General equilibrium models for assessing parental leave policies in a Canadian context

By

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An essay submitted to the Department of Economics

in partial fulfillment of the requirements for

the degree of Master of Arts

Queen's University

Kingston, Ontario, Canada

July 2014

Acknowledgements

I would like to acknowledge and extend my gratitude to all those that made this essay possible:

Dr. Chris Ferrall, for his valuable insight and advice, and especially for encouragement that was crucial to the continuation of this project,

To the members of the Queen's Economics Department, who could not have provided a better environment,

And lastly, to the many economists with whom I have no association, but who provided inspiration for this work.

Abstract

This paper provides an overview of general equilibrium models of marriage, divorce, labour and child care, with a focus on their potential for analyzing parental leave mandates in a Canadian context. Relevant literature on both parental leave and general equilibrium models are provided. Critiques of current models and suggestions for future models are given. Lastly, a benchmark general equilibrium model is laid out and some numerical results are presented. These first results illustrate some of the limitations of the model presented, provide insight as to how to manipulate parameters to match certain features of the Canadian economy, and gives clues as to which adjustments should be made in future work.

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1 Introduction

The issue of how policy can help to reconcile family and work is an issue of increasing importance to policy-making around the world, primarily due to the increase in female labour force participation. Particularly in the 1980s and 1990s, labour force growth was substantially higher for women than for men in nearly every part of the world, with women remaining in the labour force for longer than before, even throughout their childbearing years[1]. While it is difficult to say if this is generally by choice, a survey conducted in the European Union revealed than only 1 in 10 couples prefer a family structure with the male as the only income earner (Jumont, 2003). Bearing this in mind, it appears that any previous debate over whether women should participate in the labour force is over. Instead, research must take female labour force participation as given and focus on mitigating any negative impacts of this participation.

Despite the increasing need for policies that ease any negative impacts of recent changes in family structure, there is little consensus over the impact of parental leave and other such family friendly policies[2]. Arguments for parental leave may involve children, and the fear that they are harmed by parental absence in their first months. Other arguments are based on equality, as women may be disadvantaged in the labour market if they bear more of the burden of raising children. Others still may be purely economic, citing the efficiency losses of forcing parents to leave the work force for long periods of time. Still, policies can be criticized over cost and intention. For example, some might believe that women should not be encouraged to return to work because parental care is preferable to other care arrangements, such as daycare.

[1] Lim (2002) states that Africa is the only region where this has not occurred. The OECD Economics Department (2004) provides detailed statistics, but only for OECD countries.
[2] Other policies may encourage paid childcare, increase flexibility of hours, or provide tax relief.

Clearly, one of the difficulties in assessing family friendly policies is that so many outcomes need to be considered. It will be shown in the literature review that several papers have attempted to assess aspects of various family friendly policies through methods relying little on economic theory. The results of such studies are sometimes in conflict. Still, these studies have contributed a great deal of evidence in regards to the effects of parental leave policies; they do not however place much emphasis on the process by which the effects occur. This can make policy recommendations difficult. For example, suppose that a region implements a parental leave policy with the intended effect of increasing female employment and closing an observed gender wage gap, only to find a few years later that the policy has had the opposite effect. Empirical analyses have limited value in this case because there is no regional data on the effects of varying the eligibility criteria, duration or benefit level of the parental leave. In this case, a model that is more theoretical in nature can complement empirical work by showing how the undesired effect is created. In short, much of the existing literature is better at explaining what happened than it is in explaining why it happened.

This is precisely what motivates a general equilibrium model of marriage and divorce. By incorporating labour markets, marriage markets, and childhood human capital investments, researchers can analyze many of the impacts of family friendly policies. An added benefit to the GE model is that it is well suited for long-term predictions. This paper does not use a GE model to analyze a specific policy; rather, it has two main goals. The first goal is to provide a critique of past models, and offer some possible suggestions for improvement. The second goal is to provide and calibrate a benchmark GE model that if modified, could be used to analyze parental leave policy. For now, the focus is on fitting the benchmark model to Canadian data. The model laid out follows a framework devised by Aiyagari et al. (2000) (hereafter AGG). It should be

known that there are at least two other published studies using models built on this framework, but both of these studies concern US policies and as a result, utilize US data[3]. Unfortunately, the results of these studies may not be generalizable to other countries due to various policy interactions.

Take for example, the problem of using the results of these models to inform Canadian policy. While it would be easiest to assume that the introduction or adjustment of a family friendly policy will have the same effects in any country, differences between Canada and the US may dampen the positive effects found in other studies. Canada, while it has a more generous parental leave scheme than the US, has some of the weakest policy supports for families with young children (Doucet and Merla, 2007; Henderson and White, 2004; Lewis, 1992). In fact, Canada meets the fewest of UNICEF's recommendations for early childcare services of any of the 25 OECD countries ranked (UNICEF, 2008). Policymakers would be justified in wondering if these weak supports would dampen the positive effects of a more generous parental leave scheme.

Policymakers may argue that family friendly policy adjustments have little effect for other reasons. One argument against increasing supports is that Canada already has one of the highest levels of female labour force participation in OECD nations (OECD, 2005), despite the weak policy supports referenced above. This study should illustrate that many outcomes should be considered before making the assumption that policy adjustments are not necessarily. In some countries, it is possible for changes to be made to parental leave without parents in mind. In Canada, parental leave benefits are tied

[3] The two known to the author are Greenwood, Guner and Knowles (2003) and Bernal and Fruttero (2008). These are discussed in more detail further in this document. Erosa et al. (2010) offer a similar model, but without childhood investment.

into the employment insurance system, so changes to the overall system can have unintended consequences for parents.

Take for example, parental leave changes brought on by the Government of Canada's Working While on Claim (WWC) Pilot Project[4]. The project came into effect in August 2012 and will continue through July 2015. It does not affect the duration, eligibility criteria or even the initial benefit calculations; this means that benefits continue to be calculated as 55% of the weekly average of insurable earnings over the past year, and the requirement that individuals must have worked at least 600 insurable hours within the 52 weeks is still in place[5]. Mothers are still entitled to 17 weeks of maternity leave, and parents have 35 additional weeks of parental leave to share between them. What the project does affect is the manner by which individuals are penalized for receiving earnings while on leave. By extension, it affects the number of parents who will access parental leave and the extent to which they will claim benefits.

Looking at the details of the project makes it clear why this is. Under the previous pilot project, those claiming parental leave could earn 25% of their weekly benefit or $50 (whichever was higher) without penalty, but earnings above that level would be deducted on a dollar-for-dollar basis. Under the new benefit scheme, benefits are reduced at a rate of 50% for each dollar earned, up until 90% of the weekly earnings used to calculate the claim. At this point, benefits are deducted dollar-per-dollar. It seems that the intention of the new system is to make it so that the unemployed are not harmed for accepting a job where the earnings constitute a significant portion of previous earnings. In the case of parental leave however, the claimants are not deciding whether to accept a job; rather, parents decide if they should reduce hours and to which extent.

[4] Service Canada (n.d.a) gives the details of the program
[5] Service Canada (n.d.b) outlines basic employment insurance regulations

Given that they do reduce hours, they decide if and when to increase them again. The WWC Pilot Project has two effects. Parents who might have accepted occasional work from an employer while on leave will no longer want to do so because of the penalty. Another set of parents might actually reduce hours less than they would have under the old scheme, because there is no longer a dollar-for-dollar reduction cut off. If policymakers insist on having those claiming parental leave benefits follow the same rules as those claiming general employment insurance benefits, they should be aware that changes to the employment insurance program affect the uptake of parental leave. If they are willing to make changes that affect the uptake of parental leave, they should be aware of the long-term effects of parental leave, since policies that change access essentially diminish or amplify these effects.

To clarify, it is uncertain how effective a GE model would be at analyzing a change as precise as that made by the WWC Pilot Project. These models are intended as a complement, and not a replacement, for existing methods. One issue with modelling this change in a GE framework would be that in practice, parents cannot perfectly adjust hours. Researchers using a reduced form approach have some freedom here; they may ignore this fact, offer a discussion about how it may change the results, or incorporate it into a sensitivity analysis in some way. Researchers using a general equilibrium are forced to make a modelling decision, whether they explicitly discuss it or not. Should agents be allowed to perfectly adjust hours? If not, how is this rigidity represented in the model?

Since rigidity of hours is just one of the many design decisions that needs to be made with this type of model, the literature review that follows addresses some of those decisions. Restrictions on family structure, labour market outcomes and childhood investment are discussed, as well as the plausibility of these restrictions. While it would

be ideal to discard any restrictions that are deemed unrealistic, tractability of the model demands otherwise. Thus the review also offers some discussion of which compromises should be made, which compromises have been made in existing papers, and the possible consequences of these compromises.

2 Related literature

Before delving into model design, it is appropriate to discuss some of the results from studies of parental leave in general. While it might be ideal not to have any bias, knowledge of past results is helpful when deciding which compromises to make. For example, a researcher that makes fertility exogenous for simplicity may argue that an increase in fertility has not been observed previously, and is not a concern of policymakers. In fact, this may have been an observation made by Aiyagari et al. (2000), the designers of the first published GE model of marriage, labour and child investment; fertility is exogenous in this model. Since neither this paper nor its expansion (Greenwood et al, 2003) (hereafter GGK) discuss parental leave, there is limited overlap between this literature review and the literature reviews contained in those papers. Another expansion by Bernal and Fruttero (2008), does focus on parental leave; this literature review seeks to expand upon the brief one provided in that paper, particularly by providing more Canadian sources.

2.1 Parental leave policies

The Canadian maternity leave benefit (MLB) was introduced in 1971 and was adjusted twice that decade, and adjusted three times in the three decades following that[6]. The maximum MLB was for 15 weeks and remained that way until the year 2000. A major

[6] Dykeman and Williams (2014) provide a thorough analysis of the motivation for and circumstances of various changes to Canadian parental leave laws. Mahon and Brennan (2013) discuss how the institutional configuration of Canada has the development of parental leave.

adjustment came in 1983 when adoptive mothers were included in the benefit. In 1989, a paternity leave benefit was introduced so that fathers could take 10 weeks on top of the 15 weeks the mother was allowed. In 2000, the structure was changed so that 15 weeks were entitled to the mother, and an additional 35 weeks could be allocated between the two parents.

By far, information about parental leave policies and labour market outcomes was the easiest to obtain out of the three areas of concern. This is likely influenced by the fact that data on labour market outcomes is easier to obtain than that on childhood investment. Information on the effect of parental leave on fertility was by far the sparsest, even though fertility rates should be fairly easy to obtain. This may indicate a lack of interest in the topic relative to the other two areas. One possible reason for this is that limiting fertility rates is not a major concern in most OECD countries. Policymakers may view augmenting labour market outcomes as a priority, and view any increases in the fertility rate as a desirable side effect, given that so many OECD countries currently have fertility rates well below the replacement rate[7]. In all three categories, the vast majority of studies use basic statistics or reduced form analysis to make arguments. Of these studies, most use micro data, with the exception of Ruhm (1998) and Thévenon and Solaz (2013). Many of the studies focus on women's outcomes even if the country of study is one in which both parents can take leave, primarily because male participation in parental leave programs has traditionally been low in these countries (Marshall, 2008).

Even though Canada already has a fairly generous maternity and parental leave policy, it is still useful to look at studies that analyze either the introduction of parental

[7] In 2009, only four OECD nations had fertility rates above the replacement rate (OECD, 2009).

leave or an increase in the duration of parental leave benefits. As previously mentioned, these studies inform modelling decisions. Additionally, these studies show that though paid leave has been present in Canada and other countries for quite a while, its effects are still not clear in some areas. It is important to understand the effects of paid parental leave so that policy decisions that encourage or discourage leave taking can be made in good conscience.

2.1.1 Effects on labour market outcomes

It cannot be assumed that countries where women have the greatest ability to reconcile work and family life are the same countries where women have the highest labour participation rates. OECD (2005) for example notes that despite having the some of the strongest parental supports in any of the OECD nations, Finland maintains a relatively low maternal employment rate, and among mothers who do work, there is a high incidence of temporary employment. The report states that this is in part due to cultural expectations about women as primary caregivers. It also argues that a large factor in whether women will return to work after leave is the availability of part-time work, postulating that the relative commonness of part-time positions in Canada is one of the major factors behind its high maternal employment rate. The implication of this is that parental leave policies with the same specifications do not have the same effects in different countries.

Another issue with assuming that more generous supports result in favourable labour market outcomes is that there is some evidence that long leaves do the opposite. While Ruhm (1998) finds that availability of parental leave increased female labour market participation in nine OECD countries, there is a great deal of evidence that long leaves have a negative effect on career mobility and future earnings (Edin and Gustavsson, 2008; Ruhm, 1998; OECD, 2005). This effect is especially great when a

woman is not guaranteed to be able to return to her previous job. In this case, women who are low-skilled are the least likely to be able to return to the labour market after leave (Oudrich et al., 1998). Some studies measure the length of the wage penalty endured when returning from paid parental leave. One in Germany (Schönberg and Ludsteck, 2007) and one in France (Lequien, 2012) find penalties that last eight and six years, respectively, while a study from Austria finds no wage penalty at all (Lalive et al., 2011).

Though several US studies suggest that the introduction of parental leave can be associated with increases in employment and wages, Ruhm (1998) notes that studies including leave voluntarily provided by employers are biased, due to common traits between the types of companies that voluntarily provide leave. He also notes that studies based on state mandates have shown mixed results, but does show that parental leave mandates were associated with increases in employment in nine European countries. Thévenon and Solaz (2013) analyze paid parental leave mandates across 30 OECD countries and find that extensions of paid leave lengths have a small positive effect on female employment rates as long as the period of leave is no longer than two years, but the presence of any paid leave widens the gender earnings gap. Overall, researchers agree that female employment rates are improved, but find conflicting information on wage penalties from leave, both paid and unpaid.

Studies focussing specifically on Canada are no different. Baker and Milligan (2008) examine the introduction and expansion of the maternity leave benefit and find that short entitlements (17-18 weeks), do not change behaviour, while longer leaves do encourage job continuity with the pre-birth (or pre-adoption) employer. They avoid the selection bias issue by limiting their study to mandated leave, rather than leave that is provided by voluntarily by employers. As for the result that mothers suffer a wage gap,

O'Connell (1990) uses National Longitudinal Study of Children and Youth (NLSCY) data to find that in Canada, changing jobs after a leave results in a greater salary profile decrease than can be accounted for by lost experience. On the other hand, returning to the same employer involves only a minor penalty. Phipps et al. (2001) find the same conclusion with General Social Survey (GSS) data. Another Canadian study argues that timing of the leave makes a difference; skilled Canadian women who take leave later in their careers do not receive the same penalty as women who take leave shortly after completing their education or training (Drolet, 2002). As for the result that the wage penalty disappears over time, Zhang (2010) finds that Canadian mothers usually regain lost earnings within seven years of giving birth, consistent with the results in France and Germany, but inconsistent with US studies, which often show permanent earnings penalties.

There are several theories about why lost experience alone does not account for the reduction in earnings. One is that women who wish to have children seek family friendly jobs[8], even if it means a reduction in wages. Some believe that women who are not advancing in their careers are more likely to decide to become mothers (the endogenous fertility theory), but neither Phipps et al. nor Zhang find any evidence to support this conclusion. Ultimately, both studies are unable to completely explain the severity of the penalty incurred when a woman does not return to her previous employer.

While AGG does not look at parental leave, the one expansion that does (Bernal and Fruttero, 2008) assesses the effects of introducing paid leave for mothers and fathers. The authors find that paid leave increases the expected income of women in the model relative to a benchmark model with no leave, but the distribution of income in

[8] Formally, these jobs could have more flexible hours, on-site day care arrangements, or breast feeding stations. Informally, less competitive work places may just have a more accepting culture.

men is less favourable. This is in part because divorced and single men pay taxes but do not receive utility from time with children, so this result would not be as strong without the assumption that divorced men do not receive utility from time with children. Erosa et al. (2010) find a similar result using a GE model that does not explicitly model childhood investments. The primary effect in this case is that men face more competition from women in the labour market, but the leave policies do not benefit them.

2.1.2 Effects on children

Since the most consistent feature of parental leave policies is the increase in maternal employment, most literature in this area focusses on two categories of effects: the effects of a mother being separated from children for employment, and the effects of the extra income from said employment. More than the other two areas, much of the relevant literature is found in psychology and sociology journals. Bernal and Keane (2007) provide a thorough summary of psychology literature examining maternal employment and cognitive outcomes, only to find that a third of papers report positive effects, a third report negative effects, and a third report insignificant or mixed effects. The Canadian contribution to this debate shows that even though a recent leave expansion induced women to spend between 48 and 58 percent more time not working in the first year of their child's life, changes in cognitive outcomes were close to zero (Baker and Milligan, 2010). Most puzzlingly, improvements in temperament were observed amongst all children, and not just the children of the mothers who took more time off work. The major drawback of this study is that outcomes were only observed for children up until the age of two years old.

Variation of results within countries can be explained by differences in empirical methods, while some of the variation between countries may also be caused by the quality and availability of non-parental care. Various studies have suggested that formal

care, such as daycare, is associated with the greatest increases in problem behaviour, while others have shown that high-quality child care can in fact improve cognitive and behavioural outcomes[9]. In 2012, there were only enough regulated, licensed, funded child care spaces to accommodate 22.5 percent of children from the ages of 0-5 years, up from 14.9 percent in 2001 (Canadian Child Care Federation, 2013). This means that traditionally Canadian families have relied heavily on alternate arrangements, such as the care of relatives, to meet their child care needs. It might be the case that moving a child from non-parental relative care to parental care does not have much of an effect.

The results regarding paid leaves and the physical health of children are not mixed. Macroeconomic studies find that across OECD countries, leave durations are negatively correlated with perinatal mortality, neonatal deaths and low birth weight, and strongly negatively correlated with post-neonatal deaths (McGovern et al., 1997; Ruhm, 2000; Tanaka, 2005). Two U.S. studies have found that employer provided paid leave creates the same effects (Berger et al., 2005; Rossin, 2011), though both note that unpaid leave can exacerbate differences in child investments because only wealthier parents can afford to use it. The former study also finds that short maternity leave entitlements (the minimum in America is 12 weeks), can worsen child health outcomes. This is likely because the policy makes it so that women are less likely to become stay-at-home mothers, but the short leave is not enough to have any significant health benefits. Women with short leaves may also avoid using part of the leave before the baby is born. Thankfully, maternity leave entitlements are relatively long in Canada, so this is not an issue. Despite this, Baker and Milligan (2005) find that while an increase in parental

[9] Baker and Milligan (2010) give a review.

leave entitlements was negatively correlated with several child health outcomes, the effects were not as significant as they tend to be in studies based on other countries.

2.1.3 Effects on fertility

As mentioned previously, there was not a significant amount of literature on the effect of parental leave on fertility rates. At least one US study (Averett and Whittington, 2001) finds that access to 12 weeks of unpaid leave increases the probability of higher order births. Since this leave is only guaranteed at firms with greater than 50 employees, selection bias may be a concern. Lalive and Zweimüller (2005) find that the extension of paid parental leave in Austria caused an increase in second order births. They also find that women spaced births closer together after the reform, though it should be noted that benefits can be automatically renewed while the mother is still on leave from the first child. In Canada, this is not the case. Carneiro, Løken and Salvanes (2009) find the same effects in Norway as Lalive and Zweimüller did in Austria, even though parental leave cannot be automatically renewed. Duvander, Lappegård and Andersson (2010) find similar effects in Sweden. Only the Canadian study (Phipps, 2000) finds that the availability of leave benefits does not increase the probability of births of any order. Timing of births was not examined, though the data set is such that exact timing is not known. A Canadian paper that looked at timing of births could not be found.

As for the American general equilibrium papers, Bernal and Fruttero (2008) leave fertility as exogenous, so they cannot make any conclusions in this area. Erosa et al. (2010) allow fertility to vary and find that the fertility effects differ depending on education level. Except for the women in the lowest education group, longer paid leave entitlements are associated with higher fertility. Leave entitlements increase the value of a future value of a job and encourage unemployed women to postpone fertility until they find a match. Since the unemployment rate is highest amongst the least educated

women, leave entitlements have a negative effect on fertility. In other groups, this effect is dominated by the fact that the leave policy subsidizes fertility for workers who are already employed. Amongst the most educated group, longer paid leave entitlements are associated with the largest increases in fertility.

2.2 Designing a model

While the Erosa et al. model is discussed in the previous section, this section focusses on models with the same basic structure as AGG. This means that adults are only fertile for one period, and investment in children is modelled explicitly. The three models are AGG, Greenwood et al. (2003), which allows endogenous fertility, and Bernal and Fruttero (2008), which adds labour market frictions.

The basic structure of these models is as follows. There exists a continuum of agents. Women and men live for two periods as adults and are endowed some level of productivity in the first period of their adult life, which influences their draw from the labour market. Single agents take a draw from the marriage market and decide whether to be married, and whether to invest in any children they may have. It is up to the developer of the model to decide whether the distribution of male and female productivity is the same, whether all agents receive a job offer, and whether agents decide on the number of children, among other things. Children stay children for two periods upon which they receive a level of productivity that is influenced by the amount of parental investment they receive.

The type of question that the researcher can answer is restricted by the small number of periods. In particular, delays in fertility cannot be observed if adults are only fertile for one period, even if fertility is endogenous. Since the literature review did find that maternity and parental leave influence the timing of births, and the Erosa et al. model found the same result by allowing many fertile periods, this is a shortcoming of

the basic structure. It should be noted however that the AGG model is more complex in other ways and explicitly models investment in children, something that would be difficult in the Erosa et al. framework.

2.2.1 Labour market

The labour market aspects of the AGG and GGK models are simple. An agent gets a job offer at the beginning of adult life and is paid his or her productivity as a wage. Each agent can spend as much or as little of their time on that job as they please. Agents do not change jobs, and there is no job loss. Since the literature review showed that changes in earnings profile were most severe when women were not able to return to their previous job after a period of leave, this choice could cause considerable inaccuracy in the results.

The Bernal and Fruttero model complicates the labour market slightly but there is still no job loss. The complication is that individuals face an exogenous probability of receiving a job offer. While all workers facing the same probability of finding a job is likely not realistic, it at least adds some involuntarily unemployment to the model. Exogenous probability is especially unrealistic when an agent rejects their job offer in the first period and still faces the same probability in the second period. Another friction added by Bernal and Fruttero is the requirement that agents must work some minimum number of hours in order to accept the job, so that individuals who would work a little if unrestricted may decline employment. Since the authors do not explain how this affects the results, it cannot be certain if and how much labour market frictions make a difference. The minimum number of hours does give a convenient way to model parental leave, as a leave policy can be seen as a reduction in the number of required hours over the period.

2.2.2 Marriage and divorce

Since there exists a continuum of agents, the marriage market exists in each period. One can think of some of these marriages as being common-law unions; it does not affect the results. Models differ on the restrictions surrounding matches. Allowing divorce seems crucial. No researcher is going to argue that an insignificant number of marriages end. As is pointed out in the AGG paper, divorce is a cause of inequality in America. In Canada in 2003, 38 percent of female-headed lone parent households and 12.6 percent of male-headed lone parent households were below the poverty line, while the poverty rate for two-parent families was only 6.6 percent (Townson, 2005). For this reason, excluding divorce from the model does not allow accurate income distributions to be generated. Appropriately, all three models discussed allow divorce.

Another common assumption between the three models is that individuals match within their generation. While this might seem unreasonable at first, consider the fact that since there are only two periods, the age gap between generations can be thought of as large. While individuals do not always marry someone within a few years of their age, they do not often marry someone significantly older than them. In the 2001 Census data, only 8 percent of Canadians were married to someone more than 10 years older than them (Boyd and Li, 2003). Since adults in these models are typically assumed not to be fertile in the second period, intergenerational marriage creates other issues with modelling. Given the low rate of occurrence of large age gaps, it does not seem worth it to allow intergenerational marriage.

While the assumptions above do not stop the model from matching the statistics, it is not always necessary that an assumption has this feature. All of the models make the assumption of no remarriages, while statistically, this is far from the truth. Canadian research has shown that while a relatively low proportion of Canadians actually remarry

after divorce, 52.5 percent of women and 69.6 percent of men are either married or living common law within 10 years (The Vanier Institute of the Family, 2010). Accurately reflecting this is complicated. Even though the rate at which remarriages occur is known, it is uncertain how step-parents should derive utility from investment in the children in their household. While AGG and GGK allow never married parents to become married in the second period, they keep this simple by assuming children always live with their mother, and that males derive no utility from investment in children. Two observations fly in the face of this assumption. First, in 2006, Canadian fathers spent two-thirds the amount of time directly caring for their children as mothers did (Ball and Daly, 2012). This is certainly not an insignificant amount of time. Second, paternal time investments made by non-resident fathers are often quite significant[10], the effect of which is discussed in greater detail later in this paper. The important thing to note now is that since making realistic assumptions on child investments and marriage complicate the model, researchers face trade-offs when deciding which assumptions to implement.

Similarly, Bernal and Fruttero allow fewer restrictions on child investments but have more restrictions on marriage. In their model, married men do make time investments in children while divorced men pay child support. A simplifying assumption is that if an agent rejects their marriage offer in the first period, they stay single forever. This eliminates step-parents from the model, meaning that there does not need to be any consideration as to how the presence of a step-parent affects children. Since families with stepchildren represent around 13 percent of couples with children in Canada, and the number is rising (Vezina, 2012), this is another assumption that prevents the data from matching the statistics.

[10] King and Sobolewski (2006) discuss some of the studies showing to which extent non-resident fathers invest in their children.

So how is it decided that is acceptable not to match the data? Some authors argue that the effect of whatever it is that they are excluding is not that large. For the AGG model that allows no investment from fathers, it might be argued that fathers rarely invest in children, but since this is not the case, it might be argued instead that paternal investments do not influence child outcomes. For the Bernal and Fruttero model that allows no step-parents, it might be argued that child wellbeing is not significantly different in families with a mother and stepfather than it is in families with a lone mother, so the results would turn out approximately the same. But what if these things actually do matter? Can we still make a case for allowing these restrictive assumptions? Yes, if it is believed that the effects of parental leave can be estimated without loss of generality.

Even admitting that fathers and step-parents may play a large role in child investment, a model with both of the simplifying assumptions discussed above could still be used to make predictions about labour force participation or child wellbeing in general. It would not however capture any inequality created by differences in non-resident father investment, nor any of the inequality between lone parent families and stepfamilies. Thus relaxing these assumptions, however difficult, may be appropriate if determining how parental leave influences inequality in both income and child outcomes is a priority to the researcher.

Lastly for this section, the small amount of assortative mating in these models may also cause the simulated data to show less inequality than exists in the real world. Assortative mating describes a tendency of individuals to marry someone similar to themselves. For example, people in the United States have become increasingly likely to marry someone with the same education level as their own (Greenwood et al., 2013).

Since agents get a single draw from the marriage market each period, they do not have the opportunity to search for someone with a similar productivity to their own. It is true that agents in the model are reluctant to marry agents with low productivity, especially if the agent's own productivity is high, so there is a small degree of assortative mating. In a more realistic models, individuals would be more likely to receive a marriage offer from someone with a productivity level similar to their own and the amount of assortative mating in the model would be greater. Since this would be relatively more complicated to implement and many questions about the public policy being modelled can be answered without it, it is no wonder that none of the models have this feature.

2.2.3 Household decision-making

Decision-making in lone-parent households is made via straightforward constrained optimization in all three models. In the AGG and Bernal and Fruttero models, decision-making in married households is made via a non-cooperative Nash game, where each partner takes the actions of his partner as given. Though this view of marriage is a little harsh, there is some evidence that spouses make decisions in a non-cooperative manner. Bernal and Fruttero observe that multiple studies have shown that the transition from unilateral to bilateral divorce laws increased divorce rates; this would not be true if couples made decisions cooperatively. Additionally, GGK observe that assuming unitary decisions in this type of model results in a very low marriage rate. This occurs because agents know that they will be forced to act altruistically in their household, so marriage is not as attractive. GGK also observe that males are more likely to decline the opportunity to marry when unilateral decision making is assumed. This is because marriage is riskier for males since they have to pay child support if they become divorced.

While differences in attitudes of each spouse could be captured with Nash bargaining, AGG note that the analysis is much simpler using a non-cooperative Nash game. Despite this, GGK uses Nash bargaining in the model with endogenous fertility. The authors provide such a long and convincing argument for doing so that it seems more prudent to reference it than to replicate it; it can be found on page 828 and 829 of their paper.

2.2.4 Fertility

At first, it may seem essential to incorporate a fertility decision in to these models. It would seem that the decisions to get married and have children are linked. Firstly, as argued by GGK, families with low-incomes tend to have more children, and single mothers tend to have more children than married ones, so that time investments are diluted in low income families. This may help to explain the frequent observation that average IQ descends with birth order[11]. Secondly, policymakers might be curious to know if a particular policy encourages fertility.

The first point is more important in the GGK paper than it is in this one, since the GGK paper is very much focussed on the transmission of human capital. The second point is also more important in their paper because they look at welfare policies, which have been shown to have a larger effect on fertility than parental leave policies (Moffitt, 1992). While not allowing a fertility decision in the parental leave analysis may result in less inequality, it is unlikely that including it would show significant increases in fertility. Thus it should still capture the effects of changes in parental leave policy without loss of generality. As in, endogenous fertility should be present in a model that attempts to account for everything, but adding it into this model is of low priority.

[11] Kanazawa (2012) goes over the theories of why this may be and provides some of the literature supporting each theory.

2.2.5 Childhood investments

The largest difference in childhood investment assumptions between the models looked at in this section is that AGG and GGK assume that only women invest in children, while Bernal and Fruttero do not. It is not likely that the authors of the first two models thought that paternal involvement does not matter for children; there is an overwhelming body of evidence suggesting that paternal involvement influences all aspects of child development[12]. The assumption exists to simplify the model. It may also matter much less than in the Canadian case because parental leave, as opposed to maternity leave, is not widely available in the United States, so a model where only women take leave matches the data closely. In Canada, leave is available to fathers and their use of it is increasing, with 20 percent of fathers receiving paid leave in 2006, up from 3 percent in 2000 (Marshall, 2008). At this time, only 2 in 5 mothers with an eligible spouse who did not claim benefits reported that it was their preferred arrangement, suggesting that men might prefer to spend more time with children if the situation was more favourable[13].

Even if we are willing to accept that mothers provide most of the child care in two parent homes, the issue of fathers not caring about their children becomes more complicated in the case of non-resident fathers. In the past, researchers have been undecided on whether non-resident father contact benefits children, with a 1999 review stating that only 15 of 32 studies found that contact was significantly associated with children's wellbeing (Amato and Gilbreth). More recently, it has been clarified that non-

[12] Allen and Daly (2007) provide a review of over 150 studies on the impact of paternal involvement on child, maternal, and paternal outcomes.
[13] Marshall (2008) finds that many women gave finances as a reason, but a greater percentage simply said that "it was impossible for their spouse to take time off of work", reflecting that men often perceive logistical problems in taking time off.

resident father contact is beneficial if non-resident fathers are willing and allowed to spend time actually parenting children (disciplining and teaching) as opposed to just participating in leisure activities (shopping, going to dinner, or seeing movies together) (Allen and Daly, 2007). Other research suggests that time spent with non-resident fathers improves behaviour and emotional wellbeing the most when the quality of the mother-child relationship is low (King and Sobolwski, 2006).

For the reasons above, a model that allows maternal and parental investments may have the advantage, such as that of Bernal and Fruttero, though they still make the assumption that divorced fathers do not invest time in children. They do have divorced fathers pay child support, so that children of divorced parents are better off than children of never married parents. Since their model does not allow marriage in the second period, there is no opportunity for step-parents to invest in children, since there are no step-parents. It is unclear how much this would change the results. Families with stepchildren represent under 13 percent of couples with children in Canada (Vezina, 2012). Some research has shown that child wellbeing does not differ significantly between stepfamilies and lone mother families (McLanahan and Sandefur, 1994; Brown, 2004) while newer research has found child wellbeing in stepfamilies to be significantly better (Vanassche et al., 2013), and some research has actually found that involvement by resident stepfathers is just as beneficial as involvement by resident biological fathers (Bzostek, 2007). This ties back to our discussion of restrictions on marriage. Given the uncertainty surrounding the benefits of step-parents, it is better to allow single parents to marry in the second period even if the step-parent does not invest in the child directly. This at least reflects the effects brought on by the extra income of the step-parent.

2.2.6 Summary

Since these types of models are new and relatively few, the current contributions make many assumptions and there is a great deal of opportunity for expansion. The GGK expansion of the original AGG model adds endogenous fertility, while the Bernal and Fruttero expansion adds labour market frictions and paternal investments from resident fathers, but removes second period marriages. Given the information above, modifying the marriage market or labour market of the AGG model seems to be of low priority. Endogenous fertility may be important, but is more essential in a model concerned with welfare payments than it is in one concerned with parental leave. An expansion for use in analyzing parental leave policy might instead allow non-monetary involvement from non-resident fathers, and/or allow some involvement by resident stepfathers, both of which can make a large difference in outcomes.

3 Model

While it might have been ideal to relax some of the assumptions on child investments, this is not done in this paper due to time restrictions. Instead, the model presented closely follows the AGG model. This means that unlike the other model used to analyze parental leave (Bernal and Fruttero), second period marriages are retained. Accommodating both second period marriages and paternal time investments is left for future work.

3.1 Environment

The economy is populated by male and female agents and at any point in time, there exists a continuum of children and a continuum of adults. Each adult in the economy is associated with a productivity level. It is not necessary that males and female productivities are contained within the same sets. Each adult female has one male child

and one female child in the first period of her life, whether or not she chooses to marry. If she chooses not to marry, the children stay with her. This keeps things simple and reflects the fact 8 out of 10 lone parent households are headed by females (Statistics Canada, 2012).

Each agent receives a draw from the marriage market in the first period of adult life. Agents compare the expected lifetime utility of being married with the expected lifetime utility of being single, and accept or reject their offer. If two matched agents accept, they are married. If an agent remains single, they get another draw in the second period. An agent who gets divorced in the second period however does not get another draw. Since agents assume they will have children, they take this into account when deciding whether to get married. Females know that they will have to allocate time towards children. Similarly, males know that they will have to help provide for future children should they become married. They also know that if they become married and then get divorced they will have to pay child support. For simplicity, agents only match within generation.

Adult agents are endowed with one unit of non-sleeping time each period. Children do not need to be endowed any time as they do not make any decisions. Females divide time between work, child care and leisure. Males only divide their time between work and leisure, whether they are single or married. The human capital of children is determined by the level of family consumption and the child care time spent by the mother. Single females and both types of married agents derive utility from human capital investment in children, but single males do not. In each period the oldest adult males and the oldest adult females are replaced by the oldest children, who then enter the marriage market.

3.1.1 Preferences

Married agents derive utility from love, leisure, and the level of human capital investment in children, so that married females and males have similar utility functions. Let c be consumption of household production, which is a public good for the family, e be the level of human capital investment in children, l the fraction of time allocated towards labour and t the fraction of time allocated towards child care. Then females have the following utility function:

$$F(c, e, 1 - l - t) = \ln c + \delta_1 \ln e + \delta_2 \ln(1 - l - t)$$

Married males have the following utility function:

$$M(c, e, 1 - l) = \ln c + \theta_1 \ln e + \theta_2 \ln(1 - n)$$

Notice that males and females do not necessarily derive the same utility from or leisure or human capital investment in children. Single and divorced males have the following utility function:

$$M(c, 0, 1 - n) = \ln c + \theta_2 \ln(1 - n)$$

The c in the utility function of the divorced and single males are not of the same form because divorced men must pay child support while single men do not.

3.1.2 Household Production

Let x be the productivity (and therefore wage) of a type x female. Let z be the productivity (and therefore wage) of a type z male. Let γ be a discrete random variable signifying the quality of match. It is drawn upon entering the marriage market and may be negative or positive in value. Then household production for a married couple is given by:

$$c = (xl + zn) - \gamma$$

Clearly the household production of a single female is xl and the household production of a single male is given by zn. Note that if γ is positive then household consumption is

reduced, so that a positive value signifies a bad match. A negative value on the other hand signifies a good match and increases consumption.

3.1.3 Human capital formation

Human capital investment in children is given by:

$$e = t^{\alpha} c^{1-\alpha}$$

Observe that the level of human capital investment is dependent upon the time the female spends with the child and the level of consumption in the household. The future productivity of children is influenced by, but does not depend completely on these two factors. Let e_{-1} and e_{-2} denote the education level received in each period of childhood. Then females draw their productivity from the distribution

$$\Xi(x_i | e_{-2} + e_{-1}) = \Pr[x = x_i | e_{-2} + e_{-1}).$$

Males draw their productivity from the distribution

$$\Lambda(z_j | e_{-2} + e_{-1}) = \Pr[z = z_j | e_{-2} + e_{-1}).$$

Both distributions are increasing stochastically in e_{-1} and e_{-2}. Both distributions are represented by a discrete approximation to a lognormal distribution in the style of Tauchen (1986).

After the first period of adulthood the productivity level of females evolve according to the transition function

$$X(x_j | x_i) = \Pr[x' = x_j | x = x_i].$$

The productivity level of males evolve according to the transition function

$$Z(z_j | z_i) = \Pr[z' = z_j | z = z_i].$$

Let X and Z be discrete approximations to the stochastic processes, again in the style of Tauchen (1986), then

$$\ln x' = (1-\rho_x)\mu_x + \rho_x \ln x + \sigma_x \text{sqrt}(1-\rho_x^2)\xi \text{ with } \xi \sim N(0,1), \text{ and,}$$

$$\ln z' = (1-\rho_z)\mu_z + \rho_z \ln z + \sigma_z \text{sqrt}(1-\rho_z^2)\zeta \text{ with } \zeta \sim N(0,1).$$

One thing to note is that the productivity of the agent is not influenced by their experience on the job; this would remove the static nature of the time allocation decisions. Under alternate assumptions, female agents would have to be concerned with the decrease in productivity associated with taking time off work. Also note that the model is set up in such a way that parents care about the level of investment in their child care but never see the result of this investment. In reality, parents might adjust time investments based on the results they observe from these investments.

3.2 Decision making

The notation in this section follows that of AGG but greater explanation is given throughout. As before, modelling decisions preserve the static nature of the agents' choices. Married couples reach decisions via a non-cooperative Nash game rather than Nash bargaining. This means that couples take the decisions of their spouse as given. Alternatively, as in GGK, couples could make simultaneous demands via Nash bargaining. Since the logical threat point of each agent would be the value to him or her of being single, this would create a dynamic optimization problem for young agents.

3.2.1 Matching

Agents gain utility from marriage through three mechanisms. First, as shown before, household production is increased when the match quality is good (but decreased if the match quality is poor). Second, household consumption is specified as a public good so that there are economies of scale; this is logical as a larger household can take advantage of bulk discounts on perishable goods and can share some durable goods. Third, men can only receive utility from children when married.

Since agents in the model have the same utility and consumption functions as other agents of the same type, initial heterogeneity is achieved through match quality and productivity of the two types. In fact, productivity is a large enough factor that

depending on the parameters set, agents may still marry when the match quality is poor. In the AGG benchmark simulation, the authors find that the most productive men always marry, even if the woman is in the least productive category. Since there is diminishing marginal utility of income and all women are assumed to be equally good at taking care of children, a rich man has much to gain by marrying a woman that can invest heavily in children because he gains utility from the level of human capital investment in the household. Women on the other hand do not need marriage to realize utility from children, so a productive woman does not gain much from marrying an unproductive man. It should be expected that they are less likely to marry than productive men.

The odds of drawing a single age j female of type x_i is represented by

$\Phi_j(x_i)$, where $\Phi_j(x_i) \geq 0 \; \forall \; x_i$ and $\sum_n \Phi_j(x_i) = 1$,

while the odds of drawing a single age j male of type z_i be represented by:

$\Omega_j(z_i)$, where $\Omega_j(z_i) \geq 0 \; \forall \; z_i$ and $\sum_n \Omega_j(z_i) = 1$.

Given a match, denote the expected lifetime utility of a match in marriage for a male and female as $W_i(x, z, \gamma)$ and $H_i(x, z, \gamma)$, respectively. Denote the value to the male and female of remaining single as $G_i(x)$ and $B_i(z)$. Then a single female desires to be married if and only if $\sum_h \Gamma(\gamma_h) W_i(x, z, \gamma) \geq G_i(x)$ and a single male desires to be married if and only if $\sum_h \Gamma(\gamma_h) H_i(x, z, \gamma) \geq B_i(z)$, where $\Gamma(\gamma_h)$ is the distribution function of match quality. A married female desires to remain married if and only if $W_i(x, z, \gamma) \geq G_i(x)$ and a married male desires to remain married if and only if $H_i(x, z, \gamma) \geq B_i(z)$. Define four indicator functions summarizing the matching decisions of single adults, by

$$I^s{}_1(x, z; \Phi_1, \Omega_1) = 1 \text{ if } \sum_h \Gamma(\gamma_h) H_1(x, z, \gamma_h) \geq B_1(z; \Phi_1, \Omega_1) \text{ and 0 otherwise,}$$

$$I^s{}_2(x, z) = 1 \text{ if } \sum_h \Gamma(\gamma_h) H_2(x, z, \gamma_h) \geq B_2(z) \text{ and 0 otherwise,}$$

$$J^s{}_1(x, z; \Phi_1, \Omega_1) = 1 \text{ if } \sum_h \Gamma(\gamma_h) W_1(x, z, \gamma_h) \geq G_1(x; \Phi_1, \Omega_1) \text{ and 0 otherwise,}$$

$$J^{s}{}_{2}(x, z) = 1 \text{ if } \sum_{h}\Gamma(\gamma_h) W_2(x, z, \gamma_h) \geq G_2(x) \text{ and 0 otherwise.}$$

Also define four indicator functions summarizing the decisions of married adults, by

$$I^{m}{}_{2}(x, z) = 1 \text{ if } H_2(x, z, \gamma_h) \geq B_2(z) \text{ and 0 otherwise,}$$

$$J^{m}{}_{2}(x, z) = 1 \text{ if } W_2(x, z, \gamma_h) \geq G_2(x) \text{ and 0 otherwise.}$$

Note that not only do the decisions for the old not depend on the distribution of match quality; they do not depend on the distribution of productivities either. If agents could remarry this would not be the case. As is, couples get divorced if either of them can gain more utility in the period on their own than in the marriage.

The value function for a married male in the first period of adulthood is

$$H_1(x_i, z_j, \gamma_h) = M^{m}(x_i, z_j, \gamma_h) + \beta\sum_{k}\sum_{l} \max\{H_2(x_k, z_l, \gamma_h) J^{m}{}_{2}(x_k, z_l, \gamma_h), B_2(z_l)\} X(x_k | x_i) Z(z_l | z_j),$$

where $X(x_k | x_i) Z(z_l | z_j)$ is the probability that a married couple will move from state (x_i, z_j) to state (x_k, z_l). Similarly, the value function for a married female in the first period of adulthood is

$$W_1(x_i, z_j, \gamma_h) = F^{m}(x_i, z_j, \gamma_h) + \beta\sum_{k}\sum_{l} \max\{ W_2(x_k, z_l, \gamma_h) I^{m}{}_{2}(x_k, z_l, \gamma_h), G_2(z_l)\} X(x_k | x_i) Z(z_l | z_j)\}.$$

The value function for a single male in the first period of adult life is

$$B_1(z_j; \Phi_1, \Omega_1) = M^{s}(z_j) + \beta\sum_{k}\sum_{l} \max\{\sum_{h}\Gamma(\gamma_h) H_2(x_k, z_l, \gamma_h) J^{s}{}_{2}(x_k, z_l), B_2(z_l)\} \times Z(z_l | z_j) \Phi'_2(x_k).$$

with $(\Phi'_2, \Omega'_2) = P(\Phi_1, \Omega_1)$, where $Z(z_l | z_j)\Phi'_1(x_k)$ is the probability that the male will transition to the productivity level of z_l and meet a single female of type x_k. Similarly, the value function for a single female in the first period of adult life is

$$G_1(x_i;\ \Phi_1,\ \Omega_1) \quad = \quad F^s(x_i) \quad + \beta\Sigma_k\Sigma_l$$

$$\max\{\textstyle\sum_h \Gamma(\gamma_h)\, W_2(x_k,\ z_l,\ \gamma_h) I^s{}_2(x_k,\ z_l),\ G_2(x_k)\} \times X(x_k \mid x_i)\ \Omega'_2(z_l).$$

3.2.2 Married agents

As mentioned, married agents take the optimal choices of their partner as given. Starting with the female agent, denote her husband's labour supply as $n = N^m(x, z, \gamma)$. Then a type x female married to a type z male solves the problem:

$$F^m(x, z, \gamma) = \max \ln c + \delta_1 \ln e + \delta_2 \ln(1 - l - t)$$

subject to

$$c = (xl + z\, N^m(x, z, \gamma)) - \gamma$$

and

$$e = t^{\alpha} c^{1-\alpha}.$$

Denote the solutions to this problem as $l = L^m(x, z, \gamma)$ and $t = T^m(x, z, \gamma)$. Then the male solves the problem:

$$M^m(x, z, \gamma) = \max \ln c + \theta_1 \ln e + \theta_2 \ln(1 - n)$$

subject to

$$c = (x\, L^m(x, z, \gamma) + zn) - \gamma$$

and

$$e = T^m(x, z, \gamma)^{\alpha} c^{1-\alpha}$$

Denote the equilibrium level of human capital investment in this family by $e = E^m(x, z, \gamma)$. Though closed form solutions exist to these problems, they are somewhat messy. It should be noted that if the match quality variable is allowed a very large range consumption might be negative and the objective value will be undefined. As long as the match quality variable is on a reasonable range and the parameters are positive, the derivatives of the equilibrium values make logical sense. The more an agent's spouse

works, the less they want to work. It is also true that all else equal, a couple with higher match quality will not work as much as couple with lower match quality. High productivity compels agents to work more because they are paid a higher wage, while a higher value of the parameter α causes women to substitute towards teaching and men to substitute towards leisure.

3.2.3 Single and divorced agents

A single type x female solves the problem:

$$F^m(x) = \max \ln c + \delta_1 \ln e + \delta_2 \ln(1 - l - t)$$

subject to

$$c = xl$$

and

$$e = t^{\alpha} c^{1-\alpha}.$$

Denote the solutions to this problem as $l = L^s(x)$ and $t = T^s(x)$. When welfare is in the model, she compares the utility generated from this solution to the utility generated by setting $l = 0$ and receiving a fixed welfare payment. Again, higher productivity makes labour hours more attractive, while an increase in the parameter a causes a substitution towards teaching hours. The single male's problem is simpler:

$$M^m(x, z, \gamma) = \max \ln c + \theta_2 \ln(1 - n)$$

subject to

$$c = zn.$$

Then his solution depends only on his productivity and his preference for leisure. Denote the solution to this problem as $n = N^s(z)$.

Note that the problems for divorced agents are nearly identical to these. In the female's case, she has a little more income because she receives child support. In the

male's case, he has a little less income because he pays a fraction of his income as child support.

4 Numerical Exercises

The first numerical exercise is an attempt to replicate the results found in AGG. The parameter values used in that paper are given below in Table 1.

Table 1
PARAMETER VALUES

Tastes	$\delta_1 = .5$, $\delta_1 = .9$, $\theta_1 = .1$, $\theta_2 = .7$, $\beta = .67$
Technology	$\alpha = .4$, $\varepsilon = 4.2$, $c_2 = 4.2$
Stochastic structure	$\mu_{x\|e} = \varepsilon e$, $\mu_{z\|e} = c_2 + \varepsilon e$, $\sigma_{x\|e} = .4$ $\sigma_{z\|e} = .4$
	$\rho_x = .7$, $\rho_z = .7$
	$\Gamma(\gamma_1) = .5$, $\Gamma(\gamma_2) = .5$, $\gamma_1 = 2.6$ $\gamma_2 = 0$
	$n = 17$, $m = 2$
Policy Variables	$a = .10$, $w = .22$, $\tau = .03$

Here, a is the child support rate, w is the welfare rate, and τ is the tax rate. There is not much explanation given for these values. Unfortunately, the equilibrium diverges with these values, since every generation becomes more productive than the last. When ε and c_2 are reduced, equilibrium attains. Unfortunately, the original code could not be obtained so it is impossible to rule out typos or bugs in the code. It could also be that there is some unspoken difference between the AGG code and the code used for this paper; many parts of the AGG paper are ambiguous. To make matters worse, few summary statistics are reported, so there are few hints as to what could be going wrong.

The AGG choice for the distribution of the match quality variable is somewhat peculiar. The paper states that a positive value indicates hate while a negative value indicates love, but it appears from the table that no negative values are allowed. When the model as coded in Ox is run with these values (with the other adjustments and with n reduced to save computation time), the divorce rate ends up being much higher than in the AGG paper. It is uncertain how they find such a high marriage rate and such a low

divorce rate when none of the matches are of good quality. It may be because the productivities generated by the AGG code are quite high, so it is often beneficial to marry, even when the match is bad.

Table 2
AGG BENCHMARK VERSUS REPLICATIONS

	AGG Benchmark		(1) Replication n=6		(2) Replication n=6 $\gamma_1 = -2.6$		(3) Replication n=6 $\gamma_1 = -0.5, \gamma_2 = 0$	
	Young	Old	Young	Old	Young	Old	Young	Old
Married	72	83	88	7	49	73	67	57
Single	28	8	12	16	41	17	33	13
Divorced	-	9	-	77	-	10	-	29

Note: Due to differences in computational power, not as many points were used to approximate the relevant distributions. This could also account for some of the error.

Replacing 2.6 with -2.6 generates results similar to that of AGG, as seen in Table 2. It is clear from this table that the divorce rate can be manipulated significantly by improving the quality of matches. While the divorce rate drops from 77 percent to 9 percent with the modification, the first period marriage rate only drops from 88 percent to 61 percent. Experimenting with different ranges on [-2.6, 2.6], a first period marriage rate less than 60 percent was never found. Ultimately, productivity plays too large of a role for the marriage rate to be manipulated only using match quality. It is important to note that the gap between γ_1 and γ_2 is important for both the marriage rate and the divorce rate. For example, setting $\gamma_1 = -0.5$ and $\gamma_2 = 0$ resulted in a higher first period marriage rate, even though the distribution seems less favourable. This is because agents care about lifetime expected utility. In the second replication, agents who draw the low γ know that they have a chance of finding a much better match in the second period. In the third replication, agents who draw the low γ get no marriage quality bonus to household consumption, but they know that they cannot do much better in the second

period. Since the marriages in the third replication are of lower quality, more of them break up when productivity evolves, resulting in a higher divorce rate.

It is difficult to say what constitutes a good match to the data. The problem is considerably more complicated than looking up the percentage of married individuals and divorced individuals. While in reality a large portion of young people are single, it is also true that many of these individuals do not have children, while in this model all women have children. It seems more appropriate to determine the approximate percentage of households with a single parent. In Canada, this is currently about 16% (Statistics Canada, 2013). AGG states that about 22.5 percent of households are lone-parent households. Since there is an equal number of males and females in each period and an equal number of young and old adults, it is not difficult to determine how this number is calculated. Replications 1, 2 and 3 give the percentage of lone parent households as 52.5 percent, 34 percent, and 37.5 respectively. So while all are too high, replication 2 fits the best in this regard. Another feature of the data should be that the percentage of adults who are married is higher in the second period. In Canada in 2013, a full 42 percent of Canadians ages 25 through 34 were single, while only 19 percent Canadians ages 35 through 44 were single. Replication 2 is better than replication 3 in this respect. Only 5 percent of Canadians ages 34 through 44 were divorced, indicating that it is not that common for Canadians young enough to still have children living at home to be divorced[14]. (Statistics Canada, 2013) Replication 2 is again better.

Unfortunately the AGG paper does not report the percentage of woman on welfare, so this statistic could not be compared. In replication 1, with the original AGG

[14] One disclaimer here is that Canada counts an individual who was formally in a common-law marriage as single. Thus, the divorce statistic may underestimate the broken families with older parents. On the other hand, not all individuals in the census have children. It is uncertain if these effects would cancel each other out.

parameters, no young single women claim welfare. This is not a huge surprise as most women are married. It is however inconsistent with the AGG results, as they find some (unknown) level of welfare participation. In replication 2, the replication with no bad matches, 18 percent of young single women claim welfare. Far fewer women are married in this replication and it is the least productive women who are the most likely to be rejected by their match. Average income is higher in the economy with no good matches, but only 1.05 times higher. Unsurprisingly, the level of human capital investment in children is much lower in the economy with no good matches. In the first period, there are many matched couples with poor marriage quality, so many families have a penalty to consumption. In the second period, very few marriages are intact so mothers cannot specialize in time with children. Though the AGG paper does not provide detailed statistics, this is at least consistent with the result that divorced working women spend much less time on child care than married women do (9 percent of their time versus 14 percent of their time in the AGG benchmark).

5 Conclusions

Though the original intent of this paper was to study parental leave, considerable adjustments should be made to the code before that stage. As is, discrepancies exist between the results presented in AGG and the results generated by the code used for this paper. It is difficult to tell why because of some ambiguity in the paper, and difficult to tell how many discrepancies exist because not much data is reported. Manipulating parameters does lead to similar results in the marriage market, but there is more experimentation that could be done. Mainly, some thought may need to be given as to how to increase the first period marriage rate without increasing the second period divorce rate. Currently, the trade-off between the two makes it difficult to make the

number of lone-parent households in the economy small enough to fit Canadian statistics. Once these issues are resolved, the model could be easily adapted, perhaps using the same method as that of Bernal and Fruttero, to accommodate parental leave experiments.

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