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Duopoly Dynamics with Private Information

by

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"It requires some understanding of how an industry works. And then the reasoning is very much based on game theory."

- Jean Tirole

Abstract

This paper considers a two-period duopoly model with private information. The firms play a quantity-setting game and their own cost are private information. In the first period, each firm has some belief regarding its rival's cost type. Each firm chooses its output depending on its cost and its belief over the rival's quantity. At the end of the first period, the price is observed by the firms but the rival's quantity remains unknown. Depending on the observed price and their own quantity, each firm updates its belief regarding its rival's cost. In the second period, each firm produces again depending on its updated beliefs. The results indicate that a high cost firm would produce more when its rival's cost is unknown than when it is known. In contrast, a low cost firm will produce less.

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Contents

I. Introduction

In this paper, I develop and analyze a model in which two firms produce a homogeneous good and compete in a quantity-setting game. The game is played over two periods and is dynamic, because the action of the firms in the first period affects their second period choices. The firms are price-takers and their costs are private information. Prior to each period, each firm has some belief regarding its rival's cost type. After first period production, price is revealed, but due to demand uncertainty firms cannot determine their rival's output. At the end of the first period, firms update their beliefs regarding their rival's cost based on the observed price and their own output. This updating of beliefs is an important part of this dynamic game. By incorporating private information to a dynamic setup, I analyze how these actions vary, given the type of the firm. This paper also compares output in a static one period game with private information game to a static one period game with full information game.

In the classic Cournot model, a firm chooses its output after taking into consideration its rival's output and the demand function. Some papers extend the classic model to incorporate multiple periods in an asymmetric information environment. The results of these models vary depending on the type of asymmetry. Asymmetry can be due to private cost information, own demand curve parameters, or incomplete market demand parameters. Most of these models predict the emergence of sub-game Stackelberg equilibria, and collusion by the firms. Also in certain dynamic models, at the end of the first period, firms are able to realize the value of the market demand parameters. This results in a full information game in the second period.

This paper incorporates cost asymmetries and private information into a dynamic setting in an attempt to evaluate Mailath's (1989) conjecture that 'under quantity competition, each firm expands output in an attempt to make its competitors believe its costs are low.'

This modeling of incomplete information regarding costs and randomness in prices makes the model more realistic as to how firms compete with each other. For instance, consider a commodity such as gasoline with two main players, Shell and Esso. Both firms are involved in refining of gasoline; however, they are unaware of the cost structure of their competitor to produce the same product. The market determines the price and an overproduction by either firm would possibly drive down the price and thereby affect the profits of both the firms. In such a scenario, how do the firms decide optimal production quantities in order to be competitive in the market as well as to maximize their profits?

In the model, each firm's cost is either high or low. At the beginning of the first period, each firm believes that its rival's cost is either high or low with equal probability. At the end of the period, firms are unware of their rival's output and the total output of the firms affects the probability of the realized price. Based on the price observed, the firms update their initial belief. The updating of these beliefs is based on Bayes formula. In the second period, firms choose their production based on their updated belief. Price is revealed at the end of the second period.

I first evaluate a static version of the model with incomplete information and no updating. In a static one period model, the cost information could be private or public. When the cost information is private, the high cost firm produces more compared to the scenario when costs are common information. In contrast, the low cost firm reduces its output.

Further, when the firms compete in a dynamic two period game with private cost information, the high cost firm increases its output, in comparison to the static one period game with private cost information. In contrast, the low cost firm reduces its output. In a dynamic private information game, the fact the firms can affect the belief of its rival plays an important factor in determining the quantity produced. This results in the high cost firm producing more quantity when moving from a static private information game to a dynamic private information game.

The outline of the reminder of the paper is as follows. The next section discusses related literature, and the contribution of this paper. Section III describes the model framework, and the technique used to solve the model. Section IV & V details the numerical analysis findings and conclusion respectively.

II. Related Literature

Firm's interactions in a static or a dynamic setup have been widely studied in economics. The study of non-cooperative oligopolistic competition falls under two broad categories - quantity competition or price competition. The study of quantity competition originated with the influential work of Cournot, while Bertrand explored the latter.

In a Cournot model, two or more firms produce a homogeneous product, they have market power and the firms do not engage in any sort of collusive behavior. This model predicts that, the aggregate output of the firms will be greater than that of a monopoly. In a Bertrand model, two or more firms produce a homogeneous product as well, but due to price competition, the model predicts that a duopoly is sufficient to drive down the prices to the marginal cost of the firms.

Since then, extensions of those models have often focused on different aspects of dynamism in both quantity as well as price-setting games. The results of these dynamic models are dependent on the type of competition. In the case of quantity competition, firms exhibit characteristics of a collusive behavior as the firms produce lower output than a static Cournot model. In a multi-period Bertrand model, it is possible for firms to reach an equilibrium price greater than the marginal cost, once again exhibiting characteristics of a collusive behavior.

Another extension of these models include asymmetric information in dynamic environments. Various types of asymmetric information have been incorporated in multi-period models. At times, asymmetry arises due to firms' own demand curve, cost structure or demand uncertainty in the market place. Once again, asymmetry can be split based on Cournot or Bertrand models.

In the case of the latter, Caminal (1990) investigates the impact of asymmetric information on price competition in a two-period duopoly environment. His paper models firms with individual demand functions. The asymmetric information arises due to a firm's own demand uncertainty. In the model, firms first privately learn the realization of an idiosyncratic shock to their demand, and then set prices in the first period. The main result of the paper suggests that, the firms collude and set prices in all sequential equilibria[1].

Another multi-period Bertrand model with asymmetric information is discussed by Mailath (1989). His paper models n firms with private cost information, selling differentiated goods in a two-period model. The firms choose the prices simultaneously for each period, and the pricing decision signals its rivals. Depending on the type of goods, whether they are substitutes or complements, the firm's expectation of its rivals' prices varies. In the case of substitutes, firms would like its rivals to have a high cost, whereas for complementary goods, firms would like its rivals to have a low cost. Mailath's paper suggests that in a separating equilibria[2], there is an incentive for each firm to assume that the rival has a high cost, and this leads to an upward pressure on the prices, higher than that of the profit maximizing prices.

1. Sequential equilibrium is a refinement of Nash equilibrium for extensive form games. A sequential equilibrium specifies not only a strategy for each of the players but also a belief for each of the players.
2. Separating equilibrium: An equilibrium in which private information is observed/transmitted end of each period.

This paper examines a multi-period Cournot model with asymmetric information. The key difference in the Cournot model in comparison to the above-mentioned papers is quantity competition in a dynamic setup. Some of the models in this category have contributions by Arvan (1985), Saloner (1987), Riordan (1985), Robson & McMillan (1984) and Mirman et al. (1993).

Arvan (1985) analyzes a two-period quantity game with inventory, which allows firms to inventorize goods in the first period. The firms are symmetric with constant marginal costs, and inventory is used for sale in the second period. The results rule out symmetric in favor of asymmetric equilibria. Symmetric equilibrium suggests, when both firms carry inventory from the first period to the second, then both firms would be using inventory to act as a leader. This is not possible when the firms do not produce in the second period, either when inventory is exhausted or redundant. It is also not possible when there is production in the second period, since marginal costs are constant. Alternatively, the favored result of asymmetric equilibria suggests that, in the second period, one of the firms will utilize its inventory to influence the outcome. For this to occur, storage costs are very small, and the firm plays the inventory-leader strategy.

Saloner (1987) models a two period Cournot model which allows for two production periods. Firms plan their output simultaneously in the first period. At the end of the first period, firms' respective output is common knowledge. However, the price is not revealed at the end of the first period. In the next period, firms once again choose how much to produce. The total quantity produced during both the periods determines the realized price at the end of the second period. The result suggests that Stackelberg outcomes are sustainable as subgame perfect Nash equilibria.

Riordan (1985) considers a symmetric oligopoly with n firms. Each firm has the same constant unit cost of production across the two periods. In each period, firms decide the output after which the demand parameters are realized. An interesting result of the paper is that, in equilibrium, each firm is able to recover the realized value of the demand parameters directly from the market price. Hence, each firm's output in the second period is as if, each firm was perfectly informed about the demand parameters. Therefore, in equilibrium, each firm perceives itself in the second period to be facing a single period decision problem, which is exactly the situation in a static Cournot model.

Robson & McMillan (1984) introduces demand uncertainty in a dynamic model. Each firm observes last period's own output and the price. The strategy of the firm is contingent on the observed information, and it ignores its rival's output in the preceding period. Similar to Riordan's model, each firm produces before the realization of the demand shock. Hence, the dynamic duopoly equilibrium is simply the repetition of a one period equilibrium game. Moreover, no collusive behavior is expected due to demand uncertainty.

Alternatively, certain dynamic asymmetric models discuss various signal-jamming techniques. Mirman et al. (1993) models a repeated game framework and classifies them as four types of signal jamming models. In the first type, agents can make verifiable announcements regarding their private information. The second type examines transmission of private information through perfectly observed action. The third type discusses models in which the signal jammer knows the subject of signal jamming, and the fourth type discusses models in which the firms are uncertain concerning the intercept of a linear demand curve with known slope.

This paper presents a model which falls in the third type. This is where firms are aware of the subject of signal jamming, which is the rival's belief of the firm's own cost structure. Yet, it varies from the pertinent literature in this type[3]. In this paper, the private information of firm's own cost, and their different cost

3. For example Gal-Or (1988), Mailath (1989)

structure is the reason for asymmetry. The quantity produced by the firms is dependent on its cost structure. Each firm has some belief regarding its rival's cost structure, and updates its initial belief contingent on the price realized, the firm's own output, and its assessment of rival's output. Additionally, the firm's output is not carried over to the next period, and the aggregate quantity is sold for either low price or high price.

In this paper, firms cannot observe their rival's action. Additionally, since the prices are noisy functions of the aggregate quantity, firms receive imprecise inferences from the price regarding the rival's action. This results in demand uncertainty. Due to this uncertainty, the firms have an incentive to affect the realized price at the end of the first period, in a bid to signal jam the belief of its rival.

For instance, prior to the first period, a high cost firm believes that, there is a 50% probability, its rival is also of high cost. If the high cost firm realizes a low price at the end of the first period, this 50% probability that its rival has a high cost decreases. In fact in this model, due to demand uncertainty, it is possible that, the two high cost firms could produce a low aggregate quantity, and this could result in a low price realization at the end of the period. Alternatively, the two low cost firms could produce a high aggregate quantity, and this could result in a high price realization at the end of the period.

The fact that firms don't observe their rival's action and preserves private information end of each period results in a *non-separating equilibrium*[4]. In addition, the outcome of this paper shows that *pooling equilibrium* isn't possible[5]. Gal-Or (1986) states that the pooling of private information about unknown costs has the effect of revealing more accurate information regarding rival's cost type. It also

4. Non-separating equilibrium: An equilibrium in which private information isn't observed/-transmitted end of each period.
5. Pooling equilibrium: An equilibrium in which action of the agents is the same in equilibrium irrespective of its type. (Proof by contradiction in Appendix H)

reduces strategies for firms of different cost types. This increase in accuracy will lead to positive effect on the payoff of the firm.

The next section discusses the model, first by describing the environment, followed by the analytical results of the second period and the first period respectively.

III. The Model

III.1 The Environment

Consider two non-cooperative firms, firm 1 and 2, competing in a Cournot quantity-setting duopoly. They produce a homogenous good. Firms know their own cost, but do not know their rival's cost. The marginal cost of the firm of type θ is given by c_θ for $\theta \in$ {L,H} where $c_L < c_H$. We refer to a firm with marginal cost c_L as a low type firm, and a firm with marginal cost c_H as a high type firm. There are no fixed costs.

There are two periods indexed by $t \in$ {1,2} and the firms' cost are constant across periods. Price is revealed at the end of each period. Price is either P_L or P_H where $P_L < P_H$. Prices have a random component. P_H occurs in period t with a probability σ^t. σ^t has an inverse relation with the total output Q^t. In period t, Q^t is the sum of q_1^t and q_2^t, where q_i^t is the quantity produced by firm i in period t. In particular, $\sigma^t = a - bQ^t$, where $a, b > 0$.

This formulation of σ^t implies that the rival's quantity cannot be known, thereby ensuring the cost information of the firms to be private throughout the game. This results in demand uncertainty in the model.

The output of firm i in each period is not carried over to the next period. This implies, each firm's output is sold irrespective of the price observed, either P_L or P_H. Additionally by design, $c_H < P_L$. This negates the situation whereby, the marginal cost of the high type firm is more than the low price. This would prevent the scenario which deters the high type firm from not producing. This also eliminates the probability of the low type firm's incentive, not to produce in either period, and implies that both the types of firms are price-takers.

We now describe the beliefs of firms regarding their rival's cost. Let ρ denote firm i's belief that the probability of the rival is high type. At the start of the first period, firms believe its rival's cost is *50% high type, 50% low type.* Based on this assumption, in the beginning of the first period, each firm produces an output dependent on its cost and the expected price. At the end of the first period, the price is observed. Both firms then update their belief given the observed price as follows. Firm i with cost c_θ produces $q^1_{i\theta}$ and observes the first period price as either high or low. Firm i's belief that the rival is of high type probability is updated at the end of the first period. This is based on Bayes rule. The following is the probability-updating analytical expression when high price is observed:

$$\rho^2_{(q^1_\theta;P_H)} = \frac{\rho^1 \cdot (a - b(q^1_\theta + q^1_{rH}))}{\rho^1 \cdot (a - b(q^1_\theta + q^1_{rH})) + \rho^1 \cdot (a - b(q^1_\theta + q^1_{rL}))} \tag{3.1}$$

The probability-updating analytical expression when low price is observed is:

$$\rho^2_{(q^1_\theta;P_L)} = \frac{\rho^1 \cdot (1 - (a - b(q^1_\theta + q^1_{rH})))}{\rho^1 \cdot (1 - (a - b(q^1_\theta + q^1_{rH}))) + \rho^1 \cdot (1 - (a - b(q^1_\theta + q^1_{rL})))} \tag{3.2}$$

Where:

ρ^1: 0.5, i.e., the first period probability that the rival is of high type is 50%

q^1_θ: is the quantity produced in the first period by firm of type θ.

q^1_{rL}: is the quantity produced in the first period by the rival firm if it is of low type.

q^1_{rH}: is the quantity produced in the first period by the rival firm if it is of high type.

In the beginning of the second period, firms produce based on their updated belief, their own cost and the expected price. The firm's output varies depending on the price observed end of the first period, which is reflected in the updated beliefs. At the end of the second period, price is observed and the game ends.

Given how the game unfolds, the firm's objective at the beginning of the game is to maximize its expected lifetime profits. Analytically,

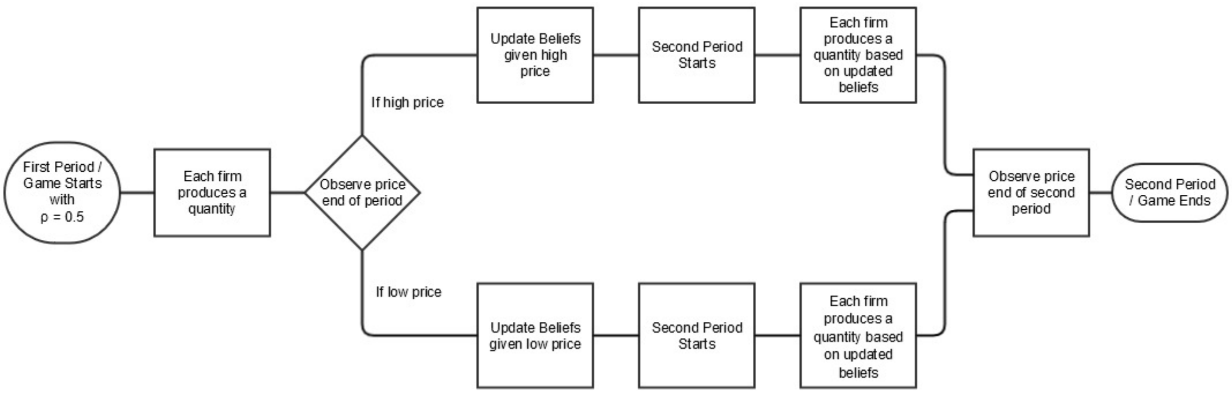
$$\max_{q_\theta^1} \ \mathbb{E}\left[\pi_\theta^1 + \mathbb{E}\left[\pi^2_{\theta\ (q_\theta^1)}\right]\right] \tag{3.3}$$

$$\equiv \max_{q_\theta^1} \ \mathbb{E}\left[\left((\sigma^1 \cdot P_H + (1-\sigma^1)\cdot P_L) - c_\theta\right)\cdot q_\theta^1\right] + \mathbb{E}\left[\sigma^1 \cdot \pi^2_{\theta\ (q_\theta^1; P_H)} + (1-\sigma^1)\cdot \pi^2_{\theta\ (q_\theta^1; P_L)}\right] \tag{3.4}$$

Where the outer expectation operator is for the rival's quantity.

The solution concept used in this paper is Perfect Bayesian Equilibrium.

Following is the flowchart of the game:



Both firms know the details of all the exogenous parameters and how the game unfolds. Given this information and Bayesian updating, if a low cost firm observes a high price at the end of the first period, then its belief that the rival is of high type increases. Alternatively, if a high cost firm observes a low price at the end of the first period, its belief that the rival is of high type decreases.

The following describes how I have solved the model and it is based on the principle of *Backward Induction.* First, the second period problem is assessed which is similar to a static one period problem. Firms calculate their expected second period profits based on the second period price realization. Firms then evaluate the impact on the second period equilibrium quantities, depending on variations in the expected first period price.

Firms then assess how quantity deviation from its equilibrium path affects the prices observed end of the first period, its rival's belief and its own second period quantity and profits. Eventually, firms maximize their expected lifetime profits at the start of the game.

III.2 Period Two

In the beginning of the second period, firms choose the quantity to produce, taking into consideration the expected price at the end of the second period, and the updated belief regarding its rival's type. Firms maximize their expected profits in the second period:

$$\max_{q_\theta^2} \ \mathbb{E}[(\mathbb{E}[P^2] - c_\theta) \cdot q_\theta^2] \tag{3.5}$$

Where the outer expectation operator is for the quantity produced by the rival in the second period, and the inner expectation operator is for the price to be realized at the end of the period.

The following is the firm's best response function:

$$q_\theta^2(q_r^2) = \frac{a}{2b} - \frac{\mathbb{E}[q_r^2]}{2} + \frac{P_L - c_\theta}{2b(P_H - P_L)} \tag{3.6}$$

The inverse relation between the quantities produced by the two firms indicates that they are strategic substitutes, which is in accordance with the Cournot model.

Hence the Nash equilibrium quantities in period two, and the expected profits for each type are as follows:
For the low type firm-

$$q_L^{2*} = \frac{a}{3b} - \frac{1}{3} \cdot \left[\left(\frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1)} - \rho^2_{(q_L^1)}\right]} \right) \cdot \left(\rho^2_{(q_L^1)} \right) \right] + \frac{P_L - c_L}{3b(P_H - P_L)} \tag{3.7}$$

$$\pi_L^{2*} = (q_L^{2*})^2 \cdot b \cdot (P_H - P_L) \tag{3.8}$$

For the high type firm-

$$q_H^{2*} = \frac{a}{3b} - \frac{1}{6} \cdot \left[\left(\frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1)} - \rho^2_{(q_L^1)}\right]} \right) \cdot \left(3\rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right) \right] + \frac{2P_L + c_L - 3c_H}{6b(P_H - P_L)} \tag{3.9}$$

$$\pi_H^{2*} = (q_H^{2*})^2 \cdot b \cdot (P_H - P_L) \tag{3.10}$$

In the second period, the profit equations 3.8 and 3.10 are for the low type firm and high type firm respectively. This conveys the fact that the profit equations of the firms are dependent on its own cost type. The only way a firm can affect its rival's second period quantity, and in turn profit, is through the first period price.

Knowing this fact, would a low type firm produce less to drive the price up, in an attempt to make its rival believe that it is of high type? Alternatively, would a high type firm produce more to drive the price down, in an attempt to make its rival believe that it is of low type?

III.3 Period One

Before firms choose their output in the first period, they calculate their expected profits in the second period conditional on the observed price in the first period.

For a firm of type $\theta \in \{L, H\}$

$$\pi_\theta^{2*} = \begin{cases} (q_\theta^{2*}(P_L^1))^2 \cdot b \cdot (P_H - P_L), & \text{if } P^1 = P_L^1 \\ (q_\theta^{2*}(P_H^1))^2 \cdot b \cdot (P_H - P_L), & \text{if } P^1 = P_H^1 \end{cases} \tag{3.11}$$

Where $q_\theta^{2*}(P_L^1)$ is the quantity produced by the firm of type θ in period two, after observing low price in the first period. Similarly, $q_\theta^{2*}(P_H^1)$ is the quantity produced by the firm of type θ in period two, after observing high price in the first period.

With all the prior information, firms decide how much to produce given their own cost. Their action in the first period can affect the probability σ^1 and in turn affect the realized price at the end of the period. The objective of firm i of type θ in the first period is to maximize their expected life time profits -

$$\max_{q_\theta^1} \mathbb{E}\left[((\sigma^1 \cdot P_H + (1-\sigma^1) \cdot P_L) - c_\theta) \cdot q_\theta^1\right] + \mathbb{E}\left[\sigma^1 \cdot \pi^2_{\theta\ (q_\theta^1; P_H)} + (1-\sigma^1) \cdot \pi^2_{\theta\ (q_\theta^1; P_L)}\right] \tag{3.12}$$

Where the outer expectation operator is for the rival's quantity.

The following are the first order conditions of the firms after imposing symmetry. The symmetry can be imposed in this model due to the fact that there are two firms, and two possible cost structures. This ascertains the fact that the action taken by the firms of the same type are alike, in spite of the fact that the firms are unaware of their rival's cost structure. This implies, in the presence of private information, two low type firms or two high type firms' actions are the same.

For the low type firm -

$$\begin{aligned}
&(a-2.5bq_L^1-0.5bq_H^1)\cdot(P_H-P_L)+P_L-c_L \\
&-b\cdot\pi^2_{L\ (q_L^1;P_H)}+(a-1.5bq_L^1-0.5bq_H^1)\cdot\left[\frac{\partial\pi^2_{L\ (q_L^1;P_H)}}{\partial q_L^2}\cdot\left[\frac{\partial q_L^2}{\partial\rho^2_{(q_H^1;P_H)}}\cdot\frac{\partial\rho^2_{(q_H^1;P_H)}}{\partial q_L^1}+\frac{\partial q_L^2}{\partial\rho^2_{(q_L^1;P_H)}}\cdot\frac{\partial\rho^2_{(q_L^1;P_H)}}{\partial q_L^1}\right]\right] \\
&+b\cdot\pi^2_{L\ (q_L^1;P_L)} \\
&+(1-(a-1.5bq_L^1-0.5bq_H^1))\cdot\left[\frac{\partial\pi^2_{L\ (q_L^1;P_L)}}{\partial q_L^2}\cdot\left[\frac{\partial q_L^2}{\partial\rho^2_{(q_H^1;P_L)}}\cdot\frac{\partial\rho^2_{(q_H^1;P_L)}}{\partial q_L^1}+\frac{\partial q_L^2}{\partial\rho^2_{(q_L^1;P_L)}}\cdot\frac{\partial\rho^2_{(q_L^1;P_L)}}{\partial q_L^1}\right]\right]=0
\end{aligned} \tag{3.13}$$

For the high type firm -

$$\begin{aligned}
&(a-2.5bq_H^1-0.5bq_L^1)\cdot(P_H-P_L)+P_L-c_H-b\cdot\pi^2_{H\ (q_H^1;P_H)} \\
&+(a-1.5bq_H^1-0.5bq_L^1))\cdot\left[\frac{\partial\pi^2_{H\ (q_H^1;P_H)}}{\partial q_H^2}\cdot\left[\frac{\partial q_H^2}{\partial\rho^2_{(q_H^1;P_H)}}\cdot\frac{\partial\rho^2_{(q_H^1;P_H)}}{\partial q_H^1}+\frac{\partial q_H^2}{\partial\rho^2_{(q_L^1;P_H)}}\cdot\frac{\partial\rho^2_{(q_L^1;P_H)}}{\partial q_H^1}\right]\right] \\
&+b\cdot\pi^2_{H\ (q_H^1;P_L)} \\
&+(1-(a-1.5bq_H^1-0.5bq_L^1))\cdot\left[\frac{\partial\pi^2_{H\ (q_H^1;P_L)}}{\partial q_H^2}\cdot\left[\frac{\partial q_H^2}{\partial\rho^2_{(q_H^1;P_L)}}\cdot\frac{\partial\rho^2_{(q_H^1;P_L)}}{\partial q_H^1}+\frac{\partial q_H^2}{\partial\rho^2_{(q_L^1;P_L)}}\cdot\frac{\partial\rho^2_{(q_L^1;P_L)}}{\partial q_H^1}\right]\right]=0
\end{aligned} \tag{3.14}$$

I have been unable to solve the Equations 3.13 and 3.14 analytically[6]. Hence, I use numerical analysis to evaluate the two non-linear simultaneous equations. The next section discusses the procedure and the findings.

6. The details of each of the terms of the two equations are part of Appendix A

IV. Numerical Analysis

This section is divided into two parts; the first part presents the graphs of the numerical results. These results are also compared to the graphs of both a static one-period game with private information, as well as a static one-period game with full information. The second part of the analysis focuses on comparative statics; the effects of the parameters are identified by changing their values, ceteris paribus.

IV.1 Dynamic Model Analysis

The parameter values that are chosen for the simulation of the model are as follows:

a = 0.6

b = 0.01

Marginal cost of low type firm = $c_L = 2$

Marginal cost of high type firm = $c_H = 3$

$P_L \in [8, 10]$, in increments of 0.1

$P_H \in [39, 41]$, in increments of 0.1

Using the above-mentioned parameters values, the simultaneous non-linear equations, Equations 3.13 and 3.14, were solved using *lsqnonlin* function in Matlab. The function lsqnonlin solves non-linear least-squares curve fitting problems with lower and upper bounds. In the model, lower and upper bounds comes from the fact that, in each period, probability σ is bound by a range [0,1].

The primary reason for using the specified values is due to the fact that, for each iteration of the non-linear least square solver, local minima is found. It is also worth noting that it is the global minima as well, since the results didn't differ when the initial estimates were changed. Moreover, these values satisfy the constraints on σ in both periods. Additionally, comparative statics was plausible. The low type firm has marginal cost lower than a high type firm. The low price is more than the cost of the high type firm, which prevents the scenario of high type firm not producing. Lastly, the difference between the low price and the marginal cost is high enough that it encourages competitive behavior for both, the low type firm and the high type firm.

Comparing the first period quantity graphs of the two types - B.1 and C.1; in the first period, given any combination of low price and high price within the range as defined in the parameter values, the quantity produced by the low type firm is always more than that of the high type firm. This is in accordance with the cost structure of the firm, i.e., the low cost firm will produce more in comparison to a high cost firm.

What is not certain is the shape of the graph. Why do the quantity irrespective of the type of the firm, have a direct relationship with low price and indirect relation with high price? In other words, when low price increases, quantity increases and when high price increases, quantity decreases.

For instance, if P_H is fixed at 39, one may notice as P_L increases from 8 to 10, the quantity of the firm increases. Alternatively, for instance, if P_L is fixed at 8, as P_H increases from 39 to 41, the quantity of the firm decreases.

Mathematically, the relation between quantity and prices are appropriate as the derivative of quantity with respect to low price is positive; whereas, the derivative with respect to high price is negative. Intuitively, as low price increases, firms can produce more. The reasoning is from the fact that the relation between price and quantity is similar to that of the supply curve in classic demand theory.

As low price increases, firms are willing to produce more quantity since aggregate quantity is sold for either low price or high price, and it is *not carried over* to the next period. Hence, as low price increases, from a firm's point of view supply should increase. In case of an increase in high price, firms expect that the demand may soften. This forces the firms to decrease the quantity produced.

These results are in accordance with first period profits (B.2, C.2) for both types of firms. In the first period profit graphs, profits are seen increasing irrespective of increase in either price type. The action taken by the firms in the first period for either low price or high price are appropriate.

One may notice similarities when comparing the first period quantity and profits of the dynamic two period game to that of a static one period game with private information[7]. In both the games, the quantities produced by the low type firm is always higher than that of the high type firm, for any combination of prices. A further look reveals the fact that the profits in the first period from the dynamic game are higher compared to the static game.

The interesting fact is that the quantity of the low type firm in the dynamic game is *lower* than the quantity of the low type firm from the static game, ceteris paribus. In contrast, the quantity of the high type firm in the dynamic game is *higher* than the quantity from that of the static game.

This is due to the nature of the dynamic game. The low type firm is already in an advantageous position given its cost, unlike the high type firm. It is probable that the high type firm tries to produce more in the first period, in order to affect its rival's belief that it is *not* of high type.

7. Refer to Figure B.3 B.4 for low type firm, C.3 C.4 for high type firm.

On further comparison to a static one period game with private information vs with no private information[8], the high type firm produces more in a static game with private information in comparison with static game with no private information. It produces even more in a dynamic two period game with private information.

In comparison, the low type firm decreases its quantity as the games change from a full information game to a static with private information, and decreases even more in a dynamic game with private information.

Analytically:

$q_H^{DP} > q_H^{SP} > q_H^{SF}$ for the high type firm.
$q_L^{DP} < q_L^{SP} < q_L^{SF}$ for the low type firm.

Where:

q_θ^{DP}: is the quantity produced by a firm of type θ in the first period in a dynamic game with private information.
q_θ^{SP}: is the quantity produced by a firm of type θ in a static game with private information.
q_θ^{SF}: is the quantity produced by a firm of type θ in a static game with full information.

This action seems to increase the profits of a high type when moving across games. Whereas the profits of the low type seems to dip when moving from a full information static game to a static game with private information, and increases when moving to the dynamic game.

8. Refer to Figure B.3 B.5 for low type firm, C.3 C.5 for high type firm.

This re-iterates the fact that with private information, the high type firm seems to pretend that it has in fact a lower cost by producing more, thereby making higher profits. But how does this first period action affect the second period in the dynamic model?

Prior to the second period, firms first update its initial probability that their rival is 50% high type, given the price observed at the end of the first period and its own quantity. Intuitively, when high price is observed, for a low type firm the probability that the rival is of high type is *higher*. Similarly, when low price is observed, for a high type firm the probability that the rival is of high type is *lower*. The model predicts this behavior irrespective of any combination of low price and high price[9].

Irrespective of the *price-realized* at the end at the first period, the second period quantity of the low type firm is always greater than that of the high type firm. Irrespective of the *type* of the firm, if high price is observed end of the first period, the quantity produced is more than the quantity produced by the *same* type of firm, if low price is observed at the end of the first period[10].
Analytically, $q_\theta^2(P_H^1) > q_\theta^2(P_L^1)\ \forall\theta = L, H$.

This increase in production, in the second period, after observing high price in the first period is possibly due to the fact that the firms expect a high price in the second period as well. This expectation of the firms is further strengthened by the fact that the quantity produced in the first period is much higher than that of the second period. Since the model is set up in such a way that the prices in first period do not necessarily affect the prices in the second period, it is likely that the model exhibits characteristics of a Markov Chain[11].

9. Refer to Figure F.3 F.4 for high price scenario, Figure F.1 F.2 for low price scenario.
10. Refer to Figure D.1 D.3 for low type firm, Figure E.1 E.3 for high type firm.
11. A Markov Chain is a stochastic model describing a sequence of possible events in which probability of each event depends only on the state attained in the previous event.

Lastly, one may notice that the second period quantity is considerably less than that of the first period as are the profits. The reasoning is from the expected second period profit equations (Equations 3.8 and 3.10). If low price is realized at the end of the second period, firms irrespective of their cost, receive a lower profit. In order to prevent this scenario, one may call it collusion, the firms act by reducing their production, thus increasing the possibility of high price being realized. This may also be the reason, if firms realize high price even after producing comparatively higher quantity in the first period, then in the second period, firms produce more in comparison to the scenario where low price is realized at the end of the first period.

IV.2 Comparative Statics

The benchmark parameters chosen for this section are as follows[12]:

a = 0.6

b = 0.01

Marginal cost of low type firm = $c_L = 2$

Marginal cost of high type firm = $c_H = 3$

To analyze the effects of low price, P_L is fixed at 9 and $P_H \in [39, 41]$

To analyze the effects of high price, P_H fixed at 40 and $P_L \in [8, 10]$

In this section, analysis is based on changing one of the exogenous parameter value. This is to measure its effect in the equilibrium quantities and the profits of both the firms in all periods, ceteris paribus.

12. Note: The parameter values for this section are same as the previous sub-section.

IV.2.1 Increase in the cost of a firm

When the cost of the high type firm increases from 3 to 5, the quantity produced by the high type firm decreases. This aligns with the industrial organization theory, which states that, when cost of the firm increases, the quantity produced decreases[13].

When comparing the benchmark Figures G.1 - G.12 to the Figures G.25 - G.36, when the cost of the high type firm increases, it reduces its own quantity. In contrast, the quantity produced by the low type firm increases. This is true in both periods. Moreover, in both periods, the profits of the high type firm decreases, and the profits of the low type firm increases. This further asserts the fact that the two goods are strategic substitutes. Similar results are true if the cost of the low type firm increases.

Alternatively, when the cost of the firm decreases, the quantity produced by the firm increases, while the quantity of the rival firm decreases. The profits move in the same direction as that of the quantity produced.

IV.2.2 Increase in parameter 'a'

The parameter 'a' can be interpreted as the probability that high price is realized at the end of the period, if no quantity is produced. In the benchmark scenario, 'a' has a value of 0.6. This can be translated as 60% probability that the price is going to be high when no quantity is produced[14]. For comparative analysis, graphs are compared when 'a' is 0.6 and 0.8.

13. The graphs for this case is shown when low price is fixed and high price varies. The same results are true when high price is fixed and low price varies.
14. This is a hypothetical scenario as in the model firms can produce any quantity and earn a positive profit which is better than not producing and earning zero profits.

When comparing the Figures G.1 - G.12 to Figures G.37 - G.48, as the parameter 'a' increases, the quantity produced by both types of firms increases in the first period, thereby increasing the first period profits. This is likely due to the fact that the increase in 'a' increases the probability that the realized price is going to be high at the end of the first period. A higher price is more beneficial for the low type firm, than to a high type firm. As a result, when 'a' increases, the difference in quantities produced between the low type and high type firm is more. This increase in quantity also increases the first period profits for both the firms.

What is worth noting is that, irrespective of the type of the firm, the realized price in the first period does not significantly impact the quantity produced in the second period[15]. This is likely due to the fact that as 'a' increases, firms expand production in the first period. Theoretically, the updated probability has similar value irrespective of the first period price.

IV.2.3 Increase in parameter 'b'

The parameter 'b' can be interpreted as the slope of the probability σ. For an increase in one unit quantity by a firm, the probability drops by 'b' units. The following analysis compares the benchmark scenario, 'b' has a value of 0.01 to the case when it is increased to 0.05[16].

When comparing the Figures G.1 - G.12 to Figures G.49 - G.60, as the parameter 'b' increases, the quantity produced by both types of firms decreases drastically in the first and second period, resulting in drop in profits.

So why do quantities fall drastically when 'b' increases? Since 'a' has a value of 0.6 and $0 \leq \sigma \leq 1$, i.e., $0 \leq a - bQ \leq 1$, when 'b' increases from 0.01 to 0.05,

15. In the order of 1000^{th} decimal unit
16. The analysis is the same if P_L is fixed and P_H varies or P_H is fixed and P_L varies

every additional quantity produced drops σ by 5%. Since σ is bounded by a range [0,1] for the model to be consistent, it excessively decreases the quantity produced by the firms in both periods.

IV.2.4 Increase in low price (P_L)

As stated previously, there exists a positive relation with quantity and profits when there is an increase in low price[17]. The relation between low price and quantity is similar to that of a supply curve of a firm. As a result, when P_L increases from 9 to 10, the quantities produced by the firms increases in both periods; thereby, the profits as well[18]. This increase in quantity, irrespective of the type of the firm, is due to the underlying fact that when low price increases, firms believe that they can expect a higher minimum price for their good.

For instance, in the scenario of Shell and Esso, suppose each firm produces 100 litres when the market price is $100. Since these firms are price-takers, an increase in the market price to $120, there is an incentive for the firms to produce a quantity greater than 100 litres.

IV.2.5 Increase in high price (P_H)

Comparing Figures G.13 - G.24 to Figures G.73 - G.82, as high price increases, the quantity produced by both the firms decreases in both periods[19]. This decrease in quantity produced is due to the belief that the demand may soften with increase in high price.

17. This case also covers the scenario when the difference between low price and high price decreases.
18. Refer to Figures G.1 - G.12 to Figures G.60 - G.72
19. This case also covers the scenario when the difference between low price and high price increases.

In the model, the firms expect to realize either low price or high price at the end of the period. Low price is a minimum value that the firms expect to realize. Whereas, the realization of high price is solely dependent on the demand for the good. When firms expect softening of the demand, they accordingly reduce their output. As a consequence, irrespective of the type of the firm, the profits decrease.

V. Conclusion and Extensions

This paper highlights the behavior of firms in a quantity competition with private cost information, and how the firm's action differs depending on the cost structure of the firm. The paper compares the behavior of firms with high cost type and low cost type, both in a static one-period game as well as in a dynamic two-period game. In the dynamic game, it also highlights the updated belief of the firms when the price observed is either high or low and how the updated belief affects the second period output of the firm. Moreover, due to demand uncertainty, the firms cannot estimate the rivals output, thereby, ensuring the cost information is private at the end of each period.

This paper details three important aspects to be considered by the firms when increasing its output in the first period. First, how a firm can affect the realized price at the end of the first period. Second, how the price affects the updated probability of the firm's own belief regarding the rival's type. Lastly, rival's belief regarding the firm itself.

The outcome of this paper suggests that, there are incentives for a high type firm to produce more in an attempt to disrupt the belief of its rival, when cost are private in comparison to a full information game. In contrast, under private information, a low cost firm decreases its output. The related literature have analyzed, how in the presence of demand uncertainty, firms strategize the next period's output based on its own output and the price observed in the last period. This paper takes into account the added information of firm's belief regarding the rival's output. This updating of beliefs, provides the firms with a more precise estimation of the rival's output, in turn, cost structure.

Although the basic foundation has been laid for a dynamic model with private information, further extensions to this model are possible. First and foremost, this paper deals with discrete price structure - either high or low. One could extend the paper by having price as continuous random variable, with a probability density function. Similar extensions are also possible to the cost structure of the firm.

Another interesting extension would be if the game is played for n *periods*. In this scenario, a non-homogenous Markov chain would be beneficial for updating the beliefs. Moreover, it is likely that as the game is played over multiple periods, the firms would be able to backtrack and realize the true information regarding their rival's type. This issue can be minimized if the private costs are normally distributed using a rectified Gaussian distribution.

VI. Bibliography

Arvan, Lanny. 'Some Examples of Dynamic Cournot Duopoly with Inventory' *The RAND Journal of Economics* Vol.16 (1985): 569-578.

Caminal, Ramon. 'A Dynamic Duopoly Model with Asymmetric Information' *The Journal of Industrial Economics* Vol.38 (1990): 315-333.

Gal-Or, Esther. 'Information Transmission - Cournot and Bertrand Equilibria' *Review of Economic Studies* Vol.53 (1986): 85-92.

Gal-Or, Esther. 'The Advantages of Imprecise Information' *RAND Journal of Economics* Vol.19 (1988): 266-275.

Mailath, George J. 'Simultaneous Signaling in an Oligopoly Model' *The Quarterly Journal of Economics* Vol.104 (1989): 417-427.

Mirman Leonard J., Samuelson, Larry & Urbano, Amparo. 'Duopoly Signal Jamming' *Economic Theory* Vol.3 (1993): 129-149.

Riordan, Michael H. 'Imperfect Information and Dynamic Conjectural Variations' *The RAND Journal of Economics* Vol.16 (1985): 41-50.

Robson, Arthur J. & McMillan, John. 'Dynamic Duopoly under Deman Uncertainty' *The Canadian Journal of Economics* Vol.17 (1984): 695-698.

Saloner, Garth. 'Cournot Duopoly with Two Production Periods' *Journal of Economic Theory* Vol.42 (1987): 183-187.

A. Analytical

Approaching the problem via Backward Induction

A.1 Solving equilibrium quantities of period two

Note: Superscript denotes the period; Subscript θ and 'r' denotes the type and rival respectively

For symmetry, following are the assumptions:

$$\rho^2_{i\ (q^1_{i\theta})} = \ \rho^2_{j\ (q^1_{j\theta})} = \ \rho^2_{(q^1_\theta)}$$
$$\rho^1_i = \rho^1_j = \rho^1$$

In the start of the second period, a firm maximizes its profits irrespective of whether the price observed end of the second period is high or low.
A firm expects P_H with a probability σ^2; similarly P_L with a probability $1 - \sigma^2$

Note: The expectation operator is for the rival's type

$$\max_{q^2_\theta}\ \mathbb{E}[\pi^2_{\theta\ (q^2_\theta)}]$$
$$\equiv \max_{q^2_\theta}\ \mathbb{E}[(\sigma^2 \cdot P_H + (1-\sigma^2) \cdot P_L - c_\theta) q^2_\theta]$$
$$\equiv \max_{q^2_\theta}\ \mathbb{E}[(\sigma^2 \cdot (P_H - P_L) \cdot q^2_\theta) + (P_L \cdot q^2_\theta) - (c_\theta \cdot q^2_\theta)]$$

We know that

$$\sigma^2 = a - bQ^2 = a - b(q^2_\theta + q^2_r) = a - bq^2_\theta - bq^2_r$$

Substituting σ^2

$$\equiv \max_{q^2_\theta}\ \mathbb{E}[((a - bq^2_\theta - bq^2_r) \cdot (P_H - P_L) \cdot q^2_\theta) + (P_L \cdot q^2_\theta) - (c_\theta \cdot q^2_\theta)]$$

$$\equiv \max_{q_\theta^2} \ \mathbb{E}[((a \cdot q_\theta^2 - b(q_\theta^2)^2 - bq_r^2 \cdot q_\theta^2) \cdot (P_H - P_L)) + (P_L \cdot q_\theta^2) - (c_\theta \cdot q_\theta^2)]$$

Taking First Order Conditions,

$$\mathbb{E}[(a - 2bq_\theta^2 - bq_r^2) \cdot (P_H - P_L) + P_L - c_\theta] = 0$$

$$\equiv \mathbb{E}[(a - 2bq_\theta^2 - bq_r^2) \cdot (P_H - P_L)] + P_L - c_\theta = 0$$

$$\equiv 2bq_\theta^2 \cdot (P_H - P_L) = a \cdot (P_H - P_L) - b\mathbb{E}[q_r^2] \cdot (P_H - P_L) + P_L - c_\theta$$

Best Response Function,

$$\longrightarrow \boldsymbol{q_\theta^{2*} = \frac{a}{2b} - \frac{\mathbb{E}[q_r^2]}{2} + \frac{P_L - c_\theta}{2b(P_H - P_L)}}$$

where $\mathbb{E}[q_r^2]$ depends on firm's own type; for instance,
For firm of type L, $\mathbb{E}[q_r^2] = \rho^2_{(q_L^1)} \cdot q_{rH}^2 + \left(1 - \rho^2_{(q_L^1)}\right) \cdot q_{rL}^2$
For firm of type H, $\mathbb{E}[q_r^2] = \rho^2_{(q_H^1)} \cdot q_{rH}^2 + \left(1 - \rho^2_{(q_H^1)}\right) \cdot q_{rL}^2$

Hence Best Response Function for Type L,

$$q_L^{2*} = \frac{a}{2b} - \frac{\mathbb{E}[q_r^2]}{2} + \frac{P_L - c_L}{2b(P_H - P_L)}$$

Similarly, Best Response Function for Type H,

$$q_H^{2*} = \frac{a}{2b} - \frac{\mathbb{E}[q_r^2]}{2} + \frac{P_L - c_H}{2b(P_H - P_L)}$$

Substituting $\mathbb{E}[q_r^2]$ for each type respectively and imposing symmetry:

$q_\theta^2 = q_r^2 = q_\theta^{2*};\ \forall\theta = L, H$

$$\longrightarrow q_L^{2*} = \frac{a}{2b} - \frac{\rho^2_{(q_L^1)} \cdot q_H^{2*} + \left(1 - \rho^2_{(q_L^1)}\right) \cdot q_L^{2*}}{2} + \frac{P_L - c_L}{2b(P_H - P_L)} \tag{1}$$

$$\longrightarrow q_H^{2*} = \frac{a}{2b} - \frac{\rho^2_{(q_H^1)} \cdot q_H^{2*} + \left(1 - \rho^2_{(q_H^1)}\right) \cdot q_L^{2*}}{2} + \frac{P_L - c_H}{2b(P_H - P_L)} \tag{2}$$

Simplifying Equation (1),

$$q_L^{2*} = \frac{a}{2b} - \frac{\rho^2_{(q_L^1)} \cdot (q_H^{2*} - q_L^{2*})}{2} - \frac{q_L^{2*}}{2} + \frac{P_L - c_L}{2b(P_H - P_L)}$$

$$\Longrightarrow \boldsymbol{q_L^{2*} = \frac{a}{3b} - \frac{1}{3} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(\rho^2_{(q_L^1)}\right)\right] + \frac{P_L - c_L}{3b(P_H - P_L)}} \tag{3}$$

Simplifying Equation (2) and Substituting Equation (3),

$$q_H^{2*} = \frac{a}{2b} - \frac{\rho^2_{(q_H^1)} \cdot (q_H^{2*} - q_L^{2*})}{2} - \frac{1}{2} \cdot \left[\frac{a}{3b} - \frac{1}{3} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(\rho^2_{(q_L^1)} \right) \right] + \frac{P_L - c_L}{3b(P_H - P_L)} \right] + \frac{P_L - c_H}{2b(P_H - P_L)}$$

$$\Longrightarrow \boldsymbol{q_H^{2*} = \frac{a}{3b} - \frac{1}{6} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(3\rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right) \right] + \frac{2P_L + c_L - 3c_H}{6b(P_H - P_L)}} \tag{4}$$

Subtracting Equation (3) from (4)

$$\begin{aligned} q_H^{2*} - q_L^{2*} = \frac{a}{3b} - \frac{1}{6} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(3\rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right) \right] + \frac{2P_L + c_L - 3c_H}{6b(P_H - P_L)} \\ - \left[\frac{a}{3b} - \frac{1}{3} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(\rho^2_{(q_L^1)} \right) \right] + \frac{P_L - c_L}{3b(P_H - P_L)} \right] \end{aligned}$$

Simplifying,

$$\begin{aligned} q_H^{2*} - q_L^{2*} = -\frac{1}{6} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(3\rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right) \right] + \frac{2P_L + c_L - 3c_H}{6b(P_H - P_L)} + \frac{1}{6} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(2\rho^2_{(q_L^1)} \right) \right] \\ - \left[\frac{2P_L - 2c_L}{6b(P_H - P_L)} \right] \end{aligned}$$

$$\equiv q_H^{2*} - q_L^{2*} = \frac{3c_L - 3c_H}{6b(P_H - P_L)} - \frac{1}{6} \cdot \left[(q_H^{2*} - q_L^{2*}) \cdot \left(3\rho^2_{(q_H^1)} - 3\rho^2_{(q_L^1)} \right) \right]$$

$$\equiv (q_H^{2*} - q_L^{2*}) \cdot \frac{\left[2 + \rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right]}{2} = \frac{c_L - c_H}{2b(P_H - P_L)}$$

$$\longrightarrow \boldsymbol{q_H^{2*} - q_L^{2*} = \frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right]}} \tag{5}$$

Substituting (5) in (3) and (4) respectively,

The following are the equilibrium quantities in period two:

$$\Longrightarrow \underline{\underline{\boldsymbol{q_L^{2*} = \frac{a}{3b} - \frac{1}{3} \cdot \left[\left(\frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right]} \right) \cdot \left(\rho^2_{(q_L^1)} \right) \right]}}} \\ \underline{\underline{\boldsymbol{+ \frac{P_L - c_L}{3b(P_H - P_L)}}}} \tag{6}$$

$$\Longrightarrow \underline{\underline{\boldsymbol{q_H^{2*} = \frac{a}{3b} - \frac{1}{6} \cdot \left[\left(\frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right]} \right) \cdot \left(3\rho^2_{(q_H^1)} - \rho^2_{(q_L^1)} \right) \right]}}} \\ \underline{\underline{\boldsymbol{+ \frac{2P_L + c_L - 3c_H}{6b(P_H - P_L)}}}} \tag{7}$$

A.2 Expected profits of period two

In the beginning of the second period, firm's are not aware of prices observed end of the period; but have updated their probabilities regarding the rival's type (depending on the observed price end of the first period and firm's own quantity)

Firm of type L's expected profits in the second period:

$$\rho^2_{(q^1_L)} \cdot \left[(\pi^2_L | q^{2*}_L, q^{2*}_{rH})\right] + \left[\left(1 - \rho^2_{(q^1_L)}\right) \cdot (\pi^2_L | q^{2*}_L, q^{2*}_{rL})\right]$$

$$\rho^2_{(q^1_L)} \left[(\pi^2_L | q^{2*}_L, q^{2*}_{rH}) - (\pi^2_L | q^{2*}_L, q^{2*}_{rL})\right] + (\pi^2_L | q^{2*}_L, q^{2*}_{rL}) \tag{8}$$

where

$$\left[(\pi^2_L | q^{2*}_L, q^{2*}_{rH}) - (\pi^2_L | q^{2*}_L, q^{2*}_{rL})\right] = q^{2*}_L \cdot b(P_H - P_L) \cdot (q^{2*}_{rL} - q^{2*}_{rH})$$
$$\pi^2_L | q^{2*}_L, q^{2*}_{rL} = [(a - b(q^{2*}_L + q^{2*}_{rL})) \cdot (P_H - P_L) + (P_L - c_L)] \cdot q^{2*}_L$$

Substituting in Equation (8)

$$\Rightarrow \rho^2_{(q^1_L)} \left[q^{2*}_L \cdot b(P_H - P_L) \cdot (q^{2*}_{rL} - q^{2*}_{rH})\right]$$
$$+ \left[[(a - b(q^{2*}_L + q^{2*}_{rL})) \cdot (P_H - P_L) + (P_L - c_L)] \cdot q^{2*}_L\right]$$

$$\equiv q^{2*}_L \left[a(P_H - P_L) - b(P_H - P_L) \cdot \left[\rho^2_{(q^1_L)}(q^{2*}_{rH} - q^{2*}_{rL}) + q^{2*}_L + q^{2*}_{rL}\right] + (P_L - C_L)\right]$$

where by symmetry, $q^{2*}_L = q^{2*}_{rL}$, $q^{2*}_H = q^{2*}_{rH}$

Substituting q^{2*}_L, q^{2*}_H; multiplying and dividing by $b(P_H - P_L)$,

$$\Rightarrow q^{2*}_L \cdot b(P_H - P_L) \cdot \left[\frac{a}{3b} + \frac{P_L - c_L}{3b(P_H - P_L)} - \rho^2_{(q^1_L)} \frac{q^{2*}_H - q^{2*}_L}{3}\right]$$

$$\Rightarrow (q^{2*}_L)^2 \cdot b(P_H - P_L)$$

Similarly, Firm of type H's expected profits in the second period:

$$\rho^2_{(q^1_H)} \left[(\pi^2_H | q^{2*}_H, q^{2*}_{rH}) - (\pi^2_H | q^{2*}_H, q^{2*}_{rL})\right] + (\pi^2_H | q^{2*}_H, q^{2*}_{rL})$$

$$\Rightarrow (q^{2*}_H)^2 \cdot b(P_H - P_L)$$

Hence low type firm's expected profits in the second period is $\underline{\underline{\boldsymbol{(q_L^{2*})^2 \cdot b(P_H - P_L)}}}$ ***and that of high type firm's is*** $\underline{\underline{\boldsymbol{(q_H^{2*})^2 \cdot b(P_H - P_L)}}}$

Note: This is an important result as the firm's expected profits are dependent on its own type (equilibrium quantity), and the only way the rival can affect its profits is by the first period quantiy which in turn affect the updated beliefs which results in change in equilibrium quantity in the second period.

A.3 Deviation

Suppose firm of type L deviates in period one and produces $q_L^1 \neq q_L^{1*}$, this results in different beliefs $\rho^2_{(q_L^1)} \neq \rho^2_{(q_L^{1*})}$ and rival plays on equilibrium path,

Hence firm's best response in period two:

$$q_L^2 = q_L^{2*} - \frac{q_{rH}^{2*} - q_{rL}^{2*}}{2} \cdot \left(\rho^2_{(q_L^1)} - \rho^2_{(q_L^{1*})}\right)$$

Similarly Suppose firm of type H deviates in period 1 and produces $q_H^1 \neq q_H^{1*}$, this results in different beliefs $\rho^2_{(q_H^1)} \neq \rho^2_{(q_H^{1*})}$ and rival plays on equilibrium path,

Hence firm's best response in period two:

$$q_H^2 = q_H^{2*} - \frac{q_{rH}^{2*} - q_{rL}^{2*}}{2} \cdot \left(\rho^2_{(q_H^1)} - \rho^2_{(q_H^{1*})}\right)$$

Expected profits after deviation for low type firm, given P_H^1: $b(P_H - P_L) \cdot (q_L^2 | P_H^1)^2$

Expected profits after deviation for low type firm, given P_L^1: $b(P_H - P_L) \cdot (q_L^2 | P_L^1)^2$

Similarly, Expected profits after deviation for high type firm, given P_H^1: $b(P_H - P_L) \cdot (q_H^2 | P_H^1)^2$

Expected profits after deviation for high type firm, given P_L^1: $b(P_H - P_L) \cdot (q_H^2 | P_L^1)^2$

A.4 Probabilities

In the begining of period one, a firm has some belief ρ^1 regarding the probability of their rival's type either high or low; since there are only 2 possible type, one can assume that firm gives equal weightage to the rivals type, i.e. $\rho^1 = 50\%$ (In other words, 50% High type, 50% Low type)

In the beginning of second period, firms aware of the price observed end of the first period and firm's own quantity produced; as per Bayes' Theorem, firms' update their belief regarding the rival's type,

When P_H,

$$\rho^2_{(q^1_\theta;P_H)} = \frac{\rho^1 \cdot (a - b(q^1_\theta + q^1_{rH}))}{\rho^1 \cdot (a - b(q^1_\theta + q^1_{rH})) + \rho^1 \cdot (a - b(q^1_\theta + q^1_{rL}))}$$

When P_L,

$$\rho^2_{(q^1_\theta;P_L)} = \frac{\rho^1 \cdot (1 - (a - b(q^1_\theta + q^1_{rH})))}{\rho^1 \cdot (1 - (a - b(q^1_\theta + q^1_{rH}))) + \rho^1 \cdot (1 - (a - b(q^1_\theta + q^1_{rL})))}$$

Simplifying the probabilities:
When P_H,

$$\rho^2_{(q^1_\theta;P_H)} = \frac{0.5 \cdot (a - b(q^1_\theta + q^1_{rH}))}{a - bq^1_\theta - 0.5b(q^1_{rH} + q^1_{rL})}$$

Similarly, when P_L,

$$\rho^2_{(q^1_\theta;P_L)} = \frac{0.5 \cdot (1 - a + b(q^1_\theta + q^1_{rH}))}{1 - a + bq^1_\theta + 0.5b(q^1_{rH} + q^1_{rL})}$$

For firm of type H, following are the probabilities:
When P_H,

$$\boldsymbol{\rho^{2}_{(q^{1}_{H};P_{H})} = \frac{0.5 \cdot \big(a - b(q^{1}_{H} + q^{1}_{rH})\big)}{a - bq^{1}_{H} - 0.5b(q^{1}_{rH} + q^{1}_{rL})}} \tag{9}$$

Similarly, when P_L,

$$\boldsymbol{\rho^{2}_{(q^{1}_{H};P_{L})} = \frac{0.5 \cdot \big(1 - a + b(q^{1}_{H} + q^{1}_{rH})\big)}{1 - a + bq^{1}_{H} + 0.5b(q^{1}_{rH} + q^{1}_{rL})}} \tag{10}$$

For firm of type L, following are the probabilities:
When P_H,

$$\boldsymbol{\rho^{2}_{(q^{1}_{L};P_{H})} = \frac{0.5 \cdot \big(a - b(q^{1}_{L} + q^{1}_{rH})\big)}{a - bq^{1}_{L} - 0.5b(q^{1}_{rH} + q^{1}_{rL})}} \tag{11}$$

Similarly, when P_L,

$$\boldsymbol{\rho^{2}_{(q^{1}_{L};P_{L})} = \frac{0.5 \cdot \big(1 - a + b(q^{1}_{L} + q^{1}_{rH})\big)}{1 - a + bq^{1}_{L} + 0.5b(q^{1}_{rH} + q^{1}_{rL})}} \tag{12}$$

A.5 Solving period one

In period one, firms have to maximize their lifetime profits,

$$\max_{q_\theta^1} \ \mathbb{E}\left[\pi_\theta^1 + \mathbb{E}\left[\pi^2_{\theta\ (q_\theta^1)}\right]\right]$$

$$\equiv \max_{q_\theta^1} \ \mathbb{E}\left[\pi_\theta^1\right] + \mathbb{E}\left[\mathbb{E}\left[\pi^2_{\theta\ (q_\theta^1)}\right]\right]$$

$$\equiv \max_{q_\theta^1} \ \mathbb{E}\left[\pi_\theta^1\right] + \mathbb{E}\left[\mathbb{E}\left[\pi^2_{\theta\ (q_\theta^1)}\right]\right]$$

$$\equiv \max_{q_\theta^1} \ \mathbb{E}\left[((\sigma^1 \cdot P_H + (1-\sigma^1)\cdot P_L) - c_\theta)\cdot q_\theta^1\right] + \mathbb{E}\left[\sigma^1 \cdot \pi^2_{\theta\ (q_\theta^1;P_H)} + (1-\sigma^1)\cdot \pi^2_{\theta\ (q_\theta^1;P_L)}\right]$$

$$\equiv \max_{q_\theta^1} \ \mathbb{E}\left[((a-bQ^1)\cdot(P_H - P_L)) + P_L - c_\theta)\cdot q_\theta^1\right]$$
$$+ \mathbb{E}\left[(a-bQ^1)\cdot \pi^2_{\theta\ (q_\theta^1;P_H)} + (1-(a-bQ^1))\cdot \pi^2_{\theta\ (q_\theta^1;P_L)}\right]$$

$$\equiv \max_{q_\theta^1} \ \mathbb{E}\left[((a - bq_\theta^1 - bq_r^1)\cdot(P_H - P_L)) + P_L - c_\theta)\cdot q_\theta^1\right]$$
$$+ \mathbb{E}\left[(a - bq_\theta^1 - bq_r^1)\cdot \pi^2_{\theta\ (q_\theta^1;P_H)} + (1-(a - bq_\theta^1 - bq_r^1))\cdot \pi^2_{\theta\ (q_\theta^1;P_L)}\right]$$

Note: $\rho^1 = 0.5$ i.e In Period I Firm's probability of Rival is of Type H is 50 %

Applying Expectation, $\mathbb{E}[q_r^1] = \rho^1 \cdot q_{rH}^1 + (1-\rho^1)\cdot q_{rL}^1 = 0.5q_{rH}^1 + 0.5q_{rL}^1$

Substituting,

$$\equiv \max_{q_\theta^1} \ ((a - bq_\theta^1 - b(0.5q_{rH}^1 + 0.5q_{rL}^1))\cdot(P_H - P_L) + P_L - c_\theta)\cdot q_\theta^1$$
$$+(a - bq_\theta^1 - b(0.5q_{rH}^1 + 0.5q_{rL}^1))\cdot \pi^2_{\theta\ (q_\theta^1;P_H)} + (1-(a - bq_\theta^1 - b(0.5q_{rH}^1 + 0.5q_{rL}^1)))\cdot \pi^2_{\theta\ (q_\theta^1;P_L)}$$

$$\equiv \max_{q_\theta^1} \ ((a - bq_\theta^1 - b(0.5q_{rH}^1 + 0.5q_{rL}^1)\cdot(P_H - P_L))\cdot q_\theta^1 + (P_L - c_\theta)\cdot q_\theta^1)$$
$$+(a - bq_\theta^1 - b(0.5q_{rH}^1 + 0.5q_{rL}^1))\cdot \pi^2_{\theta\ (q_\theta^1;P_H)} + (1-(a - bq_\theta^1 - b(0.5q_{rH}^1 + 0.5q_{rL}^1)))\cdot \pi^2_{\theta\ (q_\theta^1;P_L)}$$

Taking First Order Conditions w.r.t q^1_θ,

$$\longrightarrow \Big\langle (a - 2bq^1_\theta - b(0.5q^1_{rH} + 0.5q^1_{rL}) \cdot (P_H - P_L) + P_L - c_\theta$$
$$- b \cdot \pi^2_{\theta\ (q^1_\theta;P_H)} + (a - bq^1_\theta - b(0.5q^1_{rH} + 0.5q^1_{rL})) \cdot \left[\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_H)}}{\partial q^1_\theta}\right]$$
$$+ b \cdot \pi^2_{\theta\ (q^1_\theta;P_L)} + (1 - (a - bq^1_\theta - b(0.5q^1_{rH} + 0.5q^1_{rL}))) \cdot \left[\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_L)}}{\partial q^1_\theta}\right] \Big\rangle = 0$$

Substituting the following in the above equation,

$$\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_H)}}{\partial q^1_\theta} = \left[\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_H)}}{\partial q^2_\theta} \cdot \left[\frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_H;P_H)}} \cdot \frac{\partial \rho^2_{(q^1_H;P_H)}}{\partial q^1_\theta} + \frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_L;P_H)}} \cdot \frac{\partial \rho^2_{(q^1_L;P_H)}}{\partial q^1_\theta}\right]\right]$$

$$\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_L)}}{\partial q^1_\theta} = \left[\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_L)}}{\partial q^2_\theta} \cdot \left[\frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_H;P_L)}} \cdot \frac{\partial \rho^2_{(q^1_H;P_L)}}{\partial q^1_\theta} + \frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_L;P_L)}} \cdot \frac{\partial \rho^2_{(q^1_L;P_L)}}{\partial q^1_\theta}\right]\right]$$

$$\longrightarrow \Big\langle (a - 2bq^1_\theta - b(0.5q^1_{rH} + 0.5q^1_{rL}) \cdot (P_H - P_L) + P_L - c_\theta - b \cdot \pi^2_{\theta\ (q^1_\theta;P_H)}$$
$$+(a-bq^1_\theta-b(0.5q^1_{rH}+0.5q^1_{rL}))\cdot\left[\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_H)}}{\partial q^2_\theta} \cdot \left[\frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_H;P_H)}} \cdot \frac{\partial \rho^2_{(q^1_H;P_H)}}{\partial q^1_\theta} + \frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_L;P_H)}} \cdot \frac{\partial \rho^2_{(q^1_L;P_H)}}{\partial q^1_\theta}\right]\right]$$
$$+ b \cdot \pi^2_{\theta\ (q^1_\theta;P_L)}$$
$$+(1-(a-bq^1_\theta-b(0.5q^1_{rH}+0.5q^1_{rL})))\cdot\left[\frac{\partial \pi^2_{\theta\ (q^1_\theta;P_L)}}{\partial q^2_\theta} \cdot \left[\frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_H;P_L)}} \cdot \frac{\partial \rho^2_{(q^1_H;P_L)}}{\partial q^1_\theta} + \frac{\partial q^2_\theta}{\partial \rho^2_{(q^1_L;P_L)}} \cdot \frac{\partial \rho^2_{(q^1_L;P_L)}}{\partial q^1_\theta}\right]\right] \Big\rangle$$
$$= 0$$

Imposing symmetry:
$q^1_\theta = q^1_{r\theta} = q^{1*}_\theta;\ \forall\theta = L, H$

Period one final equations-
For Type L:

$$\longrightarrow \Big\langle (a - 2.5bq^1_L - 0.5bq^1_H)\cdot(P_H - P_L) + P_L - c_L$$
$$-b\cdot\pi^2_{L\ (q^1_L;P_H)} + (a-1.5bq^1_L-0.5bq^1_H)\cdot\left[\frac{\partial\pi^2_{L\ (q^1_L;P_H)}}{\partial q^2_L}\cdot\left[\frac{\partial q^2_L}{\partial\rho^2_{(q^1_H;P_H)}}\cdot\frac{\partial\rho^2_{(q^1_H;P_H)}}{\partial q^1_L} + \frac{\partial q^2_L}{\partial\rho^2_{(q^1_L;P_H)}}\cdot\frac{\partial\rho^2_{(q^1_L;P_H)}}{\partial q^1_L}\right]\right]$$
$$+ b\cdot\pi^2_{L\ (q^1_L;P_L)}$$
$$+(1-(a-1.5bq^1_L-0.5bq^1_H))\cdot\left[\frac{\partial\pi^2_{L\ (q^1_L;P_L)}}{\partial q^2_L}\cdot\left[\frac{\partial q^2_L}{\partial\rho^2_{(q^1_H;P_L)}}\cdot\frac{\partial\rho^2_{(q^1_H;P_L)}}{\partial q^1_L} + \frac{\partial q^2_L}{\partial\rho^2_{(q^1_L;P_L)}}\cdot\frac{\partial\rho^2_{(q^1_L;P_L)}}{\partial q^1_L}\right]\right]\Big\rangle = 0 \tag{13}$$

Where:

$$q^{2*}_L;P_H = \frac{a}{3b} - \frac{1}{3}\cdot\left[\left(\frac{c_L - c_H}{b(P_H-P_L)\cdot\left[2+\rho^2_{(q^1_H;P_H)} - \rho^2_{(q^1_L;P_H)}\right]}\right)\cdot\left(\rho^2_{(q^1_L;P_H)}\right)\right] + \frac{P_L - c_L}{3b(P_H-P_L)}$$

$$\pi^2_{L\ (q^1_L;P_H)} = (q^{2*}_L;P_H)^2\cdot b(P_H-P_L)$$

$$\frac{\partial\pi^2_{L\ (q^1_L;P_H)}}{\partial q^2_L} = \frac{\partial(q^{2*}_L;P_H)^2\cdot b(P_H-P_L)}{\partial q^2_L;P_H} = 2\cdot(q^{2*}_L;P_H)\cdot b(P_H-P_L)$$

$$\frac{\partial q^2_L}{\partial\rho^2_{(q^1_H;P_H)}} = \frac{1}{3}\cdot\frac{(c_L-c_H)\cdot\rho^2_{(q^1_L;P_H)}}{b(P_H-P_L)\cdot(2+\rho^2_{(q^1_H;P_H)} - \rho^2_{(q^1_L;P_H)})^2}$$

$$\frac{\partial\rho^2_{(q^1_H;P_H)}}{\partial q^1_L} = \frac{0.25b(a-2bq^1_H)}{(a-1.5bq^1_H+0.5bq^1_L)^2}$$

$$\frac{\partial q^2_L}{\partial\rho^2_{(q^1_L;P_H)}} = \frac{-1}{3}\cdot\frac{(c_L-c_H)\cdot(2+\rho^2_{(q^1_H;P_H)})}{b(P_H-P_L)\cdot(2+\rho^2_{(q^1_H;P_H)} - \rho^2_{(q^1_L;P_H)})^2}$$

$$\frac{\partial \rho^2_{(q_L^1;P_H)}}{\partial q_L^1} = \frac{-0.5b}{a - 1.5bq_L^1 - 0.5bq_H^1} + \frac{0.75b(a - bq_L^1 - bq_H^1)}{(a - 1.5bq_L^1 - 0.5bq_H^1)^2}$$

$$q_L^{2*}; P_L = \frac{a}{3b} - \frac{1}{3} \cdot \left[\left(\frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)} \right]} \right) \cdot \left(\rho^2_{(q_L^1;P_L)} \right) \right] + \frac{P_L - c_L}{3b(P_H - P_L)}$$

$$\pi^2_{L\ (q_L^1;P_L)} = (q_L^{2*}; P_L)^2 \cdot b(P_H - P_L)$$

$$\frac{\partial \pi^2_{L\ (q_L^1;P_L)}}{\partial q_L^2} = \frac{\partial (q_L^{2*}; P_L)^2 \cdot b(P_H - P_L)}{\partial q_L^2; P_L} = 2 \cdot (q_L^{2*}; P_L) \cdot b(P_H - P_L)$$

$$\frac{\partial q_L^2}{\partial \rho^2_{(q_H^1;P_L)}} = \frac{1}{3} \cdot \frac{(c_L - c_H) \cdot \rho_{(q_L^1;P_L)}}{b(P_H - P_L) \cdot (2 + \rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)})^2}$$

$$\frac{\partial \rho^2_{(q_H^1;P_L)}}{\partial q_L^1} = \frac{0.25b \cdot (1 - a + 2bq_H^1)}{(1 - a + 0.5bq_H^1 - 0.5bq_L^1)^2}$$

$$\frac{\partial q_L^2}{\partial \rho^2_{(q_L^1;P_L)}} = \frac{-1}{3} \cdot \frac{(c_L - c_H) \cdot (2 + \rho^2_{(q_H^1;P_L)})}{b(P_H - P_L) \cdot (2 + \rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)})^2}$$

$$\frac{\partial \rho^2_{(q_L^1;P_L)}}{\partial q_L^1} = \frac{0.5b}{1 - a + 0.5bq_L^1 - 0.5bq_H^1} - \frac{0.25b \cdot (1 - a + bq_L^1 + bq_H^1)}{(1 - a + b0.5bq_L^1 - 0.5bq_H^1)^2}$$

For Type H:

$$\longrightarrow \Big\langle (a-2.5bq_H^1-0.5bq_L^1)\cdot(P_H-P_L)+P_L-c_H-b\cdot\pi^2_{H\ (q_H^1;P_H)}$$
$$+(a-1.5bq_H^1-0.5bq_L^1))\cdot\left[\frac{\partial\pi^2_{H\ (q_H^1;P_H)}}{\partial q_H^2}\cdot\left[\frac{\partial q_H^2}{\partial\rho^2_{(q_H^1;P_H)}}\cdot\frac{\partial\rho^2_{(q_H^1;P_H)}}{\partial q_H^1}+\frac{\partial q_H^2}{\partial\rho^2_{(q_L^1;P_H)}}\cdot\frac{\partial\rho^2_{(q_L^1;P_H)}}{\partial q_H^1}\right]\right]$$
$$+b\cdot\pi^2_{H\ (q_H^1;P_L)}$$
$$+(1-(a-1.5bq_H^1-0.5bq_L^1))\cdot\left[\frac{\partial\pi^2_{H\ (q_H^1;P_L)}}{\partial q_H^2}\cdot\left[\frac{\partial q_H^2}{\partial\rho^2_{(q_H^1;P_L)}}\cdot\frac{\partial\rho^2_{(q_H^1;P_L)}}{\partial q_H^1}+\frac{\partial q_H^2}{\partial\rho^2_{(q_L^1;P_L)}}\cdot\frac{\partial\rho^2_{(q_L^1;P_L)}}{\partial q_H^1}\right]\right]\Big\rangle=0 \tag{14}$$

Where:

$$q_H^{2*};P_H=\frac{a}{3b}-\frac{1}{6}\cdot\left[\left(\frac{c_L-c_H}{b(P_H-P_L)\cdot\left[2+\rho^2_{(q_H^1;P_H)}-\rho^2_{(q_L^1;P_H)}\right]}\right)\cdot\left(3\rho^2_{(q_H^1;P_H)}-\rho^2_{(q_L^1;P_H)}\right)\right]+\frac{2P_L+c_L-3c_H}{6b(P_H-P_L)}$$

$$\pi^2_{H\ (q_H^1;P_H)}=(q_H^{2*};P_H)^2\cdot b(P_H-P_L)$$

$$\frac{\partial\pi^2_{H\ (q_H^1;P_H)}}{\partial q_H^2}=2(q_H^{2*};P_H)\cdot b(P_H-P_L)$$

$$\frac{\partial q_H^2}{\partial\rho^2_{(q_H^1;P_H)}}=\frac{(c_L-c_H)\cdot\left(-2+\frac{2}{3}\rho^2_{(q_L^1;P_H)}\right)}{b(P_H-P_L)\cdot(2+\rho^2_{(q_H^1;P_H)}-\rho^2_{(q_L^1;P_H)})^2}$$

$$\frac{\partial\rho^2_{(q_H^1;P_H)}}{\partial q_H^1}=\frac{-b}{a-1.5bq_H^1-0.5bq_L^1}+\frac{0.75b\cdot(a-2bq_H^1)}{(a-1.5bq_H^1-0.5bq_L^1)^2}$$

$$\frac{\partial q_H^2}{\partial\rho^2_{(q_L^1;P_H)}}=\frac{2}{3}\cdot\frac{(c_L-c_H)\cdot\left(1-\rho^2_{(q_H^1;P_H)}\right)}{b(P_H-P_L)\cdot(2+\rho^2_{(q_H^1;P_H)}-\rho^2_{(q_L^1;P_H)})^2}$$

$$\frac{\partial\rho^2_{(q_L^1;P_H)}}{\partial q_H^1}=\frac{-0.5b}{a-1.5bq_L^1-0.5bq_H^1}+\frac{0.25b\cdot(a-bq_L^1-bq_H^1)}{(a-1.5bq_L^1-0.5bq_H^1)^2}$$

$$q_H^{2*}; P_L = \frac{a}{3b} - \frac{1}{6} \cdot \left[\left(\frac{c_L - c_H}{b(P_H - P_L) \cdot \left[2 + \rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)}\right]} \right) \cdot \left(3\rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)}\right) \right] + \frac{2P_L + c_L - 3c_H}{6b(P_H - P_L)}$$

$$\pi^2_{H\ (q_H^1;P_L)} = (q_H^{2*}; P_L)^2 \cdot b(P_H - P_L)$$

$$\frac{\partial \pi^2_{H\ (q_H^1;P_L)}}{\partial q_H^2} = 2(q_H^{2*}; P_L) \cdot b(P_H - P_L)$$

$$\frac{\partial q_H^2}{\partial \rho^2_{(q_H^1;P_L)}} = \frac{(c_L - c_H) \cdot \left(-2 + \frac{2}{3}\rho^2_{(q_L^1;P_L)}\right)}{b(P_H - P_L) \cdot (2 + \rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)})^2}$$

$$\frac{\partial \rho^2_{(q_H^1;P_L)}}{\partial q_H^1} = \frac{b}{1 - a + 0.5bq_H^1 - 0.5bq_L^1} - \frac{0.25b \cdot (1 - a + 2bq_H^1)}{(1 - a + 0.5bq_H^1 - 0.5bq_L^1)^2}$$

$$\frac{\partial q_H^2}{\partial \rho^2_{(q_L^1;P_L)}} = \frac{2}{3} \cdot \frac{(c_L - c_H) \cdot \left(1 - \rho^2_{(q_H^1;P_L)}\right)}{b(P_H - P_L) \cdot (2 + \rho^2_{(q_H^1;P_L)} - \rho^2_{(q_L^1;P_L)})^2}$$

$$\frac{\partial \rho^2_{(q_L^1;P_L)}}{\partial q_H^1} = \frac{0.5b}{1 - a + 0.5bq_L^{-}0.5q_H^1} + \frac{0.25b \cdot (1 - a + bq_L^1 + bq_H^1)}{(1 - a + 0.5bq_L^{-}0.5q_H^1)^2}$$

The following are the updated probability equation 9, 10, 11 and 12 after imposing symmetry,
Price H -

$$\rho^2_{(q^1_H;P_H)} = \frac{0.5 \cdot (a - 2bq^1_H)}{a - 1.5bq^1_H - 0.5bq^1_L}$$

$$\rho^2_{(q^1_L;P_H)} = \frac{0.5 \cdot (a - bq^1_L - bq^1_H)}{a - 1.5bq^1_L - 0.5bq^1_H}$$

Price L -

$$\rho^2_{(q^1_H;P_L)} = \frac{0.5 \cdot (1 - a + 2bq^1_H)}{1 - a + 1.5bq^1_H + 0.5bq^1_L}$$

$$\rho^2_{(q^1_L;P_L)} = \frac{0.5 \cdot (1 - a + bq^1_L + bq^1_H)}{1 - a + 1.5bq^1_L + 0.5bq^1_H}$$

After substituting all the equations back in equation 13 and 14, I have 2 non-linear equations and 2 unknowns.

B. First period graphs of the low type firm

The extreme numerical values are mentioned with each graph, clockwise starting when P_L=8 & P_H=39.

B.1 First period quantity and profit graphs for the low type firm in a dynamic two period game with private information

FIGURE B.1: Quantity produced in the first period by the low type firm.
(26.6877, 29.4558, 28.8329, 26.2686)

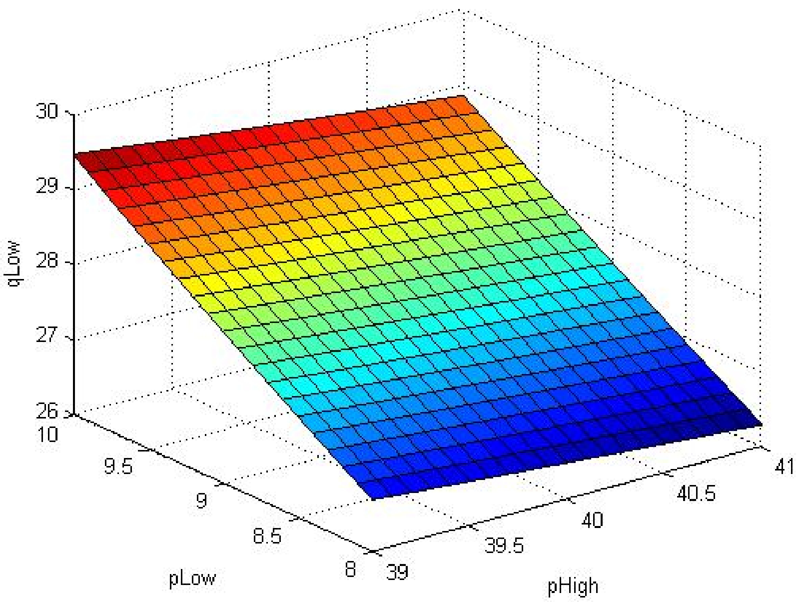


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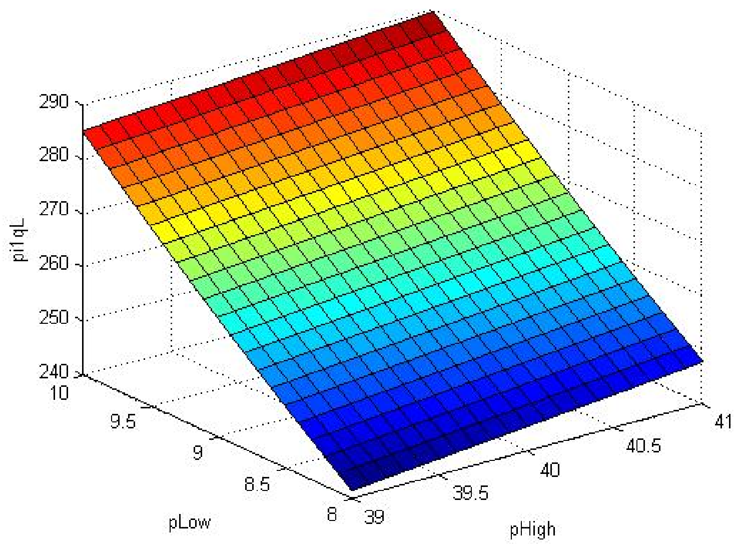
FIGURE B.2: First period profits for the low type firm.
(241.4190, 284.9278, 289.3839, 247.6322)



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B.2 Quantity and profit graphs for the low type firm in a static one period game with private information

FIGURE B.3: Quantity produced in the first period by the low type firm.
(26.7204, 29.4827, 28.8709, 26.3131)

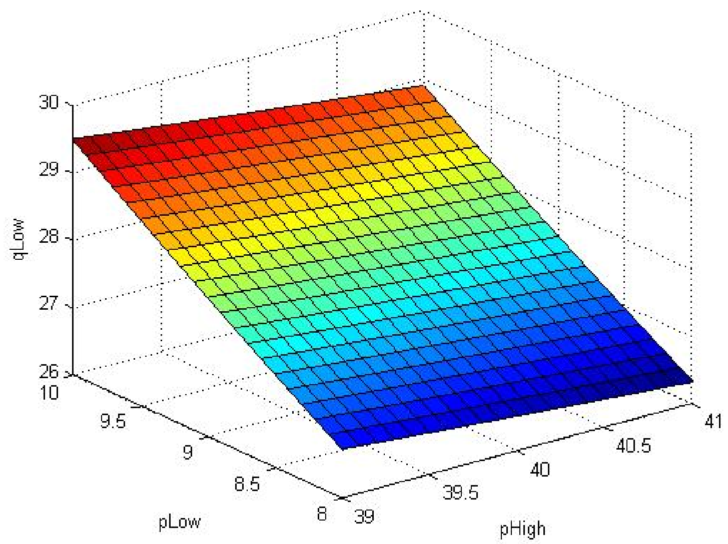

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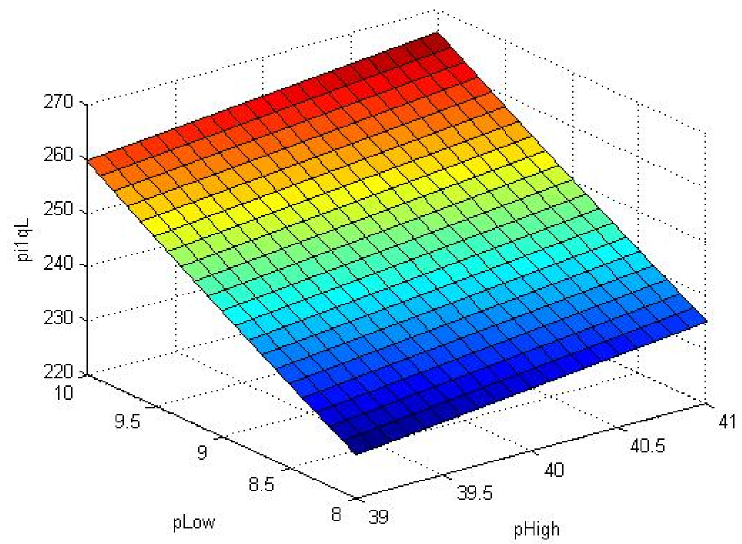
FIGURE B.4: Profits for the low type firm.
(228.01433, 259.4482, 265.6129, 235.06397)



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B.3 Quantity and profit graphs for the low type firm in a static one period game with no private information

FIGURE B.5: Quantity produced in the first period by the low type firm.
(27.5269, 30.3448, 29.6774, 27.0707)

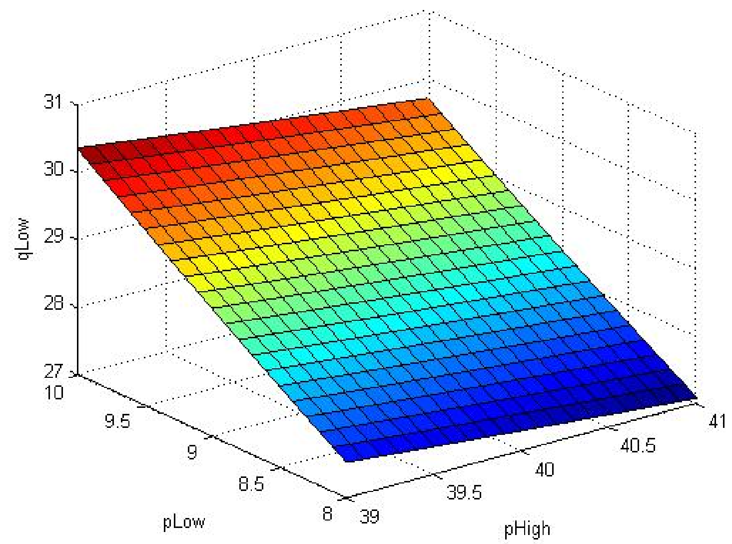

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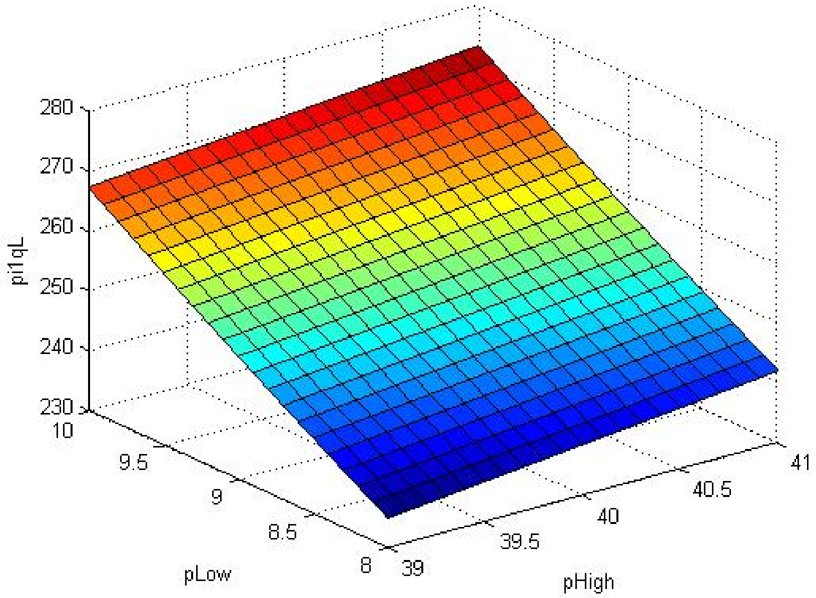
FIGURE B.6: Profits for the low type firm.
(234.8961, 267.0345, 273.0323, 241.8316)



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C. First period graphs of the high type firm

The extreme numerical values are mentioned with each graph, clockwise starting when P_L=8 & P_H=39.

C.1 First period quantity and profit graphs for the high type firm in a dynamic two period game with private information

FIGURE C.1: Quantity produced in the first period by the high type firm. (25.3531, 27.8302, 27.3285, 24.8655)

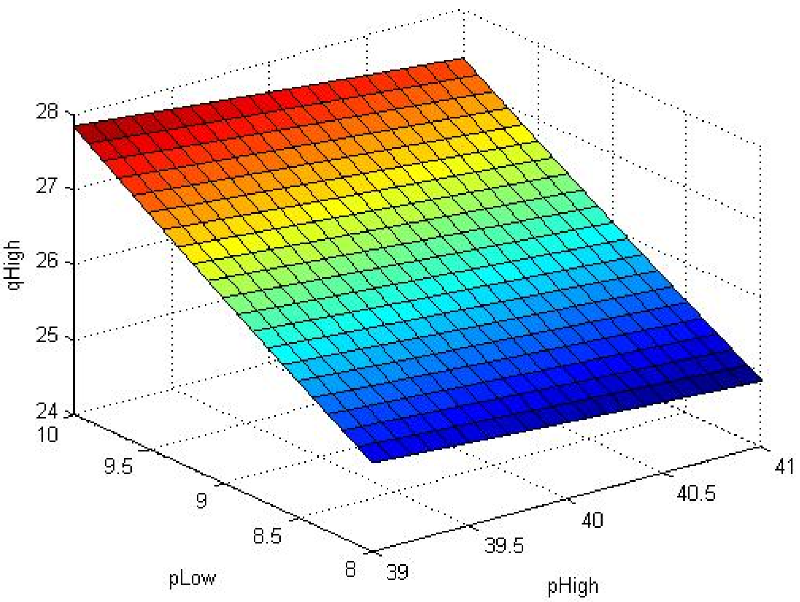

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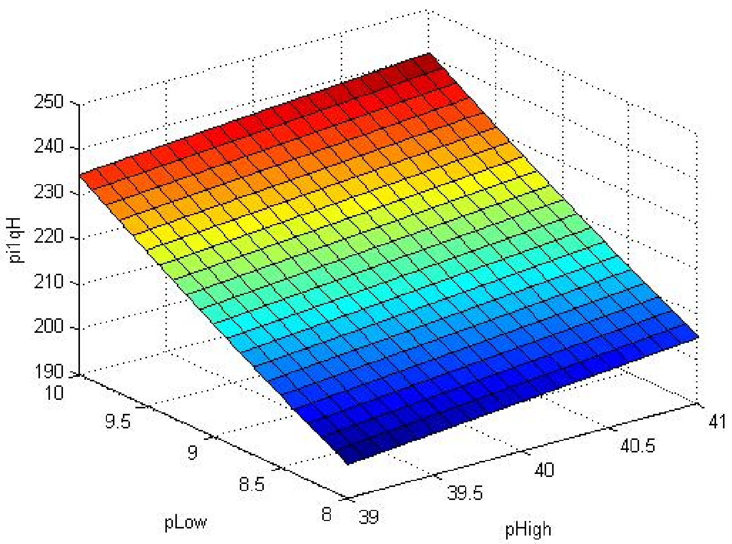
FIGURE C.2: First period profits for the high type firm.
(197.4717, 234.1772, 240.1564, 204.5565)



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C.2 Quantity and profit graphs for the high type firm in a static one period game with private information

FIGURE C.3: Quantity produced in the first period by the high type firm.
(25.1075, 27.7586, 27.2580, 24.7979)

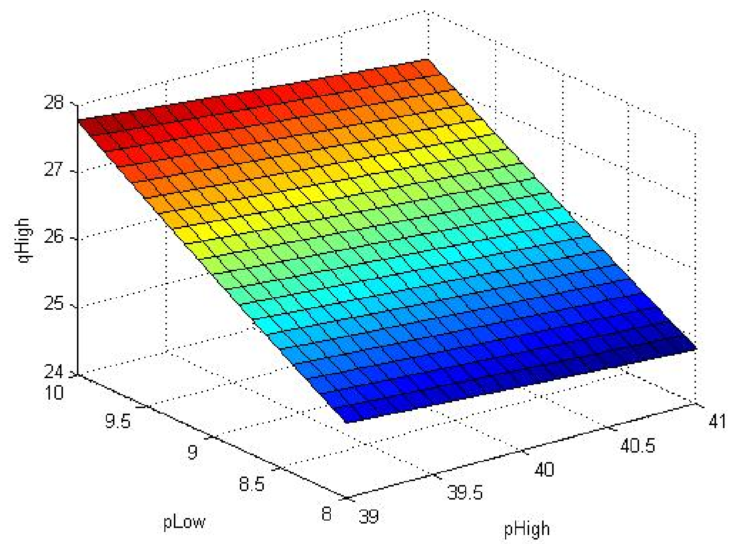

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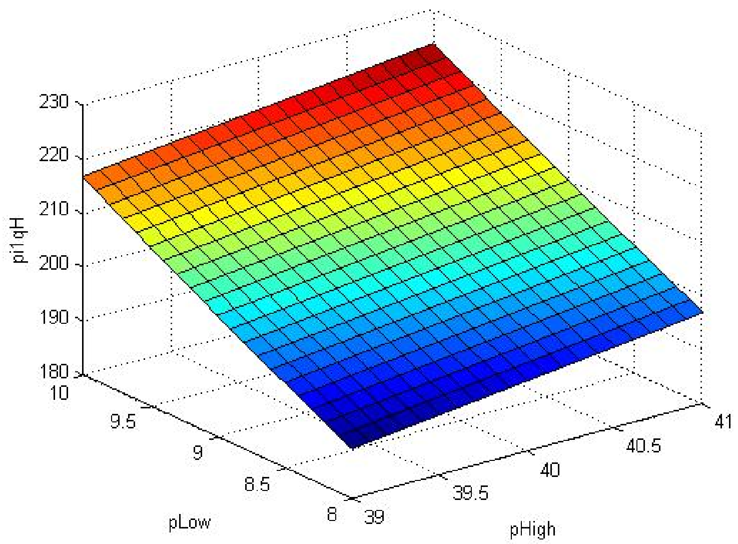
FIGURE C.4: Profits for the high type firm.
(189.1433, 216.5172, 223.5161, 196.7306)



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C.3 Quantity and profit graphs for the high type firm in a static one period game with no private information

FIGURE C.5: Quantity produced in the first period by the high type firm.
(24.3011, 26.8966, 26.4516, 24.0404)

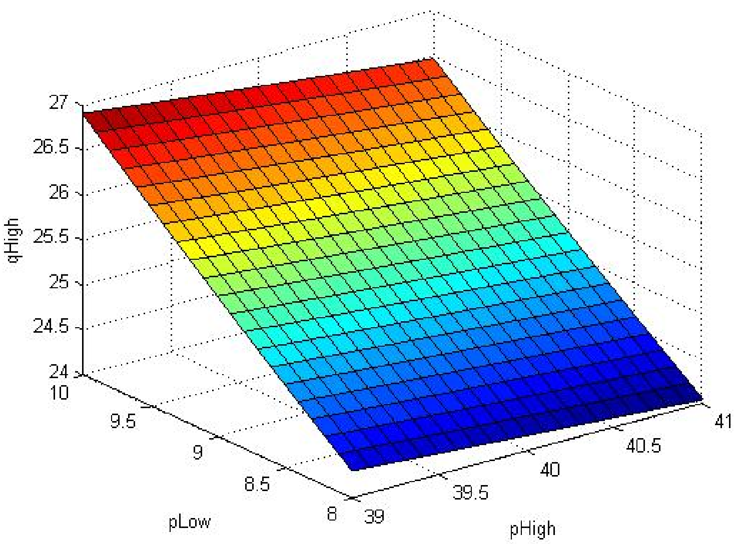

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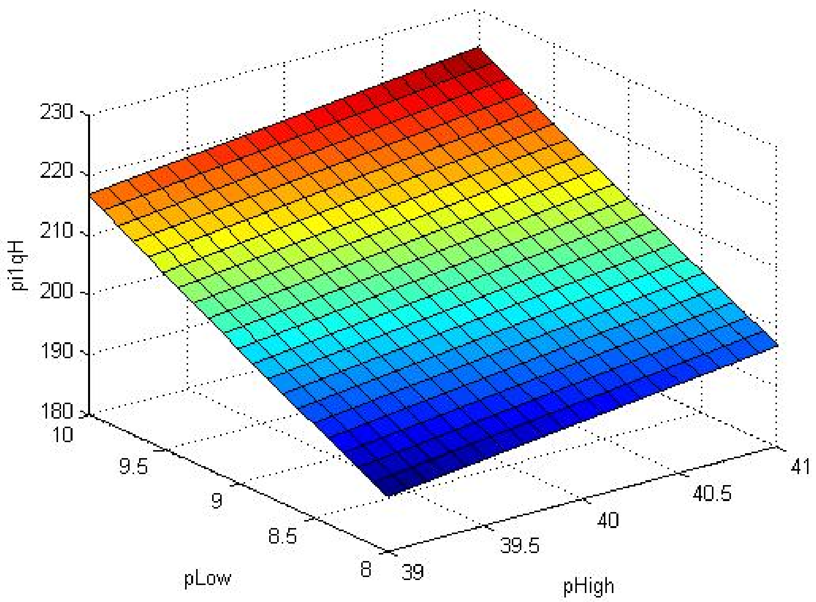
FIGURE C.6: Profits for the high type firm.
(183.0681, 209.7931, 216.9032, 190.7205)



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D. Second period graphs of the low type firm

The extreme numerical values are mentioned with each graph, clockwise starting when P_L=8 & P_H=39.

D.1 Second period quantity and profit graphs for the low type firm in a dynamic two period game with private information if low price is observed in the first period.

FIGURE D.1: Quantity produced in the second period by the low type firm if P_L is observed end of first period.
(6.7203, 9.4824, 8.8709, 6.3132)

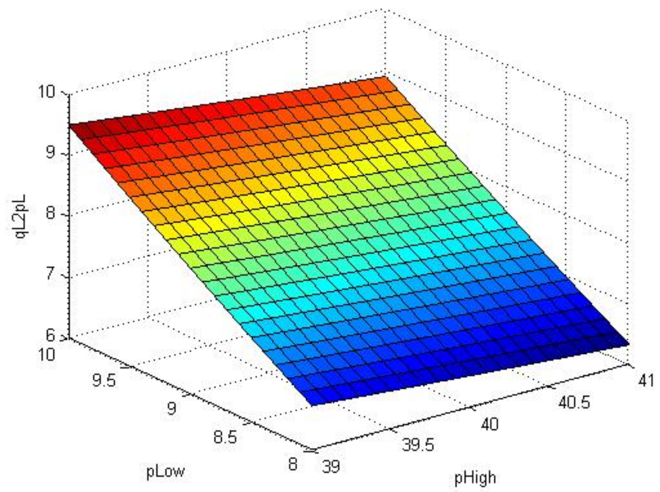

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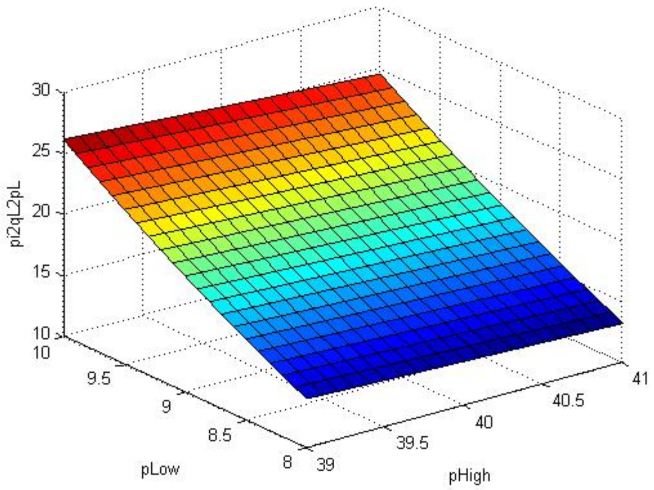
FIGURE D.2: Second period profits for low type firm if P_L is observed end of first period.
(14.0001, 26.0755, 24.3947, 13.1526)



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D.2 Second period quantity and profit graphs for the low type firm in a dynamic two period game with private information if high price is observed in the first period.

FIGURE D.3: Quantity produced in the second period by low type firm if P_H is observed end of first period.
(6.7512, 9.6289, 8.9453, 6.3380)

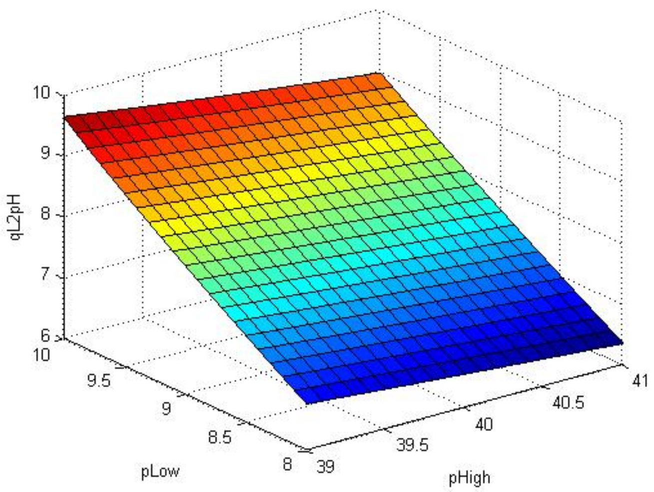

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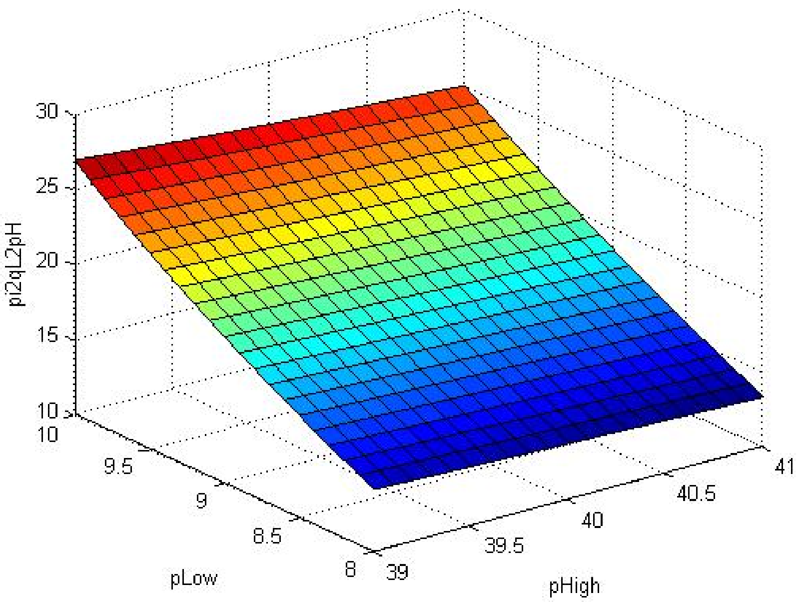
FIGURE D.4: Second period profits for low type firm if P_H is observed end of first period.
(14.1293, 26.8874, 24.8057, 13.2562)



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E. Second period graphs of the high type firm

The extreme numerical values are mentioned with each graph, clockwise starting when P_L=8 & P_H=39.

E.1 Second period quantity and profit graphs for the high type firm in a dynamic two period game with private information if low price is observed in the first period.

FIGURE E.1: Quantity produced in the second period by high type firm if P_L is observed end of first period.
(5.1073, 7.7582, 7.2579, 4.7980)

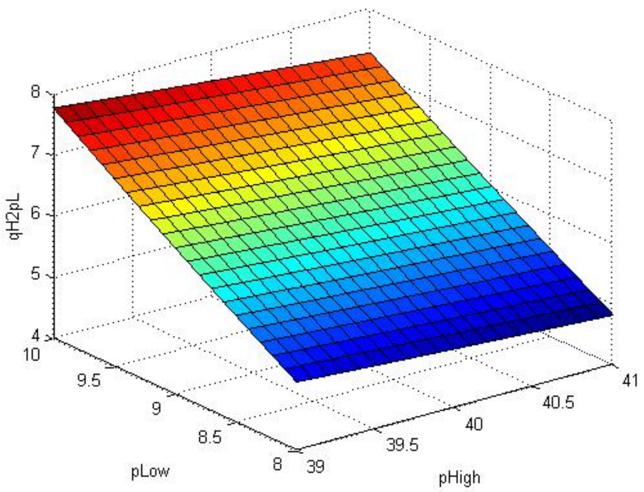

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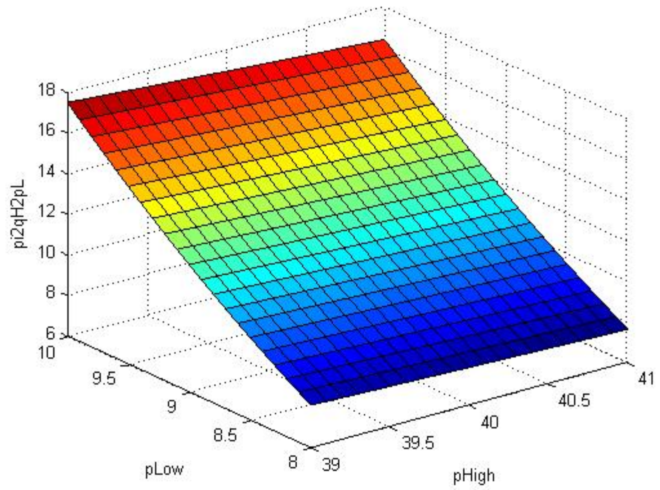
FIGURE E.2: Second period profits for high type firm if P_L is observed end of first period.
(8.0862, 17.4550, 16.3301, 7.5968)



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E.2 Second period quantity and profit graphs for the high type firm in a dynamic two period game with private information if high price is observed in the first period.

FIGURE E.3: Quantity produced in the second period by high type firm if P_H is observed end of first period.
(5.1313, 7.8154, 7.2995, 4.8180)

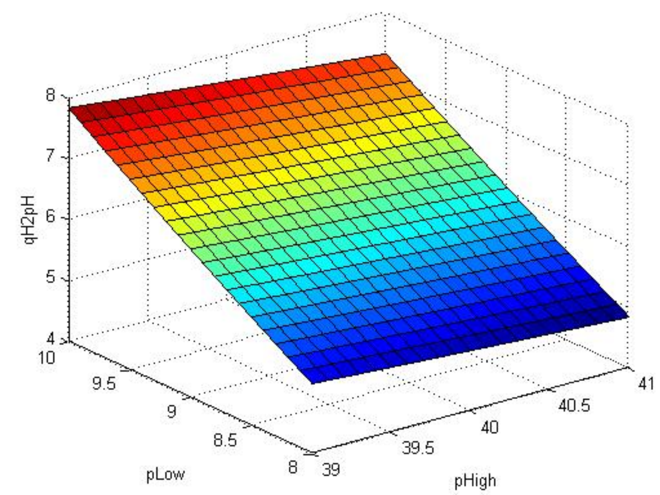

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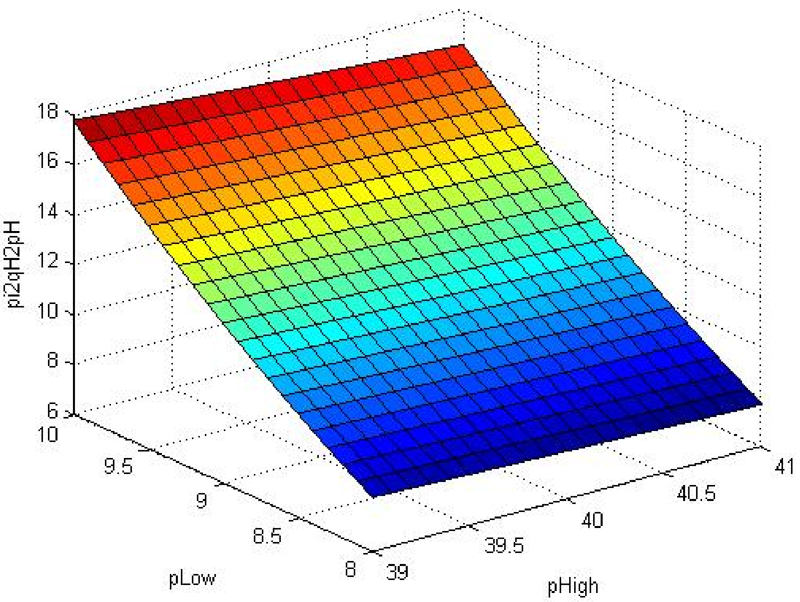
FIGURE E.4: Second period profits for high type firm if P_H is observed end of first period.
(8.1623, 17.7133, 16.5178, 7.6602)



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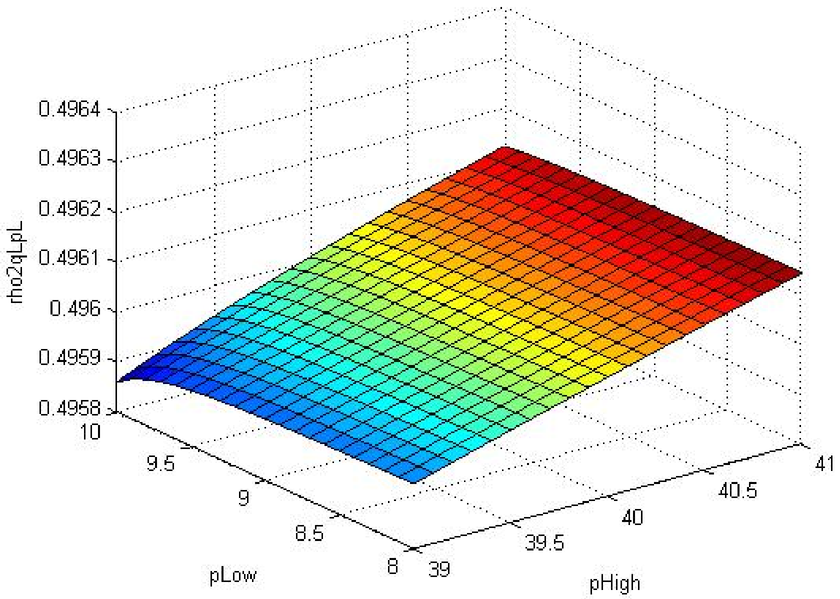
F. Graphs for the updated probabilities in the second period

The extreme numerical values are mentioned with each graph, clockwise starting when P_L=8 & P_H=39.

F.1 If low price is observed at the end of the first period

F.1.1 For low type firm

FIGURE F.1: Second period probability that the rival is of high type, given the firm i produces q_L^1 and observes P_L^1
(0.495929, 0.495857, 0.496119, 0.496140)



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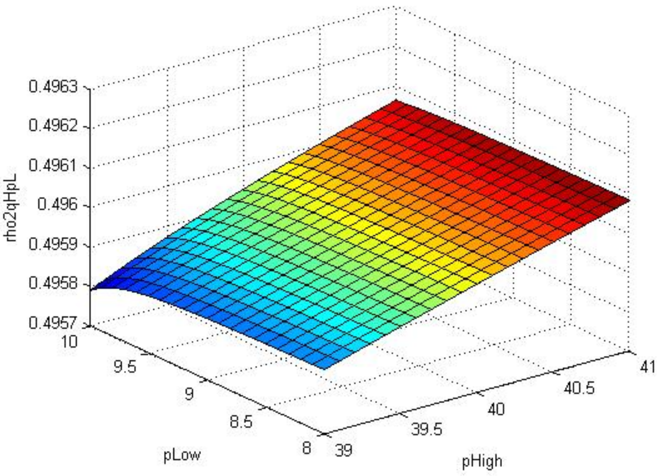
F.1.2 For high type firm

FIGURE F.2: Second period probability that the rival is of high type, given the firm i produces q_H^1 and observes P_L^1
(0.495862, 0.495787, 0.496057, 0.496079)



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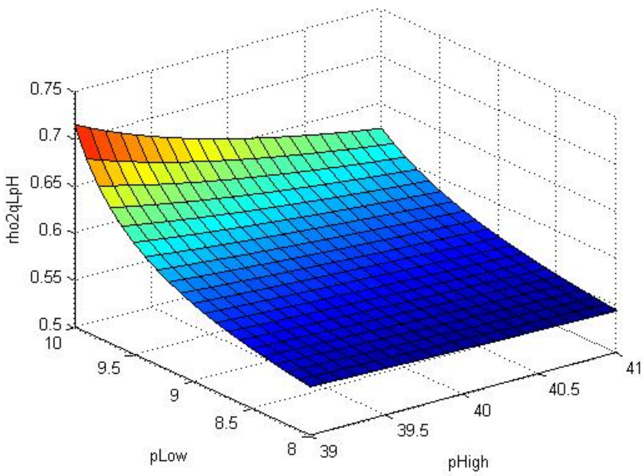
F.2 If high price is observed at the end of the first period

F.2.1 For low type firm

FIGURE F.3: Second period probability that the rival is of high type, given the firm i produces q_L^1 and observe P_H^1
(0.5511, 0.7138, 0.6219, 0.5435)



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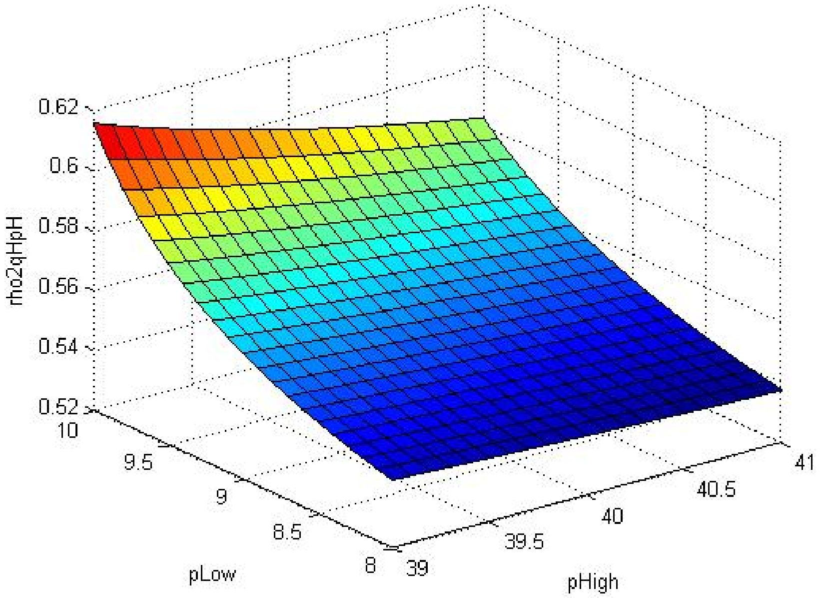
F.2.2 For high type firm

FIGURE F.4: Second period probability that the rival is of high type, given the firm i produces q_H^1 and observe P_H^1
(0.5424, 0.6152, 0.5819, 0.5371)



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G. Graphs - Comparative Statics

G.1 Graphs when low price is fixed at 9 and high price varies between 39 to 41

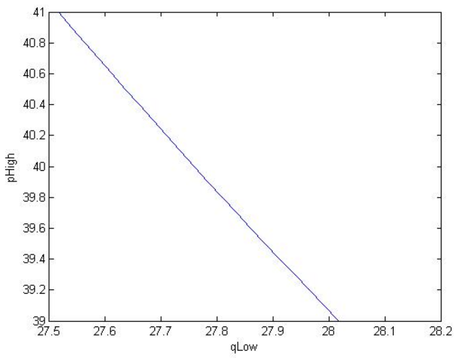

FIGURE G.1: Quantity produced in the first period by Low Type

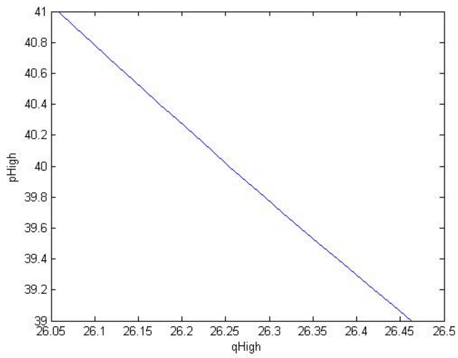

FIGURE G.2: Quantity produced in the first period by High Type

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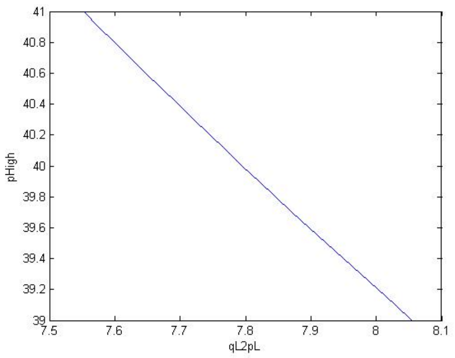

FIGURE G.3: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

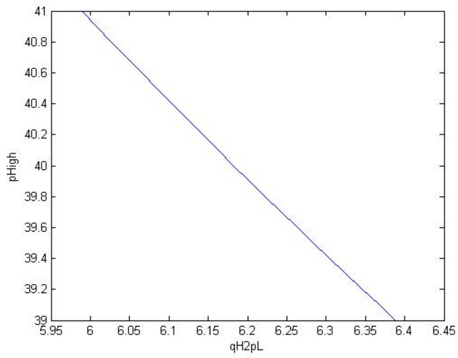

FIGURE G.4: Quantity produced in the second period by High Type firm when P_L is observed end of first period

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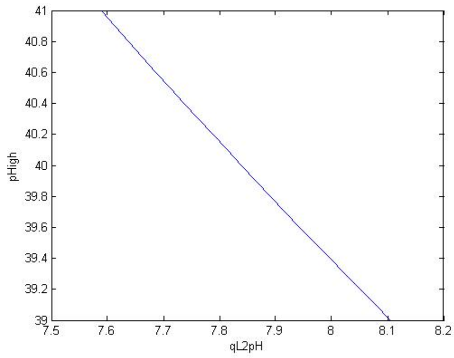

FIGURE G.5: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

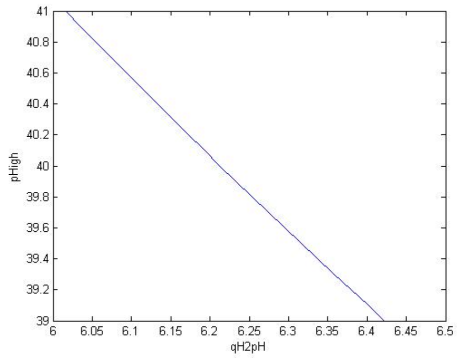

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FIGURE G.6: Quantity produced in the second period by High Type firm when P_H is observed end of first period

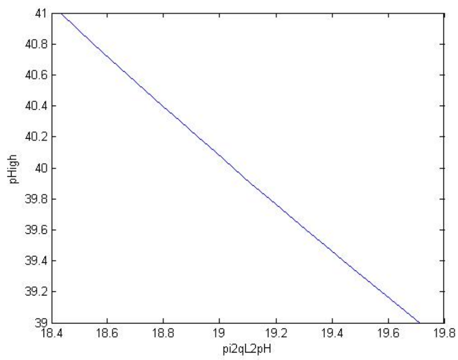

FIGURE G.7: II Period profits for Low Type firm when P_H is observed end of first period

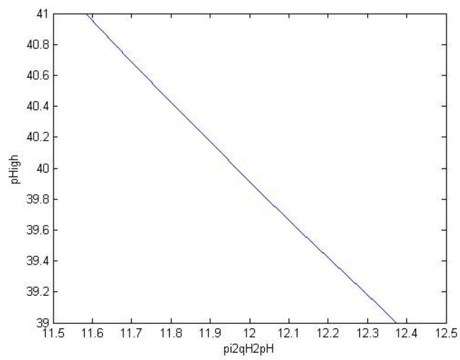

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FIGURE G.8: II Period profits for High Type firm when P_H is observed end of first period

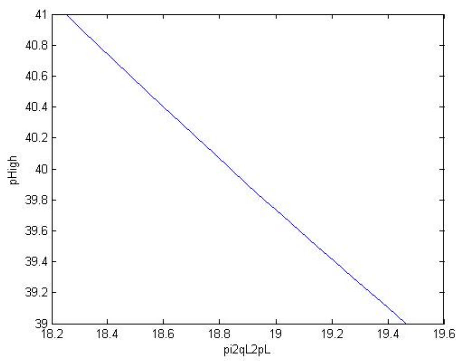

FIGURE G.9: II Period profits for Low Type firm when P_L is observed end of first period

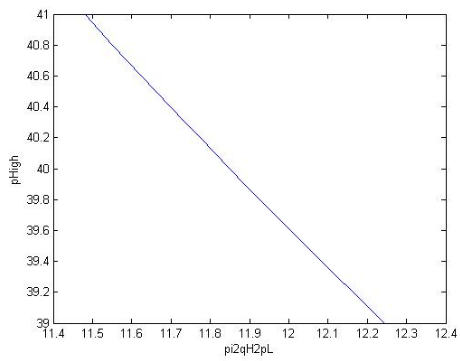

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FIGURE G.10: II Period profits for High Type firm when P_L is observed end of first period

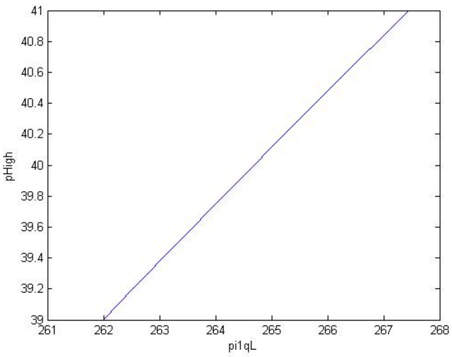

Figure G.11: I Period profits for Low Type firm

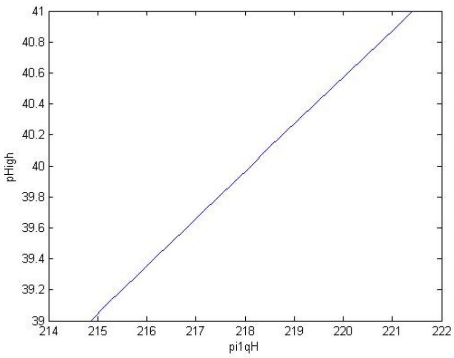

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Figure G.12: I Period profits for High Type firm

G.2 Graphs when high price is fixed at 40 and low price varies between 8 to 10

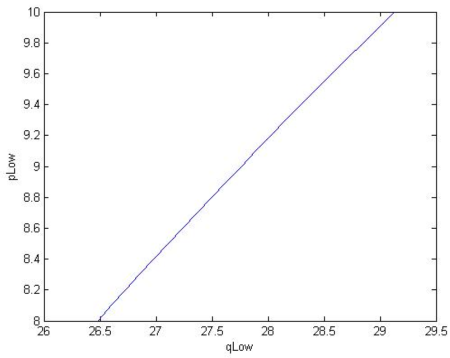

Figure G.13: Quantity produced in the first period by Low Type

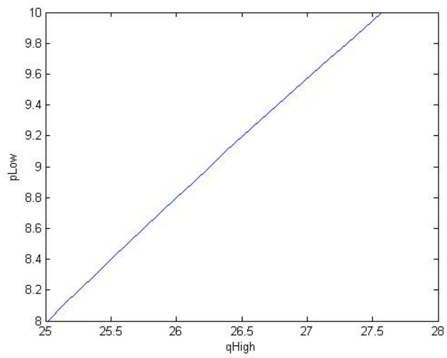

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Figure G.14: Quantity produced in the first period by High Type

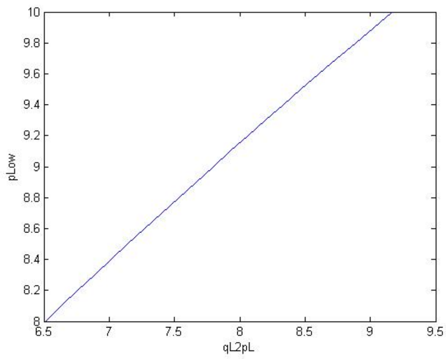

Figure G.15: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

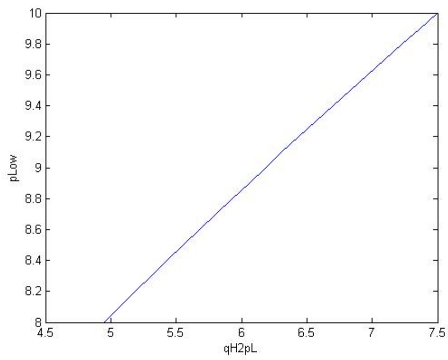

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Figure G.16: Quantity produced in the second period by High Type firm when P_L is observed end of first period

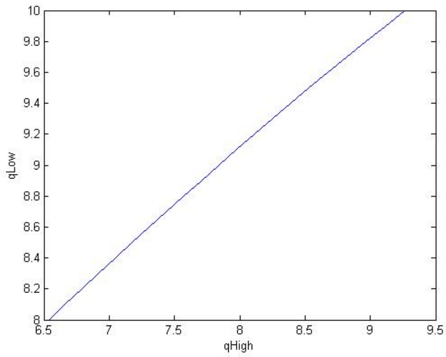

FIGURE G.17: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

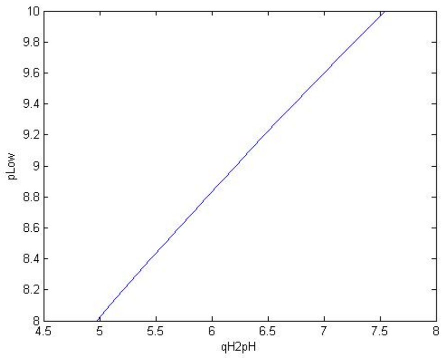

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FIGURE G.18: Quantity produced in the second period by High Type firm when P_H is observed end of first period

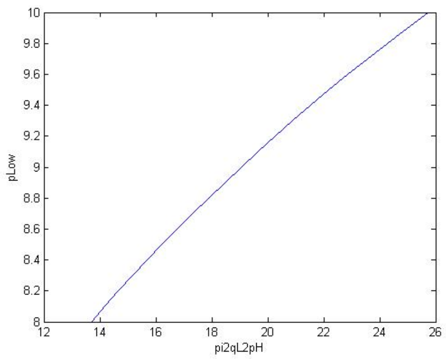

FIGURE G.19: II Period profits for Low Type firm when P_H is observed end of first period

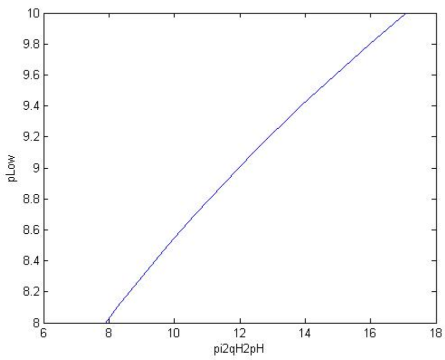

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FIGURE G.20: II Period profits for High Type firm when P_H is observed end of first period

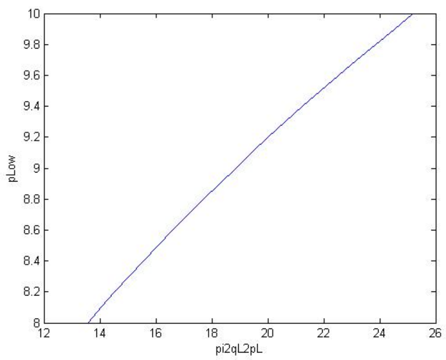

FIGURE G.21: II Period profits for Low Type firm when P_L is observed end of first period

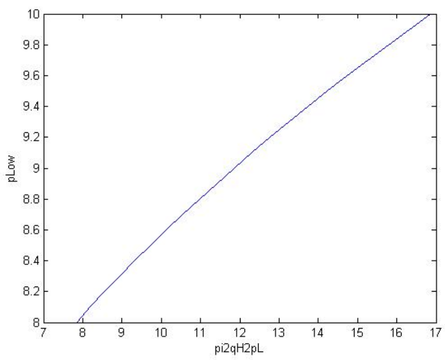

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FIGURE G.22: II Period profits for High Type firm when P_L is observed end of first period

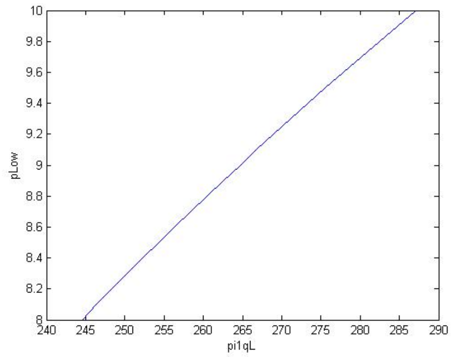

FIGURE G.23: I Period profits for Low Type firm

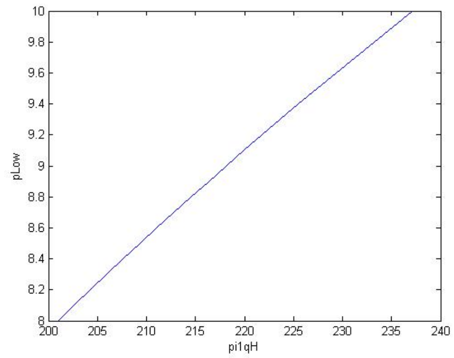

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FIGURE G.24: I Period profits for High Type firm

G.3 Graphs when cost of high type firm increases from 3 to 5

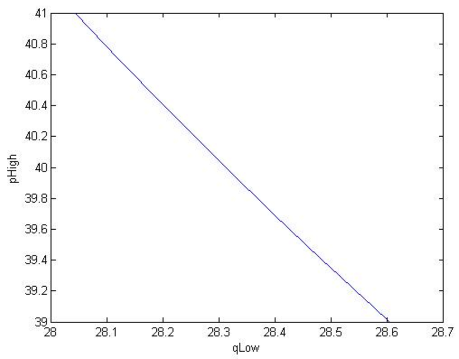

FIGURE G.25: Quantity produced in the first period by Low Type

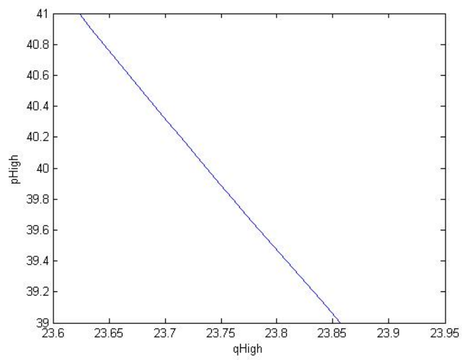

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FIGURE G.26: Quantity produced in the first period by High Type

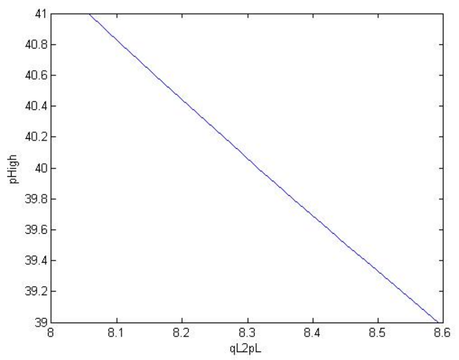

FIGURE G.27: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

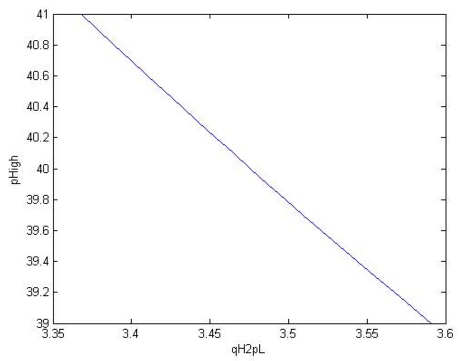

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FIGURE G.28: Quantity produced in the second period by High Type firm when P_L is observed end of first period

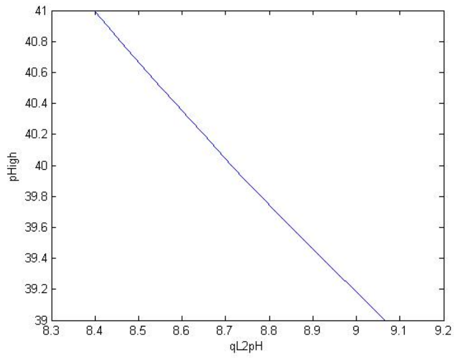

Figure G.29: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

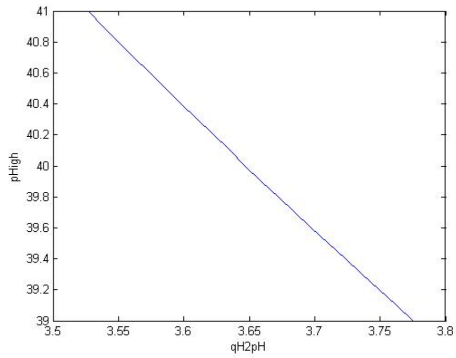

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Figure G.30: Quantity produced in the second period by High Type firm when P_H is observed end of first period

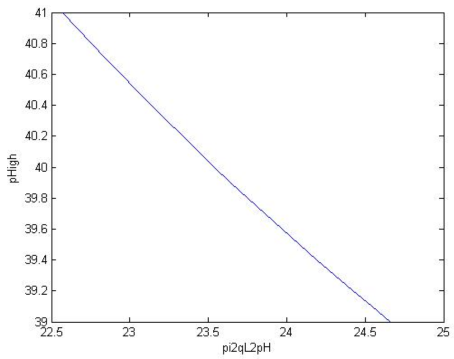

Figure G.31: II Period profits for Low Type firm when P_H is observed end of first period

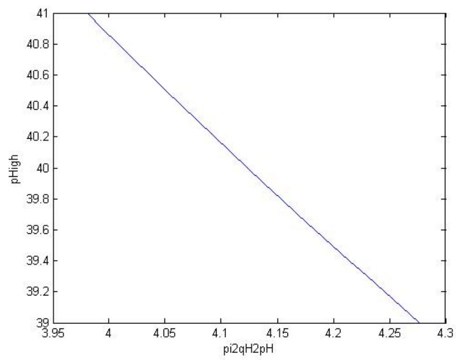

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Figure G.32: II Period profits for High Type firm when P_H is observed end of first period

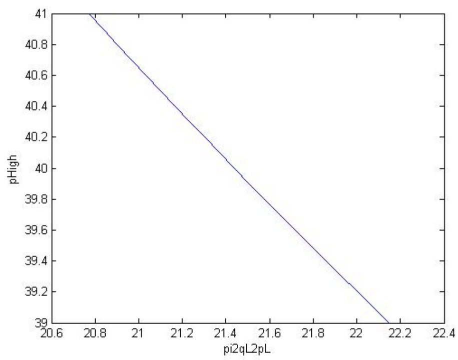

Figure G.33: II Period profits for Low Type firm when P_L is observed end of first period

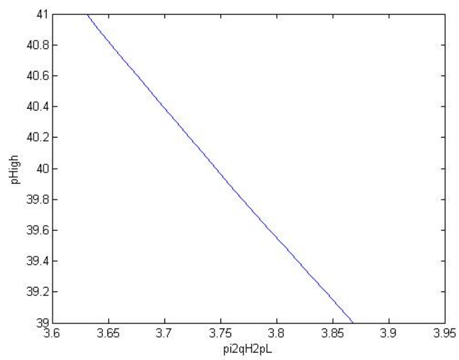

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Figure G.34: II Period profits for High Type firm when P_L is observed end of first period

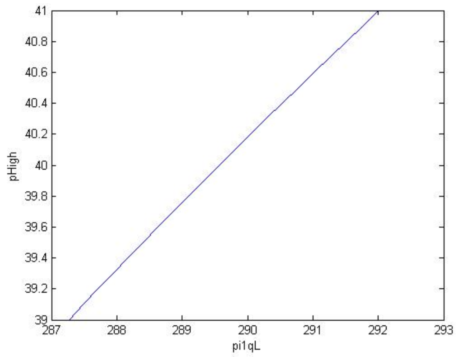

FIGURE G.35: I Period profits for Low Type firm

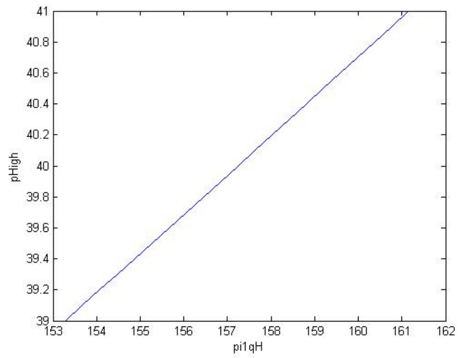

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FIGURE G.36: I Period profits for High Type firm

G.4 Graphs when parameter 'a' changes from 0.6 to 0.8

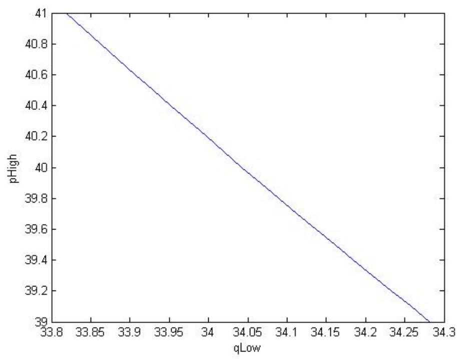

FIGURE G.37: Quantity produced in the first period by Low Type

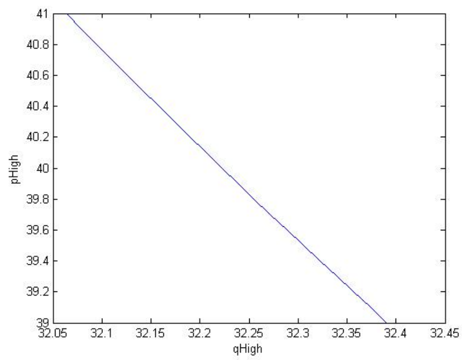

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FIGURE G.38: Quantity produced in the first period by High Type

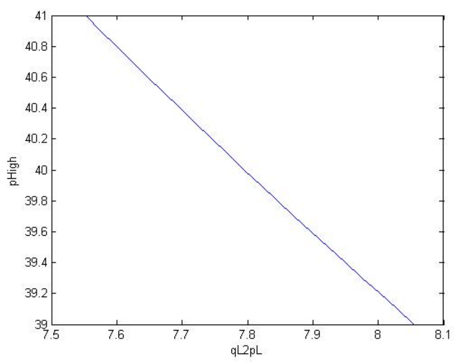

FIGURE G.39: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

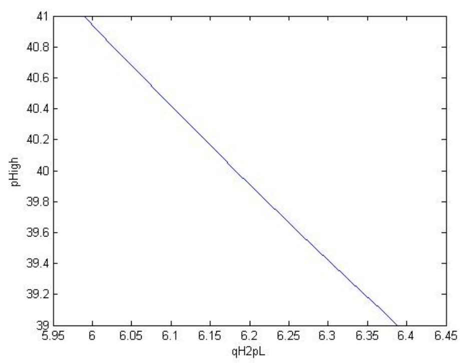

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FIGURE G.40: Quantity produced in the second period by High Type firm when P_L is observed end of first period

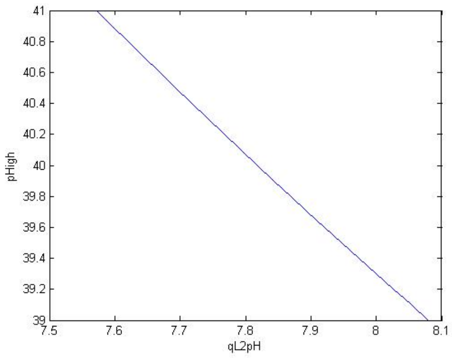

FIGURE G.41: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

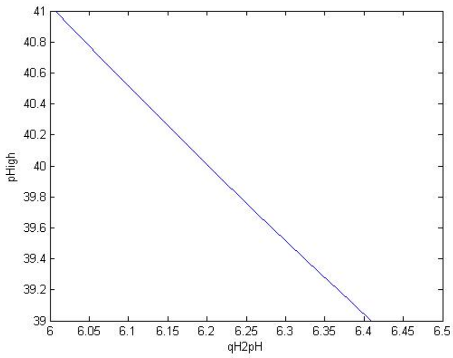

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FIGURE G.42: Quantity produced in the second period by High Type firm when P_H is observed end of first period

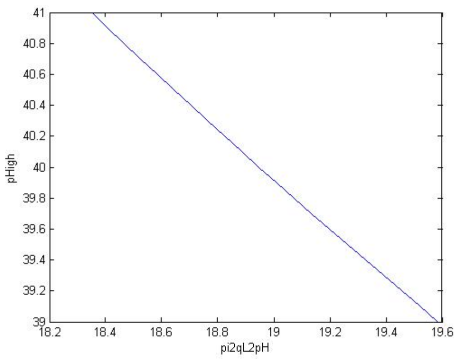

FIGURE G.43: II Period profits for Low Type firm when P_H is observed end of first period

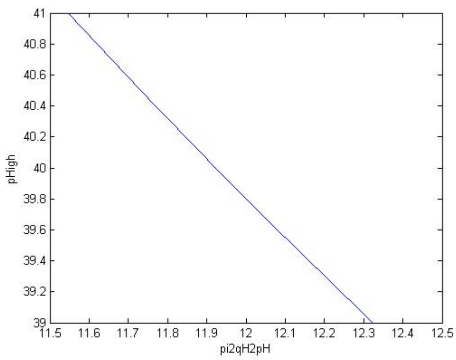

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FIGURE G.44: II Period profits for High Type firm when P_H is observed end of first period

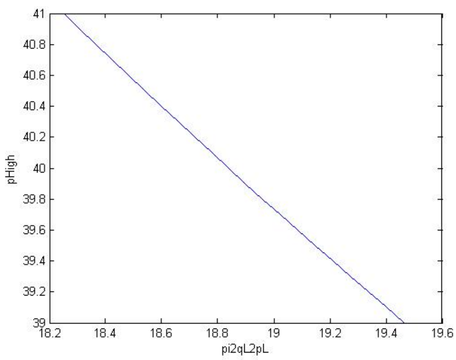

FIGURE G.45: II Period profits for Low Type firm when P_L is observed end of first period

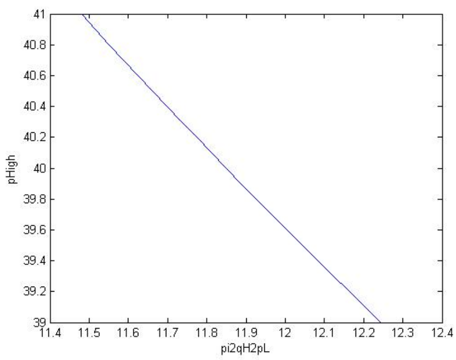

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FIGURE G.46: II Period profits for High Type firm when P_L is observed end of first period

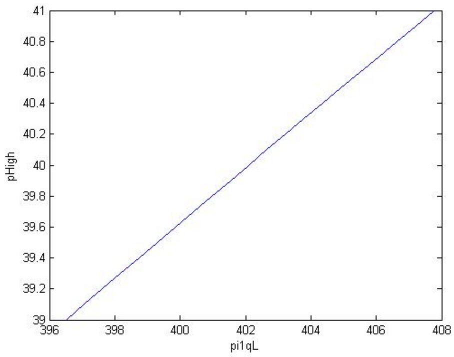

FIGURE G.47: I Period profits for Low Type firm

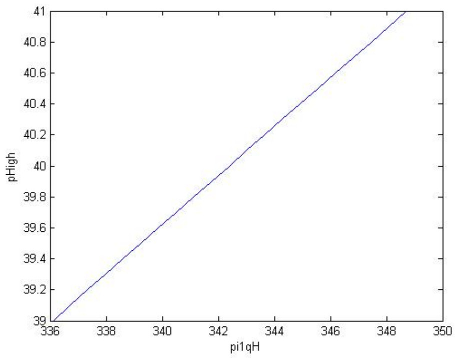

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FIGURE G.48: I Period profits for High Type firm

G.5 Graphs when parameter 'b' changes from 0.01 to 0.05

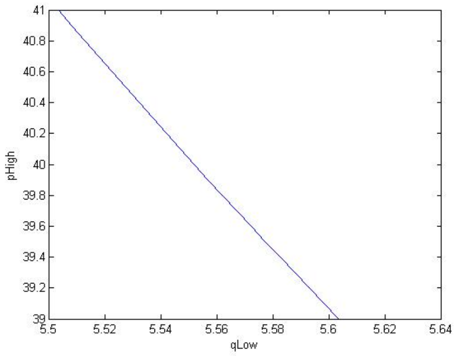

FIGURE G.49: Quantity produced in the first period by Low Type

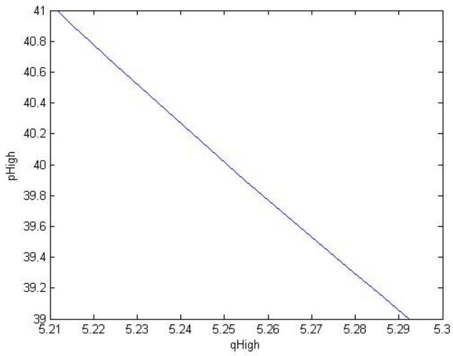

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FIGURE G.50: Quantity produced in the first period by High Type

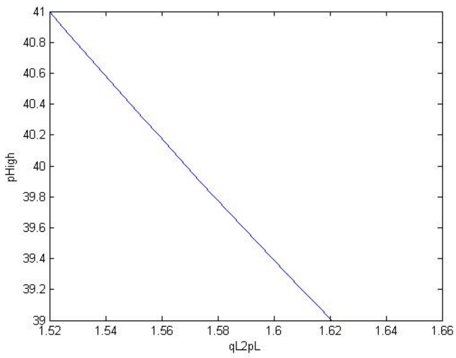

FIGURE G.51: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

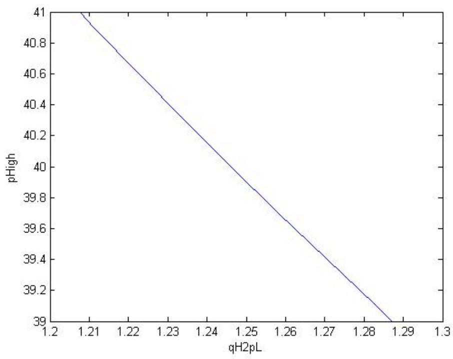

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FIGURE G.52: Quantity produced in the second period by High Type firm when P_L is observed end of first period

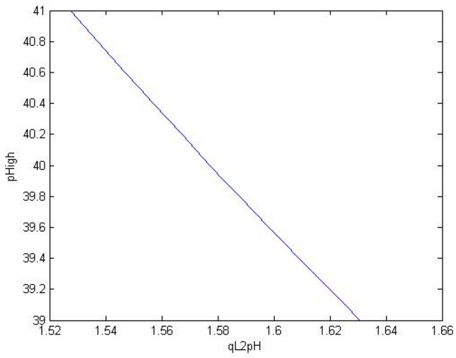

FIGURE G.53: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

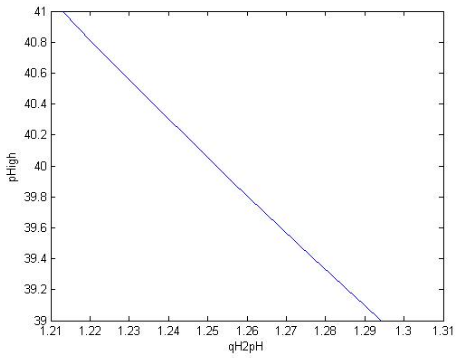

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FIGURE G.54: Quantity produced in the second period by High Type firm when P_H is observed end of first period

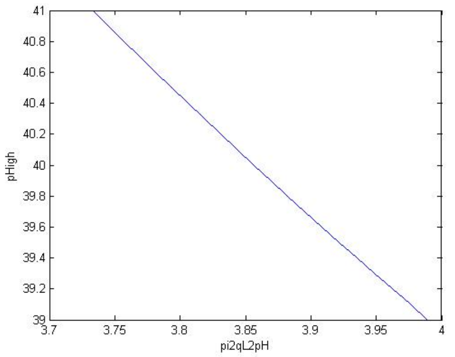

FIGURE G.55: II Period profits for Low Type firm when P_H is observed end of first period

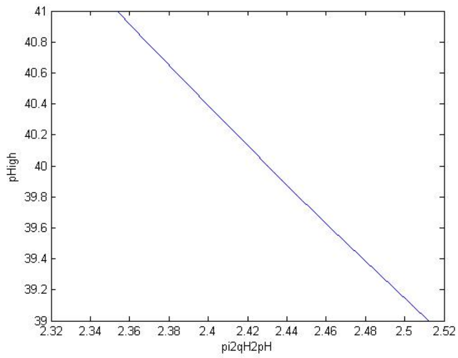

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FIGURE G.56: II Period profits for High Type firm when P_H is observed end of first period

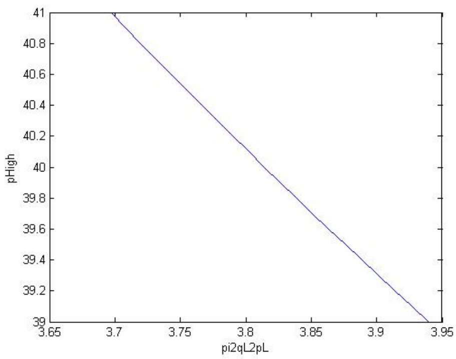

FIGURE G.57: II Period profits for Low Type firm when P_L is observed end of first period

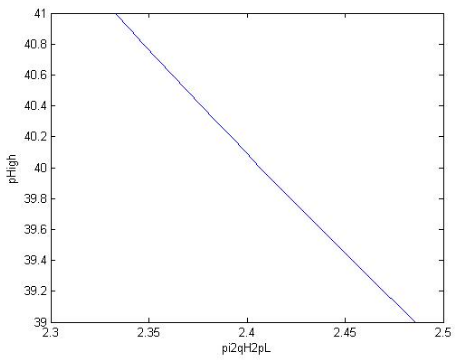

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FIGURE G.58: II Period profits for High Type firm when P_L is observed end of first period

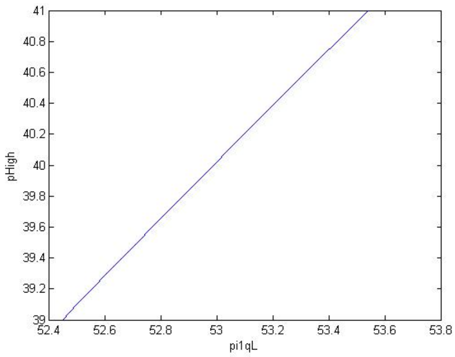

FIGURE G.59: I Period profits for Low Type firm

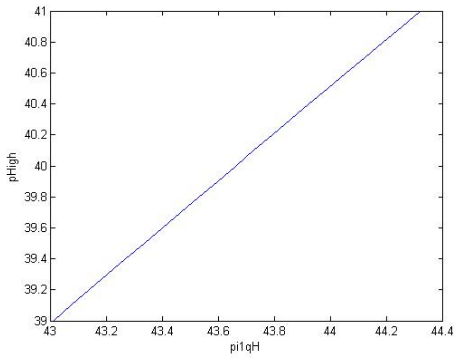

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FIGURE G.60: I Period profits for High Type firm

G.6 Graphs when parameter P_L changes from 9 to 10

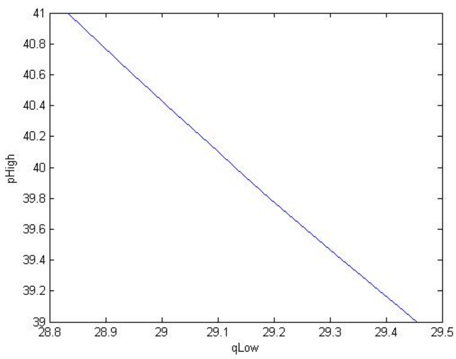

FIGURE G.61: Quantity produced in the first period by Low Type

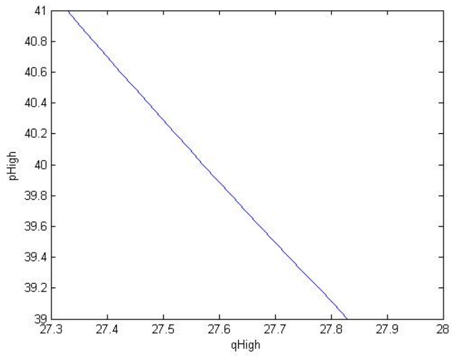

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FIGURE G.62: Quantity produced in the first period by High Type

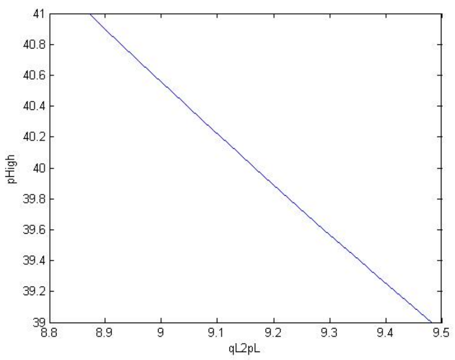

FIGURE G.63: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

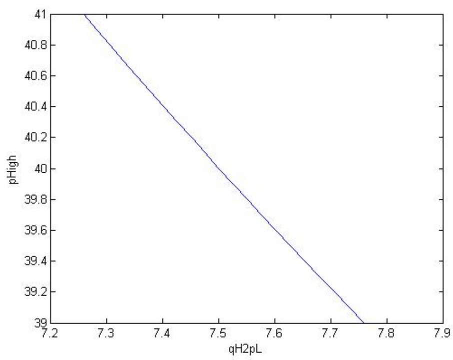

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FIGURE G.64: Quantity produced in the second period by High Type firm when P_L is observed end of first period

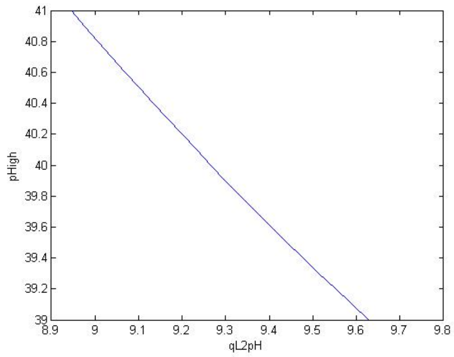

FIGURE G.65: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

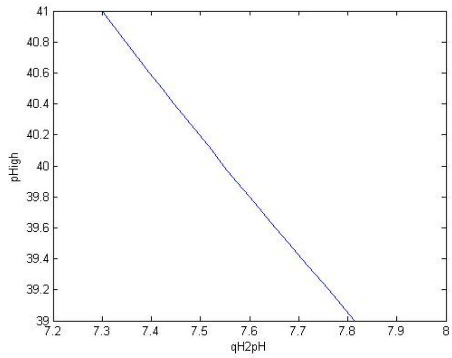

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FIGURE G.66: Quantity produced in the second period by High Type firm when P_H is observed end of first period

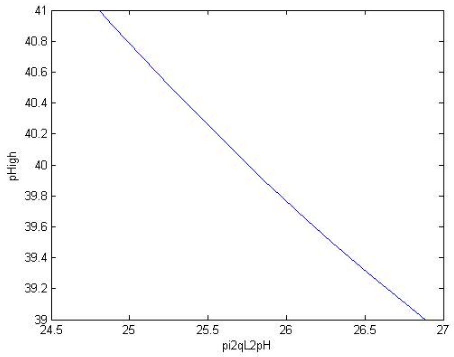

FIGURE G.67: II Period profits for Low Type firm when P_H is observed end of first period

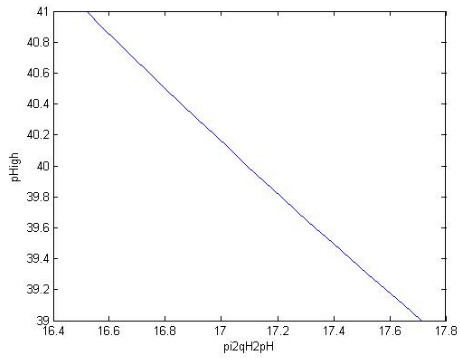

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FIGURE G.68: II Period profits for High Type firm when P_H is observed end of first period

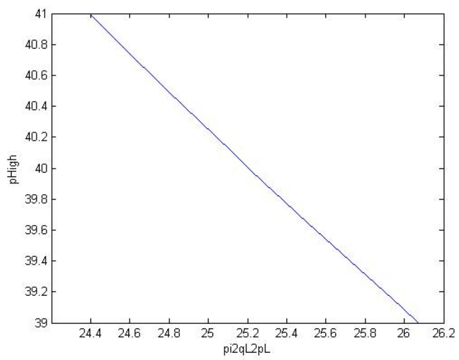

FIGURE G.69: II Period profits for Low Type firm when P_L is observed end of first period

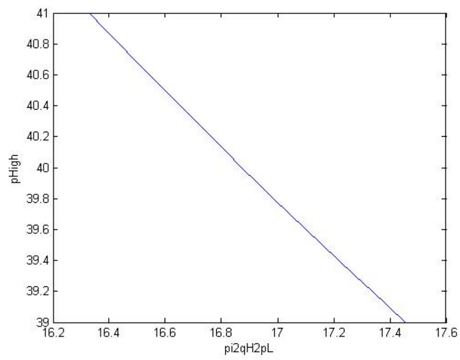

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FIGURE G.70: II Period profits for High Type firm when P_L is observed end of first period

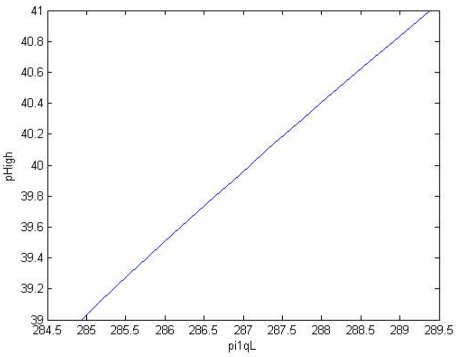

FIGURE G.71: I Period profits for Low Type firm

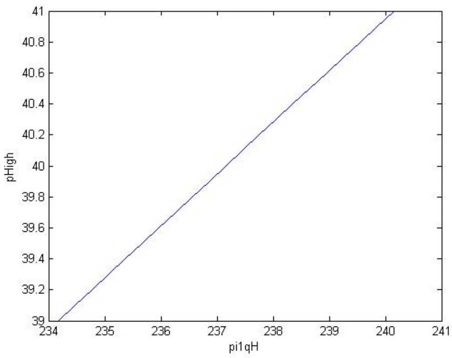

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FIGURE G.72: I Period profits for High Type firm

G.7 Graphs when parameter P_H changes from 40 to 41

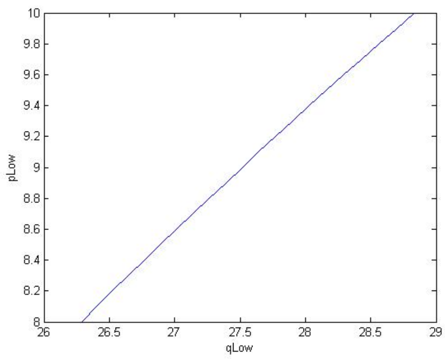

FIGURE G.73: Quantity produced in the first period by Low Type

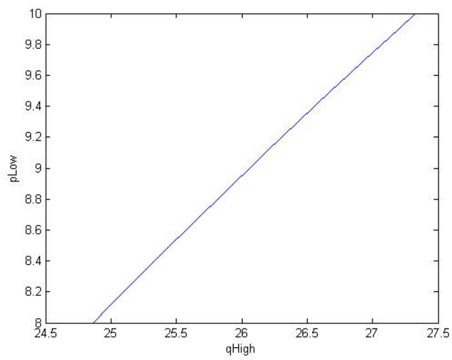

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FIGURE G.74: Quantity produced in the first period by High Type

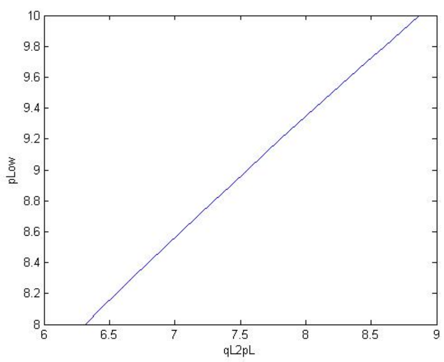

FIGURE G.75: Quantity produced in the second period by Low Type firm when P_L is observed end of first period.

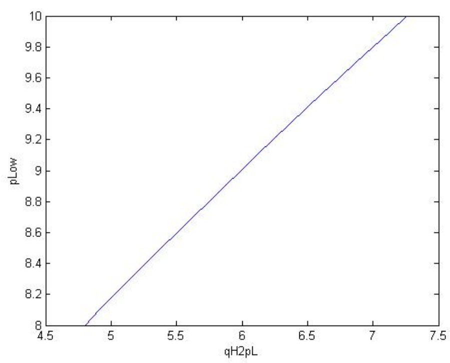

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FIGURE G.76: Quantity produced in the second period by High Type firm when P_L is observed end of first period

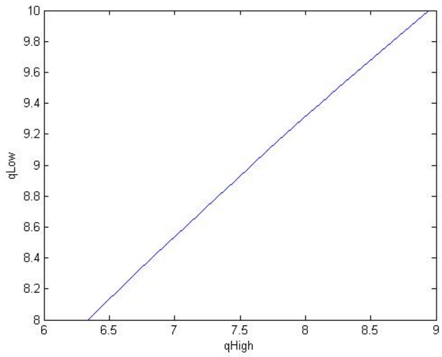

FIGURE G.77: Quantity produced in the second period by Low Type firm when P_H is observed end of first period.

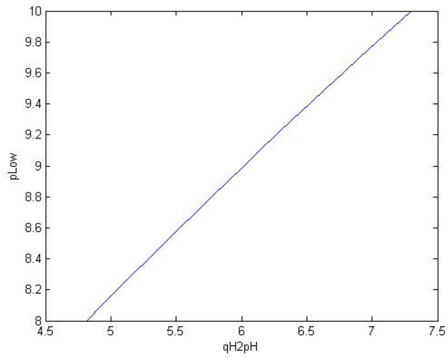

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FIGURE G.78: Quantity produced in the second period by High Type firm when P_H is observed end of first period

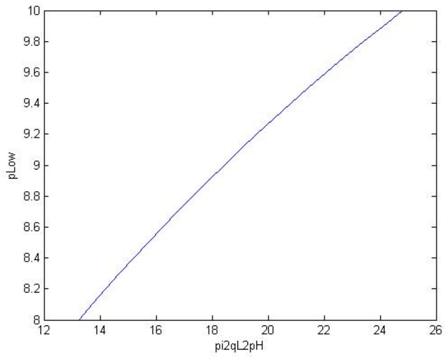

FIGURE G.79: II Period profits for Low Type firm when P_H is observed end of first period

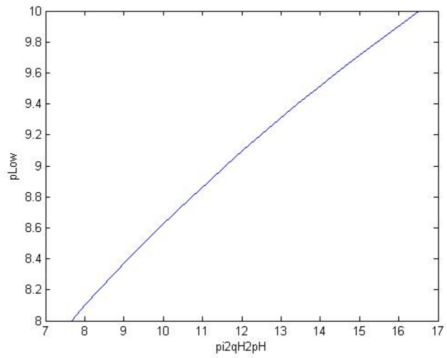

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FIGURE G.80: II Period profits for High Type firm when P_H is observed end of first period

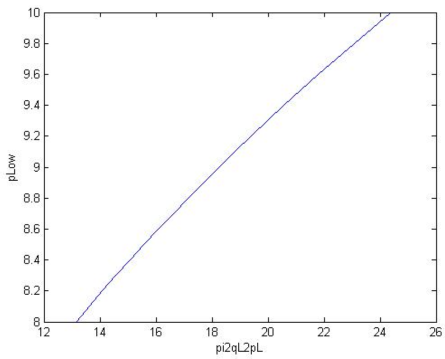

FIGURE G.81: II Period profits for Low Type firm when P_L is observed end of first period

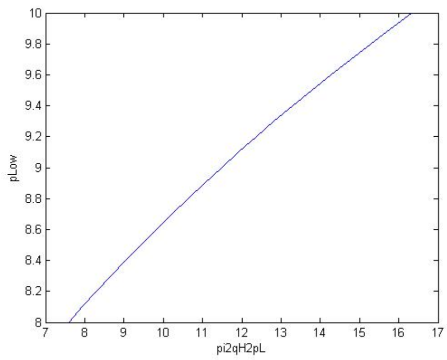

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FIGURE G.82: II Period profits for High Type firm when P_L is observed end of first period

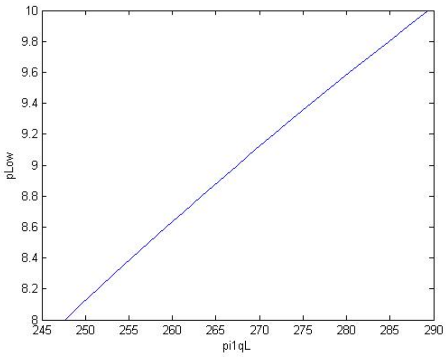

FIGURE G.83: I Period profits for Low Type firm

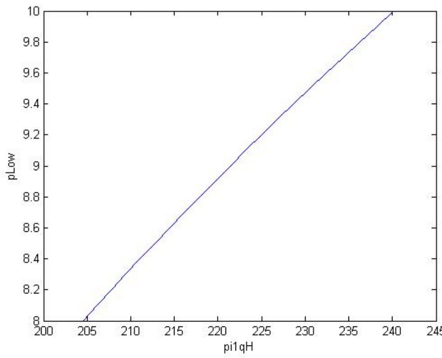

FIGURE G.84: I Period profits for High Type firm

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H. Pooling Equilibrium

Proposition: Pooling equilibrium doesn't exist in period one.
Proof (by contradiction): If pooling equilibrium exist, then $q_L^1 = q_H^1 = q^1$

$$\Rightarrow \rho^2_{q_L^1} = \rho^2_{q_H^1} = \rho^2 = \rho^1$$

$$\Rightarrow q_L^2 - q_H^2 = \frac{c_L - c_H}{2b(P_H - P_L)}$$

After P_H^1 & P_L^1, by pooling,

$$q_L^{2*} = \frac{a}{3b} + \frac{P_L - c_L}{3b(P_H - P_L)} - \frac{c_L - c_H}{6b(P_H - P_L)} \cdot \rho^1$$

$$q_H^{2*} = \frac{a}{3b} + \frac{2P_L - 3c_L + c_H}{6b(P_H - P_L)} - \frac{c_L - c_H}{6b(P_H - P_L)} \cdot \rho^1$$

If first period price observed is P_L^1, profits:

$$\Rightarrow \pi^2_{L\ (q_L^1;P_L^1)} = \pi^2_{H\ (q_H^1;P_L^1)} = b(P_H - P_L)(q_H^{2*};P_L^1)^2 = b(P_H - P_L)(q_L^{2*};P_L^1)^2$$

If first period price observed is P_H^1, profits:

$$\Rightarrow \pi^2_{L\ (q_L^1;P_H^1)} = \pi^2_{H\ (q_H^1;P_H^1)} = b(P_H - P_L)(q_H^{2*};P_H^1)^2 = b(P_H - P_L)(q_L^{2*};P_H^1)^2$$

Expected profits of period two when solving period one maximization:

$$\sigma^1 \cdot \left(\pi^2_{\theta\ (q_\theta^1;P_H^1)} - \pi^2_{\theta\ (q_\theta^1;P_L^1)}\right) + \pi^2_{\theta\ (q_\theta^1;P_L^1)} = b(P_H - P_L)(q_\theta^{2*})^2$$

FOC in period one wrt q_θ^1:

$$\Rightarrow -b(P_H - P_L)q_\theta^1 + (a - bq_\theta^1 - b\mathbb{E}[q_{rival}^1])(P_H - P_L) + (P_L - c_\theta) = 0$$

Hence unless $c_H = c_L$ we will have $q_L^1 \neq q_H^1$
No Pooling Equilibrium. Q.E.D