CO_2 Equivalent Market Versus a Multi-Pollutant Market

by

Alexandra Hartley

An essay submitted to the Department of Economics in partial
fulfillment of the requirements for the degree of Master of Arts

Queen's University
Kingston, Ontario, Canada
June 2017

Acknowledgments

To Professor Devon Garvie, my sincerest thanks and appreciation for your guidance and support throughout the process of writing this paper. You have been a mentor and friend and it was your interest in my education at Queen's that strongly influenced my enthusiasm for environmental economics.

Table of Contents

Acknowledgements **2**

Section 1: Introduction **4**

Section 2: Greenhouse Gases and Their Properties **7**

Section 3: Controlling Greenhouse Gases **14**

Section 4: Greenhouse Gas Metrics **27**

Section 5: Related Literature on GHG Metrics and Trading Systems **33**

Section 6: Greenhouse Gas Trading Systems **37**

Section 7: Conclusion **55**

References **58**

1. Introduction

Since the United Nations Climate Change Conference in Paris during November of 2015, greenhouse gas regulation has been a popular topic of political conversation across the globe. Governments from around the world signed the Paris Agreement with the commitment to limit the increase of Earth's temperature to 1.5 degrees above pre-industrial levels (European Commission, 2015). Fulfilling this commitment will require leaders to select and implement a policy to significantly reduce greenhouse gases (GHG).

A cap and trade system is one of the most common policies used by policymakers today to control emissions. It involves creating a market for polluting entities to buy the right to pollute in the form of "allowances". In this system, the policymaker chooses a limit on emissions (also called an emissions cap or target), and then creates a number of allowances equal to that cap. Polluters are only permitted to produce GHG emissions equal to the amount of allowances they own. Allowances are either distributed through auctions where firms must bid for the number of allowances they desire, or through free distribution.

The main design feature of a cap and trade system that policymakers must decide upon is the type of carbon market. The first type of carbon market is called a Multi-pollutant Market, and is comprised of separate markets and allowances for each GHG. Thus, there will exist separate emissions targets and allowances for each type of gas. In each gas' market, one allowance is equal to one tonne of emissions. Trading between each market can occur by implementing a trading ratio between each type of allowance. The trading ratio compares the impact that each gas has on the planet, and indicates the optimal ratio at which emissions from different GHGs can be exchanged. The second market type is called a CO_2 Equivalent Market, and is the most commonly used market for cap and trade systems today. It involves just one allowance market for all GHGs, and allowances are measured in units of CO_2 equivalents (CO_2 eq). A CO_2 eq is equal to the impact that one tonne of CO_2 emissions will have on the planet. In this market, a type of metric is used to quantify the impact that each GHG has on the planet's well being, relative to that of CO_2. This metric compares the harmfulness of each type of GHG. Furthermore, this metric is used to convert the emissions of a GHG into their CO_2 eq number, which specifies how many tonnes of CO_2 are equivalent to one ton of the GHG. As a result, the number of allowances required to cover one tonne of emissions differs for each type of gas.

Scientists and economists have created a plethora of possible metrics to use in CO_2 equivalent markets and to compare the destructiveness of greenhouse gases. There is much controversy over which metric should be used in practice, given that there are unique benefits and pitfalls to each. The four most common metrics are: Global Warming Potential (GWP), Global Temperature Potential (GTP), Global Cost Potential (GCP), and Global Damage Potential (GDP). The GWP is the most used metric around the world to compare GHGs within cap-and-trade systems.

When selecting and designing GHG regulation, governments are striving to reduce emissions to a level at which the harmful effects of global warming are curbed, while minimizing overall abatement costs. Significantly reducing GHG emissions is very costly to polluters and society as a whole. It requires technological and behavioural changes by producers and consumers in industries such as agriculture, transportation, energy, mining, manufacturing, etc. Therefore, a policy's cost-efficiency level is a key indicator of its success and its value to society.

There is a great deal of literature surrounding the Multi-Pollutant Market. This literature develops theoretical models for this type of market, and concludes that it is a cost-effective form of GHG regulation. However, because of its complexity, the Multi-Pollutant market is rarely used in reality. On the other hand, the CO_2 Equivalent Market is very commonly used today, but there is an absence of literature that sets up a model for this type of market. Most of the literature surrounding CO_2 Equivalent Markets solely analyzes the different types of metrics that may be used to compare GHGs. This paper will set up a theoretical model for a GWP CO_2 Equivalent Market, and analyze its cost-efficiency in comparison to a Multi-Pollutant Market. I have chosen the GWP metric for this market, because it is the most widely used metric for cap and trade regulations in the world today.

With the goal of setting up and analyzing a GWP CO_2 Equivalent Market, this paper is set up as follows. Section 2 will provide background information on the greenhouse effect and key characteristics of GHGs. Section 3 illustrates the two economically efficient methods for selecting an emissions target. Section 4 introduces and compares the four main GHG metrics. The relevant literature on GHGs metrics and markets is highlighted in section 5. Section 6 displays the Multi-Pollutant market model, and sets up the model for CO_2 Equivalent Market. In addition, Section 6 analyzes and compares the cost-effectiveness of the two markets. Lastly, section 7 concludes and summarizes the CO_2 Equivalent Market and this paper's findings.

2. Greenhouse Gases and Their Properties

2.1 The Greenhouse Effect

Global warming can be defined as the increase in the annual global average temperature, and is caused by the accumulation of greenhouse gases in the atmosphere. The sun radiates energy through short wavelengths, which is mostly comprised of ultraviolet and visible light radiation. When the sun's radiation reaches the Earth, 31% of it is reflected back into space by both the atmosphere and the Earth's surface, 20% is absorbed by the atmosphere, and the remaining 49% is absorbed by the oceans and land (Government of Canada, 2015, para 1-4). As the Earth absorbs the sun's energy, the Earth radiates approximately that same amount of energy back into the atmosphere. The Earth is a great deal cooler than the Sun, so the Earth radiates out longer wavelengths. Greenhouse gases in the atmosphere absorb some of the outgoing radiation, and the rest escapes into space (Le Treut et al, 2007, p. 97). This is called the "The Greenhouse Effect", and it is critical to keep the Earth at a liveable temperature. However, as GHG levels are growing dramatically, the Earth's temperature is also continuing to rise to dangerous levels.

2.2 Types of Greenhouse Gases

Greenhouse gases exist in the atmosphere due to both natural occurrences, and human activities. The three most prevalent anthropogenic GHGs are carbon dioxide (CO_2), methane (CH_4), and nitrous oxide (N_2O). Although these GHGs do occur naturally, humans also emit a large proportion of these gases. In 2015, CO_2 accounted for 79% of Canada's total emissions, and is mainly emitted from the combustion of fossil fuels. (Government of Canada, 2017, para 18)

Emissions of CH_4 made up 14% of Canada's total GHGs released in 2015, and N_2O accounted for 5.4%. These two gases are emitted primarily from agriculture processes, soil management, and landfills (Government of Canada, 2017, para 18). The remaining GHGs are called Fluorinated Gas (F-Gas), which include Hydrofluorocarbons (HFCs), Perfluorcarbons (PFCs), Cholrofluorocarbons (CFCs), and Sulphur Hexafluoride (SF_6). These gases are only emitted through human activities, and exist only in small quantities (Solomon et al, 2007, p.23). In this paper I will restrict my analysis to CO_2, CH_4, and N_2O, since they account for 98.4% of the greenhouse gases emitted in Canada.

2.3 Stock Accumulation

Greenhouse gases are uniformly mixed pollutants (UMP), which means they do not only impact the area immediately surrounding the emission source. These gases disperse evenly throughout the atmosphere, and have roughly the same atmospheric concentrations across the world (United States Environmental Protection Agency, 2017, "Overview of Greenhouse Gases", para 7). This means the increase in Earth's temperature from GHGs is independent of the location of emissions.

Greenhouse gases are also considered to be "stock pollutants" because emissions remain in the atmosphere for a long period of time. GHGs are slowly removed from the atmosphere due to a variety of physical and chemical process (Solomon et al, 2007, p. 23). The rate at which a GHG dissipates from the atmosphere is known as a "decay rate". Each GHG gas has its own unique decay rate, and thus remains in the atmosphere for a different period of time (Le Treut et al, 2007, p.77). At any given time, a GHG has a “stock level” or “atmospheric accumulation level” that signifies the amount of the GHG that is present in the atmosphere. The stock level at a given time is equal to the accumulation of emissions in the past, minus the amount of the gas that has decayed over time. The following model demonstrates how the stock level of a GHG may be calculated[1].

Suppose there are n number of GHGs and one GHG is denoted by g, where g=1,….,n. The decay rate for GHG g is denoted by α_g, where $0 \leq \alpha_g \leq 1$. Time is represented by t, and the level of gas g emissions released at time t is shown by E_{gt}. Let S_{gt} represent the stock level for gas g at time period t. The change in stock level between time t+1 and t is equal to the emissions released at time t+1 minus the amount of the time t stock level that has decayed:

$$S_{gt+1} - S_{gt} = E_{gt+1} - \alpha_g S_{gt} \tag{1}$$

By using recursive substitution, the stock level in equation (1) can be represented solely in terms of past and current emissions, as well as the decay rate. This can be shown in equation (2):

$$S_{gt} = \sum_{s=1}^{t} \left(1 - \alpha_g\right)^{t-s} E_{gs} \qquad for\ \ s < t \tag{2}$$

If the decay rate α_g equals zero, the GHG remains in the atmosphere forever, which means that the stock level equals the sum of all past emissions. A decay rate equal to one signifies that the GHG leaves the atmosphere immediately after it is emitted. Decay rates between zero and one indicate that the GHG stock decreases

---

[1] This model is from Devon Garvie’s “Stock Pollution Control Model” PowerPoint for course Econ 443 at Queen’s University in 2014 & “Global Warming: Efficient Policies in the Case of Multiple Pollutants” by Peter Michaelis in 1992

over time. The smaller the decay rate, the more emissions from any arbitrary period s will remain in the atmosphere in period t.

Equation (3) shows the derivative of the S_{gt} equation with respect to E_{gs}, which shows how one unit of emissions from period s impacts the stock level in period t:

$$\frac{\partial S_{gt}}{\partial E_{gs}} = \left(1 - \alpha_g\right)^{t-s} \tag{3}$$

The derivative in equation (3) calculates the amount of one unit of a GHG g emitted in period s that will remain in period t. This is known as the "emission pulse", and is represented by $\left(1 - \alpha_g\right)^{t-s}$. The emission pulse is determined by the decay rate, as well the length of time between the initial date of the emissions (period s) and the emission pulse measuring date (period t).

Table 1: Emission Pulses for the three main GHGs

Emission Pulse $\left(1 - \alpha_g\right)^t$.			
Year (t)	**CH_4**	**CO_2**	**N_2O**
0	1	1	1
1	0.92	0.88	0.99
10	0.43	0.43	0.92
50	0.02	0.36	0.65
100	0.00024	0.33	0.42
200	0.00000006	0.30	0.17

Note: Adapted from "Stock Pollution Control Model" PowerPoint by D. Garvie, 2014

Table 1 presents the emission pulses of the three main GHGs. Year 0 is used as time period s, which makes the emission pulse equal to $\left(1 - \alpha_g\right)^t$. Thus, the emission pulses in table 1 demonstrate the amount of GHG that remains in period t from one unit of emission released at year 0 (Garvie, Stock Pollution Control Model, 2014). Table 1 illustrates that after 200 years, only 0.00000006% of the one unit of CH_4 discharged in year 0 remains, while 17% of N_2O and 30% of CO_2 is left. Therefore, CH_4 decays very quickly, where as CO_2 decays at a much slower pace.

A GHG's decay rate and emission pulse can be used to calculate the length of time a GHG remains in the atmosphere. The emission pulse is an exponential decay function, and has an asymptote at zero. This makes it impossible to calculate the point at which an emission pulse is equal to zero and the gas is completely removed from the atmosphere. Instead, scientists calculate the number of years it takes the emission pulse from one unit of a GHG to reach 0.5, which indicates the number of years it takes for half of a tonne of gas to decay. Scientists have named this calculation the "lifetime" of a GHG. Table 2 displays the decay rates and lifetimes for CO_2, N_2O, and CH_4.

Table 2: The decay rate and lifetime of the top three GHGs

Greenhouse Gas	**Decay Rate (α)**	**Lifetime (in years)**
CH_4	0.07996	12.4
CO_2	Varying	5-200
N_2O	0.0087	121

Note: Adapted from "Anthropogenic and Natural Radiative Forcing" (p. 731) in the IPCC's Climate Change 2013: The Physical Science Basis by Myhre et al, NY, Cambridge University Press

Unlike most GHGs, CO_2 does not decay exponentially. Carbon dioxide is constantly travelling through the atmosphere, land, and oceans, and removing carbon from the atmosphere requires many complex processes, each with its own time frame. This phenomenon is known as the carbon cycle (Le Treut et al, 2007, p.24). As a result, CO_2 does not have one specific lifetime, and instead, can remain in the atmosphere for a range of possible years.

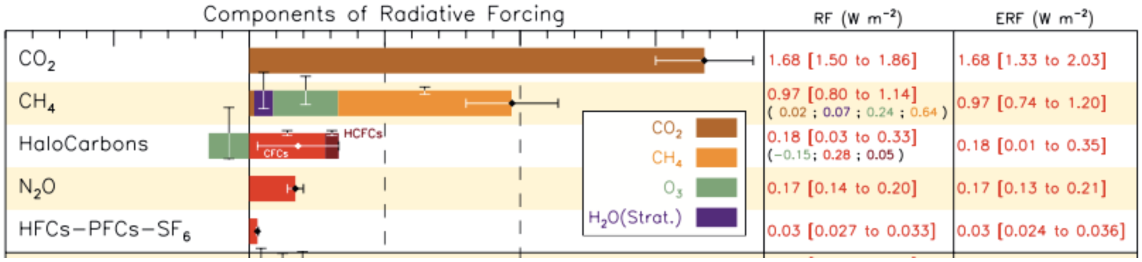
The length of time that a GHG remains in the atmosphere indicates the amount of time that it will contribute to climate change. Thus, decay rates are a very important feature to use when comparing gases. However, GHGs also differ in their ability to cause warming within the atmosphere. This will be discussed in the next subsection.

2.4 Radiative Forcing

Greenhouse gases cause an increase in the global temperature by absorbing the sun's outgoing energy. In order to compare how each GHG individually contribute to global warming, scientists study how they affect energy in the atmosphere, through a metric called radiative forcing. Radiative forcing (RF) is defined as the "net change in the energy balance of the Earth system due to same imposed perpetration" (Myhre et al, 2013, p. 664). This metric is measured "in watts per square meter (W/m^2) averaged over a particular period of time, and quantifies the energy imbalance that occurs when the imposed change takes place" (p. 664). A positive RF signifies warming of the global temperature and a negative RF demonstrates cooling of Earth's climate.

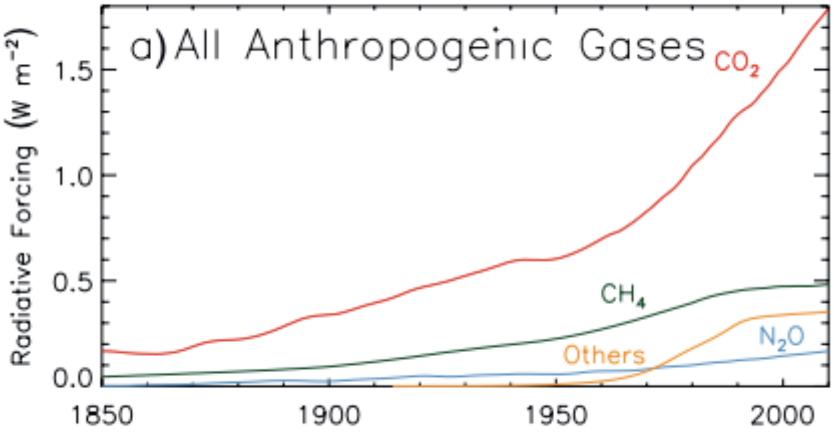
Scientists working for the International Panel on Climate Change (IPCC) have calculated the RF for the primary GHGs between the year 1750 and 2011. These RF calculations measure how both the chemical composition and changes in atmospheric concentration for each gas has impacted the global average temperature. The total RF for well-mixed GHGs between 1750 and 2011 was 3.00 W/m^2 (IPCC, 2013, p. 13). Figure 1 demonstrates the RF from 1750 to 2011 of each well-mixed GHG. CO_2 is the largest contributor with an RF of 1.68 W/m^2, and CH_4 is second largest with an RF of 0.97 W/m^{-2}. Together CO_2 and CH_4 make up 88% of the total RF for GHGs between 1750 and 2011.

Figure 1: Radiative Forcing of Well Mixed GHGs measured from 1750 to 2011



Note: Reprinted from "Technical Summary" (p. 57) in IPCC's Climate Change 2013: The Physical Science Basis by Stocker et al, NY, Cambridge University Press

Figure 2: Radiative Forcing of Well Mixed GHGs measured from 1850 to 2011



Note: Reprinted from "Anthropogenic and Natural Radiative Forcing" (p. 677) in IPCC's Climate Change 2013: The Physical Science Basis by Myhre et al, NY, Cambridge University Press

The stock level of a GHG is positively correlated with its RF. Thus, as the amount of GHGs found in the atmosphere has grown since the Industrial Revolution, their RFs has increased, meaning they are more capable of causing warming. Figure 2 illustrates the increase in RF of the main GHGs from the year 1850 to 2011. Since 1750, the atmospheric concentration of CO_2 has increased by 40%, CH_4 has increased by 150%, and N_2O has increased by 20% (IPCC, 2013, p.11). This increase in GHG levels has caused the RF of GHGs to rise between from the industrial revolution era to today.

Decay rates and radiative efficiency dictate how GHGs contribute to global warming. Thus, understanding and utilizing these properties is necessary in creating a policy to regulate GHG emissions. The next section will illustrate how these two properties are used to effectively control GHGs.

3. Controlling Greenhouse Gases

3.1 Setting an Emissions Target

When implementing a cap and trade system, the government must choose an emissions target or cap. The government then creates a number of allowances equal to that target. There are two economically efficient methods to choose a GHG emission target: cost-benefit analysis or cost-effectiveness analysis. Both of these methods are presented below in this section.

3.2 Cost-Benefit Analysis

Cost-benefit analysis is considered the first best economical approach to controlling emissions, because it involves in depth analysis of the damages and costs associated with GHG emissions. This analysis defines the optimal emissions level as the point where net social costs are minimized. The net social costs are equal to the sum of abatement costs and the damage associated with emitting a certain level of GHGs. Let's set up the model[2]. Suppose there exists a social planner who wants to stabilize GHG emissions by time T. There are a finite number of time periods, and

---

[2] This model is taken from Devon Garvie's "Social Cost of Carbon" PowerPoint for the course Econ 443 at Queen's University in 2014

time is denoted by t, where t=1,...,T. There are n possible GHGs, and g denotes the type of GHG, where g=1,…,n. Emissions for GHG g at time period t is represented by E_{gt}.

Over time, global warming will have detrimental impacts to the planet, such as a rise in sea level, poor air quality, loss of species, extreme weather events, etc. The damage from climate change is ultimately determined by how much atmospheric warming is caused by GHGs. Thus, the damage function of a GHG is based on its warming capability factor, W, which measures the increase in temperature caused by a GHG relative to that of CO_2. There are two main factors that contribute to a gas' warming capability factor – atmospheric stock level and radiative efficiency. Equation (2) defines the stock level of GHG g in time period t as $S_{gt} = \sum_{s=1}^{t} \left(1-\alpha_g\right)^{t-s} E_{gs}$. The radiative forcing for one tonne of gas g is denoted by β_g and is calculated from time 1 to time T:

$$\beta_g = \ RF_{gT} \tag{4}$$

Multiplying S_{gt} by β_g transforms the stock level of GHG g into the atmospheric warming capability factor of that gas. The total warming capability of all GHGs at time t is equal to a summation of stock levels weighted by the radiative forcing across all gases:

$$W_t = \sum_{g=1}^{n} \beta_g \, S_{gt} \tag{5}$$

Substituting equation (2) for S_{gt} into equation (5) yields a more detailed version of the total warming capability equation in period t:

$$W_t = \sum_{g=1}^{n} \sum_{s=1}^{t} \beta_g \left[\left(1 - \alpha_g\right)^{t-s} E_{gs} \right] \tag{6}$$

The damage function is associated with GHGs is a function of this warming capability factor. It is denoted by $D_t(W_t)$, and is concave up ($D'_t > 0 \; and \; D''_t > 0$). There are also costs to society associated with a certain level of emissions. Releasing low levels of GHGs is more expensive than emitting high levels because in order to reduce emissions polluters often have to invest in clean technology, change to less-polluting processes, etc. The emission cost for GHG g at time t is represented by $C_{gt}\left(E_{gt}\right)$, and is a concave up function ($C'_{gt} > 0 \; and \; C''_{gt} < 0$).

The social planner's problem is to choose a level of emissions that minimizes the net present value of social costs from reducing emissions, while placing a constraint on the total atmospheric warming capability. The net present value of social cost is the sum of the emissions costs and the damage costs, discounted at the rate ρ. The emissions target is being selected for time period i, where i=1,…,T. Thus, the net present value of social costs must be minimized with respect to the emissions of period i. In this minimization problem the social planner places a constraint on the warming capability factor, so that emissions do not exceed a dangerous level. The social planner's problem is:

$$\min_{E_{gi}} \sum_{g=1}^{n} \sum_{t=i}^{T} \frac{1}{(1+\rho)^{t-i}} \left[C_{gt}\left(E_{gt}\right) + \; D_t(W_t) \right]$$

$$subject\ to\ W_t = \sum_{g=1}^{n} \sum_{s=1}^{t} \beta_g \left[\left(1 - \alpha_g\right)^{t-s} E_{gs} \right] \tag{7}$$

The first condition with respect to E_{gi} can be arranged to equal:

$$-C'_{gi} = \beta_g \sum_{t=i}^{T} \left(\frac{1-\alpha_g}{1+\rho}\right)^{t-i} D'_{gt} \tag{8}$$

Equation (8) characterizes the optimal level of emissions in cost-benefit analysis. In this equation, $-C'_{gi}$ is the marginal abatement cost for GHG g at time i, which is the cost of reducing each consecutive unit of emissions. The negative sign in equation (8) turns the marginal costs of emissions (C'_{gi}) into the marginal cost of abatement ($-C'_{gi}$). This curve is concave down ($-C'_{gi} < 0$ and $-C''_{gi} > 0$).

In equation (8), the D'_{gt} represents the present value of the damage incurred as a result of one unit of emissions in period i across the current and all future periods. The expression $\left(\frac{1-\alpha_g}{1+\rho}\right)^{t-i}$ represents the rate that period t marginal damage is transformed into the period i present value marginal damage. In addition, the radiative forcing constant (β_g) accounts for the warming that this specific gas will create. The right–hand-side of equation (8) represents the present value of marginal damages from period i emissions. This is also known as the social cost of gas g at time i (SCG_i).

Overall, equation (8) states that the optimal emission level for GHG g in period i is the point at which the present value of the marginal cost of abatement equals the present value of all current and future marginal damage. It is considered the optimal emissions level because total costs and damages associated with gas g's emissions are minimized. Therefore, this level of emissions is the social cost minimization emissions target.

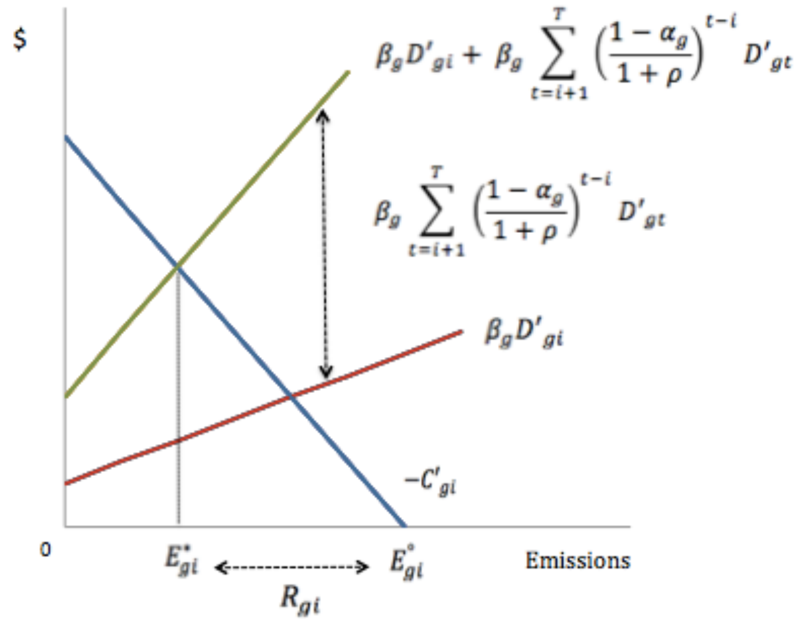
When time t equal time i, the term $\left(\frac{1-\alpha_g}{1+\rho}\right)^{t-i}$ is equal to 1, which means that equation (8) for optimal emissions can alternatively be written as:

$$-C'_{gi} = \beta_g D'_{gi} + \beta_g \sum_{t=i+1}^{T} \left(\frac{1-\alpha_g}{1+\rho}\right)^{t-i} D'_{gt} \tag{9}$$

In equation (9), the RHS is still equal to the SCG. However, it is divided up into two terms: the marginal damage for emissions at time i ($\beta_g D'_{gi}$) and the negative inter-temporal externality ($\beta_g \sum_{t=i+1}^{T} \left(\frac{1-\alpha_g}{1+\rho}\right)^{t-i} D'_{gt}$). The negative inter-temporal externality is the present value of marginal damage experienced in the future from period i emissions. This externality exists because GHGs are stock pollutants that contribute to global warming for a long time. Figure 3 on the next page illustrates the optimal solution for cost-benefit analysis. The graph shows that the negative inter-temporal externality creates a wedge between the total present value marginal damage curve and the marginal damage curve in period i.

In Figure 3, the blue curve represents the marginal cost of abatement, and the green curve represents the marginal damage curve. The point where these two curves intersect is the optimal level of emissions for GHG g in period i, which is denoted by E^*_{gi}. The unregulated emission level is symbolized by $\overset{\circ}{E}_{gi}$. There is no reduction of emissions at the unregulated level, so the marginal cost of abatement is zero. Reducing emissions from the unregulated level to the optimal level is equal to R_{gi} = $\overset{\circ}{E}_{gi} - E^*_{gi}$, and is graphically illustrated by the leftward movement from $\overset{\circ}{E}_{gi}$ to E^*_{gi}.

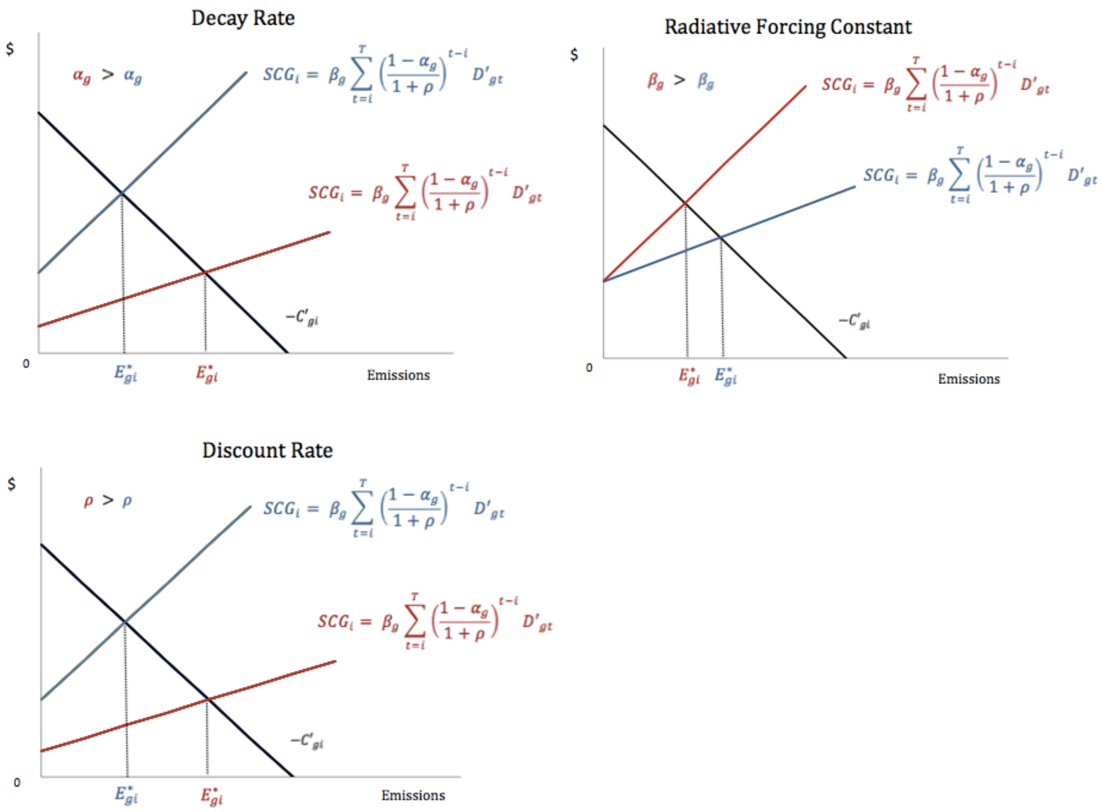
Figure 3: The optimal level of emissions for GHG g in period i in Cost-Benefit Analysis



Note: Adapted from “Social Cost of Carbon” PowerPoint by D. Garvie, 2014

The SCG_i is dependent on the following parameters: decay rate (α_g), discount rate (ρ), and the radiative forcing constant (β_g). This paragraph will explain how these parameters impact the SCG_i and the optimal level of emissions. Firstly, a high decay rate (α_g), indicates that the GHG leaves the atmosphere very quickly, while a low decay rate signifies that the GHG remains in the atmosphere for a long time. Figure 4 illustrates that reducing the decay rates causes the SCG_i curve to shift upward and rotate to the left, and as a result, the optimal level of emissions is lower. This occurs because emissions are now contributing to global warming for a longer period of time. Secondly, the discount value represents that value that society places on damage from GHGs today relative to the future. The higher the ρ value, the more heavily the future is discounted, and the less value people place on future damage from GHG g. A higher ρ results in a smaller $\left(\frac{1}{1+\rho}\right)^{t-i}$ term, which rotates the SCG curve to the left and moves it downward. This results in a high level of optimal of emissions. Lastly, a high value for β_g shows that gas g will cause a great deal of warming with each additional unit of emissions. Thus, with a large β_g, the SCG_i curve is very steep and the level of optimal emissions is very low. Figure 4 below illustrates how the SCG curve shifts with differing magnitudes of the three parameters.

Figure 4: How changes in parameter values affect the SCG and optimal emissions



Note: Adapted from “Social Cost of Carbon” PowerPoint by D. Garvie, 2014

3.3 Cost-Effective Analysis

Cost-effective analysis requires a social planner to choose a warming capability target for GHGs, and minimize costs associated with that level of emissions. Thus, it is a cost minimization problem with respect to a selected target warming level. Cost-effective analysis is the considered the second best approach to controlling GHGs, behind cost-benefit analysis. This is because it does not comprehensively analyze the damage associated with GHGs. Let's set up the model[3] The social planner aims to minimize emissions costs of gas g $\left(C_{gt}\left(E_{gt}\right)\right)$, which are discounted at the rate ρ. The warming capability cap is denoted by $(\bar{W})$, and must be greater than or equal to the warming capability of the stabilization in period T (W_T). The constraints placed on W_T are defined by equation (6). Therefore, the social planner's minimization problem is:

$$\min_{E_{gt}} PVC = \sum_{t=1}^{T}\sum_{g=1}^{n}\frac{C_{gt}\left(E_{gt}\right)}{(1+p)^{t-1}} \qquad Subject\ to \quad W_T \leq \bar{W} \tag{10}$$

$$W_T = \sum_{t=1}^{T}\sum_{g=1}^{n}\beta_g\, S_{gt}$$

$$S_{gt} = \sum_{s=1}^{t}\left(1-\alpha_g\right)^{t-s}E_{gs}$$

The Lagrangean for this minimization problem is:

$$L = \sum_{t=1}^{T}\sum_{g=1}^{n}\frac{C_{gt}(E_{gt})}{(1+\rho)^{t-1}} + \frac{\lambda_T}{(1+\rho)^{T-1}}\left[\sum_{t=1}^{T}\sum_{g=1}^{n}\beta_g\left(1-\alpha_g\right)^{T-t}E_{gt}\right] \tag{11}$$

---

[3] This model is taken from Devon Garvie's "Cost-effective stock pollution control" PowerPoint for the course Econ 443 at Queen's University in 2014 and Peter Michaelis' "Global Warming: Efficient Policies in the Case of Multiple Pollutants" from 1992

The first-order condition with respect to emission for GHG g in time period t (E_{gt}) is:

$$\frac{\partial L}{\partial E_{gi}} = \frac{C'_{gi}(E_{gi})}{(1+\rho)^{i-1}} + \frac{\lambda_T}{(1+\rho)^{T-1}}\left(\beta_g(1-\alpha_g)^{T-i}\right) = 0 \quad for\ i = 1, \ldots., T \qquad (12)$$

This can be arranged to equal:

$$-C'_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-i} \quad for\ i = 1, \ldots, T \qquad (13)$$

Equation (13) characterizes the optimal level of emissions for gas g in period i in the cost-effectiveness analysis. The λ_T variable denotes the shadow cost of one unit of gas g stock in period T. The emission pulse of a unit of period i emissions in period T is represented by $\left(1-\alpha_g\right)^{T-i}$. Additionally, the term $\left(\frac{1}{1+\rho}\right)^{T-i}$ is the discount factor that converts the shadow cost of period i emissions in period T to period i dollars (Garvie, 2014). Combining these three terms on the RHS of equation (13) creates a metric called the Shadow Cost of Emissions (SCE_{gi}). It is equal to the present value cost of one unit of period i emissions in the stabilization period T (Garvie, 2014). Equation (14) below demonstrates the equation for SCE_{gi}:

$$SCE_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-i} \qquad (14)$$

Overall, equation (13) illustrates that the optimal emissions level for GHG g in period i is the point at which the marginal cost of abatement equals to the SCE. This is the level of emissions that minimizes total costs to society with respect to the warming constraint. Thus, this optimal emissions level is considered the cost-effective target. However, setting an effective emission target requires choosing an emission level for each period between i and T, so that the marginal cost of abatement is equal to the SCE_g for every period (Garvie 2014). Therefore, choosing the optimal emissions target requires both static and dynamic efficiency.

i) Static Efficiency

Static efficiency requires that an optimal emissions level is selected for one given time period. In cost-effective analysis, gas g's optimal static emission level for period i is the point of intersection between $-C'_{gi}$ and the SCE_{gi}. Static efficiency also requires that the "efficient combination of abatement activities" amongst different types of greenhouse gases be selected for time period i (Michealis, 1992, p. 63). To illustrate the cost-effective combination of abatement activities, equation (15) shows the optimal emissions level conditions for example gases g and n at time i.

$$\overline{E_{g\imath}}\colon \quad -C'_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-i} \qquad \overline{E_{n\imath}}\colon \quad -C'_{ni} = \lambda_T \beta_n \left(\frac{1-\alpha_n}{1+\rho}\right)^{T-i} \tag{15}$$

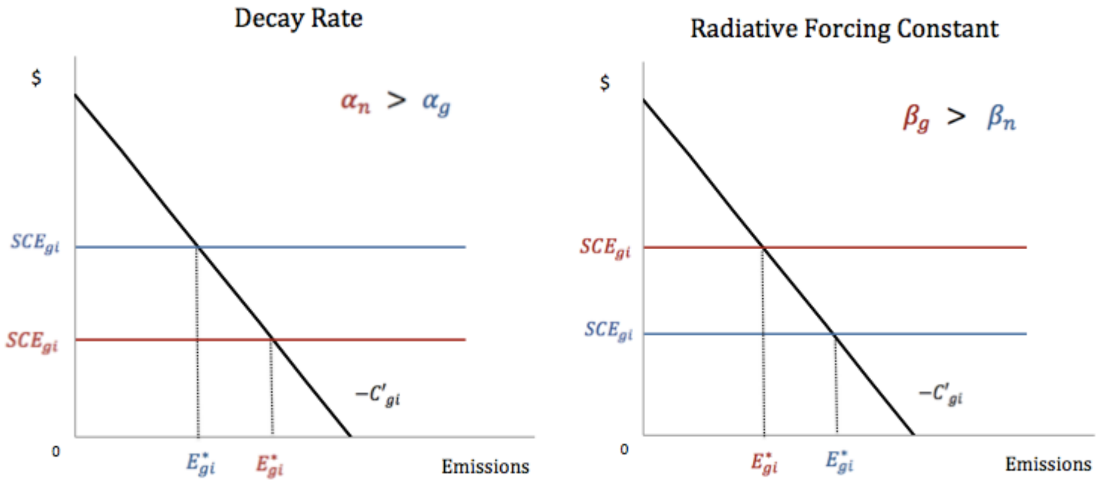
Combining these first order conditions yields the following ratio:

$$\{\overline{E_{g\imath}}, \overline{E_{n\imath}}\}\colon \ \frac{-C'_{gi}}{-C'_{ni}} = \frac{\beta_g}{\beta_n}\left(\frac{1-\alpha_g}{1-\alpha_n}\right)^{T-i} \tag{16}$$

Equation (16) indicates the cost-effective ratio of the optimal conditions between gas g and n. Suppose that the radiative forcing value for gas g is higher than gas n ($\beta_g > \beta_n$), and that the decay rate is lower $(\alpha_g < \alpha_n)$. These parameters indicate that gas g causes more atmospheric warming and lasts in the atmosphere for a longer period of time. As a result, cost-effective static efficiency requires gas g to have a larger "share of abatement activities" than gas n (Michaelis, 1992, p. 64).

Figure 5 shows how the SCE for time i changes with respect to the magnitude of the radiative forcing and decay rate.

Figure 5: How changes in parameter values affect the SCE curve in Static Efficiency



ii) Dynamic Efficiency

Dynamic efficiency in cost-effective analysis requires choosing an inter-temporal emission path so that the marginal cost of period T stock reduction over all periods equals the shadow cost of a unit of stock reduction in period T (Garvie, 2014). An inter-temporal emission path shows how the efficient level of emissions evolves over time. The inter-temporal emission path for gas g can be observed by comparing the optimal conditions for two time periods, i and k, where i < k:

$$\overline{E_{gi}}: \quad -C'_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-i} \qquad \overline{E_{gk}}: \quad -C'_{gk} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-k} \tag{17}$$

Combining these first order conditions yields the following:

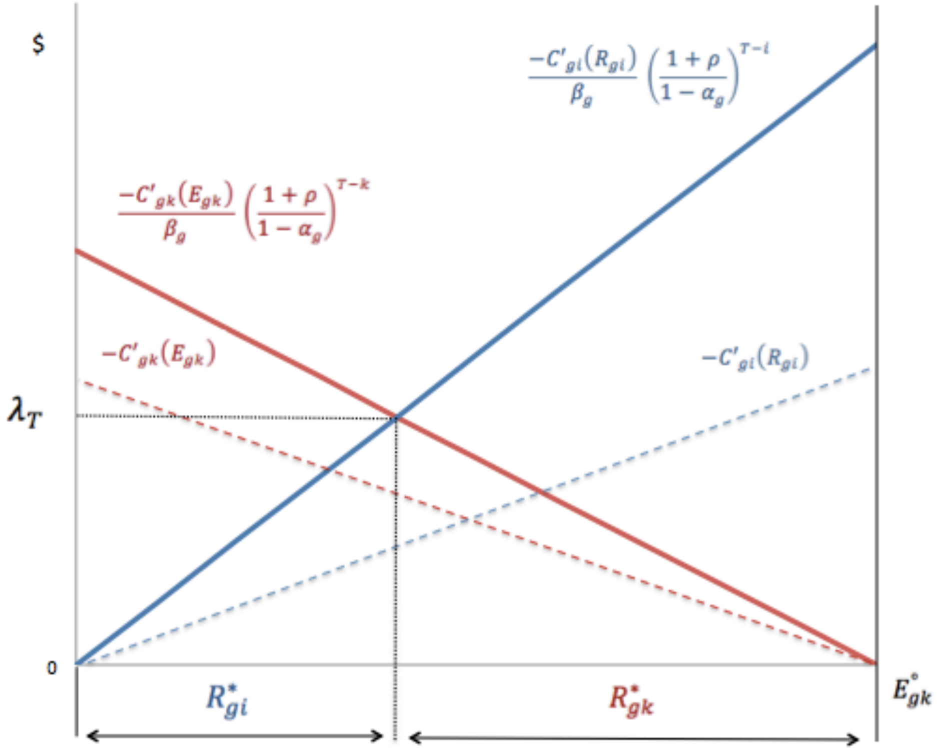
$$\{\overline{E_{gi}}, \overline{E_{gk}}\}: \quad \frac{-C'_{gk}}{-C'_{gi}} = \left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i} \qquad where\ i < k \quad and \quad i, k = 1, \dots, T \tag{18}$$

Equation (18) is the inter-temporal emission path for gas g from time i to time k, and thus shows how the cost-effective level of gas g evolves from period i to k. The LHS side of the equation is equal to the ratio of marginal abatement costs between time k and i, and is greater than 1. The term $\left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i}$ converts the time i's marginal cost of abatement to time k's marginal cost of abatement, and additionally transforms the period k marginal abatement cost into the period i dollars (Garvie, 2014). Since the RHS is greater than one, the marginal abatement cost increases from period i to period k at the rate $\left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i}$ (Michaelis, 1992, p. 64). As a result of the increasing marginal abatement costs over time $(-C'' > 0)$, the optimal emissions level decreases over time $\left(E_{gk} < E_{gi}\right)$.

In dynamic efficiency, there exists an optimal level of emissions, as well as an optimal level of emissions reduction for any given time period. Thus, there also exists an inter-temporal path for emission reduction in dynamic efficiency. Figure 7 on the next page illustrates the inter-temporal path for greenhouse gas reductions in cost-effective analysis from two adjacent time period i to k, where $i < k$.

The marginal cost of emissions reduction for period i is denoted by $-C'_{gi}(R_{gi})$, and is a concave up function. On figure 7, the dashed blue line represents this function. Multiplying $-C'_{gi}(R_{gi})$ by $\frac{1}{\beta_g}\left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i}$ transforms the marginal cost of emission reduction to the marginal cost of stock reduction in period i, which is represented by the solid blue line (Garvie, 2014). The dashed red curve represents the marginal cost of abatement for time k, and is concave down. The solid red curve is equal to $\frac{-C\prime_{gk}(E_{gk})}{\beta_g}\left(\frac{1+\rho}{1-\alpha_g}\right)^{T-k}$, which the marginal cost of stock reduction in time k.

Figure 7: Dynamic Cost-Efficiency for Emission Reduction



Note: Adapted from "Cost-Effective Pollution Control" PowerPoint by D. Garvie, 2014

Dynamic cost-efficiency requires that the marginal cost of emissions stock reduction is equal across all time periods (Garvie, 2014). Thus, the intersection point between the marginal cost of stock reduction for period i and k illustrates the optimal level of emission reduction for both time periods. Figure 7 illustrates that this intersection point occurs at the shadow cost of meeting the stabilization target, λ_T (Garvie, 2014).

For time i, the optimal reduction in emissions is measured on the x-axis from 0 to the intersection point, and is denoted by R^*_{gi}. The optimal emissions reduction for time k (R^*_{gk}), is measured from the unregulated the point $E°_{gk}$ to the intersection point. Figure 7 illustrates that the optimal reduction in emissions increases from time i to k, since $R^*_{gi} < R^*_{gk}$. As a result, the optimal emissions level decreases with time.

Cost-effective analysis is seen as a special case of cost-benefit analysis (Dueber et al, 2013, p.41). The two approaches are both seen as economically efficient methods for controlling emissions. Thus, they are the foundation upon which government should be designing their GHG regulatory policies.

4. Greenhouse Gas Metrics

Greenhouse gas metrics compare GHGs against CO_2, based on the impact they have on the atmosphere and society as a whole. These metrics are then used to compare different types of gases and compute CO_2 Equivalents. Economists and scientists have created a variety of metrics, but the four primary ones are: Global Warming Potential (GWP), Global Temperature Potential (GTP), Global Cost Potential (GCP), and Global Damage Potential (GDP). This subsection will present these metrics, and provide the benefits and consequences of using each metric.

4.1 Global Damage Potential

The GDP is derived from cost-benefit framework. It demonstrates the damage that one unit of emissions from a GHG will create, relative to the damage that one unit of CO_2 would generate (Tol et al., 2012, p. 4). Using the cost-benefit analysis from section three, we can derive the equation for GDP. The GDP is equal to the optimal condition from (8) for GHG g divided by the optimal condition for CO_2:

$$GDP_{gi} = \frac{-C'_{gi}}{-C'_{CO2i}} = \frac{\beta_g \sum_{t=i}^{T} \left(\frac{1-\alpha_g}{1+\rho}\right)^{t-i} D'_{gt}}{\beta_{CO2} \sum_{t=i}^{T} \left(\frac{1-\alpha_{CO2}}{1+\rho}\right)^{t-i} D'_{CO2t}} = \frac{\beta_g}{\beta_{CO2}} \sum_{t=i}^{T} \left(\frac{1-\alpha_g}{1-\alpha_{CO2}}\right)^{t-i} \frac{D'_{gt}}{D'_{CO2t}} \quad (19)$$

Equation (19) shows that the GDP for gas g is the ratio of damages caused by one unit of emissions from gas g to the damages from one unit of CO_2. This metric is time dependent, and so it will evolve as variable i increases. If gas g has a faster decay rate than CO_2 ($\alpha_g > \alpha_{CO2}$), the GDP metric will increase over time. This means that it is more costly for society to emit gas g closer to the stabilization period, than it is earlier on in the time horizon, relative to CO_2. This holds true because a high percentage of gas g emissions released in early years will decay by the stabilization period, while only a very small amount of gas g emissions will decay if they are released further in the future. As a result, gas g emissions in early years are less harmful to society, and thus are less expensive.

Cost-benefit analysis is considered the best method to determine to the inter-temporal path of emissions, since it uses extensive physical and economic analysis. Since the GDP is derived from cost-benefit analysis, it is known as the best metric to compare GHGs. As a result, the GDP “ensures that the trade-off between different forcing agents is efficient” (Deuber et al, p. 1).

Although the GDP is the best metric is theory, it is extremely difficult to use in reality. This is because it is very difficult to actually quantify the damages of emitting a GHG and of climate change (p.41). The consequences of climate change include rising sea level, mass flooding, loss of species, disease, etc. It is extremely difficult and controversial to value these future problems in terms of dollars. As a result, most policymakers are not able to use this metric in practice when designing GHG regulation.

4.2 Global Cost Potential

The GCP is derived from cost-effectiveness analysis, and it "is given by the ratio of two gases' [marginal costs of abatement] least cost emissions trajectory maintaining a prescribed climate target" (Deuber et al, 2013, p 41). Using the cost-effective analysis from the previous section, we can display the equation for GCP for gas g:

$$GCP_{gt} = \frac{-C'_{gt}}{-C'_{nt}} = \frac{\lambda_T \beta_g \left(\frac{1-\alpha_g}{1+p}\right)^{T-t}}{\lambda_T \beta_{CO2} \left(\frac{1-\alpha_{CO2}}{1+p}\right)^{T-t}} = \frac{\beta_g}{\beta_{CO2}} \left(\frac{1-\alpha_g}{1-\alpha_{CO2}}\right)^{T-t} \tag{20}$$

Equation (20) shows that the GCP for gas g is the ratio abatement costs caused by one unit of gas g relative to one unit of CO_2. Similar to the GDP, this metric also evolves over time. If gas g has a slower decay rate than CO_2 ($\alpha_g < \alpha_{CO2}$), the GDP metric will decrease over time. This would mean that it is cheaper for society to emit gas closer to the stabilization period, than it is earlier on in the time horizon, relative to CO_2.

The GCP is considered the second best metric behind the GDP, because although it includes economic and physical analysis, it does not account for the damage function. In reality, the GCP would be easier for regulators to use than the GDP, because they would only need to quantify the cost of emissions. However, quantifying the total cost of GHG emissions is still a very complex task, and as a result, policymakers seldom use this metric.

4.3 Global Temperature Potential

The GTP is solely physical metric, and does not involve economic analysis. The GTP is defined as the "change in global mean surface temperature at a chosen point in time in response to an emission pulse – relative to that of CO_2" (Myhre et al, 2013, p.712). It is "based on the temperature change for a selected year, t" (p. 712). The equation for GTP for GHG g at time t equals:

$$GTP_{gt} = \frac{\Delta T_{gt}}{\Delta T_{CO2t}} = \frac{AGTP(t)_g}{AGTP(t)_{CO2}} \tag{21}$$

AGTP stands for Absolute Global Temperature Potential and it shows the temperature change per unit of emissions (p. 712). This temperature change caused by a GHG is computed using radiative forcing, as well as accounting for the exchange of heat between the atmosphere and ocean (p. 712). Thus, GTP is a complex metric that uses extensive scientific analysis, and is considered a good physical metric to compare GHGs.

Table 3 below shows the GTP and AGTP values for the CO_2, CH_4, and N_2O. The 20-Year GTP value shows the atmospheric warming that would occur in year 20 caused by gas g relative to CO_2 if one tonne of each gas was emitted in year 0. CO_2 has a GTP of 1, because it is equal to $\frac{AGTP(t)_{CO2}}{AGTP(t)_{CO2}}$.

Table 3: GTP value for top three GHGs

GHG	AGTP 20-year (K kg^{-1})	**GTP 20-Year**	AGTP 100-year (K kg^{-1})	**GTP 100-Year**
CO_2	6.84e-16	**1**	5477e-16	**1**
CH_4	4.62e-14	**67**	2.34e-15	**4**
N_2O	1.89e-13	**277**	1.28e-13	**234**

Note: Adapted from "Anthropogenic and Natural Radiative Forcing" (p. 731) in IPCC's Climate Change 2013: The Physical Science Basis by Myhre et al, NY, Cambridge University Press

In comparison to the GDP or GCP, the GTP is an easier metric for regulators to use because the uncertainties associated with calculating the metric are restricted to the physical properties of greenhouse gases (Deuber, 2012, p. 41)

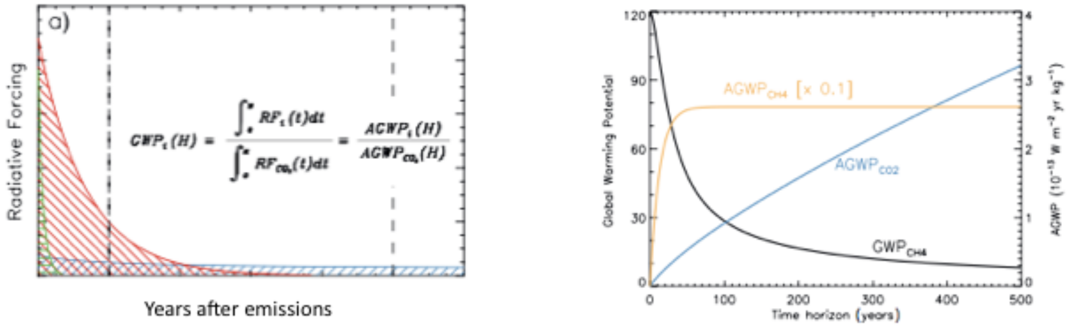
4.4 Global Warming Potential

Global Warming Potential is another purely physical metric. The GWP is the "time integrated RF due to a pulse emission of a given component relative to pulse emission of an equal mass of CO_2" (Myhre et al, 2013, p. 710). The GWP essentially shows how much energy one ton of a specific GHG will absorb, relative to the amount of energy that one ton of CO_2 can absorb over a specified time horizon (United States Environmental Protection Agency, 2017, "Understanding Global Warming Potential", para. 2). The equation for a GHG's GWP is:

$$GWP_i(H) = \frac{\int_o^H RF_i(t)dt}{\int_0^H RF_{CO_2}(t)dt} = \frac{AGWP_i(H)}{AGWP_{CO_2}(H)} = constant \tag{22}$$

In equation (22), i represents the type of GHG, t signifies time, and H represents the length of time between the time of emission and the time of measurement. AGWP stands for the Absolute Global Warming Potential, and represents the time integrated RF for a GHG over a given period. By integrating the RF in the GWP calculation, scientists are calculating the total warming that has been caused by the one ton of the GHG over a period of time, relative to that of CO_2. The GWP metric also accounts for the lifetime of a GHG. Once the one-ton of a GHG reaches its lifetime, it will no longer contribute to global warming, and thus, its GWP will stabilize.

Figure 8: GWP and AGWP



Note: Reprinted from "Anthropogenic and Natural Radiative Forcing" (p. 711) in IPCC's Climate Change 2013: The Physical Science Basis by Myhre et al, NY, Cambridge University Press

Note: Reprinted from "Anthropogenic and Natural Radiative Forcing" (p. 712) in IPCC's Climate Change 2013: The Physical Science Basis by Myhre et al, NY, Cambridge University Press

Figure 8 graphically illustrates the concept of GWP. On the left graph in figure 8, the vertical axis represents the RF, and the horizontal axis shows the number of years after release of one-ton of GHG. The blue curve represents the RF of one ton of CO_2 over time, and the shaded area underneath is equal to the time integrated RF for CO_2 ($\int_0^H RF_{CO_2}(t)dt$). The green curve is the RF of an example gas that has a lifetime of 1.5 years, and the red curve is the RF of an example gas with a lifetime of 13 years (Myhre et al, 2013, p. 711). Once again, the shaded areas under the curves represent the time integrated RFs. The GWP is the ratio of a GHG's time integrated RF to the CO_2 integrated RF. Therefore, the GWP for the red GHG is the shaded red area divided over the shaded blue area.

The right graph of Figure 8 illustrates the AGWP for one ton of emissions of both CH_4 and CO_2 over a time period of 500 years. The yellow curve shows the AGWP for CH_4 the blue curve is the AGWP for CO_2. The black curve represents the GWP for CH_4, which is calculated by dividing the $AGWP_{CH4}$ by $AGWP_{CO2}$. The GWP curve is downward sloping because CH_4 has a quick decay rate, and so the $AGWP_{CH4}$ decreases over time relative to the $AGWP_{CO2.}$

Table 4 below shows the GWP over a 20-year and 100-year time horizon for the three main GHGs, as well as, the metric necessary to calculate the GWP. CO_2 has a GWP of 1 no matter what time horizon is used, because its AGWP is used as the base metric for GWP. N_2O has the essentially the same GWP for both time horizons, while CH_4 has very different GWPs.

Table 4: GWP value for top 3 GHGs

GHG	Lifetime	Radiative Efficiency (Wm^{-2} ppb^{-1})	AGWP 20-year (Wm^{-2} yr kg^{-1})	**GWP 20-Year**	AGWP 100-year (Wm^{-2} yr kg^{-1})	**GWP 100-Year**
CO_2	5-200	1.37e-5	2.49e-14	**1**	9.17e-14	**1**
CH_4	12.4	3.63e-4	2.09e-12	**84**	2.61e-12	**28**
N_2O	121	3.00e-3	6.58e-12	**264**	2.43e-11	**265**

Note: Adapted from "Anthropogenic and Natural Radiative Forcing" (p. 731) in IPCC's Climate Change 2013: The Physical Science Basis by Myhre et al, NY, Cambridge University Press

The GWP is a simpler metric than the GTP metric, because the GWP only takes into account the RF of a gas. "By accounting for the climate sensitivity and the exchange of heat between the atmosphere and the ocean, the GTP includes physical processes that the GWP does not" (p.712). Therefore, the uncertainty risk with physical metric is very low, and as a result, the GWP is the most commonly used metric for cap and trade systems around the world today.

5. Related Literature on GHG Metrics and Trading Systems

John Dales introduces the idea of a tradable permit market in 1968 in "Pollution, Property, and Prices". Dales explains that a tradable permit system was a cost-effective method to control pollution, because each polluter selects their own level of emissions and the number of permits they purchase. In doing so, they aim to minimize their own abatement costs. This will result in aggregate abatement cost minimization in the tradable permit market.

David Montgomery's "Market in Licenses and Efficient Pollution Control Programs" from 1972 is one of the first papers outlining a theoretical model for a tradable permit system for pollution. The purpose of this paper is to set up theoretical trading models for two types of permits: a "pollution license" and an "emissions license". A pollution license allows an entity to pollute up to a rate that ensures pollution will remain under a specified level. The model for pollution licenses requires that entities choose the number of licenses to hold by minimizing their abatement costs subject to the constraint that their pollution level equals their emissions multiplied by the emission concentration level. On the other hand, an emissions license allows an entity to pollute up to a direct rate of emissions.

The model for emission licenses requires a social planner to select an efficient number of emission licenses, and then polluters purchase emission licenses as to minimize their abatement costs. The pollution license model aligns the most with the GHG tradable permit markets that are discussed and used, because it insinuates that emissions should be controlled based on their contribution to total pollution. In academic papers as well as carbon markets used by policymakers, emissions are compared and controlled by their ability to contribute to global warming. Furthermore, while setting up the models, Montgomery proves a very important characteristic of a tradable permit market. He proves that the any initial allocation of licenses would result in a cost-efficient market, because polluting firms will trade allowances to the point where total abatement costs are minimized.

Approximately two decades later, Peter Michaelis's "Global Warming: Efficient Policies in the Case of Multiple Pollutants" sets up a tradeable permit market specific to GHGs. Michealis's uses a cost-effect framework to set up his dynamic carbon market using discrete time for multiple GHGs. He first sets up the theoretical cost-effective analysis to prove what the optimal emissions level, price, and trading ratio would be over a long time horizon. Secondly, he selects parameters and provides an empirical application for his theoretical model. These empirical results show that the three GHGs that should be reduced the most are CO_2, CH_4, and N_2O, as a opposed to Fluorinated Gas. As a result, Michaelis indicates the burden of reducing GHG emissions should fall on the use of fossil fuels, as well as agriculture and livestock, since they are the largest emitters of the three main gases.

Most of the literature on the topic of carbon markets involve cost-benefit and cost-effective analysis, and set up markets using these approaches. There is a lack of literature outlining a model for a CO_2 Eq Market that uses the GWP metric, which is what this paper aims to do. Most of the academic literature surrounding the economics of GHG control is focussed on the metrics used as trading ratios for carbon markets. In the past decade, there is especially a great deal of literature analyzing the GWP metric, since it is the metric of choice for so many policymakers and GHG initiatives around the world today.

Johansson, Persson, Azar (2006) estimate the economic loss from using GWPs as a trading ratio, compared to using the global cost potential. For their cost-effective analysis portion, they used a temperature cap of 2°C above pre-industrial levels. They concluded that for CO_2, CH_4, and N_2O, the loss from using GWPs is equal to 3.8 percent of the overall costs of meeting the target with cost-effective analysis, which is equivalent to \$ US_{2000} 1 billion.

"Physico-economics evaluation of climate metrics: A conceptual framework" by Deuber, Luderer, and Edenhofer from 2013 presents the popular GHG metrics (GDP, GCP, GWP, and GTP), and analyzes how these popular metrics perform with uncertainty. Firstly, Deuber et al. explained that the GCP, GWP, GTP can be considered as variants of the GDP. Secondly, they determined that the advantages of using the GWP metrics are: (1) uncertainty associated with normative judgements are very small because the only variables that policymakers need to select is the time horizon, (2) there is minor scientific uncertainty, and (3) "scenario uncertainty" is eliminated because future states are considered to be the same as the present (p.43). However, the disadvantages of GWP are it has "low policy relevance" because there is not a direct link from RF to global warming damages, and the future atmospheric conditions will not remain constant (p.43). Lastly, the authors provided insight into the benefits and pitfalls into using physcio-economic metrics, over purely physical metrics. Dueber et al concluded that physcio-economics metrics, such as GCP and GDP, have very low "structural uncertainty", which means they are very relevant to policymakers and real world trade-offs (p.37). However, unlike physical metrics, GDP and GCP have extremely high scientific uncertainty.

6. Greenhouse Gas Trading Systems

6.1 Multi-Pollutant Market

A multi-pollutant trading system involves separate markets for each GHG. In each gas' market, the government sets a cap on emissions and then creates enough allowances for that gas to cover the cap. Trading between these markets can occur by implementing an exchange rate between the allowances in each GHG's market. This subsection will set up a model for a multi-pollutant model that follows cost-effective analysis [4]. Although cost-benefit analysis is considered to be first best approach for controlling emissions, the damage from GHGs and global warming is very difficult to quantify. Cost-effective analysis is a more realistic approach to control emissions because policymakers need less information about greenhouse gases and global warming. With this method policymakers are not required to quantify damages, and only need insight into abatement costs for polluters.

In this market, the policymaker's problem is to select optimal amount of allowances while minimizing overall costs. Allowances for GHG g at time i is shown by L_{gi}. The number of allowances that the policymaker chooses to make available in the market is known as the allowance supply. The allowance supply for gas g at time is represented by $\overline{L_{g\iota}}$. In this model, an allowance equals one tonne of emissions. Thus, the amount of emissions in the market equals the number of allowances available. Since $E_{gi} = L_{gi}$, the policymaker shares their cost-minimization problem with the cost-effective analysis from section 3.3. The policymaker's goal is to

---

[4] This model was taken from Devon Garvie's "Mitigation Policy" PowerPoint Slides for the Course Econ 443 at Queen's University in 2014

minimize the cost of allowances, with the constraint placed on the warming capability as gas g. Thus, the equation (13) from cost-effective analysis provides the condition for the optimal allowance supply.

Suppose there are a finite number of profit-maximizing polluters for each GHG market. The representative polluter for gas g is denoted by j, where j = 1,….,n. Let L_{gij} represent the allowances purchased by polluter j for GHG g at time i. Suppose that the cost of reducing emissions for gas g is the same for all polluters. Thus, the costs faced by polluter j of meeting a specific level of emissions can be written as a function of allowances, and is denoted by $C_{gi}\left(L_{gij}\right)$. Furthermore, each polluter has their own allowance demand. Polluter j's allowance demand for gas g at time i is denoted by $L_{gij}\left(p_{gi}\right)$, and is a function of the price of emissions (p_{gi}).

There are few assumptions that are made with this multi-pollutant market. Firstly, I am assuming that the government distributes allowances solely through auctions, and thus, there is no free distribution. Changes in the initial allowance distribution will have no impact on the efficiency of the carbon system, but only the distribution of costs (Montgomery, 1972, p. 408). In addition, I assume that there is a competitive market for allowances with no barriers to entry. Thus, every polluter is a price taker, which means the price of allowances is the same for each polluter. Lastly, I am assuming that there are no transaction costs associated with purchasing or trading allowances.

When choosing how many allowances to purchase, each firm is aiming to minimize their total costs. Their total costs are equal to their emission costs plus the amount of money they spend allowances, which is equal to the price of emissions multiplied by the number of emissions they purchase. Thus, polluter j's total cost minimization problem is:

$$\underset{L_{gij}}{\text{Min}}\ TC_{gij}\left(L_{gij}, p_{gi}\right) = C_{gij}\left(L_{gij}\right) + p_{gi}\left(L_{gij}\right) \tag{23}$$

The first order condition with respect to their allowance demand $L_{gij}\left(p_{gi}\right)$ is:

$$-C'_{gij} = p_{gi} \tag{24}$$

Equation (24) signifies that in any period i firm j should purchase allowances up to the point where the marginal abatement cost of gas g is equal to the price of allowances. Every polluter emitting gas g in period i has the same first order condition for his or her cost minimization problem. Therefore, we can say that the aggregate marginal cost of abatement in minimized when it equals the price:

$$-C'_{gi} = p_{gi} \tag{25}$$

Equation (25) sets up the optimal condition for marginal abatement costs from a social planner's perspective, and equation (13) sets up the optimal condition for the marginal abatement costs from the viewpoint of the polluters. Since they both minimize cost of reducing emissions, and set up an optimal equation for marginal abatement costs, the two equations can be equated:

$$-C'_{gi} = p_{gi} \quad \Rightarrow \quad -C'_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-i} = SCE_{gi} \tag{26}$$

Equating the two equation shows that the equilibrium price of an allowance for gas g at time i will equal the SCE_{gi}:

$$p_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+\rho}\right)^{T-i} = SCE_{gi} \tag{27}$$

The optimal condition for the social planner's cost minimization problem is $-C'_{gi} = SCE_{gi}$. Since the equilibrium price is equal to the SCE, the emissions target will be met at the least-cost for gas g. Therefore, this greenhouse tradable permit market is considered to be cost-effective.

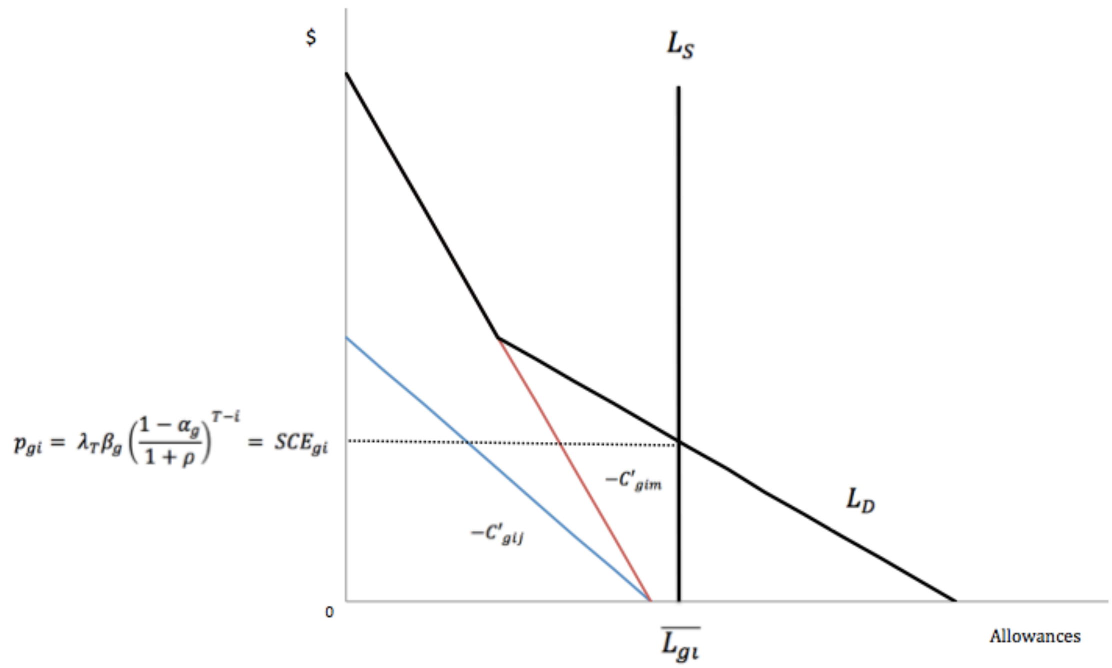
i) Market Static Efficiency

In the market for gas g, the allowance demand from polluters must equal the allowance supply. The allowance demand for polluter j is denoted by $L_{gij}(p_{gi})$. The number of allowances that a polluter will purchase is dependent on their marginal cost of abatement. Thus, the allowance demand of a polluter is represented by the marginal abatement curve. Horizontally summing the allowance demand curves for all polluters yields the total allowance demand for the economy, which is denoted by $L_{gi}(p_{gi})$. In order for the allowance market to clear, the allowance demand must equal supply:

$$p^*_{gi}: \quad L_{gi}(p_{gi}) = \overline{L_{gi}} \qquad \Rightarrow \qquad L_D = L_S \tag{28}$$

Figure 9 graphically illustrates the tradable allowance market at time i. There are two representative polluters in this market: polluter j and polluter m. Their individual marginal abatement cost curves are displayed by the blue and red curves. The total allowance demand is constructed by horizontally summing the two polluters' individual demand curves, and is denoted by L_D. The intersection point between the allowance supply and demand represents the optimal allowance price. This optimal price is equal to the SCE_{gi}, as shown in equation (27).

Figure 9: The Tradable Allowance Market for GHG g



In a multi-pollutant model, each GHG has its own market, with its own emission cap and price. Polluters are permitted to trade between markets by using trading ratios. To illustrate these trading ratios, we will look at two example gases: gas g and gas n. Equation (29) sets up an expression for a sum of the warming capabilities of gas g and n. The warming capability demonstrates the amount of warming caused by a certain level of emissions. Thus, the sum of the warming capabilities shows effect that the two gases have on the atmosphere.

$$\beta_g \sum_{i=1}^{T} \left(1-\alpha_g\right)^{T-i} E_{gi} + \beta_n \sum_{i=1}^{T} (1-\alpha_n)^{T-i} E_{ni} = W_{gi} + W_{ni} \tag{29}$$

Taking the total derivative with respect to emissions in equation (29) demonstrates how the warming capabilities change with respect to the emissions level for both gas g and gas n:

$$\beta_g\left(1-\alpha_g\right)^{T-i}dE_{gi} + \beta_n(1-\alpha_n)^{T-i}dE_{ni} = 0 \tag{30}$$

Re-arranging Equation (29) yields:

$$\frac{dE_{gi}}{dE_{ni}} = \frac{\beta_n(1-\alpha_n)^{T-i}}{\beta_g\left(1-\alpha_g\right)^{T-i}} \tag{31}$$

$$\frac{dE_{gi}}{dE_{ni}} = \frac{\beta_n}{\beta_g}\left(\frac{1-\alpha_n}{1-\alpha_g}\right)^{T-i} \tag{32}$$

Equation (32) shows the ratio of the derivatives of emissions for gas g and n. This equation compares the change in the atmospheric warming from one tonne of gas g relative to one tonne of gas n. Additionally, this equation ratio is equal to the ratio of the shadow cost of emissions. Since the SCE is the cost-effective price for allowances, the ratio of the SCE is equal to the cost-effective trading ratios in the market. Using this trading ratio ensures that there is no loss in efficiency from exchanging allowances. Furthermore, these cost-effective trading ratios guarantees that the emission targets across all GHG are met at least cost in the multi-pollutant market.

$$Trading\ ratio\ between\ g\ and\ n\ = \frac{\beta_n}{\beta_g}\left(\frac{1-\alpha_n}{1-\alpha_g}\right)^{T-i} = \frac{SCE_{ni}}{SCE_{gi}} = \frac{p_{ni}}{p_{gi}} \tag{33}$$

The trading ratio between gas g and gas n allowances signifies how many allowances of gas g is equal to one allowances for gas n. To demonstrate how trading ratio works, suppose that polluter j emits gas g and polluter m emits gas n. If polluter j wants to be able to emit one tonne of gas g, she will have to purchase $\frac{SCE_{gi}}{SCE_{ni}}$ number of gas n allowances from polluter m. Thus, the equation to transform one allowance of gas g to an allowance of gas n is:

$$L_{ni} = \; L_{gi} \, x \, \frac{SCE_{ni}}{SCE_{gi}} \tag{34}$$

ii) Market Dynamic Efficiency

A cost-effective allowance market requires that the emission target be met at least cost over the whole compliance period. The price of allowances is equal to the SCE, and is time dependent. Thus, the price of allowances will evolve over time. This subsection will show the time evolution of the allowance price, and the impact it has on the optimal allowances.

To demonstrate how the price of allowance evolve over time, assume there are two time periods i and k , where $i < k$. Equation (35) shows the optimal price of allowances for time i and k for gas g:

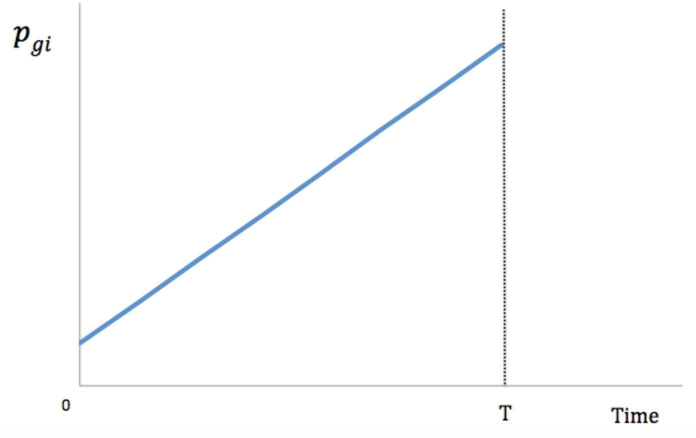
$$\overline{L_{gi}}\colon \; p_{gi} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+p}\right)^{T-i} \qquad \overline{L_{gk}}\colon \; p_{gk} = \lambda_T \beta_g \left(\frac{1-\alpha_g}{1+p}\right)^{T-k} \tag{35}$$

Combining these equation yields the following:

$$\{\overline{L_{gi}}, \overline{L_{gk}}\}\colon \quad \frac{p_{gk}}{p_{gi}} = \left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i} = \lambda_T \qquad where \quad i,k = 1, \ldots., T-1 \tag{36}$$

Combining the optimal conditions shows that the allowance price evolves from time i to time k at the rate $\left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i}$. The term $\left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i}$ converts the time i marginal cost of abatement into the time i marginal stock reduction in terms of period k dollars. The numerator $(1+\rho)^{k-i}$ converts time i dollars into time k dollars. The denominator is $\left(1-\alpha_g\right)^{k-i}$ is the amount of emissions that will decay between period i and k. Since the term $\left(\frac{1+\rho}{1-\alpha_g}\right)^{k-i}$ has a positive value, the price will increase from time i to time k.

Figure 10: Increasing permit price over time.



Section 3.3 explained that in cost-effective analysis the optimal level of emissions decreases over time because the harm of GHG emissions increases over time. As a result, the policymaker will decrease the allowance supply over time to match the decreasing optimal level of emissions. This reduced supply over time will cause an increase the equilibrium price level. Furthermore, as the allowance price increases over time, polluters will prefer to reduce their emissions further to avoid paying for expensive allowances. This will cause the allowance demand will also decrease over time. Overall, the equilibrium level of allowances will decrease over time as the equilibrium price increases over time.

The trading ratios in the multi-pollutant model are equal to the ratio of SCE's. The SCE changes over time to reflect how the atmospheric impact of emissions also evolves over time, due to its decay rates and radiative forcing. Thus, the trading ratio of SCE's accurately compares how different GHGs impact the atmosphere at each time period. Therefore, having trading ratios equal to SCE means that the emissions target at each given time period can be met cost-effectively across each GHG's market.

6.2 CO_2 Equivalent Market using the GWP metric

In a Carbon Dioxide Market the government implements one allowance market for all GHGs. In this market, one allowance is equal to one CO_2 equivalent. Emissions from each GHG are converted to CO_2 equivalents by using a metric selected by the government. This model is based on the use of the Global Warming Potential (GWP) metric.

This model assumes that there are a finite number of GHG denoted by g, where g=1,…,n. Time is denoted by i, where i=1,…,T. Time T is known as the "stabilization period" and is equal to the time period by which policymakers wish to meet their reduction goals by. The total emissions in the economy for gas g at time i is denoted by E_{gi}, and is measured in tonnes. The GWP metric with the time horizon equal to T for gas g is represented by GWP_{gT}. Total CO_2 equivalents allowances for time i is denoted by L_i. The equation to transform gas g emissions to CO_2 equivalents allowances is:

$$L_i = GWP_{gT} \; x \; E_{gi} \tag{37}$$

The equation to transform GHG emissions to the total CO_2 equivalents allowances for time i is:

$$L_i = \sum_{g=1}^{n} GWP_{gT} \; x \; E_{gi} \tag{38}$$

Thus, the conversion factor is constant across all time periods. When setting up this market, the social planner must first choose the optimal allowance target for the economy. In doing so, their objective is to minimize the present value of the total cost to society of reducing emissions. The total cost to society of meeting an allowance target at time i is denoted $C_i(L_i)$. This allowance target cost is discounted at the rate, ρ. The warming capability factor is denoted by W_T, and measures the amount of warming that may occur from time i to the stabilization period, T. In this model, the warming capability factor is equal to summation of emissions multiplied by GWP for each GHG from time i to T. From equation (39), we can see that the warming capability is equal to the total CO_2 eq allowances summed over all time i to T.

$$W_T = \sum_{i=1}^{T} \sum_{g=1}^{n} GWP_{gT} \times E_{gi} = \sum_{i=1}^{T} L_i \tag{39}$$

The constraint of the social planner's problem is the level of atmospheric warming that he will permit the economy to create, from time i to time T. This specific level is known as the target warming level is denoted by $\bar{W}$.

The social planner's problem is the minimization of allowance costs discounted at rate ρ, constrained by the warming factor:

$$\operatorname*{Min}_{L_i} \sum_{i=1}^{T} \frac{C_i(L_i)}{(1+\rho)^{i-1}} \qquad \textit{subject to} \quad W_T \leq \bar{W} \tag{40}$$

$$W_T = \sum_{i=1}^{T} L_i$$

The Lagrangean for the problem is:

$$L = \sum_{i=1}^{T} \frac{C_i(L_i)}{(1+\rho)^{i-1}} + \frac{\lambda_T}{(1+\rho)^{T-1}} \left(\sum_{i=1}^{T} L_i - \bar{W} \right) \tag{41}$$

The first order condition with respect to L_i is:

$$\bar{L_i}\text{:}\ -C'_i = \lambda_T \left(\frac{1}{1+\rho} \right)^{T-i} \tag{42}$$

The LHS of equation (42) is equal to the marginal cost of abatement at time i. The RHS is the present value shadow cost of a unit of CO_2 eq stock in the stabilization period. This represents the Shadow Cost of CO_2 Equivalent Emissions (SCCEE) for this market. The point where the marginal cost of abatement equals the term $\lambda_T \left(\frac{1}{1+\rho}\right)^{T-i}$ indicates the optimal number of allowances. This optimal allowance level will minimize costs in the CO_2 Equivalent Market.

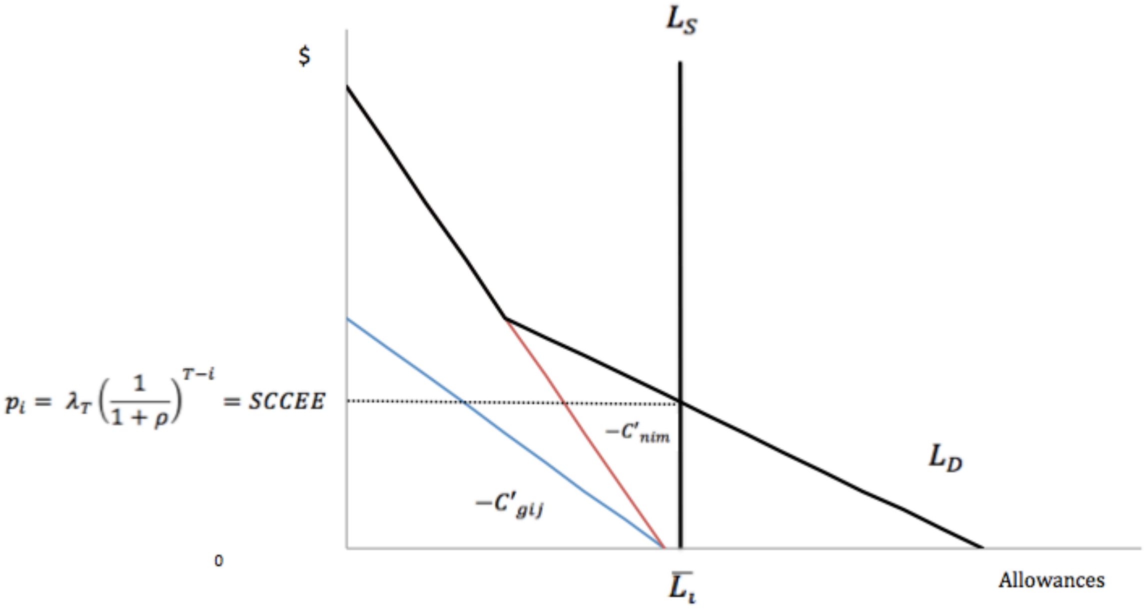
In addition to the assumptions made in the Multi-Pollutant Market, I am also assuming that the ratio of polluters to GHGs is one-to-one. Thus, each polluter only emits one type of gas, and there is only one polluter for each gas. Let's set up the allowance selection problem for the representative polluter, polluter j. Suppose polluter j solely emits GHG g, and is the only polluter that emits this gas. Polluter j has the same cost minimization problem as in the Multi-Pollutant Market, which is shown in equation (23). The first order condition of polluter j's problem is equal to equation (24), and the condition for minimized aggregate emissions can be shown by equation (25). The equilibrium price of allowances is the point is equal to the marginal cost of abatement. Thus, the equilibrium price is equal to the SCCEE from the policymaker's cost minimization problem.:

$$p_i = \lambda_T \left(\frac{1}{1+\rho}\right)^{T-i} = SCCEE \tag{43}$$

i) Market in Static State

Suppose that there are two representative polluters: polluter j and polluter m. Polluter j only emits gas g and polluter m emits gas n. The total allowance demand for the economy is equal to the horizontal summation of the abatement cost curves of polluter j and m, and is denoted by L_D. In order for the CO$_2$ Eq Market to clear, the allowance supply and demand must be equated ($L_D = L_S$). The intersection point of the supply and demand represents the optimal level of emissions, at the equilibrium price equal to SCCEE. Figure 11 illustrates the CO$_2$ Eq Market.

Figure 11: The CO$_2$ Equivalent Market



Each polluter will purchase CO$_2$ eq allowances at the same price. However, they must transform their GHG emissions into CO$_2$ eq allowances by using the GWP metric specific to each gas, as demonstrated in equation (37). Thus, each polluter must purchase a different amount of allowances to cover one tonne of emissions of their unique GHG. This creates trading ratios between GHGs. To demonstrate the trading ratios between gases, equation (44) sets up an expression for the sum of the warming capabilities of gas g and n. The warming capabilities for each gas compare the impact that each gas will have on the atmosphere, and the calculation that each polluter use to determine the CO_2 eqs for their gas.

$$GWP_{gT}E_{gi} + GWP_{nT}E_{ni} = W_{gT} + W_{nT} \tag{44}$$

Taking the total derivative yields:

$$GWP_{gT}dE_{gi} + GWP_{nT}dE_{ni} = 0 \tag{45}$$

Re-arranging:

$$\frac{dE_{gi}}{dE_{ni}} = \frac{GWP_{nT}}{GWP_{gT}} \tag{46}$$

Equation (46) is the ratio of warming capability derivatives of gas g to gas n, which is equal to the ratios of warming potentials. In this market, the GWP metric is used to compute the impact that each GHG will have on the atmospheric temperature. Therefore, the trading ratio in the CO_2 Equivalent Market is equal to the ratio of GWPs.

$$Trading\ ratio\ between\ g\ and\ n = \frac{GWP_{nT}}{GWP_{gT}} \tag{47}$$

Trading ratios convert emissions from one gas to another. Equation (47) shows that emissions from gas g are converted to emissions of gas n at a rate equal to the ratio of GWPs. Thus, for every one tonne of gas n that polluter m emits, polluter j can emit $\frac{GWP_{nT}}{GWP_{gT}}$ of gas g.

$$E_{ni} = E_{gi}\ x\ \frac{GWP_{nT}}{GWP_{gT}} \tag{48}$$

To illustrate how the trading ratio would operate in the market, assume that the GWP_{gT} is equal to 3 and GWP_{nT} is equal to 4. If polluter j would like to emit one tonne of gas g, she has to purchase 3 CO_2 eq allowances. Polluter m has to purchase

4 allowances to emit one tonne of gas n. This means that one CO_2 eq permits polluter j to emit $^1/_3$ tonnes of gas g, and polluter m to emit $^1/_4$ tonnes of gas n. Thus, one tonne of gas n is equivalent to $\frac{GWP_{nT}}{GWP_{gT}} = \frac{4}{3}$ tonnes of gas g.

ii) Market in Dynamic State

The social planner aims to choose an emissions target that minimizes abatement costs from time 1 to T. The inter-temporal path for allowances can be shown by the ratio of the conditions for the social planner's cost minimization problem:

$$\{\overline{L_\iota}, \overline{L_k}\}: \ \frac{-C'_k}{-C'_i} = (1+\rho)^{k-i} \tag{49}$$

Equation (49) shows that the marginal cost of abatement increases at the rate $(1+p)^{u-i}$. The discount rate, ρ, measures the value that society places on the future, and so the term $(1+p)^{u-i}$ converts the marginal abatement costs of time i into time u dollars. The increasing marginal abatement costs indicate that the cost to society of abating emissions is greater as time get closer to the stabilization period. As costs increase, the optimal allowance supply decreases over time.

The equilibrium price is equal to the marginal cost of abatement, thus the inter-temporal path of price is also to:

$$\frac{p_k}{p_i} = (1+\rho)^{k-i} \qquad where\ i < k \ \ and\ \ i,k = 1, \ldots., T \tag{50}$$

Equation (50) shows that the price increases from time i to time k at the rate $(1+p)^{k-i}$. The term $(1+p)^{k-i}$ converts the time i dollars into time k dollars. This increasing price will cause polluters to reduce their demand for allowances, because over time polluters will prefer to reduce emissions rather than purchasing the expensive allowances.

The trading ratio between GHGs is a ratio of their GWP metrics. The GWP is calculated for a given time horizon, where the time horizon should be equivalent to number of years between time 1 and the stabilization period. This GWP represents the amount of the warming that the gas will cause in the atmosphere from time 1 to time T. Although the GWP does take into account the length of the time horizon, it is a constant metric from time 1 to T. As a result, the trading ratios are also constant over time.

6.3. Trading Ratios Comparison and Analysis

The Multi-Pollutant Model and the CO_2 Equivalent Market are variations of cost-effectiveness analysis, because in both markets the social planner selects an optimal allowance level by minimizing abatement costs with respect to an atmospheric warming constraint. Neither market can achieve global cost minimization, as this can only be attained by cost-benefit analysis. However, the two markets do differ in their ability to meet their emissions in a cost-efficient manner. The Multi-Pollutant Market is more cost-efficient than the CO_2 Equivalent Market, and this is largely to do with the difference in trading ratios. This subsection will compare the trading ratios between the Multi-Pollutant Market and the CO_2 Equivalent Market, and examine their impact on market cost-efficiency.

i) Inter-Temporal Path of Trading Ratios

As explained earlier in this section, the optimal emissions target decreases and the optimal price increases over time, in both the Multi-Pollutant Market and the CO_2 Equivalent Market. However, the Multi-Pollutant Market trading ratio increases over time, while the CO_2 Equivalent Market trading ratio stays static. The inter-temporal path of a trading ratio shows how it evolves across time. I will use CH_4 and CO_2 to illustrate the difference in the inter-temporal path for the two trading ratios.

The Multi-Pollutant Market trading ratio between gas g and gas n is defined as $\frac{SCE_{ni}}{SCE_{gi}} = \frac{\beta_n}{\beta_g}\left(\frac{1-\alpha_n}{1-\alpha_g}\right)^{T-i}$. CH_4 will represent gas n and CO_2 will represent gas g. Table 5 defines the parameters used to calculate the trading ratio between these two gases. By using CH_4's lifetime from the IPCC 's "Anthropogenic and Natural Radiative Forcing" in Climate Change 2013: The Physical Science Basis", I was able to calculate its decay rate. The lifetime of CO_2 can be a range of values from 5 to 200, because of the complexity of the carbon cycle. For the CO_2 decay rate calculation, I chose a lifetime of 150 years. In Michealis' "Global Warming: Efficient Policies in the Case of Multiple Pollutants", he provides empirical analysis of his cost-effective model. In his analysis, Michealis uses a value of 58 for the ratio of radiative forcing between CH_4 relative to CO_2, which he obtained from "Radiative Forcing" in the IPCC's 1990 Scientific Assessment. I have used the same as Michaelis for the term $\frac{\beta_n}{\beta_g}$.

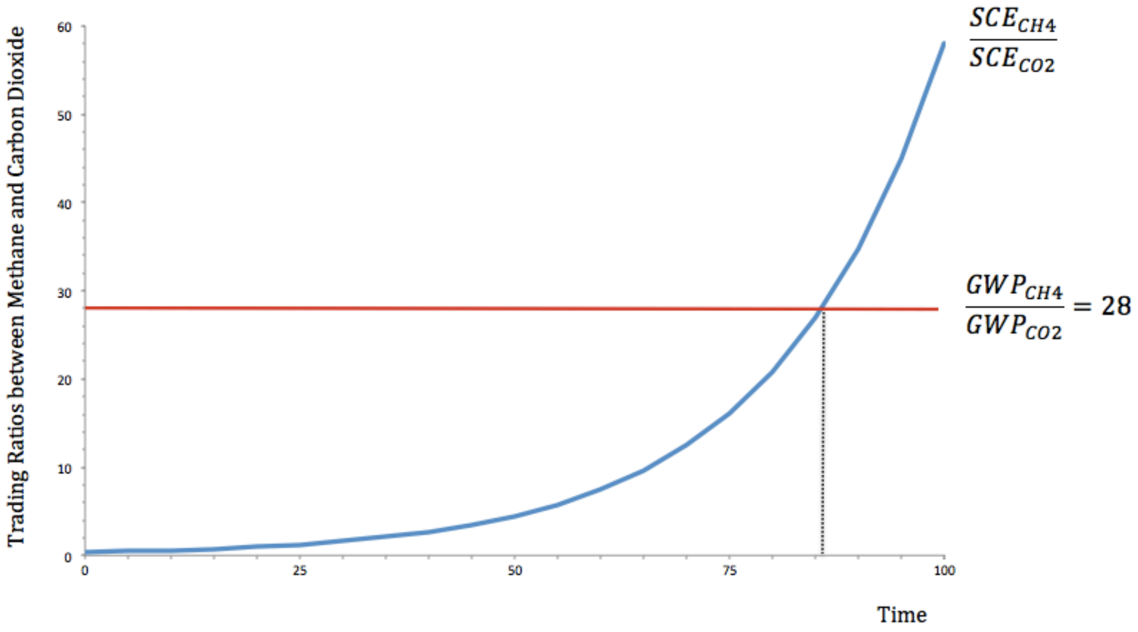
The trading ratio for the CO_2 Equivalent Market is equal to $\frac{GWP_{nT}}{GWP_{gT}}$. Table 5 also displays the 100-year GWP values for both CH_4 and CO_2. These values were obtained from the IPCC's "Anthropogenic and Natural Radiative Forcing" in Climate Change 2013: The Physical Science Basis" by Myhre et al.

Table 5: Trading Ratio Parameters

GHG	Decay Rate (α)	Radiative Forcing Ratio (β_n/β_g)	GWP
CH_4	0.054365328	58	28
CO_2	0.004610321		1

The parameters outlined in Table 5 were used to generate the inter-temporal paths for the Multi-Pollutant Market trading ratio between CH_4 and CO_2. Figure 12 illustrates how the CH_4 to CO_2 trading ratio evolves over a 100-year time horizon. At time zero, the trading ratio is equal to just above zero, and it evolves to a value of 58 at year 100. In addition, Figure 12 also shows that the CO_2 Equivalent Market trading ratio between CH_4 and CO_2. This ratio is equal to $\frac{GWP_{CH4(100)}}{GWP_{CO2(100)}} = \frac{28}{1} = 28$. The SCE trading ratio lies below the GWP trading ratio, until approximately year 85 when the SCE ratio surpasses the GWP ratio.

Figure 12: Trading Ratios Comparison



ii) Cost-Efficiency of Trading Ratios

Only the Multi-Pollutant Market outlines the cost-effective emission target and price for a carbon market. As a result, the SCE trading ratio is the cost-effective path for a trading ratio to ensure that the emission target is met at approximately least cost. Any deviations away from this intertemporal trading ratio path results in cost inefficiency. Thus, using the GWP as a trading ratio will ultimately create cost inefficiencies within the allowance market.

In an allowances trading market, trading ratios determine the rate at which polluters can trade allowances between gases. In doing so, they ultimately dictate the amount of each gas that can be emitted in the economy. For example, since the CO_2 Equivalent Market trading ratio between CH_4 and CO_2 is 28, one tonne of CH_4 can be traded for 28 tonnes of CO_2. As a result, a polluter needs just one allowance to emit one ton of CO_2, but is required to purchase 28 allowances to emit one ton of methane. One hundred CO_2 Equivalent allowances can translate into 100 tonnes of CO_2, 3.5 tonnes of CH_4, or a combination of both gases. Thus, in the CO_2 Equivalent Market, the higher the trading ratio between a GHG and the base gas (CO_2), the less that gas will be emitted by polluters. The Multi-Pollutant Market follows the same logic – a high trading ratio between two GHGs signifies that the gas in the numerator of the ratio will be emitted less than the gas in the denominator.

Figure 12 illustrates that between year 0 and 85, the GWP trading ratio for CH_4 is higher than the SCE trading ratio. As a result, using the CO_2 Equivalent Market will result in over-control of CH_4 from year 0 to 85. This means that polluters are reducing their CH_4 emissions more than is required by the cost-effective SCE trading ratio. This period of over-control will require polluters to drastically reduce their emissions starting immediately at time 0. To do this, polluters will need to invest in clean technology, and alter their emitting processes, which can be very expensive.

From years 85 to 100, the GWP is below the SCE trading ratio for CH_4. Consequently, the GWP ratio is under-controlling CH_4 during this time period. The period of under-control will result in more atmospheric warming than is optimal so close to the stabilization period. As a result, society will incur costs from increased damage from climate change, such as extreme weather events, rise in sea level, loss of species, etc. Therefore, the periods of over and under control of methane emission in the CO_2 Equivalent Market result in cost-inefficiencies.

7. Conclusion

In an allowances trading market, trading ratios not only determine the rate at which polluters can trade allowances between gases, but also compare each gas's marginal impact to society. There are four main greenhouse gas metrics: Global Damage Potential (GDP), Global Cost Potential (GCP), Global Temperature Potential (GTP), and Global Warming Potential (GWP). As previously explained, the GDP metric is the first-best metric to use in trading ratios because it involves the most in depth economic and physical analysis of GHGs. The use of this metric in a GHG market will result in global cost minimization. Second, the GCP is known as the second-best because it does not involve thorough analysis of the damage cause by increasing GHGs level and global warming. However, using this metric as a trading ratio will allow regulators to meet their emissions target in a cost-effective manner.

The GTP is the third best metric. Although it does not include economic analysis, it does entail thorough scientific analysis. It evaluates how the heat exchange between the air and ocean impact the overall global temperature, and measures how the GHG impacts the atmospheric temperature for a select period of time. Lastly, the GWP is known as the least comprehensive metric. It is generated from solely physical analysis of GHGs, and looks at the impact a gas has the atmospheric temperature over along time horizon.

In the Multi-Pollutant Market, the SCE trading ratio between CH_4 and CO_2 is a ratio of the GCP's for CH_4 and CO_2. This ratio includes physical analysis of GHGs through the decay rate and radiative forcing. It also evolves economic analysis, by examining how society values the future, through the discount factor. Therefore, the SCE trading ratio compares the impact that each GHGs will have on the economic and atmospheric well-being of society. On the other hand, the GWP trading ratio in the CO_2 Equivalent Market is only based on the physical analysis, and implicitly accounts for the decay rate and radiative forcing. Thus, the GWP trading ratio only compares the marginal impact that GHGs have on the atmosphere.

The Multi-Pollutant Market is a cost-effective method of controlling GHGs. However, the GCP metric requires a lot of physical and economic analysis. As a result, this market is considered to be very complex and uncertain. It is almost impossible for a policymaker to implement this type of market today. The GWP CO_2 Equivalent Market on the other hand, does not result in cost-efficiency. However, because the GWP is a purely physical metric with little scientific uncertainty, this market is seen as being much simpler and more reliable. Consequently, governments around the world use the GWP CO_2 Equivalent Market to combat increasing GHGs level and the issue of climate change.

References

Dales, J. (1968) *Pollution, Property, and Prices*. Toronto, Canada: University of Toronto Press

Deuber, O., Luderer, G., Edenhofer, O. (2013). Physico-economic evaluation of climate metrics: A conceptual framework*. Environmental Science and Policy*, Volume 29, pp. 37 – p.45. Retrieved from: http://www.sciencedirect.com/science/article/pii/S146290111300021X

European Commission. (2015) Paris Agreement. *Climate Agreement*. Retrieved from https://ec.europa.eu/clima/policies/international/negotiations/paris_en

Garvie, D. (2014). Cost-effective Pollution Control Powerpoint Presentation. Econ 443 Course. Queen's University.

Garvie, D. (2014). Mitigation Policy Powerpoint Presentation. Econ 443 Course. Queen's University.

Garvie, D. (2014). Stock Pollution Powerpoint Presentation. Econ 443 Course. Queen's University.

Government of Canada (2015). *Greenhouse Effect*. Retrieved from https://www.canada.ca/en/environment-climate-change/services/climate-change/greenhouse-effect.html

Government of Canada (2017). *National Inventory Report 1990-2015: Greenhouse Gas Sources and Sinks in Canada – Executive Summary*. Retrieved from https://www.ec.gc.ca/ges-ghg/default.asp?lang=En&n=662F9C56-1#ghgemissions

IPCC, 2013: Summary for Policymakers. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Johansson, D.J.A, Persson, U.M., and Azar, C. (2006). The cost of using global warming potentials: analyzing the trade off between CO_2, CH_4, and N_2O. *Climate Change*, Volume 77, pp. 291-309. Doi: 10.1007/s10584-006-9054-1

Le Treut, H., R. Somerville, U. Cubasch, Y. Ding, C. Mauritzen, A. Mokssit, T. Peterson and M. Prather, 2007: Historical Overview of Climate Change. In: Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change [Solomon, S., D. Qin, M. Manning, Z. Chen, M. Marquis, K.B. Averyt, M. Tignor and H.L. Miller (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Michaelis, P. (1992). Global Warming: Efficient Policies in the Case of Multiple Pollutants. *Environmental and Resource Economics*, Volume 2 (Issue 1), pp. 61 – pp. 77. Doi: 10.1007/BF00324689

Montgomery, D. (1972). Markets in Licenses and Efficient Pollution Control Programs. *Journal of Economic Theory*, Volume 5 (Issue 3), pp. 395-418.

Myhre, G., D. Shindell, F.-M. Bréon, W. Collins, J. Fuglestvedt, J. Huang, D. Koch, J.-F. Lamarque, D. Lee, B. Mendoza, T. Nakajima, A. Robock, G. Stephens, T.

Takemura and H. Zhang (2013) Anthropogenic and Natural Radiative Forcing. In: *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Roberts, M., and Spence, M (1976). Effluent Charges and Licenses Under Uncertainty. Journal of Public Economics, Volume 5, pp.193-208

Sine, K.P., Derwent, R.G., Wuebbles, D.J, Morcrette, J.J. (1990). Radiative Forcing of Climate. In: Scientific Assessment of Climate Change 1990. *Contribution of Working Group I to the First Assessment Report of the Intergovernmental Panel on Climate Change.* [Houghton, J.T., G.J Jenkins, and J.J. Ephramus (eds.)] Cambridge University Press, Cambridge, Great Britain, New york, NY, USA and Melbourne, Australia

Solomon, S., D. Qin, M. Manning, R.B. Alley, T. Berntsen, N.L. Bindoff, Z. Chen, A. Chidthaisong, J.M. Gregory, G.C. Hegerl, M. Heimann, B. Hewitson, B.J. Hoskins, F. Joos, J. Jouzel, V. Kattsov, U. Lohmann, T. Matsuno, M. Molina, N. Nicholls, J. Overpeck, G. Raga, V. Ramaswamy, J. Ren, M. Rusticucci, R. Somerville, T.F. Stocker, P. Whetton, R.A. Wood and D. Wratt, 2007: Technical Summary. In: Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change [Solomon, S., D. Qin, M. Manning,

Z. Chen, M. Marquis, K.B. Averyt, M. Tignor and H.L. Miller (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Stocker, T.F., D. Qin, G.-K. Plattner, L.V. Alexander, S.K. Allen, N.L. Bindoff, F.-M. Bréon, J.A. Church, U. Cubasch, S. Emori, P. Forster, P. Friedlingstein, N. Gillett, J.M. Gregory, D.L. Hartmann, E. Jansen, B. Kirtman, R. Knutti, K. Krishna Kumar, P. Lemke, J. Marotzke, V. Masson-Delmotte, G.A. Meehl, I.I. Mokhov, S. Piao, V. Ramaswamy, D. Randall, M. Rhein, M. Rojas, C. Sabine, D. Shindell, L.D. Talley, D.G. Vaughan and S.-P. Xie, 2013: Technical Sum- mary. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assess- ment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Tol, R.S.J., Berntsen, T.K., O'Neill, B.C., Fuglestvedt, J.S., Shine, (2012). A unifying framework for metric for aggregating the climate effect of different emissions. *Environmental Research Letters*, Volume 7 (Issue 4)

United States Environmental Protection Agency (2017). *Overview of Greenhouse Gases*. Retrieved from https://www.epa.gov/ghgemissions/overview-greenhouse-gases

United States Environmental Protection Agency (2017). *Understanding Global Warming Potential*. Retrieved from https://www.epa.gov/ghgemissions/understanding-global-warming-potentials