The Societal Costs of Servicing Public Debt

By

Daniel Brouillard

An essay submitted to the Department of Economics

in partial fulfillment of the requirements for

the degree of Master of Arts

Queen's University

Kingston, Ontario, Canada

Abstract

This paper will introduce a model to ascertain the magnitude of the negative effects financing public expenses through debt has on society. It will split the negative effects into two distinct issues, namely the cost to society of paying interest to foreign entities that own the debt and the within society distributional effects of the concentration of ownership of the debt of the top quintile. By estimating the ownership rates along with carrying cost of the debt we find that there are important costs to perpetually financing public expenses through debt that run counter to policy objectives and therefore should be considered when determining how governments finance their expenses.

Acknowledgements

I would like to take this opportunity to thank my supervisor professor Sumon Majumdar for his advice while working on this paper and for allowing me the freedom to go about it in the way that I did. I would also like to thank my other professors who have taught me both at Queen's University during my masters and at Concordia University during my undergrad as without them, I would not have enough economic knowledge or drive to complete this essay. I would like to thank my fellow students who have kept me motivated during trying times and who I hope I continue to grow and develop with over the years to come. I would also be remiss if I didn't take this opportunity to thank my parents for their years of support and guidance.

Table of Contents Page

Introduction

When looking at financing public expenses through debt, one can immediately see the advantage of being able to delay taxes to future dates but one might not realise there are immediate and important issues of growth and distribution that arise depending on how the debt ownership is comprised. There are 4 main owners of public debt, namely, other government accounts, foreign entities, the central bank and private individuals. Since from an accounting point of view, other government accounts are part of the overall government budget and debt owned in these accounts are both government liabilities and assets, the public debt is normally divided into two subsections, publicly held debt and intra governmental debt. Since the focus of this paper is the implications of financing public expenses through debt, intra governmental debt which from a budgetary stand point is neutral falls outside the scope of this paper. That is to say, this paper will break down the effects of debt ownership rates of the central bank, foreign entities and domestic private investors and the impact of this break down on growth and equity.

Why look at the implications of financing public policy through debt? Since 1948 the US federal government has financed some of its yearly expenses through debt in all but 7 of those years and in the past 55 years the government has financed some of its expenses by debt in 53 of those years[1] (Office of Management and Budget, 2016). Not only that but the amount of public expenses that is financed through debt has become more and more substantial.

This increase in financing public expenses through debt has come about through two channels. The outlays of the government have been steadily increasing as a percentage of GDP since 1948, while the receipts the government collects have been steadily declining.[2] This means that the

[1] See Figure 1 for visual representation
[2] See Figure 2 for visual representation

cause of increased government deficits comes both from lower revenues and higher expenses, part of the deficits incurred by the US government represent both tax breaks and increases in social spending.

Looking at the revenue side we see that from 1983 the tax rates on the each of the 4 bottom quintiles have dropped by more than 5% while the tax rates on the top quintile have remained relatively stable with only a small decrease.[3] This has meant an increase in the progressivity of the tax system and indicates that societal equity has been an important policy goal over the past 30+ years.

On the expense side a similar story presents itself. Most expense categories have remained relatively stable while national defense has decreased and human resources has increased dramatically. Human resources expenses are the main social expenses countries incur including things like education, social services, health, income and social security along with veteran benefits and services. That is to say most of the growth in expenses has been in social programs indicating the priority of the federal government providing these services to society.[4]

However, decreasing revenue while increasing expenses has lead to an explosion of debt, most notable in recent years. Since the great recession in 2008 the debt held by the public has more then doubled to sit at over 13 trillion dollars in 2015 (Office of Management and Budget, 2016).[5] This ballooning of the debt has become a highly publicized and researched topic but little is done to ascertain what effect this has and will have on public policy. The big number of 13 trillion tends to make good headlines in research and news coverage but the focus should instead be on the interest payments on the debt that effects the government and society.

[3] See Figure 3 for visual representation
[4] See Figure 4 for visual representation
[5] See Figure 5 for visual representation

When looking at net interest payments we see that the growth before 1999 was much faster relative to the growth of debt held by the public. While in recent years the growth of net interest payments has been much slower.[6] This is because net interest payments have 2 variables effecting them, the size of the debt and the interest rate paid to carry the debt.

Although the debt held by the public has ballooned over the past several years, the interest rate the government has had to pay to carry the debt has been decreasing from its peak of 9.2% in 1982 to sit at 2.66% in 2015.[7] This has allowed interest payments to remain relatively modest compared to what they would have been if the interest rate had not decreased so drastically in the last 30 years. This means that when policy makers finance policy through deficit spending how much debt they take on is not the only issue they must consider. They must also consider what the carrying cost of that debt will be.

This paper will focus on two main parts of the carrying cost, the distributional impact within society of the interest and the societal cost of money leaving the economy. To do this, this paper will create a model that quantifies the total cost of financing public policy through debt by normalizing the investment for the policy, determining the cost of carrying the debt accounting for the interest rate paid upon the debt and dividing that cost by the various ownership rates of the debt that this paper will estimate. To do this, first we will review literature that motivates this research, we will then explain the model, followed by calibrating the model through estimation from US data. We will then discuss the results obtained from the model and then conclude by summarizing the findings, discussing future applications of the model and suggest future research that could be done to improve the model proposed and public policy in general.

[6] See Figure 6 for visual representation
[7] See Figure 7 for visual representation

Literature Review

The focus of papers trying to model the implications of deficit financing have been mostly focused on two streams of thought. The conventional train of thought focusing on increases in aggregate demand in the short run and crowding out effect of investment (therefore lower growth) in the long run and the other being the notion of Ricardian equivalence that argues neutrality of effects. A good paper to see the thought and models in this field of literature is from a paper written by Douglas W. Elmendorf and N. Gregory Mankiw entitled Government Debt. This paper surveys the literature on the subject and pronounces that most economists follow conventional train of thought that using debt to finance public spending causes a temporary increase in aggregate demand leading to a short term boost in the output of the economy but then crowds out private investment by having individuals invest in financing the public debt instead. The paper discusses quantifies the cost of the crowding out effect by surmising that if the debt was instead transformed into productive capital (assuming a one to one crowding out effect), the capital would increase net output by about 3%. The paper then discusses how the assumption of a one to one crowding out effect relates to estimations by others in the field from a paper by Blanchard called Debt, Deficits, and finite horizons and a paper by Auerbach and Kotlikoff called Dynamic Fiscal Policy that come to similar conclusions using an intergenerational model and a general equilibrium model respectively.

The Ricardian equivalence camp views public debt as being neutral and having no short term or long term effects as individuals act perfectly rational viewing an increase in public debt as an increase in future tax liabilities. The reason this has not really taken hold and the conventional stream holds more sway for most economists and policy makers is that the postponement of the taxes usually has such a long standing delay to them that the individuals that are going to have to

pay these increased taxes aren't even alive today to save the money that the public spending (or tax cut) generated in the economy and therefore the Ricardian equivalence is unlikely to hold.

This paper also brings forth the view that most of the literature surrounding public debt hold, i.e. "The debt-service payments themselves are not a cost to society as a whole, but, leaving aside any payments to foreigners, merely a transfer among members of society" (Mankiw, 1998). This is where my paper attempts to fill a gap in the current literature by estimating the ownership rates of the debt and trying to determine both a society cost through foreign ownership rate and a within society cost through the distribution of ownership of the debt-service payments.

Since there is little research that has been done on the debt-service payments cost to society, the rest of this literature review will focus on papers that motivate and recommend methodology to use for this paper. Since the paper is on public finance, we will look at some papers that discuss the benefits that public policies can achieve (mainly growth and equality), review the literature on the sustainability of public debt, review literature on public policy rules as they relate to growth and equality and then conclude the review with papers that this paper draws some of its methodology from.

The first important issue to motivate this paper is to look at the advantages of having a progressive tax system. In the paper Mobility, Income, and Taxation by Sami Babi, Jean-Yves Duclos and Abdelkrim Araar, they are motivated into looking at the impact of a progressive taxation policy on mobility. To do this they discuss that mobility has two main effects on people's permanent incomes. First it creates income uncertainty across time for individuals. Meaning that because an individual can move from one income to another, individuals cannot be certain of what their income will be tomorrow. This creates a difficulty in determining their lifetime incomes creating budgetary uncertainties that negatively impact their lifetime welfare.

The second impact of mobility they look at is that it tends to equalize permanent income when compared to periodic income across individuals. That is to say the variation between individuals' periodic incomes tend to vary to a larger extent that individuals' permanent income. This means that permanent incomes are more equal than periodic incomes because of mobility and thus mobility creates more equality.

The paper then tests what a progressive tax system does to these two effects concluding that it reduces the variability of periodic incomes to permanent incomes (reducing uncertainty). This is done by taxing individuals more when they are making more and less when they are earning less. Meaning that there post tax periodic incomes are less variable then there pre tax periodic incomes.

A progressive tax system also makes permanent post tax welfare more equal then permanent pre tax welfare across individuals. It does this by taxing higher earning individuals more over their lifetime while taxing lower earning individuals less. Thus the paper concludes that a progressive tax system helps avert some of the negative effects of mobility while strengthening its positive effects.

The paper inequality and debt in a model with heterogeneous agents by Fredrico Ravanna and Nicolas Vincent discusses one of the negative impacts of having high inequality. The paper makes a link between the rising level of debt of the lowest income quintile and income inequality in America. They do this by creating a model with two types of individuals, namely entrepreneurs and workers. They then introduce two different shocks into their model, one being a positive 10% shock on the top quintile incomes and the other being a government policy shock of a lump sum transfer from the lowest quintile to the top quintile by the same amount. Comparing the two shocks that see that income growth has a much smaller impact than tax policy change as the

feedbacks for the first are negative since entrepreneurs' demand for workers increases so workers receive higher wages and thus some of the benefit while the tax policy change has no feedback effects.

From their model and experiment, they surmise that the rise in debt for the bottom quintile in the US is not due to growth differentials or easier financial intermediation but due to the poor borrowing more from the rich (who have more and more to lend). They argue that this is why even though income inequality has risen rapidly, consumption inequality has risen at a much slower rate. They also argue that this large increase in debt has lead to economic fragility leading to more financial crises.

The next topic of papers that is important to get an understanding of for this paper is papers on the sustainability of public debt. When conducting a literature review on papers pertaining to the sustainability of debt it is important to note that many papers have been written about this issue and that most conclude that there is a sustainable deficit level that can be maintained indefinitely. This sustainable debt level usually depends on various factors most notably, the amount of debt that is currently held and the value of public goods in the economy.

In the paper Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting, Akira Yakita investigates the sustainability of deficit spending under the assumptions that the government can control the public capital/GDP ratio, the finance ratio of public investment is held constant and the tax rate is endogenously determined to fulfill the government budget. The paper finds that there is an initial level of debt threshold that is increasing in public assets, where indefinite deficit spending is sustainable. This is due to the growth of the economy out-passing the growth of the debt financing expenditure allowing government revenues to raise fast enough to enable a continuously growing debt. While this

literature proves theoretically that government deficits are sustainable, they do not deal with the economic implications of debt ownership that this paper will model. The papers in this topic do give rise to a steady state of deficit spending allowing governments at least theoretically to be able to sustain perpetual deficits.

This leads into the literature on public finance and welfare. The papers in this area tend to focus on comparing over arching rules that the fiscal authority would follow and how they impact policy and welfare. These papers tend to compare the golden rule of public finance with other alternatives, normally using the dynamic government budget constraint (DGBC) as the base rule. For analysis, the golden rule is normally described as government expenditures for public consumption, transfer payments, and interest payments should not exceed tax revenue (Yakita, 2008). That is to say borrowing should only be used to finance government investment. While the DGBC's only constraint is that the intertemporal budget constraint must hold. The papers then compare various fiscal policies rules long run growth implications and conclude that less restrictive policies do not necessarily lead to higher long run growth.

An example of a paper in this section of literature is Endogenous growth, welfare and budgetary regimes by Sugata Ghosh and Iannis A. Mourmouras. It compares the welfare implications of using the golden rule against using the dynamic governmental budget constraint. They find that the ratio of public to private capital is lower under the golden rule. They argue that this means the inefficiency associated with over-investment in public capital is lower. They make this argument by suggesting that because the DGBC has less constraints the government will end up borrowing more and thus the return on government bonds will be higher so that real interest rate and growth rate are both higher under the golden rule than DGBC. They conclude that even though the DGBC has fewer restrictions, this fiscal policy approach leads to more crowding out and higher implied taxes and lower welfare.

All of these papers tend to focus solely on growth implications of public finance and do not deal with distributional effects of the various regimes. This issue has been a hotly contested one with one side of the argument stating that the public debt is not regressive as those that own it pay most of the taxes while the other side tends to make the argument that the debt redistributes the taxes to an elite few who own the debt. There is very little recent empirical work done to deal with this divide with the only notable exception being in the paper What happened to the bondholding class? Public debt, power and the top one percent by Sandy Brian Hager that estimates (through similar means this paper will use to do its estimation of debt ownership) that the top 1% own 45% of the debt and argues that this constitutes a powerful bondholding class that has influence on policy. While little quantitative work is done to back up this argument, the qualitative argument makes a link between recent policy favoring the rich and the concentration of ownership of the debt. While this paper seems to be the only modern attempt at formulating ownership rates of the debt, the paper doesn't necessarily attempt to formulate a model to see the distributional impact of the debt.

Model

There are two main effects of financing government programs through debt. One is the societal cost of having to pay future interest payments to those not in the society and the other is the distributional impact of future payments to the members of society that own the debt. It is easiest to look at these two effects separately especially since one is a growth issue while the other is a distributional one. We will also model two potential types of policies and their inherent costs. One will be a one-time cost or investment. An example of this type of policy would be a one-time investment in infrastructure. The other type of policy this paper will focus on is a long standing repeating policy. This type of policy represents policies like welfare payments or tax credits. This

means that in total this paper will look at 4 implications of various policies and the effects financing them using debt will have.

Since this paper is attempting to create a model about how financing public policy through debt effects distribution and growth, this paper will focus on the cost side and leave the full cost benefit analysis to papers focussing on public policy in general. If the policy in question is a one-time investment financed by debt and we wanted to see the cost to society we would simply have to look at the compounded interest over time, determine the payoff of the investment, and look at who the carrying cost of the debt goes to. The societal cost of debt financing has to do with the amount of foreign ownership of the debt as that is the part of the interest payments that the society is losing. This will be calculated as follows:

$$Interest\ cost\ of\ carrying\ debt\ for\ a\ one\ time\ investment: (Intial\ investmen\ x\ interest\ rate)^{years\ carried}$$

$$minus\ intial\ investment. Interest\ cost\ of\ carrying\ debt$$ will be abbreviated to IC henceforth.

$$Cost\ to\ society: interest\ cost\ of\ carrying\ debt * percent\ of\ debt\ held\ by\ foreign\ entities$$

$$which\ we\ will\ simplify\ as: C = IC * FO$$

While this simple model will be complicated by the calculation of the investment benefit, including this model in the cost benefit analysis could be illuminating about the interim costs of financing an investment through the use of debt and lead to better public policy practices.

The model for the distributional effects will rely on the distribution of within society ownership of the debt. To determine the ownership makeup of the debt within society, first a distribution must be chosen to divide the society and then we must determine by estimation what part of the distribution gets what from the interest payments. To simplify the analysis of this paper we will divide society into two segments, those above a certain threshold and those below it but the model can be modified to incorporate any division of society that is of interest to the policy maker

or researcher. We will use the same calculation to calculate interest cost as in the societal cost but will change the impact calculation as follows:

$$Distributional\ Impact: Interest\ Cost * (percent\ owned\ by\ above\ threshold - percent\ owned\ by\ below\ threshold)$$

$$DI: IC * (AT - BT)$$

We could also alter the above model to calculate distributional effects of debt financing on any number of distributions. For example, we could calculate it by quintile, top 1% vs bottom 99% or ethnic or educational background.

These simple models can also be adapted to incorporate yearly investment into programs to look at the impact of a repeating program such as tax breaks, or welfare payments or simply repeating deficits. We would then have repeating debts that get compounded and the model equations would look like:

$$C = (IC + l.IC + l2.IC \ldots) * FO, where\ l\ is\ a\ lag\ operator$$

$$and\ would\ be\ used\ for\ as\ many\ years\ as\ we\ want\ to\ look\ at$$

$$and\ similarly: DI: (IC + l.IC + l2.IC \ldots) * (AT - BT)$$

The lagging interest costs would be that the first year the one-time investment and repeating investment would be equal, the second year you would have the same as one-time investment in year 2 + year 1 interest cost, with the third year being year 3 + year 2 +year 1 interest cost and this would proceed until the analysis was over.

The model could be modified to enable calculation of varying interest cost's year after year, changing interest rates or any number of any alternative modifications but for the purpose of introducing the model and keeping the overall analysis done is this paper reasonable we will look at the simplest versions of these models.

Calibration

Since this paper is not about specific policies but is intended to begin to model the cost of financing public policy through debt we will normalize all initial investments to 1. We will also calibrate the model using US data to show how future researchers or policy makers could go about estimating these difficult but important variables. The interest rate of public debt is the most important measure in the model and varies significantly depending on timing and which instrument is used to finance the debt. Since we have already estimated the weighted average interest rate of the debt we will use those estimates to calibrate 3 different interest rates to use in our model. In recent years, the interest rate to carry the debt has been steadily declining. At the moment our estimate says it currently sits at 2.66%. Since it is decreasing it is likely that new debt taken on over the past several years has had an interest rate lower then this number so to compensate for that, our low bench marker for the interest rate we will use will be 2%. The interest rate to carry the debt however is expected to rise in future years. For our high end benchmark, we will use 5% which is just under the long term average estimate that we found to be 5.3%. While it is possible that the cost of carrying the debt may one day rise to levels above this number, this is unlikely to happen in the near future and it would be a gross overstatement to use any number much above this benchmark. To see how the costs that the model estimates vary over different interest rates, we will use the midpoint between our low and high benchmark. The results from using 3.5% as our middle benchmark will be an indication of how quickly the costs rise with respect to interest rates as these are compounded interest rates and results will not change in a linear fashion.

When looking at the cost to society, the part of the debt that is owned by foreign entities is what is of real interest. According to data provided by Federal Reserve Economic Data, in 2013 Q4, 47% of the debt held by the public was owned by foreign entities (Fred Graph, 2015). Given that

foreign debt ownership used to be as little as 30% a little over 10 years ago using a wide range of foreign ownership values in our analysis seems to be rather appropriate. We will use 47% as our mid point and ±10 as our extreme points to see what effect more and less foreign ownership will have on the results.

The distributional effect of debt relies on the part of the debt not owned by foreign investors and the federal reserve system. This comes to 35% of the debt (Fred Graph, 2015). We then have to estimate the distribution of the ownership of that 35%. Combining information of financial asset ownership by quintile from the survey of consumer finance tables based on internal data from table 6 13 and table 6 13 means, we estimate that the top quintile owns between 40% and 86% of most financial assets (Board of Governors of the Federal Reserve System, 2014). We do this by taking the percentage of respondents in quintile that own assets (table 6 13) and multiply that by the median values of the assets (table 6 13 means).[8] By doing this we determine the means of each quintile including those that own none of the asset class and can determine how much of each asset class the top quintile owns.

Once we get ownership rates of each asset class, we then go the to the flow of funds released by the Board of Governors of the Federal reserve and match asset ownership rates to each class of assets that is public debt. The final estimate we get when combining these two information sets is 66.39% (Board of Governors of the Federal Reserve System, 2016). This estimate does not account for each asset class having different payoff structures and use's the lowest ownership rate of asset class for bonds as the survey does not allow us to estimate ownership rates for that estimate. Because of these factors, our estimate is likely underestimated because riskier assets that have higher payoff tend to be owned more by the top quintile as they can afford the risk.[9]

[8] We do this for each asset class in the survey and the results we obtain are in appendix A table 1.
[9] As seen in the ownership rates of the asset classes in the survey

Even considering this, we will use this estimation as our midpoint and include ±5 as our other benchmarks. Since distributional impact is an above threshold – below threshold calculation, using 5 instead of 10 like we did for foreign ownership will keep the differences between the estimates similar. In our analysis we will not consider changing the 35% societal ownership rates and leave this to future research.

Results

Our results section will be divided into 6 subsections. In the first section we will discuss the carrying cost of the debt, i.e. the debt carrying costs. Then we will have a subsection for 1-time investment societal cost, repeated investment societal cost, 1-time investment distributional impact and finally repeated investment distributional cost. Then we will briefly discuss the results as a whole.

Debt Carrying Costs:

Since the US federal government is a habitual user of deficit spending to finance its budgets, on the margin, any program that is net negative in terms of revenue and cost can be seen as being financed through debt. Moreover, since the deficits are so constant, once a dollar is financed through debt it can be seen as permanent debt that will accrue a carrying cost compounded indefinitely. Therefore, when doing policy analysis to determine the value of any policy on the margin, it is important to include the effects of the policies financing on the long term implications of that policy.

Below is a table representing the yearly costs of two types of policies that are commonly financed through debt:

Yearly Cost	Interest Cost (One Time Investment)			Interest Cost (Repeated Investment)		
Interest Rate	2%	3.5%	5%	2%	3.5%	5%

1	0.02	0.035	0.05	0.02	0.035	0.05
2	0.0404	0.071225	0.1025	0.0604	0.106225	0.1525
3	0.061208	0.108718	0.157625	0.121608	0.214943	0.310125
4	0.082432	0.147523	0.215506	0.20404	0.362466	0.525631
5	0.104081	0.187686	0.276282	0.308121	0.550152	0.801913
6	0.126162	0.229255	0.340096	0.434283	0.779408	1.142009
7	0.148686	0.272279	0.4071	0.582969	1.051687	1.549109
8	0.171659	0.316809	0.477456	0.754628	1.368496	2.026564
9	0.195093	0.362897	0.551328	0.949722	1.731393	2.577893
10	0.218994	0.410599	0.628895	1.168716	2.141993	3.206787

From the table we can see that for a one-time investment that is financed through debt at a 2% interest rate the compounded cost quickly adds up. For the first year the cost is 2% of the initial investment and the yearly costs double every year to sit at 21.9% of the investment by year 10. If financed with an interest rate of 5% (which is below the long term average interest rate of the debt) after 10 years, it costs 62.9% of the initial investment in that year alone to continue to carry the debt.

The last 3 columns of the table are meant to represent repeating policies that are financed through debt. The most common policy example of this are yearly tax credits or welfare programs but can be used to represent deficits themselves if one would be interested in performing analysis on their impacts. The way the model used in this paper was created was more in line with the first two because they tend to have a more constant cost which this simplistic version of the model assumes. While the basic premise is the same as the single investment the numbers increasing so rapidly might be a little misleading at first. Let us for example use a tax credit that cost the government 40 billion dollars a year.

By year 10 and assuming a 2% interest rate, it will cost the government about 46.4 billion dollars to carry the cost of financing the program through deficit spending in year 10 but the government

will have kept 400 billion dollars of potential revenue in tax payers hands. However, by year 10 the government will have spent around 184 billion dollars (40x4.6) on carrying cost over those 10 years and that is why in most sections that follow we will not only look at the yearly cost but also the cumulative cost of carrying the debt.

Cumulative Cost	Interest Cost (One Time Investment)			Interest Cost (Repeated Investment)		
Interest Rate	2%	3.5%	5%	2%	3.5%	5%
1	0.02	0.035	0.05	0.02	0.035	0.05
2	0.0604	0.106225	0.1525	0.0804	0.141225	0.2025
3	0.121608	0.214943	0.310125	0.202008	0.356168	0.512625
4	0.20404	0.362466	0.525631	0.406048	0.718634	1.038256
5	0.308121	0.550152	0.801913	0.714169	1.268786	1.840169
6	0.434283	0.779408	1.142008	1.148453	2.048193	2.982178
7	0.582969	1.051687	1.549109	1.731421	3.099881	4.531286
8	0.754628	1.368496	2.026564	2.486049	4.468377	6.55785
9	0.949721	1.731393	2.577892	3.435772	6.199769	9.135743
10	1.168715	2.141992	3.206787	4.604488	8.341763	12.34253

You can see that from the table the cumulative cost of the one-time investment is identical to the yearly cost of the repeating program.[10] This is unsurprising given how the repeating model was derived by simply lagging and adding the single investment model but because in the analysis that follows both yearly and cumulative cost implications will be looked at it was important at this point to explicitly state the interest costs and see this pattern.

These tables come from basic accounting principles that have been known for generations. There is nothing new or novel about compounding interest. The government borrows money from current tax payers and uses future taxes from future tax payers to pay the principle plus interest. What is new and novel in this paper is that we will look at the breakdown of the ownership rates of the debt and look at implications on society and within society about using public debt as a financial instrument to finance expenses. In the sections that follow this paper will break down

[10] Differences are due to rounding issues

the interest costs into two distinct subsections that have important policy implications that are new and novel and should be considered when creating a government budget because of the impacts carrying costs have.

Societal Impact:

The first implication we will look at is the cost to society of financing policy through debt by the foreign ownership of the debt that see's society having to pay interest to people outside the society. This effect runs counter to most policy initiatives that aim to grow the economy so society as a whole will be better off. Normally policy makers want to avoid raising taxes because taxes distort people's decisions leading to inefficiency and lower maximum welfare. Foreign ownership of the debt has similar consequences. Foreign ownership of debt leads to the society losing capital in the form of money that leaves the economy lowering maximum welfare.

One-Time Investment:[11]

In our model using the calibration that is now facing the US economy of 2% carrying cost of new debt and 47% foreign ownership rate of the debt we can see that this cost society 0.9% of every dollar borrowed year 1 and goes up to 10.29% by year 10. This means that for a policy to have a net benefit in year 10, it has to have a return of over 10.29% in that year. This is a rather high number given that the average arithmetic return on the S&P 500 over the span from 1966-2015 was at just 11.01% with the geometric mean being 9.61% and in the years 2006-2015 the average as been as low as 9.03% with the geometric mean even lower (Damodaran, 2016). That is to say that for the society to be benefitting from the investment, the government has to be outperforming the S&P 500 from year 10 on.

[11] For the tables with the information used for this analysis see appendix B page 30

This issue would be greatly exacerbated if foreign ownership increased by just 10% as it would cost 12.48% of the investment by year 10. The interest rate paid by the government can also not be expected to remain so low as most experts predict that we will see a rise in borrowing costs in the near future. An increase to 3.5% and the same foreign ownership rate of 47% would see the yearly cost almost double by year 10 to 19.3%. From the analysis above we can see that societal costs are much more sensitive to interest rate rises as they are to foreign ownership increases, which comes from the compounding effect of interest rate while transitions of more or less foreign ownership are linear.

Most government projects of one time investments tend to have significant delays to see payoff. That is why many policy analysts may be more interested in the cumulative costs to society of foreign ownership. Again at 2% interest and 47% foreign ownership, by year 10 it will have cost the society 54.93% of the investment in money leaving the economy. At 5% interest rate (and same foreign ownership rate) that cost would have ballooned to 150.72%. All this to say that carrying public debt when a large portion of that debt is owned by foreign entities cost society dearly and although raising taxes to finance public expenses carries a cost to society so does financing those same expenses through debt.

To combat this issue, policy makers need to take into account the cost of having deficits year after year. If an investment that grew the economy and therefore tax revenues was implemented and financed through borrowing, and that extra growth was then used to pay back the initial loan, a proper and diligent investment program could be implemented that improved societies well being. However, society being perpetually in debt to outside forces has negative effects that counter act any long term societal benefit derived from the programs and can make government programs ineffective means of creating long term growth.

Repeated Investment:[12]

Unsurprisingly repeated investments have a similar story. A tax cut or welfare program financed through debt has important societal costs that add up quickly. A tax cut may spurn economic growth but for it to payoff long term for society, that growth must be substantial and sustainable. By adding the cost of financing programs into governments cost-benefit analyses, policy makers can get a better understanding of what the overall societal impact will be of the proposed policy.

Through the data used to calibrate the model, the economic payoff that society must see is 0.94%. A modest and easily achievable number that means that using temporary tax cuts to spurn economic growth are viable economic strategies when the economy is underperforming. Tax cuts that are permanent deficit creating measures however, tend to quickly cost society more than they are worth. During more trying economic times, investors tend to favor less risky assets and flock to safe investments like purchasing government bills or bonds. This decreases the cost of borrowing by lowering interest rates right when the government needs it the most as revenues will be down and expenses are likely to be higher then normal. This creates the ideal market for governments to give tax breaks to spurn economic growth at a low cost to society. However, if the government does not end these tax cuts when the economy is recovered and investors require a higher payoff to lend to the government. This is when government policies tend to have the least amount of impact and cost the most. By including the model brought forth in this paper, it will become easier for policy makers to justify when tax cuts need to be put in place and when their societal cost out way their benefits.

During the times the economy is performing well, the government can expect to pay higher interest rates raising the necessary payoff of any program. If the interest rate grows to 3.5%, the

[12] For the tables with the information used for this analysis see appendix B page 31

cost increases to 1.65% and at 5% the cost increases to 2.35%. Compounding means that keeping these tax cuts in place during good times quickly costs society more and more. By year 5, the yearly costs equate to 14.48%, 20.36% and 37.69% for 2%, 3.5% and 5% respectively.

Distributional Impact:

The societal impact is not the only issue to consider. Many have debated whether or not financing government policy through debt has a redistributive effect within society. The redistributive argument relies on how much of the debt is owned by people with certain affluence. In this paper we have took the benchmark level to be the top quintile. That is to say we looked at the comparative ownership rates of the top 20% of the income distribution vs the bottom 80%.

One-Time Investment:[13]

Surprisingly only about 35% of the US federal debt is owned by private US citizens and of this share 66% of it is owned by the top quintile. That means that the top quintile owns approximately 23.1% of the debt while the bottom 4 quintiles owns 11.9%. The difference between them is only 11.2% leaving little room for this to have large distributional impacts compared to the 47% foreign ownership rate.

Using the 2% interest rate and the ownership rate described above we get the yearly distributional impact of transferring wealth to the top 20% of the population of any one-time policy only comes to 2.66% after 10 years. Even when our model considers an interest rate at 5% and increases the amount owned by the top quintile to 71% the yearly cost only increases to 9.24%. That is to say there is a negligible distributional impact of a one-time investment financed through debt and should not be an important policy concern since if we account for the fact that the top 20%

[13] For the tables with the information used for this analysis see appendix B page 32

provides around 68% of the revenues that the federal government receives it can probably be argued that the debt has a positive distributional effect (Congressional Budget Office, 2016).

This trend holds true for our model if we increase the ownership rate of the debt by private individuals to 65% (instead of the actual rate of 35%) and keep the distribution the same. We only see a modest increase that for 2% and 66% ownership rate we get the distributional impact to be 4.5%. Since at this threshold ownership rates are not particularly concentrated the impact is not nearly as severe as the societal impacts.

Cumulative costs are little more concerning for policy makers as after 10 years in the US calibration we see the costs rise to 13.09% meaning that if a 100-billion-dollar project was undertaken and financed fully by debt, after 10 years because of the carrying cost of the debt the government would have transferred 13.09 billion dollars more to the top quintile compared to the bottom 80%. This of course increases with the interest rate along with the concentration of ownership of the debt. Although these impacts are small compared to the overall effects on income inequality that the tax and transfer system has, it is an area of research that may be of interest to see how this issue has evolved over time and how the issue will continue to evolve since these effects tend to work in the opposite direction as the tax and transfer system governments aim to have.

Repeated Investment:[14]

The distributional impact of financing policy through debt may be much more interesting for repeated investment programs since this are normally the programs governments put into place to combat income inequality, alleviate poverty and minimize variation of periodic income around

[14] For the tables with the information used for this analysis see appendix B page 33

permanent income. Government transfers to the rich tend to undermine all these policy objectives.

A program financed through debt, put in place to alleviate some of the financial woes of the bottom 4 quintiles will see by the tenth year, 13.09% of its impact on income inequality undone that year. If we continue at 2% interest by the 27th year, the policy will have a negative impact on income inequality. This issue only gets exacerbated by higher interest rates and more concentration of debt ownership. At 5% interest rate with the debt ownership rates as have been estimated in this paper, it only takes 17 years to have the yearly effect of this policy be a growth in income inequality and by year 22, this policy will be transferring twice as much to the top quintile as it would be to the bottom 4.

Even more worrisome is when we account for population differences in our two populations. The bottom 4 quintiles contain 4/5ths of the population meaning that the policies will be causing growth in income inequality once the threshold of 0.20 is surpassed. This occurs at year 13 in the calibrated model to match US data and in year 8 when we take the long term average interest rate of 5%.

All this to say that financing policies through debt quickly tend to actually increase income inequality and cost society lots of income. And because of perpetual deficits all policies that cost more than they generate can be technically viewed as financed through public debt. Public policy makers must keep this is mind if they are to create policy to improve the lives of their citizens.

Conclusion

The papers goal was to introduce a model that could capture the costs of financing public expense through debt. We did this using basic accounting techniques along with using various publicly available information to estimate ownership rates of the public debt and carrying costs of the

debt. While the model is simplistic and one sided, as it does not take into account benefits or policy alternatives, it does give insight into potential issues with having perpetual deficits because of the way government expenses are financed. The novelty of the paper comes from the introduction of calculating debt servicing fees, which is lacking in the body of literature relating to this topic.

Although in this paper no hard conclusion will be made about any policy or debt financing, in general, this paper introduces a model that should be a starting point to beginning to fully understand if/when governments should be using debt to finance public policy. The basic results of the paper are that negative (counter effects to policy objectives) occur when using debt to finance public expenses. Foreign ownership rates of the debt cost society by forcing society to pay interest to foreign entities and the distributional makeup of within society ownership create distributional effects. As the debt ownership make up stands now in the US, these effects have important impacts on policy as they run counter to policy goals that governments aim to accomplish. These effects that take place because of debt financing are rather sensitive to interest rate costs and less so to the actual breakdown of the ownership rates.

While this paper only focuses on the debt makeup in one year in the US, future research may be interested in determining trends in ownership rates over time and across countries. Policy makers may also want to use this model to determine whether financing through debt is the appropriate financial instrument to use or if creating budgets that are more balanced may be a better policy alternative. While this paper is a good introduction to the cost of financing public expenses through debt, much more research into this important topic is needed to make any real policy suggestions.

Bibliography

Bibi, S., Duclos, J.-Y., & Araar, A. (2013). Mobility, Taxation and Income. *Social Choice and Welfare*, 503-527.

Blanchard, O. J. (1985). Debt, Deficits, and Finite Horizons. *Journal of Political Economy*, 223-247.

Board of Governors of the Federal Reserve System. (2014, April 9). *Survey of Consumer Finances.* Récupéré sur Board of Governors of the Federal Reserve System: http://www.federalreserve.gov/econresdata/scf/files/scf2013_tables_internal_real.xls

Board of Governors of the Federal Reserve System. (2016, June 6). Flow of Funds, Balance Sheets, and Integrated Macroeconomic Accounts. *Financial Accounts of the United States*, p. 112. Récupéré sur https://www.federalreserve.gov/releases/z1/current/annuals/a2005-2015.pdf

Congressional Budget Office. (2016). *The Distribution of Household Income and Federal Taxes, 2013 (supplemental data).* Washington: Congressional Budget Office.

Damodaran, A. (2016, January 5). *Annual Returns on Stock, T.Bonds and T.Bills: 1928 - Current*. Récupéré sur NYU, Stern: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html

Fred Graph. (2015, July 8). Récupéré sur Economic Research Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/graph/?g=ALy#0

Ghosh, S., & Mourmouras, I. A. (2004). Endogenous growth, welfare and. *Journal of Macoeconomics*, 623-635.

Hager, S. B. (2014). What Happened to the Bondholding Class? Public Debt, Power and the Top One Per Cent. *New Political Economy*, 155-182. Récupéré sur http://www.tandfonline.com/doi/abs/10.1080/13563467.2013.768613

Hager, S. B. (Future). Corporate Ownership of the Public Debt, Mapping the New Aristocracy of Finance. *Socio-Economic Review*.

Kotlikoff, A. J., & Auerbach, L. J. (1987). Dynamic Fiscal Policy. *Cambridge: Cambridge University Press*.

Mankiw, D. W. (1998). Government Debt. *Handbook of Macroeconomics*.

Office of Management and Budget. (2016). *Historical Tables.* Washington: Office of Management and Budget. Récupéré sur White House: https://www.whitehouse.gov/sites/default/files/omb/budget/fy2017/assets/hist.zip

Ravenna, F., & Vincent, N. (2014). Inequality and debt in a model with heterogeneous agents. *Economics Letters*, 177-182.

Yakita, A. (2008). Sustainability of public debt, public capital formation, and. *Journal of Public Economics*, 897-914.

Figures

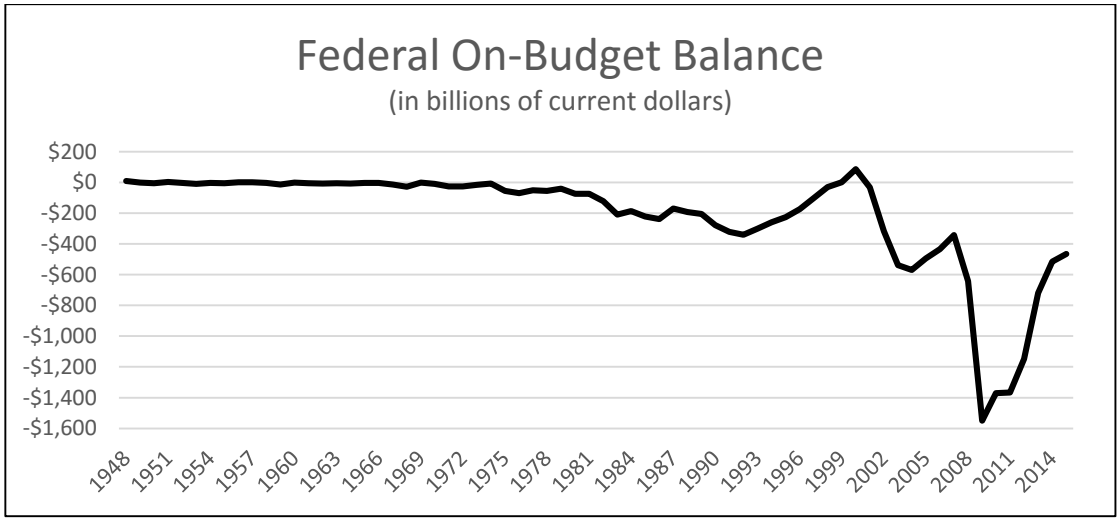

Figure 1 Created Using data from Office of Budget and Management Historical Tables 14.6

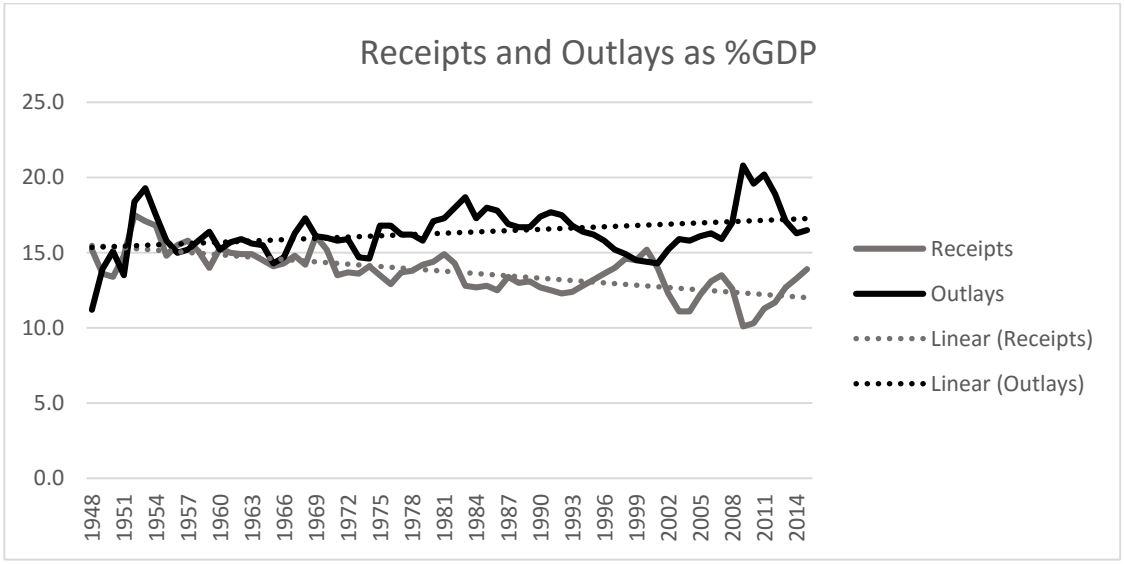

Figure 2 Created Using data from Office of Budget and Management Historical Tables 1.2

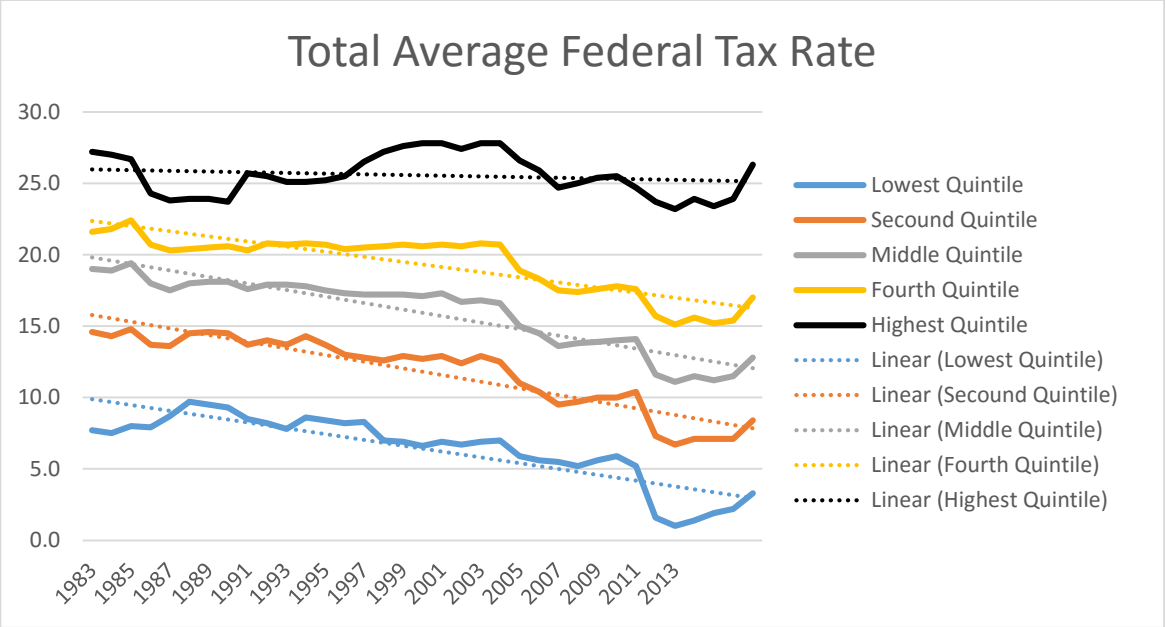

Figure 3 Created using supplemental data from CBO publication The Distribution of Household Income and Federal Taxes supplemental data https://www.cbo.gov/sites/default/files/114th-congress-2015-2016/reports/51361-SupplementalData-2.xlsx

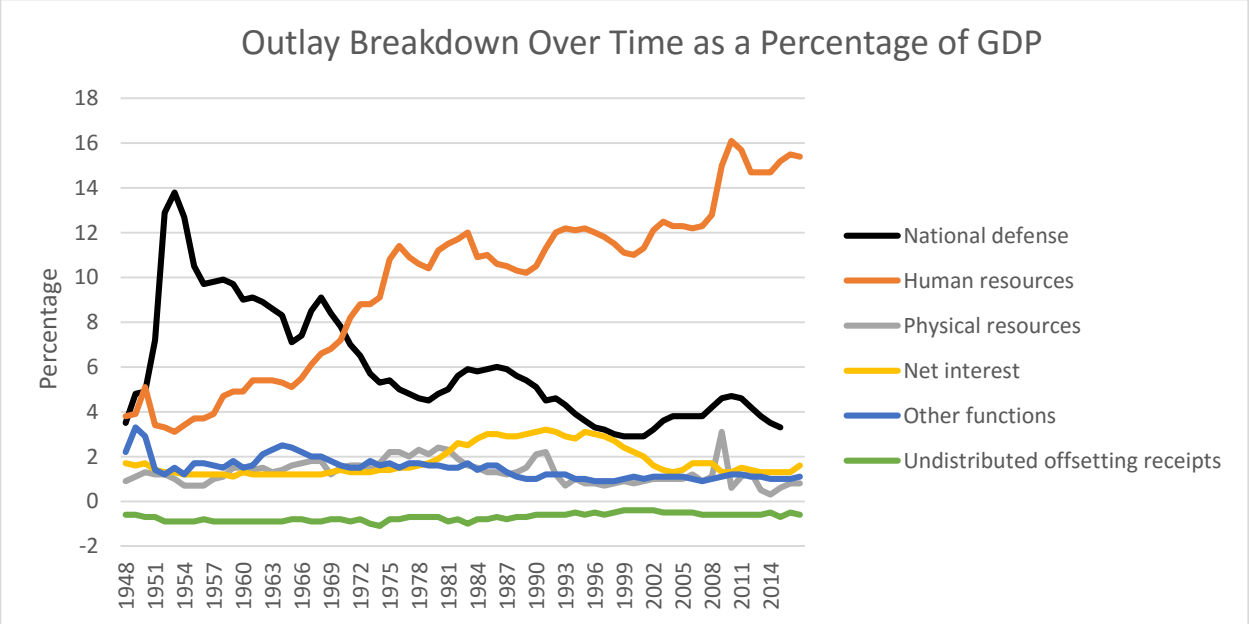

Figure 4 Created Using data from Office of Budget and Management Historical Tables 3.1

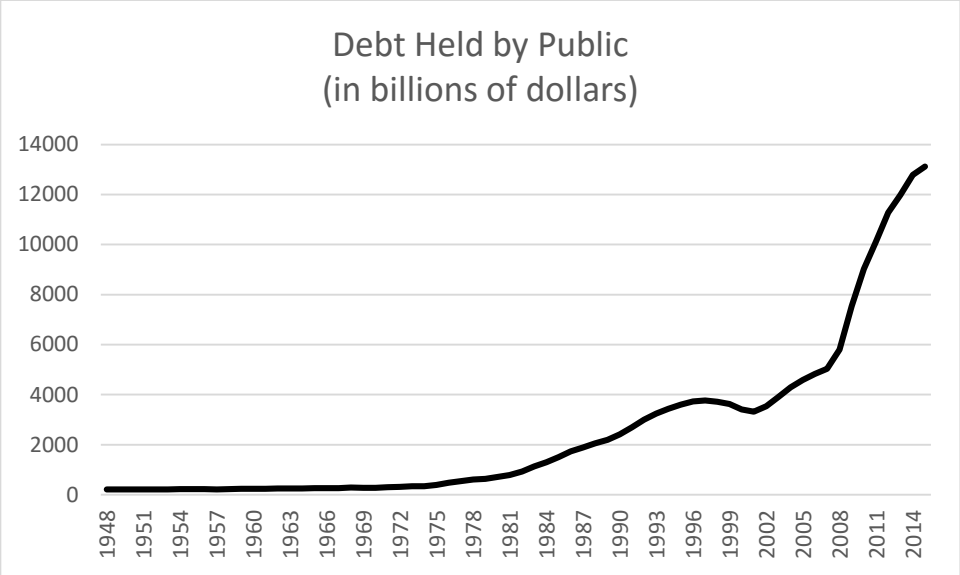

Figure 5 Created Using data from Office of Budget and Management Historical Tables 7.1

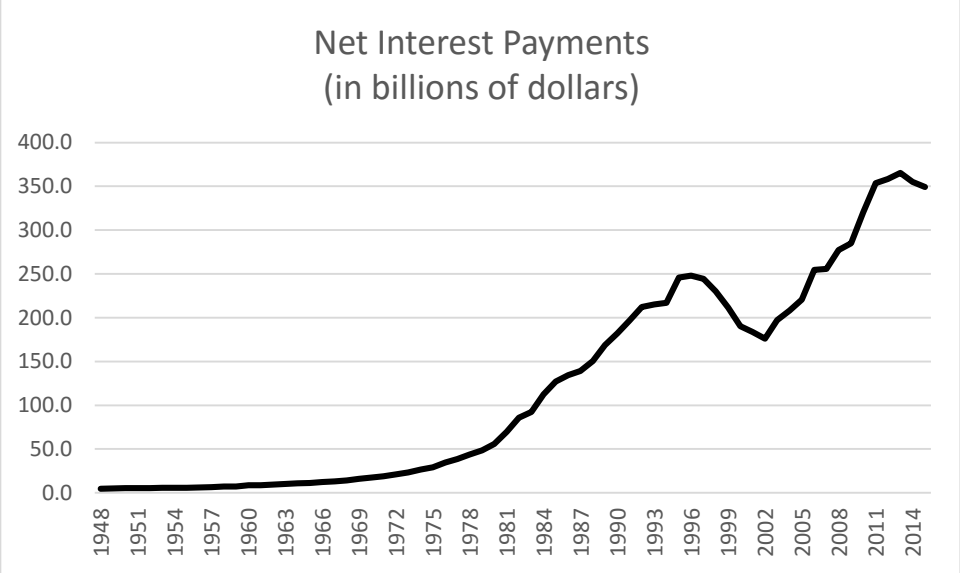

Figure 6 Created Using data from Office of Budget and Management Historical Tables 14.4

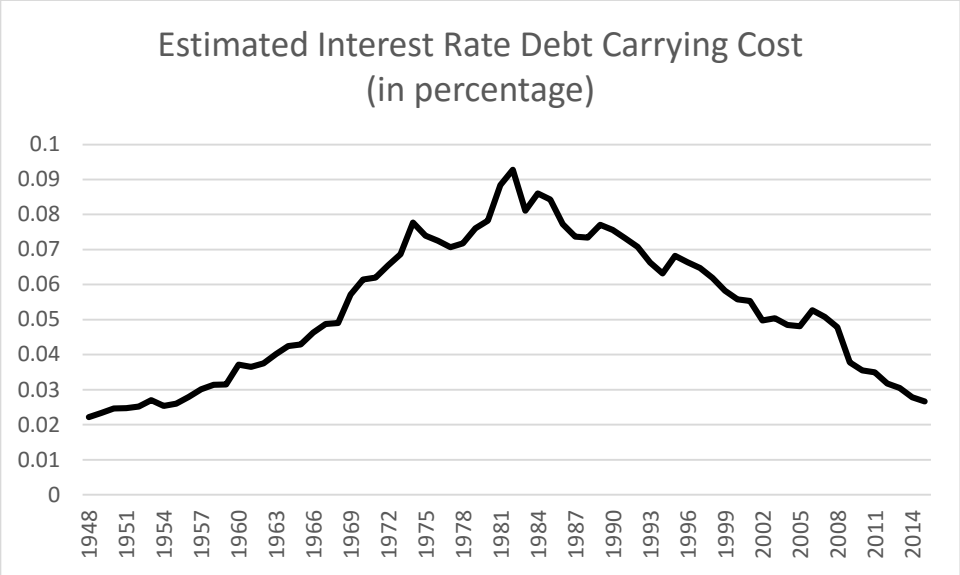

Figure 7 Estimated using data on debt held by public table 7.1 and net interest payment table 14.4

Appendix A

Quintiles	Transaction accounts	Certificates of deposit	Savings bonds	Bonds	Stocks	Pooled investment funds	Retirement accounts	Cash value life insurance	Other managed assets	Other	Any financial asset
Less than 20	4.588	3.046	0.172	*	2.331	4.690	3.807	0.859	1.232	4.891	25.603
20–39.9	9.342	3.254	0.260	2.030	3.537	3.164	11.551	1.868	3.534	2.205	40.788
40–59.9	12.442	3.967	0.464	0.835	7.912	6.637	36.139	3.220	4.291	0.966	76.812
60–79.9	23.189	4.586	0.749	1.746	14.297	11.101	86.151	4.916	11.573	2.080	160.379
80–100	119.606	10.133	1.563	*	174.157	163.780	357.654	22.979	76.252	8.993	971.872
Estimated Percentage Owned by Top Quintile	0.707	0.406	0.487	*	0.861	0.865	0.722	0.679	0.787	0.470	0.762

	Household Assets	Nonfinancial Businesses	Banks	Property Insurance	Life Insurance	Pension Fund	State Government Ownership	Money Market Mutual Funds	Mutual Fund	Other	Total
Value	1144.9	94	304.4	102.4	168.7	299.9	814.9	496.1	472.8	296.2	4194.3
Estimated Ownership Rates	0.406	0.861	0.861	0.787	0.679	0.722	0.722	0.865	0.865	0.470	**0.664**
Owned by Top Quintile	464.297	80.950	262.139	80.595	114.549	216.556	588.434	429.057	408.906	139.209	2784.690

Appendix B

Societal Impact:

One-Time Investment:

Interest Rate ->	Single Investment Foreign Ownership Cost								
	2%			3.5%			5.0%		
Year	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%
1	0.0074	0.0094	0.0114	0.0130	0.0165	0.0200	0.0185	0.0235	0.0285
2	0.0149	0.0190	0.0230	0.0264	0.0335	0.0406	0.0379	0.0482	0.0584
3	0.0226	0.0288	0.0349	0.0402	0.0511	0.0620	0.0583	0.0741	0.0898
4	0.0305	0.0387	0.0470	0.0546	0.0693	0.0841	0.0797	0.1013	0.1228
5	0.0385	0.0489	0.0593	0.0694	0.0882	0.1070	0.1022	0.1299	0.1575
6	0.0467	0.0593	0.0719	0.0848	0.1078	0.1307	0.1258	0.1598	0.1939
7	0.0550	0.0699	0.0848	0.1007	0.1280	0.1552	0.1506	0.1913	0.2320
8	0.0635	0.0807	0.0978	0.1172	0.1489	0.1806	0.1767	0.2244	0.2721
9	0.0722	0.0917	0.1112	0.1343	0.1706	0.2069	0.2040	0.2591	0.3143
10	0.0810	0.1029	0.1248	0.1519	0.1930	0.2340	0.2327	0.2956	0.3585

Interest Rate ->	Cumulative Single Investment Foreign Ownership Cost								
	2%			3.5%			5.0%		
Year	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%
1	0.0074	0.0094	0.0114	0.0130	0.0165	0.0200	0.0185	0.0235	0.0285
2	0.0223	0.0284	0.0344	0.0393	0.0499	0.0605	0.0564	0.0717	0.0869
3	0.0450	0.0572	0.0693	0.0795	0.1010	0.1225	0.1147	0.1458	0.1768
4	0.0755	0.0959	0.1163	0.1341	0.1704	0.2066	0.1945	0.2470	0.2996
5	0.1140	0.1448	0.1756	0.2036	0.2586	0.3136	0.2967	0.3769	0.4571
6	0.1607	0.2041	0.2475	0.2884	0.3663	0.4443	0.4225	0.5367	0.6509
7	0.2157	0.2740	0.3323	0.3891	0.4943	0.5995	0.5732	0.7281	0.8830
8	0.2792	0.3547	0.4301	0.5063	0.6432	0.7800	0.7498	0.9525	1.1551
9	0.3514	0.4464	0.5413	0.6406	0.8138	0.9869	0.9538	1.2116	1.4694
10	0.4324	0.5493	0.6662	0.7925	1.0067	1.2209	1.1865	1.5072	1.8279

Repeated Investment:

Interest Rate ->	Repeated Investment Foreign Ownership Cost								
	2%			3.5%			5.0%		
Year	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%
1	0.0074	0.0094	0.0114	0.0130	0.0165	0.0200	0.0185	0.0235	0.0285
2	0.0223	0.0284	0.0344	0.0393	0.0499	0.0605	0.0564	0.0717	0.0869
3	0.0450	0.0572	0.0693	0.0795	0.1010	0.1225	0.1147	0.1458	0.1768
4	0.0755	0.0959	0.1163	0.1341	0.1704	0.2066	0.1945	0.2470	0.2996
5	0.1140	0.1448	0.1756	0.2036	0.2586	0.3136	0.2967	0.3769	0.4571
6	0.1607	0.2041	0.2475	0.2884	0.3663	0.4443	0.4225	0.5367	0.6509
7	0.2157	0.2740	0.3323	0.3891	0.4943	0.5995	0.5732	0.7281	0.8830
8	0.2792	0.3547	0.4301	0.5063	0.6432	0.7800	0.7498	0.9525	1.1551
9	0.3514	0.4464	0.5413	0.6406	0.8138	0.9869	0.9538	1.2116	1.4694
10	0.4324	0.5493	0.6662	0.7925	1.0067	1.2209	1.1865	1.5072	1.8279

Interest Rate ->	Cumulative Repeated Investment Foreign Ownership Cost								
	2%			3.5%			5.0%		
Year	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%	FO 37%	FO 47%	FO 57%
1	0.0074	0.0094	0.0114	0.0130	0.0165	0.0200	0.0185	0.0235	0.0285
2	0.0297	0.0378	0.0458	0.0523	0.0664	0.0805	0.0749	0.0952	0.1154
3	0.0747	0.0949	0.1151	0.1318	0.1674	0.2030	0.1897	0.2409	0.2922
4	0.1502	0.1908	0.2314	0.2659	0.3378	0.4096	0.3842	0.4880	0.5918
5	0.2642	0.3357	0.4071	0.4695	0.5963	0.7232	0.6809	0.8649	1.0489
6	0.4249	0.5398	0.6546	0.7578	0.9627	1.1675	1.1034	1.4016	1.6998
7	0.6406	0.8138	0.9869	1.1470	1.4569	1.7669	1.6766	2.1297	2.5828
8	0.9198	1.1684	1.4170	1.6533	2.1001	2.5470	2.4264	3.0822	3.7380
9	1.2712	1.6148	1.9584	2.2939	2.9139	3.5339	3.3802	4.2938	5.2074
10	1.7037	2.1641	2.6246	3.0865	3.9206	4.7548	4.5667	5.8010	7.0352

Distributional Impact:

One-Time Investment:

Interest Rate ->	Single Investment Distribution Cost								
	2%			3.5%			5.0%		
Year	61%	66%	71%	61%	66%	71%	61%	66%	71%
1	0.0015	0.0022	0.0029	0.0027	0.0039	0.0051	0.0039	0.0056	0.0074
2	0.0031	0.0045	0.0059	0.0055	0.0080	0.0105	0.0079	0.0115	0.0151
3	0.0047	0.0069	0.0090	0.0084	0.0122	0.0160	0.0121	0.0177	0.0232
4	0.0063	0.0092	0.0121	0.0114	0.0165	0.0217	0.0166	0.0241	0.0317
5	0.0080	0.0117	0.0153	0.0145	0.0210	0.0276	0.0213	0.0309	0.0406
6	0.0097	0.0141	0.0185	0.0177	0.0257	0.0337	0.0262	0.0381	0.0500
7	0.0114	0.0167	0.0219	0.0210	0.0305	0.0400	0.0313	0.0456	0.0598
8	0.0132	0.0192	0.0252	0.0244	0.0355	0.0466	0.0368	0.0535	0.0702
9	0.0150	0.0219	0.0287	0.0279	0.0406	0.0533	0.0425	0.0617	0.0810
10	0.0169	0.0245	0.0322	0.0316	0.0460	0.0604	0.0484	0.0704	0.0924

Interest Rate ->	Cumulative Single Investment Distribution Cost								
	2%			3.5%			5.0%		
Year	61%	66%	71%	61%	66%	71%	61%	66%	71%
1	0.0015	0.0022	0.0029	0.0027	0.0039	0.0051	0.0039	0.0056	0.0074
2	0.0047	0.0068	0.0089	0.0082	0.0119	0.0156	0.0117	0.0171	0.0224
3	0.0094	0.0136	0.0179	0.0166	0.0241	0.0316	0.0239	0.0347	0.0456
4	0.0157	0.0229	0.0300	0.0279	0.0406	0.0533	0.0405	0.0589	0.0773
5	0.0237	0.0345	0.0453	0.0424	0.0616	0.0809	0.0617	0.0898	0.1179
6	0.0334	0.0486	0.0638	0.0600	0.0873	0.1146	0.0879	0.1279	0.1679
7	0.0449	0.0653	0.0857	0.0810	0.1178	0.1546	0.1193	0.1735	0.2277
8	0.0581	0.0845	0.1109	0.1054	0.1533	0.2012	0.1560	0.2270	0.2979
9	0.0731	0.1064	0.1396	0.1333	0.1939	0.2545	0.1985	0.2887	0.3790
10	0.0900	0.1309	0.1718	0.1649	0.2399	0.3149	0.2469	0.3592	0.4714

Repeated Investment:

Interest Rate ->	Repeated Investment Distribution Cost								
	2%			3.5%			5.0%		
Year	61%	66%	71%	61%	66%	71%	61%	66%	71%
1	0.0015	0.0022	0.0029	0.0027	0.0039	0.0051	0.0039	0.0056	0.0074
2	0.0047	0.0068	0.0089	0.0082	0.0119	0.0156	0.0117	0.0171	0.0224
3	0.0094	0.0136	0.0179	0.0166	0.0241	0.0316	0.0239	0.0347	0.0456
4	0.0157	0.0229	0.0300	0.0279	0.0406	0.0533	0.0405	0.0589	0.0773
5	0.0237	0.0345	0.0453	0.0424	0.0616	0.0809	0.0617	0.0898	0.1179
6	0.0334	0.0486	0.0638	0.0600	0.0873	0.1146	0.0879	0.1279	0.1679
7	0.0449	0.0653	0.0857	0.0810	0.1178	0.1546	0.1193	0.1735	0.2277
8	0.0581	0.0845	0.1109	0.1054	0.1533	0.2012	0.1560	0.2270	0.2979
9	0.0731	0.1064	0.1396	0.1333	0.1939	0.2545	0.1985	0.2887	0.3790
10	0.0900	0.1309	0.1718	0.1649	0.2399	0.3149	0.2469	0.3592	0.4714

Interest Rate ->	Cumulative Repeated Investment Distribution Cost								
	2%			3.5%			5.0%		
Year	61%	66%	71%	61%	66%	71%	61%	66%	71%
1	0.0015	0.0022	0.0029	0.0027	0.0039	0.0051	0.0039	0.0056	0.0074
2	0.0062	0.0090	0.0118	0.0109	0.0158	0.0208	0.0156	0.0227	0.0298
3	0.0156	0.0226	0.0297	0.0274	0.0399	0.0524	0.0395	0.0574	0.0754
4	0.0313	0.0455	0.0597	0.0553	0.0805	0.1056	0.0799	0.1163	0.1526
5	0.0550	0.0800	0.1050	0.0977	0.1421	0.1865	0.1417	0.2061	0.2705
6	0.0884	0.1286	0.1688	0.1577	0.2294	0.3011	0.2296	0.3340	0.4384
7	0.1333	0.1939	0.2545	0.2387	0.3472	0.4557	0.3489	0.5075	0.6661
8	0.1914	0.2784	0.3654	0.3441	0.5005	0.6569	0.5050	0.7345	0.9640
9	0.2646	0.3848	0.5051	0.4774	0.6944	0.9114	0.7035	1.0232	1.3430
10	0.3545	0.5157	0.6769	0.6423	0.9343	1.2262	0.9504	1.3824	1.8144