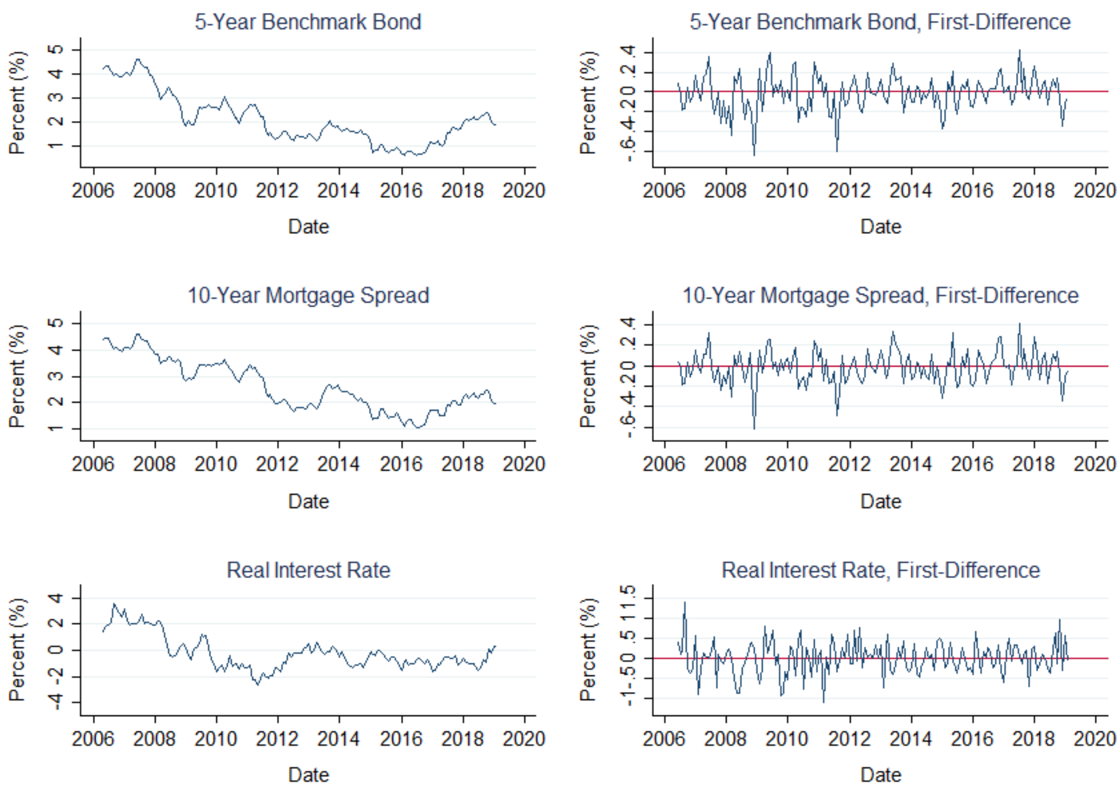

Figure B.3: 5-Year and 10-Year Mortgage Rate Dynamics, January 2009–February 2019

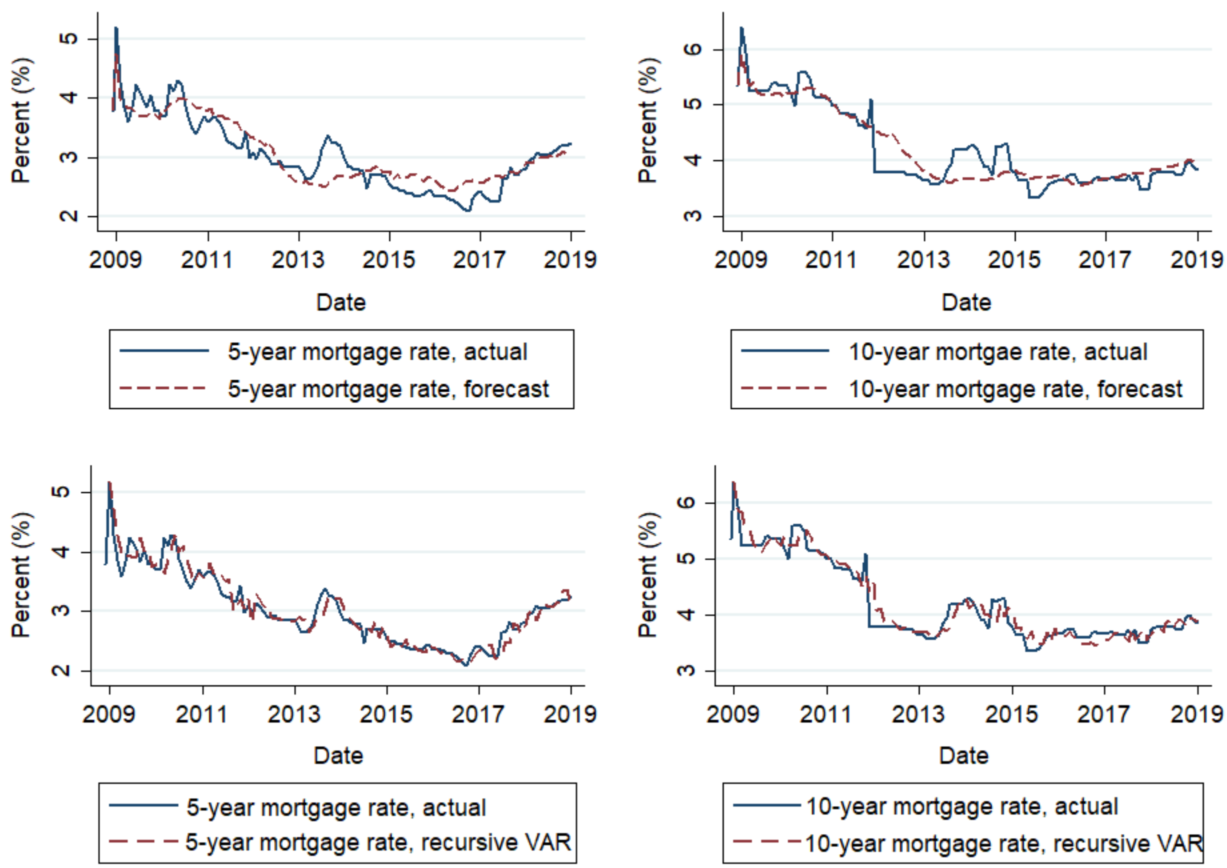

Figure B.4: Spread Dynamics, January 2009–February 2019

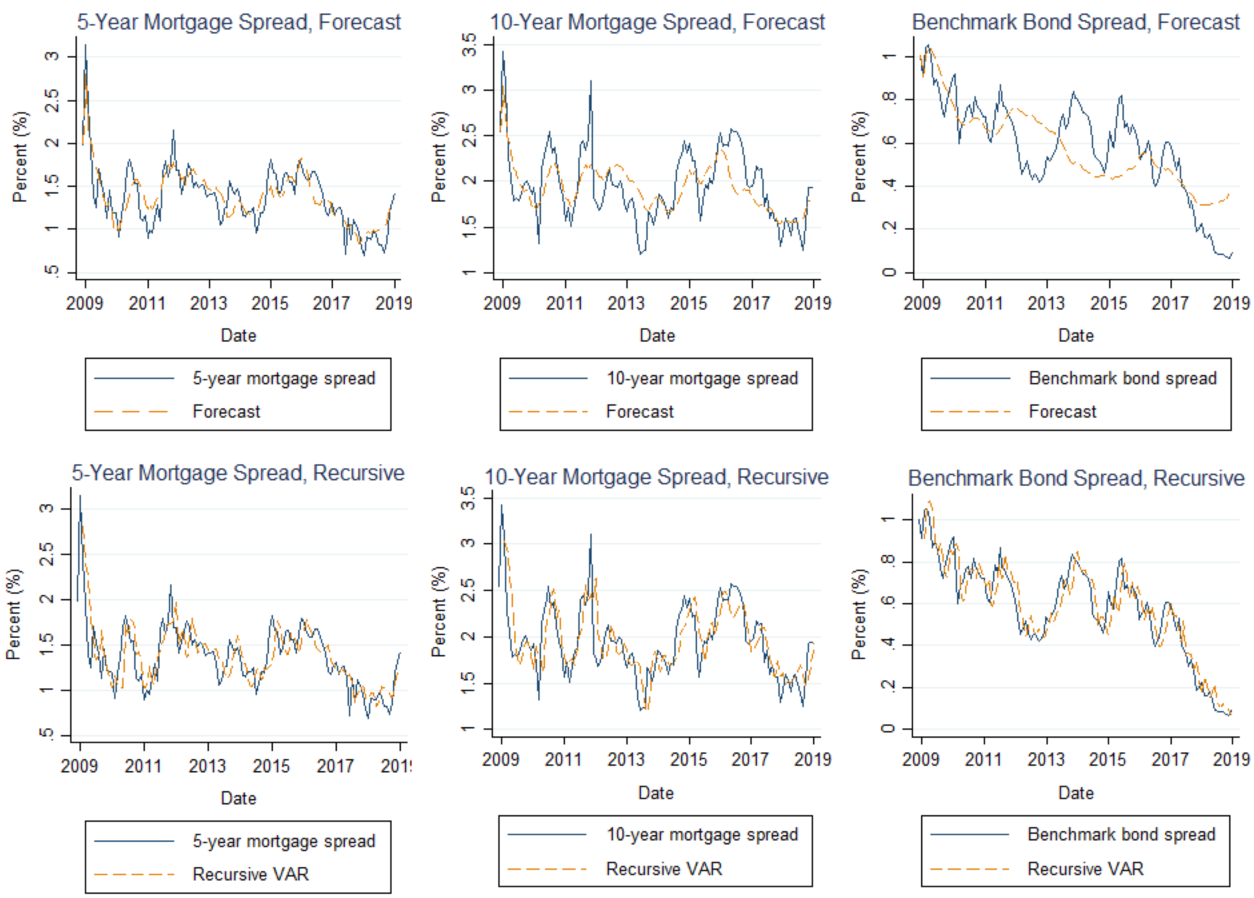

Figure B.5: Government of Canada 5-Year and 10-Year Benchmark Bond Yield Dynamics, January 2009–February 2019