Contingent Convertible Bonds: Hedging, Credit Default Swaps, and Sensitivity to Subjective Market Opinions

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Abstract

In response to the Great Recession, contingent convertible bonds have been pitched as the only debt instrument providing fully loss-absorbing going-concern capital to financial institutions. Legitimized by Basel III, they have rapidly become the hallmark of financial stability. Detractors have however raised concerning points about hedging and price stability, including the so-called death spiral, which is a self-fulfilling collapse in the underlying stock price as a result of delta hedging. Those points call into question the validity of this new security as a stabilizing force to the financial system. Seeking to address those concerns, this research begins with a thorough review of the literature on the current market environment, the structure and design of contingent convertible bonds and the various methods developed to price this hybrid security. It then expands the scope of the current literature, proceeding with an analysis of the hedging dynamics, the introduction of credit default swaps, and the sensitivity of the price to shifts in market opinions. Overall, this research finds that while significant, the risks and repercussions of a death spiral have been overblown. The strong emphasis on this particular issue in the press overshadows many of their other concerning features, including the distorted market-clearing price that currently prevails, the counterparty risk that could arise from the creation of a market for credit default swaps, and their high sensitivity to subjective market opinions.

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Chapter 1

Introduction

Contingent convertible bonds (CoCos) are relatively new to financial markets, and yet supply and demand for those untested securities is growing rapidly. Pitched as the only debt instrument providing fully loss-absorbing going-convern capital and legitimized by Basel III, CoCos have become the poster child for financial stability. Like any other innovation, their growth has attracted as many detractors as supporters. From the death spiral to concerns about the market underestimating the risk they provide, criticism in the financial press abound. While fear of change often drives the initial skepticism, the concerns being raised in this case are based on structural characteristics of contingent convertible bonds and the markets in which they are traded.

As the tool created to address financial instability, the design of CoCo bonds and their behaviour in times of crisis deserve further attention to ensure they are the stabilizing force that they are claimed to be. This research hence reviews the literature on contingent convertible bond design, pricing and hedging, and then delves into the concerns of delta hedging and the death spiral, overpricing, and finally regulatory triggering and its effect on pricing.

Chapters 2, 3 and 4 are a critical review of the literature on the current market environment, the structure and design of contingent convertible bonds and the various methods developed to price this hybrid security. Chapters 5 and 6 expand beyond the scope of the current literature to study the dynamics of hedging, the introduction of credit default swaps on CoCos, and the sensitivity of the price to the various triggers through a trinomial tree.

More specifically, Chapter 2 presents the context and the regulatory environment that set the stage for the eclosion of the CoCo market and that define the characteristics of the CoCos being issued. The taxation of financial institutions that varies widely across jurisdictions and the slightly differently implementation of Basel III by each national regulator has a given rise to broad range of features that may seem like minor variations but that can significantly affect the price. Chapter 3 hence defines the potential characteristics of contingent convertible bonds and analyses their effect on pricing.

Chapter 4 presents the three main pricing models: the structural, credit derivatives, and equity derivatives models. It highlights the assumptions that underlie their construction as well as their respective strengths and weaknesses. All three models are found to rely on unobservable variables and must hence be used with the observed market price to solve for an implied input variable such as the equivalent market trigger. Overall the equity derivatives model is found to be the most practical for empirical applications and the weaknesses it suffers from are no worse than the ones afflicting the structural and credit derivatives models.

Chapter 5 quickly reviews the hedging of contingent convertible bonds and then investigates the death spiral that can result from delta hedging. It expands the analysis to include the effect of volatility on delta hedging. The scope is then expanded to include the effects of gamma and vega hedging on market dynamics. The higher volatility that is often associated with market instability or times of crisis is shown to reduce delta and could hence mitigate the risks of a death spiral. The effect of gamma and vega hedging is found to be limited to liquidity concerns as derivatives market merely redistribute risk.

The chapter concludes with an analysis of the impact of the creation of a market for credit default swaps on contingent convertible bonds. Although there are no regulatory restrictions on shorting CoCos, the current market structure constrains shorting to a great extent. Credit default swaps are hence shown to relax this constraint and potentially restore the competitive equilibrium that could not prevail as a result of the combination of the shorting constraint and the investors' wide range of subjective opinions about the conversion or write-down risk.

Chapter 6 describes how a trinomial tree used for pricing options on equity can be adapted to pricing contingent convertible bonds. The tree is then used to gauge the sensitivity of CoCo prices to changes in the market-clearing trigger and compare the range against an empirical range of implied market triggers for similar CoCos issued by the same financial institution. The tree is then extended to include an exogenous regulatory trigger and test for the sensitivity to shifts in the subjective probability of regulatory conversion or write-down.

Ultimately, this research finds that while significant, the risks and repercussions of a death spiral have been overblown and that the strong emphasis on this particular issue in the press overshadows many other concerning features of contingent convertible bonds. Although the creation of a market for credit default swaps is expected to restore the competitive equilibrium, it does so at the expense of creating exposure to counterparty risk. With any concentration of ownership in the CDS market, the absence of other hedges could create unexpected exposure in times of crisis. Finally, using a trinomial tree, the price of contingent convertible bonds is shown to be highly sensitive to shifts in subjective expectations of both the equivalent market trigger, and the regulatory trigger. This high sensitivity destabilizes the equilibrium and jeopardizes hedging.

Chapter 2

Context and Regulatory Environment

The Financial Crisis and Great Recession of 2008 brought financial risk management back to the top of everyone's mind. Throughout the early 2000s, the economic effervescence and the low volatility of both output and inflation that had started in the mid-1980s, which came to be known as the Great Moderation (Bernanke, 2004), seemed to have deluded many into thinking that financial instability was an ailment of the past. The global contagion that followed the collapse of Lehman Brothers and Bear Sterns and forced the US government to rush through the US$700 billion troubled assets purchase program known as the 2008 Emergency Economic Stabilization Act (University of California, Berkeley, 2011) made reform of banking capital regulation an urgent priority.

The Bank for International Settlements (BIS) had already begun reviewing the weaknesses of Basel II when Lehman Brothers failed (BIS, 2013b, p. 4), but the crisis made it clear that there was a global systemic shortage of loss-absorbing capital (Croft, 2009). Making the balance sheets of financial institutions more loss-absorbing hence became the heart of the Basel III accord, and contingent convertible bonds took the center stage as a financial product that can quickly recapitalize banks without resorting to equity markets.

2.1 The Market for Contingent Convertible Bonds

Contingent convertible bonds may owe their popularity to Basel III, but they are not its creation. They made their entry on financial markets in November 2009, when Lloyd's Banking Group offered holders of £16 billion of their existing hybrid debt the possibility of exchanging their hybrids for contingent convertible bonds (Sakoui, 2009). Many other financial institutions have followed suit to shore up their balance sheets as they face the prospect of stricter regulatory capital ratios, including Credit Suisse, Rabobank, Bank of Cyprus, Canton of Zurich, UBS, Macquarie Group, Barclays, KBC, Banco Bilbao Vizcaya Argentari, Société Générale, Credit Agricole, Banco Popular, Federative Republic of Brazil, Nomura Holding, People's Republic of China, and the Royal Bank of Canada[1]. Indeed, Standard & Poor pegged the potential supply of contingent convertible securities at over US$1 trillion over the next decade (Standard & Poor's, 2010) until the Basel committee rejected CoCos as loss-absorbing capital for the systemically important financial institutions (SIFI) surcharge in Basel III (Louis, 2011). On the demand side, investors' hunt for higher yields driven by the world's central banks' policies of record low interest rates has sent CoCo bond prices soaring (Thompson, 2014a).

2.2 Regulation of Banking Capital

The regulation of banking capital falls under the purview of national regulators in each country. The Basel Committee does not impose any requirements on bank capital directly, but rather relies on its member countries to implement the recommendations elaborated by the Secretariat (de Spiegeleer et al., 2014, p. 90). Its role is to propose "general supervisory principles" for "the creation of a level playing field amongst the internationally active banks" (de Spiegeleer et al., 2014, p. 90). Although the Basel agreements are not directly binding, most national regulators base their legislation on the essence of the Basel recommendations, and so the next three sections cover the evolution of capital ratio regulation through Basel I, II and III to better understand the role of contingent convertible bonds on the balance sheets of modern banks.

[1] See sources and the full list of issued CoCos in appendix A.

2.2.1 Basel I

The Basel I recommendations were published in 1988 by the Basel Committee on Banking Supervision (BCBS) and are the result of negotiations by central bankers that sought to address the concerns brought to light by the 1980's Latin American debt crisis (BIS, 2013b, p. 2). The accord imposed a minimum Cooke Ratio of 8%, which was to be calculated as (de Spiegeleer et al., 2014, p. 91):

$$\text{Cooke Ratio} = \frac{\text{Total Regulatory Capital}}{\text{Risk-Weighted Assets}} \tag{2.1}$$

The denominator, risk-weighted assets, is calculated by applying a risk-weight that depends not only on the source of the asset, but also on whether the loan is to a borrower within or outside the OECD (de Spiegeleer et al., 2014, p. 91):

Assets	**Risk-Weights**
Cash and OECD Government Debt	0%
Loans to Domestic Public-Sector Entities	10%
Loans to Banks in the OECD	20%
Loans Secured by a Residential Property	50%
Other Loans	100%

(de Spiegeleer et al., 2014, p. 91)

This regulatory structure provided a basis point around which closer harmonization of global banking regulation could be achieved, but it was eventually made obsolete by financial innovation. As the portfolio of financial derivatives held by banks grew, the regulatory structure became more and more vulnerable to being arbitraged (de Spiegeleer et al., 2014, p. 91).

2.2.2 Basel II

The Basel II recommendations were published in 2004 by the BCBS and are meant to address the concerns of improper evaluation of risk by the Basel I framework (de Spiegeleer et al., 2014, pp. 91-92). The accord maintains the minimum Cooke Ratio of 8% imposed by

Basel I, but redefines risk-weighted assets to account for credit risk, market risk and operational risk, while also eliminating the OECD vs non-OECD discrimination (de Spiegeleer et al., 2014, p. 92). The risk-weights under Basel II depend on the rating agencies' credit rating as well as the borrower's type (de Spiegeleer et al., 2014, p. 92):

	Risk Weights		
Credit Rating	**Sovereign**	**Banks**	**Corporates**
AAA to AA-	0%	20%	20%
A+ to A-	20%	50%	50%
BBB+ to BBB-	50%	50%	100%
BB+ to B-	100%	100%	100%
Below B-	150%	150%	150%
Unrated	100%	50%	100%

(de Spiegeleer et al., 2014, p. 92)

Although this framework addressed many of the issues of Basel I, it also reduced the risk-weighting of AAA-rated mortgage-backed securities from 50% to 20% (de Spiegeleer et al., 2014, p. 92). This change has been criticized for potentially playing a role in the crisis of 2008 (de Spiegeleer et al., 2014, p. 92).

2.2.3 Basel III

"Higher global minimum capital standards for commercial banks" as well as a "capital and liquidity reform package" were published in 2010 and form the basis for what is now known as Basel III (BIS, 2013b, p. 4). According to BIS:

> "The objective of the reforms is to improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spillover from the financial sector to the real economy... The Committee's comprehensive reform package addresses the lessons of the financial crisis. Through its reform package, the Committee also aims to improve risk management and governance as well as strengthen banks' transparency and disclosures. Moreover, the reform package includes the Committee's efforts to strengthen the resolution of systemically significant cross-border banks." (BIS, 2011a, p. 1)

In essence, the BIS is recommending a move towards loss-absorbing capital to reduce the probability of future bank failures, and more importantly, to prevent a potential systemic contagion through large global financial institutions. The changes to capital requirements once Basel III is fully implemented in 2019 (BIS, 2013a):

Common Equity Tier 1

The ratio of common equity tier 1 to risk-weighted assets is to be raised to 4.5% (BIS, 2011a, p. 12). Common equity tier 1 is comprised mostly of common shares and retained earnings (BIS, 2011a, p. 13). It is the most loss-absorbing form of capital.

Additional Tier 1

The total ratio of tier 1 capital to risk-weighted assets, including common equity and additional tier 1, must be 6% (BIS, 2011a, p. 12), which implies a minimum of 1.5% of additional tier 1 capital if the bank has only 4.5% of common equity tier 1. The requirements for recognition as additional tier 1 capital ensure that it is going-concern capital and include subordination to depositors, creditors and subordinated debt; perpetual maturity; full discretion on payment of dividends or coupons; restrictions on callability and repayment so that such an action does not jeopardize the capitalization of the bank; and principal loss absorption through conversion to common shares or write-down (BIS, 2011a, pp. 15-17). The features just listed describe characteristics of many contingent convertible bonds and it is hence in this capital tier that many CoCos are issued.

Tier 2

The total ratio of tier 1 and tier 2 capital to risk-weighted assets, including common equity tier 1, additional tier 1 and tier 2, must be 8% (BIS, 2011a, p. 12), which implies a minimum of 2% of tier 2 capital if the bank has only 6% of tier 1 capital. There are few changes to tier 2 capital under Basel III, with its primary focus on gone-concern loss absorption (BIS, 2011a, p. 18). It holds the bank's debt that is "subordinated to depositors and general creditors", but that does not satisfy the criteria for inclusion in additional tier 1 capital (BIS, 2011a, p. 17-18). Contingent convertible bonds that, for example, do not have perpetual maturity or full discretion on payment of coupons are included in this capital tier.

Capital Conservation Buffer

The capital conservation buffer is an innovation of Basel III and requires an additional 2.5% of common equity tier 1 capital (BIS, 2011a, p. 55). Falling below this threshold does not affect the normal operation of the bank but imposes constraints on capital distribution through for example dividend payments or share buybacks (BIS, 2011a, p. 55-56). Contingent convertible bonds cannot be used to fulfill this requirement.

Countercyclical Buffer

The countercyclical buffer is also an innovation of Basel III and requires up to 2.5% in additional common equity tier 1 capital, as determined by the national regulator, to reduce systemic risk stemming from excess credit growth (BIS, 2011a, p. 58). Contingent convertible bonds cannot be used to fulfill this requirement.

Systemically Important Financial Institutions (SIFI) Surcharge

The SIFI surcharge was published shortly after the initial Basel III recommendations and became a part of the Basel III package of reform. It imposes a capital surcharge ranging from 1 to 3.5% to internationally active banks to mitigate the moral hazard caused by the perverse incentive to take more risk, as their role in the global economy provides them with an implicit insurance from national governments (BIS, 2011b, pp. 1-2, 15). The BIS committee chose to require common equity tier 1 capital for this surcharge, judging in particular that contingent convertible bonds "are new instruments" with "uncertainty around their operation and whether they would be triggered as designed" and that their complexity creates "considerable uncertainty about how price dynamics will evolve or how investors will behave, particularly in the run-up to a stress event" (BIS, 2011b, pp. 18-19). This decision significantly reduced the potential market supply of contingent convertible bonds.

Figure 2.1 summarizes the phase-in timeline of Basel III for a systemically important bank subject to a 2.5% SIFI surcharge. With the Basel committee's decision to require common equity for the SIFI surcharge, it seems as though Basel III leaves little room for contingent

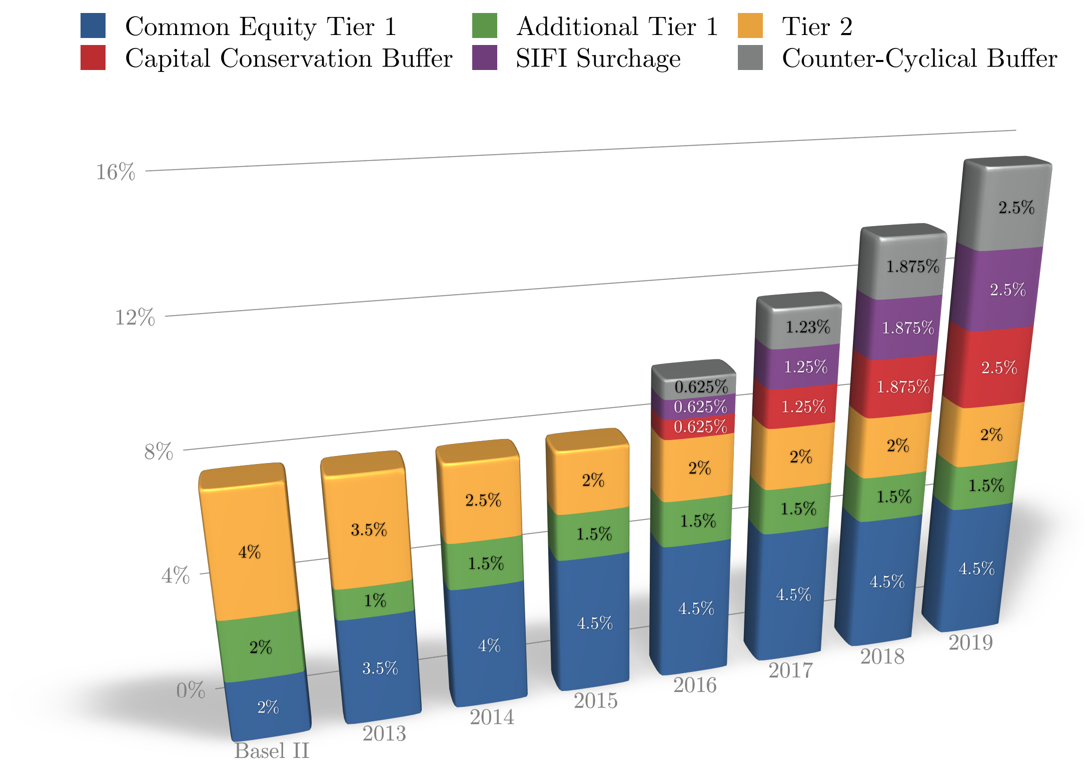

Figure 2.1: Basel III Phase-In Timeline (de Spiegeleer et al., 2014, p. 98)

convertible bonds. Although the decision did severely limit the size of the CoCo market (Louis, 2011), the sheer scale of the increase in capital requirements magnifies the attractiveness of contingent convertible bonds in the additional tier 1 and tier 2 categories of capital.

Chapter 3

Definitions

Before defining a model to price contingent convertible bonds, it is important to understand their various characteristics and how they affect the structural design of this financial security. This section hence defines conversion to equity, the conversion price, principal write-down, as well as the various conversion or write-down triggers.

3.1 Conversion to Equity

Conversion to equity is the feature common to both the more traditional hybrid securities and the first contingent convertible bonds issued. What sets contingent convertible bonds apart from bail-in securities is their focus on providing going-concern capital, which may avoid the bankruptcy resolution process (von Furstenberg, 2011, p. 1). Relative to convertible bonds, which provide optional or mandatory conversion to equity when the share price rises to a certain level, CoCos are subject to mandatory conversion when a pre-specified trigger is breached (Wilkens and Bethke, 2014, p. 60). In essence, if there is no bankruptcy, convertible bonds provide the potential for unlimited upside and limited downside, while contingent convertible bonds provide limited upside and limited, but more significant, downside (Wilkens and Bethke, 2014, p. 60).

3.1.1 Conversion Price

The price at which a contingent convertible bond converts into equity when the trigger is breached is one of the most important feature in pricing the CoCo as it determines the number of shares received in exchange for the bond, and consequently the share of the loss absorbed by the bondholder and the dilution imposed on shareholders (Shang, 2013, pp. 18-19). As shown in equation 3.1, the conversion price can be defined as a function of the face value of the bond and the conversion ratio, which is the number of shares received upon conversion (de Spiegeleer et al., 2014, p. 79):

$$C_P = \frac{N}{C_r} \tag{3.1}$$

The conversion price then feeds directly into the calculation of the loss upon conversion of the contingent convertible bonds as in equation 3.2 (de Spiegeleer and Schoutens, 2012, p. 30):

$$\text{Loss}_{CoCo} = N\left(1 - \frac{S^*}{C_P}\right) \tag{3.2}$$

3.1.1.1 Fixed Conversion Price

A contingent convertible bond with a fixed conversion price has a conversion price that is predetermined in its prospectus. It is generally set equal to the share price at the time the CoCos are issued, such that $C_P = S_0$ (de Spiegeleer and Schoutens, 2012, p. 29), but there is no reason why another share price could not be used. Adopting a fixed conversion price not only limits dilution, but also makes potential dilution known at the time of issue (de Spiegeleer and Schoutens, 2012, p. 29). Many CoCos were issued using a fixed conversion price, including the first CoCos issued by Lloyd's Banking Group, which adopted a fixed conversion price of £0.592093 based on an average of the market price of the underlying shares at the time of issue (Lloyd's Banking Group, 2009).

3.1.1.2 Floating Conversion Price

A contingent convertible bond has a floating conversion price if the conversion price is the market price of the underlying stock at the time of conversion, i.e. $C_P = S^*$ (Girolamo et al., 2012, p. 7). A floating conversion price has the potential for unbounded dilution as the number of new shares created converges to infinity as the share price upon conversion converges to zero (Girolamo et al., 2012, p. 7-8). No contingent convertible bonds have been issued using a floating conversion price yet, likley because of the potential for significant dilution. Most of the current research, including Girolamo et al. (2012, p. 7) and de Spiegeleer et al. (2014, p. 80), argue that a floating conversion price leads to full recovery upon conversion, which means no loss from conversion to CoCo bondholders. Such a statement, however, implicitly assumes perfect information and glosses over the potential for conversion of CoCo bonds to serve as a market signal. Section 3.1.2 briefly describes the building blocks needed to relax the assumption of perfect information.

3.1.1.3 Floored Conversion Price

A contingent convertible bond has a floored conversion price if it is the maximum of a floating and a fixed conversion price (Girolamo et al., 2012, p. 8), the latter of which is set lower than the stock price at the time the security is issued[2]. In terms of loss to CoCo holders and dilution of shareholder ownership, it is a compromise between a fixed and a floating conversion price (Girolamo et al., 2012, p. 8). Credit Suisse has favoured this option, proceeding with a floored conversion price of $\max\{S^*$, CHF20, \$20$\}$ (Ineke et al., 2011, p. 30).

3.1.2 Conversion as a Market Signal

Full recovery in the case of a floating conversion price implicitly depends on the assumption of perfect information. Relaxing the assumption of perfect information allows modelling the conversion event as a signal that provides adverse information about the financial institution

[2]Choosing a conversion price floor equal to or larger than the price of the underlying stock at the time of issue would guarantee that the price floor is binding. It would hence be equivalent to adopting a fixed conversion price.

to market participants. In such a situation, conversion would cause a drop in the stock price. Defining S^*_{pre} and S^*_{post} as the stock prices pre and post-conversion respectively, equation 3.3 describes the loss to CoCo bondholders of conversion:

$$\text{Loss}_{CoCo} = C_r \left(\frac{S^*_{pre} - S^*_{post}}{S^*_{pre}} \right) \tag{3.3}$$

In the context of imperfect information, there exists a range of stock prices, $(\underline{S}, \overline{S})$, that cover the full range of market-implied probabilities of conversion of the CoCo. After conversion, the value S^*_{pre} relative to the range $(\underline{S}, \overline{S})$ describes the ex-ante market expectation of conversion. Consistent with the efficient market's hypothesis, S^*_{post} should equal $\underline{S}$. Further research could focus on the information that can be inferred from market events around CoCos, such as issuance or conversion, as well as the evolution of their market prices against the information already reflected in the stock price.

3.2 Principal Write-Down

In contrast to contingent convertible bonds which convert to equity, principal write-down CoCos suffer a full, partial, or staggered write-down of their face value when the trigger is breached (de Spiegeleer et al., 2014, p. 81-82). As shown in figure 3.1, principal write-down contingent convertible bonds have been growing in importance since the first CoCos were issued in 2009. This shift towards write-down CoCos has been driven by several competing reasons, which all make write-down a more attractive option than conversion to equity:

Privately Held Banks

Privately held banks, such as Rabobank from the Netherlands, have no publicly traded shares and therefore cannot offer conversion to equity (de Spiegeleer et al., 2014, p. 81).

Majority Ownership

Some banks are tightly controlled by a group of shareholders who own a majority stake, such as KBC, and offering contingent convertible bonds that convert to equity would expose the owners to dilution that could jeopardize their majority owner-

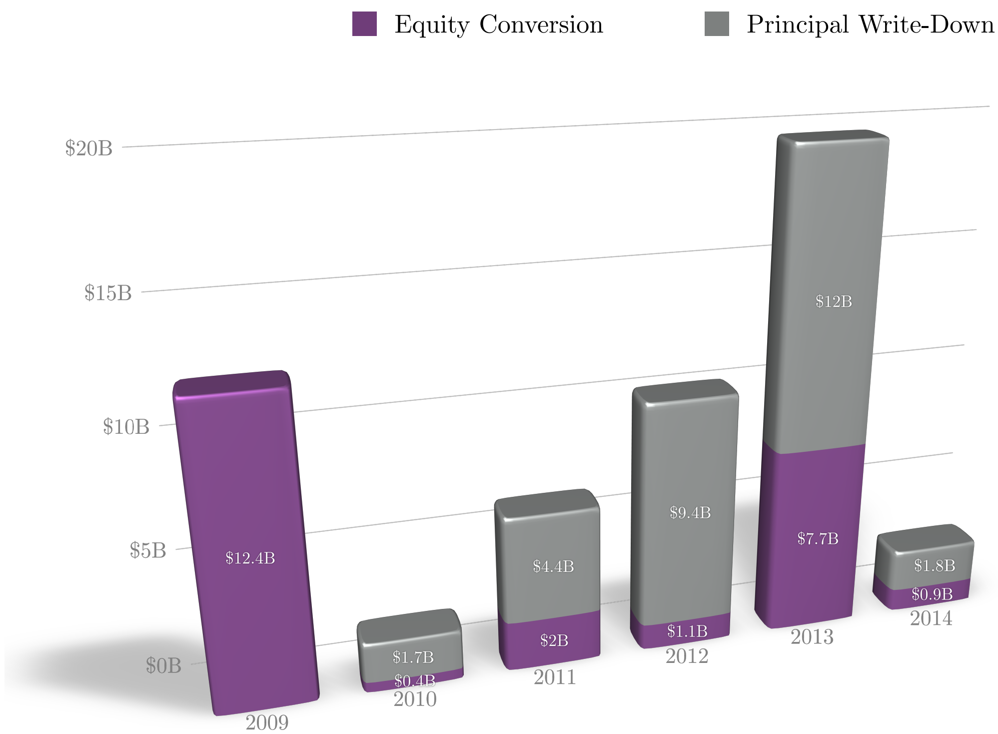

Figure 3.1: Contingent Convertible Bonds Issued Since 2009 (see data in Appendix A)

ship (de Spiegeleer et al., 2014, p. 81).

Bond Funds

Some bond funds are not allowed to own securities that have the possibility of converting to equity, which limits the potential market for conversion to equity CoCo and may make the issuance of write-down CoCos more profitable (de Spiegeleer et al., 2014, p. 81).

If the trigger is breached, the write-down process is much simpler than conversion to equity, but different write-down CoCos still have their own peculiarities. The current forms of write-down used in the CoCos issued are full, partial and staggered write-down (de Spiegeleer et al., 2014, p. 82). Full and partial write-down is simply reducing the entire or a predetermined portion of the face value of the bond (de Spiegeleer et al., 2014, p. 82). Partial write-down can worsen the situation in which the bank finds itself, as the terms of some partial write-down CoCos such as the ones issued by Rabobank include an immediate cash repayment of the share not written down, which in Rabobank's case is 25% (Pitt et al., 2011, p. 17). Such a cash outflow at a time in which the financial institution is struggling with the strength of its balance sheet could create a liquidity crisis (Pitt et al., 2011, p. 10). Staggered write-down is an innovation of the Swiss ZKB bank and it entails writing-down the face value of the bond in tranches of 25% until the measure of capitalization is back above the level at which the trigger was breached (de Spiegeleer et al., 2014, p. 82).

Mathematically, a full write-down bond is a simple extension of the definition used for the conversion to equity CoCos. As shown in equation 3.4, the conversion price for a write-down CoCo can be found by taking the limit of equation 3.1 as the conversion ratio goes to zero, and is simply equal to infinity:

$$\lim_{C_r \to 0} C_P = \lim_{C_r \to 0} \frac{N}{C_r} = \infty \tag{3.4}$$

3.3 Trigger

The definition of the trigger used for conversion or write-down is perhaps the most important characteristic affecting the pricing of contingent convertible bonds as it has a profound impact on the probability of the trigger being breached. Although the trigger can take various forms, almost all contingent convertible bonds issued have made use of accounting ratios as their primary trigger, and in some cases, a secondary regulatory trigger.

3.3.1 Accounting Trigger

The accounting trigger is the most obvious choice for contingent convertible bonds as it is directly tied to the capitalization of the bank and it is a good measure of the health of its balance sheet. The vast majority of the contingent convertible bonds issued since 2009 use the core equity tier 1 capital ratio with a trigger ranging from 5 to 8% (de Spiegeleer and Schoutens, 2014). Although the accounting trigger seems the perfect choice for appropriate recapitalization, it has some important downsides. First, the accounting ratios are backward-looking and are not continuously observable (Shang, 2013, p. 17) as the balance sheet is published only quarterly. This was made painfully obvious during the financial crisis, when Bear Sterns and Lehman Brothers failed while still reporting capital ratios above the regulatory minimum (de Spiegeleer and Schoutens, 2012, p. 28). Finally, an accounting trigger may also expose investors to manipulation of accounting ratios as GAAP and IFRS allow some leeway in reporting (Shang, 2013, p. 17).

3.3.2 Market Trigger

A market trigger could for example be based on the underlying stock falling below a predetermined value. It resolves many of the issues that arise out of the use of an accounting ratio, as market prices are forward-looking and continuously observable, but it is not as perfectly correlated with the level of capitalization of the bank. It may also worsen the concerns of manipulation (Shang, 2013, p. 17) as market participants with competing interests could take highly levered positions to provoke the desired outcome. No contingent convertible bonds have been issued using a market trigger, most likely because of the concerns of market manipulation, but it remains important academically as the credit and equity derivatives model presented in chapter 4 use an equivalent market trigger as a proxy for the accounting trigger.

3.3.3 Regulatory Trigger

The regulatory trigger is quite simply shifting the power of conversion or write-down away from the pre-determinedness of an accounting ratio or a stock price to instead rely on the judgement of the national regulator. No contingent convertible bonds have adopted it as a primary trigger, but it is gaining popularity as a secondary trigger. In fact, the Canadian national regulator, the Office of the Superintendent of Financial Institutions, is requiring that all contingent convertible bonds issued by Canadian banks include a regulatory trigger in addition to their primary trigger (OSFI, 2011, p. 2).

3.3.4 Other Triggers and Combined Triggers

A broad variety of other measures can be used as a triggering mechanism, which further obscures the market for contingent convertible bonds. Each new issue requires an analysis of its characteristics and the potential agency costs that may arise as a result of how they interact with market dynamics. An example of such an alternative trigger is Swiss Re's 2013 issuance of contingent convertible bonds, which use a 125% score on the Swiss Solvency Test as their trigger (de Spiegeleer et al., 2014, p. 85). It is also possible to combine triggers such that multiple triggers need to be satisfied simultaneously to set off conversion or write-down. No contingent convertible bonds have been issued using combined triggers, but adopting this structure has been recommended by the 2009 Squam Lake Working Group on Financial Regulation. They argue that requiring both the financial institution to be insolvent and the national regulator to declare a systemic financial crisis would restore "an important disciplining force for management" that arises from the presence of debt that can cause bankruptcy on the balance sheet (Squam Lake Working Group on Financial Regulation, 2009).

Chapter 4

Pricing

With a deeper understanding of the regulatory environment of banking capital and the numerous characteristics of the various contingent convertible bonds, we can now turn to the three main pricing models: the structural, credit derivatives, and equity derivatives model.

4.1 Structural Model

The structural model for pricing contingent convertible bonds originates in Pennacchi (2011) and is based on the work of Merton (1974) for pricing corporate debt. The approach is structural as it directly models the evolution of the main components of the balance sheet, such as deposits and assets. Contrarily to the credit and equity derivatives models covered in sections 4.2 and 4.3 respectively, the structural model incorporates a jump process when modelling the evolution of the bank's asset (Pennacchi, 2011, p. 3). This provides a more realistic framework in which to price contingent convertible bonds, as their downside risk, which is the path that significantly draws down their price, is likely to take place during a systemic crisis characterized by downward jumps in the bank's assets.

4.1.1 Change in Deposits

Based on empirical evidence that banks adapt their borrowing and lending activity to remain at their target capital ratios, Pennacchi (2011, p. 7) models deposits as a non-stochastic mean-reverting process of time as shown in equation 4.1:

$$\frac{dD_t}{D_t} = g(x_t - \hat{x})dt \tag{4.1}$$

With the following definitions (Pennacchi, 2011, p. 7):

$$A_t = \text{assets} \tag{4.2}$$

$$D_t = \text{deposits} \tag{4.3}$$

$$x_t = \frac{A_t}{D_t} \tag{4.4}$$

$$\hat{x}_t = \text{target asset-to-deposit ratio} > 1 \tag{4.5}$$

$$g = \text{positive constant} \tag{4.6}$$

Modelling deposits as a function of only time is not only consistent with the empirical data, but it is also theoretically logical, as deposits are non-stochastic liabilities. In spite of the strength of this process at describing the evolution of deposits, it is wholly inadequate for modelling the liabilities of the bank as a whole. Banks have evolved away from simply serving as lenders and deposit-takers, and now have a considerable portion of their operations devoted to derivatives markets. Options are classified either as assets or liabilities from the start, depending on whether the bank is the buyer or the writer, while other derivatives such as forwards can swing back and forth throughout their life (Valdivia-Velarde, 2012, p. 34). Modelling such liabilities hence requires a stochastic jump process. Since the structural model makes no attempt at modelling liabilities other than deposits, it must be assumed that they are modelled as negative assets using the stochastic difference equation describing the evolution of assets over time.

4.1.2 Change in Assets

Contrarily to deposits, assets are assumed to follow a stochastic process with a Brownian motion and a Poisson jump process as in equation 4.7 (Pennacchi, 2011, p. 32):

$$\frac{dA_t}{A_t} = \underbrace{\left[(r_t - \lambda k) - (r_t + h_t)\frac{D_t}{A_t} - c_t b_t \frac{D_t}{A_t}\right] dt}_{\text{Drift}} + \underbrace{\sigma dz}_{\text{Brownian Motion}} + \underbrace{(Y_{q_{t-}} - 1)dq_t}_{\text{Poisson Jump}} \tag{4.7}$$

The first term in the drift, $r_t - \lambda k$, includes the rate of return on the bank's asset, r_t and the negative mean of the Poisson process, as λ is the probability of a Poisson jump and $k_t \equiv E_t^Q \left[Y_{q_{t-}} - 1\right]$ (Pennacchi, 2011, p. 5). The second term in the drift, $-(r_t + h_t)\frac{D_t}{A_t}$, reduces the assets over time by the rate of return paid to depositors, $r_t + h_t$, where h_t is the deposit credit risk premium that compensates depositors for a potential loss (Pennacchi, 2011, p. 15-16). The last term in the drift, $-c_t b_t \frac{D_t}{A_t}$, accounts for the decline in assets as a result of coupon payments to bondholders (Pennacchi, 2011, p. 8).

4.1.3 Solving the Model

With processes defining the evolution of assets and deposits, the change in the asset to deposit ratio can be written as the change in assets minus the change in deposits as in equation 4.8 (Pennacchi, 2011, p. 32):

$$\frac{dx_t}{x_t} = \frac{dA_t}{A_t} - \frac{dD_t}{D_t} \tag{4.8}$$

The evolution of the asset to deposit ratio can be used to price the contingent convertible bond by generating a number of sample paths through a Monte Carlo simulation (Wilkens and Bethke, 2014, p. 62). Coupon payments are then discounted along the sample paths and the discounted values of all the sample paths are averaged to find the value of the CoCo bond (Wilkens and Bethke, 2014, p. 62). Along the sample path, the coupon is paid if the asset to deposit ratio has exceeded the trigger level, $x_c > 1$, since issuance[3] (Wilkens and Bethke, 2014, p. 73).

[3] A financial institution with an asset to deposit ratio of $x < 1$ is in bankruptcy (Wilkens and Bethke, 2014, p. 73).

4.1.4 Analysis of the Model

The strength of the structural model comes from its underlying pricing process, which is based directly on the accounting ratio driving conversion or write-down of contingent convertible bonds. This focus on the fundamental dynamic behind conversion or write-down of contingent convertible bonds knits the CoCo price and the accounting ratio to an extent that is unparalleled by the credit and equity derivatives models covered in sections 4.2 and 4.3 respectively. Conversely, the model is complex and makes use of numerous semi-observable and nonobservable variables and parameters (Wilkens and Bethke, 2014, p. 64). The nonobservable parameters include the deposit mean-reversion speed (g), the asset volatility (σ), and the jump intensity (λ) (Wilkens and Bethke, 2014, p. 67). Semi-observable variables include accounting data, which is only available quarterly with the publication of the balance sheet (Wilkens and Bethke, 2014, p. 64). The solution implemented by Wilkens and Bethke (2014, pp. 64, 67) is to use the stock price multiplied by the number of outstanding shares as a proxy for how the equity portion of the balance sheet moves over time which, in combination with the difference equation for deposits, lets them infer the change in assets in periods between financial reporting. This extension makes the model useful for pricing in the real world, as having an updated price only every three months would barely provide any value. It however defeats the purpose of having a pricing model that is based on the underlying accounting variable driving conversion or write-down, and makes the simpler and more insightful equity derivatives model a more serious contender.

4.2 Credit Derivatives Model

The credit derivatives model for pricing contingent convertible bonds is a simple extension of the models already being used for pricing credit derivatives. Its usefulness and appeal comes from the ease with which current market participants can adapt their current operations to pricing CoCos.

4.2.1 Description of the Model

In credit derivatives, the underlying bond is assumed to default according to the hazard rate λ (de Spiegeleer and Schoutens, 2012, p. 30). It is important to keep in mind that the latter is not a true statistical probability of default, but rather a Martingale probability of default. Using the credit triangle, the spread of a corporate bond relative to the risk-free rate can then be found by multiplying the loss from default by the Martingale probability of default (White, 2013):

$$\text{Spread} = \lambda(\text{Loss }\%) \tag{4.9}$$

de Spiegeleer and Schoutens (2012, p. 30) build on this approach by defining Spread_{CoCo} as the spread in basis points for which a contingent convertible bond sells above a corporate bond of the same bank. The Martingale probability of default, λ, becomes the Martingale probability of conversion or write-down, λ_{CoCo}, and similarly, (Loss %) becomes $(\text{Loss }\%)_{CoCo}$, such that equation 4.9 can be written as (de Spiegeleer and Schoutens, 2012, p. 30):

$$\text{Spread}_{CoCo} = \lambda_{CoCo}(\text{Loss }\%)_{CoCo} \tag{4.10}$$

de Spiegeleer and Schoutens (2012, p. 31) fail to clearly specify that the loss to CoCo holders in equation 4.10 needs to be the percentage loss and then they mistakenly write out the total loss as part of their definitions, but their final equation clearly makes use of the percentage loss. The percentage loss is simply the total loss to CoCo holders described in equation 3.2 divided by the face value of the bond:

$$(\text{Loss }\%)_{CoCo} = 1 - \frac{S^*}{C_P} \tag{4.11}$$

The Martingale probability of conversion or write-down during the life of the CoCo, p^*, can be inferred from option prices using Black-Scholes by assuming that there exists a stock price that is a market trigger equivalent to the CoCo's accounting trigger (de Spiegeleer and Schoutens, 2012, p. 31):

$$p^* = \Phi\left(\frac{log\left(\frac{S^*}{S}\right) - \mu\Delta T}{\sigma\sqrt{\Delta T}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{log\left(\frac{S^*}{S}\right) + \mu\Delta T}{\sigma\sqrt{\Delta T}}\right) \tag{4.12}$$

With the following definitions (de Spiegeleer and Schoutens, 2012, p. 31):

$$\mu = r - q - \frac{\sigma^2}{2} \tag{4.13}$$

$$q : \text{Continuous dividend yield} \tag{4.14}$$

$$r : \text{Continuous interest rate} \tag{4.15}$$

$$\sigma : \text{Volatility} \tag{4.16}$$

$$\Delta T : \text{Maturity of the contingent convertible } (\Delta T = T - t) \tag{4.17}$$

$$S : \text{Current share price} \tag{4.18}$$

The Martingale probability of conversion or write-down every period can then be calculated from the one for the life of the CoCo derived using Black-Scholes (de Spiegeleer and Schoutens, 2012, p. 31):

$$\lambda_{CoCo} = -\frac{ln(1 - p^*)}{T} \tag{4.19}$$

Equations 4.10, 4.11 and 4.19 combine to form an equation for the spread on contingent convertible bonds as a function of the Black-Scholes implied probability of conversion or write-down, the price of the stock at the time of conversion and the conversion price:

$$\text{Spread}_{CoCo} = \left(-\frac{ln(1 - p^*)}{T}\right)\left(1 - \frac{S^*}{C_P}\right) \tag{4.20}$$

4.2.2 Solving and Analysis

Solving the model empirically is not as trivial as inputting the market data into equation 4.20 to find the appropriate spread of a contingent convertible bond, as the equivalent market trigger S^* is unknown. The model can however be used with the spread as an input to solve for a implied market trigger (de Spiegeleer and Schoutens, 2012, p. 31). The model's inability to produce results without first observing the market price at which contingent convertible bonds are trading makes it ineffective at identifying under or overpricing. The model is however useful for comparing similar CoCos, as once the adjustments to account for their differences are made, they should have the same implied market trigger.

A key weakness of the model, however, is that it overlooks the loss of the stream of coupon payments that results from conversion or write-down (de Spiegeleer and Schoutens, 2012, p. 32). Serjantov (2011) tweaked the model to account for coupon payments by weighing the face value and the coupon payments according to the cumulative Martingale probabilities (Wilkens and Bethke, 2014, p. 62). The extension significantly complicates the model and makes the equity derivatives model covered in section 4.3 a more appealing choice.

Using the stock price and the CoCo prices from March 14th 2014, de Spiegeleer and Schoutens (2013) applied the credit derivatives pricing model to a series of contingent convertibles bonds issued by Credit Suisse between 2011 and 2013, and found a series of implied market triggers ranging from CHF 2.33 to CHF 9.02. Credit Suisse's stock was trading for CHF 27.35 at the time. The wide range of implied market triggers illustrates the model's sensitivity to its various inputs, and may be the result of de Spiegeleer and Schoutens (2012) approach, which overlooks coupon payments.

4.3 Equity Derivatives Model

The equity derivatives model suffers from many of the same weaknesses as the previous two models, but it is structured in a way that simplifies its implementation as well as its use in hedging. The equity derivatives approach starts with a riskless bond at its core and adds the contingent convertible bonds' equity-like features using exotic options. This reliance on option theory allows for easy derivation of the CoCos' greeks, which provide unparalleled insights into the dynamics of hedging contingent capital.

4.3.1 Description of the Model

Conversion or write-down is the defining feature of contingent convertible bonds and any model that can appropriately describe their behaviour must take into account this binary set of outcome. Using an indicator function, **1**, as the binary representation of whether the trigger has been breached lets de Spiegeleer and Schoutens (2012, p. 32) write the payoff of the contingent convertible bond at maturity in a single equation as shown in equation 4.21:

$$P_T = N + C_r(S_T - C_P)\mathbf{1}_{\text{triggered}} = \begin{cases} C_r S_T & \text{if triggered} \\ N & \text{if not triggered} \end{cases} \tag{4.21}$$

Equation 4.21 is in general terms, but "triggered" can easily be replaced with the appropriate condition, such as $\min(\text{CET1}_t) \leq 7\%$ for an accounting trigger that is breached when the core equity tier 1 ratio falls below 7%. The coupons can be added to the contingent convertible bond in the same way. Each coupon payment is assigned its own indicator function, which indicates whether coupons are paid or not (de Spiegeleer and Schoutens, 2012, p. 32).

With a mathematical representation of the payoffs of contingent convertible bonds in hand, de Spiegeleer and Schoutens (2012) develop a valuation technique which offers a similar set of payoffs. Much like in the credit derivatives approach and the empirical application of the structural approach, an unknown market trigger that is equivalent to the accounting trigger is assumed to exist. de Spiegeleer and Schoutens (2012, p. 32) then make use of this market trigger to replace the indicator function with exotic knock-in options that only become active when the market trigger is breached as shown in equation 4.22:

$$\begin{aligned} \text{CoCo Price} = {} & \text{Riskless Bond} + C_r(\text{Down and In Forwards}) \\ & - \sum(\text{Binary Down and In Options}) \end{aligned} \tag{4.22}$$

The down and in forwards for the face value are constructed using a long down and in call and a short down and in put, while the down and in options for the coupons are short binary options such that they only have a fixed negative payoff if the trigger is breached, offsetting the foregone coupons that are part of the riskless bond (de Spiegeleer and Schoutens, 2012, p. 32). Based on the Black-Scholes fundamental partial differential equation, de Spiegeleer and Schoutens (2012) find the closed form solution written out in appendix B.1.

4.3.2 Solving and Analysis

As in the credit derivatives model, the equivalent market trigger is unknown. Empirical applications of the model must hence make use of observed market prices for contingent convertible bonds to solve for an implied market trigger. Relative to other models, the equity derivatives model thus provides no additional insights into under or overpricing of CoCo bonds.

By using synthetic forwards in the construction of the contingent convertible bond, the model ignores any potential dividends that may be paid out after conversion (de Spiegeleer et al., 2014, p. 309). de Spiegeleer et al. (2014, p. 309) argue that the former is insignificant as a bank having just gone through conversion of its CoCos is unlikely to have a balance sheet healthy enough to pay dividends. The argument is consistent with Basel III's capital conservation buffer for low trigger CoCos, but extending it to high trigger CoCos may be overly simplistic.

Another weakness of using synthetic forwards to reproduce the payoff of the stock is that it ignores the value of voting rights (de Spiegeleer et al., 2014, p. 309). de Spiegeleer et al. (2014, p. 309) simply choose to ignore it as voting rights cannot easily be assigned a financial value. Although prepaid forwards typically sell for the stock price minus the present value of the dividends (McDonald, 2013, p. 127), in other words ignoring the value of voting rights, making the same assumption when it comes to contingent convertible bonds may be overlooking the complex dynamics that surround the collapse of a bank.

If a low conversion price is chosen, CoCo holders may end up having considerable weight in choosing the institution's turnaround strategy. In such a situation, voting rights would not only be valuable to purchasers of contingent convertible bonds, but may also expose both the stock and CoCos to manipulation. Contingent convertible bonds on the edge of conversion could provide a large pool of voting rights for activist investors looking for control of the board of directors. Further research could focus on the potential agency costs that come with the shift in voting power that accompanies conversion.

Overall, the equity derivatives approach's key weakness is common to all models, and is the imperfect relationship between the contingent convertible bond's accounting trigger and the model's implied market trigger. Its elegant construction and its ease of use for hedging, which is the topic of the next chapter, makes it the best compromise.

4.3.3 Closed-Form Solution for Write-Down CoCos

With the growing importance of write-down contingent convertible bonds highlighted in figure 3.1, making the model consistent with their characteristics is crucial. Adapting the equity derivatives model written out in appendix B.1 to pricing full write-down CoCos simply requires taking the limit as $C_r \to 0$. With $K = C_P$, $C_r = \frac{N}{C_P}$, and keeping in mind that, as shown in equation 3.4, $C_P \to \infty$ as $C_r \to 0$, the model collapses to the following set of equations:

$$\mathbf{P} = \mathbf{A} + \mathbf{B} + \mathbf{C} \tag{4.23}$$

$$\mathbf{A} = Ne^{-r(T-t)} + \sum_{i=1}^{k} c_i e^{-r(t_i-t)} \tag{4.24}$$

$$\mathbf{B} = -Ne^{-r(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) - Ne^{-r(T-t)}\Phi\left(-x_1 + \sigma\sqrt{T-t}\right) \tag{4.25}$$

$$\mathbf{C} = -\sum_{i=1}^{k} c_i e^{-r(t_i-t)} \left[\Phi\left(-x_{1i} + \sigma\sqrt{t_i-t}\right) + \left(\frac{S^*}{S}\right)^{2\lambda-2} \Phi\left(y_{1i} - \sigma\sqrt{t_i-t}\right)\right] \tag{4.26}$$

Full write-down CoCos are just a specific case of more general write-down CoCos with the recovery rate set equal to zero. Applying the concept to more general write-down CoCos, including partial write-down such as the ones issued by Rabobank, can be done by writing the conversion price and the conversion ratio as a function of the recovery rate (R) and the stock price at the time of write-down:

$$C_P = \frac{S^*}{\mathrm{R}} = K \tag{4.27}$$

$$C_r = \frac{N(R)}{S^*} \tag{4.28}$$

The model then generalizes to all write-down CoCos in the following way:

$$\mathbf{P} = \mathbf{A} + \mathbf{B} + \mathbf{C} \tag{4.29}$$

$$\mathbf{A} = Ne^{-r(T-t)} + \sum_{i=1}^{k} c_i e^{-r(t_i-t)} \tag{4.30}$$

$$\begin{aligned}\mathbf{B} = &\left(\frac{N(R)}{S^*}\right) Se^{-q(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda} \Phi\left(y_1\right) - Ne^{-r(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\ &- Ne^{-r(T-t)} \Phi\left(-x_1 + \sigma\sqrt{T-t}\right) + \left(\frac{N(R)}{S^*}\right) Se^{-q(T-t)} \Phi(-x_1)\end{aligned} \tag{4.31}$$

$$\mathbf{C} = -\sum_{i=1}^{k} c_i e^{-r(t_i-t)} \left[\Phi\left(-x_{1i} + \sigma\sqrt{t_i - t}\right) + \left(\frac{S^*}{S}\right)^{2\lambda-2} \Phi\left(y_{1i} - \sigma\sqrt{t_i - t}\right)\right] \tag{4.32}$$

The model not only adapts nicely to the characteristics of write-down contingent convertible bonds, but is also devoid of some of the weaknesses that came about when applying it to conversion CoCos, including voting rights and dividend payments after conversion.

Chapter 5

Hedging

The hedging dynamics of contingent convertible bonds are at the heart of the concerns around how those new securities behave in times of crisis. From impediments on hedging (Whittall, 2014) to risks of a self-fulfilling death spiral (Atkins, 2014), potential complications abound. Concerns by market participants and the financial press always arise when untested products enter the market, and in this case even more so as CoCos are aimed at stabilizing the financial system in times of crisis. The crucial difference this time around is that those concerns are justified by the mathematical models. This chapter begins with an analysis of how contingent convertible bonds can be hedged, and then delves into the market dynamics of such hedging to gain a deeper understanding of the potential pitfalls that exists for CoCo holders, stockholders and the issuing financial institution. The chapter concludes with an evaluation of credit default swaps, and how the creation of a market for CoCo CDS could alleviate or worsen the hedging concerns for the various market participants.

5.1 Delta Hedging

In equity derivatives, the delta of a financial derivative is the partial derivative of the security with respect to the underlying stock price. Using the equity derivatives approach at pricing contingent convertible bonds with an equivalent market trigger, the price of the CoCo is constructed using exotic options which derive their value from the underlying stock. It is hence possible to calculate how the price of the contingent convertible bond evolves as the

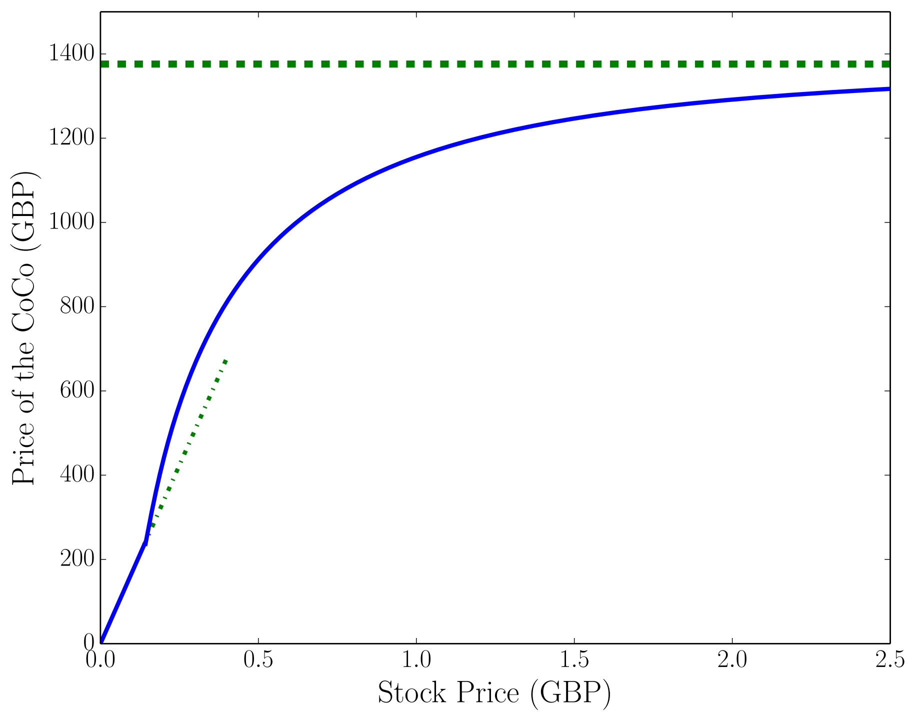

Figure 5.1: Price of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price

price the stock changes. Figure 5.1 shows the price of a contingent convertible bond issued by Lloyd's Banking Group in 2009 as a function of the bank's underlying stock price[4]. As the stock price goes to infinity, the price of the CoCo asymptotically approaches the green dotted line just below £1400, which is the maximum value that the bond can take. At that price, the return on the CoCo would be the riskless rate of return. Conversion happens at the inflection point, which is just below 14 pence.

Figure 5.1 also provides important insights into the delta and the gamma of the security. As the first derivative with respect to the stock price, the delta is the slope and the gamma the convexity of figure 5.1. Delta is hence very low at high stock prices, it increases as the stock price declines, and becomes linear and equal to the conversion ratio C_r when the CoCo converts. The gamma is negative throughout the life of the CoCo and zero after conversion. Figures 5.2 and 5.3 concur with this analysis.

With a better understanding of how the delta of a contingent convertible bond evolves

[4]The parametrization is done with data from November 16th 2013 using Lloyd's Banking Group's contingent convertible bond XS0459093364 (ISIN) (de Spiegeleer and Schoutens, 2013).

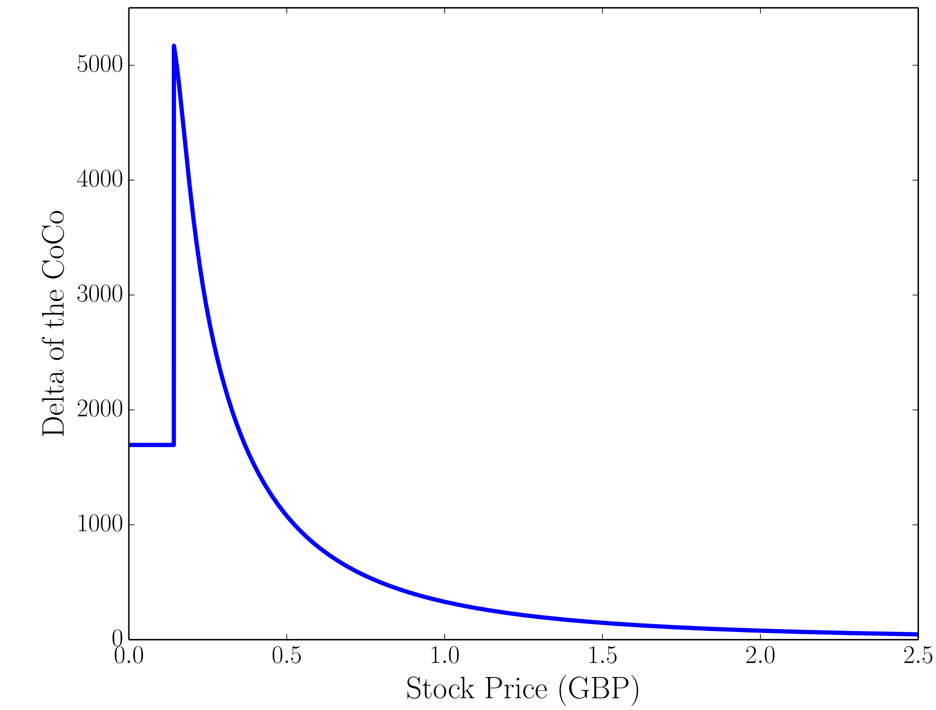

Figure 5.2: Delta of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price

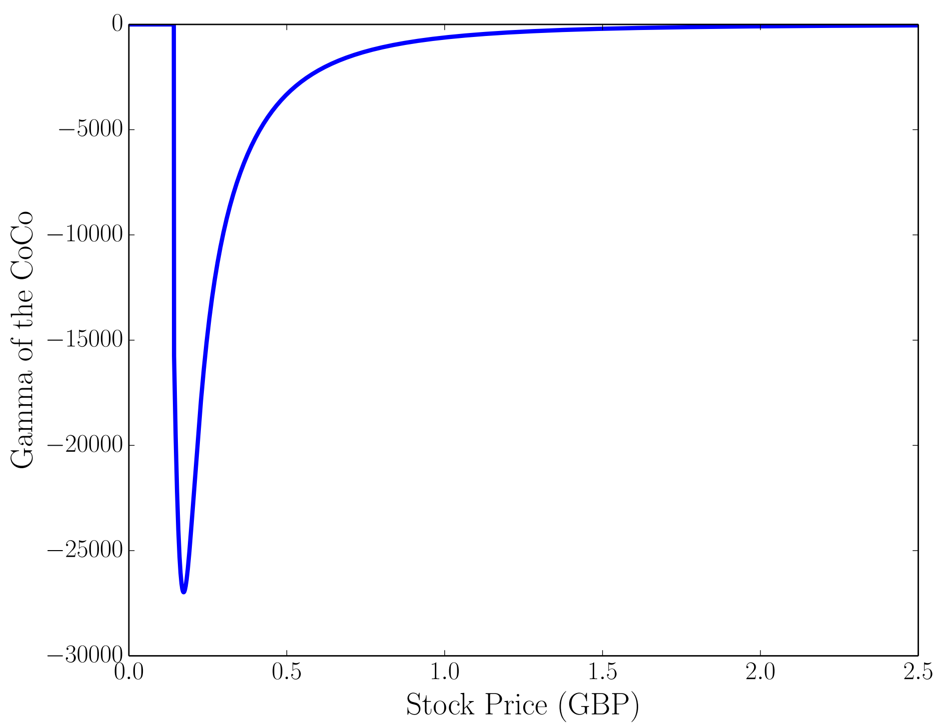

Figure 5.3: Gamma of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price

with the underlying stock price, let's now turn to how delta hedging is done in practice. In its simplest form, delta hedging can be done by taking the offsetting position in the underlying stock as in equation 5.1 (Jarrow and Turnbull, 2000, p. 273):

$$-n_1\Delta_{CoCo} = n_2\Delta_{Stock} = n_2(1) \tag{5.1}$$

Since the delta of the stock is one, i.e. $\Delta_{Stock} = 1$, a simple delta hedge is achieved by shorting $n_1\Delta_{CoCo}$ shares of the underlying stock. As highlighted by figure 5.2, the delta of the CoCo changes significantly as the price of the underlying stock drifts, and so a perfect hedge relies on continuous trading and the evolution of the stock price being driven by geometric Brownian motion (Jarrow and Turnbull, 2000, p. 294). Since the evolution of the stock price likely incorporates Poisson jumps, and transactions costs make it impossible to rebalance in continuous time, a perfect hedge is impossible to achieve.

In spite of those limitations, delta hedging is commonly used by market participants and so its effect on the market as a whole and the financial institution issuing the contingent convertible bond is important. In normal times, delta hedging by CoCo holders should have a fairly muted effect on the underlying stock price. By the efficient market's hypothesis, any downward pressure on the stock caused by hedging should be offset by arbitrageurs. In times of crisis, however, a collapse in the stock price can rapidly increase the delta that needs to be hedged by CoCo holders. This effect is known as the death spiral and is the topic of the next section.

5.1.1 Death Spiral and Gamma Concavity

As seen in the previous section, the death spiral is the result of accelerating delta hedging by CoCo holders in difficult times. The delta of CoCos increases as the underlying stock decreases because of the concavity of the price of CoCos. As shown on figure 5.3, the concavity of the price of CoCos is reflected by the negative gamma as the bond approaches conversion. Even more interestingly, the gamma is getting more negative as it heads towards conversion, which means that concavity is increasing as the stock price drops. All of those features combine to create an environment that is conducive to a self-reinforcing collapse in the stock price in times of crisis. The hedging activity puts downward pressure on the stock price, and the ensuing price drop entices further shorting to maintain the delta hedge.

In a world with perfect information, the downward pressure on the stock price caused by delta hedging should have no effect on the stock price, as this trading activity provides the market with no information about the health of the bank. Even in cases where the latter argument does not hold and the stock price drops because of delta hedging, the price of the contingent convertible bond should not drop and CoCo holders should not readjust their delta hedge because the change in the stock price reflects no fundamental change in the health of the financial institution. In the current market environment, however, not only does the information asymmetry that prevails makes such foresight next to impossible, but market illiquidity may also severely limit market efficiency, which is precisely why the death spiral is so concerning.

Despite the grim outlook painted by the death spiral, it is important to keep in mind that a self-reinforcing collapse in the stock price caused by CoCo hedging cannot itself cause conversion. The vast majority of contingent convertible bonds issued have an accounting trigger based on the core equity tier 1 ratio, which means that, in a vacuum, the death spiral cannot trigger conversion. In the context of a financial crisis, however, a death spiral can spread panic, alter perceptions, and change the behaviour of the various stakeholders. These changes in perceptions and behaviour can potentially spread throughout the financial system and cause the very problems that contingent convertible bonds seek to address: liquidity issues and capitalization problems.

5.1.2 Volatility and the Vanna

The effects of volatility on delta hedging and the death spiral have been overlooked by the vast majority of the literature on contingent convertible bonds. In the equity derivatives pricing model, volatility of the underlying stock is crucially important to the probability of conversion, and ergo the price of CoCos. Figure 5.4 illustrates the evolution of the price of the contingent convertible bonds issued by Lloyd's Banking Group as a function of the bank's underlying stock price and volatility. The white dotted line indicates the point at which the equivalent market trigger is breached. The CoCo hence converts to equity at

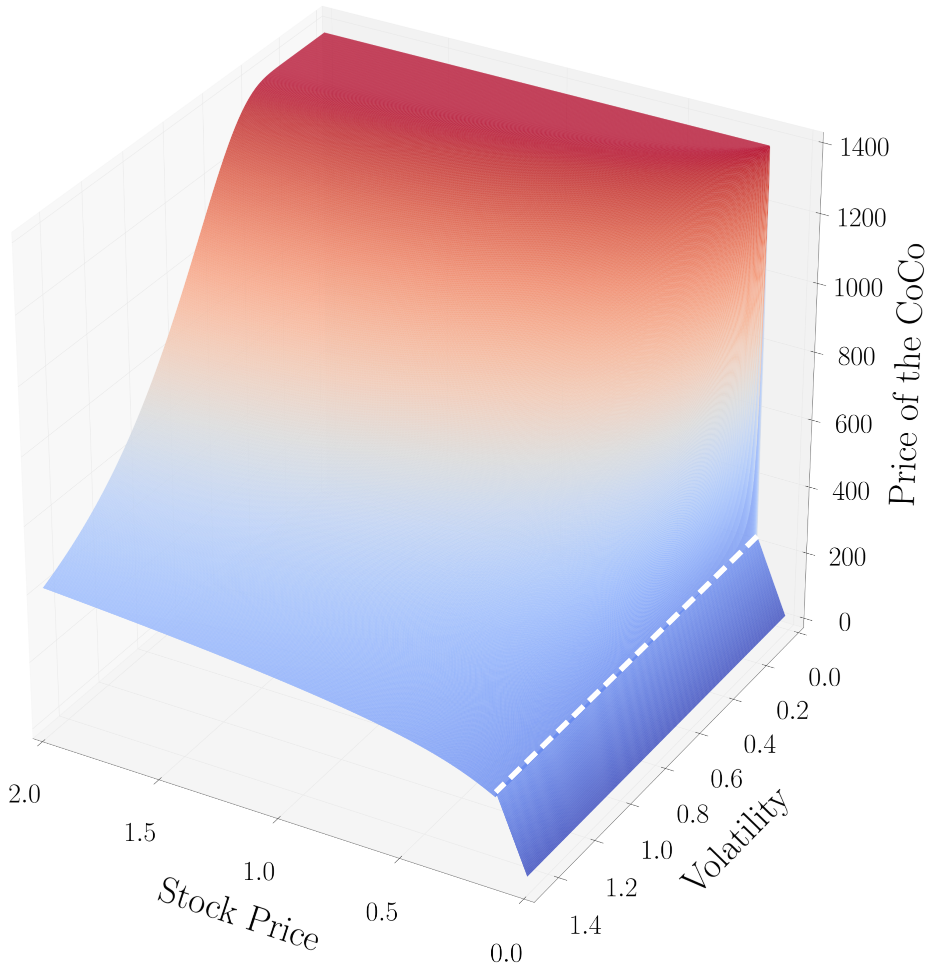

Figure 5.4: Price of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price and Volatility

lower stock prices. As expected, the value of the contingent convertible bond below that line is linearly dependent on the underlying stock price. Above the conversion trigger, the value of the CoCo peaks at low volatility and a high stock price as the probability of conversion approaches zero. At high volatility and a stock price approaching the equivalent market trigger, the price of the CoCo approaches its post-conversion value. The interesting characteristic highlighted by figure 5.4 is the change in the slope of the price of the CoCo relative to the stock price, which is the delta, as volatility increases. At high stock prices, the very low value of delta increases with volatility as the price of the CoCo begins dropping from its maximum value. Conversely, at stock prices close to conversion, delta declines with

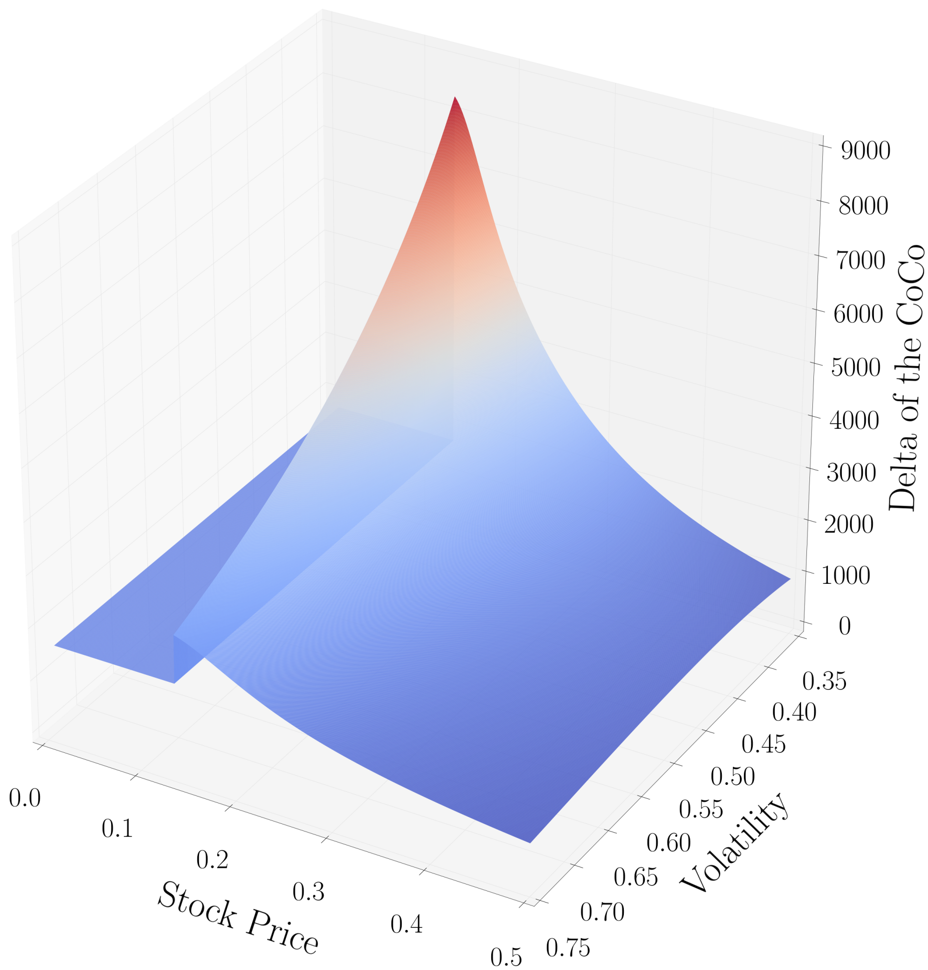

Figure 5.5: Delta of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price and Volatility. Note: the axes have been changed relative to figure 5.4 to make the surface easier to analyze.

an increase in volatility. Figure 5.5 illustrates the delta as a function of the underlying stock price and volatility at stock prices close to conversion and at a range of volatilities that brackets the volatility as of November 16th 2013. It is clearly consistent with the observations of how delta varies with volatility from figure 5.4.

Figure 5.5 adds to our analysis the impact on delta of a change in volatility holding the stock price constant. This effect is a less well-known second-order greek called vanna (Haug,

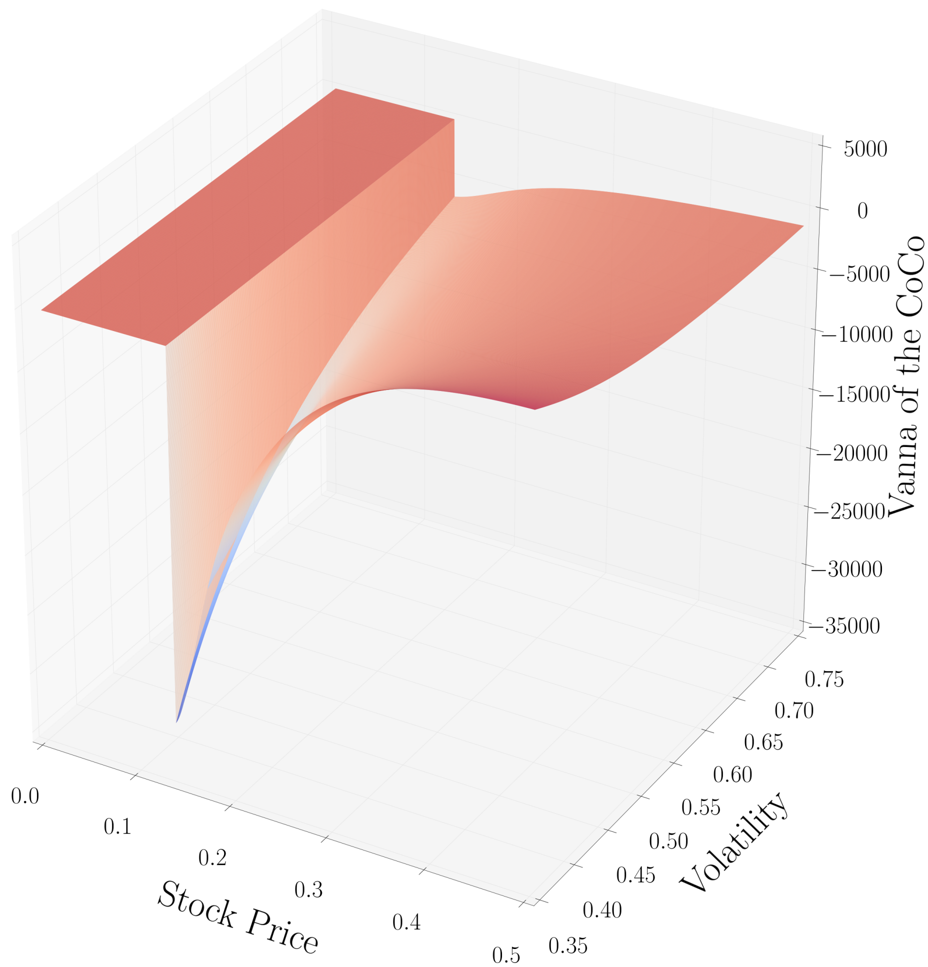

Figure 5.6: Vanna of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price and Volatility. Note: the axes have been changed relative to figure 5.4 and 5.5 to make the surface easier to analyze.

2006, p. 32):

$$\text{Vanna} = \frac{\partial^2 P}{\partial \sigma \partial S} = \frac{\partial \Delta}{\partial \sigma} = \frac{\partial \nu}{\partial S} \tag{5.2}$$

Figure 5.6 illustrates the vanna for the same range of stock prices and volatility as in figure 5.5. As expected, it is negative throughout the life of the contingent convertible bond, and zero once the bond has converted. It plunges at low volatility and at stock prices approaching conversion. Holding the stock price constant just above conversion, the strong negative vanna at volatility levels consistent with normal times shows how quickly the delta drops if volatility suddenly spikes. Vanna increases fairly rapidly as volatility increases, but it remains negative.

In times of crisis, volatility is expected to rise as a response to the uncertainty of the situation. This increase in volatility is an increase in the probability of conversion at every stock prices, which lowers the value of the contingent convertible bond. Ceteris paribus, the drop in the value of the CoCo caused by the increase in volatility lowers the potential downside of the CoCo while the potential downside on the underlying stock remains constant. This change in the relative downside lowers the delta of the CoCo. In essence, adding volatility to the analysis of delta hedging in times of crisis mitigates the consequences of a potential death spiral.

5.2 Delta-Gamma-Vega Hedging

The analysis of the hedging dynamics in times of crisis can be expanded to include the hedging of gamma and vega, as they are likely to be hedged by many market participants. Hedging of gamma and vega must be done through the options market as the underlying stock has no gamma or vega. Delta-gamma-vega hedging can be achieved with three linearly independent securities that depend on the underlying stock. Using the underlying stock and two options on the underlying stock, solving for the quantities of each securities to hold is a simple exercise in linear algebra (Jarrow and Turnbull, 2000, p. 293):

$$\begin{bmatrix} -n_1\Delta_{CoCo} \\ -n_1\Gamma_{CoCo} \\ -n_1\nu_{CoCo} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_3 & \Delta_4 \\ 0 & \Gamma_3 & \Gamma_4 \\ 0 & \nu_3 & \nu_4 \end{bmatrix} \begin{bmatrix} n_2 \\ n_3 \\ n_4 \end{bmatrix} \tag{5.3}$$

The system of equations can be made smaller for delta-gamma or delta-vega hedging, as both cases require only two linearly independent securities. Any linearly independent options on the underlying stock can be used to achieve this hedge, but to limit extreme positions, long put options of different strikes or maturities provide the best mix of greeks. Contingent convertible bonds have positive delta, and negative gamma and vega, while put options

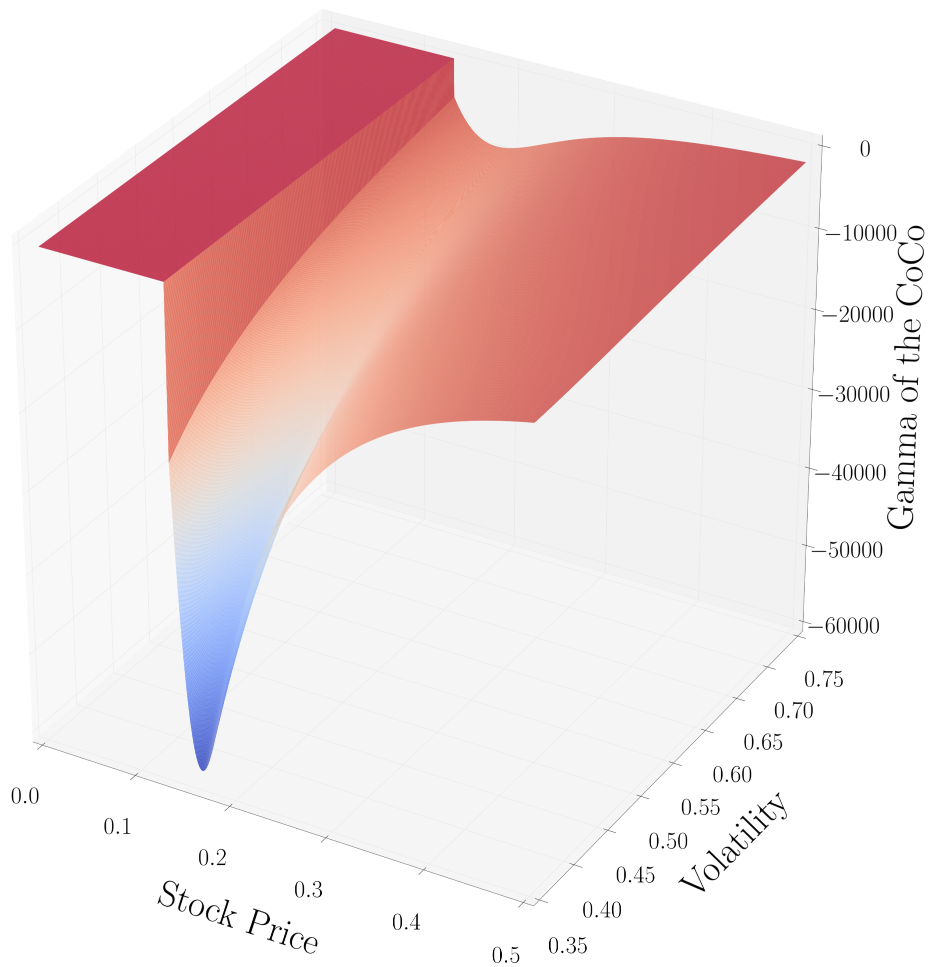

Figure 5.7: Gamma of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price and Volatility. Note: the axes have been changed relative to figure 5.4 and 5.5 to make the surface easier to analyze.

have negative delta, and positive gamma and vega.

Figure 5.7 and 5.8 illustrate the gamma and the vega of the contingent convertible bond issued by Lloyd's Banking Group respectively. Much like the decrease in delta observed when the volatility increases, holding the stock price constant, gamma becomes less negative as volatility increases. This means that delta decreases less and less as volatility increases, and that the mitigating effect of an increase in volatility on the death spiral gets smaller as volatility increases. Consistent with the vanna shown in figure 5.6, the negative vega steeply converges to zero as the stock price approaches conversion. In essence, the probability

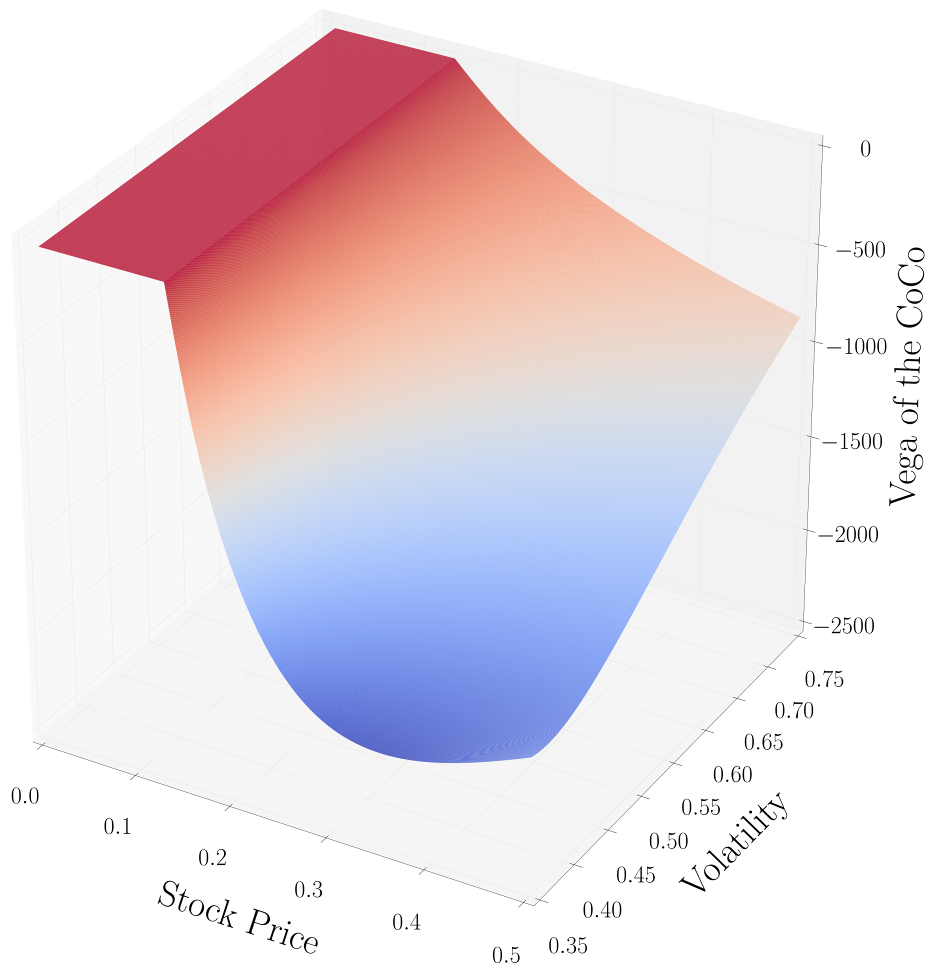

Figure 5.8: Vega of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a Function of the Underlying Stock Price and Volatility. Note: the axes have been changed relative to figure 5.4 and 5.5 to make the surface easier to analyze.

of conversion becomes so close to one that the value of the contingent convertible bond approaches its post-conversion value and volatility has almost no impact on the price.

The impact of delta-gamma-vega hedging on markets and the issuing bank seems difficult to predict because the rebalancing needed to keep a contingent convertible bond fully delta-gamma-vega hedged depends on the derivative securities chosen by the holders to construct the hedge. In reality, the choice of securities is in many ways irrelevant as derivative markets only redistribute risk. Holding the market's willingness to hold risk constant, any excess delta will be hedged with the underlying stock, which brings us back to the results from section 5.1.

5.3 Credit Default Swaps

With contingent convertible bonds gaining broader acceptance in financial markets, demand for hedging the conversion and write-down risk is growing. Hedging the securities' greeks is at best imperfect in normal times, and completely inadequate in cases of conversion or write-down. Creating a market for credit default swaps (CDS) on contingent convertible bonds would provide CoCo holders with an ability to purchase insurance against conversion or write-down. In addition, nothing would prevent investors wanting to bet against CoCos to purchase CDS without owning the underlying contingent convertible bond.

The demand for better hedges has grown so strong that the International Swaps & Derivatives Association is planning to create such a market in September 2014 (Moses, 2014). The creation of a market for CDS on CoCos spurs questions about its impact on pricing, hedging and agency costs. With all pricing models unable to identify over or underpricing, this section focuses on the impact of creating credit default swaps for CoCos on market completeness and their implications for overpricing.

5.3.1 Market Completeness

The creation of financial derivatives is generally ineffective at expanding the market. As a matter of fact, the use of the Modigliani and Miller (1958) theorem through the creation of a replicating portfolio is key to most option pricing models. Applying the same concepts to the pricing of a credit default swap on a full write-down CoCo, a replicating portfolio can be constructed using the short contingent-convertible bond and a long risk free bond. The credit default swap being priced by this replicating portfolio would pay off the present value of the loss to CoCo holders if written down or if the financial institution defaults.

In reality, credit default swaps on contingent convertible bonds are quite different from financial derivatives on common equity because of the constraints on shorting CoCos imposed by their market structure. There are no regulatory restrictions on shorting CoCos, but as thinly traded over-the-counter securities, they are difficult to short effectively. Under those circumstances, the addition of credit default swaps can result in a Pareto-superior expansion of the market. The effect of this market expansion on the equilibrium price of contingent convertible bonds and the arbitrage-free relationship between credit default swaps and the replicating portfolio is the subject of the next section.

5.3.2 Overpricing of Contingent Convertible Bonds and Arbitrage-Pricing of Credit Default Swaps

Modelling the difficulties in shorting as a binding constraint on shorting provides insights into the equilibrium that prevails under those circumstances. In the current environment, many market participants are concerned that contingent convertible bonds are overpriced and thus do not adequately compensate investors for the risk they provide (Thompson, 2014b). The constraint on shorting may partially explain this perceived overpricing as it reduces the downward pressure on CoCo prices. In essence, market participants value contingent convertible bonds with different subjective probabilities, but only those with a bullish outlook are allowed to participate in the market.

Empirically, Boehme et al. (2006) tested Miller (1977) theory that markets with constraints on short-selling in which uncertainty creates divergence of opinion are characterized by overvaluation. Using data from 1988 to 2002, they found that “the most short-sale constrained, high dispersion stocks” significantly underperformed relative to average returns (Boehme et al., 2006, p. 485). They also find that a high cost of short selling is necessary for divergence of opinion to cause overpricing (Boehme et al., 2006, p. 485).

The results found by Boehme et al. (2006) can easily be applied to the current state of the market for contingent convertible bonds. Not only are CoCos short-sale constrained, but divergence of opinion is inherent to their design, which depends on a subjective probability of conversion or write-down. Figure 5.9 illustrates the effect theoretically in an Edgeworth box with two consumers and two assets. As in Milne (2003, pp. 37-38), the dashed box represents the boundaries imposed by eliminating short-selling, and point E the endowments of assets 1 and 2 that each consumer owns. The indifference curves are not linear, as the two assets are not perfect substitutes, i.e. they are not linearly dependent. The contingent convertible bond is represented by asset 1.

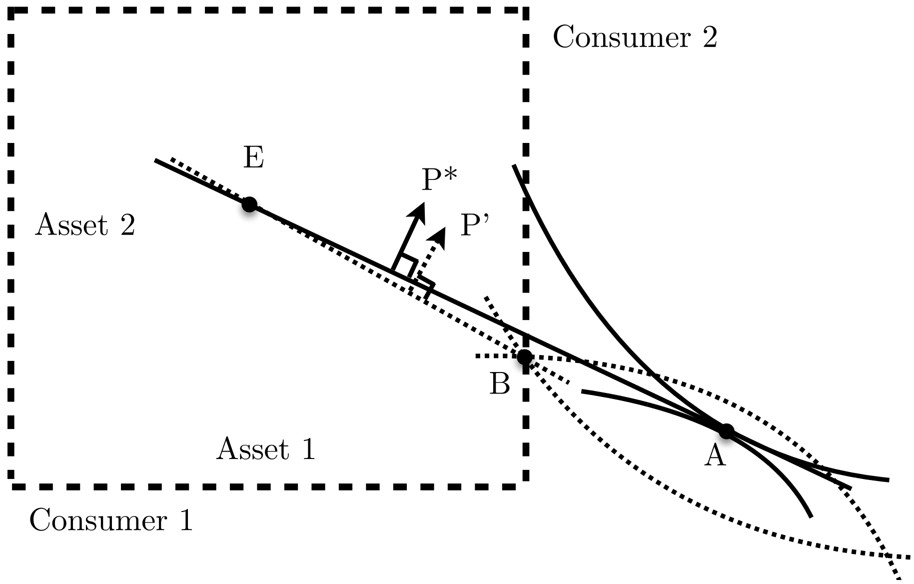

Figure 5.9: Edgeworth Box

In the competitive equilibrium, at point A, consumer 2 wishes to short the CoCo while consumer 1 is willing to own more than the two consumers' total endowment of the CoCo. The relative price of the two assets is $P^* = \frac{P_2}{P_1}$. When imposing restrictions on short-sales, the equilibrium moves to point B, with consumer 2 owning no CoCos. The relative price of the two assets decreases to P'. Since the relative price is defined as the price of asset 2 over the price of asset 1, the constraint on shorting causes an increase in the relative price of the contingent convertible bond.

The addition of credit default swaps to this market eliminates the effective restrictions on shorting and restores the competitive equilibrium. Having credit default swaps does not make shorting of the underlying CoCo easier, but they can be used to create a replicating portfolio that reproduces shorting. The CDS market soaks up the demand for shorting and a portion of the long demand for CoCos transfers from the underlying contingent convertible bonds to the CDS market. This transfer in long demand takes place to restore the arbitrage-free pricing relationship between the two markets.

The inability to short in the underlying CoCo market does not prevent the pricing of credit default swaps by arbitrage. Even though the short replicating portfolio cannot be reproduced using the underlying CoCo, the reproducibility of the long replicating portfolio ensures that the relationship holds. As argued in the previous paragraph, the CDS market absorbs the entire demand for short positions. Buyers of contingent convertible bonds will then maintain the arbitrage-free relationship by buying the cheapest of the underlying CoCo and the long replicating portfolio.

Overall, the expansion of the market caused by the creation of a market for credit default swaps on contingent convertible bonds can address concerns of overpricing by restoring the competitive equilibrium, which reflects the market's full range of subjective probabilities of conversion or write-down. The return to the competitive equilibrium also restores the no-arbitrage relationship between contingent convertible bonds and the replicating portfolios, and hence makes it possible to price the credit default swaps using linear algebra.

5.3.3 Hedging Credit Default Swaps and Privately-Held Banks

Although the inability to short in the underlying CoCo market is not an impediment to the restoration of the competitive equilibrium in normal times, it can have disastrous consequences in times of crisis. By preventing the creation of a short replicating portfolio, it limits CDS writers' ability to effectively hedge their positions and can leave both the CoCo and the CDS markets extremely vulnerable in times of crisis.

If the writing of credit default swaps on contingent convertible bonds suffers from concentration of issuance like credit default swaps on mortgage-backed securities did with AIG in 2008, CoCo holders having purchased insurance through the CDS market could be exposed to significant counterparty risk. The constraints on shorting the underlying CoCos leaves a firm having written credit default swaps on CoCos with purchasing CDS as its only hedge. The latter option is likely to be prohibitively expensive for a large insurer, such as AIG pre-2008, and would potentially force it into bankruptcy.

The effect is even more pronounced for CoCos issued by privately-held banks. With no publicly traded stock against which to hedge, credit default swaps provide one of the only ways of hedging contingent convertible bonds. As the only hedge, the CDS market for privately-held banks can take on more dramatic proportions relative to the size of those institutions.

Chapter 6

Trinomial Trees and Trigger Sensitivity

The sensitivity of the price of a contingent convertible bond to the various conversion or write-down triggers provides a picture of the stability of the market-clearing equilibrium. With all pricing models relying on an unknown implied market trigger, demand for CoCos can vary widely with swings in the subjective opinion of market participants. The prospect of conversion or write-down triggered by the national regulator adds to the uncertainty. Using a trinomial tree for pricing contingent convertible bonds allows the straightforward addition of an exogenous regulatory trigger, which can then be used for sensitivity analysis. This chapter hence begins with the creation of a trinomial tree for pricing contingent convertible bonds, and then proceeds with applying the trinomial to an analysis of the price sensitivity to the equivalent market trigger and the subjective probability of regulatory conversion or write-down.

6.1 Path-Dependent Trinomial Tree

Pricing using a trinomial tree provides the best compromise between rapid convergence to the continuous time price and the flexibility necessary for adding exogenous shocks, such as conversion or write-down imposed by the national regulator. Applying a trinomial tree to the pricing of contingent convertible bonds is not trivial as the process driving

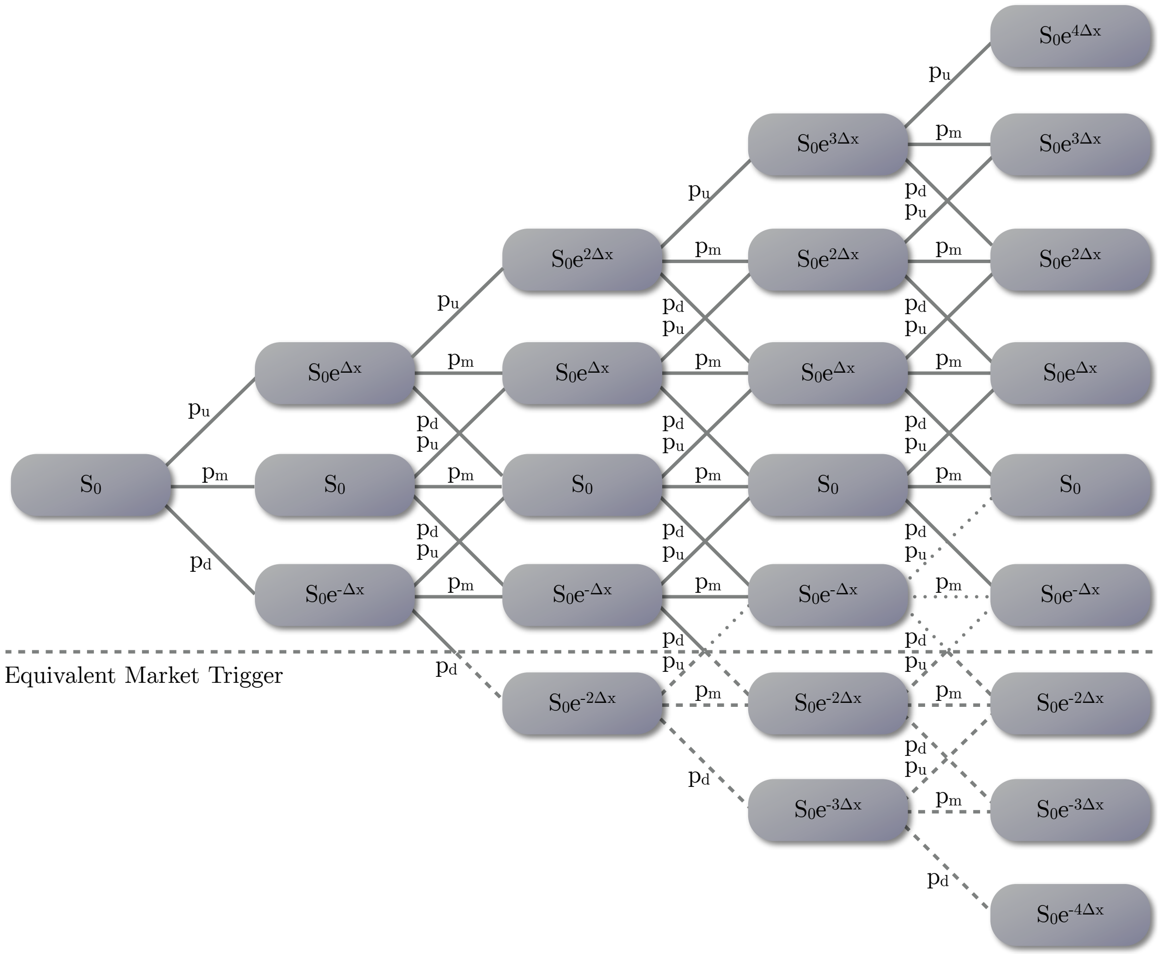

Figure 6.1: Trinomial Tree Modelling the Evolution of the Underlying Stock Price

the underlying stock price can be modelled in a reconnecting tree, but the payoff of the contingent convertible bond at each end node depends on whether the price of the underlying stock has dipped below the equivalent market trigger during the life of the CoCo. Figure 6.1 illustrates the evolution of the underlying stock price in a four period trinomial tree. In this example, the equivalent market trigger lies somewhere between $S_0e^{-2\Delta x}$ and $S_0e^{-\Delta x}$. Nodes linked by dashed lines represent states in which the CoCo has converted to equity or been written down, as the trigger has been breached. Nodes linked by dotted lines represent states in which the CoCo may or may not have converted to equity or been written down, depending on whether the underlying stock price has dipped below the equivalent market trigger in the past. It is hence a reconnecting path-dependent trinomial tree.

The trinomial tree is constructed under the assumption that the underlying stock price follows geometric Brownian motion such that (Clewlow and Strickland, 1999, p. 52):

$$dS = (r - \delta)Sdt + \sigma Sdz \tag{6.1}$$

Applying Ito's Lemma to equation 6.1 to obtain dx, where $x = ln(S)$ (Clewlow and Strickland, 1999, p. 52):

$$dx = vdt + \sigma Sdz \tag{6.2}$$

$$v = r - \delta - \frac{1}{2}\sigma^2 \tag{6.3}$$

The trinomial tree can then be parametrized in the following way (Clewlow and Strickland, 1999, pp. 52-53):

$$\Delta x = \sigma\sqrt{3\Delta t} \tag{6.4}$$

$$p_u = \frac{1}{2}\left(\frac{\sigma^2\Delta t + v^2\Delta t^2}{\Delta x^2} + \frac{v\Delta t}{\Delta x}\right) \tag{6.5}$$

$$p_m = 1 - \frac{\sigma^2\Delta t + v^2\Delta t^2}{\Delta x^2} \tag{6.6}$$

$$p_d = \frac{1}{2}\left(\frac{\sigma^2\Delta t + v^2\Delta t^2}{\Delta x^2} - \frac{v\Delta t}{\Delta x}\right) \tag{6.7}$$

Pricing a contingent convertible bond with this trinomial tree is done by finding the time zero Martingales of each potential payoff. First, the terminal nodes that are above the equivalent market trigger are assigned a payoff of one, while the ones below are assigned a payoff of zero for write-down CoCos and S_t/C_P for conversion CoCos. The method is repeated for all coupon payments but with zero in states below the equivalent market trigger even in the case of conversion CoCos. The challenge with intermediate coupon payments is that it is not clear to which node they belong. This problem is resolved by choosing the closest of the node before or after the coupon payment. The approximation becomes exact when taking the limit as the $\#\ of\ nodes \rightarrow \infty$. Evidently, practical applications can only have a limited number of nodes, but using a large number of nodes is sufficient to achieve a reasonable approximation. The constraint preventing the selection of an arbitrarily large number of nodes is computational power, and this is made especially relevant in the case of a computationally intensive path-dependent tree such as this one.

With the appropriate payoffs at the nodes at which the face value and coupons are paid, it is now possible to look into how to step back through the tree while taking the CoCo's path dependency into account. Individually for the face value and for each coupon payments, the payoffs at the time of payment t are brought back node by node to time t = 0. At each node, the undiscounted expected payoff is calculated by weighting the payoff of the child nodes according to the Martingale probabilities of an up, middle or down movement (p_u, p_m and p_d) if the stock price at that node is above the equivalent market trigger. Otherwise, the payoff is set to S_t/C_P for the final payment of conversion CoCos and zero for coupon payments or for the final payment of write-down CoCos. Ensuring that the stock price has not dipped below the equivalent market trigger at each node as we step back through the tree fulfills the path dependency requirement of CoCo bonds. Having stepped back all the way to time t = 0, all that is left is to multiply the face value and each coupon payments by their respective time zero undiscounted expected payoff and the risk free discount factor. Summing those discounted expected payoffs yields the price of the contingent convertible bond. The computational implementation of this trinomial tree is in appendix C.2.1.

6.2 Sensitivity to the Equivalent Market Trigger

Figure 6.2 illustrates the price of a CoCo issued by Lloyd's Banking Group as a function of the equivalent market trigger[5]. It shows the sensitivity of the price of the CoCo to the implied market trigger that is consistent with the other parameters. As described in section 5.3.2 on overpricing resulting from constraints on shorting, markets participants have a wide range of subjective market triggers that they believe to be equivalent to the accounting trigger. Shifts in the distribution of market opinion, or structural changes such as relaxing the constraint on shorting are likely to change the implied market trigger and can hence have a significant impact on the price of the CoCo.

The implied market trigger of the CoCo used for the parametrization of figure 6.2 calculated using the observed market price of the CoCo is £0.1380. Using the same parametriza-

[5] Figure 6.2 was generated using the computational implementation of the trinomial tree described in the previous section.

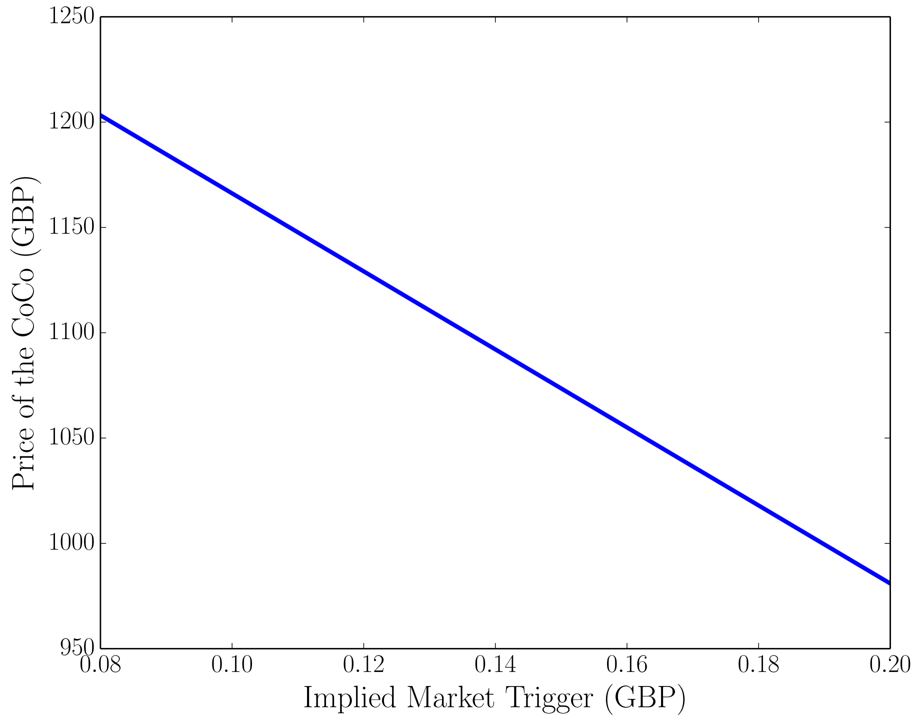

Figure 6.2: Price of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a function of the Equivalent Market Trigger

tion and solving for the price of the CoCo with a range of implied market triggers from £0.08 to £0.20 yields a range of CoCo prices ranging from less than £1000 to just over £1200. While this extremely large range of CoCo prices may at first seem to be the result of having inputted a wide range of implied market triggers, de Spiegeleer and Schoutens (2013) calculations of the arbitrage-free equilibrium for twenty-seven of Lloyd's Banking Group contingent convertible bonds yields implied market triggers ranging from just under 11 pence to almost 17 pence[6]. Although the twenty-seven CoCos have slightly different features, those peculiarities should already be taken into account by the pricing models. As long as all the contingent convertible bonds being compared are issued by the same financial institution and have the same trigger, their implied market triggers should be identical. In this case, all twenty-seven CoCos have a conversion trigger that is breached if the core equity tier 1 ratio crosses 5%. The wide range of implied market triggers for those twenty-seven CoCos is indicative of either model misspecification, or the breadth of

[6]The derivation of the implied market triggers is done based on market data from November 16th 2013.

subjective probabilities of conversion that prevail amongst market participants. Either of those cases combined with the high sensitivity of the price of the CoCo to changes in the implied market trigger may lead to instability of the equilibrium.

6.3 Path-Dependent Trinomial Tree with a Subjective Probability of Regulatory Conversion or Write-Down

Conversion or write-down triggered by the national regulator has been characterized by some investors as "handing over a blank cheque to the government" (de Spiegeleer and Schoutens, 2011, p. 5). Modelling this uncertainty as a function of investors' subjective probability of the national regulator stepping in to trigger conversion or write-down is hence crucial to understanding the dynamics of the contingent convertible bond market and the stability of the equilibrium. Extending the trinomial tree from the previous section to include a subjective probability of conversion or write-down by the national regulator simply requires adding a probability $\lambda(S_t)$ of conversion or write-down at each node. Figure 6.3 illustrates this extension.

The subjective probability of conversion or write-down $\lambda(S_t)$ is written as function of S_t as it can evolve with the path taken by the underlying stock price. Consequently, pricing a contingent convertible bond using this framework not only requires making a subjective assumption about the probability of regulatory conversion or write-down, but also about the evolution of this probability at each node. Computationally, the undiscounted payoff at each node is multiplied by $1 - \lambda(S_t)$. As we can see from figure 6.3, the nodes below the equivalent market trigger have no probability of regulatory triggering as conversion or write-down has already taken place. Including it computationally is irrelevant as the undiscounted payoffs at those states is already zero.

The simplest implementation of the subjective probability of regulatory conversion or write-down is the assumption that $\lambda(S_t)$ is constant throughout the tree, or in other words, that $\lambda(S_t) = \lambda$. While this may seem like an unrealistic assumption, the dependence on S_t is already taken into account by the Martingales of the trinomial tree. In this case, regulatory conversion or write-down is an exogenous shock that could for example be the result of

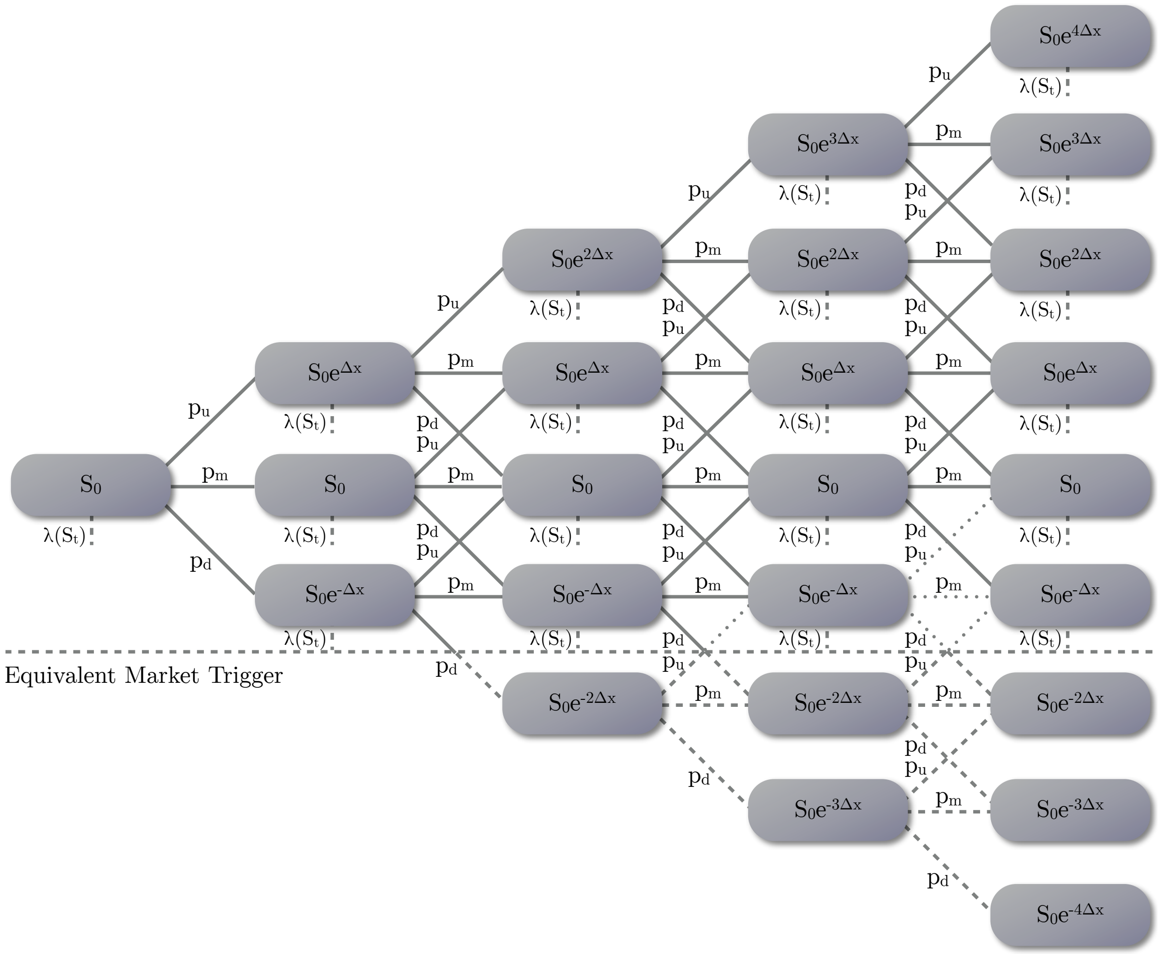

Figure 6.3: Trinomial Tree with a Subjective Probability of Regulatory Conversion or Write-Down $\lambda(S_t)$

political turmoil, regulatory changes, or balance sheet manipulation. Regulatory changes can have an enormous impact on the capitalization of a financial institution. For example, according to JP Morgan data from March 2013 (as cited in de Spiegeleer et al., 2014, p. 84), the phasing-in of Basel III reduces UBS' reported core equity tier 1 capital from 19% to 9.8%. It is therefore very unlikely that the subjective probability of regulatory conversion or write-down will remain constant over the life of the CoCo, but it likely depends on factors other than the price of the underlying stock. The computational implementation of this trinomial tree is in appendix C.2.2.

6.4 Sensitivity to the Subjective Probability of Regulatory Conversion or Write-Down

The path-dependent trinomial tree with a subjective probability of regulatory conversion or write-down presented in the previous section can be used along with a range of subjective probabilities to generate a range of CoCo prices and gain some insights into how sensitive this price is to shifts in market expectations. Figure 6.4 illustrates the range of prices of a contingent convertible bond issued by Lloyd's Banking Group that is consistent with a range of yearly subjective probabilities of conversion from 0% to 3.5%.

The subjective probability of conversion or write-down is inputted as a yearly probability p^* as, unlike the intensity λ which depends on the number of nodes in the trinomial tree, it has a meaningful interpretation for researchers and market participants. Converting the yearly subjective probability to an instantaneous intensity that depends on the number of nodes in the tree is done by looking at the probability of having no regulatory conversion over the period of a year:

$$(1-\lambda)^{\frac{nodes}{T-t}} = 1 - p^* \tag{6.8}$$

$$\lambda = 1 - (1-p^*)^{\frac{T-t}{nodes}} \tag{6.9}$$

As shown in figure 6.4, the relationship between the price of the contingent convertible bond and the yearly subjective probability of regulatory conversion is linear because λ was assumed to be constant throughout the trinomial tree. Relaxing this assumption by making λ dependent on S_t would likely induce a non-linear relationship.

The results in figure 6.4 are not as easy to analyze as the ones for the implied market trigger, as there is no empirical distribution of the subjective probability of regulatory conversion or write-down that can be used for comparison. The results nonetheless indicate that the price of contingent convertible bonds is very responsive to shifts in subjective probabilities of regulatory triggering. Much like changes in the distribution of expectations of implied market triggers, changes in the distribution of subjective probabilities of regulatory triggering could also lead to instability of the equilibrium.

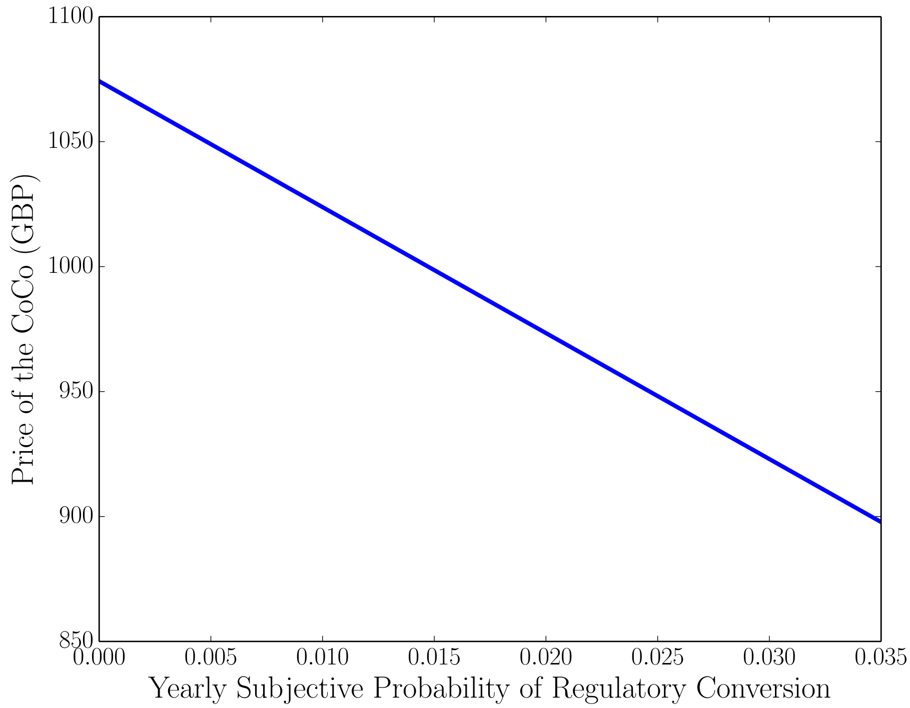

Figure 6.4: Price of a Contingent Convertible Bond Issued by Lloyd's Banking Group as a function of the Yearly Subjective Probability of Regulatory Conversion (p^*)

6.5 Hedging

Allowing for potential changes in market opinions about the equivalency of the implied market trigger to the fundamental accounting trigger as in section 6.2, as well as for the eventuality of conversion or write-down by the national regulator as a fourth branch on the trinomial tree as in section 6.4 raises questions as to the hedging of those potential outcomes. As explained in section 5.1 on delta hedging, the hedging techniques covered in chapter 5 depend on continuous trading for portfolio rebalancing, and a trinomial tree which evolves according to geometric Brownian motion. Although continuous trading is a generally unrealistic presumption, the assumption of geometric Brownian motion is where the hedge truly breaks down.

In spite of the fact that the evolution of the underlying stock price in the trinomial tree from figure 6.1 and 6.3 follows geometric Brownian motion, changes in market opinion about the equivalency of the market trigger or the expected probability of regulatory conversion or write-down breaks the fundamental relationship between the underlying stock price and the price of the CoCo bond. The latter is the key difference between using trinomial trees for pricing options and contingent convertible bonds. As a result, the price of the contingent convertible bond incorporates jump processes even though the underlying security is unaffected. The presence of subjective opinions that can shift in a similar fashion to a jump process may expose CoCo holders to unexpected risk for which there is no reliable hedge.

The absence of adequate hedges can have disastrous repercussions on market participants and may even adversely affect the market as a whole. The presence of only imperfect hedges complicates the unwinding of large positions, especially in times of market instability. Unwinding a position is done by either taking the offsetting position, in other words acquiring a perfect hedge, or offloading the securities on the open market or to private buyers. The former is made impossible by the absence of perfect hedges, while the latter is undermined by the unattractiveness of the market to market makers.

Market makers rely on hedging to limit their exposure regardless of the depth of their holdings. With only imperfect hedges at their disposal, the cost of providing liquidity to the market increases significantly. Market makers will therefore enter the market only in cases where the bid-ask spread is wide enough to provide compensation for this increased risk. The result is an effective reduction in volume and market liquidity, which can be characterized as an increase in transaction costs.

Chapter 7

Conclusion

Issuance of contingent convertible bonds has grown at a staggering rate since Basel III made them a cornerstone of banking capital in an attempt to shore up the balance sheets of banks with more going-concern loss-absorbing capital. While this shift away from traditional tier 2 debt is meant to stabilize the financial system, it has been met by many with a great deal of skepticism. The greatest concern is the so-called death spiral, which is a self-fulfilling collapse in the underlying stock price as a result of delta hedging by CoCo holders.

This research finds that the increase in volatility that tends to accompany a worsening of a bank's capitalization reduces the delta of a contingent convertible bond, and hence mitigates the risks and repercussions of a death spiral. In terms of overpricing, theoretical and empirical research finds that divergence of opinion combined with constraints on shorting lead to overvaluation. This research hence argues that the constraints on shorting contingent convertible bonds lead to the perceived overvaluation. The creation of a market for credit default swaps on CoCos is shown to effectively relax the constraints on shorting and eventually lead to a restoration of the competitive equilibrium.

Conversely, the creation of a market for CDS on CoCos may also add counterparty risk if there is concentration of underwriting. Such a situation could leave hedged CoCo holders exposed to unanticipated risk. If systemically important institutions were to hold those hedged positions, the triggering of conversion or write-down of a large enough issue of CoCos could ripple through the financial system and create generalized instability.

Finally, the pricing models' reliance on a trigger that is implied from observed market prices makes the pricing of those securities dependent on the subjective opinion of market participants. Empirically, a wide range of implied market triggers were observed for similar CoCos, which is indicative of either model misspecification or an unstable equilibrium caused by a wide distribution of subjective opinions. Adding the regulatory trigger to the model shows that the price of contingent convertible bonds is very sensitive to changes in market's subjective probability of regulatory triggering, which also contributes to the instability of the market-clearing equilibrium.

Further research could add the time dimension to evaluate how the reduction in delta from increasing volatility, and the sensitivity of the price of CoCos to the implied market trigger and the regulatory trigger varies with changes in time to maturity. Other aspects of contingent convertibles bonds that merit further research include the agency costs that arise out of the creation of a CDS market, such the empty creditor problem (Bolton and Oehmke, 2011), and the conversion or write-down of contingent convertible bonds as a market signal in the presence of asymmetric information.

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Appendices

Appendix A

Contingent Convertible Bonds Issued

Year	Financial Institution	Total Face Value (USD)	Loss Absorption
2009	Lloyd's Banking Group	$12.42B	Equity Conversion
2010	Lloyd's Banking Group	$408M	Equity Conversion
2010	Rabobank	$1.66B	Partial Write-Down
2011	Rabobank	$4B	Partial Write-Down
2011	Credit Suisse	$2B	Equity Conversion
2011	Bank of Cyprus	$40M	Equity Conversion
2011	Nomura Holdings	$197M	Full Write-Down
2011	People's Republic of China	$232M	Full Write-Down
2012	Canton of Zurich	$629M	Staggered Write-Down
2012	UBS	$4B	Full Write-Down
2012	Credit Suisse	$800M	Equity Conversion
2012	Macquarie Group	$250M	Equity Conversion
2012	Barclays	$3B	Full Write-Down
2012	Federative Republic of Brazil	$1.75B	Full Write-Down
2013	Barclays	$1B	Full Write-Down
2013	KBC Groep	$1B	Full Write-Down
2013	Banco Bilbao Vizcaya Argentari	$1.50B	Equity Conversion
2013	Credit Suisse	$4.47B	Full Write-Down
2013	UBS	$1.50B	Full Write-Down
2013	Société Générale	$3B	Full Write-Down
2013	Crédit Agricole	$1B	Full Write-Down
2013	Barclays	$3.33B	Equity Conversion
2013	Banco Popular Espanol	$664M	Equity Conversion
2013	Credit Suisse	$2.25B	Equity Conversion
2014	Crédit Agricole	$1.75B	Full Write-Down
2014	Royal Bank of Canada	$930M	Equity Conversion

Sources: data from de Spiegeleer and Schoutens (2014) except the Royal Bank of Canada's contingent convertible bond, which is from Gutscher (2014). For bonds denominated in foreign currencies, the total face value was converted to USD using the Bank of Canada (2009, 2010, 2011, 2012, 2013) yearly average exchange rates.

Appendix B

Mathematical Derivations

The derivations in this section are based de Spiegeleer and Schoutens (2012) equity derivatives model, which is written out in section B.1. The derivations of the Greeks in sections B.1.1 to B.1.5 are not part of the existing literature and are hence done exclusively for this research.

B.1 Equity Derivatives Model

The equity derivatives model prices CoCo bonds (P) as the sum of a corporate bond (A), down-and-in forwards for the final payment (B), and binary down-and-in options for the coupon payments (C) (de Spiegeleer and Schoutens, 2012, p. 33):

$$\mathbf{P} = \mathbf{A} + \mathbf{B} + \mathbf{C} \tag{B.1}$$

$$\mathbf{A} = Ne^{-r(T-t)} + \sum_{i=1}^{k} c_i e^{-r(t_i-t)} \tag{B.2}$$

$$\begin{aligned}\mathbf{B} = C_r \Bigg[& Se^{-q(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda} \Phi\left(y_1\right) - Ke^{-r(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\ & - Ke^{-r(T-t)} \Phi\left(-x_1 + \sigma\sqrt{T-t}\right) + Se^{-q(T-t)} \Phi(-x_1) \Bigg]\end{aligned} \tag{B.3}$$

$$\mathbf{C} = -\sum_{i=1}^{k} c_i e^{-r(t_i-t)} \left[\Phi\left(-x_{1i} + \sigma\sqrt{t_i - t}\right) + \left(\frac{S^*}{S}\right)^{2\lambda-2} \Phi\left(y_{1i} - \sigma\sqrt{t_i - t}\right) \right] \tag{B.4}$$

Where:

$$K = C_P \tag{B.5}$$

$$C_r = \frac{N}{C_P} \tag{B.6}$$

$$x_1 = \frac{ln\left(\frac{S}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \tag{B.7}$$

$$y_1 = \frac{ln\left(\frac{S^*}{S}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \tag{B.8}$$

$$x_{1i} = \frac{ln\left(\frac{S}{S^*}\right)}{\sigma\sqrt{t_i-t}} + \lambda\sigma\sqrt{t_i-t} \tag{B.9}$$

$$y_{1i} = \frac{ln\left(\frac{S^*}{S}\right)}{\sigma\sqrt{t_i-t}} + \lambda\sigma\sqrt{t_i-t} \tag{B.10}$$

$$\lambda = \frac{r-q+\frac{\sigma^2}{2}}{\sigma^2} \tag{B.11}$$

B.1.1 Delta

The Delta of the CoCo bond is the derivative of the price of the CoCo with respect to the underlying stock price:

$$\Delta = \frac{\partial P}{\partial S} = \frac{\partial A}{\partial S} + \frac{\partial B}{\partial S} + \frac{\partial C}{\partial S} \tag{B.12}$$

$$\frac{\partial A}{\partial S} = 0 \tag{B.13}$$

$$\begin{aligned}\frac{\partial B}{\partial S} = C_r \Bigg[& (1-2\lambda)e^{-q(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi(y_1) - \frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1) \\ & + \left(\frac{K}{S^*}\right)e^{-r(T-t)}(2\lambda-2)\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_1-\sigma\sqrt{T-t}) \\ & + \frac{Ke^{-r(T-t)}}{S\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_1-\sigma\sqrt{T-t}) + \frac{Ke^{-r(T-t)}}{S\sigma\sqrt{T-t}}\Phi'(-x_1+\sigma\sqrt{T-t}) \\ & + e^{-q(T-t)}\Phi(-x_1) - \frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\Phi'(-x_1)\Bigg]\end{aligned} \tag{B.14}$$

$$\begin{aligned}\frac{\partial C}{\partial S} = -\sum_{i=1}^{k} c_i e^{-r(t_i-t)} \Bigg[& \frac{-\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})}{S\sigma\sqrt{t_i-t}} - \frac{(2\lambda-2)}{S^*}\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_{1i}-\sigma\sqrt{t_i-t}) \\ & - \left(\frac{S^*}{S}\right)^{2\lambda-2}\frac{\Phi'(y_{1i}-\sigma\sqrt{t_i-t})}{S\sigma\sqrt{t_i-t}}\Bigg]\end{aligned} \tag{B.15}$$

Simplifying equation B.14 can be done by proving that portions of some terms are equal. Let's first prove by contradiction that $e^{-q(T-t)}\Phi'(-x_1) = \frac{S^*}{S}e^{-r(T-t)}\Phi'(-x_1+\sigma\sqrt{T-t})$. Assuming it is not true:

$$e^{-q(T-t)}\Phi'(-x_1) \neq \frac{S^*}{S}e^{-r(T-t)}\Phi'(-x_1+\sigma\sqrt{T-t}) \tag{B.16}$$

$$-q(T-t) - \frac{(-x_1)^2}{2} \neq ln\left(\frac{S^*}{S}\right) - r(T-t) - \frac{(-x_1+\sigma\sqrt{T-t})^2}{2} \tag{B.17}$$

$$2\left(ln\left(\frac{S}{S^*}\right) - q(T-t) + r(T-t)\right) \neq (-x_1)^2 - (-x_1+\sigma\sqrt{T-t})^2 \tag{B.18}$$

$$2\left(ln\left(\frac{S}{S^*}\right)-q(T-t)+r(T-t)\right)\neq(-x_1+x_1-\sigma\sqrt{T-t})(-x_1-x_1+\sigma\sqrt{T-t}) \tag{B.19}$$

$$2\left(ln\left(\frac{S}{S^*}\right)-q(T-t)+r(T-t)\right)\neq(-\sigma\sqrt{T-t})\left(-\frac{2ln\left(\frac{S}{S^*}\right)}{\sigma\sqrt{T-t}}-2\lambda\sigma\sqrt{T-t}+\sigma\sqrt{T-t}\right) \tag{B.20}$$

$$2\left(ln\left(\frac{S}{S^*}\right)-q(T-t)+r(T-t)\right)\neq 2ln\left(\frac{S}{S^*}\right)+2\lambda\sigma^2(T-t)+\sigma^2(T-t) \tag{B.21}$$

$$2\left(ln\left(\frac{S}{S^*}\right)-q(T-t)+r(T-t)\right)\neq 2\left(ln\left(\frac{S}{S^*}\right)-q(T-t)+r(T-t)\right) \tag{B.22}$$

Which is a contradiction and proves that $e^{-q(T-t)}\Phi'(-x_1)=\frac{S^*}{S}e^{-r(T-t)}\Phi'(-x_1+\sigma\sqrt{T-t})$. Analogously, it is possible to prove by contradiction that $e^{-q(T-t)}\Phi'(y_1)=\frac{S}{S^*}e^{-r(T-t)}\Phi'(y_1-\sigma\sqrt{T-t})$. Assuming it is not true:

$$e^{-q(T-t)}\Phi'(y_1)\neq\frac{S}{S^*}e^{-r(T-t)}\Phi'(y_1-\sigma\sqrt{T-t}) \tag{B.23}$$

$$-q(T-t)-\frac{(y_1)^2}{2}\neq ln\left(\frac{S}{S^*}\right)-r(T-t)-\frac{(y_1-\sigma\sqrt{T-t})^2}{2} \tag{B.24}$$

$$2\left(ln\left(\frac{S^*}{S}\right)-q(T-t)+r(T-t)\right)\neq(y_1)^2-(y_1-\sigma\sqrt{T-t})^2 \tag{B.25}$$

$$2\left(ln\left(\frac{S^*}{S}\right)-q(T-t)+r(T-t)\right)\neq(y_1-y_1+\sigma\sqrt{T-t})(y_1+y_1-\sigma\sqrt{T-t}) \tag{B.26}$$

$$2\left(ln\left(\frac{S^*}{S}\right)-q(T-t)+r(T-t)\right)\neq\sigma\sqrt{T-t}\left(\frac{2ln\left(\frac{S^*}{S}\right)}{\sigma\sqrt{T-t}}+2\lambda\sigma\sqrt{T-t}-\sigma\sqrt{T-t}\right) \tag{B.27}$$

$$2\left(ln\left(\frac{S^*}{S}\right)-q(T-t)+r(T-t)\right)\neq 2ln\left(\frac{S^*}{S}\right)+2\lambda\sigma^2(T-t)-\sigma^2(T-t) \tag{B.28}$$

$$2\left(ln\left(\frac{S^*}{S}\right)-q(T-t)+r(T-t)\right)\neq 2\left(ln\left(\frac{S^*}{S}\right)-q(T-t)+r(T-t)\right) \tag{B.29}$$

Which is a contradiction and proves that $e^{-q(T-t)}\Phi'(y_1)=\frac{S}{S^*}e^{-r(T-t)}\Phi'(y_1-\sigma\sqrt{T-t})$. Using those two proofs to simplify $\frac{\partial B}{\partial S}$:

$$\begin{aligned}\frac{\partial B}{\partial S}=C_r\Bigg[&(1-2\lambda)e^{-q(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi(y_1)+\frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1)\left(\frac{K}{S}\left(\frac{S^*}{S}\right)^{-1}-1\right)\\&+\left(\frac{K}{S^*}\right)e^{-r(T-t)}(2\lambda-2)\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_1-\sigma\sqrt{T-t})+e^{-q(T-t)}\Phi(-x_1)\\&+\frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\Phi'(-x_1)\left(\frac{K}{S}\left(\frac{S^*}{S}\right)^{-1}-1\right)\Bigg]\end{aligned} \tag{B.30}$$

$$\frac{\partial B}{\partial S} = C_r \Bigg[(1-2\lambda)e^{-q(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda} \Phi(y_1) + \frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}} \left(\frac{S^*}{S}\right)^{2\lambda} \Phi'(y_1) \left(\frac{K}{S^*}-1\right)$$
$$+ \left(\frac{K}{S^*}\right) e^{-r(T-t)}(2\lambda-2) \left(\frac{S^*}{S}\right)^{2\lambda-1} \Phi(y_1-\sigma\sqrt{T-t}) + e^{-q(T-t)}\Phi(-x_1)$$
$$+ \frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\Phi'(-x_1)\left(\frac{K}{S^*}-1\right)\Bigg] \tag{B.31}$$

Summing equations B.13, B.31 and B.15 gives us an equation for Δ:

$$\Delta = C_r \Bigg[(1-2\lambda)e^{-q(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda} \Phi(y_1) + \frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}} \left(\frac{S^*}{S}\right)^{2\lambda} \Phi'(y_1) \left(\frac{K}{S^*}-1\right)$$
$$+ \left(\frac{K}{S^*}\right) e^{-r(T-t)}(2\lambda-2) \left(\frac{S^*}{S}\right)^{2\lambda-1} \Phi(y_1-\sigma\sqrt{T-t}) + e^{-q(T-t)}\Phi(-x_1)$$
$$+ \frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\Phi'(-x_1)\left(\frac{K}{S^*}-1\right)\Bigg] - \sum_{i=1}^{k} c_i e^{-r(t_i-t)} \Bigg[\frac{-\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})}{S\sigma\sqrt{t_i-t}}$$
$$-\frac{(2\lambda-2)}{S^*}\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_{1i}-\sigma\sqrt{t_i-t}) - \left(\frac{S^*}{S}\right)^{2\lambda-2}\frac{\Phi'(y_{1i}-\sigma\sqrt{t_i-t})}{S\sigma\sqrt{t_i-t}}\Bigg] \tag{B.32}$$

B.1.2 Gamma

The Gamma of the CoCo bond is the second derivative of the price of the CoCo with respect to the underlying stock price:

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 P}{\partial S^2} \tag{B.33}$$
$$\Gamma = C_r \Bigg[\frac{(1-2\lambda)(-2\lambda)e^{-q(T-t)}}{S}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi(y_1)$$
$$-\frac{(1-2\lambda)e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1) - \frac{2\lambda e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\left(\frac{K}{S^*}-1\right)\Phi'(y_1)$$
$$+\frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\underbrace{\Phi'(y_1)(-y_1)}_{=\Phi''(y_1)}\frac{-1}{\sigma\sqrt{T-t}}\left(\frac{K}{S^*}-1\right)$$
$$-\left(\frac{K}{S^*}\right)\frac{e^{-r(T-t}(2\lambda-2)(2\lambda-1)}{S}\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_1-\sigma\sqrt{T-t})$$
$$-\left(\frac{K}{S^*}\right)\frac{e^{-r(T-t}(2\lambda-2)}{S\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi'(y_1-\sigma\sqrt{T-t})$$
$$-\frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\Phi(-x_1) + \frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\underbrace{\Phi'(-x_1)(-x_1)}_{=\Phi''(-x_1)}\frac{-1}{\sigma\sqrt{T-t}}\left(\frac{K}{S^*}-1\right)\Bigg]$$

$$
\begin{aligned}
&-\sum_{i=1}^{k}\left[\frac{\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})}{S^2\sigma\sqrt{t_i-t}}-\frac{\overbrace{\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})(-x_{1i}+\sigma\sqrt{t_i-t})}^{=\Phi''(-x_{1i}+\sigma\sqrt{t_i-t})}}{(S^2\sigma\sqrt{t_i-t})(\sigma\sqrt{t_i-t})}\right.\\
&+\frac{(2\lambda-1)(2\lambda-2)}{S^2}\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi(y_{1i}-\sigma\sqrt{t_i-t})\\
&+\frac{2\lambda-2}{S^2\sigma\sqrt{t_i-t}}\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\\
&+\frac{2\lambda-1}{S^2\sigma\sqrt{t_i-t}}\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\\
&\left.+\left(\frac{S^*}{S}\right)^{2\lambda-2}\frac{\overbrace{\Phi'(y_{1i}-\sigma\sqrt{t_i-t})(y_{1i}-\sigma\sqrt{t_i-t})}^{=\Phi''(y_{1i}-\sigma\sqrt{t_i-t})}}{(S^2\sigma\sqrt{t_i-t})(\sigma\sqrt{t_i-t})}\right]
\end{aligned}
\tag{B.34}
$$

Combining terms to simplify equation B.34, Gamma becomes:

$$
\begin{aligned}
\Gamma = C_r &\left[\frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\Phi'(-x_1)\left(\frac{x_1}{\sigma\sqrt{T-t}}\left(\frac{K}{S^*}-1\right)-1\right)-\frac{2\lambda(1-2\lambda)e^{-q(T-t)}}{S}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi(y_1)\right.\\
&+\frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'\left(\left(\frac{K}{S^*}-1\right)\left(\frac{y_1}{\sigma\sqrt{T-t}}-2\lambda\right)-1\right)\\
&\left.-\left(\frac{K}{S^*}\right)\frac{e^{-r(T-t)}(2\lambda-2)}{S}\left(\frac{S^*}{S}\right)^{2\lambda-1}\left(\frac{\Phi'(y_1-\sigma\sqrt{T-t}}{\sigma\sqrt{T-t}}+(2\lambda-1)\Phi(y_1-\sigma\sqrt{T-t}\right)\right]\\
&-\sum_{i=1}^{k}c_ie^{-r(t_i-t)}\left[\frac{\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})}{S^2\sigma\sqrt{t_i-t}}\left(2-\frac{x_{1i}}{\sigma\sqrt{t_i-t}}\right)\right.\\
&+\frac{(2\lambda-1)(2\lambda-2)}{S^2}\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi(y_{1i}-\sigma\sqrt{t_i-t})\\
&\left.+\left(\frac{S^*}{S}\right)\frac{\Phi'(y_{1i}-\sigma\sqrt{t_i-t})}{S^2\sigma\sqrt{t_i-t}}\left(4\lambda-4+\frac{y_{1i}}{\sigma\sqrt{t_i-t}}\right)\right]
\end{aligned}
\tag{B.35}
$$

B.1.3 Vega

The Vega of the CoCo bond is the derivative of the price of the CoCo with respect to volatility:

$$
\nu = \frac{\partial P}{\partial \sigma} \tag{B.36}
$$

$$
\begin{aligned}
\nu = C_r &\left[Se^{-q(T-t}\left(\frac{S^*}{S}\right)^{2\lambda} ln\left(\frac{S^*}{S}\right)2\left(\frac{-2(r-q)}{\sigma^3}\right)\Phi(y_1)\right.\\
&+Se^{-q(T-t}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1)\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{T-t}}+\left(\frac{1}{2}-\frac{r-q}{\sigma^2}\right)\sqrt{T-t}\right)
\end{aligned}
$$

$$-Ke^{-r(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda-2} ln\left(\frac{S^*}{S}\right)2\left(\frac{-2(r-q)}{\sigma^3}\right)\Phi(y_1-\sigma\sqrt{T-t})$$

$$-Ke^{-r(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_1-\sigma\sqrt{T-t})\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{T-t}}+\left(-\frac{1}{2}-\frac{r-q}{\sigma^2}\right)\sqrt{T-t}\right)$$

$$-Ke^{-r(T-t)}\Phi'(-x_1+\sigma\sqrt{T-t})\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}-\left(-\frac{1}{2}-\frac{r-q}{\sigma^2}\right)\sqrt{T-t}\right)$$

$$+Se^{-q(T-t)}\Phi'(-x_1)\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}-\left(\frac{1}{2}-\frac{r-q}{\sigma^2}\right)\sqrt{T-t}\right)\Bigg]$$

$$-\sum_{i=1}^{k}c_ie^{-r(t_i-t)}\Bigg[\Phi'(-x_{1i}+\sigma\sqrt{t_i-t}\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{t_i-t}}-\left(-\frac{1}{2}-\frac{r-q}{\sigma^2}\right)\right)$$

$$+\left(\frac{S^*}{S}\right)^{2\lambda-2} ln\left(\frac{S^*}{S}\right)2\left(\frac{-2(r-q)}{\sigma^3}\right)\Phi(y_{1i}-\sigma\sqrt{t_i-t})$$

$$+\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{t_i-t}}+\left(-\frac{1}{2}-\frac{r-q}{\sigma^2}\right)\sqrt{t_i-t}\right)\Bigg] \tag{B.37}$$

Combining terms to simplify equation B.37, Vega becomes:

$$\nu = C_r\Bigg[Se^{-q(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda}\left(ln\left(\frac{S^*}{S}\right)\left(\frac{4(q-r)}{\sigma^3}\Phi(y_1)-\frac{\Phi'(y_1)}{\sigma^2\sqrt{T-t}}\right)+\left(\frac{1}{2}+\frac{q-r}{\sigma^2}\right)\right.$$

$$\left.\Phi'(y_1)\sqrt{T-t}\right)+Ke^{-r(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda-2}\left(ln\left(\frac{S^*}{S}\right)\left(\frac{\Phi'(y_1-\sigma\sqrt{T-t})}{\sigma^2\sqrt{T-t}}-\frac{4(q-r)}{\sigma^3}\right.\right.$$

$$\left.\left.\Phi(y_1-\sigma\sqrt{T-t})\right)-\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\Phi'(y_1-\sigma\sqrt{T-t})\right)+Ke^{-r(T-t)}\Phi'(-x_1+\sigma\sqrt{T-t})$$

$$\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}-\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{T-t}\right)+Se^{-q(T-t)}\Phi'(-x_1)\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}-\left(\frac{q-r}{\sigma^2}\right.\right.$$

$$\left.\left.+\frac{1}{2}\right)\sqrt{T-t}\right)\Bigg]-\sum_{i=1}^{k}c_ie^{-r(T-t)}\Bigg[\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{t_i-t}}-\left(\frac{q-r}{\sigma^2}\right.\right.$$

$$\left.\left.-\frac{1}{2}\right)\sqrt{t_i-t}\right)+\left(\frac{S^*}{S}\right)^{2\lambda-2} ln\left(\frac{S^*}{S}\right)\frac{4(q-r)}{\sigma^3}\Phi(y_{1i}-\sigma\sqrt{t_i-t})$$

$$-\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\left(\frac{ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{t_i-t}}-\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{t_i-t}\right)\Bigg] \tag{B.38}$$

B.1.4 Volga

The Volga of the CoCo bond is the second derivative of the price of the CoCo with respect to volatility:

$$Volga = \frac{\partial \nu}{\partial \sigma} = \frac{\partial^2 P}{\partial \sigma^2} \tag{B.39}$$

$$
\begin{aligned}
Volga = C_r \Bigg[& Se^{-q(T-t)} \left(\frac{S^*}{S}\right)^{2\lambda} \left(ln\left(\frac{S^*}{S}\right) \left(\frac{4(q-r)}{\sigma^3}\right) \left(ln\left(\frac{S^*}{S}\right) \left(\frac{4(q-r)}{\sigma^3}\Phi(y_1) \right.\right.\right. \\
& -\frac{\Phi'(y_1)}{\sigma^2\sqrt{T-t}}\right) + \left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\Phi'(y_1)\sqrt{T-t}\right) + ln\left(\frac{S^*}{S}\right)\left(-\frac{12(q-r)}{\sigma^4}\Phi(y_1)\right. \\
& +\frac{2\Phi'(y_1)}{\sigma^3\sqrt{T-t}}\right) - \frac{2(q-r)}{\sigma^3}\Phi'(y_1)\sqrt{T-t} + \left(\frac{-ln\frac{S^*}{S}}{\sigma^2\sqrt{T-t}} + \left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\sqrt{T-t}\right) \\
& \left(ln\left(\frac{S^*}{S}\right)\left(\frac{4(q-r)}{\sigma^3}\Phi'(y_1) + \frac{\Phi'(y_1)(y_1)}{\sigma^2\sqrt{T-t}}\right) - \left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\Phi'(y_1)(y_1)\sqrt{T-t}\right)\Bigg) \\
& +Ke^{-r(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda-2}\left(ln\left(\frac{S^*}{S}\right)\frac{4(q-r)}{\sigma^3}\left(ln\left(\frac{S^*}{S}\right)\left(\frac{\Phi'(y_1-\sigma\sqrt{T-t}}{\sigma^2\sqrt{T-t}}\right.\right.\right. \\
& -\frac{4(q-r)}{\sigma^3}\Phi(y_1-\sigma\sqrt{T-t})\right) - \left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\Phi'(y_1-\sigma\sqrt{T-t})\right) + \left(ln\left(\frac{S^*}{S}\right)\right. \\
& \left(-\frac{2\Phi(y_1-\sigma\sqrt{T-t}}{\sigma^3\sqrt{T-t}} + \frac{12(q-r)}{\sigma^4}\Phi(y_1-\sigma\sqrt{T-t})\right) + \frac{2(q-r)}{\sigma^3}\Phi'(y_1-\sigma\sqrt{T-t})\right) \\
& +\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{T-t}} + \left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\right)\left(ln\left(\frac{S^*}{S}\right)\left(\frac{\Phi'(y_1-\sigma\sqrt{T-t})}{\sigma^2\sqrt{T-t}}(-y_1+\sigma\sqrt{T-t})\right.\right. \\
& -\frac{4(q-r)}{\sigma^3}\Phi'(y_1-\sigma\sqrt{T-t})\right) - \left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\Phi'(y_1-\sigma\sqrt{T-t}(-y_1+\sigma\sqrt{T-t})\right)\Bigg) \\
& +Ke^{-r(T-t)}\Phi'(-x_1+\sigma\sqrt{T-t})\left((-x_1+\sigma\sqrt{T-t})\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}} - \left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\right.\right. \\
& \left.\sqrt{T-t}\right)^2 - \frac{2ln\left(\frac{S}{S^*}\right)}{\sigma^3\sqrt{T-t}} + \frac{2(q-r)}{\sigma^3}\sqrt{T-t}\right) + Se^{-q(T-t)}\Phi'(-x_1)\left((-x_1)\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}\right.\right. \\
& \left.-\left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\sqrt{T-t}\right)^2 - \frac{2ln\left(\frac{S}{S^*}\right)}{\sigma^3\sqrt{T-t}} + \frac{2(q-r)}{\sigma^3}\sqrt{T-t}\right)\Bigg] \\
& -\sum_{i=1}^{k} c_i e^{-r(T-t)}\Bigg[\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})\left((-x_{1i}+\sigma\sqrt{t_i-t})\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{t_i-t}} - \left(\frac{q-r}{\sigma^2}\right.\right.\right. \\
& \left.-\frac{1}{2}\right)\sqrt{t_i-t}\right) - \frac{2ln\left(\frac{S}{S^*}\right)}{\sigma^3\sqrt{t_i-t}} + \frac{2(q-r)}{\sigma^3}\sqrt{t_i-t}\right) + \left(\frac{S^*}{S}\right)^{2\lambda-2} ln\left(\frac{S^*}{S}\right) \\
& \left(\left(\frac{4(q-r)}{\sigma^3}\right)^2 ln\left(\frac{S^*}{S}\right)\Phi(y_{1i}-\sigma\sqrt{t_i-t}) - \frac{12(q-r)}{\sigma^4}\Phi(y_{1i}-\sigma\sqrt{t_i-t})\right. \\
& \left.+\frac{4(q-r)}{\sigma^3}\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{t_i-t}} + \left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{t_i-t}\right)\right) \\
& -\left(\frac{S^*}{S}\right)^{2\lambda-2}\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\left(\frac{4(q-r)}{\sigma^3}ln\left(\frac{S^*}{S}\right)\left(\frac{ln\left(\frac{S^*}{S}\right)}{\sigma^3\sqrt{t_i-t}} - \left(\frac{q-r}{\sigma^2}\right.\right.\right. \\
& \left.-\frac{1}{2}\right)\sqrt{t_i-t}\right) - (y_{1i}+\sigma\sqrt{t_i-t})\left(\frac{ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{t_i-t}} - \left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{t_i-t}\right)^2
\end{aligned}
$$

$$-\frac{2ln\left(\frac{S^*}{S}\right)}{\sigma^3\sqrt{t_i-t}}+\frac{2(q-r)}{\sigma^3}\sqrt{t_i-t}\Bigg)\Bigg]\tag{B.40}$$

B.1.5 Vanna

The Vanna of the CoCo bond is the derivative of the Delta with respect to volatility. By Young's Theorem, it is also the derivative of the Vega with respect to the stock price. Deriving Vanna:

$$Vanna=\frac{\partial\Delta}{\partial\sigma}=\frac{\partial\nu}{\partial S}\tag{B.41}$$

$$Vanna=C_r\Bigg[-\frac{4(q-r)}{\sigma^3}e^{-q(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi(y_1)+(1-2\lambda)e^{-q(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda}ln\left(\frac{S^*}{S}\right)$$
$$\left(\frac{4(q-r)}{\sigma^3}\right)\Phi(y_1)+(1-2\lambda)e^{-q(T-t)}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1)\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{T-t}}+\left(\frac{q-r}{\sigma^2}\right.\right.$$
$$\left.\left.+\frac{1}{2}\right)\sqrt{T-t}\right)-\frac{e^{-q(T-t)}}{\sigma^2\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1)\left(\frac{K}{S^*}-1\right)+\frac{e^{-q(T-t)}}{\sigma^2\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}$$
$$ln\left(\frac{S^*}{S}\right)\left(\frac{4(q-r)}{\sigma^3}\right)\Phi'(y_1)\left(\frac{K}{S^*}-1\right)+\frac{e^{-q(T-t)}}{\sigma^2\sqrt{T-t}}\left(\frac{S^*}{S}\right)^{2\lambda}\Phi'(y_1)(y_1)$$
$$\left(\frac{ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{T-t}}-\left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\sqrt{T-t}\right)\left(\frac{K}{S^*}-1\right)+\left(\frac{K}{S^*}\right)e^{-r(T-t)}\left(\frac{4(q-r)}{\sigma^3}\right)$$
$$\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_i-\sigma\sqrt{T-t})+\left(\frac{K}{S^*}\right)e^{-r(T-t)}(2\lambda-2)\left(\frac{S^*}{S}\right)^{2\lambda-1}ln\left(\frac{S^*}{S}\right)$$
$$\left(\frac{4(q-r)}{\sigma^3}\right)\Phi(y_i-\sigma\sqrt{T-t})+\left(\frac{K}{S^*}\right)e^{-r(T-t)}(2\lambda-2)\left(\frac{S^*}{S}\right)^{2\lambda-1}$$
$$\Phi'(y_i-\sigma\sqrt{T-t})\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{T-t}}+\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{T-t}\right)+e^{-q(T-t)}\Phi'(-x_1)$$
$$\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}-\left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\sqrt{T-t}\right)-\frac{e^{-q(T-t)}}{\sigma^2\sqrt{T-t}}\Phi'(-x_1)\left(\frac{K}{S^*}-1\right)$$
$$\frac{e^{-q(T-t)}}{\sigma\sqrt{T-t}}\Phi'(-x_1)(-x_1)\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{T-t}}-\left(\frac{q-r}{\sigma^2}+\frac{1}{2}\right)\sqrt{T-t}\right)\left(\frac{K}{S^*}-1\right)\Bigg]$$
$$-\sum_{i=1}^{k}c_ie^{-r(t_i-t)}\Bigg[\frac{\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})}{S\sigma^2\sqrt{t_i-t}}-\left(\frac{\Phi'(-x_{1i}+\sigma\sqrt{t_i-t})(-x_{1i}+\sigma\sqrt{t_i-t})}{S\sigma\sqrt{t_i-t}}\right)$$
$$\left(\frac{ln\left(\frac{S}{S^*}\right)}{\sigma^2\sqrt{t_i-t}}-\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{t_i-t}\right)-\left(\frac{4(q-r)}{S^*\sigma^3}\right)\left(\frac{S^*}{S}\right)^{2\lambda-1}\Phi(y_{1i}-\sigma\sqrt{t_i-t})$$
$$-\frac{2\lambda-2}{S^*}\left(\frac{S^*}{S}\right)^{2\lambda-1}ln\left(\frac{S^*}{S}\right)\left(\frac{4(q-r)}{\sigma^3}\right)\Phi(y_{1i}-\sigma\sqrt{t_i-t})-\frac{2\lambda-2}{S^*}\left(\frac{S^*}{S}\right)^{2\lambda-1}$$
$$\Phi'(y_{1i}-\sigma\sqrt{t_i-t})\left(\frac{-ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{t_i-t}}+\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{t_i-t}\right)+\left(\frac{S^*}{S}\right)^{2\lambda-2}$$

$$
\frac{\Phi'(y_{1i}-\sigma\sqrt{t_i-t})}{S\sigma^2\sqrt{t_i-t}} + \left(\frac{S^*}{S}\right)^{2\lambda-2} ln\left(\frac{S^*}{S}\right)\left(\frac{4(q-r)}{S\sigma^4\sqrt{t_i-t}}\right)\Phi'(y_{1i}-\sigma\sqrt{t_i-t})
$$

$$
-\left(\frac{S^*}{S}\right)^{2\lambda-2}\frac{\Phi'(y_{1i}-\sigma\sqrt{t_i-t})}{S\sigma\sqrt{t_i-t}}(-y_{1i}+\sigma\sqrt{t_i-t})\left(\frac{ln\left(\frac{S^*}{S}\right)}{\sigma^2\sqrt{t_i-t}}\right.
$$

$$
\left.\left. -\left(\frac{q-r}{\sigma^2}-\frac{1}{2}\right)\sqrt{t_i-t}\right)\right] \tag{B.42}
$$

Appendix C

Computational Work in Python

This appendix includes the functions programmed in Python that were needed to produce this research. The functions make use of the numpy and the scipy packages:

```python
import numpy as np
import scipy.stats as ss
```

All the functions are designed to model the characteristics of Lloyd's Banking Group's contingent convertible bond XS0459093364, but can easily be adapted to other CoCo bonds.

C.1 Equity Derivatives Model

This section includes the computational implementation of the equity derivatives model as well as functions for its delta, gamma, vega, volga and vanna.

C.1.1 Price

```python
def price(sig = 0.49, S = 0.75380, N = 1000.0, q = 0.01, r = 0.021456939952621, trig = 0.1379985950200, CP = 0.59, T = 2020.649315, t = 2013.876712, ci = 39.345):

    '''
    This function prices a contingent convertible bond using the
        equity derivatives model.

    Parameters:
    sig    - volatility
    S      - current price of the underlying stock
    N      - face value of the CoCo bond
    q      - dividend yield
    r      - riskless interest rate
    trig   - equivalent market trigger
    CP     - conversion price
    T      - time at maturity
    t      - current time
    ci     - coupon payment
```

```
    Output:
    0        - price of the contingent convertible bond
    '''

    #Calculating lambda
    l = (r - q + ((sig**2)/2))/sig**2

    #Creating a list that includes the time t of each coupon payment. Change this segment of code for pricing CoCos with a different coupon payment schedule.
    ti = np.zeros(14)
    ti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            ti[i] = ti[i-1] + (184.0/365.0)
        else:
            ti[i] = ti[i-1] + (181.0/365.0)

    #Calculating the inputs of the model
    K = CP
    Cr = N/CP
    x1 = (np.log(S/trig)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    y1 = (np.log(trig/S)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    x1i = np.zeros(14)
    y1i = np.zeros(14)
    for i in range(0, 14):
        x1i[i] = (np.log(S/trig)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)
        y1i[i] = (np.log(trig/S)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)

    ### Pricing the bond ###

    #Calculating the time t value of the final payment at the riskless rate
    A1 = N * np.exp(-r*(T-t))

    #Calculating the time t value of the coupon payments at the riskless rate
    A2 = 0
    for i in ti:
        A2 = A2 + ci * np.exp(-r*(i-t))

    #Calculating the value of the synthetic down and in forward for the final payment
    B = Cr*(S*np.exp(-q*(T-t))*((trig/S)**(2*l))*ss.norm(0,1).cdf(y1) - K*np.exp(-r*(T-t))*((trig/S)**(2*l-2))*ss.norm(0,1)
```

```
.cdf(y1 - sig*np.sqrt(T-t)) - K*np.exp(-r*(T-t))*ss.norm(0,1).cdf(-x1 + sig*np.sqrt(T-t))+S*np.exp(-q*(T-t))*ss.norm(0,1).cdf(-x1))

    #Calculating the value of the synthetic down and in forward for the coupons
    C = 0
    for i in range(0, 14):
        C = C - ci*np.exp(-r*(ti[i]-t))*(ss.norm(0,1).cdf(-x1i[i]+sig*np.sqrt(ti[i]-t)) + ((trig/S)**(2*l-2))*ss.norm(0,1).cdf(y1i[i]-sig*np.sqrt(ti[i]-t)))

    #Returning the price of the bond
    return A1+A2+B+C
```

C.1.2 Delta

```
def delta(sig = 0.49, S = 0.75380, N = 1000.0, q = 0.01, r = 0.021456939952621, trig = 0.1379985950200, CP = 0.59, T = 2020.649315, t = 2013.876712, ci = 39.345):

    '''
    This function calculates the delta of a contingent convertible bond using the equity derivatives model.

    Parameters:
    sig    - volatility
    S      - current price of the underlying stock
    N      - face value of the CoCo bond
    q      - dividend yield
    r      - riskless interest rate
    trig   - equivalent market trigger
    CP     - conversion price
    T      - time at maturity
    t      - current time
    ci     - coupon payment

    Output:
    0      - delta of the contingent convertible bond
    '''

    #Calculating lambda
    l = (r - q + ((sig**2)/2))/sig**2

    #Creating a list that includes the time t of each coupon payment. Change this segment of code for pricing CoCos with a different coupon payment schedule.
    ti = np.zeros(14)
```

```
    ti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            ti[i] = ti[i-1] + (184.0/365.0)
        else:
            ti[i] = ti[i-1] + (181.0/365.0)

    #Calculating the inputs of the model
    K = CP
    Cr = N/CP
    x1 = (np.log(S/trig)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    y1 = (np.log(trig/S)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    x1i = np.zeros(14)
    y1i = np.zeros(14)
    for i in range(0, 14):
        x1i[i] = (np.log(S/trig)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)
        y1i[i] = (np.log(trig/S)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)

    ### Calculating delta ###

    #Calculating the delta of the synthetic down and in forward for the final payment
    delta1 = Cr * ((1-2*l)*np.exp(-q*(T-t))*((trig/S)**(2*l))*ss.norm(0,1).cdf(y1) + ((K/trig)-1)*((np.exp(-q*(T-t)))/(sig*np.sqrt(T-t)))*(((trig/S)**(2*l))*ss.norm(0,1).pdf(y1)+ss.norm(0,1).pdf(-x1))+trig*np.exp(-r*(T-t))*(2*l-2)*((trig**(2*l-2))/(S**(2*l-1)))*ss.norm(0,1).cdf(y1-sig*np.sqrt(T-t))+np.exp(-q*(T-t))*ss.norm(0,1).cdf(-x1))

    #Calculating the delta of the synthetic down and in forward for the coupons
    delta2 = 0
    for i in range(0, 14):
        delta2 = delta2 - ci * np.exp(-r*(ti[i]-t)) * (ss.norm(0,1).pdf(-x1i[i]+sig*np.sqrt(ti[i]-t))*(-1/(S*sig*np.sqrt(ti[i]-t))) + (-(2*l-2))*((trig**(2*l-2))/(S**(2*l-1)))*ss.norm(0,1).cdf(y1i[i]-sig*np.sqrt(ti[i]-t)) + ((trig/S)**(2*l-2))*ss.norm(0,1).pdf(y1i[i]-sig*np.sqrt(ti[i]-t))*(-1/(S*sig*np.sqrt(ti[i]-t))))

    #Returning the delta of the bond
    return delta1+delta2
```

C.1.3 Gamma

```python
def gamma(sig = 0.49, S = 0.75380, N = 1000.0, q = 0.01, r =
    0.021456939952621, trig = 0.1379985950200, CP = 0.59, T =
    2020.649315, t = 2013.876712, ci = 39.345):

    '''
    This function calculates the gamma of a contingent
        convertible bond using the equity derivatives model.

    Parameters:
    sig     - volatility
    S       - current price of the underlying stock
    N       - face value of the CoCo bond
    q       - dividend yield
    r       - riskless interest rate
    trig    - equivalent market trigger
    CP      - conversion price
    T       - time at maturity
    t       - current time
    ci      - coupon payment

    Output:
    0       - gamma of the contingent convertible bond
    '''

    #Calculating lambda
    l = (r - q + ((sig**2)/2))/sig**2

    #Creating a list that includes the time t of each coupon
        payment. Change this segment of code for pricing CoCos
        with a different coupon payment schedule.
    ti = np.zeros(14)
    ti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            ti[i] = ti[i-1] + (184.0/365.0)
        else:
            ti[i] = ti[i-1] + (181.0/365.0)

    #Calculating the inputs of the model
    K = CP
    Cr = N/CP
    x1 = (np.log(S/trig)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    y1 = (np.log(trig/S)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    x1i = np.zeros(14)
    y1i = np.zeros(14)
    for i in range(0, 14):
        x1i[i] = (np.log(S/trig)/(sig*np.sqrt(ti[i]-t))) + l*sig*
            np.sqrt(ti[i]-t)
```

```
        y1i[i] = (np.log(trig/S)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)

    ### Calculating gamma ###

    #Calculating the gamma of the synthetic down and in forward for the final payment
    gamma1 = Cr * (((-2*l*(1-2*l)*np.exp(-q*(T-t)))/S)*((trig/S)**(2*l))*ss.norm(0,1).cdf(y1)+((np.exp(-q*(T-t)))/(S*sig*np.sqrt(T-t)))*((trig/S)**(2*l))*ss.norm(0,1).pdf(y1)*(((K/trig)-1)*((y1/(sig*np.sqrt(T-t)))-2*l)-1)-(K/trig)*((np.exp(-r*(T-t))*(2*l-2))/S)*((trig/S)**(2*l-1))*(((ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t)))/(sig*np.sqrt(T-t)))+(2*l-1)*ss.norm(0,1).cdf(y1-sig*np.sqrt(T-t)))+((np.exp(-q*(T-t)))/(S*sig*np.sqrt(T-t)))*ss.norm(0,1).pdf(-x1)*(((x1)/(sig*np.sqrt(T-t)))*((K/trig)-1)-1))

    #Calculating the gamma of the synthetic down and in forward for the coupons
    gamma2 = 0
    for i in range(0, 14):
        gamma2 = gamma2 - ci * np.exp(-r*(ti[i]-t)) * (((ss.norm(0,1).pdf(-x1i[i]+sig*np.sqrt(ti[i]-t)))/((S**2)*sig*np.sqrt(ti[i]-t)))*(2-((x1i[i])/(sig*np.sqrt(ti[i]-t))))+(((2*l-1)*(2*l-2))/(S**2))*((trig/S)**(2*l-2))*ss.norm(0,1).cdf(y1i[i]-sig*np.sqrt(ti[i]-t))+((trig/S)**(2*l-2))*((ss.norm(0,1).pdf(y1i[i]-sig*np.exp(ti[i]-t)))/((S**2)*sig*np.sqrt(ti[i]-t)))*(4*l-4+(y1i[i]/(sig*np.sqrt(ti[i]-t)))))

    #Returning the gamma of the bond
    return gamma1+gamma2
```

C.1.4 Vega

```
def vega(sig = 0.49, S = 0.75380, N = 1000.0, q = 0.01, r = 0.021456939952621, trig = 0.1379985950200, CP = 0.59, T = 2020.649315, t = 2013.876712, ci = 39.345):

    '''

    This function calculates the vega of a contingent convertible bond using the equity derivatives model.

    Parameters:
    sig    - volatility
    S      - current price of the underlying stock
    N      - face value of the CoCo bond
    q      - dividend yield
```

```
    r       - riskless interest rate
    trig    - equivalent market trigger
    CP      - conversion price
    T       - time at maturity
    t       - current time
    ci      - coupon payment

    Output:
    0       - vega of the contingent convertible bond
    '''

    #Calculating lambda
    l = (r - q + ((sig**2)/2))/sig**2

    #Creating a list that includes the time t of each coupon payment. Change this segment of code for pricing CoCos with a different coupon payment schedule.
    ti = np.zeros(14)
    ti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            ti[i] = ti[i-1] + (184.0/365.0)
        else:
            ti[i] = ti[i-1] + (181.0/365.0)

    #Calculating the inputs of the model
    K = CP
    Cr = N/CP
    x1 = (np.log(S/trig)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    y1 = (np.log(trig/S)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    x1i = np.zeros(14)
    y1i = np.zeros(14)
    for i in range(0, 14):
        x1i[i] = (np.log(S/trig)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)
        y1i[i] = (np.log(trig/S)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)

    ### Calculating vega ###

    #Calculating the vega of the synthetic down and in forward for the final payment
    vega1 = Cr * (S*np.exp(-q*(T-t))*((trig/S)**(2*l))*(np.log(trig/S)*(((4*(q-r))/(sig**3))*ss.norm(0,1).cdf(y1)-(ss.norm(0,1).pdf(y1)/((sig**2)*np.sqrt(T-t)))))+((1/2)+((q-r)/(sig**2)))*np.sqrt(T-t)*ss.norm(0,1).pdf(y1))+K*np.exp(-r*(T-t))*((trig/S)**(2*l-2))*(np.log(trig/S)*(((ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t)))/((sig**2)*np.sqrt(T-t)))
```

```
        -((4*(q-r))/(sig**3))*ss.norm(0,1).cdf(y1-sig*np.sqrt(T-T)
        ))-(((q-r)/(sig**2))-(1/2))*ss.norm(0,1).pdf(y1-sig*np.
        sqrt(T-t)))+K*np.exp(-r*(T-t))*ss.norm(0,1).pdf(-x1+sig*np
        .sqrt(T-t))*(((np.log(S/trig))/((sig**2)*np.sqrt(T-t)))
        -(((q-r)/(sig**2))-(1/2))*np.sqrt(T-t))+S*np.exp(-q*(T-t))
        *ss.norm(0,1).pdf(-x1)*(((np.log(S/trig))/((sig**2)*np.
        sqrt(T-t)))-(((q-r)/(sig**2))+(1/2))*np.sqrt(T-t)))

    #Calculating the vega of the synthetic down and in forward
        for the coupons
    vega2 = 0
    for i in range(0, 14):
        vega2 = vega2 - ci * np.exp(-r*(ti[i]-t)) * (ss.norm(0,1)
            .pdf(-x1i[i]+sig*np.sqrt(ti[i]-t))*(((np.log(S/trig))
            /((sig**2)*np.sqrt(ti[i]-t)))-(((q-r)/(sig**2))-(1/2))
            *np.sqrt(ti[i]-t))+((trig/S)**(2*l-2))*np.log(trig/S)
            *((4*(q-r))/(sig**3))*ss.norm(0,1).cdf(y1i[i]-sig*np.
            sqrt(ti[i]-t))-((trig/S)**(2*l-2))*ss.norm(0,1).pdf(
            y1i[i]-sig*np.sqrt(ti[i]-t))*(((np.log(trig/S))/((sig
            **2)*np.sqrt(ti[i]-t)))-(((q-r)/(sig**2))-(1-2))*np.
            sqrt(ti[i]-t)))

    #Returning the vega of the bond
    return vega1+vega2
```

C.1.5 Volga

```
def volga(sig = 0.49, S = 0.75380, N = 1000.0, q = 0.01, r =
    0.021456939952621, trig = 0.1379985950200, CP = 0.59, T =
    2020.649315, t = 2013.876712, ci = 39.345):

    '''
    This function calculates the volga of a contingent
        convertible bond using the equity derivatives model.

    Parameters:
    sig    - volatility
    S      - current price of the underlying stock
    N      - face value of the CoCo bond
    q      - dividend yield
    r      - riskless interest rate
    trig   - equivalent market trigger
    CP     - conversion price
    T      - time at maturity
    t      - current time
    ci     - coupon payment

    Output:
```

```
    0       - volga of the contingent convertible bond
    '''

    #Calculating lambda
    l = (r - q + ((sig**2)/2))/sig**2

    #Creating a list that includes the time t of each coupon
       payment. Change this segment of code for pricing CoCos
       with a different coupon payment schedule.
    ti = np.zeros(14)
    ti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            ti[i] = ti[i-1] + (184.0/365.0)
        else:
            ti[i] = ti[i-1] + (181.0/365.0)

    #Calculating the inputs of the model
    K = CP
    Cr = N/CP
    x1 = (np.log(S/trig)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    y1 = (np.log(trig/S)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    x1i = np.zeros(14)
    y1i = np.zeros(14)
    for i in range(0, 14):
        x1i[i] = (np.log(S/trig)/(sig*np.sqrt(ti[i]-t))) + l*sig*
           np.sqrt(ti[i]-t)
        y1i[i] = (np.log(trig/S)/(sig*np.sqrt(ti[i]-t))) + l*sig*
           np.sqrt(ti[i]-t)

    ### Calculating volga ###

    #Calculating the volga of the synthetic down and in forward
       for the final payment
    volga1 = Cr * (S*np.exp(-q*(T-t))*((trig/S)**(2*l))*(np.log(
       trig/S)*((4*(q-r))/(sig**3))*(np.log(trig/S)*(((4*(q-r))/(
       sig**3))*ss.norm(0,1).cdf(y1)-(ss.norm(0,1).pdf(y1)/((sig
       **2)*np.sqrt(T-t))))+(((q-r)/(sig**2))+(1/2))*ss.norm(0,1)
       .pdf(y1)*np.sqrt(T-t))+np.log(trig/S)*(-((12*(q-r))/(sig
       **4))*ss.norm(0,1).cdf(y1)+((2*ss.norm(0,1).pdf(y1))/((sig
       **3)*np.sqrt(T-t))))-((2*(q-r))/(sig**3))*ss.norm(0,1).pdf
       (y1)*np.sqrt(T-t)+(((-np.log(trig/S))/((sig**2)*np.sqrt(T-
       t)))+(((q-r)/(sig**2))+(1/2))*np.sqrt(T-t))*(np.log(trig/S
       )*(((4*(q-r))/(sig**3))*ss.norm(0,1).pdf(y1)+((ss.norm
       (0,1).pdf(y1)*y1)/((sig**2)*np.sqrt(T-t))))-(((q-r)/(sig
       **2))+(1/2))*ss.norm(0,1).pdf(y1)*(y1)*np.sqrt(T-t))) + K*
       np.exp(-r*(T-t))*((trig/S)**(2*l-2))*(np.log(trig/S)*((4*(
       q-r))/(sig**3))*(np.log(trig/S)*(((ss.norm(0,1).pdf(y1-sig
```

```
*np.sqrt(T-t)))/((sig**2)*np.sqrt(T-t)))-((4*(q-r))/(sig
**3))*ss.norm(0,1).cdf(y1-sig*np.sqrt(T-t)))-(((q-r)/(sig
**2))-(1/2))*ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t)))+np.log
(trig/S)*(((-2*ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t)))/((
sig**3)*np.sqrt(T-t)))+((12*(q-r))/(sig**4))*ss.norm(0,1).
cdf(y1-sig*np.sqrt(T-t)))+((2*(q-r))/(sig**3))*ss.norm
(0,1).pdf(y1-sig*np.sqrt(T-t)))+(((-np.log(trig/S))/((sig
**2)*np.sqrt(T-t)))+(((q-r)/(sig**2))-(1/2)))*(np.log(trig
/S)*(((ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t)))/((sig**2)*np
.sqrt(T-t)))*(-y1+sig*np.sqrt(T-t))-((4*(q-r))/(sig**3))*
ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t)))-(((q-r)/(sig**2))
-(1/2))*ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t))*(-y1+sig*np.
sqrt(T-t)))+K*np.exp(-r*(T-t))*ss.norm(0,1).pdf(-x1+sig*np
.sqrt(T-t))*((-x1+sig*np.sqrt(T-t))*((((np.log(S/trig))/((
sig**2)*np.sqrt(T-t)))-(((q-r)/(sig**2))-(1/2))*np.sqrt(T-
t))**2)-((2*np.log(S/trig))/((sig**3)*np.sqrt(T-t)))+((2*(
q-r))/(sig**3))*np.sqrt(T-t)) + S*np.exp(-q*(T-t))*ss.norm
(0,1).pdf(-x1)*((-x1)*((((np.log(S/trig))/((sig**2)*np.
sqrt(T-t)))-(((q-r)/(sig**2))+(1/2))*np.sqrt(T-t))**2)
-((2*np.log(S/trig))/((sig**3)*np.sqrt(T-t)))+((2*(q-r))/(
sig**3))*np.sqrt(T-t)))

    #Calculating the volga of the synthetic down and in forward
        for the coupons
    volga2 = 0
    for i in range(0, 14):
        volga2 = volga2 - ci * np.exp(-r*(ti[i]-t)) * (ss.norm
            (0,1).pdf(-x1i[i]+sig*np.sqrt(ti[i]-t))*((-x1i[i]+sig*
            np.sqrt(ti[i]-t))*((((np.log(S/trig))/((sig**2)*np.
            sqrt(ti[i]-t)))-(((q-r)/(sig**2))-(1/2))*np.sqrt(ti[i
            ]-t))**2)-((2*np.log(S/trig))/((sig**3)*np.sqrt(ti[i]-
            t)))+((2*(q-r))/(sig**3))*np.sqrt(ti[i]-t)) + ((trig/S
            )**(2*l-2))*np.log(trig/S)*((((4*(q-r))/(sig**3))**2)*
            np.log(trig/S)*ss.norm(0,1).cdf(y1i[i]-sig*np.sqrt(ti[
            i]-t))-((12*(q-r))/(sig**4))*ss.norm(0,1).cdf(y1i[i]-
            sig*np.sqrt(ti[i]-t))+((4*(q-r))/(sig**3))*ss.norm
            (0,1).pdf(y1i[i]-sig*np.sqrt(ti[i]-t))*(((-np.log(trig
            /S))/((sig**2)*np.sqrt(ti[i]-t)))+(((q-r)/(sig**2))
            -(1/2))*np.sqrt(ti[i]-t))))
        volga4 = volga4 - ci * np.exp(-r*(ti[i]-t)) * (-((trig/S)
            **(2*l-2))*ss.norm(0,1).pdf(y1i[i]-sig*np.sqrt(ti[i]-t
            ))*(((4*(q-r))/(sig**3))*np.log(trig/S)*(((np.log(trig
            /S))/((sig**2)*np.sqrt(ti[i]-t)))-(((q-r)/(sig**2))
            -(1/2))*np.sqrt(ti[i]-t))-(-y1i[i]+sig*np.sqrt(ti[i]-t
            ))*((((np.log(trig/S))/((sig**2)*np.sqrt(ti[i]-t)))
            -(((q-r)/(sig**2))-(1/2))*np.sqrt(ti[i]-t))**2)-((2*np
            .log(trig/S))/((sig**3)*np.sqrt(ti[i]-t)))+((2*(q-r))
            /(sig**3))*np.sqrt(ti[i]-t)))
```

```

    #Returning the volga of the bond
    return volga1+volga2
```

C.1.6 Vanna

```
def vanna(sig = 0.49, S = 0.75380, N = 1000.0, q = 0.01, r =
    0.021456939952621, trig = 0.1379985950200, CP = 0.59, T =
    2020.649315, t = 2013.876712, ci = 39.345):

    '''
    This function calculates the vanna of a contingent
        convertible bond using the equity derivatives model.

    Parameters:
    sig     - volatility
    S       - current price of the underlying stock
    N       - face value of the CoCo bond
    q       - dividend yield
    r       - riskless interest rate
    trig    - equivalent market trigger
    CP      - conversion price
    T       - time at maturity
    t       - current time
    ci      - coupon payment

    Output:
    0       - vanna of the contingent convertible bond
    '''

    #Calculating lambda
    l = (r - q + ((sig**2)/2))/sig**2

    #Creating a list that includes the time t of each coupon
        payment. Change this segment of code for pricing CoCos
        with a different coupon payment schedule.
    ti = np.zeros(14)
    ti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            ti[i] = ti[i-1] + (184.0/365.0)
        else:
            ti[i] = ti[i-1] + (181.0/365.0)

    #Calculating the inputs of the model
    K = CP
    Cr = N/CP
    x1 = (np.log(S/trig)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
```

```
    y1 = (np.log(trig/S)/(sig*np.sqrt(T-t))) + l*sig*np.sqrt(T-t)
    x1i = np.zeros(14)
    y1i = np.zeros(14)
    for i in range(0, 14):
        x1i[i] = (np.log(S/trig)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)
        y1i[i] = (np.log(trig/S)/(sig*np.sqrt(ti[i]-t))) + l*sig*np.sqrt(ti[i]-t)

    ### Calculating vanna ###

    #Calculating the vanna of the synthetic down and in forward for the final payment
    vanna1 = Cr * (((-4*(q-r))/(sig**3))*np.exp(-q*(T-t))*((trig/S)**(2*l))*ss.norm(0,1).cdf(y1)+(1-2*l)*np.exp(-q*(T-t))*((trig/S)**(2*l))*np.log(trig/S)*((4*(q-r))/(sig**3))*ss.norm(0,1).cdf(y1)+(1-2*l)*np.exp(-q*(T-t))*((trig/S)**(2*l))*ss.norm(0,1).pdf(y1)*(((-np.log(trig/S))/((sig**2)*np.sqrt(T-t)))+(((q-r)/sig**2)+(1/2))*np.sqrt(T-t))-((np.exp(-q*(T-t)))/((sig**2)*np.sqrt(T-t)))*((trig/S)**(2*l))*ss.norm(0,1).pdf(y1)*((K/trig)-1)+((np.exp(-q*(T-t)))/(sig*np.sqrt(T-t)))*((trig/S)**(2*l))*np.log(trig/S)*((4*(q-r))/(sig**3))*ss.norm(0,1).pdf(y1)*((K/trig)-1)+((np.exp(-q*(T-t)))/(sig*np.sqrt(T-t)))*((trig/S)**(2*l))*ss.norm(0,1).pdf(y1)*(-y1)*(((-np.log(trig/S))/((sig**2)*np.sqrt(T-t)))+(((q-r)/sig**2)+(1/2))*np.sqrt(T-t))*((K/trig)-1)+(K/trig)*np.exp(-r*(T-t))*((4*(q-r))/sig**3)*((trig/S)**(2*l-1))*ss.norm(0,1).cdf(y1-sig*np.sqrt(T-t))+(K/trig)*np.exp(-r*(T-t))*(2*l-2)*((trig/S)**(2*l-1))*np.log(trig/S)*((4*(q-r))/(sig**3))*ss.norm(0,1).cdf(y1-sig*np.sqrt(T-t))+(K/trig)*np.exp(-r*(T-t))*(2*l-2)*((trig/S)**(2*l-2))*ss.norm(0,1).pdf(y1-sig*np.sqrt(T-t))*(((-np.log(trig/S))/((sig**2)*np.sqrt(T-t)))+(((q-r)/(sig**2))-(1/2))*np.sqrt(T-t))+np.exp(-r*(T-t))*ss.norm(0,1).pdf(-x1)*(((np.log(S/trig))/((sig**2)*np.sqrt(T-t)))-(((q-r)/(sig**2))+(1/2))*np.sqrt(T-t)) - (np.exp(-q*(T-t))/((sig**2)*np.sqrt(T-t)))*ss.norm(0,1).pdf(-x1)*((K/trig)-1)+(np.exp(-q*(T-t))/(sig*np.sqrt(T-t)))*ss.norm(0,1).pdf(-x1)*(-x1)*(((np.log(S/trig))/((sig**2)*np.sqrt(T-t)))-(((q-r)/(sig**2))+(1/2))*np.sqrt(T-t))*((K/trig)-1))

    #Calculating the vanna of the synthetic down and in forward for the coupons
    vanna2 = 0
    for i in range(0, 14):
        vanna2 = vanna2 - ci * np.exp(-r*(ti[i]-t)) * ((ss.norm(0,1).pdf(-x1i[i]+sig*np.sqrt(ti[i]-t))/(S*(sig**2)*np.sqrt(ti[i]-t))) - ((ss.norm(0,1).pdf(-x1i[i]+sig*np.
```

```
                sqrt(ti[i]-t))*(-x1i[i]+sig*np.sqrt(ti[i]-t)))/(S*sig*
                np.sqrt(ti[i]-t)))*(((np.log(S/trig))/((sig**2)*np.
                sqrt(ti[i]-t)))-(((q-r)/sig**2)-(1/2))*np.sqrt(ti[i]-t
                ))-(2/trig)*((2*(q-r))/sig**3)*((trig/S)**(2*l-1))*ss.
                norm(0,1).cdf(y1i[i]-sig*np.sqrt(ti[i]-t)) - ((2*l-2)/
                trig)*((trig/S)**(2*l-1))*np.log(trig/S)*((4*(q-r))/
                sig**3)*ss.norm(0,1).cdf(y1i[i]-sig*np.sqrt(ti[i]-t))
                - ((2*l-2)/trig)*((trig/S)**(2*l-1))*ss.norm(0,1).pdf(
                y1i[i]-sig*np.sqrt(ti[i]-t))*(((-np.log(trig/S))/(S*(
                sig**2)*np.sqrt(ti[i]-t)))+(((q-r)/sig**2)-(1/2))*np.
                sqrt(ti[i]-t))+((trig/S)**(2*l-2))*(ss.norm(0,1).pdf(
                y1i[i]-sig*np.sqrt(ti[i]-t))/(S*(sig**2)*np.sqrt(ti[i
                ]-t))) - ((trig/S)**(2*l-2))*np.log(trig/S)*((4*(q-r))
                /sig**3)*(ss.norm(0,1).pdf(y1i[i]-sig*np.sqrt(ti[i]-t)
                )/(S*sig*np.sqrt(ti[i]-t))) - ((trig/S)**(2*l-2))*(ss.
                norm(0,1).pdf(y1i[i]-sig*np.sqrt(ti[i]-t))/(S*sig*np.
                sqrt(ti[i]-t)))*(-y1i[i]-sig*np.sqrt(ti[i]-t))*(((np.
                log(trig/S))/((sig**2)*np.sqrt(ti[i]-t)))-(((q-r)/sig
                **2)-(1/2))*np.sqrt(ti[i]-t)))

    #Returning the vanna of the bond
    return vanna1+vanna2
```

C.2 Trinomial Trees

This section includes the computational implementation of the path-dependent trinomial trees presented in sections 6.1 and 6.3.

C.2.1 Path-Dependent Tree

```
def trinomial(sig = 0.49, S0 = 0.75380, N = 1000.0, q = 0.01, r =
    0.021456939952621, trig = 0.1379985950200, CP = 0.59, T =
   2020.649315, t = 2013.876712, ci = 39.345, nodes = 2500):


    '''
    This function calculates the price of a contingent
        convertible bond using a path-dependent trinomial tree.

    Parameters:
    sig     - volatility
    S0      - current price of the underlying stock
    N       - face value of the CoCo bond
    q       - dividend yield
    r       - riskless interest rate
    trig    - equivalent market trigger
    CP      - conversion price
    T       - time at maturity
```

```
    t       - current time
    ci      - coupon payment
    nodes   - nodes in the trinomial tree

    Output:
    0       - price of the contingent convertible bond
    '''

    #Creating a list that includes the time t of each coupon payment. Change this segment of code for pricing CoCos with a different coupon payment schedule.
    cti = np.zeros(14)
    cti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            cti[i] = cti[i-1] + (184.0/365.0)
        else:
            cti[i] = cti[i-1] + (181.0/365.0)

    #Creating a list that includes the time t of each node
    ti = np.zeros(nodes+1)
    for i in range(0,nodes+1):
        ti[i] = t + ((i*(T-t))/nodes)

    #Assigning a node to each coupon payment
    cnodes = np.zeros(14)
    for i in range(0,len(cti)):
        for j in range(0,nodes):
            if cti[i] < ti[j+1] and cti[i] > ti[j]:
                if ti[j+1] - i > i - ti[j]:
                    cnodes[i] = j
                else:
                    cnodes[i] = j+1
    cnodes[-1] = nodes

    #Calculating the inputs of the model
    dt  = (T-t)/nodes
    nu  = r - q - 0.5 * (sig ** 2)
    dxu = sig * np.sqrt(3 * dt)
    dxd = - dxu
    pu  = ((nu * dt * dxu) + ((sig ** 2) * dt) + ((nu ** 2) * (dt **2 ))) / (2 * (dxu ** 2))
    pd  = (((sig ** 2) * dt) + ((nu ** 2) * (dt ** 2)) - (nu * dt * dxu)) / (2 * (dxu ** 2))
    pm  = 1 - pu - pd

    #Calculating the underlying stock price at each node at time T
```

```
    St = np.zeros(shape=(nodes*2+1, nodes+1))
    St[0][nodes] = S0 * np.exp(nodes * dxu)
    for i in range(0, nodes*2):
        St[i+1][nodes] = St[i][nodes] * np.exp(dxd)

    #Stepping back through the tree, calculating the underlying stock price at each node
    for i in range(nodes-1, -1, -1):
        for j in range(nodes-i, nodes+1+i):
            St[j][i] = St[j][i+1]

    #Calculating the undiscounted expected payoff of the final payment
    martingales = np.zeros(nodes*2+1)
    for i in range(0, nodes*2+1): #At terminal nodes
        if St[i][nodes] > trig:
            martingales[i] = 1
        else:
            martingales[i] = St[i][nodes]/CP #Replace with martingales[i] = 0 for write-down CoCos
    for i in range(nodes-1, -1, -1): #Stepping back through the tree
        k = 0
        for j in range(nodes-i, nodes+1+i):
            if St[j][i] > trig:
                martingales[k] = pu*martingales[k] + pm*martingales[k+1] + pd*martingales[k+2]
            else:
                martingales[k] = St[j][i]/CP #Replace with martingales[k] = 0 for write-down CoCos
            k = k + 1
    fpmartingale = martingales[0]

    #Calculating the undiscounted expected payoffs of the coupon payments
    cmartingales = np.zeros(14)
    for i in range(0, 14):
        martingales = np.zeros(cnodes[i]*2+1)
        k = 0
        for j in range(nodes-int(cnodes[i]), nodes+int(cnodes[i])+1): #At the nodes at which the coupon is paid
            if St[j][cnodes[i]] > trig:
                martingales[k] = 1
            k = k + 1
        for j in range(int(cnodes[i])-1,-1,-1):
            l = 0
            for k in range(nodes-j, nodes+j+1):
                if St[k][j] > trig:
```

```
                    martingales[l] = pu*martingales[l] + pm*martingales[l+1] + pd*martingales[l+2]
                else:
                    martingales[l] = 0
                l = l + 1
            for k in range(l, int(cnodes[i])*2+1):
                martingales[l] = 0
                l = l + 1
        cmartingales[i] = martingales[0]

    #Discounting back to time t
    price = N * fpmartingale * np.exp(-r*(T-t))
    for i in range(0, 14):
        price = price + (ci * cmartingales[i] * np.exp(-r*(cti[i]-t)))

    #Returning the price of the bond
    return price
```

C.2.2 Path-Dependent Tree with a Subjective Probability of Regulatory Conversion or Write-Down

```
def trinomial_regulatory_trigger(sig = 0.49, S0 = 0.75380, N = 1000.0, q = 0.01, r = 0.021456939952621, trig = 0.1379985950200, CP = 0.59, T = 2020.649315, t = 2013.876712, ci = 39.345, nodes = 2500, probrt = 0.01):

    '''
    This function calculates the price of a contingent convertible bond with a regulatory trigger using a path-dependent trinomial tree.

    Parameters:
    sig    - volatility
    S0     - current price of the underlying stock
    N      - face value of the CoCo bond
    q      - dividend yield
    r      - riskless interest rate
    trig   - equivalent market trigger
    CP     - conversion price
    T      - time at maturity
    t      - current time
    ci     - coupon payment
    nodes  - nodes in the trinomial tree
    probrt - yearly subjective probability of regulatory conversion or write-down

    Output:
```

```
    0       - price of the contingent convertible bond
    '''

    #Creating a list that includes the time t of each coupon payment. Change this segment of code for pricing CoCos with a different coupon payment schedule.
    cti = np.zeros(14)
    cti[0] = 2014.153425
    for i in range(1,14):
        if i % 2 == 0:
            cti[i] = cti[i-1] + (184.0/365.0)
        else:
            cti[i] = cti[i-1] + (181.0/365.0)

    #Creating a list that includes the time t of each node
    ti = np.zeros(nodes+1)
    for i in range(0,nodes+1):
        ti[i] = t + ((i*(T-t))/nodes)

    #Assigning a node to each coupon payment
    cnodes = np.zeros(14)
    for i in range(0,len(cti)):
        for j in range(0,nodes):
            if cti[i] < ti[j+1] and cti[i] > ti[j]:
                if ti[j+1] - i > i - ti[j]:
                    cnodes[i] = j
                else:
                    cnodes[i] = j+1
    cnodes[-1] = nodes

    #Calculating the inputs of the model
    dt  = (T-t)/nodes
    nu  = r - q - 0.5 * (sig ** 2)
    dxu = sig * np.sqrt(3 * dt)
    dxd = - dxu #symmetric jump sizes
    pu  = ((nu * dt * dxu) + ((sig ** 2) * dt) + ((nu ** 2) * (dt **2 ))) / (2 * (dxu ** 2))
    pd  = (((sig ** 2) * dt) + ((nu ** 2) * (dt ** 2)) - (nu * dt * dxu)) / (2 * (dxu ** 2))
    pm  = 1 - pu - pd

    #Calculating the probability of regulatory triggering at each node
    lmbda = 1-(1-probrt)**((T-t)/nodes)

    #Calculating the underlying stock price at each node at time T
    St = np.zeros(shape=(nodes*2+1, nodes+1))
```

```
    St[0][nodes] = S0 * np.exp(nodes * dxu)
    for i in range(0, nodes*2):
        St[i+1][nodes] = St[i][nodes] * np.exp(dxd)

    #Stepping back through the tree, calculating the underlying stock price at each node
    for i in range(nodes-1, -1, -1):
        for j in range(nodes-i, nodes+1+i):
            St[j][i] = St[j][i+1]

    #Calculating the undiscounted expected payoff of the final payment
    martingales = np.zeros(nodes*2+1)
    for i in range(0, nodes*2+1): #At terminal nodes
        if St[i][nodes] > trig:
            martingales[i] = 1
        else:
            martingales[i] = St[i][nodes]/CP #Replace with martingales[i] = 0 for write-down CoCos
    for i in range(nodes-1, -1, -1): #Stepping back through the tree
        k = 0
        for j in range(nodes-i, nodes+1+i):
            if St[j][i] > trig:
                martingales[k] = (1-lmbda) * (pu*martingales[k] + pm*martingales[k+1] + pd*martingales[k+2])
            else:
                martingales[k] = (1-lmbda) * St[j][i]/CP #Replace with martingales[k] = 0 for write-down CoCos
            k = k + 1
    fpmartingale = martingales[0]

    #Calculating the undiscounted expected payoffs of the coupon payments
    cmartingales = np.zeros(14)
    for i in range(0, 14):
        martingales = np.zeros(cnodes[i]*2+1)
        k = 0
        for j in range(nodes-int(cnodes[i]), nodes+int(cnodes[i])+1): #At the nodes at which the coupon is paid
            if St[j][cnodes[i]] > trig:
                martingales[k] = 1
            k = k + 1
        for j in range(int(cnodes[i])-1,-1,-1):
            l = 0
            for k in range(nodes-j, nodes+j+1):
                if St[k][j] > trig:
```

```
                    martingales[l] = (1-lmbda) * (pu*martingales[l] + pm*martingales[l+1] + pd*martingales[l+2])
                else:
                    martingales[l] = 0
                l = l + 1
            for k in range(l, int(cnodes[i])*2+1):
                martingales[l] = 0
                l = l + 1
        cmartingales[i] = martingales[0]

    #Discounting back to time t
    price = N * fpmartingale * np.exp(-r*(T-t))
    for i in range(0, 14):
        price = price + (ci * cmartingales[i] * np.exp(-r*(cti[i]-t)))

    #Returning the price of the bond
    return price
```