PRODUCTIVITY SHOCKS AND MONETARY POLICY IN A SMALL OPEN ECONOMY: AN UPDATED ASSESSMENT OF GALI AND MONACELLI

by

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ABSTRACT

This paper revisits the seminal work by Galí and Monacelli (2002) and evaluates the model with a more recent sample period. A comparison between the original calibration of the model and the new calibration with updated data helps assess the robustness of the GM model with an updated sample. Furthermore, a comparison of volatility of the output gap and inflation is conducted to assess optimal monetary policy.

Beyond the original evaluation of three simple monetary policy rules included in Galí and Monacelli (2002), this paper extends the evaluation of the effect of a domestic and a foreign technology shock to include also a Taylor Rule.

The responses to these shocks under both calibrations show the same sign but, in some cases, lower initial response magnitude and higher persistence of the shocks. The comparison of volatility deemed domestic inflation targeting as the optimal policy, as it did in the original paper, but it uncovered some advantages to the use of the Taylor rule.

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TABLE OF CONTENTS

1 INTRODUCTION

This paper presents an extension and updated calibration of Galí and Monacelli (2002). The objective of this extension and re-calibration is to evaluate robustness of the original findings of the paper with respect to optimal monetary policy and to include the Taylor rule in the study, since it is one of the most widely used rule in current monetary policy. Finally, this paper aims to compare the effects of domestic and foreign productivity shocks in the Galí and Monacelli economy under the original calibration and a re-calibration based on a more recent sample. The aim of this exercise is to verify the validity of the model's findings when the calibration sample is updated.

New Keynesian macroeconomic models aim to explain the reaction of key variables to shocks to the economy and/or policy shocks. New Keynesian models follow the Keynesian school's tradition while incorporating microeconomic foundations, namely frictions in wages and prices (Gordon 1990, 1115).

Much of recent literature in monetary economics is based on New Keynesian models. Their effectiveness has made them popular as tools for monetary policy analysis, and many central banks model their respective economies in the form of New Keynesian models of various degrees of complexity. After dramatic instances of stagflation in the 1970s and structural changes that granted more independence to central banks in developed countries, many studies used New Keynesian models to identify the optimal monetary policy rule. Studies have continued to be conducted to assess the validity of monetary policy rules in the current economic context. One such study was conducted by Galí and Monacelli (2002). The paper evaluates the welfare implications of three simple policy rules - Domestic Inflation Targetting (DIT), CPI Inflation Targeting (CIT), and an Exchange Rate Peg (PEG). The model is shocked via domestic technology, foreign technology, and both shocks combined.

2 LITERATURE REVIEW

Small open economy DSGE models are the basis for much of recent research in monetary economics and trade. This type of model is widely used to portray Canada's economic role in the global setting. Furthermore, simple two-economy models can be used in Canadian research since trade occurs primarily with the United States, which allows the two-economy model (in which US data is used to proxy for the world economy) to be sufficiently accurate to be informative.

Central bank researchers in other small, developed economies, have used this type of model to assess optimal monetary policy. Einarsson (2002) compares a monetary union with a floating exchange rate regime for Iceland, Kollmann (2004) compares a currency peg to a floating rate for the US and Europe, and Ambler, Dib, and Rebei (2004) compare Taylor Rules for Canada.

The assessment of optimal monetary policy is based on welfare analysis. Several papers base this analysis on a welfare loss function. Paoli (2003) uses a utility-based loss function which penalizes volatility in domestic inflation, the output gap, and the real exchange rate. Ravenna and Walsh (2009) find a second-order approximation of a welfare function that is affected by inflation and current and lagged unemployment. Benigno and Woodford (2004) derive a quadratic loss function consisting of a weighted average of squared deviations of inflation and squared deviations of log of output.

Beyond assessment of optimal monetary policy, small open economy New Keynesian models help examine the role of specific factors on welfare and economic stability. Dib (2008) develops a substantially more complex version of a New Keynesian model to examine welfare effects of volatility in the price level and in the exchange rate. This model assumes a particular monetary policy rule and thus does not cover the optimal policy area of Galí and Monacelli, but it does use the basic framework to assess the reaction of the economy to shocks. As the GM paper, Dib models two economies (one small, one large). Similarly, Gourio, Siemer, and Verdelhan (2010) use the same two-country structure to study the response of the economy and of exchange rates in particular to a shock in the volatility at the international level. Ambler, Dib, and Rebei (2004) also use a similar structure to evaluate alternative Taylor Rules specifically. The role of Taylor rules will be discussed later in the paper.

The Galí and Monacelli (2002) paper has been significantly influential in the literature. The model has served as basis for both application and testing of monetary theory. On the applications, Malin et al (2006) use the model to conduct a Bayesian estimation on the euro area. Auray, Eyquem and Poutineau (2009) use the framework to evaluate potential welfare gains from trade integration, also specific to the euro area. In terms of testing theory, Beltran and Draper (2008) use maximum likelihood and Bayesian methods to test the feasibility and reliability of parameter identification and inference. Barkbu et al (2005) use the case of the United States and Europe to test the New Keynesian Phillips Curve - defined and described in later sections - for aggregation bias, and for stability and robustness of estimation results.

Despite their significant contribution to monetary analysis, New Keynesian models are subject to critique and feature their own set of challenges. Chari, Kehoe, and McGrattan (2008) show many characteristic features of this class of model are not consistent with microeconomic data, and thus deem them unreliable as a tool for policy analysis. For models with wage rigidity in addition to price rigidity, Basu and House (2015) present challenges to justify wage rigidities and reconcile this model feature with the data. Having said this, Galí and Monacelli's model has been scrutinized in specific. Ried (2009) tests the model against the *the Six Major Puzzles in International Macroeconomics*. The model holds with significant robustness at replicating the puzzles, but it is important to note that Ried adds trade costs to make this possible.

What appears necessary for this model to continue to be used for policy analysis in the current context is an update of the calibration of the model to test its robustness out of sample and the addition of a Taylor rule, since the latter has gained an overwhelming presence in contemporary monetary policy and its analysis must be included if the model is to be informative in the present context. The extension of adding the Taylor rule and the updating of the sample are contributions that this paper will make to allow for a better understanding of this model. Enhancing understanding of the model will in turn allow for its continued use in policy analysis.

3 THE MODEL

This model has several key characteristics:

1. It is a small open economy: this means the economy trades with other economies, but it does not have power over the pricing of the goods it trades, it is otherwise known as a price taker. Note that traditionally, small open economy models assume the interest rate is determined in the aggregate world economy and that the small country is also a price taker in that it takes interest rate as given. This is not the case in this model. In the GM model, not only is the domestic economy small, but it is part of a continuum of small economies.

2. It includes two reference economies: for the purpose of analysis, the model focuses on one domestic economy and one foreign economy.

3. I features *á la Calvo* price stickiness: this is a common feature of NK models. Price stickiness refers to the notion that prices are not adjusted immediately in reaction to a change in economic conditions. Calvo pricing in particular illustrates this stickiness as producers' inability to adjust prices every period. Producers can adjust their prices in period t with a given probability, in this case $(1-\theta)$, and is stuck with their $t-1$ prices with probability θ.

4. Money is included implicitly: Money appears implicitly through the interest rate rule. It does not appear explicitly in the budget constraint or the utility function.

3.1 Households

The Galí and Monacelli model observes a continuum of households of measure one. This means an infinite number of households that, by normalizing, can be thought of as adding up to one. Each household maximizes utility of the form:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\varphi}}{1+\varphi}\right]\right] \tag{1}$$

where $t = 0, 1, 2, ...$, N_t denotes labour measured in hours and C_t is total consumption by the household.

Households' optimization is subject to a budget contraint of the form:

$$\int_0^1 [P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i)]di + E_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_t Nt + T_t \quad (2)$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of domestic and foreign goods.

Galí and Monacelli assume households access a complete set of internationally traded contingent claims. Households hold an investment portfolio at time t, which includes shares in firms, and get nominal payoff D_{t+1} in period t+1, this payoff is scaled by the the stochastic discount factor Q_{t+1}. Finally, W_t represents a household's nominal wage, and T_t represents lump-sum payments or transfers when positive and taxes when negative.

Consumption is divided into domestic and foreign in the form:

$$C_t = [(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \quad (3)$$

where $C_{H,t}$ and $C_{F,t}$ represent consumption domestic and foreign goods, respectively and $\eta > 0$. The consumption of each household is aggregated using the following constant elasticity of substitution (CES) formulas:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (4)$$

$$C_{F,t} = \left(\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (5)$$

where $\varepsilon > 1$.

Galí and Monacelli parameterize consumption in the form:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{6}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{7}$$

such that $P_t \equiv (1-\alpha)P_{H,t} + \alpha P_{F,t}$ is the Consumer Price Index (CPI) and α is the proportion of domestic consumption that corresponds to foreign goods (imports). The Lagrangean for this optimization problem is as follows:

$$\mathscr{L} = \mathbb{E}_0\left[\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\right]\right] + \lambda_t\left(D_t + W_tN_t + T_t - P_tCt - \mathbb{E}_t\left[\left[Q_{t+1}D_{t+1}\right]\right]\right) \tag{8}$$

The first order conditions with respect to C_t, C_{t+1}, and D_{t+1} are, respectively:

$$\beta^t C_t^{-\sigma} - P_t\lambda_t = 0 \tag{9}$$

$$\beta^{t+1} C_{t+1}^{-sigma} - P_{t+1}\lambda_{t+1} = 0 \tag{10}$$

$$\lambda_{t+1} - \lambda_t Q_{t+1} = 0 \tag{11}$$

these result in the following expressions:

$$Q_{t+1} = \frac{\lambda_t}{\lambda_t} \tag{12}$$

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \frac{P_{t+1}\lambda_{t+1}}{P_t\lambda_t} \tag{13}$$

combining these expressions yields:

$$\beta\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{P_t}{P_{t+1}} = Q_{t+1} \tag{14}$$

taking conditional expectation of each side and defining $R_t^{-1} \equiv \mathbb{E}_t\left[Q_{t+1}\right]$ results in the

Euler equation:

$$\beta R_t \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{P_t}{P_{t+1}} \right] = 1 \tag{15}$$

finally, he MRS condition is:

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{16}$$

The last two equations will be used to solve the model in their log-linearized form:

$$c_t = \mathbb{E}_t \left[c_{t+1} \right] - \frac{1}{\sigma} (r_t - \mathbb{E}_t \left[\pi_{t+1} \right] - \rho) \tag{17}$$

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{18}$$

where lowercase letters are defined as the natural logarithm of their uppercase counterparts. The world economy (a representative foreign economy) face the same optimization problem for households.

In equation (17), $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation. Log-linearizing around the steady state where $P_{H,t} = P_{F,t}$ yields:

$$p_t \equiv (1 - \alpha) p_{H,t} + \alpha p_{F,t} \tag{19}$$

$$= p_{H,t} + \alpha s_t \tag{20}$$

where $s_t \equiv p_{F,t} - p_{H,t}$ represents the terms of trade in logarithmic form, or the real price of foreign goods measured in units of domestic goods. 3.19 implies that domestic and CPI inflation are linked in the following way:

$$p_{t+1} - p_t = p_{H,t+1} - p_{H,t} + \alpha(s_{t+1} - st) \tag{21}$$

$$\pi_{H,t} \equiv p_{H,t+1} - p_{H,t} \tag{22}$$

$$\pi_t = \pi H, t + \alpha \Delta st \tag{23}$$

that is, the gap between CPI and domestic inflation is proportional to the change in terms of trade with coefficient α, the degree of 'openness' of the domestic economy.

A key assumption by Galí and Monacelli is that, because the small economy is negligible in size with respect to the world economy, the world can be treated as a closed economy. One implication of this assumption is that the foreign price index P_t^* is equal to the foreign currency price of foreign goods, P_t^*. This implies $\pi_t = \pi_{F,t}$, where * denotes foreign variables.

Another important assumption is that the *law of one price* holds. In other words, prices in the domestic and foreign economies differ exclusively according to the exchange rate, or:

$$[p_{F,t}(i) = e_t + p_{F,t}^*(i) \forall i \in [0,1] \tag{24}$$

where e_t is the log of the nominal exchange rate, or the price of the foreign currency in units of the domestic currency.

The terms of trade then become:

$$s_t \equiv e_t + p_t^* - p_{H,t} \tag{25}$$

The log of the real exchange rate is defined as $q_t \equiv e_t + p_t^* - p_t$. It is related to the terms of trade as follows:

$$q_t = s_t + p_{H,t} - p_t \tag{26}$$

$$q_t = (1-\alpha)s_t \tag{27}$$

which holds to a first order approximation. Note that this expression, as equation (20), establishes a connection to the terms of trade as a function of the degree of openness - in this case a relationship between s_t and the real exchange rate.

As mentioned above, Galí and Monacelli assume households have access to complete securities markets, which must implies the first order condition in equation (14) must also hold for foreign consumers:

$$\beta\left(\frac{C_t^*}{C_{t+1}^*}\right)^{\sigma}\frac{P_t^*}{P_{t+1}^*}\frac{e_t}{e_{t+1}} = Q_{t+1} \tag{28}$$

Combining this with equation (14) and the definition of Q_t results in:

$$C_t = \vartheta C_t^* Q_t^{1/\sigma} \forall t \tag{29}$$

where ϑ is a constant that depends on initial conditions in the form $C = Y = \vartheta Y^*$.

Taking logs of both sides, using equation (22), and omitting the constant yields:

$$c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t \tag{30}$$

which holds exactly.

Thus, assuming complete international securities markets yields a simple relationship between domestic and foreign consumption and the terms of trade. This illustrates the concept of *international risk sharing*.

This assumption also implies the equilibrium domestic price of a riskless bond denominated in foreign currency is $\frac{exp(e_t)}{R_t^*} = \mathbb{E}_t\left[Q_{t,t+1} exp(e_t)\right]$. Combining this with the domestic bond pricing definition as above, $R_t^{-1} \equiv \mathbb{E}_t\left[Q_{t+1}\right]$, produces the *uncovered interest parity condition*:

$$\mathbb{E}_t\left[Q_{t,t+1}\left[R_t - R_t^*\left(\frac{exp(e_{t+1})}{exp(e_t)}\right)\right]\right] = 0 \tag{31}$$

which can be log-linearized around a deterministic steady state to obtain:

$$r_t - r_t^* = \mathbb{E}_t\left[\Delta e_{t+1}\right] \tag{32}$$

This expression produces the following first order stochastic difference equation for the terms of trade:

$$s_t = \left(r_t^* - \mathbb{E}_t\left[\pi_{t+1}^*\right]\right) - \left(r_t - \mathbb{E}_t\left[\pi_{H,t+1}\right]\right) + \mathbb{E}_t\left[s_{t+1}\right] \tag{33}$$

Finally, Galí and Monacelli prove that $lim_{T\to\infty}\mathbb{E}_t\left[s_T\right]=0$, which implies *Purchasing Power Parity* (PPP) -price of goods in terms of local currency are equal at the international level, such that differences in prices are only a function of the exchange rate- holds in the long run. This last result allows equation (26) to be solved forward to obtain:

$$s_t = \mathbb{E}_t\left[\sum_{k=0}^{\infty}[(r^*_{t+k}-\pi^*_{t+k+1})-(r_{t+k}-\pi_{H,t+k+1})]\right] \tag{34}$$

3.2 Firms

Firms produce differentiated goods with linear technology given by production function:

$$Y_t(i)=A_tN_t(i) \tag{35}$$

or in log-linearized form:

$$y_t(i)=a_t+n_t(i) \tag{36}$$

where a_t follows the AR(1) process:

$$a_t=\rho_a a_{t-1}+\varepsilon_t \tag{37}$$

Nominal marginal cost is then given by:

$$mc_t=-\nu+w_t-a_t \tag{38}$$

$$\nu=-log(1-\tau) \tag{39}$$

where τ is an employment subsidy, and mc_t is equal for all firms.Output from all firms is aggregated as:

$$Y_t\equiv\left[\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{40}$$

Similarly, labour in the different firms is aggregated to:

$$N_t \equiv \int_0^1 N_t(i)di = \frac{Y_t \int_0^1 \frac{Y_t(i)}{Y_t} di}{A_t} \tag{41}$$

Galí and Monacelli prove that up to a first order approximation, there exists the following aggregate relationship:

$$y_t = n_t + a_t \tag{42}$$

The same problem and aggregate relationship is assumed to hold for firms in the world economy. It is of particular interest to specify that foreign technology follows the process:

$$a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_t^* \tag{43}$$

where ε_t^* is white noise and may be correlated with ε_t. This correlation is calibrated in following sections. The two productivity shocks are the main tool Galí and Monacelli use to examine potential welfare loss under alternative monetary policy rules.

3.2.1 Price Setting

In line with New Keynesian tradition, this model features sticky prices. In particular, firms are assumed to set prices *á la Calvo*, that is, a measure $(1-\theta)$ of randomly selected firms are allowed to revise their prices each period. They can react to changes in productivity, demand, and other relevant factors by adjusting their price up or down. With probability θ, however, the firms are stuck with their price from the previous period. Note that the probability of a firm reoptimizing that is, revising its price if market conditions change is independent of whether it was able to reoptimize last period and also of the amount of time passed since it was last able to reoptimize. The firm must therefore take into account the possibility that it will not be able to reoptimize at each future period, so it will set prices taking into account average expected future marginal costs instead of only current marginal cost. Its optimal strategy can be approximated in log-linear form as:

$$\overline{p_{H,t}} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t\left[mc_{t+k}^n\right] \tag{44}$$

where $\overline{p_{H,t}}$ newly chosen price and $\mu \equiv log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the log of the steady state gross markup, or the optimal markup under flexible prices. The same pricing equation is assumed to hold in the world economy.

Note that a limitation to Calvo pricing is that there is a fraction of firms that cannot adjust their pricing since the first period. This fraction is represented by $\varsigma \to 0$, but $\varsigma > 0 \forall t$. This limitation is sometimes dealt with by indexing, but the Galí and Monacelli model does not include this feature. Thus, there is potential for improvement of the model in this aspect.

The original model also includes the assumption that $\theta = \theta^*$, where θ^* is the foreign Calvo pricing parameter. Finally note that, at the flexible price limit $(\theta \to 0)$, the markup rule returns to the usual $\overline{p_{H,t}} = \mu + mc_t^n$.

3.3 Equilibrium

3.3.1 Aggregate Demand and Output

In the world economy, the consumer optimization problem is identical to that in the domestic economy except for the negligible role of imports resulting from the treatment of the world as a closed economy. The log-linearized Euler equation and market clearing, $y_t^* = c_t^*$, imply:

$$y_t^* = \mathbb{E}_t\left[y_{t+1}^*\right] - \frac{1}{\sigma}(r_t^* - \mathbb{E}_t\left[\pi_{t+1}^* - \rho\right]) \tag{45}$$

Foreign demand for domestic good i is denoted by $C_{H,t}^*(i)$. A necessary condition for market clearing in the small open economy is:

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + C^*_{H,t}(i) \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\eta} \varepsilon \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)C_t + \left(\frac{P_{H,t}}{exp(e_t)P^*_t}\right)^{-\eta}\alpha^* Y^*_t\right] \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\eta} \varepsilon\vartheta Y^*_t \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)Q_t^{\frac{1}{\sigma}} + \left(\frac{P_{H,t}}{exp(e_t)P^*_t}\right)^{-\eta}\alpha^*\right] \end{aligned} \tag{46}$$

$\forall i \in [0,1]$ and $\forall t$ where the last equality is based on $\frac{\alpha^*}{\vartheta} = \alpha$, a necessary condition for zero trade balance in steady state. This expression can be plugged into the definition of aggregate output $Y_t \equiv \left[\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ to obtain:

$$Y_t = \vartheta Y^*_t S^\eta_t \left[(1-\alpha)Q_t^{\frac{1}{\sigma}-\eta} + \alpha\right] \tag{47}$$

which can be first-order approximated by:

$$y_t = y^*_t + \frac{\omega_\alpha}{\sigma} s_t \tag{48}$$

where $\omega_\alpha \equiv 1 + \alpha(2-\alpha)(\sigma\eta - 1) > 0$. In the special case covered in the paper, where $\sigma\eta = 1$, the following linear relationship holds exactly:

$$y_t = y^*_t + \eta s_t \tag{49}$$

An alternative approximation for domestic consumption as a weighted average of domestic and foreign output is:

$$c_t = \Phi_\alpha y_t + (1-\Phi_\alpha)y^*_t \tag{50}$$

where $\Phi_\alpha \equiv \frac{1-\alpha}{\omega_\alpha} > 0$. In the case of a closed economy, where $\alpha = 0$, $\omega_0 = 1$ and $\Phi_0 = 1$. Therefore $c_t = y_t \ \forall t$. Furthermore:

$$c_t = (1-\alpha)y_t + \alpha y^*_t \tag{51}$$

Finally, Galí and Monacelli use these identities to derive a first order difference equation for output in the domestic economy in terms of domestic real interest rate and world output:

$$y_t = \mathbb{E}_t\left[y_t + 1\right] - \frac{\omega_\alpha}{\sigma}(r_t - \mathbb{E}_t\left[\pi_{H,t+1}\right] - \rho) + (\omega_\alpha - 1)\mathbb{E}_t\left[\Delta y^*_{t+1}\right] \tag{52}$$

Exports in terms of domestic output, expressed as a fraction of steady state output Y, are given by $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right)$. In the case where$\sigma = \eta = 1$, $P_{H,t}Y_t = P_tC_t$ $\forall t$, which implies balanced trade at every period. The first-order approximation of this expression is $nx_t \simeq y_t - c_t - \alpha s_t$. From these expressions:

$$nx_t = (1 - \Phi_\alpha)(y_t - y^*_t) - \alpha s_t \tag{53}$$

$$nx_t = \frac{\alpha[(2-\alpha)(\sigma\eta - 1) + (1-\sigma)]}{\omega_\alpha}(y_t - y^*_t) \tag{54}$$

In the special case mentioned above $nx_t = 0$ $\forall t$. When this condition is satisfied, the objective function for the monetary authority - the central bank - in the small open economy collapses to that of the closed world economy monetary authority.

3.3.2 Marginal Cost and Inflation Dynamics

The assumptions of the model imply the dynamics of foreign inflation correspond to those of a closed economy with Calvo pricing. In log-linear terms:

$$\pi^*_t = \beta\mathbb{E}_t\left[\pi^*_{t+1}\right] + \lambda\widehat{mc}^*_t \tag{55}$$

where $\widehat{mc}_t^* \equiv mc_t^* + \mu$ is the log form of marginal cost expressed as a deviation from steady state, $-\mu$, and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Foreign marginal cost is itself given by:

$$\begin{aligned} mc_t^* &= -\nu^* + (w_t^* - p_t^*) - a_t^* \\ &= -\nu^* + \sigma c_t^* + \varphi n_t^* - a_t^* \\ &= -\nu^* + (\sigma + \varphi) y_t^* - (1+\varphi) a_t^* \end{aligned} \tag{56}$$

where $\nu^* \equiv -log(1-\tau^*)$ and τ is a constant employment subsidy.

The domestic economy features analogous inflation dynamics to the world economy, thus:

$$\pi_{H,t} = \beta \mathbb{E}_t [\pi_{H,t+1}] + \lambda \widehat{mc}_t \tag{57}$$

Where the two economies differ slightly is in the determination of real marginal cost as a function of domestic output, since the domestic economy features a wedge between consumption and output generated by trade and, as a consequence, also features a difference between domestic and consumer prices. The domestic economy thus has:

$$\begin{aligned} mc_t &= -\nu + w_t - a_t - p_{H,t} \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\ &= -\nu + \sigma y_t^* + \varphi y_t + s_t - (1+\varphi) a_t \end{aligned} \tag{58}$$

and $\nu \equiv -log(1-\tau)$.

Marginal cost is thus affected by terms of trade and world output. Furthermore, changes in the terms of trade affect product wage for any given real wage. Marginal cost in terms of domestic output and productivity is then:

$$mc_t = -\nu + \left(\frac{\sigma}{\omega_\alpha} + \varphi\right) y_t + \sigma \left(1 - \frac{1}{\omega_\alpha}\right) y_t^* - (1+\varphi) a_t \tag{59}$$

3.3.3 A Canonical Representation of Equilibrium Dynamics

Define $\tilde{y}_t$ as the output gap, or the deviation of log output from its natural level $\bar{y}_t$ which is itself defined as the equilibrium output level with no nominal rigidities. Thus:

$$\tilde{y}_t \equiv y_t - \bar{y}_t \tag{60}$$

The same expression, with corresponding star notation, is used for the world economy.

Equilibrium dynamics in the world economy are identified by the following expressions. Under flexible prices - all firms can reoptimize in every period - $mc^* \equiv -\mu$ and this is used to derive the natural level of foreign output:

$$\bar{y}_t^* = \Omega_0 + \Gamma_0 a_t^* \tag{61}$$

with $\Omega_0 \equiv \frac{v^* - \mu}{\sigma + \varphi}$ and $\Gamma_0 \equiv \frac{1\varphi}{\sigma + \varphi}$.

Marginal cost is related to the output gap, in terms of deviations from steady state, by:

$$\widehat{mc}_t^* = (\sigma_\varphi)\tilde{y}_t^* \tag{62}$$

This relationship produces the New Keynesian Phillips Curve (NKPC), which represents the trade-off between output and inflation:

$$\pi_t^* = \beta \mathbb{E}_t \left[\pi_{t+1}^*\right] + \kappa_0 \tilde{y}_t^* \tag{63}$$

with $\kappa_0 \equiv \lambda(\sigma + \varphi)$. Finally, equation (34) can be expressed in terms of the output gap:

$$\tilde{y}_t^* = \mathbb{E}_t \left[\tilde{y}_{t+1}^*\right] - \frac{1}{\sigma}(r_t^* - \mathbb{E}_t \left[\pi_{t+1}^*\right] - \bar{r}r_t^*) \tag{64}$$

where $\bar{r}r_t^* \equiv -\sigma(1-\rho_a^*)\Gamma_0 a_t^* + \rho$ is the natural expected real interest rate.

In the domestic economy, equilibrium dynamics are driven by the equilibrium marginal cost, $mc_t = -\mu \ \forall t$, which implies domestic output is:

$$\bar{y}_t = \omega_\alpha + \Gamma_\alpha a_t + \Theta_\alpha y_t^* \tag{65}$$

where $\Omega_\alpha \equiv \frac{\omega_\alpha(\nu-\mu)}{\sigma+\omega_\alpha\varphi}$, $\Gamma_\alpha \equiv \frac{\omega_\alpha(1+\varphi)}{\sigma+\omega_\alpha\varphi} > 0$, and $\Theta_\alpha \equiv \frac{\sigma(1-\omega_\alpha)}{\sigma+\varphi\omega_\alpha}$.

Marginal cost and the output gap are related in the following manner:

$$\widehat{mc}_t = \left(\frac{\sigma}{\omega_\alpha} + \varphi\right)\tilde{y}_t \tag{66}$$

The NKPC for the domestic economy follows:

$$\pi_{H,t} = \beta \mathbb{E}_t\left[\pi_{H,t+1}\right] + \kappa_\alpha \tilde{y}_t \tag{67}$$

with $\kappa \equiv \lambda\left(\frac{\sigma}{\omega_\alpha}\right)$. Note that for $\alpha = 0$ the NKPC corresponds to the closed economy.

Finally, the IS equation for the domestic economy is given by:

$$\tilde{y}_t = \mathbb{E}_t\left[\tilde{y}_{t+1}\right] - \frac{\omega_\alpha}{\sigma}\left(r_t - \mathbb{E}_t\left[\pi_{H,t+1}\right] - \bar{rr}_t\right) \tag{68}$$

where $\bar{rr}_t \equiv \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma+\varphi\omega_{alpha}}a_t - \varphi\Theta_\alpha\mathbb{E}_t\left[\Delta y_{t+1}^*\right]$ is the natural rate of interest in the domestic, small open economy.

3.4 Simple Monetary Policy Rules

This subsection describes the alternative monetary policy rules originally evaluated by Galí and Monacelli. The addition of the Taylor Rule will be explained in the Extension section.

3.4.1 Domestic Inflation Targeting

This policy, abbreviated to DIT, aims to fully stabilize domestic inflation - implying $\tilde{y}_t = \pi_{H,t} = 0 \ \forall t$. That is to say that $y_t = \bar{y}_t$ and $r_t = \bar{rr}_t \ \forall t$ and the remaining variables remain at their natural levels in every period. The equilibrium dynamics of the exchange rate under this rule are described by:

$$\begin{aligned} e_t &= \frac{\sigma}{\omega_\alpha}(\bar{y}_t - \bar{y}_t^*) \\ &= \frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_\alpha}(a_t - a_t^*) \end{aligned} \tag{69}$$

so the exchange rate responds to the differential of productivity - depreciating in response to relative increases in domestic productivity and appreciating in response to analogous changes in world productivity. Variance of e_t under flexible is proportional to $(\sigma_a - \sigma_a^*)^2 + 2\sigma_a\sigma_a^*(1-\rho_{a,a^*})$ where σ_a and σ_a^* represent the standard deviation of domestic and world productivity, respectively. The equilibrium dynamics of CPI under this rule are described by:

$$\begin{aligned} p_t &= \alpha e_t \\ &= \frac{\alpha\sigma(1+\varphi)}{\sigma+\varphi\omega_\alpha}(a_t - a_t^*) \end{aligned} \tag{70}$$

Finally, the real exchange rate under this regime will be $q_t = (1-\alpha)e_t$.

3.4.2 *CPI Inflation Targeting*

This policy, abbreviated to CIT, aims to stabilize CPI inflation ($\pi_t = 0\ \forall t$). Under the assumption that the foreign monetary authority pursues an optimal monetary policy (i.e. foreign price level is constant), and setting $p_t = p_t^* = 0\ \forall t$ (which follows from the assumption that the world as a collective economy is large and has pricing power) then:

$$p_{H,t} = -\alpha s_t \tag{71}$$

Plugging this expression into equation (35) and plugging the resulting expression into equation (45) produces:

$$mc_t = -\frac{1}{\alpha}\left(1+\frac{\varphi\omega_\alpha}{\sigma}\right)p_{H,t} - (1+\varphi)(a_t - a_t^*) \tag{72}$$

From this last equality follows the second order difference equation for the dynamics of the domestic price level under CIT:

$$\gamma_c p_{H,t} = p_{H,t-1} + \beta \mathbb{E}_t [p_{H,t+1}] - \lambda (1+\varphi)(a_t - a_t^*) \tag{73}$$

where $\gamma_c \equiv 1 + \beta + \frac{\lambda}{\alpha}\left(1 + \frac{\varphi \omega_\alpha}{\sigma}\right)$. Making the simplifying assumption that $\rho_a = \rho_a^*$, the latter equation has the following unique, stationary representation:

$$p_{H,t} = \xi_c p_{H,t-1} - \zeta_c (a_t - a_t^*) \tag{74}$$

where $\xi_c \equiv \frac{1}{2\beta}\left(\gamma_c - \sqrt{\gamma_c^2 - 4\beta}\right) \in (0,1)$, and $\zeta_c \equiv \frac{\lambda \xi_c (1+\varphi)}{1 - \xi_c \beta \rho_a} > 0$.

Under this regime, equilibrium exchange rate is given by:

$$e_t = q_t = -\frac{1-\alpha}{\alpha} p_{H,t} \tag{75}$$

3.4.3 Exchange Rate Peg

This policy, abbreviated to PEG, aims to stabilize the exchange rate of the domestic currency with respect to the foreign currency by imposing a rate. In this model, this is considered equivalent to the domestic economy adopting the world currency. This policy implies $s_t = -p_{H,t}$ and $q_t = -p_t \; \forall t$. The second order difference equation for the domestic price level is:

$$\gamma_e p_{H,t} = p_{H,t-1} + \beta \mathbb{E}_t [p_{H,t+1}] - \lambda (1+\varphi)(a_t - a_t^*) \tag{76}$$

where $\gamma_e \equiv 1 + \beta + \lambda \left(1 + \frac{\varphi \omega_\alpha}{\sigma}\right)$. The unique, stationary representation is:

$$p_{H,t} = \xi_e p_{H,t-1} - \zeta_e (a_t - a_t^*) \tag{77}$$

where $\xi_e \equiv \frac{1}{2\beta}\left(\gamma_e - \sqrt{\gamma_e^2 - 4\beta}\right) \in (0,1)$, and $\zeta_e \equiv \frac{\lambda \xi_e (1+\varphi)}{(1 - \xi_e \beta \rho_a)} > 0$. CPI level is proportional to the domestic price level according to $p_t = (1-\alpha)$.

3.5 Shocks

The economy is hit by a domestic productivity shock and a world productivity shock. For the exercise described in the Results section, both shocks are assumed to be 1% increases in productivity - domestic and foreign, respectively. Both shocks are assumed to follow an AR(1) process in the following form:

$$a_t = rho_a a_{t-1} + \varepsilon_a + \sigma_{a,a^*} \varepsilon_{a^*} \tag{78}$$

$$a_t^* = rho_a a_{t-1}^* + \varepsilon_{a^*} \tag{79}$$

respectively. Notice the correlation between the shocks only affects the domestic productivity, since the negligible size of the domestic economy renders its productivity shocks inconsequential to the world economy.

3.6 Optimal Monetary Policy

While Galí and Monacelli develop a welfare loss function to assess optimal monetary policy, its numerical computation poses a significant challenge. An alternative way to evaluate these policy rules is to recognize that, as the original paper states: "[the welfare loss equation] penalizes fluctuations in domestic inflation and output gap." That is, if a policy rule features higher variance of the output gap or of domestic inflation it will be less optimal compared to an alternative with lower variance of these variables of interest.

Table 6.1 shows the comparison of volatility of output gap and inflation under each of the alternative policy rules evaluated and in response to each of the shocks. The parameters for the monetary policy rules are set according to the initial calibration by Galí and Monacelli, the updated calibration added in this paper, and recent monetary policy literature to inform the choice of parameters for the Taylor rule.

While this simple comparison does not determine optimal policy in a rigorous way, it serves as an initial step to guide further research on the matter.

3.7 Extension

In addition to the policy rules originally evaluated by Galí and Monacelli, this paper includes a fourth alternative monetary regime: a Taylor rule. This regime was characterized by John Taylor in 1993 and establishes an optimal response of the monetary policy tool - the nominal interest rate - to both the output gap *and* inflation.

The rule is based on the Taylor Principle, which states that the central bank must react to changes in inflation by adjusting the interest rate by *more* than the change in inflation. That is, if inflation goes up by 1%, the central bank must raise interest rates by more than 1% to stay on target.

So, if the Taylor rule has been deemed so useful for monetary policy, why was it not included in the Galí and Monacelli assessment of optimal policy? While the authors do not explicitly outline a reason in the original paper, one potential explanation is that they are comparing simple rules - policy rules that target only one variable. Introducing the Taylor rule in the mix may not have served the original purpose of determining which variable is most effective as a monetary policy target. However, expanding the study to include the Taylor rule is an asset in that it may uncover dynamics that only occur when variables interact in the policy rule; that is, there may be some synergies in including both inflation and the output gap in the policy rule.

The conventional Taylor Rule policy rule is of the form:

$$r = \rho_r r(-1) + \rho_\pi \pi_t + \rho_{\tilde{y}} \tilde{y}_t \tag{80}$$

where ρr is the coefficient of autocorrelation, ρ_π and $\rho_{\tilde{y}}$ are the weights assigned to CPI inflation and the output gap, respectively. These weights are calibrated from recent monetary economics literature, Côté et al (2002), as $\rho_\pi = 2$ and $\rho_{\tilde{y}} = 0.5$.

3.8 Solving the Model

To recap the process thus far, the equations representing the initial assumptions of the model and its basic structure were optimized to produce equilibrium conditions. These equilibrium conditions now form a nonlinear system of equations that needs to be solved in order to characterize the equilibrium.

The way this model is solved is by first determining the log-linearized system of equations representing the steady state. To this end, a complementary program to the Matlab statistical package called Dynare is particularly useful. Dynare takes as input the system of equations representing the model, parameter values, and names of endogenous variables. Dynare then produces the log-linearized system and calculates the steady state. Note that a log-linearization is simply a transformation of the model to make its estimation possible or more feasible. The model is transformed into log-form and the estimate results in a system of linear equations that can be easily computed.

Once Dynare has calculated the steady state, one can impose a shock. The program takes as input the exogenous variable that is to be shocked and the magnitude of the shock (e.g. 1%, 1 standard deviation, etc.). The effect of the shock is represented by impulse response functions (IRFs), which trace the effect of the shock on each variable in the system through time. The variable responses are shown as deviations from the variables' steady state values. Note that the IRFs reflect not only the effect of the initial shock, but also interactions between variables in the system. Thus, IRFs can depict more accurately the reactions for the variables of interest, since both intertemporal and intratemporal variable dynamics are taken into account.

4 DATA

The updated calibration of the Galí and Monacelli economy is based on two data series: a domestic and a foreign productivity measure.

The domestic productivity series is based on the quarterly, seasonally-adjusted Indexes of Labour Productivity for Canada. This data series is published by Statistics Canada through CANSIM, Table 383-0008. The sample is 1985Q1-2014Q3.

The foreign productivity series is based on the quarterly, seasonally-adjusted, Indexes of Early Estimate of Quarterly ULC Indicators: Total Labor Productivity for the United States. This data series is published by the Federal Reserve Bank through FRED. The sample is 1985Q1-2014Q3.

Finally, for the addition of the Taylor rule, the parameters are calibrated from Côté et al (2002), published by the Bank of Canada.

5 CALIBRATION

Both productivity data series are detrended linearly. The series are then fit to an AR(1) process to obtain the estimate of the autocorrelation coefficients and the standard errors.

The estimated processes are:

$$a_t = \rho_a a_{t-1} + \varepsilon_a \tag{81}$$

for the domestic shock and

$$a_t^* = \rho_{a^*} a_{t-1}^* + \varepsilon_{a^*} \tag{82}$$

for the foreign shock, where $\rho_a = 0.721$, $\rho_{a^*} = 0.6826$, $\sigma_a = 0.946$, and $\sigma_{a^*} = 0.9404$. However, note that the shocks imposed on the model for the purposes of this paper are set as 1% in magnitude and not as one standard deviation as is common practice. This is to ensure that the objective is to compare the responses under the two calibrations, therefore making the shock of the same magnitude makes the alternatives comparable.

It is important to note that the GM calibration used in this paper is based on a replication code. The shock AR processes used in the replicated GM calibration are $\rho_a = 0.9$, $\rho_{a^*} = 0.9$, and the shocks are set as 1% in magnitude.

In terms of parameters aside from the shock processes, note that this model is evaluated in the special case where $\sigma = 1$ (log utility) and $\eta = 1$. β, the discount factor, is pervasive in recent literature with a value of 0.99. α, the 'degree of openness' is defined by GM as the ratio of imports to GDP. The original calibration has this ratio as 0.4, the updated value used in the new calibration is 0.304.θ is kept at 0.75, consistent with an average period of one year between price adjustments. ε, the elasticity of substitution between domestic goods, is taken from Dixit-Stiglitz (1977). Finally, ϕ, the disutility from labour, is kept at 3 from GM, implying a labour supply elasticity of $\frac{1}{3}$. All other parameters are functions of the parameters mentioned above and constant terms.

6 RESULTS

The following are impulse response functions (IRFs) in reaction to a domestic productivity shock. Recall that both shocks are assumed to be a 1% increase in the respective productivity. The responses are shown as percentage deviations from their long term, steady state values.

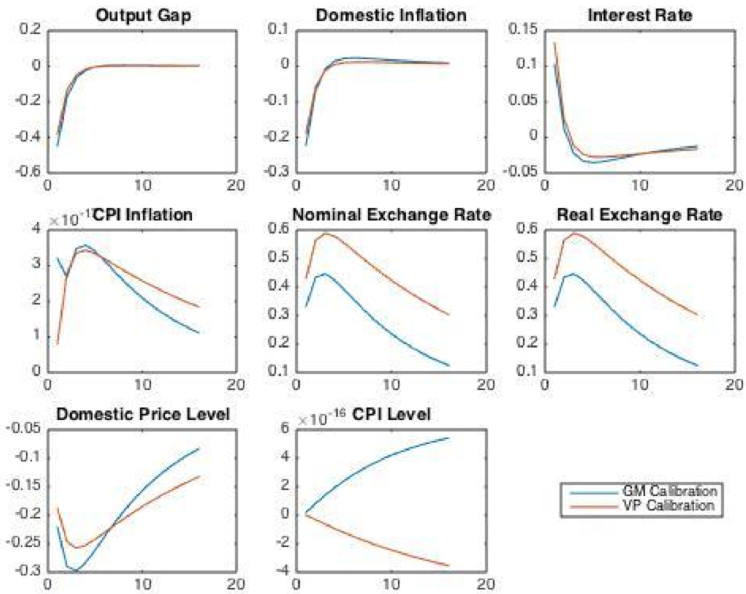

Figure 6.1: Impulse Responses to a Domestic Technology Shock under CPI Inflation Targeting - Comparison of Alternative Calibrations

As expected, a positive shock to domestic productivity lowers domestic inflation and, through the terms of trade, increases CPI inflation. However, the first difference between the results under the two alternative calibrations is the difference in sign of the reaction for the CPI level. While in the original case CPI level increases, the new calibration features a decrease in the CPI level. Furthermore, the new calibration CPI inflation jumps up considerably more than in the original calibration before transitioning to steady state. Correspondingly domestic inflation decreases by less in the new calibration, and exchange rate absorbs the shock such that it increases more under the new calibration. Do note that the target of CPI inflation, that is $\pi_t = 0$ is well approximated under both calibration.

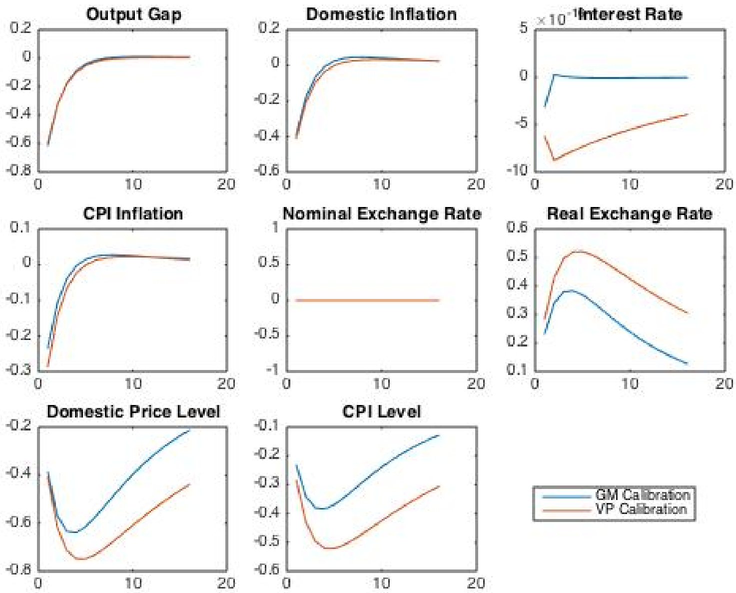

Figure 6.2: Impulse Responses to a Domestic Technology Shock under an Exchange Rate Peg - Comparison of Alternative Calibrations

In accordance with macroeconomic theory, a positive shock to domestic productivity lowers the output gap (as a consequence of higher output) and domestic inflation. Since nominal exchange rate is fixed, CPI inflation follows a very similar path to the domestic inflation one. Price levels, as expected, follow a hump-shaped pattern down and then back to steady state. This corresponds to the price level decreasing (because of the increased supply) at a decreasing rate (thus inflation jumps down initially and then gradually increases).

It is puzzling that the interest rate response has opposite signs under the two calibrations. It is possible that, because the shock under the new calibration features higher persistence and magnitude, interest rate in this case decreases further than initial jump to compensate the shock and keep exchange rate fixed. That is, since nominal exchange rate cannot absorb the shock because of the $e = q = 0$ target, movements in inflation must be counteracted in the interest rate. Perhaps the lower persistence under the original calibration does not warrant further reduction of the interest rate beyond the initial jump. However, this is still not a definitive explanation of this condition.

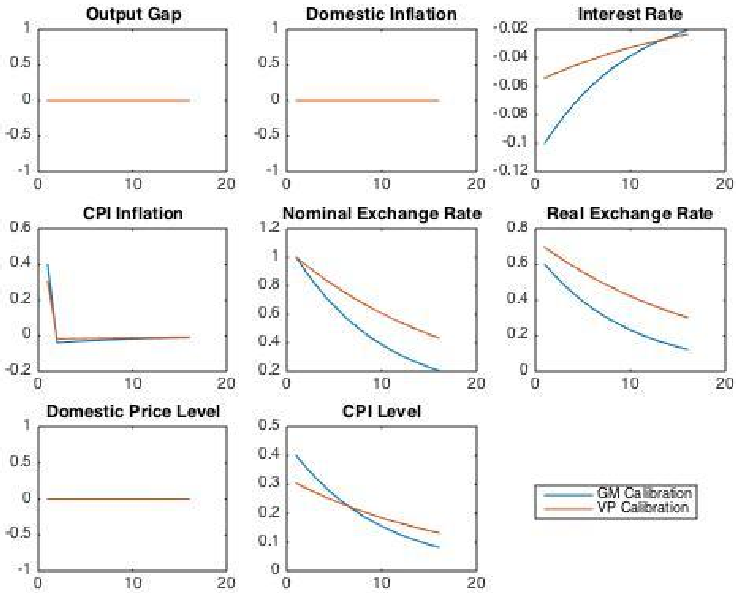

Figure 6.3: Impulse Responses to a Domestic Technology Shock under Domestic Inflation Targeting - Comparison of Alternative Calibrations

This is the case in which the policy rule - here $\pi_{H,t} = 0$ is achieved most accurately. As will be discussed later in this section, this feature contributes to the appeal of DIT as a policy. As in the CIT case, theory dictates CPI inflation increases and domestic inflation decreases. While we see the expected behaviour in CPI inflation, domestic inflation is targeted at zero and with zero growth. To maintain the target, exchange rates absorb the shock, which reflects on the interest rate.

In terms of comparing both calibrations, the direction of all responses is the same, while the persistence of certain responses differs. Both exchange rates decrease back to steady state more slowly under the new calibration and, correspondingly, interest rate also increases more slowly. Persistence is also higher for CPI level under the new calibration. Note, however, that the magnitude of the initial jump is almost identical for exchange rates, while it is lower under the new calibration for both CPI level and interest rate.

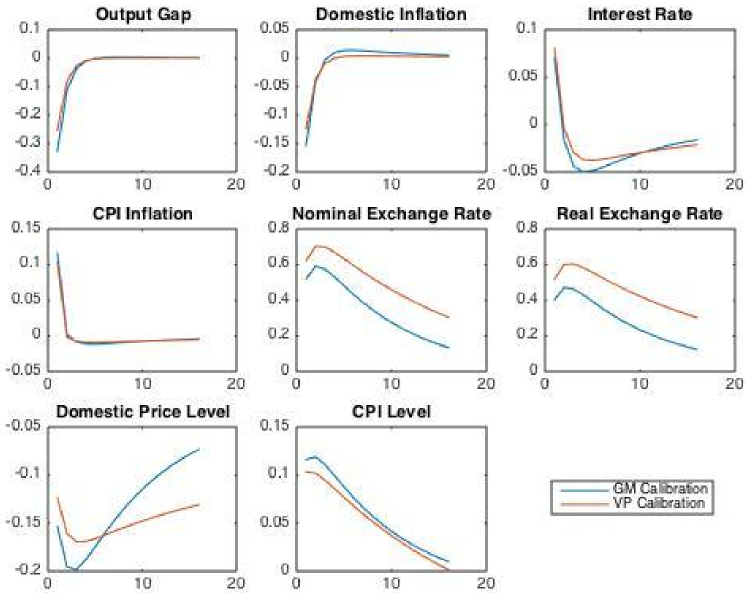

Figure 6.4: Impulse Responses to a Domestic Technology Shock under a Taylor Rule - Comparison of Alternative Calibrations

Note that, since the Taylor rule is not a simple monetary rule that fixes a single variable, all of the variables in this case respond to some extent to the shock.

Exchange rates still increase to absorb some of the shock to inflation and the output gap since monetary policy aims to stabilize them. Another variable that absorbs part of the shock is the interest rate, related to the exchange rate through the international financial market.

In comparison to the original calibration, the new one features once again higher persistence, this time for the exchange rates and most strongly for domestic price level. Though the magnitude of the initial response also differs, these differences are quantitatively minor.

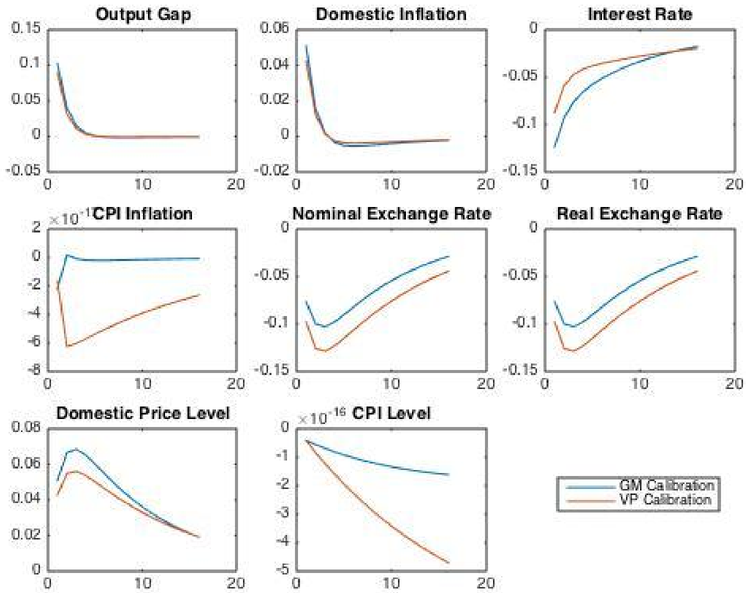

Figure 6.5: Impulse Responses to a Foreign Technology Shock under CPI Inflation Targeting - Comparison of Alternative Calibrations

As expected, a positive productivity shock in the world economy cause marginal cost to increase in the domestic economy and to decrease in the world economy. Since relative productivity has fallen, domestic inflation increases. On the contrary, CPI inflation decreases since the terms of trade act as a channel connecting the lower world prices to inflation.

Note that CPI inflation shows opposite signs of response under the two calibrations. It remains unclear why these responses have opposite signs. Also, persistence in CPI level is considerably higher under the new calibration. That said, these two variables have very small movements since the monetary target is $\pi_t = 0$.

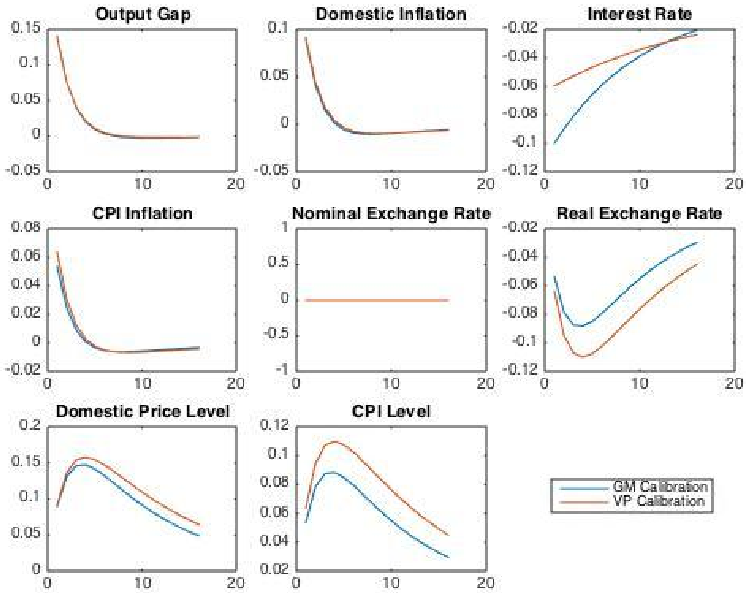

Figure 6.6: Impulse Responses to a Foreign Technology Shock under an Exchange Rate - Comparison of Alternative Calibrations

This case features considerable volatility of both the domestic and CPI price levels. As expected, a foreign shock to productivity rises CPI inflation and, since nominal exchange rate is not flexible and thus able to absorb the shock, domestic inflation also rises. This is why, though nominal exchange stays on its zero target, real exchange rate falls.

Once more, persistence under the new calibration is higher. Furthermore, in this case the max point of the CPI level and, consequently, the min point of real exchange rate are further from steady state under the new calibration.

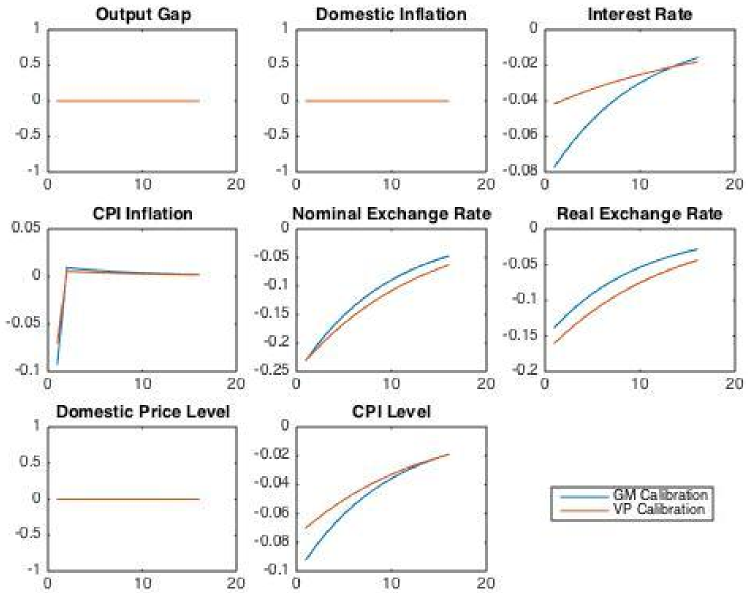

Figure 6.7: Impulse Responses to a Foreign Technology Shock under Domestic Inflation Targeting - Comparison of Alternative Calibrations

A foreign productivity shock decreases CPI inflation and, through the terms of trade, should increase domestic inflation. However, to stay on the zero target for domestic inflation, it is necessary for exchange rate to absorb the shock, thus both nominal and real exchange decrease. The interest rate also reacts to exchange rate movements via the international financial markets channel.

In this case, the main distinction is in the higher persistence and lower initial response magnitude of interest rate under the new calibration.

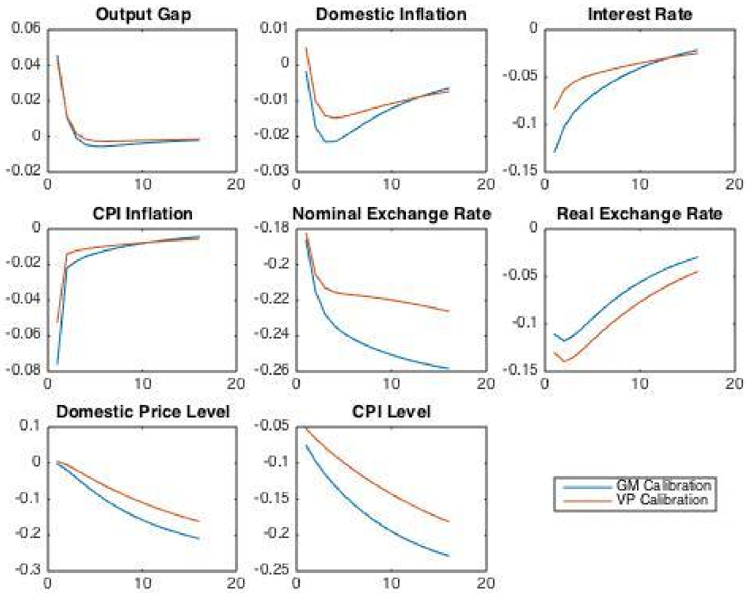

Figure 6.8: Impulse Responses to a Foreign Technology Shock under CPI Inflation Targeting - Comparison of Alternative Calibrations

Output gap shows a higher magnitude of initial than inflation, which is expected given the parameters used in the Taylor Rule, which give priority to stabilizing inflation. Notice that nominal exchange rate absorbs much of the shock, which protects inflation. This is a typical yet controversial feature of Taylor rules since international competitiveness is significantly more volatile when the exchange rate is allowed a full float and thus is used as buffer for shocks.

In terms of comparing responses under both calibrations, the most differentiated responses are for nominal exchange rate and CPI level, which both have lower magnitude of response though are comparable in terms of persistence across calibrations.

Volatility and Optimal Monetary Policy

Table 6.1 shows the variance of the two variables of interest, output gap and domestic inflation, in determining optimal monetary policy. In the original model, Galí and Monacelli find domestic inflation targeting to be the optimal policy rule since it minimizes variance of these two variables. Note that, while this paper does not include a full computation of the welfare loss function used in the GM paper, the original optimal monetary policy analysis exposition is clear that these two variances negatively affect welfare and are the two main elements in the loss function. This means the comparison of these policy rules based on the two variances here mentioned is a useful exercise to provide initial insight regarding optimal policy before incurring the computational cost of solving the full loss function.

The same ultimate result of DIT minimizing variance also holds under the new calibration, variance for most policy rules is lower for both output gap and inflation (PEG is the exception). However, it is also informative to compare the sub-optimal policy rules. The exchange rate peg yields by far the highest volatility in both variables of interest. Notice that, though the Taylor rule features higher volatility than CPI inflation targeting in response to a foreign shock under the new calibration, it features the lowest output gap volatility in response to a foreign shock under the original calibration, and in response to a domestic shock under the new calibration - excluding the DIT case.

So why do central banks use the Taylor Rule if domestic inflation targeting seems to hold as the optimal monetary policy rule? The answer lies in the limitations of this model. This is a very stylized model with considerable potential for aggregation bias on since sectors are aggregated and firms and households are assumed to be homogeneous. A more complex model would be more appropriate to evaluate the advantages of the Taylor rule.

As a final note, it is important to note that, quantitatively, the variation between these alternative policy rules under the new calibration is very small.

Table 6.1: Output Gap and Domestic Inflation Under Alternative Rules

	CIT			
	Domestic Shock		Foreign Shock	
	Output Gap	Domestic Inflation	Output Gap	Domestic Inflation
GM Calibration	0.013878439	0.003803966	0.000734169	0.00020123
VP Calibration	0.009736408	0.002604474	0.000540051	0.000137287

	PEG			
	Domestic Shock		Foreign Shock	
	Output Gap	Domestic Inflation	Output Gap	Domestic Inflation
GM Calibration	0.029238971	0.013355249	0.001546742	0.000706493
VP Calibration	0.026939472	0.014534625	0.00149469	0.000744335

	DIT			
	Domestic Shock		Foreign Shock	
	Output Gap	Domestic Inflation	Output Gap	Domestic Inflation
GM Calibration	0	0	0	0
VP Calibration	0	0	0	0

	TAY			
	Domestic Shock		Foreign Shock	
	Output Gap	Domestic Inflation	Output Gap	Domestic Inflation
GM Calibration	0.000158717	3.45948E-05	0.007166977	0.001751371
VP Calibration	0.000131882	2.16734E-05	0.004221183	0.001052011

7 CONCLUSION

This paper revisited the Galí and Monacelli (2002) model with a re-calibration to compare a more recent sample and an extension to evaluate the Taylor rule against the three simple policy rules originally listed in the GM paper.

The updating of the model did not change the sign of most of the responses of the variables of interest, however the magnitude and persistence of some of the responses varied significantly. The distinction between model under the different calibrations was, in several cases negligible. The most common features of distinction were higher persistence and lower magnitude of initial response for the new calibration.

In terms of optimal policy, both calibrations lead to the domestic inflation targeting as the option that minimizes volatility of the output gap and domestic inflation. In the original paper, this minimization of volatility corresponds to a minimization of welfare loss in response to the shock(s), which is considered to identify optimal monetary policy. While the assertion of optimal monetary policy cannot be made in the scope of this paper, the result that DIT is still the rule that yields the least volatility in output gap and inflation is a basis for further research in which the welfare loss is fully computed. In addition, a more complex economy could more accurately represent the requirements for an optimal policy rule.

This exercise leaves room for further potential research that shocks to other variables, interactions between shocks, or a more complex structure of the economy in which the monetary policy rules can have an impact that more closely represents the data.

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LIST OF FIGURES