Risk Fluctuations in A Small Open Economy:

Do They Matter in Canada?

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Abstract

This paper considers the agency cost model by Carlstrom and Fuerst (1997) in a small open economy context to analyze whether fluctuations in risk are a significant source of business cycles in Canada. I find that risk fluctuations have little impact on the Canadian economy while external shocks such as terms of trade shocks, foreign real interest rate shocks play an important role for Canadian real business cycles. This also further suggests that the recent episodes of recessions in Canada are more likely to be incurred via external channels rather than internal factors.

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1 Introduction & Literature Review

Ever since the sudden start of the Great Recession in the late 2007, people have been wondering why the traditional macroeconomic models and econometric models fail to predict and explain the strong economic downturn following the turbulences on financial markets. In an attempt to explain the mechanism that triggers the Great Recession, the main efforts have been devoted to incorporating financial sector and different types of shocks to financial variables in the dynamic stochastic general equilibrium (DSGE) framework. In particular, two types of shocks have been studied extensively in the macroeconomics literature. The first type of shocks deals with the sudden change in the wedge between return to capital and the risk-free rate. As Hall (2010), Gilchrist et al. (2009) and others point out, this type of financial shocks can explain the key features of the Great Recession. The other type of shock is the shock on marginal efficiency of investment (MEI). MEI shocks affect the economic fluctuations by hitting the investment-to-capital transformation process. Specifically, Justiniano et al. (2009) establishes that the MEI shocks act as the driving force for the U.S. business cycle including the Great Recession period. In addition to this, by estimating a structural time-varying parameter VAR model, Prieto et al. (2013) show that the contribution of financial shocks constitutes more than 50% during the Great Recession period empirically.

Apart from the two types of shocks discussed above, a few number of prominent researchers look into the role of uncertainty in the Great Recession. In particular, two specific types of changes in uncertainty are studied mostly by researchers in the literature. The first type refers to the changes in risk. For example, Bloom (2009) defines uncertainty as the cyclical variation in the cross-sectional standard deviation of firm-level stock returns and it is shown that uncertainty increases drastically during the Great Recession period. Recently, Christiano, Motto and Rostango (2014) introduce the concept of idiosyncratic risk shocks into the literature. In the work of Christiano et al. (2014), they include the risk shocks into an otherwise standard financial accelerator framework proposed by Bernanke, Gertler and Gilchrist (1999). By comparing the marginal likelihood of different versions of the proposed baseline DSGE model based on Bayesian estimation, Christiano, Motto and Rostango (2014) (CMR hereafter) find that the fluctuations in risk are the most important shocks driving the business cycles. The second type of changes in uncertainty refers to the change in the preferences or individual risk-aversion. For instance, Benchimol (2014) finds that the risk-aversion shocks in the Eurozone becomes relatively more important in the period between 2006 and 2011 than other periods. The recent research conducted by Guiso, Sapienza and Zingales (2013) also shows that investors' risk-aversion increases following the 2008 financial crisis.

Risk shocks have been examined thoroughly in the closed economy context but there seems to be a lack of literature about risk shocks within other environments. It was only until recently that some researchers started picking up this topic. In particular, Letendre and Wagner (2016) point out that the effects of risk shocks are too small to significantly impact the dynamics in a two-country model embedded with the agency cost feature as in the Carlstrom and Fuerst (1997) model . Hence, we may ask ourselves, do risk shocks really matter in a small open economy like Canada? The first attempt to study on this topic is done recently by Mendicino and Zhang (2016). They develop their model related to the works by a few other researchers such as Ambler, Dib and Rebei (2004). Then they introduce risk shocks as suggested by the CMR model. However, differently from the CMR model, they consider risk shocks to be sector-specific and therefore the risk shocks faced by entrepreneurs in the tradable sector are different from those in the non-tradable sector. They find that risk shocks still contribute significantly to the output fluctuations in a small open economy.

However, what if we try to model risk shocks from a different aspect? How important will risk shocks be within a different model? To answer these specific questions, in this paper, we continue to investigate the role of risk shocks, based on a small open economy model. For the analysis, we introduce the agency cost setting proposed by Carlstrom and Fuerst (1997) into an otherwise standard real business cycle model in a small open economy with one good. Hence, I make contributions to the existing macroeconomic literature by examining risk shocks in the aforementioned context for the first time. The model I develop is mostly related to the work of Carmichael and Samson (2002). However, unlike Carmichael and Samson (2002), we do not incorporate non-tradable sector into our model. Instead, we simplify our model by only considering one tradable good in the model economy.

The results of our model are different from the findings by Mendicino and Zhang (2016). Our results suggest that risk shocks are too little to make an impact on the Canadian economy. Instead, external shocks such as terms of trade shocks or foreign real interest rate shocks constitute a significant proportion of the output fluctuations in Canada. Moreover, we find that the terms of trade shocks are the main driving force for the Canadian real business cycles in the long run. However, if we take the correlation between terms of trade and foreign real interest rates into account, our theoretical model suggests that the foreign real interest rate shocks become the most influential for the Canadian real business cycles in both the long run and short run instead.

The rest of the paper proceeds in the following way: Section 2 describes the small open economy model setting. Section 3 discusses the calibration of the model. Section 4 presents the benchmark results. Section 5 presents the alternative results based on Bayesian Estimation. Section 6 considers the simple reduced form VAR estimations. Section 7 summarizes the results and concludes the paper with final remarks and implications for future work.

2 The Small Open Economy Model

In our small open economy model, the essence of the model of Carlstrom and Fuerst (1997) is preserved. We introduce entrepreneurs and a partial equilibrium mechanism of financial contracts into an otherwise standard real business cycle model in a small open economy setting. We also introduce the risk shocks into the model and observe how they influence the business cycles in the Carlstrom and Fuerst (1997) framework.

The introductory description of our model is demonstrated as follows: Our small open economy consists of three types of agents, consumers/lenders, entrepreneurs and firms. Also there exists only 1 type of composite consumption good in the world of our model. The consumers/lenders maximize their lifetime utility and their lifetime wealth coming from their capital, their labour supply as well as their lending/borrowing in the international market. The consumers are also the lenders because they invest in a domestic capital mutual fund (CMF) that finances the entrepreneurs. We explain the mechanism of the mutual fund in greater detail below. Firms maximize their profits and produce the composite consumption goods using a constant return to scale technology subjected to exogenous technology shocks.

The third type of agent is the entrepreneur. The entrepreneurs use their net worth and the borrowing from the CMFs to finance their purchase of domestic and imported investment goods. No direct external borrowing is allowed in our model.[1] In order to make our model tractable, as in Carlstrom and Fuerst (1997), we assume that entrepreneurs' financial transactions are carried out through a capital mutual fund and are limited to intra-period transactions.

Our small open economy interacts with the rest of the world in the following way. The entrepreneurs need both domestic investment good and imported investment good to produce capital goods. The economy is small so that the terms of trade is

[1] Carmichael and Samson (2002) explains that this is a reasonable feature motivated by the assumption that monitoring costs in the foreign mutual funds are too high.

exogenously given and the domestic real interest rate is pinned down by the exogenous world real interest rate and the aggregate external debt. Furthermore, the consumers can borrow from or lend to the rest of the world. No outside borrowing or lending is made by the CMF.

2.1 The household

The household's utility function is based on the one suggested by Stockman and Tesar (1995). However, due to the absence of non-tradable sector, we make a few adjustments and simplify the utility function by removing the tradable-consumption components and the relevant parameters. As a result, we assume that the representative household has the following expected lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\frac{1}{1-\epsilon}({c_t^d}^{\theta} {c_t^f}^{1-\theta})^{1-\epsilon} + \psi ln(1-h_t)] \tag{1}$$

where the household consumption of domestic good is denoted by c_t^d, consumption of foreign good by c_t^f and individual labour supply by h_t. The discount factor is denoted by β and ϵ signifies the elasticity of inter-temporal substitution for consumption by the household, while θ measures the degrees of relative preference over domestic and foreign goods and ψ is a parameter that determines household's preference on leisure relative to consumption. The representative household must satisfy the following budget constraint

$$c_t^d + tot_t c_t^f + q_t i_t + b_{t-1} \leq w_t h_t + r_t k_t + b_t R_t \tag{2}$$

The left hand side of the constraint reflects the fact that the households use their income to consume, invest or borrow/lend in the international market. In particular, b_t denotes the amount of financial assets the household owns in the international market at period t. The terms of trade (TOT) is given by tot_t.[2] In addition, the households can increase their capital stock by purchasing the investment good i_t. Hence we assume the representative household has the following capital accumulation process:

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{3}$$

where δ is the depreciation rate. The representative household chooses c_t^d, c_t^f, b_t, and i_t to maximize their lifetime utility (1) subject to equations (2) through (3).

2.2 Firms

We assume all the firms produce identical goods with the following Cobb-Douglas technology:

$$F_t = A_t K_t^{\alpha} H_t^{1-\alpha} \tag{4}$$

where F_t denotes the aggregate domestic production and A_t denotes the exogenous technology parameter, the aggregate capital stock is signified by K_t , the aggregate supply of labour of household is given by H_t. We assume perfect competition in the market, which further implies that the wage and the rental rate of capital are equal to their respective marginal product.

2.3 The Entrepreneurs

In our small open economy, entrepreneurs account for a small proportion of the population. Each time when an entrepreneur attempts to accumulate their capital through purchasing i_t^e units of investment good, he has to make a random draw of his efficiency of investment, ω, which is assumed to be independently and identically distributed

[2] In Carmichael and Samson (2002), for convenience, they define terms of trade as the ratio of import price index to export price index, which is the opposite of the usual definition of the terms of trade. For this paper, we also decide to follow this definition for convenience and the ease of notation

across time and entrepreneurs, with distribution Φ, density ϕ, a non-negative support, a mean of unity and a standard deviation σ_t^ω. Moreover, we define σ_t^ω as the idiosyncratic risk. The resources required by entrepreneurs to fund the investment projects come from their own net worth n (internal funds), plus the external funds borrowed from the CMFs.

We introduce the agency cost into our model by assuming that each entrepreneur's stochastic investment-to-capital efficiency, ω, is private information. If a CMF wants to monitor an entrepreneur's level of productivity, a cost of μi will be applied to the CMFs, which is measured in terms of the units of investment goods. The moral hazard problem between borrowers and lenders arises due to the asymmetric information as described. However, as in the Carlstrom and Fuerst (1997), the optimal contract is formed in such a way that the entrepreneurs never intend to lie about their true value of the stochastic productivity ω. More detail about the optimal contract will be discussed in the next subsection.

We assume each entrepreneur is risk neutral and maximizes their expected lifetime utility

$$E_0 \sum_{t=0} (\beta\gamma)^t (ef_t + ed_t) \tag{5}$$

where $0 < \gamma < 1$ is a discount factor and this implies that we implicitly assume entrepreneurs are relatively more impatient than households. We denote the individual entrepreneurial total consumption by c_t^e. The level of entrepreneurial foreign consumption is denoted by ef_t and the level of domestic entrepreneurial consumption is given by ed_t. Here we assume that the foreign consumption by the entrepreneur is a proportion of the entrepreneur's total consumption c_t^e. That is,

$$ef_t = \lambda c_t^e \tag{6}$$

$$ed_t = (1 - \lambda)c_t^e \tag{7}$$

where $0 < \lambda < 1$ is the proportion parameter. Now we switch our focus onto the budget constraint of the representative entrepreneur. At the beginning of period t, an entrepreneur rents out his current capital stock k_t^e to a local producers, which generates rental income $r_t k_t^e$. Then, the entrepreneur sells off all of their remaining capital stock $(1-\delta)k_t^e$ to the local CMF, which pays $q_t(1-\delta)k_t^e$ units of consumption goods to the entrepreneur. As a result, the net worth constraint for each entrepreneur is given by

$$n_t = r_t k_t^e + q_t(1 - \delta)k_t^e \tag{8}$$

The entrepreneur's risk neutrality and the high internal return imply that the entrepreneur will always spend all his net worth on the loan contract. At the end of the period, the entrepreneur finances consumption out of the returns from the investment project. As a result, the law of motion for the entrepreneur's capital accumulation is given by:

$$k_{t+1}^e = \frac{rif_t n_t}{q_t} - \frac{[(1-\lambda) + \lambda tot_t]c_t^e}{q_t} \tag{9}$$

where $rif_t = \dfrac{q_t f(\bar{\omega}_t) i_t}{n_t}$ is the expected return on internal funds. In the next subsection, more detail of rif_t is discussed. Combining equation (8) and equation (9), we have the following equation:

$$k_{t+1}^e = \frac{rif_t[k_t^e r_t + (1-\delta)q_t k_t^e]}{q_t} - \frac{[(1-\lambda) + \lambda tot_t]c_t^e}{q_t} \tag{10}$$

At the end of each period, those solvent entrepreneurs choose c_t^e and k_t^e to maximize their lifetime expected utility function (5) subject to equation (10), for every period t. In other words, we can set up the following optimization problem:

$$\max_{c_t^e, k_t^e} \quad E_0 \sum_{t=0} (\beta\gamma)^t c_t^e$$

$$\text{subject to} \quad k_{t+1}^e = \frac{rif_t[k_t^e r_t + (1-\delta) q_t k_t^e]}{q_t} - \frac{[(1-\lambda) + \lambda tot_t] c_t^e}{q_t}, \ \forall t.$$

Assuming the existence of an interior solution, we are able to derive the following Euler equation for the entrepreneurs:

$$\gamma\beta E_t[(r_{t+1} + (1-\delta)q_{t+1}) \frac{rif_{t+1}}{(1-\lambda) + \lambda tot_{t+1}}] = \frac{q_t}{(1-\lambda) + \lambda tot_t} \tag{11}$$

2.4 The Optimal Financial Contracts

In this subsection, we introduce the financial contract in the partial equilibrium setting. It is worth mentioning that the partial equilibrium setting from the CF model essentially follows the framework of the costly state verification (CSV) model by Townsend (1979). In order to introduce the asymmetric information problem, we assume that net worth is sufficiently small so that entrepreneurs would like to to receive some external financing from firms.

Unlike the original C-F model, it is assumed that the composite investment good i_t, consists of the domestic investment good i_t^d and imported investment good i_t^d. More specifically, we assume that i_t is a Leontief function of i_t^d and i_t^f, for every period t. That is,

$$i_t = min(\kappa_d i_t^d, \kappa_f i_t^f), \ \forall t \tag{12}$$

This further implies that $i_t^d = \frac{\kappa_f}{\kappa_d} i_t^f$ at the optimal condition. We denote the characteristic interest rate in the financial contract by r_t^k. Therefore, an entrepreneur who borrows the amount $(1 + \frac{\kappa_f}{\kappa_d}) i_t - n_t$ agrees to repay $(1 + r_t^k)(i_t - n_t)$ capital goods to the lender. The entrepreneur chooses to default if the stochastic efficiency technology $\omega < \frac{(1+r^k)(i-n_t)}{i}$, where $\bar{\omega} = (1+r^k)(i-n)/i$ is the cut-off value of ω for entrepreneur to decide whether to default or not. The lender will monitor the project outcome only if the entrepreneur defaults. The contract is therefore characterized by i and ω. Under the contract, expected entrepreneurial income is given by

$$q[\int_{\bar{\omega}_t}^{\infty} \omega i_t \Phi(\omega) d\omega - (1-\Phi)(1+r^k)(i_t - n_t)] \tag{13}$$

Substituting $\bar{\omega}_t i_t/(i_t - n_t) = (1 + r^k)$, we have:

$$q_t i_t f(\bar{\omega}_t) = q_t i_t [\int_{\bar{\omega}_t}^{\infty} \omega \Phi(\omega) d\omega - (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t] \tag{14}$$

where $f(\bar{\omega}_t)$ is interpreted as the fraction of the expected net capital output received by the entrepreneur. Similarly, we can write the expected income of a lender on the financial contract as $q_t i_t g(\bar{\omega}_t)$, where we define $g(\bar{\omega}_t)$ as

$$g(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} \omega \Phi(\omega) d\omega - \Phi(\bar{\omega}_t)\mu + (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t \tag{15}$$

which is interpreted as the fraction of the expected net capital output received by the lender. Therefore, we have

$$f(\bar{\omega}_t) + g(\bar{\omega}_t) = 1 - \Phi(\bar{\omega}_t)\mu \tag{16}$$

The equation above basically tells us that $\Phi(\bar{\omega}_t)\mu$ is the fraction of capital wasted by monitoring while the remaining capital is split by the entrepreneur/borrower and the lender.

From one of the many potential contracts, there exists an optimal contract that maximizes the expected income to the entrepreneur while leaving the lender indifferent between lending or retaining the necessary funds. Therefore, the optimal contract is given by choosing the i_t and $\bar{\omega}_t$ that maximize $q_t i_t f(\bar{\omega}_t)$ subject to $q_t i_t g(\bar{\omega}_t) \geq (i_t - n_t)$.

Alternatively, we can rewrite the optimization problem as the following:

$$\max_{i_t^d, \bar{\omega}_t} \quad q_t \kappa_d i_t^d f(\bar{\omega}_t)$$
$$\text{subject to} \quad (1 + \frac{\kappa_d}{\kappa_f} tot_t) i_t^d - n_t \leq q_t \kappa_d i_t^d g(\bar{\omega}_t), \ \forall t.$$

Solving the optimization problem above, assuming the existence of interior solutions, we can derive the following two first order conditions:

$$q_t \kappa_d i_t^d f'(\bar{\omega}_t) = -m_t q_t \kappa_d i_t^d g'(\bar{\omega}_t) \tag{17}$$

$$q_t \kappa^d f(\bar{\omega}_t) = -m_t (1 + \frac{\kappa_d}{\kappa_f} tot_t - q_t \kappa^d g(\bar{\omega}_t)) \tag{18}$$

where m_t is the Lagrangian multiplier/shadow price. Using the definition of $f(\bar{\omega}_t)$ and $g(\bar{\omega}_t)$, we can rewrite condition (17) as:

$$\frac{1}{m_t} = 1 - \frac{\phi(\bar{\omega}_t)}{1 - \Phi(\bar{\omega}_t)} \mu \tag{19}$$

Substituting equation (19) into the the first order condition (18) to eliminate m_t, we obtain the following desired equation:

$$q_t \kappa_d \{1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu [\frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)}]\} = 1 + \frac{\kappa_d}{\kappa_f} tot_t \tag{20}$$

Moreover, rewriting the binding constraint $(1 + \frac{\kappa_d}{\kappa_f} tot_t) i_t^d - n_t = q_t \kappa_d i_t^d g(\bar{\omega}_t)$ in terms of i_t^d, we have:

$$i_t^d = \{\frac{1}{[1 + \frac{\kappa_d}{\kappa_f} tot_t - q_t g(\bar{\omega}_t)]}\} n_t \tag{21}$$

Equation (20) defines an implicit function $\bar{\omega}_t(q_t)$, with $\bar{\omega}_t$ increasing in q_t. Substituting this function into (21), we have the implicit function $i_t(q_t, n_t)$, which represents the amount of consumption goods placed into the capital technology. Together, this implies that investment supply is an increasing function of the price of capital, q_t, of net worth, n_t, and a decreasing function of the terms of trade, tot_t. Thus, differently from the original CF model, the investment supply also depends on the exogenous terms of trade.

It is also worth noticing that the expected return on internal funds rif_t is given by

$$rif_t = \frac{q_t f(\bar{\omega}) \kappa_d i_t^d}{n_t} = \frac{\kappa^d f(\bar{\omega}_t) q_t}{1 + \frac{\kappa^d}{\kappa^f} tot_t - \kappa^d g(\bar{\omega}_t) q_t} \tag{22}$$

2.5 The General Equilibrium

In this section, we provide a detailed description of the general equilibrium of our model. The general equilibrium involves the resolution of the firm's problem, the optimal contract problem, the consumer/lender problem, the entrepreneur's problem together with the market clearing conditions.

Aggregate production is normalized to unity, with a continuum of agents divided between η entrepreneurs and $(1-\eta)$ consumers. Therefore, the market clearing conditions of the labour market is:

$$H_t = (1-\eta) h_t \tag{23}$$

Secondly, the economy's trade balance, TB_t must reflect the difference between exports and imports. That is,

$$TB_t = F_t(K_t, H_t) - (1-\eta)(cd_t + tot_t cf_t) - \eta(ed_t + i_t^d + tot_t(ef_t + i_t^f)) \tag{24}$$

To obtain the equation above, notice that $F_t(K_t, H_t) - (1-\eta)cd_t - \eta ed_t - \eta i_t^d$ is the amount of exported consumption goods and investment goods while $\eta(tot_t(ef_t + i_t^f) + (1-\eta)i_t^d$ is the amount of imported consumption goods and investment goods. The difference between these two terms yields the trade balance. Then we can rewrite the difference to obtain the equation above. Consequently, we define our aggregate output as

$$Y_t = C_t + I_t + TB_t \tag{25}$$

where $C_t = (1-\eta)(c_t^f + c_t^d) + \eta(c_t^e)$ is the aggregate consumption and $I_t = \eta i_t$ is the aggregate level of investment. Next, we also need to take into account the law of motion for capital accumulation and the aggregate resource constraint:

$$K_{t+1} = (1-\eta)K_t + \eta[1 - \Phi(\bar{\omega})\mu]i_t \tag{26}$$

$$C_t + I_t + B_{t-1} = w_t H_t + r_t K_t + B_t R_t \tag{27}$$

It is well documented in the literature that external debt in a small open economy is indeterminate when β and the world interest rate are exogenous. In a deterministic setting, agents would borrow or lend indefinitely depending on whether $\beta < R$ or $\beta > R$ while the small country international indebtedness would stay constant at its exogenously given initial value if $\beta = R$. To avoid this feature of the model and obtain a determinate level for the country's external debt, we follow Carmichael and Samson (2002) and make the *adhoc* but reasonable assumption that the implicit interest rate is determined by the following formulation:

$$R_{t+1} = R_t^* e^{-\zeta B_{t-1} - \kappa[B_t - B_{t-1}]} \tag{28}$$

where $R_t^* = \dfrac{1}{1 + r^* + v_t^r}$ is the world benchmark discount factor and r^* is the associated world level real interest rate whereas v_t^r is an AR(1) shock process which will be given more detail below. The formulation above states that the interest rate at which individual consumers can borrow internationally depends negatively on the world benchmark discount factor R^* and positively on the level and the change in the country's aggregate outstanding debt $B_t = (1-\eta)b_t$. With this assumption, the world benchmark factor is only available to consumers in countries with no outstanding debt ($B_t = 0$) and zero current aggregate borrowing ($B_t - B_{t-1} = 0$), for all the states where the variables I solve for with these aforementioned equations.

2.6 Shocks

In this section, we introduce different types of shocks into our model economy. First off, the productivity shocks driving the TFP of consumption goods are assumed to follow a stationary first-order autoregression process given by:

$$log(A_t) = \rho_a log(A_{t-1}) + \epsilon_t^a \tag{29}$$

where ϵ_t^a is the error term that has mean 0 and standard deviation σ^a The TOT shocks are also assumed to follow an AR(1) process, which is given by:

$$log(tot_t) = \rho_p log(tot_{t-1}) + \epsilon_t^p \tag{30}$$

The concept of risk shocks is now introduced. We relax the assumption that the idiosyncratic risk is fixed in the CF model and assume that it is time-varying and the risk shock process has the following specification:

$$\sigma_t^\omega = \sigma^\omega + v_t^s \tag{31}$$

where ϵ_t^s is the associated disturbance and σ^ω is the steady-state level dispersion of ω. Another shock process we consider is the the wealth shock proposed by Carlstrom and Fuerst (1997). They consider a one-time 0.1 percent redistribution of capital from household to entrepreneur. We would like to assume that the wealth shocks v_t^n are independently and identically distributed as a white noise process with mean 0 and standard deviation σ^n. [3] The wealth shock term v_t^n is added to the net worth constraint equation (8).

Lastly comes the introduction of the foreign real interest rate shocks v_t^r. It is assumed that the external interest rate shocks v_t^r have the following specification :

$$v_t^r = \rho_r v_{t-1}^r + \epsilon_t^r \tag{32}$$

where ϵ_t^r denotes the disturbance term that has mean 0 and standard deviation σ^r.

3 Calibration

In this section, we provide the methodology of the calibration of the model parameters. We calibrate most of the parameters to match the Canadian economic data in the long run, whereas alternatively we also estimate the parameters of the shock processes based on Bayesian estimation corresponding to each suggested time period. The subsections below discuss the data description as well as the descriptions of calibrations in turn.

3.1 Data Description

In this paper, we utilize the time series data of 7 main variables to facilitate the calibration and estimation. These 7 variables are real output Y_t, the terms of trade

[3] Another reason is because we use TSX composite index as an indicator for net worth. Stock prices usually follow a random walk and have low to zero persistence. We therefore prefer not to assume it is an AR(1) process

tot_t, the total factor productivity (TFP), the world level real interest rate, the net worth n_t, the world interest rate r_t^* , as well as the idiosyncratic risk σ_t^ω.

For the variable Y_t, we use the quarterly Canadian real GDP obtained from the CANSIM database. Since the data of A_t are not available, we use the labour productivity data obtained from the CANSIM database as a proxy for TFP instead. The entrepreneurial net worth is also not available. In Christiano (2014), the Dow Jones Wilshire 5000 index is served as the indicator for the American entrepreneurial net worth. For the Canadian counterpart, we would like to use the TSX/SP500 composite index. As for the world level real interest rate, the U.S. real interest rate can be a robust proxy. We calculate the desired data series based on the quarterly federal funds rate retrieved from the St. Louis Fred database.[4] For the Canadian real interest rate, we calculate the quarterly data series by using the monthly 3-month T-bill rate, which is available on CANSIM as well. Last but not least, we compute the quarterly implied volatility of the TSX composite index as the proxy for the idiosyncratic risk.

3.2 Benchmark Calibration

We try to fix all the parameters in the benchmark calibration case. Alternatively, we would also estimate the parameters of the the shock processes based on Bayesian technique. We would like to see how the importance of the shocks evolve in different suggested time periods. The parameters we fix are β, ϵ, α, θ, ψ, ζ, κ, σ^ω, μ, γ, κ_d, κ_f , η, r^*, and λ .

First of all, we fix the discount factor $\beta = 0.9916$ to reflect the average quarterly real interest of around 0.08% in Canada from 1992-Q1 to 2016-Q1. The risk aversion coefficient is set at $\epsilon = 1$. The capital income share α is chosen to be 0.36. The values for β, ϵ and α are well within the acceptable range found in the literature. The parameter θ measures the household's preference over domestic and foreign con-

[4] we collect the monthly data on inflation and then convert them into quarterly frequency, then we apply Fisher equation to approximate the U.S. real interest rate

sumption goods. Following an approach similar to Corsetti et al. (2008), we calibrate $1-\theta$ so that it matches the average value of Canadian imports of goods and services as percentage of GDP from 2000 to 2015. [5] As a result, we obtain $1-\theta = 0.33125$ so $\theta = 0.66875$. The depreciation rate δ is fixed at 0.025.

Some of parameters are selected so that the steady state equilibrium is compatible with some stylized facts. We set $\psi = 1.77$ so that it complies with the consensus in the literature that household generally contribute around 33% of their lifetime to labour. As for the value of ζ and κ, Carmichael and Samson (2002) set ζ equal to 0.004 to reflect the ratio of net foreign debt to GDP reported by Macklem (1993). However, the net foreign debt to GDP ratio has changed significantly in the recent years in Canada. Due to the fact that the net foreign debt to GDP ratio has been much smaller in the recent years, we choose $\zeta = 0.017$ so that it approximately reflects the average net foreign investment position as percentage of GDP in Canada in the past 15 years.[6] With regard to the quantity of κ, we follow Carmichael and Samson (2002) to set it at 0.1.

We specify the distribution $\Phi(\omega)$ to be log-normal. Since $\Phi(\omega)$ has a mean of unity, the corresponding CDF and density function have the following forms:

$$\phi(\omega) = \frac{1}{\omega\sqrt{2\pi S_t^2}} e^{-\dfrac{(ln(\omega)-\mu_\omega)}{2(\sigma_t^\omega)^2}} \tag{33}$$

$$\Phi(\omega) = \frac{1}{2} + \frac{1}{2} ERF[\frac{ln(\omega)-\mu_\omega}{\sqrt{2(\sigma_t^\omega)^2}}] \tag{34}$$

When an entrepreneur's realized productivity is below the threshold $\bar{\omega}_t$, he defaults on the contract. This triggers the monitoring by the CMF. At the aggregate level, the monitoring costs is equal to $\mu q_t \Phi(\bar{\omega}_t)$. Given the fact that there is a broad possible

[5]The data for imports of goods and services as percentage of GDP can be found on World Bank website: http://data.worldbank.org/indicator/NE.IMP.GNFS.ZS?locations=CA

[6]Quarterly data for net foreign investment position are available on CANSIM

range for the monitoring cost μ, we adopt the benchmark value reported in Carlstrom and Fuerst (1997), setting μ equal to 0.25.

Furthermore, we choose σ^ω to match the average quarterly bankruptcy rate from 2005 to 2009 in Canada. The average quarterly bankruptcy rate in Canada is around 0.8 percentage point.[7] Accordingly, we set σ^ω equal to 0.1943.[8] For the value of γ, we set it equal to the inverse of the internal rate of return, or equivalently,

$$\frac{qf(\bar{\omega})i}{n} = \frac{1}{\gamma} \tag{35}$$

Accordingly, we approximate the value and set $\gamma = 0.952$. Now we are left with the calibrations of κ_d, κ_f, λ η, r^* for the group of fixed parameters.

We follow Carmichael and Samson (2002) to set $\kappa_d = \kappa_f = 2$ due to numerical restrictions.[9] As for λ, we assume half of the entrepreneurial consumption comes from imported goods and therefore we fix λ at 0.5. We assume the entrepreneurs account for 10% of the population in our model economy and therefore we set the scale parameter $\eta = 0.1$. Lastly, most people calibrate r^* to be 0.01 to reflect a conventional benchmark world interest rate of 4% in the past DSGE literature. However, the real interest rates have dropped drastically over the decades globally and therefore we decide to choose r^* to be equal to 0.00661 to reflect an average annual "world" interest rate of 2.64% estimated by King and Low (2014).[10]

Next, we would like to fix the parameters of the shock processes for the benchmark calibration. We fit the terms of trade data with an AR(1) process based on the sample period 1992Q1 to 2016Q1. The estimated persistence parameter ρ^p is 0.98 and the associated standard deviation of the innovation σ^p is approximately equal to 0.02.

[7] Data source for annual business bankruptcy rate in Canada: https://www.ic.gc.ca/eic/site/bsf-osb.nsf/eng/br01821.html

[8] This is an approximated value. We approximate this value based on a similar method shown by Salyer et al. (2008).

[9] For further details of the numerical issues, see Carmichael and Samson (2002).

[10] We take the average of all the estimated values across all the periods ranging from 1992 to 2013

We calibrate the parameters of the productivity shocks by using the labour productivity data from 1992Q1 to 2016Q1. First, we obtain the residuals from fitting the labour productivity with a linear trend. Then we fit the residuals with an AR(1) process. The estimated persistence parameter ρ^a is equal to 0.948 whereas the estimated standard deviation of the disturbance is 0.0076.

As for the standard deviation of the risk shock process σ_s, the implied volatility of the return on TSX composite index is used to measure this quantity. We choose its quantity to match the standard deviation of the quarterly implied volatility from 1992Q1 to 2016Q1, which is equal to 0.003948.

Penultimately comes the benchmark calibration of the wealth shock process. We detrend the quarterly TSX composite index by taking the first difference. The standard deviation of the wealth shock process σ_n, is set to match the standard deviation of the first-differenced series from 1992-Q1 to 2016-Q1, which is equal to around 0.007.

We close this section with the calibration of the external interest rate shocks. We detrend the quarterly real federal funds rate data series by taking the first difference. Subsequently, we fit it with an AR(1) process, the estimated persistence parameter ρ_r is equal to around 0.6. The estimated standard deviation of the innovation is equal to 0.001.

3.3 Bayesian Estimation

In addition to the traditional calibration method, we also explore the use of Bayesian technique to estimate the parameters of the shock processes.

Among all the parameters, $\{\rho_a, \rho_p, \rho_r, \sigma_a, \sigma_p, \sigma_s, \sigma_r, \sigma_n\}$ are the parameters we are mostly interested in estimating based on Bayesian technique. We are interested in estimating the parameters of the shock processes to see how they vary across different periods.

The Bayesian estimation is executed based on the Markov Chain Monte Carlo

Table 1: Benchmark Calibration

Parameter	Value	Parameter	Value
β	0.9916	η	0.1
α	0.36	λ	0.5
ϵ	1	r^*	0.00661
θ	0.66875	ρ_p	0.98
ψ	1.77	σ^p	0.02
ζ	0.017	ρ^a	0.948
κ	0.1	σ^a	0.0076
μ	0.25	σ^s	0.003948
σ^ω	0.1943	σ^n	0.007
γ	0.952	ρ^r	0.6
κ_d	2	σ^r	0.001
κ_f	2		

(MCMC) simulation method. The algorithm we choose to use is the Metropolis Hastings Algorithm. Considering the complexity of our model, we set the number of iterations for performing the algorithm at 50000 in order to ensure the convergence of the MCMC chains. We compute the posterior mode using the default optimization routine in Matlab. We set the burn-in rate to be 0.3 and we allow two parallel chains for Metropolis-Hastings algorithm. Besides, we adjust the scale parameter of the jumping distribution each time for different estimation periods, in order to get close to the optimal acceptance rate of 0.234 suggested by Roberts, Gelman & Gilks (1997).

The observable data we use are the real GDP per capita, TSX/SP 500 composite index, and the Canadian terms of trade index. All observable variables are first-differenced in logarithmic forms and demeaned.

Following the standard convention of the prior distribution assignment, we assign a Beta distribution for parameters that fall between zero and one, such as the persistence parameters. We use an Inverse Gamma distribution for parameters that are strictly positive, such as the standard errors of the shock processes. All the standard error parameters are assigned a mean of 0.01 and a standard deviation of 2 whereas the means of the prior distributions of all the persistence parameters are set at 0.75

with a standard deviation of 0.15. A table containing detailed description of the prior distributions can be found below.

Table 2: Prior Distribution

Parameter	Distribution	Mean	STD
ρ_a	Beta	0.75	0.15
ρ_p	Beta	0.75	0.15
ρ_r	Beta	0.75	0.15
σ_a	Inv.Gamma	0.01	2
σ_p	Inv.Gamma	0.01	2
σ_s	Inv.Gamma	0.01	2
σ_n	Inv.Gamma	0.01	2
σ_r	Inv.Gamma	0.01	2

4 Benchmark Results

An analysis of the model based on benchmark calibration is ready to be revealed in this section. The analysis can be dichotomized into two different aspects: 1. Impulse Response Functions (IRFs) and 2. Unconditional and conditional Forecast Error Variance Decompositions (FEVDs) for output. We will engage in deeper discussion in the following subsections.

Figure 13 to 17 in Appendix report the results of the IRFs of the productivity shocks, TOT shocks, wealth shocks, risk shocks and external interest rate shocks respectively. Figure 1 and 2 in the following subsections unravel the results of the FEVDs. We use the unconditional variance decomposition to analyze the explanatory power of different types of shocks in the long run. For the short run analysis, we turn to the conditional variance decompositions of output conditioning on the first period.

4.1 Impulse Response Functions (IRFs)

4.1.1 Productivity Shocks and Wealth Shocks

For the IRFs of the productivity shocks, we can refer to Figure 13. First of all, we can see that the impacts of the productivity shocks in our model are able to replicate almost the exact same dynamics as the ones in Carlstrom and Fuerst (1997). A 0.76% positive increase in productivity enables a roughly 1.27% initial increase in output and the impulse dies out gradually after its peak due to the autoregressive nature of the shock process. One distinct feature of the agency cost, as documented by Carlstrom and Fuerst (1997), is that it entails the hump-shape response function for investment, which leads to the "reverse hump" in household consumption after its initial increase.

Other than the similarities as described in the original CF model, one interesting feature of the dynamics can be observed through the IRFs. Unlike the CF model, the aggregate consumption decreases initially due to the productivity shocks but increases rapidly and imitates the dynamics in CF model thereafter. The initial decrease in aggregate consumption might be counter-intuitive. The straightforward answer for this phenomenon can be explained by the IRFs of the household consumption and the entrepreneurial consumption in Figure 13. It can be observed that a 0.4% decrease in entrepreneurial consumption dominates the 0.22% increase in household consumption initially. The underlying numerical explanations for this counter-intuitive result are twofold. Firstly, it is worth recalling our aggregate resource constraint, equation (24), is different in our small open economy context in that the introduction of external debt mitigates the impact of productivity shocks on the household consumption. Secondly, the absence of the household consumption of tradable goods in our model economy results in the loss of the responsiveness of household consumption to any kinds of shock processes.[11] Since our main focus of this paper is to investigate the explanatory

[11] We try to incorporate tradable sector but due to the complexity of the model, we decide to stick

power of the proposed types of shocks, this minor counter-intuitive distinction from the CF model is not a main concern.

The wealth shocks mimic the features from the CF model as we can see from Figure 16. As mentioned previously, the investment supply curve is a function of net worth. We can see from Figure 14 that a sudden 0.7% increase in net worth shifts the investment supply curve, hence a 0.19% initial increase in investment and a 0.00041685% initial decrease in the price of capital. Since the households invest more, they are supposed to reduce their consumption initially. However, it should be noted that the wealth shocks in our model are slightly different from the wealth shocks in the CF model. Our wealth shocks are a one-time increase in the entrepreneurial wealth whereas the wealth shocks in the CF model are the redistribution from households' capital to the entrepreneurial net worth. Hence, in contrast to dynamics of the household consumption in the CF model, the household consumption actually increases and in turn the household's labour supply decreases. It should also be recalled that the resource budget constraint is different in the small open economy. A tiny 0.4 basis points decrease in price of capital is associated with a 2.6 % decrease in the domestic interest rate. It further translates into a lower demand for international assets B_t. The reduction in B_t also boosts up the household consumption. In particular, the external debt reduces by 0.0024508% and the household consumption increases by 0.0071069% initially.

4.1.2 Terms of Trade shocks

The next type of shock we would like to investigate is the TOT shock. The TOT shocks in our model basically replicate the results from Carmichael and Samson (2002). A 1% increase in TOT can either be interpreted as a 1% increase in im-

with the current model.

port price index or a 1% decrease in export price index in the context of this paper.[12] Figure 14 in the Appendices provides the graphical representations of the IRFs for TOT shocks.

A positive shock to the terms of trade makes the composite investment good more expensive. As a result, the entrepreneurs purchase less investment goods and therefore produce less capital. The lower demand for investment goods drives the price of capital downward by 0.94%. Due to the lower capital price, the risk neutral nature of the entrepreneur also implies that they would consume more in the present and hold less investment goods in the future. A 2% increase in terms of trade results in a 0.49% increase in the entrepreneurial consumption immediately as the TOT shocks come into effect.

The net worth makes an increase initially due to the temporal high price of capital. The fall in investment, however, creates a channel through which the TOT shocks induce a negative impact on the net worth subsequently due to the lower capital stock. The rise in capital price also stimulates consumption spending and discourages capital accumulation. Given the nature of the AR(1) process, the impulses eventually die out and most of the variables return to the steady state level. Particularly, a 2% increase in the terms of trade results in a 0.97% initial increase in net worth, a 0.24% initial decrease in investment, a 0.94% increase in capital price and a 0.83% decrease in output.

4.1.3 Risk Shocks

One of our main objectives in this paper is to study how idiosyncratic risk shocks influence the dynamics in a small open economy like Canada. Figure 15 highlights the IRFs under the impact of risk shocks in our model economy.

A sudden 0.3948% increase in idiosyncratic risk means a higher dispersion of

[12] Once again, it should be reminded that in this paper, the terms of trade is defined as ratio of import price index to export price index, which is the opposite of the traditional definition.

the entrepreneurial productivity. The increased uncertainty discourages some of the lenders from lending out their capital, which means a decline in $g(\bar{\omega})$. A fall in $g(\bar{\omega})$ in turn reflects a rise in $f(\bar{\omega})$. As a result, higher proportion of the net capital goods are received by entrepreneurs/borrowers, which leads to the increase in entrepreneurial wealth. The capital price makes a tiny rise initially as the investment supply decreases. The drop in investment can be explained by the fact that higher productivity uncertainty discourages entrepreneurs from purchasing investment goods.

The decrease in entrepreneurial consumption can be analyzed via equation(16) and equation (21). The decrease in investment counteracts the positive increase in $f(\bar{\omega})$. Due to the Leontief function of the investment goods and the risk neutral preference, The entrepreneurs also reduce their foreign investment and foreign consumption, which induces a slight increase in balance of trade at the first period. The initial minor increase in output is therefore attributed to the improved trade balance in the first period. As a consequence of the higher capital price as well as the higher rental rate of capital due to the diminishing return of capital , the real domestic interest rate is lower according to the first order conditions of the household. Hence, this encourages household to purchase more of the external asset and therefore they reduce both of their domestic and foreign consumption. As a result, the output decreases following the decrease in aggregate consumption and aggregate investment. In particular, a 0.3948% increase in risk is associated with a 0.019% initial increase in output and a -0.00985% deviation from the steady state level of output in the second period. It also results in a 0.49% decrease in investment, which implies that investment is very elastic towards the change in idosyncratic risks in our model since 1% increase in risk leads to more than a 1% initial decrease in investment.

4.1.4 External Interest Rate Shocks

We end this subsection with the analysis of the impact of the external real interest rate (EIR) shocks.

As we can see from Figure 17, the sudden increase in external real interest rate entails the subsequent increase in domestic real interest rate in a small open economy. Meanwhile, the price of capital decreases as the domestic real interest rate increases. The entrepreneur reduce their purchase of investment and entrepreneurial consumption due to their initial decrease in net worth. Investment is also lower due to the increase in domestic real interest rate. The initial decrease in net worth results from the decreased share of net capital income received by the entrepreneurs. On the other hand, the consumers reduce their consumption and lower foreign interest rate attract them to purchase more of the external debt. As a consequence, the aggregate consumption drops initially, responding to the foreign real interest rate shocks. However, The aggregate output increases initially due to the improved trade balance. More specifically, a 0.01% increase in real exchange rate is associated with a 0.12% initial increase in output, a 0.14% decrease in net worth, a 0.03% decrease in consumption and a 0.4% initial decrease in investment.

4.2 Variance Decomposition

In this subsection, we analyze the explanatory power of the shocks in driving the output variation. This is done by examining the variance decomposition. Later in the next section, we would present the results of the variance composition based on Bayesian Estimation of the DSGE model and the empirical reduced form vector autoregressive model.

The first thing we need to look at is the unconditional variance decomposition. The unconditional variance decomposition is the asymptotic variance decomposition based on infinite horizons. It shows the fraction of the variance of each variable that each shock would explain in an infinitely long simulation of our model economy. Hence, we regard the unconditional variance decomposition as the long run variance decomposition. The second thing we need to look at is conditional variance decomposition. The conditional variance decomposition is based on finite horizons, conditional on the present period. In the analysis below, we define the short-run variance decompositions as the variance decompositions conditioned on the first period.

The results of the long-run variance decomposition are visualized in the figures below. On one hand, we can see that the TOT shocks are the most influential shocks in the long run business cycle, explaining more than 50% of the output variance. The productivity shocks come the second with around 38.48% . On the other hand, it is interesting to see that our model predicts that both the wealth shocks and the risk shocks have extremely low power in affecting the output fluctuations. The risk shocks account for 0.01% of the output variance in the long run whereas the wealth shocks have almost 0.01% explanatory power in output variance. Besides, The real interest rate shocks explain a tiny 0.03% of the output variance.

The low contribution of the foreign real interest rate shocks can be due to the orthogonality of the shocks. Empirically, terms of trade is usually highly correlated with foreign real interest rate. An alternative approach would be to assume correlation between the TOT shocks and EIR shocks. We will discuss this topic more in the last subsection.

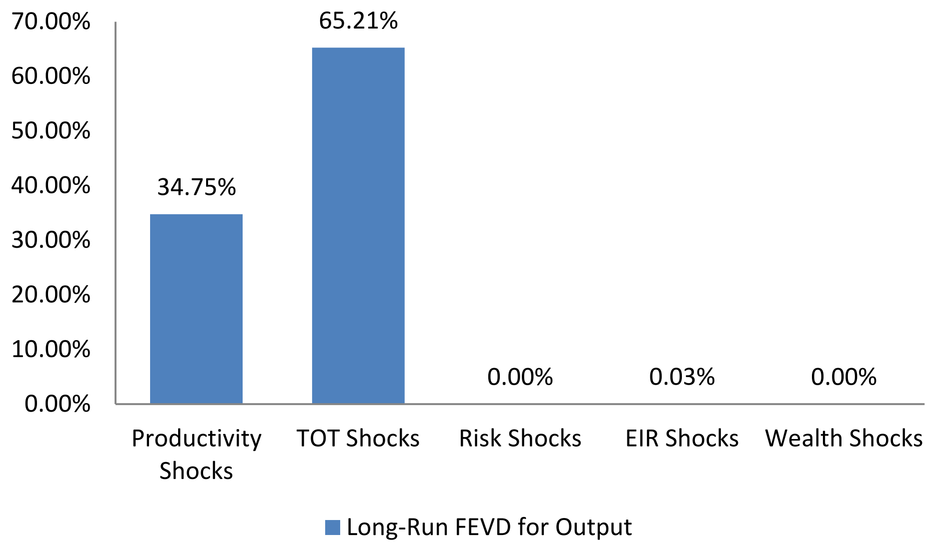

Figure 1: Long-run Variance Decomposition

For the results of the short-run variance decomposition, the story becomes different. The productivity shocks become the most influential source of output variation in the short run, with almost 70% of the output fluctuation explained by the productivity shocks in the first period while the influence of the productivity shocks decrease gradually over the periods. The second most important shocks are the TOT shocks, accounting for 29.81 % of output variance. In addition to that, the amount of output variance explained by the productivity shocks is on a gradual rise over the first eight periods. The third most important shocks in the short run are the EIR shocks, which contributes a tiny 0.66% of the output variance in the first period. The wealth shocks explains almost 0% of the output variance. The risk shocks contributes a tiny 0.02% of the variance at the very first period and stay at 0.01% after period 3.

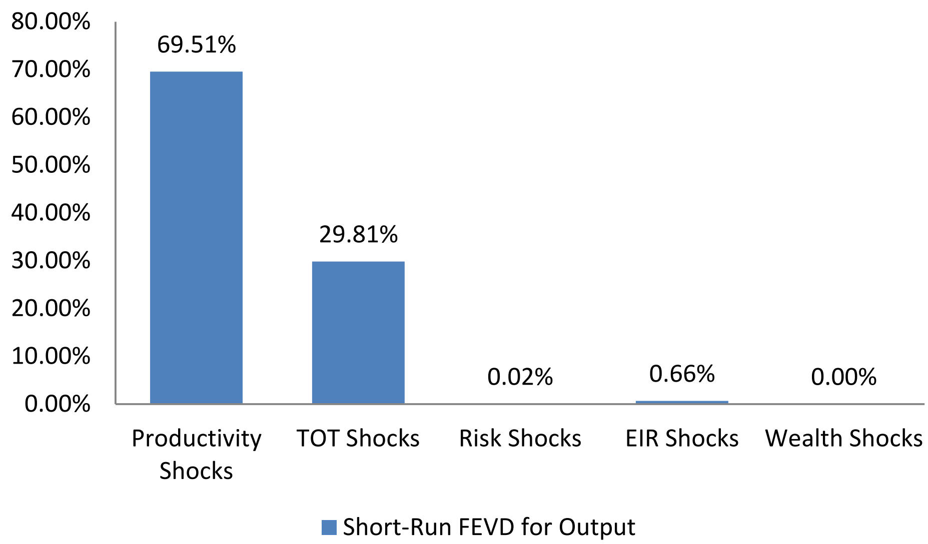

Figure 2: Short-Run Variance Decomposition

On the one hand, it is obvious from the above description that the terms of trade shocks have a much bigger impact on output fluctuations in the long run. In the short run, the impact is smaller but it is amplified as time evolves. On the other hand, the impacts of other types of shocks are bigger in the short run than in the long run. As the time period evolves, their output variance contributions are “absorbed” by the terms of trade shocks.

How relatively large should the magnitude of the risk shocks be so that they will capture 50% of the output fluctuations? To answer this question, we do a little experiment in the very end of this subsection. We set the standard deviation for each type of shock equal to 0.001 and then we increase the standard deviation of the risk shocks by 0.001 each time to see how the variance decomposition changes. In the long run, the results of the experiment suggest that the magnitude risk shocks have to be 85 times as large as other types of shocks to capture 50% of the output fluctuations. In the short run, the results of the experiment suggest that the magnitude of the risk shocks have to be 44 times as large as other types of shocks. The results also imply that risk shocks are more influential in the short run than in the long run. One possible explanation for this could be the features of the model. It is possible that the risk shocks tend to be downplayed due to the agency cost mechanism as introduced in the CF model.

4.3 Alternative Approach: Correlated Shocks

It is possible that the TOT shocks and the EIR shocks are highly correlated. In fact, the correlation between terms of trade and foreign interest rate from 1992Q2 to 2016Q1 is estimated to be as high as -0.826891! In this section, we will briefly discuss the results of the model with correlated shocks. Accordingly, we specify $Corr(\sigma^p, \sigma^r) = -0.8$.[13] Other parameters are set at the same value as in the benchmark calibration case in the section above. Dynare by default uses the Cholesky decomposition scheme to orthogonalize shocks in case of correlated shocks. Hence, the variance decompositions of correlated shocks depend on the ordering of the shocks. It is assumed that shocks to real foreign interest rate are more likely to cause shocks to the terms of trade than the other way around. We thus specify the ordering as: $\sigma^r \to \sigma^p$.

The discussion of the results of the IRFs is skipped since the only difference from the model with orthogonalized shocks is the change in magnitude of the shocks and the dynamics is exactly the same as the IRFs of the model with uncorrelated shocks. The results of variance decompositions can be seen below.

[13] Due to the time constraint of the project, I only calibrate this value instead of estimating it. Time permitting, I could have estimated the correlation by Bayesian technique.

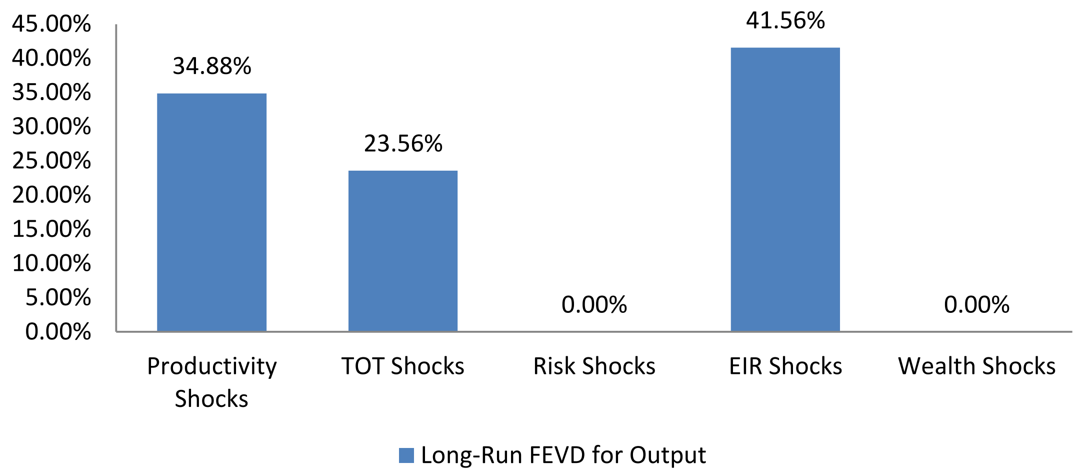

Figure 3: Short-Run Variance Decomposition: Correlated EIR and TOT Shocks

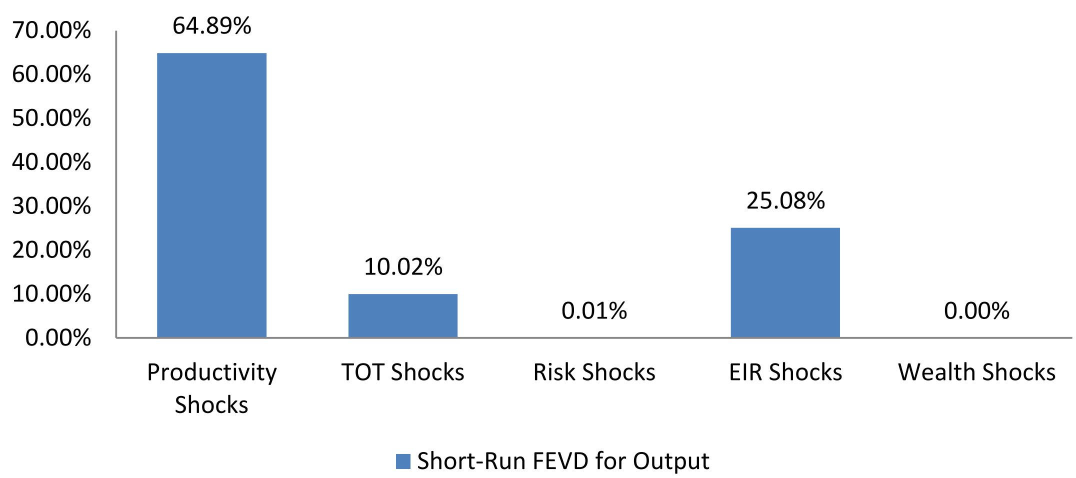

Figure 4: Short-Run Variance Decomposition: Correlated EIR and TOT Shocks

As we can see from above, once we correlate the EIR and TOT shocks, the contributions of EIR shocks and TOT shocks to output fluctuations become completely different. The EIR Shocks dominate other types of shocks by capturing 41.56% of the output fluctuations in the long run. The third place comes the TOT shocks, which pick up 23.56%. In the short run, the EIR shocks place in the second, explaining 25.08% of the output variance. In light of this result, we can tell that the correlation between foreign interest rate and terms of trade is critical to pin down the external influence in the Canadian economy.

5 Results based on Bayesian Estimation

The Bayesian estimation results tell us a different story about the variance decompositions of output. The three different sample periods our estimation uses are: 1992Q2-2000Q4, 20001Q1-2016Q1, and 1992Q2-2016Q1. There are two main reasons why we choose these time periods: Firstly, we prefer the periods after the early 90s because the inflation targeting regime was executed in Canada around that time. Secondly, ever since the early 2000s, the interest rates in many countries, including Canada, have experienced a persistent declining trend.

The tables below display all the posterior estimates of the parameters for all three different time periods we select. Due to the fact that the Bayesian IRFs replicate the exact same dynamics as the regular IRFs, we skip the discussion of the Bayesian IRFs here and jump to the discussion of the variance decomposition results.

Table 3: Posterior Estimates:1992Q2-2000Q4

Parameter	Posterior Mean	Posterior Mode	90% HPD interval
ρ_a	0.9639	0.9729	[0.9346, 0.9936]
ρ_p	0.8724	0.9012	[0.7724, 0.9846]
ρ_r	0.9265	0.9407	[0.8799, 0.9766]
σ_a	0.0053	0.0049	[0.0041, 0.0055]
σ_p	0.0054	0.0051	[0.0043, 0.0064]
σ_s	0.0085	0.0046	[0.0024, 0.0158]
σ_n	0.044	0.0427	[0.0291. 0.0604]
σ_r	0.0029	0.0026	[0.0020, 0.0038]

Table 4: Posterior Estimates:2001Q4-2016Q1

Parameter	Posterior Mean	Posterior Mode	90% HPD interval
ρ_a	0.9781	0.9709	[0.9588, 0.9976]
ρ_p	0.9372	0.9880	[0.8775, 0.9971]
ρ_r	0.9167	0.9165	[0.8735, 0.9597]
σ_a	0.0049	0.0058	[0.0038, 0.0059]
σ_p	0.0105	0.0103	[0.0090, 0.0121]
σ_s	0.0084	0.0046	[0.0023, 0.0158]
σ_n	0.0396	0.0051	[0.0198. 0.0613]
σ_r	0.0037	0.0047	[0.0026, 0.0047]

Table 5: Posterior Estimates:1992Q2-2016Q1

Parameter	Posterior Mean	Posterior Mode	90% HPD interval
ρ_a	0.9811	0.9849	[0.9659, 0.9971]
ρ_p	0.9603	0.9682	[0.9275, 0.9945]
ρ_r	0.9303	0.9364	[0.8970, 0.9635]
σ_a	0.0051	0.0050	[0.0027, 0.0040]
σ_p	0.0089	0.0088	[0.0078, 0.0099]
σ_s	0.0077	0.0046	[0.0025, 0.0137]
σ_n	0.0439	0.0433	[0.0315, 0.0562]
σ_r	0.0033	0.0032	[0.0027, 0.0040]

5.1 Esimation Period: 1992Q2-2000Q4

Unlike the benchmark results, the Bayesian estimation suggests that the productivity shocks are the major source of output variance during the 90s in Canada, accounting for 88.50% of the variance. In the same sample period, the EIR shocks place in the second, capturing 8.43% of the variance in output, while the third come the TOT shocks, explaining 3% of the variance. Both risk shocks and wealth shocks have extremely low contributions to the output variance for this period.

In the short run, the importance of foreign real interest rate fluctuations are strengthened. 20.97% of the output variance comes from the EIR shocks in the initial period even though its effects die out gradually. The productivity shocks still play the biggest role in the short run and they capture 73.25% of the output variance in the first period and the share rise steadily over the periods. Moreover, the TOT shocks make up 5.40% of the variance for the initial period in the short run. Risk shocks and wealth shocks account for 0.17% and 0.21% of the variance respectively in the first period.

5.2 Estimation Period: 2001Q1-2016Q1

During the post-millennial period, the variance decompositions tell us a different story again. The productivity shocks still play the major role in the long run, occupying 76.96% of the output variance. The TOT shocks place in the second in this period, attributing 15.23% of the variance. The EIR shocks drop to the third place, accounting 7.76% of the variance. Similarly to the 90s, both risk shocks and wealth shocks still have extremely low contributions to output variance in the 2000s.

In the short run, the productivity shocks still make up more than half (51.59%) of the output variance initially and the influence enhanced gradually. The share of the variance generated from foreign real interest rates becomes 28.74% whereas the corresponding share for TOT shocks is 19.37%. Risk shocks and wealth shocks explain 0.15% and 0.16% of the variance respectively.

5.3 Estimation Period: 1992Q2-2016Q1

We also estimate the parameters using the whole sample size. The results again are different but similar to the benchmark results. The TOT shocks become the most important shocks in addition to the productivity shocks, capturing 13.94% of the variance in the long run while the share is 79.91% for the productivity shocks. The EIR shocks only account for 6.11% in the long run. Both Risk shocks and wealth shocks have no impact on output variance.

In the short run, the productivity shocks still dominate in terms of source of variance, making up 58.80% in the initial period. The EIR shocks are the second most influential shocks in the short run, accounting for 26.54% of the variance in the first period, meanwhile the TOT shocks only explain 14.32% of the variance. The risk shocks and wealth shocks attribute 0.13% and 0.21% of the variance respectively.

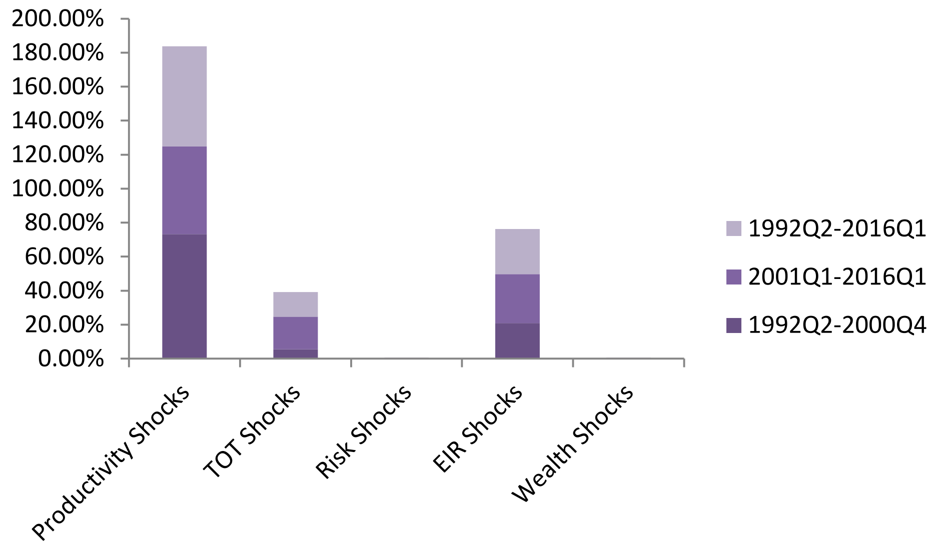

Figure 5: Bayesian Short-Run Variance Decomposition for 3 Different Estimation periods

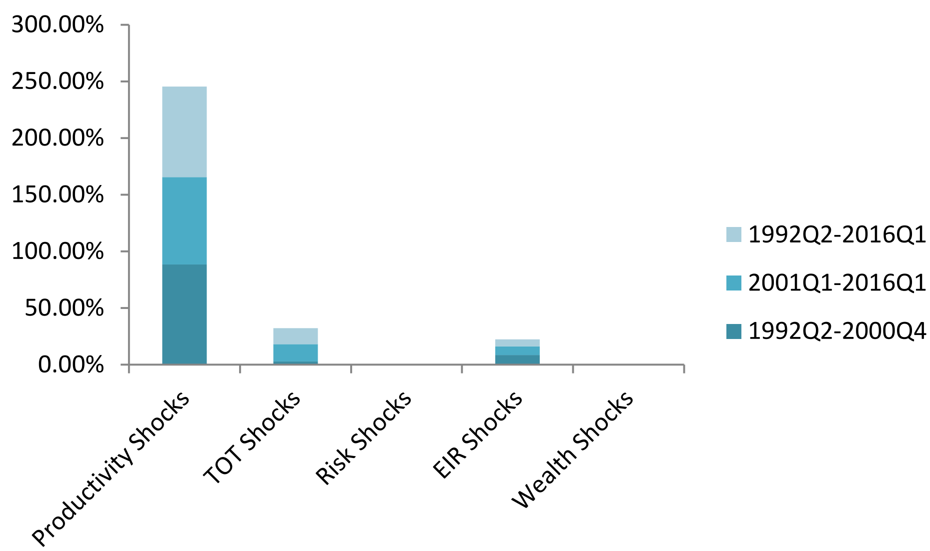

Figure 6: Bayesian Long-run Variance Decomposition for 3 Different Estimation periods

Our Bayesian estimations suggest that in the short run, the foreign interest rate is more important than the terms of trade fluctuations while in the long run the opposite holds true.

6 Evidence from VAR estimation

In order to further verify our findings, we estimate the reduced form VARs based on the 3 proposed sample periods.

The proposed variables of the VAR include output (real GDP), the inverse terms of trade, S&P/TSX composite index, and the idiosyncratic risk (quarterly implied volatility of the return on S&P/TSX composite index) as well as the foreign interest rates (the Fed Funds rate) . Since all the variables except the implied volatility are I(1),[14] to avoid misleading results generated from the possibility of spurious regression, we transform these variables by taking the logarithmic first difference respectively. Furthermore, we choose the optimal order of lags for the VAR estimation based on the conventional VAR lag order selection criteria.[15] We also do post-estimation econometric analysis sush as LM test for serial autocorrelation to guarantee the estimated VARs capture all the dynamics of the variables.

To identify the shocks in our VAR model, we choose the ordering of the Cholesky decomposition as the following: foreign real interest rate (real Fed Funds rate)$\rightarrow$ Output(real GDP) $\rightarrow$ TOT $\rightarrow$ net worth (S&P/TSX price index) $\rightarrow$ Idiosyncratic Risk (TSX Implied Volatility).

6.1 Estimation Results

As for the results, we can take a look at the IRFs of the VAR first. The IRFs of the VAR describe similar dynamics as predicted by our DSGE model, which provides

[14]We apply unit root tests to test stationarity of the variables.

[15]The criteria include Log-likelihood, Likelihood-Ratio, FPE, AIC, SIC, and HQ.

empirical support for our DSGE model.[16] A rise in the TOT growth has a negative impact on output growth.[17] An Increase in net worth growth has positive effect on output growth. A change in real interest rate growth tend to stimulate the output growth.

6.1.1 Estimation Period: 1992Q2-2016Q1

Then we can take a look at the results from the estimation period 1992Q1 to 2016Q1. Aside from the IRFs, the variance decomposition of the VAR also verifies the fact that the terms of trade is one of the most important external sources contributing to the fluctuation of output, explaining roughly 15% of the real GDP fluctuations in Canada. Unlike the variance decomposition result obtained from our DSGE model, however, the estimated VAR suggests that the wealth shocks come after TOT shocks as the second most important shocks, accounting for around 5.1% of the output fluctuation in period 10 although they contribute essentially nothing to the output volatility in the first period. It is also worth noticing that most of the variance is preserved by output itself.

6.1.2 Estimation Period: 1992Q2-2000Q4

For the Canadian economy in the 90's, most of the output fluctuation is still preserved by the output growth itself. The risk shocks become the most influential relative to other types of shocks, contributing 8.75% of the output variance. The foreign real interest rates play a significant role compared to other economic indicators, explaining 8.11% of the output fluctuation in the 10th period. the terms of trade comes the third, making up 7.54% in the 10th period. The net worth (TSX price index) contribute 2.84% in the tenth period. However, due to the small sample size of this sample

[16]However, it should be noted that the variables in the VAR is in first-difference while the variables in the DSGE model is in level

[17]The TOT here is still defined as the inverse of the conventional TOT.

period, it is better to treat the estimation results for this period with a grain of salt.

6.1.3 Estimation Period: 2001Q1-2016Q1

The VAR estimation results based on period 2001-Q1 to 2016-Q1 give us a different story. We choose the lag order of the VAR according to the econometric analysis. The shares of output variance explained by idiosyncratic risks stay at around 7.85% in the post-millennial period. However, the variance decomposition still holds the position that external shocks, including TOT shocks and EIR shocks, are still the most influential driving force for booms and busts of the Canadian economy, explaining 27.65% and 3.902% of the variance in the 10th period although the foreign real interest rates become the least important source of output fluctuation during this period. However, the foreign real interest rates dominates in the first period, accounting for 1.904% of the output variance.

7 Conclusion

In terms of both our theoretical and empirical results, we can draw some interesting conclusions. Both our theoretical and empirical analysis point to the argument that risk shocks are not that important in a small open economy like Canada. However, our reduced form VAR estimation does suggest that idiosyncratic risks play an a large role in the 90s in Canada. Moreover, both Bayesian estimation of the DSGE model and the VAR estimation suggest that, from a long run perspective, the impacts of the foreign real interest rates weaken in the 2000s relative to the impacts in the 90s . However, from a short run perspective, the impacts of the foreign real interest rates actually strengthen in the 2000s relative to the 90s. One interesting reflection generated from this result is that, how effective will the interest rate manipulations by the central banks be in the long run? In addition to the results above, both our theoretical model and empirical estimation establish that the terms of trade have a huge role to play in the Canadian real business cycle.

In light of our results, we can take a look back at the Great Recession and the mild recession in 2014 for Canada. The real culprit behind the Great Recession in Canada is not the domestic risk fluctuations. Instead, it can be more of a contagion transmitted from the U.S. through some external channels such as commodity price shocks and foreign interest rate shocks. The magnitude of domestic risk fluctuations in Canada are small thanks to its robust financial regulation. The fluctuations in the foreign real interest rate is most likely to contribute the most to the development of the Great Recession in Canada. As Beaton and Desroches (2011) point out in their empirical work that movements in U.S. financial conditions, in particular, tend to spill over to Canadian financial conditions. They also find that the short-term interest rate linkages among countries have the most significant impact on Canada's economy. Our results are actually in line with their findings. In the short run, the results of our DSGE model suggest that, the foreign real interest rate shocks are the most influential shocks in the period 2001Q1-2016Q1 other than the productivity shocks. The financial crisis also leads to decline in international trade for Canada, which in turn causes the terms of trade fluctuations. In addition, the recession in the 2014 in Canada is a different episode. It is mostly attributed to the commodity price shocks (in particular, oil price) rather than the financial spillover . This episode witnesses the greatest decline in Canadian dollar ever since 2004. Indeed, commodity price and terms of trade are highly correlated in Canada. In other words, we could argue that terms of trade shocks hold the main responsibility for the emergence of the Canadian recession in 2014.

The results of our theoretical model are, however, subject to caveat. It is worth mentioning that our model does not consider the non-tradable sector in the model economy. As a consequence, the model may overemphasize the importance of the terms of trade in the model economy. Nonetheless, we try to alleviate this underlying problem by doing Bayesian inference using actual observable data. Our Bayesian estimations and VAR estimations still produce similar results to some extent even though they do not agree with each other on everything. Another problem resulted from our model is that the steady state level trade balance is positive in our model whereas the trade balance is always negative for Canada. This is due to the calibration of the low foreign real interest rate. Our model cannot reconcile this problem when real interest rate is set extremely low nowadays. Besides, the agency cost model seems to downplay the role of risk shocks, compared with the model by Christiano et al. (2014).

There are several possible areas of future work which can be explored regarding this topic. First, in the future, we can make the model more realistic by considering the non-tradable sector. Furthermore, we can try to relax the assumption that entrepreneurs are risk-neutral in general. Candian and Dmitriev (2015) have done similar work regarding this aspect. Last but not least, it is also worth including the marginal efficiency of investment (MEI) shocks in our model since the MEI shocks have been discussed vastly in the literature.

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A Appendices

A.1 Methodology of Bayesian Estimation

Let Θ_t denote a vector of observable data and Ψ^M denote the parameters we are interested in estimating. Then applying Bayes theorem, we can obtain the posterior distribution of the structural parameters given the observable data:

$$f(\Psi^M|\Theta_t) = \frac{f(\Theta_t|\Psi^M)f(\Psi^M)}{f(\Theta_t)}$$

where $f(\Psi^M|\Theta_t)$ is the posterior distribution. The likelihood function refers to $f(\Theta_t|\Psi^M)$ and $f(\Psi^M)$ is the prior distribution whereas $f(\Theta_t)$ is the marginal density for the observable data in the model. The likelihood function can be expressed as:

$$f(\Theta_t|\Psi^M) = f(\Theta_0|\Psi^M)\prod_{t=1}^{T} f(\Theta_t|\Theta_{t-1}, \Psi^M)$$

Our perception about the distribution of the parameters forms the prior distribution $f(\Psi^M)$. Furthermore, the marginal density of the observable data can be written as:

$$f(\Theta_t) = \int f(\Theta_t, \Psi^M) d\Psi^M$$

Then we compute the posterior distribution using the Metropolis Hastings algorithm. Since the marginal density is simply a constant, hence from the Bayes theorem, we know that the posterior is always proportional to the product of the likelihood function and the prior:

$$f(\Psi^M|\Theta_t) \propto f(\Theta_t|\Psi^M)f(\Psi^M)$$

Since the posterior is difficult to compute directly, Metropolis Hastings algorithm can now be applied to sample indirectly from the log posterior kernel:

$$log(f(\Theta_t|\Psi^M)) + log(f(\Psi^M))$$

As more and more values are sampled by the Metropolis Hastings algorithm, the law of large numbers and the central limit theorem prove that the distribution of the sample would get closer and closer to the true posterior distribution.

A.2 MCMC Convergence Diagnostics

In this section, we present the Brooks and Gelman (1998) convergence diagnostics for the MCMC chains, [18] the mode check plots and other figures for Bayesian estimation

[18]For more details of the MCMC diagnostics in Dynare, please see Pfeifer (2014).

analyses.[19]

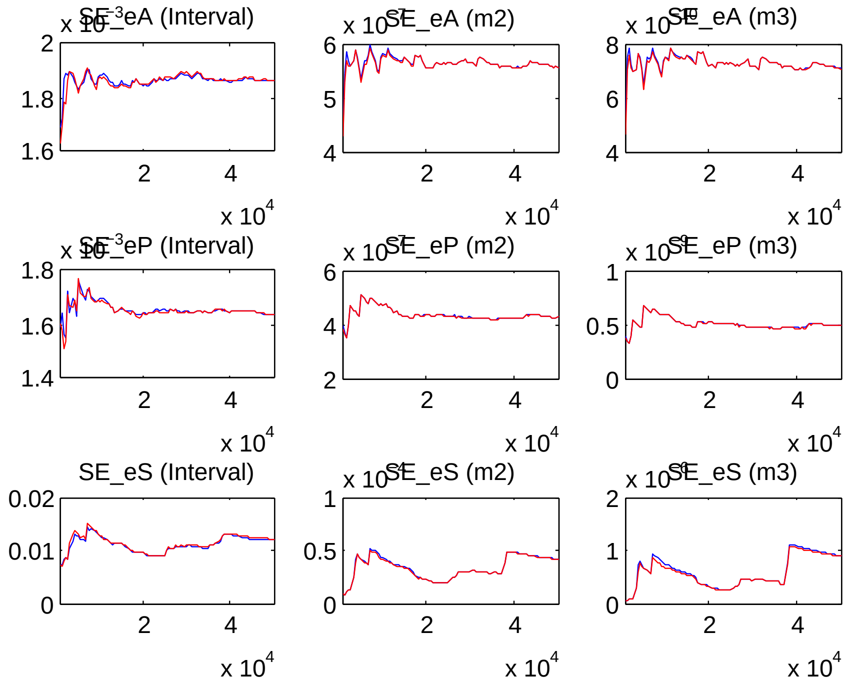

Figure 7: Univariate Convergence Diagnostics 1: 1992Q2-2000Q4

[19]Here we only present the diagnostics for the estimation period 1992Q2-2000Q4. The convergence diagnostics of other estimation periods can be obtained through running the author's Dynare codes. The Dynare codes are available upon request.

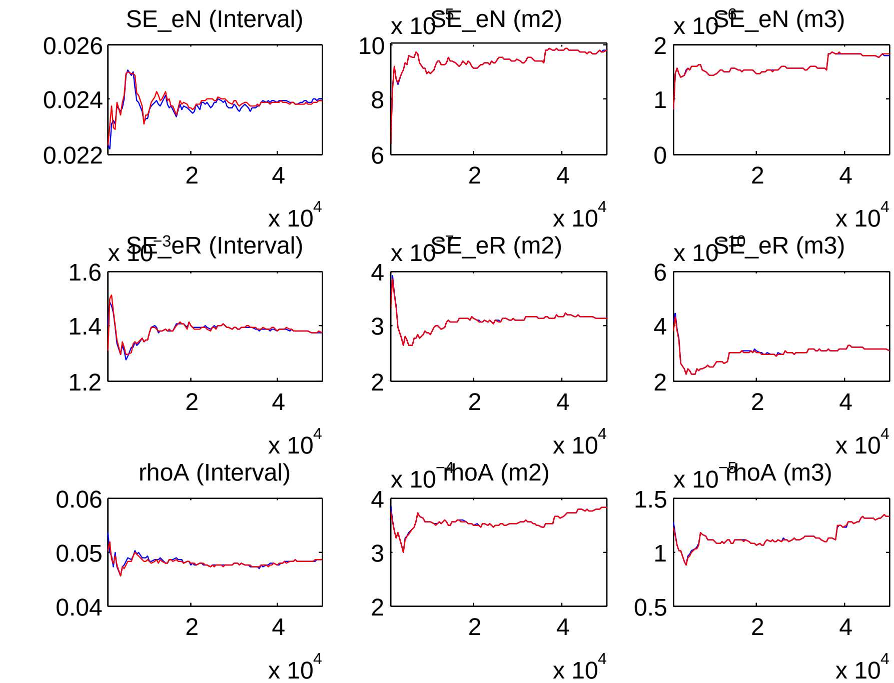

Figure 8: Univariate Convergence Diagnostics 2: 1992Q2-2000Q4

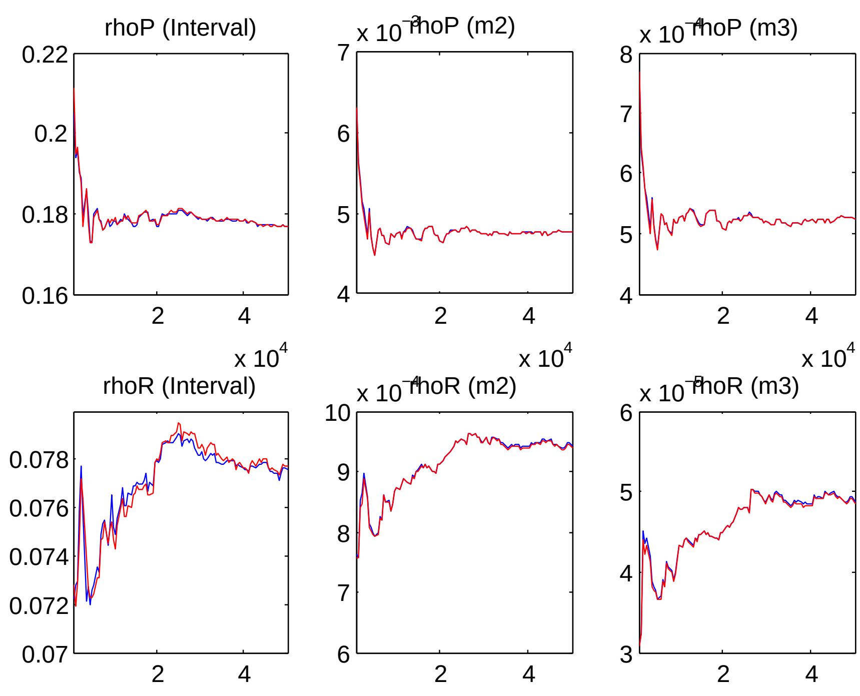

Figure 9: Univariate Convergence Diagnostics 3: 1992Q2-2000Q4

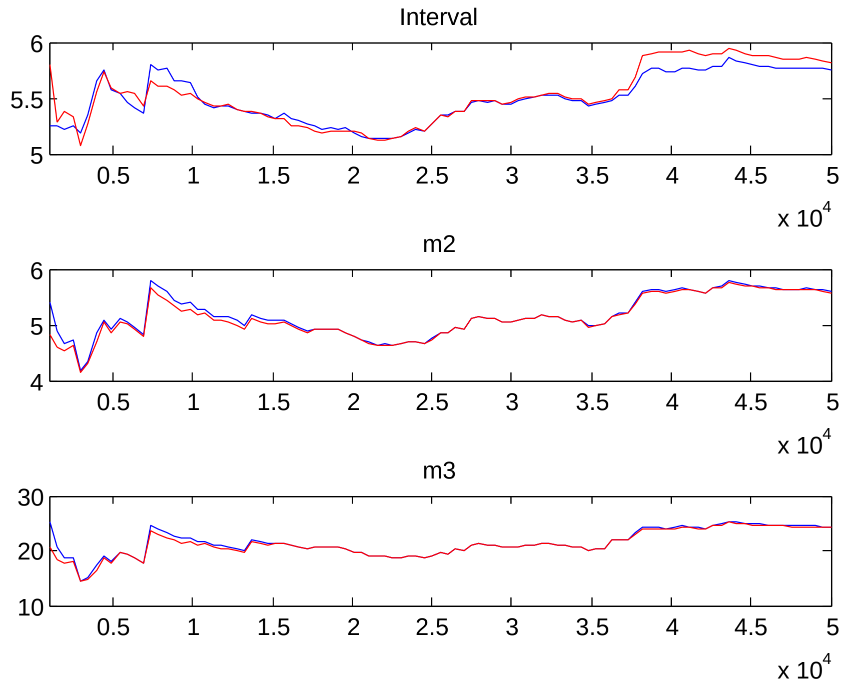

Figure 10: Multivariate Convergence Diagnostic: 1992Q2-2000Q4

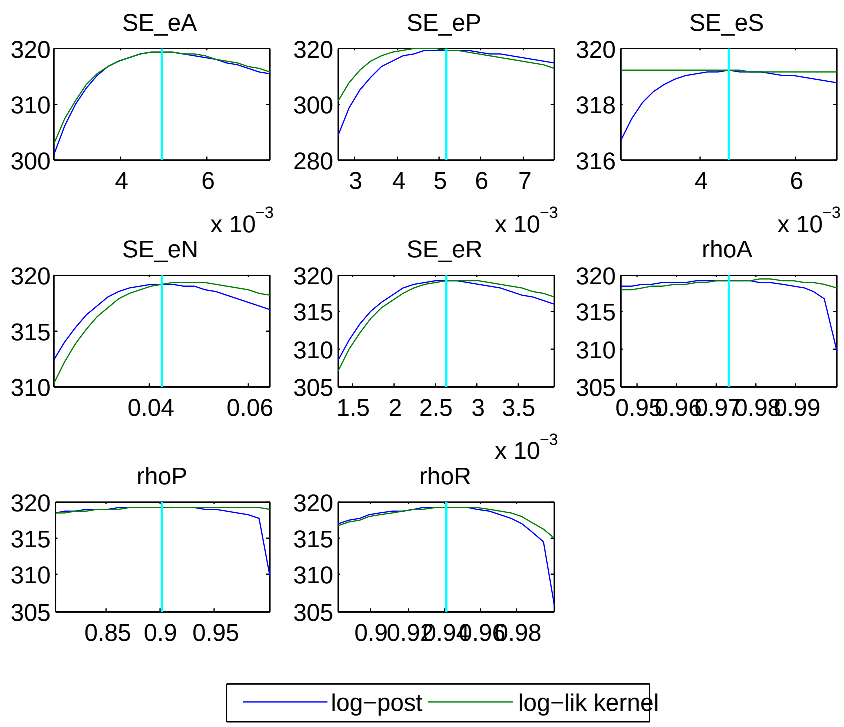

Figure 11: Mode Check Plots: 1992Q2-2000Q4

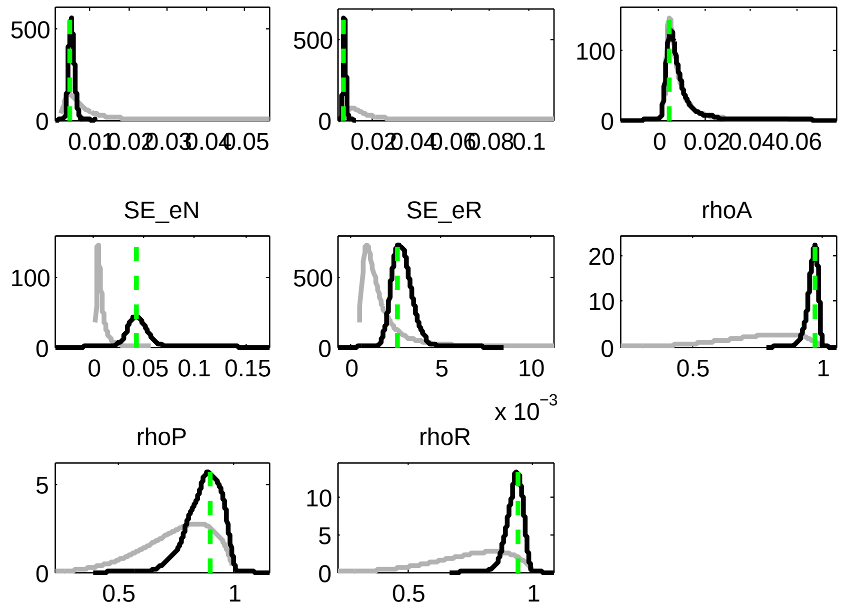

Figure 12: Prior and Posterior Distributions: 1992Q2-2000Q4

A.3 IRFs

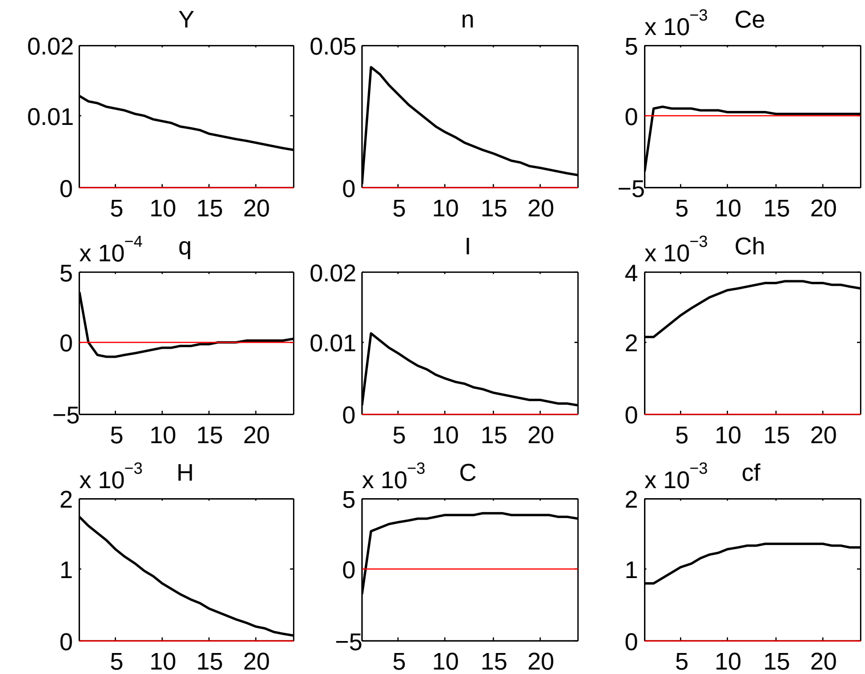

Figure 13: Impulse response function: Productivity Shocks

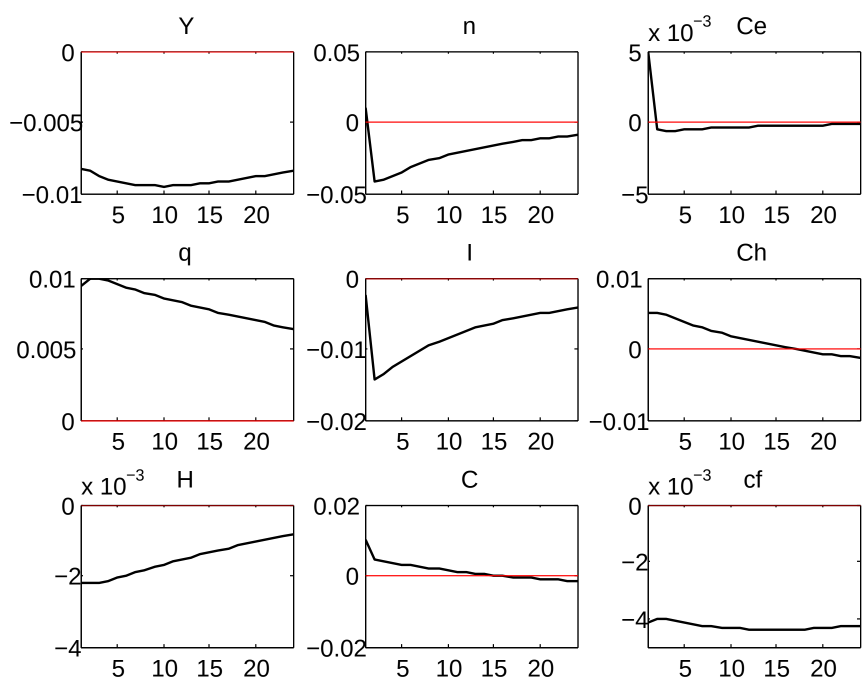

Figure 14: Impulse response function: TOT Shocks

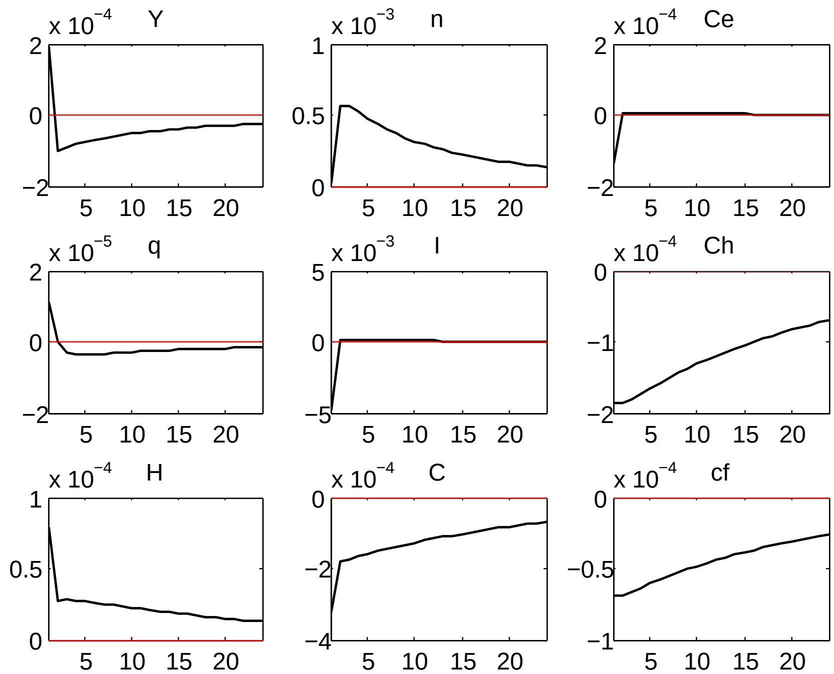

Figure 15: Impulse response function: Risk Shocks

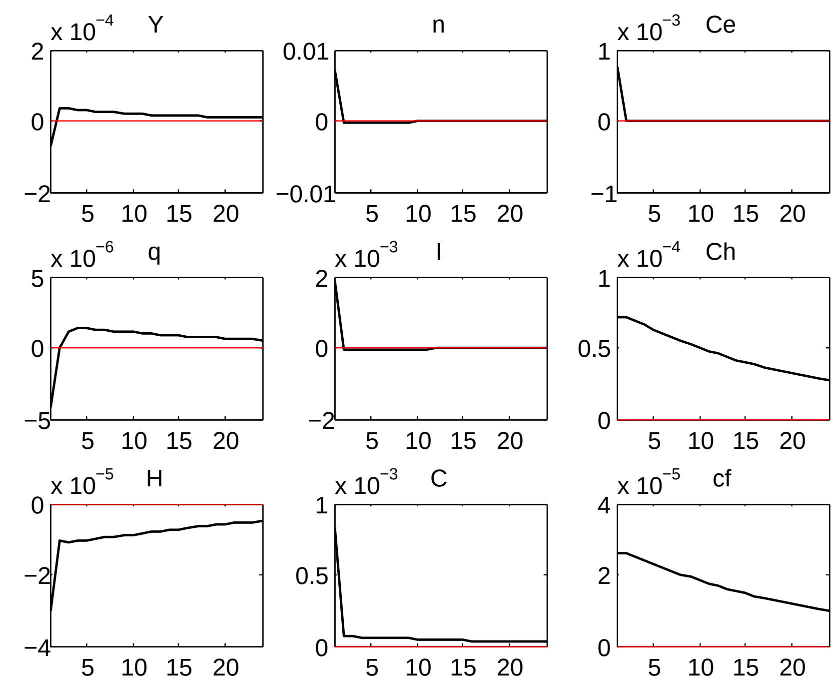

Figure 16: Impulse response function: Wealth Shocks

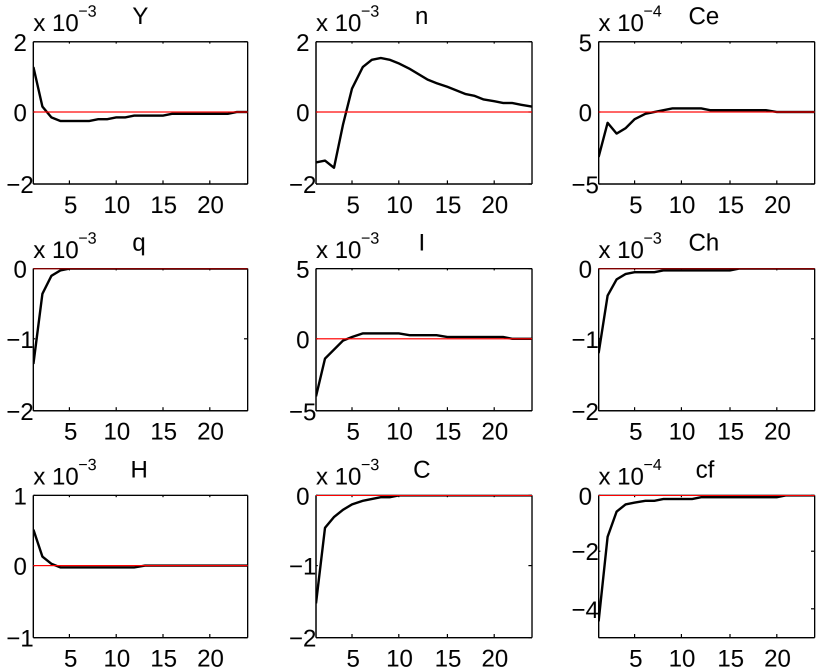

Figure 17: Impulse response function: Foreign Real Interest Rate Shocks

A.4 VAR Estimation Output

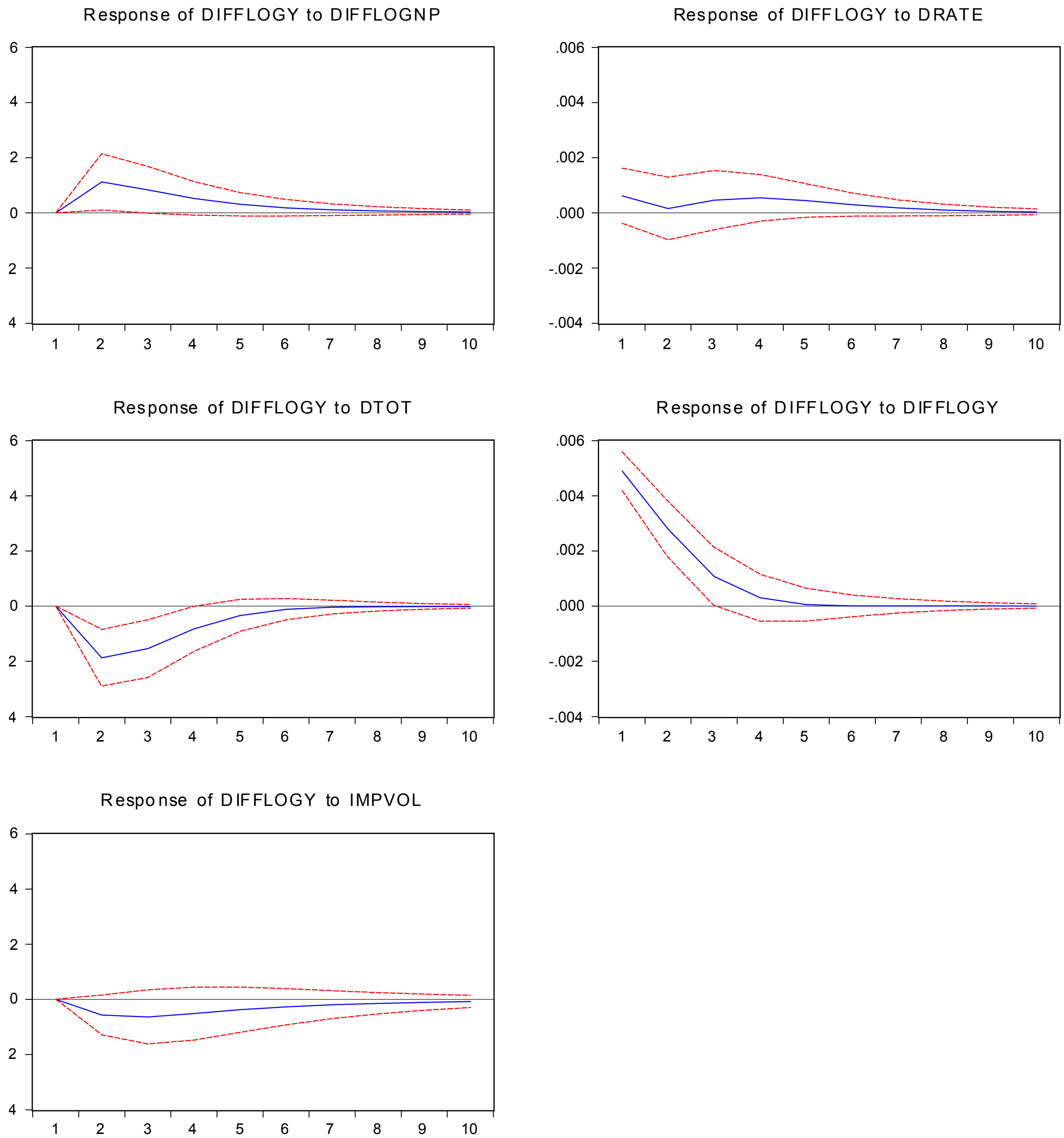

Figure 18: VAR IRF: 1992Q2-2016Q1

Period	S.E.	DTOT	DIFFLOGY	DIFFLOGNP	IMPVOL	DRATE
1	0.018854	0.000000	98.82244	0.000000	0.000000	1.177560
2	0.020500	9.700950	85.18136	3.334928	1.019205	0.763560
3	0.020790	14.41270	78.01747	4.511638	2.004502	1.053693
4	0.020866	15.48338	75.33573	4.923659	2.603239	1.653993
5	0.020886	15.54754	74.39681	5.062985	2.912851	2.079814
6	0.020892	15.50072	74.04728	5.109194	3.069279	2.273530
7	0.020893	15.47366	73.90721	5.125033	3.151064	2.343026
8	0.020894	15.46155	73.84692	5.130737	3.195090	2.365700
9	0.020894	15.45611	73.81887	5.132844	3.218876	2.373294
10	0.020894	15.45362	73.80510	5.133619	3.231534	2.376126

Cholesky Ordering: DRATE DIFFLOGY DTOT DIFFLOGNP IMPVOL

Figure 19: VAR FEVD: 1992Q2-2016Q1

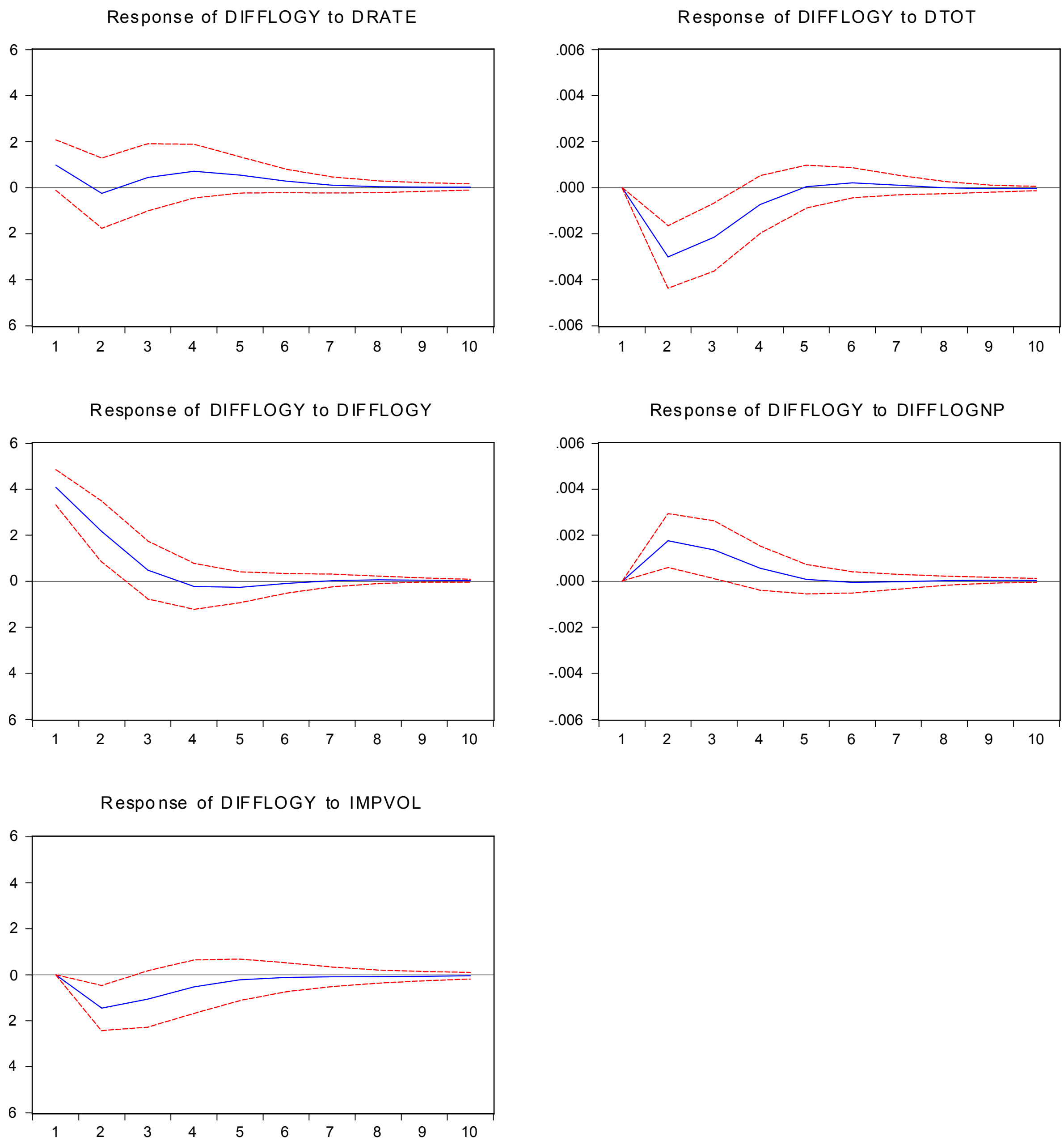

Figure 20: VAR IRF: 2001Q1-2016Q1

Period	S.E.	DTOT	DIFFLOGY	DIFFLOGNP	IMPVOL	DRATE
1	0.022152	0.000000	98.09597	0.000000	0.000000	1.904027
2	0.024503	22.67733	62.25669	8.219185	5.774385	1.072415
3	0.024962	28.16962	52.17223	10.52076	7.539878	1.597517
4	0.025162	27.98258	50.24643	11.00551	7.820082	2.945402
5	0.025247	27.68485	49.78710	11.02991	7.823803	3.674332
6	0.025267	27.66404	49.63223	11.01439	7.821686	3.867651
7	0.025271	27.66656	49.59245	11.01155	7.831385	3.898058
8	0.025272	27.66138	49.58275	11.01264	7.841804	3.901431
9	0.025272	27.65822	49.57777	11.01382	7.848290	3.901894
10	0.025272	27.65708	49.57479	11.01456	7.851340	3.902232

Cholesky Ordering: DRATE DIFFLOGY DTOT DIFFLOGNP IMPVOL

Figure 21: VAR FEVD: 2001Q1-2016Q1

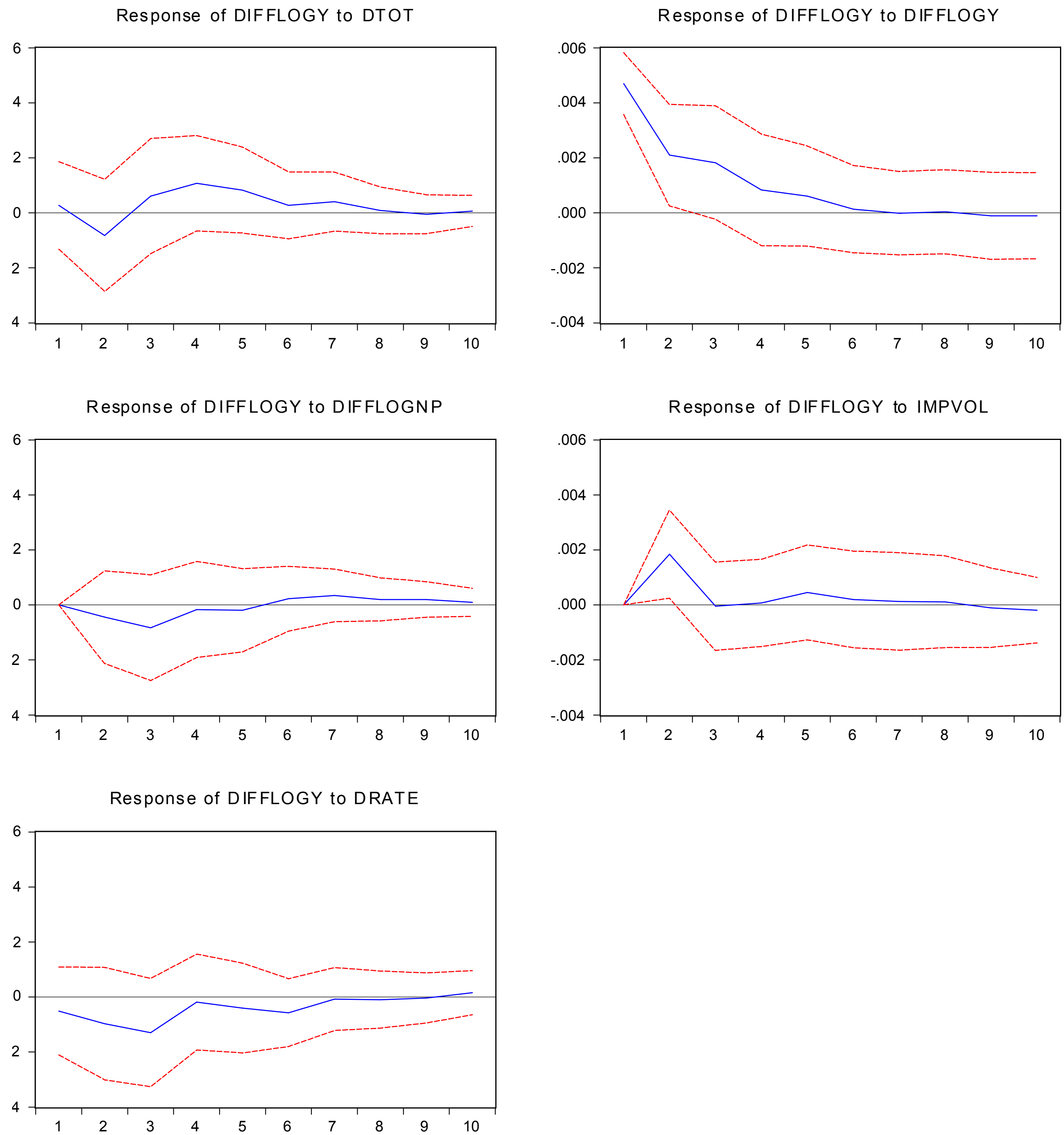

Figure 22: VAR IRF: 1992Q2-2000Q4

Period	S.E.	DTOT	DIFFLOGY	DIFFLOGNP	IMPVOL	DRATE
1	0.010708	0.331057	98.51212	0.000000	0.000000	1.156822
2	0.011537	2.355317	82.68808	0.619747	10.59448	3.742379
3	0.012177	2.947645	78.23594	2.340270	8.913407	7.562736
4	0.012298	5.688665	76.22560	2.297506	8.498753	7.289476
5	0.012529	7.128881	74.44227	2.310907	8.692680	7.425266
6	0.012918	7.216259	73.58774	2.401517	8.678544	8.115940
7	0.013222	7.553944	73.05974	2.661947	8.652398	8.071974
8	0.013485	7.559109	72.94161	2.750552	8.668024	8.080710
9	0.013677	7.554462	72.85632	2.836022	8.681158	8.072034
10	0.013778	7.549086	72.73808	2.849992	8.750087	8.112750

Cholesky Ordering: DRATE DTOT DIFFLOGY DIFFLOGNP IMPVOL

Figure 23: VAR FEVD: 1992Q2-2000Q4