ESTIMATING AN EFFORT COORDINATION GAME BETWEEN PARENTS AND THEIR CHILDREN

by

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1 Introduction

The provision of public education is a high cost endeavour and how it is done is critical to determining the success of future generations. But many questions are not clearly answered surrounding the best practices in the design of the education system. Student learning is produced when students provide effort. In the literature on early childhood education the effort decision of the child is often excluded from the optimization problem that is solved by the parent, and so the model often ignores the child's decisions as in Flinn et al. (2014) and Cobb-Clarke et al. (2016). This assumption makes sense in early childhood but is less believable as students move into high school and beyond where they are able to make their own decisions. As reviewed in Lundberg (2008), the psychology literature shows that there is a gradual transfer of decision making power from the parents to the child over the ages of 9 to 17. Few studies have looked seriously at how the student chooses their effort level especially as it relates to parent effort, and existing attempts such as Lundberg (2009) and Consconatti (2011) focus on the disciplinary aspects of the relationship.

To understand student effort choice there is a large literature on peer effects and classroom decisions, usually applied in the higher grades, where the amount of effort provided by students is completely unrelated to their parents except potentially through socioeconomic status, school or race based effects. This literature is surveyed in Epple and Romano (2011). More realistically, the effort decision for a student is the result of a combination of parental and external pressures but parents are likely to have substantial influence on the effort level chosen by their children. This paper will focus on uncovering this relationship. Bernal and Keane (2010), finds that the most productive activities for the production of learning in children are educational activities that are done with parents such as time spent reading together and time doing chores. This paper aims to contribute to the literature by estimating production parameters for parent and child effort in the child's high school years, when effort levels are chosen by the parent and child as the results of a coordination game.

Major factors that affect learning for a child include their teacher's education, teacher's ambition, the student's own effort, their parent's effort, access to libraries or computers, their peers and social groups, the school programs and curriculum design and a variety of other factors that influence a students experiences and preferences. So far, the teacher student relationship has received some attention in understanding the effort levels of students and the student-student relationships have received considerable attention in the peer effects literature, see Epple and Romano (2011), however it is easy to imagine that parent-student interaction may be a more important factor in a students decision. A parent that provides additional effort can work with their child one on one and obtain more results more quickly than a teacher in a classroom full of students. Effort from the parent also seems more likely to generate extra student effort than if the teacher is more or less engaged in the subject as the direct contact from the parent can bring the student to work more directly outside of the class hours. In addition to this, a parent is able to impose controls and monitoring on their child that reduce the quantity or value of the child's time spent with peers and in leisure activities. For this reason, the relationship between the parent and the child is likely to play a very important role in the production of learning in students. The high productivity of one on one parent-child education time is shown, for example in Fiorini and Keane (2012). Understanding the relationship between student and parent effort choices is then very important as a detailed investigation of it could indicate their relative productiveness and provide information on what resources could be provided to increase student's learning. Increasing student effort could be an effective way to increase the production of education without new teachers or large classroom restructuring.

In this essay the relationship between student and parent effort is considered as the result of a coordination game and a model structure similar to that of Todd and Wolpin (2015), which estimates effort as a coordination game between teachers and students with effort as an argument of the knowledge production function, is adopted. In the model parents and students have preferences over the knowledge level of the student, but face a cost to supply effort. Knowledge is produced by a production function which depends on the parents ability, the level of past knowledge, and the student's and parent's chosen levels of effort.

The model is estimated using data from the National Educational Longitudinal Survey, 1988 with lagged measures of test scores from the base year of this survey and all other variables from the first follow up survey in 1990. In the base year the students are in grade 8, while in the follow up year they are grade 10. Therefore, they represent the high school population of interest. The model is estimated with a latent factor structure, where unobserved past knowledge, preferences and parent ability are each determined by some background variables as well as an individual error term, which is allowed to be correlated with the error terms of the other latent variables. While these latent variables are not observed there are some noisy measures of these latent variables which are observed. These noisy measures are assumed to be a linear function of the latent variable plus noise. Using these measures along with suitable normalizations, the parameters of the knowledge production function as well as the parameters of the latent variable equations and measurement equations are estimated using simulated maximum likelihood estimation.

2 Literature Review

The economics literature studying education is large, but this essay relates mostly to two subsections. The first is the literature on early childhood education and skill formation which studies the effects of decisions made in early childhood by the parents on human capital formation and skill development. The connection of this essay with the early childhood literature is through the techniques for the estimation of the education production functions to explain the productivity of various educational inputs in producing knowledge or additional human capital. Todd and Wolpin (2003), (2007) provide a serious inquiry into estimating an education production function and review the papers estimating education production functions. They lay out basic production functions and illustrate the identifying assumptions needed to justify commonly assumed functional forms. In this essay the value added form is used, which could also have a cumulative form interpretation if data from all the life of the student rather than just one year and the lag were considered.

Heckman, Cuhnha and Schennach (2010) makes important contributions by estimating a CES production function for cognitive and non-cognitive skill formation with noisy measures and unobserved latent variables similar to the structure of the model considered here. Their paper shows this type of measurement error model with unobserved latent variables is identified under a few conditions. In the empirical section they find that the elasticity of substitution between inputs in developing cognitive skills and in developing non-cognitive skills are high in the first stage and low in the second stage of the child's life. Building on this it shows that cognitive development is more important in the first stage and emotional development in the second stage of a child's development.

Flinn DelBoca and Wiswall (2013) estimates a dynamic model of household choices, where households decide their labour supply and time and money inputs into the production of child quality. In this model, the parents have preferences over their leisure and child knowledge production and make their decisions given wage offers which evolve by some process and the current child quality. They are able to choose levels of leisure, labour, and child inputs to maximize household utility. As acknowledged in the paper, this story breaks down for the older ages where children make their own decisions and parent investment plays a different role as their model assumes that the parents are the sole decision makers.

Fiorini and Keane (2012) studies the productivity of childrens time when it is used in different ways with data from time use surveys. They find that the most productive use of time is time that is spent with parents especially reading or doing chores. This motivates the exploration of this relationship and the better understanding of how parents and children choose their effort levels together that will be explored in this paper. In another related work, Bernal and Keane (2010) performs a quasi-structural estimation of the production function using NLSY data. The mother's employment and child care decisions together with the education production function of the child are modeled using exogenous variation from welfare reforms to identify the model.

The research of Cobb-Clark et al. (2016) is also strongly connected with the second area on parenting style. The model that is developed is drawn largely from Becker (1973) but with parenting style as an input and with an additional cost of cognitive effort as well as the original costs of time and market goods. In their model, the cognitive capacity is allowed to be a function of socioeconomic status. They then develop production functions in general form for human capital production and some utility function over human capital that will be maximized subject to budget constraints on cognitive capacity, and money and time inputs. However, in this model all the decisions are made by the parents. The model is not estimated directly as the authors do not have data on all the relevant factors and so estimation would produce biased estimates. They proceed to use the youth in focus data and perform a principle component analysis to show that it may be a plausible model.

A smaller literature exists on the interaction between parents and children in decisions of effort. This literature is concerned mostly with parenting style and the way that parents encourage their children to provide effort. For example, Doepke and Zilibotti (2014) describes a theory of intergenerational preference transmission where parents effect their children by imposing restrictions and influencing the child's preferences. The model has two generations and the costs of parental restrictions come only through the reduction in child utility (which the parents care about). The equilibria in the model include authoritarian authoritative and permissive parent types depending on economic conditions. The parents take the child's best response and solve their maximization problem conditional on it.

Another similar attempt is made in Consconatti (2011) which estimates a dynamic game using NLSY97 data where the parent moves first, and then the child responds at each stage of the game. The child chooses a level of costly effort which produces future learning while he gains utility from leisure. The parent and child value the childs future knowledge, although not necessarily by the same amount, and the parent can affect the childs decision by imposing a rule which sets the minimum level of effort the child must provide. If the child doesnt provide the amount set by the parent, the model requires that the next period the parent will increase the level of strictness. While this game provides an interesting examination of the interaction it omits the fact that parent effort can increase the productivity of student effort as strategic complements, and the increasing strictness rule is, by the author's own admission, hard to square with the real world.

Lundberg et al. (2008), (2009) estimate the interaction between children and parents and are among the first to treat children as decision makers in their own right and to investigate the balance of power between children and their parents as they interact through a model of non-cooperative interaction. In this model, parents act by imposing rules and children respond by choosing a level of disobedience. The model is estimated using data from the NLSY79. This model is primarily designed to understand the decisions by children to engage in various risky behaviors and it examines the difference in their actions based on the rules set out for them. Finally, Burton et al. (2002) considers a similar problem where the parents choose a parenting style where they are uncertain over the mood of the child. In this model the child is assumed not to care about the future and the childs utility is increasing in praise and decreasing in effort. The model is estimated on the 1994 NLSCY79 and represents one of the first real attempts at looking at the parenting decisions to influence a childs effort choice.

Dauphin (2011) examines if a teenage child is a decision maker in the household using a model of bargaining with threats that the child will leave the household providing the child with bargaining power. Using data from the family expenditure survey this model is developed and tested. While the context and results of this paper are not closely related to the case considered here, the consideration of the child as a decision maker in the household spending decision and the results indicating that the child has an important role in this decision should motivate the current examination of their role in the education and study effort decisions.

Cardoso (2010) models children and parents time use with evidence from France Italy and Germany. This study considers the time spent in three categories: studying and reading, socializing, and time watching television. It finds that in all countries time watching TV by the student is closely influenced by time spent watching television by their parents. For studying and reading the role of parents is very strong in Italy less in France and quite small in Germany. Further investigation into the reasons of these cross country differences is an interesting topic for investigation.

Kuehn (2014) devotes a large section of their paper to a theoretical model for the optimal passing standard which is not relevant to the current work. However the empirical section is relevant to this work and for comparison. Using data from the CDI program in Madrid the relationship between student effort and family background is explored. They use a log linear specification with school fixed effects and controls for individual characteristics and estimate the model with OLS allowing clustering at the school level. Effort is measured as hours of homework a week and family background as the highest occupational and educational categories of the parents. Talent is controlled for using dummy variables for disabled students and students who failed grades. The results indicate that students from less advantageous backgrounds are more productive in effort, students from higher educated parents work more, and finally that there is a positive marginal productivity of effort.

Houtenville, Conway (2008) studies parental involvement in education production using the National Education Longitudinal Survey. They develop a theoretical household model in the style of Becker (1973) and then in the empirical section estimate a regression of effort and some background variables on achievement. To attempt to deal with endogeneity they also estimate one version where chores is used as an instrument which would be strongly correlated with parental effort but not with student achievement. They find a positive effect in all specifications for parent effort. In this paper the details of the effort decision and the idea of student effort are not considered.

Effort provision of students together with parents as the result of a coordination game is considered in deFraja et al. (2010). The theoretical section of their paper considers a slightly different game, where their outcome of interest is a discrete qualification level and they also include an effort choice for the school. In this essay a continuous outcome of test scores is considered rather than a discrete qualification level and the effort choice of the school is not considered. The empirical section of deFraja et al. (2010) uses data from the National Child Development Study and considers a linear regression of academic achievement on some effort measures along with some controls for demographic factors. Likewise the effort level is determined by a linear regression of the measure of effort on background variables, and the other players' effort levels. Finally they simultaneously estimate the effort equations and the production equation. The effort measures are constructed by factor analysis from many variables in the data set. They find that student effort increases with parent effort and parent effort increases in student effort and both student and parent effort increase slightly in ability which supports the consideration of a model as specified in this essay. In their approach, the education production function is estimated in a linear form. This means it will miss any nonlinearity in the effects of effort and that an increase from zero to one hour of studying produces the same increase in qualification level as an increase from 20 to 21 hours which seems unlikely. In the context considered, this choice of functional form is even more limiting as it forces student and parent effort to be perfect substitutes in the knowledge production process. To motivate the consideration in this essay of parent and student effort as complements one can imagine that parent effort alone while the child is not attentive should not produce knowledge gains for the student. This however, occurs under the perfect substitute specification where parent effort produces equal gains regardless of the child's effort level.

The technique and model structure adopted here are similar to those considered by Todd and Wolpin (2015). That paper estimates a coordination game between students and teachers in the classroom in Mexico with data from an experiment giving end of year exams to high school students. The results are then used to discuss about the level of the school curriculum and what changes might improve the amount of learning. The model here is quite similar although in their case, there are many students interacting with one teacher. However, their estimates provide an interesting comparison between the teacher side and the parent side of the game that is estimated here as well as the difference between American and Mexican education production.

The model structure applied here is chosen over direct estimation because inputs and outputs are not directly observed (only some noisy measures of them). Also, the inputs in the production equation may be endogenous and the unobservables in the input equations may be correlated. For these reasons, direct estimation of the parameters is not an appropriate technique as shown in Wolpin (2003), Cunha and Heckman (2007) and Cunha et al. (2010). Using the latent factor structure adopted here these problems can be controlled as shown in Cunha et al. (2010). A similarly structured model is used in Cunha et al (2010), Todd and Wolpin (2015) as well as Cunha and Heckman (2007), Black and Smith (2006) and Bernal et al. (2016). In these studies it has been shown that using this type of measurement structure is important to estimate the impact of unobservable characteristics, for which some noisy measures are observed, on the education produced. This model structure may also be preferable to atheoretic regression approaches as these would ignore the structure of the game of interest which has been shown to be problematic, for example, in Breitmoser (2013). One important reason for the use of the structure over atheoretic regressions is the clear interpretation of the mechanism through which changes in the effort or preferences act to influence final knowledge. That is, rather than simply finding that an increase in student enjoyment of school increases final knowledge, the approach considered here can see it does this by raising student effort which in turn leads to a higher parent effort which together produce more knowledge. It is also not clear that appropriate instruments are available that would allow for the estimation of these parameters. For a discussion of instrumental variable approaches in education production functions see Wolpin (2003). The instruments for similar cases of education production function estimation are also discussed in Keane (2010) and Bernal and Keane (2007) who consider time at home for young mothers with a change in welfare rules producing the required instrument. The approach adopted in this paper avoids these challenges, while dealing with the issues around direct estimation.

3 Model

Consider that the student and parent effort choices are the outcome of a Nash game. The students end of year knowledge depends on their initial knowledge, as well as their effort, the effort of their parents and the ability of their parents. Each student begins with knowledge $K_{t-1,n}$ and chooses effort e_n. The parent has ability a_p and chooses effort e_p. Knowledge is produced by the following production function where the functional form is assumed for estimation.

$$K_{t,n} = \delta_0 K_{t-1,n}(1 + \kappa a_p^{\gamma_0} e_n^{\gamma_1} e_p^{\gamma_2}) \tag{1}$$

This version of the production function is used throughout the paper and is written here in value added form. Where $K_{t-1,n}$ includes student ability, and is a result of their past effort choices and all other past inputs. The production function in this form could thus be rewritten as a cumulative product from past rounds of the game. Therefore the insights on estimating this function and bias from Wolpin (2003), and (2007) are relevant.

Students choose their effort level to maximize utility. Students have some preference for learning θ_n and face some cost c_n to put in effort. A fairly general functional form is also imposed for the utility of the student, as required to allow for the estimation of the model. It is assumed that the student's utility can be written as

$$U_n(e_n) = \theta_n K_n - \frac{c_n}{2} e_n^2. \tag{2}$$

Although this form makes clear the idea of the utility function, in this essay the θ_n and c_n parameters are not considered separately but instead the function is used in the form

$$U_n(e_n) = \frac{\theta_n}{c_n} K_n - \frac{1}{2} e_n^2, \tag{3}$$

where only the ratio of preferences over costs is estimated, hereafter known as $\tilde{\theta_n}$.

Similarly for the parent, the parent is assumed to have some preferences over the students knowledge level and some cost for them to provide effort giving utility

$$U_p(e_p) = \theta_p K_n - \frac{c_p}{2} e_p^2. \tag{4}$$

As above the cost and preferences are combined and only this ratio, hereafter known as $\tilde{\theta_p}$ is estimated.

The coordination game can then be solved by taking the first order conditions to obtain reaction functions:

$$e_p = (\tilde{\theta_p} \gamma_2 \delta_0 K_{t-1,n} \kappa a_p^{\gamma_0}, e_n^{\gamma_1})^{\frac{1}{2-\gamma_2}}, \tag{5}$$

$$e_n = (\tilde{\theta_n} \gamma_1 \delta_0 K_{t-1,n} \kappa a_p^{\gamma_0}, e_p^{\gamma_2})^{\frac{1}{2-\gamma_1}}, \tag{6}$$

which has a unique solution as the equilibrium effort levels of the parent and the student as a function of the model parameters.

$$e_p^* = (\gamma_1 \tilde{\theta_n})^{\frac{\gamma_1}{4-2(\gamma_1+\gamma_2)}} (\tilde{\theta_p} \gamma_2)^{\frac{2-\gamma_1}{4-2(\gamma_1+\gamma_2)}} a_p^{\frac{2\gamma_0}{4-2(\gamma_1+\gamma_2)}} (\delta_0 \kappa K_{t-1,n})^{\frac{2}{4-2(\gamma_1+\gamma_2)}} \tag{7}$$

$$e_n^* = (\gamma_2 \tilde{\theta_p})^{\frac{\gamma_2}{4-2(\gamma_1+\gamma_2)}} (\tilde{\theta_n} \gamma_1)^{\frac{2-\gamma_2}{4-2(\gamma_1+\gamma_2)}} a_p^{\frac{2\gamma_0}{4-2(\gamma_1+\gamma_2)}} (\delta_0 \kappa K_{t-1,n})^{\frac{2}{4-2(\gamma_1+\gamma_2)}} \tag{8}$$

It is however important to note that this equilibrium is not unique. In a coordination game like this, there is always another equilibrium where both sides provide zero effort. This is clear as if one side chooses zero effort than the marginal productivity of effort for the other side is zero while its cost is greater than zero. Mixed strategy equilibrium do not need to be considered as by the results of Echenique, Edlin (2004) the mixed strategy equilibria in this model are not stable. The existence of the two stable equilibria means that in the identification and estimation of the model an equilibrium selection rule must be developed. When estimating games with multiple equilibria the choice and estimation of this rule is an important problem and while many more sophisticated methods have been proposed many authors adopt the view that some equilibria, such as the Pareto efficient equilibrium are more likely to be played. For a recent review of these methods see de Paula (2013). In this paper the equilibrium selection rule is given by assuming that the Pareto dominant equilibrium is always chosen. While this is a simplifying assumption it seems reasonable especially in the case of a student/parent coordination game as parental rules and discipline could at a low cost encourage the selection of the high type equilibrium for the student. To include the selection rule as a part of the model would require the model be extended so that the choice of rules by the parents would also be included in their decision problem. Due to the low frequency with which zero effort is chosen in the data, at just 4.8 percent of the time, such an extension and the associated complication of the model is likely not justified.

4 Estimation

4.1 Latent Factor Equations

It is assumed that the student initial knowledge, parent ability, and the student and parent preference parameters are latent factors that are determined by some exogenous initial characteristics. They are affected by some individual error components which are mean zero and are orthogonal to the observed characteristics. The error components have some correlations across equations. These equations can be written as

$$K_{t-1,n} = X_n^{K_{t-1,n}} \beta^{K_{t-1,n}} + \xi^{K_{t-1,n}} \tag{9}$$

$$\tilde{\theta}_p = X_p^{\theta_p}\beta^{\theta_p} + \xi^{\theta_p} \tag{10}$$

$$\tilde{\theta}_n = X_n^{\theta_n}\beta^{\theta_n} + \xi^{\theta_n} \tag{11}$$

$$a_p = X_p^{a_p}\beta^{a_p} + \xi^{a_p} \tag{12}$$

This is a simple structure but allows for the researcher to see how a variety of exogenous factors influence the latent variables. Hereafter these exogenous factors are referred to as determinants. The interpretation of this is clear, as these latent variables are preferences or ability levels which are not exogenous but depend on things like family background, gender, age or race. When the model is estimated the β and the standard deviations and correlations of the ξ parameters are found. It is assumed that the ξ parameters are normally distributed.

4.2 Measurement Equations

These latent factors, student and parent effort levels, and the final test results are measured with error. The measurement errors of these terms are assumed to be normal, mean zero and are assumed uncorrelated with each other and uncorrelated with the latent variables. For each of the factors there are multiple measures that are observed. For the purpose of this exposition let the number of measures of each factor be M_i. Therefore the measurement equations are:

$$K_{t-1,n}^m = \alpha_0^{K_{t-1,n,m}} + \alpha_1^{K_{t-1,n,m}}, K_{t-1,n} + \zeta^{K_{t-1,n,m}}, m = 1, ..., M_K \tag{13}$$

$$\tilde{\theta}_n{}^m = \alpha_0^{\theta_{m,n}} + \alpha_1^{\theta_{m,n}}, \tilde{\theta}_n + \zeta^{\theta_{m,n}}, m = 1, ..., M_{\theta_n} \tag{14}$$

$$\tilde{\theta}_p{}^m = \alpha_0^{\theta_{m,p}} + \alpha_1^{\theta_{m,p}}, \tilde{\theta}_p + \zeta^{\theta_{m,p}}, m = 1, ..., M_{\theta_p} \tag{15}$$

$$a_p{}^m = \alpha_0^{a_{m,p}} + \alpha_1^{a_{m,p}} a_p + \zeta^{a_{m,p}}, m = 1, ..., M_a \tag{16}$$

$$e_n^m = \alpha_0^{e_{m,n}} + \alpha_1^{e_{m,n}} e_n + \zeta^{e_{m,n}}, m = 1, ..., M_{e_n} \tag{17}$$

$$e_p^m = \alpha_0^{e_{m,p}} + \alpha_1^{e_{m,p}} e_p + \zeta^{e_{m,p}}, m = 1, ..., M_{e_p} \tag{18}$$

and one measure of final performance, the final test score

$$T_n = K_n + \zeta_n^T. \tag{19}$$

In these equations the ζ are normal with mean zero and a standard deviation that is estimated. Therefore an observation for an individual is a set that contains an observed end of year score, along with M_i measures of initial knowledge, student and parent preferences and student and parent efforts. Some of the measures are continuous and bounded, and so are treated as censored, some are ordered categorical and so are treated as ordered probit with standard normalizations of the variance and intercept in the ordered probit estimation routine.

4.3 Likelihood

The model is estimated by simulated maximum likelihood. Thinking of the observable measures as the Y, and the individual determinants as X, the structure discussed above means that the maximization problem is to choose coefficients that maximize the likelihood of observing y given the x observed. Therefore the goal is to estimate

$$f(y_i|X_i) = \prod_{i=1}^{N} f(y_i|X_i), \tag{20}$$

however, because of the error terms in the latent factor equations, this cannot be estimated directly and so the researcher must first condition on a particular draw of the error terms in the latent factor equations. This gives the model the form

$$f(y_{i,m}|X_{i,m}, d_i) = \prod_{m=1}^{M} f(y_{i,m}|X_{i,m} d_i) \tag{21}$$

and so by taking the expectation over d

$$f(y_i|X_i) = \prod_{i=1}^{N} E_d[f(y_i|X_i, d_i)] \tag{22}$$

and this expectation can be expressed as an integral which can be numerically integrated using Monte Carlo integration. Therefore it is in a form which can be estimated with simulated maximum likelihood.

To perform the estimation first draw D times vectors of disturbances then transform them according to the imposed correlation structure of the current guess of parameter values and given these shocks and the other parameter values calculate the $K_{t-1,n,d}, \tilde{\epsilon_{n,d}}, \tilde{\theta_{p,d}}, a_{p,d}$ that result. Using these calculate the equilibrium effort levels. Now from the measurement equations calculate the joint density of measurement errors implied by these parameter values. In the discussion above this is $f(y_i|X_i, d)$. To do this the log likelihood contribution for observation i is calculated under the given parameter values. This gives first for the ordered probit with p categories a likelihood function of

$$ln(f_i) = \sum_{p}^{P} z_{ip} ln(\Phi(\alpha_{j+1} - X_i\beta) - \Phi(\alpha_j - X_i\beta)), \tag{23}$$

and for the censored regressions the likelihood of

$$ln(f_i) = I(y_i) ln(\frac{1}{\sigma}\phi(\frac{y_i - X_i\beta}{\sigma})) + I(y_{i_l}) ln(1 - \Phi(\frac{X_j\beta - y_l}{\sigma})) + I(y_{i_u}) ln(1 - \Phi(\frac{y_u - X_j\beta}{\sigma})), \tag{24}$$

where y_l, y_u indicate the limits for censoring and I is an indicator function. Then summing the log likelihood over all the measures and exponentiating it gives $\hat{f}(y_i|X_i, d)$.

Using this, the expectation can be calculated as the average over the D draws. It is then possible to calculate the product over the N observations to get the likelihood of the parameter vector

$$f(y_i|X_i) = \prod_{i=1}^{N} \frac{1}{D} \sum_{1}^{D} \prod_{m=1}^{M} f(y_{i,m}|X_{i,m} d_i). \tag{25}$$

This equation is a continuous function around the area of interest and can be maximized using standard methods given reasonable starting values. The standard errors are estimated as the negative inverse of the hessian matrix where from the usual hessian the f which has no closed form solution and cannot be computed here is replaced by the average f estimated from the simulations, as discussed in Cameron and Triverdi (2005) and Greene (2003) among others. As shown in Cameron and Triverdi (2005), the estimator here is asymptotically equivalent to the maximum likelihood estimator if D and N go to infinity, which gives the consistency and $\sqrt{N}/D$ goes to zero, which gives efficiency. Judging the number of simulations, D, required is a difficult problem and is usually answered informally by looking for changes in parameter estimates as the number of simulations is increased.

4.4 Data

The data used is from the National Educational Longitudinal study 1988, hereafter NELS 88. Data from the base year is used for the lagged test score measures and data from the first follow up year is used for the other parameters. Summary statistics are given in Table 1 and the correlations between measures of the same underlying variables are reported along with their significance levels in Tables 2-6.

The measures of initial knowledge are the grade 8 (base year) test scores. These are adjusted to be used in order to measure their gains over time and are estimates of the probability that person would answer a question correctly, thus even for a student with no knowledge it represents the probability of correct guesses. These measures are treated as continuous variables. For further details of this measure see Appendix H of the NELS F2 Student Component data user file. From this two of the scores are used, the science and the math score. The final test score is given by 10th grade irt scores designed for comparison with this base year score. Again the math score is used and it is again treated as a continuous variable. The determinants of this past score include the average grade the student received from grade 6-8 which is a continuous variable constructed from the average in each subject, as well as the students sex, relative age and a dummy variable for their race.

The student's preferences are measured by a question asking if the student thinks that classes are interesting, treated as continuous and bounded, coded on a scale of 1 to 4. Ideally these variables could be treated as ordered probits but continuous measures are required in order to identify the scale and be able to obtain identification of the model. The second measure of student preferences is an ordered probit indicating how much satisfaction the student has from learning at school. The determinants for the student preference latent variable are dummy variables for sex and race, as well as a measure of the rewards the teacher provides for effort and the students grade 6-8 average grades.

Parent preferences are not reliably measured in the data set. Therefore, this measure is set to one. Unfortunately this is a restriction on the model, but this is necessary due to the availability of the data.

Parent ability is measured by their level of education as an ordered probit for one parent and a continuous measure for the education level of the second parent. Forcing this to a continuous measure is not natural but is done in order to be able to identify the ξ standard deviations and more importantly the correlation of this ξ with those from the other equations. The expected education levels produced by the probit fitted values and those of a continuous regression are similar, so there may not be a big loss from imposing the continuity compared to the alternative of not identifying the correlations, which could change the production function parameter estimates.

Parent effort is measured by the number of hours a week they spend helping their child with homework as well as the frequency with which they check their child does their homework and the amount of time they spend with their child doing extra music lessons or sports. Hours a week is treated as continuous and bounded and is the normalized measure. The original measure is in terms of the frequency the parent helps the child (every day, a few times a week, once every couple weeks, or once a month). To construct a measure in hours, an hour is assigned for each time the parent helped, so that a parent helping everyday has 7 hours a week while one once a month has 0.25. This is done so that parent and student effort are on the same scale, as hours a week. The second measure of how frequently the parent checks the childs homework is also treated as bounded and continuous, coded at the midpoints of the intervals. This is done as it is necessary for identification that two measures of student and parent effort are continuous (not ordered probit or binary choice). The third measure of parent effort is an ordered probit with 4 categories of how frequently the parents bring their child for extra music lessons or sports.

The student effort is measured by hours a week spent doing homework as well as the frequency with which the student skips class. Hours a week is treated as bounded and continuous to meet the requirements imposed by the model structure. The frequency with which the student skips is also treated as bounded and continuous with the number of skips coded at the midpoints of the intervals. There is also a third measure of how often on a scale of (often sometimes never) the student goes to school without doing their homework which is treated as an ordered probit.

4.5 Identification

At this point it is not clear that all the parameters can in fact be identified given just the set of observations discussed above. In this section the normalizations that are required are discussed and a similar identification argument to Todd and Wolpin (2015) is presented to show that the remaining parameters are identified. The parameters of the latent factor equations are identified by imposing a normalization on one measure in each of the measurement equations that sets α_0^m to zero and α_1^m to one. Imposing this normalization allows for the consideration of

$$K_{t-1,n}^m = K_{t-1,n} + \zeta_{K_{t-1,n}}, \tag{26}$$

which means that

$$K_{t-1,n}^m = X_n^{K_{t-1,n}} \beta^{K_{t-1,n}} + \xi^{K_{t-1,n}} + \zeta_{K_{t-1,n}}, \tag{27}$$

$$K_{t-1,n}^m = \alpha_0^m + \alpha_1^m K_{t-1,n} + \zeta_{K_{t-1,n}}, \tag{28}$$

where the normalization in the first equation identifies the beta parameters in the latent factor equation and the other equations allow α_0^m and α_1^m to be identified for the second through M_K^{th} measures. The same argument can be made for the other latent factors and the effort levels. Therefore, with some restrictions the set of β and α parameters from the latent factor and measurement equations are identified. The covariance of the measures and the variances of the errors taken together allow the ξ and ζ to be separated in order to estimate their standard deviations and the correlations across the errors allow their correlations to be identified.

To identify the parameters of the production function, first assume there are perfect measures of $K_{t-1,n}, e_n, e_p, a_p$. Then using equation 19, the only unobservable is measurement error which is orthogonal to the parameters in the knowledge production function, and so the parameters are identified. Extending this argument to the case where the latent factors are functions of the determinants with error is done in Todd and Wolpin (2015) by using theorem 2 from Cunha, Heckman and Schennach (2010). Note that in order to identify all the parameters requires one continuous measure that establishes the scale for each of the variables as well as two continous and one additional measure for the parent and student effort equations.

In the empirical implementation $\tilde{\theta}_p$ is normalized to one, as the data does not contain a good measure of this quantity and the use of something that is not exogenous but is really determined as part of the solution to the model such as the parents expectations for the childs education would bias the results. This normalization however, means that it is no longer possible to separately identify all the production function parameters as before the normalization the equilibrium effort equations had an additional observed variable exponentiated by gamma parameters that is now gone. Therefore kappa, which rescales the knowledge produced is also set to 1. This is natural as this parameter is only a rescaling factor.

5 Results

5.1 Estimates

Estimates are provided in the tables. Tables 7a-f present the estimated parameters for the full model broken down by section for clarity. The production function parameters are given in Table 7a, the correlations of latent factors in Table 7b, the student parameters Table 7c, parent parameters Table 7d, effort parameters Table 7e and the determinant coefficients in Table 7f. It is clear that in order for the model to make sense, the different measures of the same variable should be correlated with each other. Correlation estimates for the measures are provided in Tables 2-6. All the variables are coded so the correlations are positive. All the correlations are significant at the 1 percent level. Summary statistics for the measures used are presented in Table 1. Note that it is not necessary for the measures to share the same scale as the α_1 coefficient absorbs the differences in their scales. The model contains 54 parameters that are estimated after the normalizations discussed in the previous sections that are necessary for the model to be identified.

The production function parameters in Table 7a, show that student effort is more productive than parent effort in producing knowledge and the production function is increasing in both types of effort with a decreasing marginal product of effort. This makes sense as studying more should produce more knowledge and the first hour of studying can produce a lot of learning but improving after many hours is more difficult. The delta parameter represents the amount of knowledge from the past year that is retained and here is estimated as 0.45. The γ_0 parameter estimate is -0.921 which means that in this case, the student of a parent with high ability is less productive when providing effort than a student from low ability parents.

The estimates of the β parameters from the latent factor equations depend on the normalizations chosen. The variable that is normalized determines the scale for the latent variable. As displayed in table 7f, the estimated parameters are close to the estimates from a regression of the appropriate type of the determinant variables directly on the normalized measure. The coefficients for all of the determinants are significant at the 1 percent level except for the coefficients on gender as a determinant of the past score and father's age as a determinant of parent ability which are significant at the 5 percent level, and the coefficient on race as a determinant of student preferences, which is not significant. The signs of the coefficients are the same as would be expected.

The parameters of the measurement equations in Table 7c, 7d and 7e are all significant at the 1 percent level except for the α_1 coefficient for the measure of the frequency with which the student skips class, and the α_1 coefficient for the measure of the frequency with which the parent brings the child to extra music or sports which are significant at the 5 percent level and the first cutoff in the ordered probit for the third measure of student effort, how often the student does not do their homework, which is significant only at the 10 percent level. The signs of these coefficients are all as expected.

The correlations and standard deviations of the latent factor errors are shown in Table 7b and are all significant at the 1 percent level. The size of the standard deviations depends on the scale of the latent variable as determined by the normalization. The correlation is largest between the base year test score and the parent ability latents, showing that parents with high education raise children who have higher base year scores beyond what is explained by simple exogenous determinants of race, gender age and past grades. There is also a fairly large correlation between the parents ability and the preferences of the student and a smaller but still positive and significant correlation between the level of base year knowledge and the students preferences. All of these correlations are not surprising as these variables are closely related. Having higher test scores, for example may make a student feel they are better at school which improves their preference for school or a parent that liked school more may have done more of it and may have done something that inspired that same enjoyment in their children.

5.2 Discussion

The estimates of the production parameters in this paper are slightly different from those of Todd and Wolpin (2015) which uses different data and estimates the coordination game between teachers and their students. Their γ_1 for student effort is 0.371 where this essay finds 0.559 and their estimate of teacher effort is 0.197 while here the estimate of parent effort is 0.287. In both papers the student effort is the most productive. This contrasts the result of de Fraja et al. (2010) which finds parent effort more productive. The finding that parent effort is more productive than the teacher effort is also not surprising as there is more interaction and importance of a parent then a teacher in the life of a child and because as suggested in Todd and Wolpin (2015), the estimates on Mexican data of the productivity of effort may be lower due to the Mexican education system. The big difference between the two sets of parameter estimates is in their estimate of the γ_0 at 0.52 and the estimate in this paper at -0.921. The difference here is somewhat surprising as although a teachers level of education is likely more important than a parents education in a students learning as it lets them control the classroom better and provide focus on more important concepts in the course design, it may be expected that parents with more education would also be better able to assist their children in learning than those with less education. If that was true then there would be a positive coefficient for γ_0.

It is possible that the negative coefficient observed for γ_0 is caused by parents with lower ability wanting a better life for their children and putting more effort to help them, or of them having more available time to assist their children (a lower opportunity cost in lost work), or reporting bias amongst respondents or perhaps the structure and type of game developed by this model. However if it was the case that lower ability parents put more effort then a negative correlation between parent effort and parent ability should be observed, while in the data a positive correlation is present. Similarly, the discipline story is also unlikely to be the cause of this result because there are also weak positive correlations between parent ability and a number of possible measures for the amount of restrictions and rules the child faces. It also could be the case of reporting bias, that parents with high education under report and/or those with lower education over report the amount of time they spend then maybe higher education would cause higher effort and the better scores. Such an effect cannot be ruled out. It also could be that parents with lower ability are less likely to spend time helping their children themselves as it is not as effective for them to help their children but instead decide to provide private tutors not captured in the effort measurements included here which could produce the effect of having more specialized hours, while the parent effort measured remains low and they appear more productive. Finally it could be that students from low ability parents are more focused when they spend time working, and so their effort is simply more productive, although no measure is available in the data to adjust for this as done in Todd and Wolpin (2015) for students studying while on their cellphones.

Finally their delta parameter is 0.9 while in this paper it is 0.451 but their measures of K are not of a single scale and thus their delta also represents a normalization along with the knowledge depreciation. Comparison with their other parameters is not useful due to differences in the model. In their version they were surprised at the low productivity of effort, and conclude this is at least partially due to the overly demanding curriculum and the education program in Mexico. Therefore it is not surprising to find effort of the parents and students which are more productive compared to the estimates obtained in their paper.

Another set of results that these estimates may be compared to are the results of Kuehn (2014). Using data from the CDI in Madrid, it is found that reducing a parents education level from university to incomplete compulsory reduced their childs effort on average by 21-23 percent. In the model of this essay, this would be an increase, from 3.57 hours to 14 hours. This shows that in this channel the results of the current study are not able to match their results. They find an increase of one hour of homework time for 12 year old students produces an increase of 3 percent of a standard deviation in the test score. In this essay, this increase of an hour of effort by the child would cause an increase of 2.8 in the score or 20 percent of a standard deviation and it would be accompanied by a response by the parent of 0.2h with it which contributes to the additional magnification in the result that is obtained in this essay compared to theirs. In this paper, they also provide some evidence which suggests that the effort choice may not be from a coordination game and they show that while children work more hours with higher educated parents, their working time is less productive. Our results support the lower productivity of such effort, and although it is a coordination game structure, and intuitively if their effort is less productive they want to work less, this model allows for an adjustment through the preference parameter so that despite the lower productivity they still work more and this result is also matched.

5.3 Model Fit

Table 9 reports the coefficients in the determinant equations and also includes estimates of the parameters obtained by the appropriate type of regression without the structural model of the measure which was normalized in the measurement equation on the determinants. This allows for comparison with the values obtained under the full model. These values are quite close to those obtained through estimation of the full model. Table 11 shows summary statistics for the variables generated by the model. That is, the latent variables and equilibrium solutions are calculated for a given draw of error terms. The results of this calculation show that the mean and variance of the latent variables and the equilibrium solutions match the data in means and variances for the parameter they are scaled to match, which is the one normalized in the measurement equations. The close match of the generated data as well as the similar coefficients in the determinant equations show the model is being estimated as expected and is able to reproduce some key properties of the observed data.

Following Cunha et al. (2010) and Todd and Wolpin (2015) the fraction the variance of each measure that is due to noise is estimated. The results, reported in Table 9, demonstrate the importance of measurement error in the model. The results show a large amount of noise for most of the measures. Final Math Score, Base year Math score, the amount of pleasure the student gets from school, the frequency the student skips class and the frequency a parent checks their child's homework are the least noisy measures. For these variables the latent variable is responsible for most of the variation observed. In the remaining measures there are large amounts of noise not accounted for by the latent variable, however this is similar to the results of Todd and Wolpin (2015) where in both cases the noisy measures include hours per week of study and hours per week of extra time spent helping. As noted in Todd and Wolpin (2015) the amount of the total variation accounted for by measurement error is not critical to assessing the performance of the model or in accessing its ability to explain knowledge produced and in this paper as there, the production function parameters are estimated with precision and so the variance in the latents must be sufficiently large although their measures are noisy.

Following Todd and Wolpin (2015) the model fit is also examined by estimating regressions of actual and predicted outcomes on the exogenous student and parent characteristics that are the determinants of the latent factors. The results of these regressions are reported in Table 10. These regressions do not have a clear interpretation and are really misspecified because the actual model is highly nonlinear. Given these nonlinearities, the ability of the model to capture these relationships is a demanding test as noted by Todd and Wolpin (2015), which also reports similar results for this test.

For the end of year test score, in the real data all of the coefficients are significant at the 90 percent level or better. The coefficient on student age for the real data is significant with a p-value less than 0.001, and while the coefficient for the generated data is similar, it does not fall within the tight 95 percent confidence interval of the coefficient from the data. The coefficient for student gender is not significant in the generated data and does not fall in the 95 percent confidence interval for the coefficient from the data. The Student race coefficients in the real data and the generated data are very similar and the generated coefficient falls inside the 95 percent confidence interval. The coefficients for primary school grades are both significant and the coefficient from the real data has a p-value less than 0.001 but the coefficient for the generated data again falls just outside of the 95 percent confidence interval. The coefficients for both parent traits considered fall well outside the 95 percent confidence interval for the coefficient based on the real data. Therefore of the 6 variables 1 lies within the 95 percent confidence interval for the real data and two others lie just outside, with the remaining 3 are further removed.

For the student effort, equation only two of the six variables are statistically significant when the model is estimated on the real data and so interesting conclusions can only be drawn on the match of the coefficients for these two variables. For the two significant variables the estimated coefficient on primary school grades, falls just outside the 95 percent confidence interval, while for the student gender the coefficient is well outside the confidence interval. For the parent effort, there is only one significant variable in the linear regression on the original data, and the estimated coefficient on the model data falls inside the 95 percent confidence interval. The low significance of most of the coefficients in the effort regressions is not surprising given the results in Table 9 showing the large amount of noise in these measures and the highly nonlinear model.

6 Counterfactual Impact Evaluation

Tables 11-12 display estimates of the responses in test scores predicted by the model and the corresponding responses in effort levels, to a change in initial level (by standard deviations away from the mean) of the variable of interest. The variable of interest is changed by the amount listed and the others remain at their mean values. In Table 12, the results show when the variables are increased by a standard deviation, what the outcome is for the student's and parent's effort levels and the final test scores. For an increase in a student's past score by one standard deviation the model finds that an extra 1.63 hours a week of student effort and extra 0.716 hours of parent effort and a large increase in the test score at the end of the year result.

The results from an increase in parent ability are somewhat surprising, as the estimated exponent of parent ability is negative. This means that increasing the parent's education causes a decrease in the productivity of the student's effort and a lower hours choice. Therefore, here the student chooses a lower effort and the parent as well and less knowledge is produced. As in the results of Kuehn (2014) the students from lower ability parents have a higher productivity of effort and those students work less, which is supported here in the coefficient and the positive correlation between student effort and parent ability. It is also possible that the normalization of the parent preferences to 1 is interfering with the estimated effect of parent ability as in reality there is likely to be some correlation between these two parameters.

Finally, a change in student preferences increases student effort and is accompanied by smaller increases in parental effort and test scores. The effect on test scores of an increase in student preferences is an increase of less than a third of a standard deviation. This suggests that attempting to change student preferences may not be the most effective way to encourage learning and increase test scores.

The results for decreases in Table 13 are similar and their interpretations are also similar. A decrease in parent ability has a large positive effect on a student's effort and through that effect also produces an increase in their parent's effort. Changes in student preferences produce small decreases in all the outcomes, and past scores continue to play an important role with a one standard deviation decrease in past score producing a 1.6 standard deviation decrease in the final knowledge level. Although it is not made clear by the model, because this process occurs year after year, the effect of low knowledge in one year follows the student for a long time. This model shows that improving the level of knowledge in the base year is very important for improving future knowledge.

To contrast the policy implications solved here with the implications from the estimates in de Fraja et al. (2007) it is interesting to look at the effects of the complementarity between parent and student effort and the nonlinear functional form. In their model, increasing student effort from 1 to 2 hours has the same effect as increasing the effort from 11 to 12 hours of study effort while the model this paper estimates has decreasing marginal productivity of effort which is more likely to be observed in the real world. In the model of deFraja et al. the student and parent effort are perfect substitutes in the empirical education production function while here they are complementary. In the experiments considered here the least effective intervention is an increase in the student preference parameter $\tilde{\theta}_n$. Although due to the nature of the coordination game the increase in student effort produces an increase in parent effort, the increase in final knowledge produced is larger if a smaller increase in student preferences is accompanied by an increase in the parent preferences. This differs from the results of de Fraja et al. (2007) which suggest that the biggest effect comes from improving only the variable with the largest coefficient. A better approach to improve test scores given the results of this paper would combine increases in the determinants or preferences of both the student and the parent.

7 Conclusion

This essay estimates the education production function as the result of an effort coordination game between parents and their children. To do this a latent factor model is adopted with multiple measures of the variables of interest taken from the NELS 88. This essay adds to the literature on parent and child interactions as well as the literature on estimating education production functions. This paper shows that parent effort plays an important role in the production of learning as an input in the education production function. It also shows that the level of current knowledge is very important in producing knowledge in the next period and that next period study efforts are strongly linked to the level of knowledge today. A surprising result is also produced by the model, in the fact that increasing the parental ability makes student's and parent's effort less productive. In general this essay provides evidence that students and parents play an effort coordination game to produce education for the child.

The existing data on available and appropriate measures of the latent variables and their determinants are the biggest limitations for the results of this paper. The lack of appropriate measures for the parental preference parameter mean that this parameter could not be estimated. Another limitation is the assumption that the high effort equilibrium is always selected. These limitations provide a direction for future work. The first limitation can be overcome by finding a data set with appropriate parental preference measures which could then be used to estimate the same model more completely. To solve the second limitation, it might be possible to extend the model so that it includes parental choices of rules separately from their assistive effort decisions. From this, their choice of rules could be allowed to enter into the equilibrium selection rule or be made to influence the preference parameter of the child. Another interesting extension would be to estimate the model for the development of noncognitive skills. It has been shown by Cunha et al. (2010) that noncognitive skills are very important in development and may in fact play a larger role in teenage development than cognitive skills which develop more at a younger age. Also in noncognitive skill development parents have been shown to play a very important role and so, formulating the parent student coordination game and studying their effort levels may be even more important in that context.

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Tables

Table 1: Summary statistics

Variable	Mean	Std. Dev.
Final Knowledge	46.047	13.439
Base Year Math	38.392	11.523
Base Year Science	19.746	4.72
Thinks School Interesting	2.661	0.634
Pleasure from school	2.684	0.543
Mother Education	2.913	1.454
Father Education	3.01	1.596
Student hw, h/week	6.026	6.73
Frequency Skip	2.173	0.622
Don't do Homework	2.94	0.693
Parent Help hw, h/week	2.287	2.211
Parent Checks Homework	2.587	1.035
Extra Music or Sports	1.784	1.166

Table 2: Correlation Measures: Parent Ability

Variables	Mother Education	Father Education
Mother Education	1	
Father Education	0.5874***	1

*** $p \leq 0.01$

Table 3: Correlation Measures: Initial Knowledge

Variables	Base Year Math	Base Year Science
Base Year Math	1	
Base Year Science	0.7331***	1

*** $p \leq 0.01$

Table 4: Correlation Measures: Student Preference

Variables	School is Interesting	Pleasure doing School
School is Interesting	1	
Pleasure doing School	0.5831***	1

*** $p \leq 0.01$

Table 5: Correlation Measures: Student Effort

Variables	Hours/week Homework	Skip Class	Don't do Homework
Hours/week Homework	1		
Skip Class	0.1935***	1	
Don't Do Homework 3	0.1935***	0.2675***	1

*** $p \leq 0.01$

Table 6: Correlation Measures: Parent Effort

Variables	Hours/week Help Hw	Check Hw	Extra Music/ Sports
Hours/week Help Homework	1		
Check Homework	0.495***	1	
Extra Music or Sports	0.0807***	0.1138***	1

*** $p \leq 0.01$

Table 7a: Parameter Estimates: Production Function

Production parameters	Value	Standard Error
γ_0	-0.921	0.137
γ_1	0.559	0.035
γ_2	0.287	0.018
δ	0.502	0.031
κ	1	

Table 7b: Standard Deviations and Correlations

Standard deviation	Value	Standard Error	Correlation	Value	Standard Error
σ_{ξ_K}	10.457	0.254	$\rho(\xi_K, \xi_{\theta_n})$	0.254	0.051
$\sigma_{\xi_{\theta_n}}$	0.366	0.024	$\rho(\xi_K, \xi_{a_p})$	0.993	0.004
$\sigma_{\xi_{a_p}}$	0.515	0.047	$\rho(\xi_a, \xi_{\theta_n})$	0.355	0.068

Table 7c: Parameter Estimates: Student Components

Initial Knowledge $K_{t-1,n}$	Value	Standard Error
Base Year Math		
constant	0	
slope	1	
Standard dev	4.53	0.166
Base year Science		
constant	6.44	0.422
slope	0.347	0.016
Standard dev	2.974	0.074
Student Preferences $\tilde{\theta}_n$		
Thinks school Interesting		
constant	0	
slope	1	
variance	0.516	0.014
Pleasure from doing School		
constant	0	
slope	2.748	0.292
cutoff 1	4.769	0.688
cutoff 2	6.47	0.0246
variance	1	

Table 7d: Parameter Estimates: Parent Components

Parent ability a_p	Value	Standard Error
Education Mother		
constant	0	
slope	1	
Standard dev	1.357	0.031
Education Father		
constant	0	
slope	0.714	0.095
variance	1	
cutoff 1	0.921	0.277
cutoff 2	2.18	0.06
cutoff 3	2.54	0.03
cutoff 4	2.79	0.026
cutoff 5	3.43	0.047

Table 7e: Parameter Estimates: Effort Components

Parent effort e_p	Value	Standard Error
Hours/week help Homework		
constant	0	
slope	1	
Standard dev	2.14	0.050
Check Homework		
constant	1.88	0.422
slope	0.277	0.152
Standard dev	1.667	0.054
Extra Music or Sports		
constant	0	
slope	0.211	0.106
cutoff 1	0.974	0.294
cutoff 2	1.17	0.06
cutoff 3	1.583	0.059
variance	1	
Student effort e_n		
Hours of homework/week		
constant	0	
slope	1	
variance	7.187	0.177
Skip		
constant	1.186	0.089
slope	0.107	0.054
variance	0.358	0.009
Dont do Assigned Homework		
constant	0	
slope	0.201	0.037
variance	1	
cutoff 1	-0.423	0.225
cutoff 2	0.286	0.022
cutoff 3	2.27	0.034
Final Knowledge		
Constant	0	
slope	1	
Standard dev	5.704	0.187

Table 7f: Determinant Equations

Latent Factor Determinants	Value	Standard Error	Estimate	Standard Error
K_0				
constant	26.58	1.04	17.4	0.08
race black	-5.75	0.954	-3.03	1.56
sex	-0.676	0.286	-2.48	0.63
age	2.19	0.404	3.52	0.699
old grades	2.527	0.194	5.02	1.66
$\tilde{\theta_n}$				
constant	2.038	0.071	1.93	0.09
sex	0.122	0.029	0.04	0.04
old grades	0.044	0.015	0.086	0.20
reward effort	0.16	0.021	0.19	0.027
race black	0.005	0.07	0.078	0.094
a_p				
constant	3.055	0.177	4.42	0.176
race black	-0.438	0.098	-0.8495	0.215
Father Age	-0.031	0.015	-0.351	0.041

Estimate column reports results of regression of determinants on the scaling measure without the rest of the full model considered. For K_0 estiamtes are from a standard linear regression, while for $\tilde{\theta_n}$ and a_p estimates are from tobit regressions.

Table 8: Predicted Summary Statistics

Variable	Mean	Standard Deviation
Final Math Score $K_{t,n}$	45.981	12.357
HW Hours/Week Student e_n	6.24	1.13
Hours/Week Help HW e_p	2.742	0.386
Base Year Math Score K_0	38.394	11.024
Thinks School Interesting $\tilde{\theta_n}$	2.654	0.387
Mother Education a_p	2.918	0.525

Variables scaled by normalized measure

Table 9: Fraction of Variance of Measure that is Noise

Variable	Fraction
Final Math Score	0.315
HW Hours/Week Student	0.86
Skip	0.089
Don't Do Homework	0.95
Hours/Week Help HW	0.96
Check HW	0.083
Extra Music or Sports	0.96
Base Year Math Score	0.29
Base Year Science Score	0.58
Thinks School Interesting	0.86
Pleasure from School	0.255
Mother Education	0.72
Father Education	0.78

Table 10: Model Fit: Explanatory Variables

Variable	K_t data	K_t model	e_n data	e_n model	e_p data	e_p model
Student Traits:						
Student Age	10.42***	7.64***	0.63	0.1.06**	0.26	0.49***
	(9.16,11.71)		(-0.17,1.42)		(-0.05,0.58)	
Student Gender	-2.51***	0.39	2.41***	0.24***	0.17	0.03
	(-3.95,-1.04)		(1.49,3.31)		(-0.19,0.52)	
Student Race	-5.85**	-5.41**	1.57	-0.19	-0.19	-0.10
	(-10.54,-1.16)		(-1.39,4.53)		(-1.37,0.99)	
Past Grades	7.25***	6.13***	1.41***	0.71***	0.112	0.29
	(6.55, 7.94)		(0.97,1.85)		(-0.06,0.29)	
Parent Traits:						
Father Age	0.745***	2.64***	-0.19	0.4***	0.248***	0.19***
	(0.18,1.30)		(-0.54,0.16)		(0.11,0.39)	
Father Race	3.80*	-0.98	-0.37	-0.045	-0.31	-0.001
	(-0.57, 8.17)		(-3.13, 2.38)		(-1.41,0.77)	

Effort measure for the student hours per week of homework and for the parent hours per week helping with homework. 95 percent C.I. displayed in brackets. * for $p \leq 0.1$ ** for $p \leq 0.05$, and *** for $p \leq 0.01$. Final Knowledge results from linear regression. Student effort results from tobit, lower limit 0 upper limit 20. Parent effort results from tobit lower limit 0 upper limit 7.

Table 11: Test Score Result From Changes $\mu+\sigma$

Variables Changed	Student Effort	Parent Effort	K
No change	6.37	2.799	46.579
Past score	8.002	3.515	68.097
Parent Ability	4.613	2.026	33.583
Student Preference	7.469	2.95	49.56

Student and parent effort in h/week scale, Parent ability scaled by Mother Education and Student preference scaled by student thinks school interesting

Table 12: Test Score Result From Changes μ-σ

Variables Changed	Student Effort	Parent Effort	Final Knowledge
No change	6.37	2.799	46.579
Past score	4.667	2.054	28.197
Parent Ability	11.07	4.84	101.62
Student Preference	5.21	2.62	43.20

Student and parent effort in h/week scale, Parent ability scaled by Mother Education and Student preference scaled by student thinks school interesting