Canadian Housing Prices: Macroeconomic Factors, Local Fundamentals, and Persistence

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Abstract

I analyse the behaviour of housing prices in Canada's seven largest cities using monthly data from 1999:5 to 2014:6. A dynamic factor model is utilized to extract a common component from seven Canadian house price indexes. This is interpreted as the impact of nationwide factors on local markets. Results indicate that the common component explains between 27% and 41% of the variation in house prices for four of the seven cities while for the remaining three cities it explains below 17%. Analysis conducted at the local level provides evidence of the importance of persistence across all markets, particularly Eastern Canada. Local demand-side factors (per-capita income, labour force size) are also of importance while local supply-side factors (housing completions, cost of construction) explain a relatively small amount of price fluctuations.

Contents

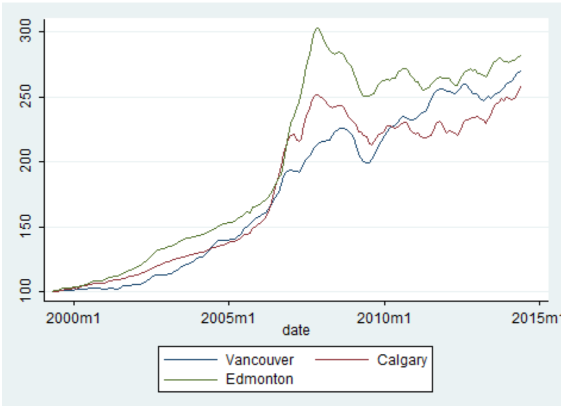
1 Introduction

A survey of the existing housing literature indicates that there is little consensus as to what factors determine house prices. Some researchers believe that house prices are determined to a large extent by nationwide fundamentals, particularly monetary policy. Others contend that prices are determined by factors which determine supply and demand conditions in specific urban markets. During housing bubbles, prices move entirely outside of the influence fundamentals and instead are driven by household expectations based on past growth. The goal of this essay is to quantify the relative importance of macroeconomic fundamentals, local factors, and persistence[1] in order to discover what has driven the steady appreciation of housing in the Canadian market since the beginning of the last decade and whether or not this growth constitutes a speculative bubble in specific urban markets.

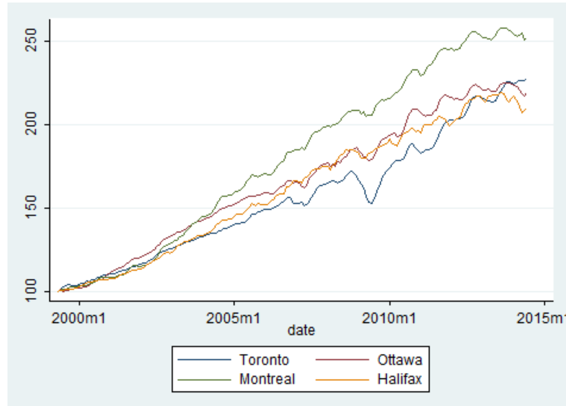
In recent times the Canadian housing market has received much attention from a variety of sources. Much of the commentary is centred around speculation as to whether or not the Canadian housing market is currently in the midst of a bubble similar in nature to that which several OECD countries experienced prior to the financial crisis of 2008. While several newspapers such as the *Financial Post* and *The Economist* have run stories to this effect, speculation has also come from market watchers at the International Monetary Fund. In a speech given in June 2014, Min Zhu, the Deputy Managing Director for the IMF, expressed his belief that there currently exists yet another worldwide housing bubble and named Canada as one of the key markets at risk, citing price-income and price-rent ratios well in excess of their historical averages in addition to the huge growth in housing prices as indicators of an overheated market. The rationale behind looking at the ratio of the price of housing to income is that there should exist a long run relationship between the two. If house prices rise too much relative to income, fewer households will be able to enter the market for housing. This shifts demand inward and brings prices back into equilibrium. Similarly, if the price of purchasing a house rises too much relative to the price of renting, demand is again depressed, prices fall, and the market is rebalanced. As Himmelberg et al. (2005) point out, such measures are often considered by market watchers to be good indicators of a housing bubble.

[1]Persistence in the time series sense can be thought of as the continuance of an effect after the cause is removed. In the context of this essay, it represents the influence of household expectations based on past growth in determining house price fluctuations.

Figure 1: Nominal Price Levels, 100=1999:5

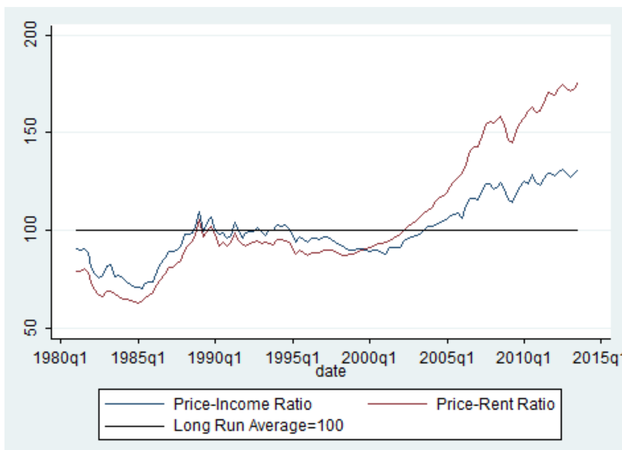


(a) Western Canada



(b) Central/Eastern Canada

Figure 2: Price-Income and Price-Rent Ratios



Source: The Economist

To give the reader a sense of where the Canadian housing market stands today, these indicators are displayed on the following page. Figure 1 plots the nominal price level of housing markets across Canada while Figure 2 plots nationwide measures of the price-income and price-rent ratios. Indeed these measures present an alarming picture. Growth in prices, a necessary condition of an overheated market, is evident for each of the seven major urban centres considered. Western Canada displays a more volatile pattern of growth compared with the East with a significant boom in prices occurring at the beginning of 2006. Each of the cities in Western Canada display nominal growth in excess of 250% while growth in the East is somewhat lower and steadier with the exception of Montreal.

Examining Figure 2, it is apparent that both ratios hovered around their historical levels between 1990:Q1 and 2000:Q1 when they both began an upward climb, although today the price-income ratio is far higher relative to its historical average compared with the price-rent ratio. While these figures certainly seem to indicate that the Canadian housing market is overvalued, Himmelberg et al. (2005) present a compelling argument to suggest that these conventional indicators of a housing bubble are inappropriate. The authors describe a housing bubble as a phenomenon in which excessive public expectations regarding the future growth of housing prices causes the market to be temporarily overvalued. Consumers are willing to pay excessive prices for houses because they expect the asset to appreciate, thereby compensating them for their initial investment. Rapidly rising prices also induce more households to enter the market by sparking the fear that many may soon become unable to afford the purchase of a home. The bubble bursts when it becomes evident to the public that prices will not continue to rise which causes demand to collapse. Essentially, a bubble in the housing market should thus be characterized by a situation in which fluctuations in prices are not readily explained by a handful of economic fundamentals related to supply and demand but rather by their own history. The price-income and price-rent ratios may therefore represent poor indicators of a bubble since they do not incorporate other fundamentals which are likely important determinates of housing prices.

Within the existing literature there exists little consensus as to the nature of these fundamentals, particularly for the Canadian market. Some researchers hold the belief that nation-wide or macroeconomic factors determine price movements while others hold that city specific factors related to the supply and demand of housing are most important.

Macroeconomic Fundamentals

There exists a substitutional body of literature which ties the behaviour of housing markets to movements in macroeconomic fundamentals. In a paper published prior to the economic crisis Himmelberg et al. (2005) attempt to ascertain whether or not the American housing market was overvalued in 2004. They argue that looking at the conventional price-income and price-rent ratios is a short-sighted way of answering such a question due to the fact that these measures treat the price of a house as if it were the same as the annual cost of owning. In fact, during periods of low interest rates, mortgage payments will be low and so will the annual cost of owning a house. In this situation, housing will be affordable even if prices are excessive.

In order to evaluate prices in the American housing market, the authors compute a measure of imputed rent for cities across the country. They define imputed rent as the annual cost to a home-owner of renting an equivalent property and in their calculations account for things such as the cost of maintaining the home, tax differences between renting and owning, and the opportunity cost of money. Using this approach they conclude that high prices in the American housing market can be attributed to an undervalued market in the 1990's, low interest rates, and high income growth. Their calculations also indicate that prices will be more sensitive to monetary policy if: 1. interest rates are already low and/or 2. if there has been substantial appreciation in the market.

Utilizing imputed rent rather than price to calculate the price-income and price-rent ratios, the authors concluded that the housing market in America was not intensely overvalued in a historical sense; in hindsight this is likely erroneous. I say likely because it may not have been an overvalued American housing market per se that necessitated the correction in prices that happened shortly after Himmelberg et. al. published their findings in 2005 but the fact that new home construction was 170% above household formation and that the household debt burden was at an all-time high[2].

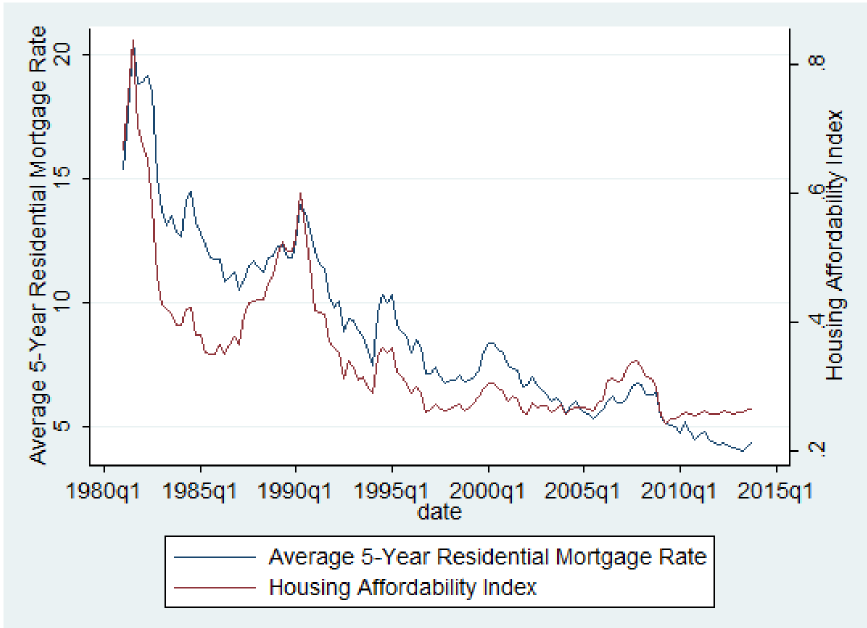
Figure 3 plots a measure of housing affordability constructed by the Bank of Canada as well as the average mortgage rate. This figure illustrates the fact that owning a house in Canada has never been more affordable. The figure also shows that the increase in affordability is closely paralleled by a decrease in the Average 5-year Residential Mortgage Rate[3] indicating that monetary policy has likely played a substantial role in this phenomenon by substantially lowering the cost of home financing. Indeed if we look at the *affordability* of housing rather than its price there appears to be little evidence in favour of a nationwide housing bubble today.

The use of vector auto regressions to link house prices to macroeconomic factors is common in the literature. For instance, Sutton (2002) examines the joint effect of national income, interest rates, and equity prices on the price of housing for six industrialized countries, including Canada, from 1995 to 2002. He argues that changes in stock prices may affect the price of housing due to the fact that they alter the wealth of households. Utilizing a vector auto regression (VAR) model he finds evidence supporting the significance of these variables. In particular he finds that house

[2](Marr, 2014)

[3]As Allen et al. (2009) point out the use of this rate is appropriate due to its high correlation with other maturities as well as the fact that over 50% of households use this term.

Figure 3: Affordability of Housing



Source: The Bank of Canada
Note: The Housing Affordability Index provides an estimate of the share of disposable income that a representative household would put towards expenses related to home-ownership. A decline in the index therefore illustrates an increase in the affordability of housing.

prices in the Canadian market are particularly sensitive to changes in the rate of interest, although his results also indicate that housing prices should have increased by far more over the sample period due to relatively low Canadian mortgage rates. Musso et al. (2011) view the housing market as a channel for the transmission of various macroeconomic shocks, including monetary policy. They adopt a structural VAR approach to analyse the differences between the American and the European housing markets and find that a positive shock to the interest rate significantly impacts real house prices and residential investment in both markets; although the effect is much more pronounced for the US than for the Euro zone. The authors attribute this difference to various dissimilarities between the two countries, in particular within the mortgage markets of the two areas. Indeed Adrian and Shin (2008) support this explanation, finding that the degree to which a country's mortgage lending sector is dominated by market-based lending institutions (as opposed to traditional commercial banks) affects the behaviour of the housing market as a transmission mechanism for monetary policy. They argue that cuts to the fed funds rate cause market-based lending institutions to expand lending; because their balance sheets are composed primarily of mortgage-related securities, investment in the housing sector is stimulated. By contrast, they find that the behaviour of commercial banks is unaffected by changes to the fed funds rate. Utilizing a structural VAR, they demonstrate that the effect of a fed funds shock on housing investment differs when the behaviour of market-based lending institutions is accounted for and when it is not.

The estimation of unobserved factors related to house price indexes is also used in the literature. Fadiga and Wang (2009) apply a multivariate state-space model to identify unobserved common trends and cycles in the price fluctuations of the four regional housing markets in the United States: the Northeast, Midwest, South, and West. They estimate two common trends and three common cycles for the data and utilize regression analysis to link these to the unemployment rate, GDP growth, and construction costs. They also incorporate a nation-wide measure of the corporate default risk and the fed funds rate. They argue that corporate default risk may be relevant due to the fact that housing represents an alternative to investment instruments such as corporate bonds and stocks. While their results concerning the impact of corporate default risk are mixed, they highlight the negative and significant impact of the fed funds rate on each of the estimated trends and cycles of their model. Similar to Himmelberg et al. (2005) they argue that increases in the fed funds rate reduce the affordability of housing by raising the effective mortgage rate and this leads to an increase in the inventory of houses. Because prices adjust to inventory levels, the build up leads to lower prices.

Local Factors

The preceding section demonstrates that there exists a number of researchers who employ a variety of techniques to successfully link house price fluctuations to movements in nationwide fundamentals, particularly monetary policy. However, there also exists a substantial literature that either fails to find evidence of such a linkage or argues in favour of local, city-specific factors as being far more important in determining the behaviour of housing markets.

In a study similar to Fadiga and Wang (2009) described above, Del Negro and Otrok (2007) utilize Bayesian methods to extract commonalities in house price fluctuations in the American housing market at the national and regional levels. Their motivation for this stems from the fact that they observe a wide-spread (though not entirely homogeneous) pattern of rapid growth in prices from 2001 to 2005 within markets at the state level. Their primary interest is to determine whether or not this phenomenon can be attributed to developments in national factors, primarily monetary policy. They argue that discovering a significant national co-movement in prices could dispel the popular notion held at the time that there were a handful of localized bubbles within the United States and that, rather than being out of step with fundamentals, the growth in housing prices could be attributed to the relatively low rate of interest prevalent at the time. While they do successfully extract a nationwide factor and link this to monetary policy, they conclude that prices were being driven primarily by regional components and factors at the state level. Moreover, a counter factual approach indicates that while monetary policy can explain a non-negligible amount of price growth over the first half of the last decade, it is quite small given the rate at which house prices were increasing. The difference in conclusion between Fadiga and Wang (2009)[4] and Del Negro and Otrok (2007) despite the fact that they employ similar methodologies can probably be attributed to the fact that the former uses a broader aggregate of housing prices (regional as opposed to state-level). While this enables them to construct a more sophisticated model, it has the consequence that individual components at the state level are likely to be lost.

To date, literature on the Canadian housing market has been somewhat limited. The literature that does exist tends to place emphasis on the heavy segmentation of local housing markets within Canada. For instance Allen et al. (2009) conduct tests of cointegration to determine if there exists a long-run relationship between the price levels of 8 major Canadian cities. Using data which spans 1981:Q1 to 2005:Q1 they find no evidence that such a relationship exists. Given this finding they also suggest that the usage of nationwide house price indexes (something which characterizes many of the studies described in the previous section) to study housing markets may not always be entirely appropriate. In the second half of the paper, the authors also employ a fully modified OLS procedure which is designed to conduct standard regression analysis when endogeneity and cointegration are present in the model. They include city-specific measures of per-capita income, labour force size, construction costs, as well as the new housing price index which reflects the substitutability between new and existing homes in their regression. They also include the 5-year average mortgage rate. This approach reveals strong evidence for the importance of local factors and limited evidence for the importance of monetary policy.

Maclean (1994) examines the degree to which inflation disparities across Canada can be at-

[4]That monetary policy is of great importance in determining house prices.

tributed to local housing markets. He argues the housing market can be characterized by inefficiency in part because the supply of housing is fixed in the short to medium-term; the market is therefore characterized by slow adjustment to localized shocks affecting fundamentals such as income or demography. Moreover houses are spatially fixed meaning that they can not be traded between markets to readily bring supply and demand into equilibrium; in other words, arbitrage is not possible. Because of this, city-to-city or region-to-region disparities in housing inflation are likely to persist for extended periods of time. Further exacerbating the difference between markets is the finding of Case and Shiller (1989) that household expectations of future home prices are not at all based on fundamentals but on the historical movements in prices[5]. In many ways the result of their study is distinctly at odds with much of the literature surveyed here. It is particularly difficult to reconcile these results with the work of Himmelberg et al. (2005) who define a bubble as a situation where prices are being determined by their own history rather than fundamentals. Such tension contributes motivation for the subject of this paper. If on the one hand, price fluctuations in all markets studied are determined to a large extent and to relatively the same degree by their own history (or their *persistence*) then Shiller's view is likely to be correct. On the other hand, if only certain markets display a large degree of persistence once all relevant fundamentals are accounted for then this would be evidence of the type of speculative bubble described by Himmelberg et al. (2005).

Motivation for this Paper, the Methodology, and Empirical Findings

The preceding review of literature provides motivation for the remainder of this essay. Each of the two bodies of literature appear to offer compelling arguments for the importance of macroeconomic and local fundamentals in driving house prices. It is therefore of interest to determine the role that each of these types of factors has played in the increase of prices in the Canadian housing market since the beginning of the last decade. Determining the role of persistence is also of interest because it may provide evidence as to the presence of speculative bubbles in local markets.

While the arguments and evidence offered by the limited body of literature dealing with the Canadian housing market indicate that nationwide fundamentals may not be important, it should

[5] Their conclusions are drawn from study which involved the distribution of questionnaires within major American cities that had recently experienced booms or busts in their housing market.

be noted that these studies focused on time periods which differ from that which is considered in this essay. In particular, such studies do not include the great recession which occurred from 2007-2009 and affected Canada on a national level. As Figure 2 demonstrates, the price indexes of all the cities being considered exhibit a significant degree of co-movement during this 2-year period. The sample period is also uniquely characterized by a high level of house price growth in conjunction with historically low rates of interest. Recalling the finding of (Himmelberg et al., 2005)[6] this should imply that the sensitivity of individual housing markets in Canada to changes in monetary policy should be greater than it ever has been.

To determine the degree to which the housing prices of seven Canadian markets share co-movement, a dynamic factor model is estimated through the use of a Kalman Filter. This type of approach estimates an unobserved (or latent) factor that describes the common movement of housing prices in each of the local markets and is similar in nature to some of the other studies which have been conducted in the housing literature[7]. Such an approach is advantageous because it allows me to quantify in percentage terms the degree to which price fluctuations in each individual market are attributable to the common factor. As Del Negro and Otrok (2007) point out, these results are of interest due to the fact that a large degree of co-movement of prices within local markets can be interpreted as evidence that monetary policy has mainly driven the run-up in prices [8]. Results indicate that the common factor is important for 4 of the 7 cities considered, explaining $27 - 41\%$ of the variation in prices of these markets.

Determining specifically which fundamentals were driving the common factor proved to be problematic and represents an unfortunate short-coming of this paper. It is instead assumed that the common factor is characterized by movements in macroeconomic fundamentals as well as similarities between cities of local supply and demand factors related to the market for housing. It is argued that macroeconomic fundamentals are also likely to affect house prices indirectly by determining to a large extent the similarity in local supply and demand conditions across markets.

The next part of this essay investigates the extent to which supply and demand factors can be

[6]The finding that house prices will be more sensitive to monetary policy if 1. interest rates are already low and 2. there is substantial growth in the market

[7]See (Del Negro and Otrok, 2007),(Fadiga and Wang, 2009), and (Qin, 2004).

[8]While there was also the possibility that quantifying the importance of the common factor could also provide evidence for speculative bubbles in local markets, particularly Vancouver and Toronto where market watchers have focused their attention, the empirical results were not so very clean cut.

related to price fluctuations at the local level. To this end the idiosyncratic movements in prices (changes in prices at the local level which are not accounted for by the common factor)are incorporated into a structural vector autoregression model that includes local factors related to supply and demand. This approach is intended to determine the importance of local factors while controlling for the influence of macroeconomic fundamentals. A forecast error variance decomposition is used to determine the importance of the local variables in driving price fluctuations within specific urban markets. Such an approach indicates that persistence in prices is of importance for all markets being considered, particularly those in Eastern Canada. This finding—that movements in prices are determined to a large extent by their own history—is consistent with the finding of Case and Shiller (1989). However fundamentals are also of significant importance for several, but not all, of the markets. Demand-side factors (per-capita income, size of a city's labour force) explain a large proportion of price fluctuations in several markets. By contrast, only a small proportion of price movements can be tied to local supply factors (housing completions, the cost of construction) which may be evidence that the demand for housing is elastic. Both local and national factors are of relatively little importance in determining the price fluctuations of housing in Toronto and Halifax over the sample period. This is interpreted as evidence of the existence of a housing bubble within these respective markets.

The remainder of this essay is organised as follows. Section 2 outlines the estimation of the dynamic factor model. It includes a brief theoretical overview of the model, the data employed, diagnostics, as well as the empirical results. Section 3 outlines the use of the structural VAR model and the results of the forecast-error variance decomposition in a manner similar to Section 2. Section 4 is reserved for the final results of the study as well as concluding remarks. An appendix is included which contains a number of materials referenced throughout the paper.

2 The Dynamic Factor Model

The dynamic factor model has a variety of economic applications and is used in general forecasting as well as in the study of finance, learning behaviour, and expectation formation. As has already been noted, such models have also been used to study housing markets[9] although it is admitted

[9] See (Del Negro and Otrok, 2007), (Fadiga and Wang, 2009), and(Qin, 2004).

that the Kalman Filter used in this paper to estimate the model is somewhat primitive compared with the more modern estimation techniques employed in this literature.

The following discussion of the model as well as the procedure used to estimate it follows Stock and Watson (1988) quite closely. Lutkepohl (2005), Qin (2004), as well as Allan Gregory's class notes were also used as additional sources.

2.1 The Theoretical Model

The Dynamic Factor Model

Let $y_{i,t}$ denote the growth rate[10] of city-specific house price index i at time t where $i = 1, ..., 7$. Nationwide shocks are hypothesized to have an affect on all seven markets such that the indexes share some degree of contemporaneous co-movement. $y_{i,t}$ is therefore comprised of two stochastic components: the common factor c_t which captures the co-movement of the indexes and is driven by macroeconomic fundamentals and the idiosyncratic component $u_{i,t}$ which represents the price fluctuations of market i that are not captured by the common factor. Idiosyncratic price fluctuations are assumed to be driven by the unique supply and demand conditions which exist at the local level. c_t and $u_{i,t}$ are also assumed to follow autoregressive processes of order p and order q respectively. The model can be expressed as

$$y_{i,t} = \gamma_i c_t + u_{i,t} \tag{1}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} .... + \phi_p c_{t-p} + \eta_t \tag{2}$$

$$u_{i,t} = d_{i,1} u_{i,t-1} + d_{i,2} u_{i,t-2} .... + d_{i,q} u_{i,t-q} + \epsilon_{i,t} \tag{3}$$

The main identifying assumption of the model is that the co-movements in the growth rates of the city-specific indexes are captured entirely by c_t. If we define U_t as an $nx1$ vector of the idiosyncratic components of each index where $n = 7$ then this implies that the covariance matrix of U_t is diagonal. It also implies that the n+1 disturbances are uncorrelated:

$$E \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix} \begin{bmatrix} \eta_t & \epsilon_t^{'} \end{bmatrix} = \Sigma = Diag(\sigma_\eta^2, \sigma_{\epsilon 1}^2, ..., \sigma_{\epsilon n}^2)$$

[10]The growth rate as calculated by Equation 11.

In order to identify the scale of c_t, $\sigma^2_{\eta t}$ is also set to unity.

State Space Representation

In order to estimate the model described by equations (4)-(6) it is necessary first to express them in state space form. Essentially the idea is that an observed time series $y = (y_1, y_2, ..., y_T)'$ depends upon an unobserved (or latent) state vector α_t comprised of the common and idiosyncratic components and their disturbances. Define Y_t, U_t, and E_t as nx1 vectors of the indexed variables $y_{i,t}, u_{i,t}$, and $\epsilon_{i,t}$ respectively.

Next define

$$c_t^* = (c_t \;\; c_{t-1}.....c_{t-p+1})' \qquad\qquad U_t^* = (U_t' \;\; U_{t-1}'.....U_{t-q+1}')'$$

Using the above notation it is possible to write the system in a condensed state space form given by the following two equations

$$Y_t = Z\alpha_{t-1} + \epsilon_t \tag{4}$$

$$\alpha_t = T_t\alpha_{t-1} + R\varsigma_t \tag{5}$$

where

$$\alpha_t = \begin{pmatrix} c_t^* \\ U_t^* \\ c_{t-1} \end{pmatrix} \qquad\qquad \varsigma_t = \begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix}$$

Equation (4) represents the measurement equation which describes the relationship between the observed data Y_t and the latent state variable α_t. Equation (5) is the transition equation and describes the evolution of the system over time. T_t represents a transition matrix while R and Z denote time-invariant selection matrices.

Estimation

Once equations (4)-(6) are cast in state space form it becomes possible to use the Kalman Filter to estimate the unobservable component c_t as a state variable. During estimation we are extracting the best estimates of this variable from the data, hence we are *filtering* out all the noise contained in the data in order to obtain some pure signal from it. The Kalman Filter is comprised of two sets of equations: the prediction equations and the updating equations both of which are applied at each point in time contained in the sample.

Let $\alpha_{t|t-1}$ be the estimate of the state variable, α_t, conditional on the information contained in $(y_1, ...y_{t-1})$ and let $P_{t|t-1}$ denote the prediction error at time t. They are denoted by

$$\alpha_{t|t-1} = T_t \alpha_{t-1|t-1} \quad (6)$$

$$P_{t|t-1} = T_t P_{t-1|t-1} T_t^{'} + R\Sigma R^{'} \quad (7)$$

where H and Σ denote the variance-covariance matrices of ϵ_t and ς_t respectively. An initial starting value is selected and the prediction equations calculate $\alpha_{t|t-1}$ and $P_{t|t-1}$ recursively forward.

The updating equations are then used to refine our estimate of the state variable by incorporating information at time t and combining it optimally with the information contained at time $t-1$. The Kalman Filter is distinguishable from a forecasting procedure due to the fact that we are using information from the current period t. Let the one-step ahead forecast of Y_t be given by $Y_{t|t-1} = Z\alpha_{t|t-1}$ so that the one step-ahead forecast error is given by $\nu_t = Y_t - Z\alpha_{t|t-1}$. The updating equations are thus described by

$$\alpha_{t|t} = \alpha_{t|t-1} + \overbrace{P_{t|t-1} Z^{'} F_t^{-1}}^{\text{Kalman Gain}} \nu_t \quad (8)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} Z^{'} F_t^{-1} Z P_{t|t-1} \quad (9)$$

Where F_t is the variance-covariance matrix of the forecast error at time t. In Equation (8) the one-step ahead forecast error is weighted by what is known as the Kalman Gain. This determines the relative weight given to information in period t and period $t-1$. If the Kalman Gain is large (small), more (less) weight is given to current information relative to past information. Note that the same type of process is occurring in Equation (9) as well. Once the estimates are properly refined they are plugged back into the the predication equations and the process begins again for the next period.

The parameters of the model are estimated by maximizing the Gaussian log likelihood $\mathcal{L}$ given by Equation (10) over the parameter space.

$$\mathcal{L} \quad = \quad \frac{1}{2}\Sigma_{t=1}^{T}\nu_t^{'}F_t^{-1}\nu_t - \frac{1}{2}\Sigma_{t=1}^{T}ln(det(F_t)) \qquad (10)$$

2.2 Data

The data set used in this section is developed by Teranet in conjunction with the Bank of Canada and consists of seven city-specific house price indexes for: Vancouver, Edmonton, Calgary, Toronto, Ottawa, Montreal and Halifax. The data is gathered on a monthly basis and spans 1999:5 to 2014:6.

The Teranet indexes are computed using the repeat sales method. Put simply, this method utilizes the observed appreciation of homes which have been sold twice or more over a selected sample period to infer the appreciation of homes that have not been sold during the same period. The repeat sales method has the advantage that it calculates appreciation based on sales of the same property and in this way avoids having to account for price differences in homes with varying characteristics, which can be problematic. However it has the limitation that it does not explicitly take into account any home renovations which may have occurred between sales; rather this issue is dealt with using statistical techniques designed to account for this bias. Two other data sets were considered for use in this section. The New Housing Price Index which had the advantage of a longer time span but was rejected on the basis that it only tracks the prices of newly completed homes and thus only measures a subset of the types of homes available on the market. The MLS Home Price Index used by Allen et al. (2009) was also considered. It has the advantage of drawing from a more complete data set which allows the application of sophisticated statistical techniques; however publicly available information for this index as of the writing of this paper only goes as far back as 2005. In addition, MLS also offers a less heterogeneous sample of cities compared with the Teranet data used here.

Manipulation of Data

The raw data obtained from Teranet had to undergo significant manipulation before it was suitable to begin estimation of the model. As Figure 1 demonstrates, all the data display a significant trend which was removed by calculating the logarithmic growth rate for each of the seven indexes:

$$y_{i,t} = (ln(x_{i,t}) - ln(x_{i,t-1})) * 100 \tag{11}$$

where $x_{i,t}$ denotes the level of index i at time t. Once first differenced all indexes displayed seasonality to varying degrees. As Camacho et al. (2012) point out, this is an issue which must be addressed by practitioners when estimating dynamic factor models. The problem of seasonality can be dealt with using one of the following two methods: the data can be corrected for seasonality prior to the estimation of the model or a seasonal component can be explicitly built into the model. As the size of the sample period was small relative to the number of indexes and because estimation of dynamic factor models tends to be computationally expensive, it made sense to keep the model as parsimonious as possible. As such each of the seven city-specific indexes were seasonally adjusted by applying the Frisch-Waugh-Lovell (FWL) theorem outlined by Lovell (1963) whereby each $y_{i,t}$ is regressed without a constant on twelve dummy variables which each indicate a month. The residuals obtained from this regression represent the deseasonalized data, with zero mean.

It is also worth noting here that the data were left in nominal terms. While some studies in the literature do deflate the house price indexes with some measure of inflation, a city-specific measure of inflation which excludes shelter is not readily available for Canada. Given the finding of Maclean (1994) that house prices drive a significant amount of the regional variation in inflation it seemed inappropriate to deflate each index with a measure of city-specific inflation which included house prices. While a national measure of inflation is available which does exclude housing, nationwide inflation pressures seem like a macro-economic fundamental which would be desirable for the model to capture. The data are thus left in nominal terms.

Lastly, it is necessary to normalize each of the city-specific indexes to have a mean of zero and variance equal to unity. If this step is neglected the resulting model will be biased because a greater weight will be given to those indexes which have a higher variance. Due to the fact that the data already had mean zero from the FWL procedure it was only necessary to divide by the standard deviation to achieve normalization.

Once the data were properly adjusted, Phillips-Perron test was carried out to ensure that the data were stationary. Table 1 reports the results of this test.

Table 1: Results of the Phillips-Perron Test of Stationarity

City	Mackinnon approximate p-value
Vamcouver	0.0001***
Calgary	0.0000***
Edmonton	0.0021***
Toronto	0.0000***
Ottawa	0.0000***
Montreal	0.0000***
Halifax	0.0000***

*** significant at the 1% critical value
** significant at the 5% critical value
* significant at the 1% critical value

Potential Source of Bias

It is important to note that if the model includes a subset of indexes that are highly correlated with one another, the model can be biased in the sense that the estimate of the common factor will be dominated by the co-movements of the correlated indexes. The contemporaneous correlations were thus computed and are included in Table 2.

Table 2: Correlations of the Seven City-Specific Price Indexes

	Vancouver	Calgary	Edmonton	Toronto	Ottawa	Montreal	Halifax
Vancouver	1.0000						
Calgary	0.5826	1.0000					
Edmonton	0.4783	0.6782	1.0000				
Toronto	0.4505	0.2220	0.2326	1.0000			
Ottawa	0.2512	0.2252	0.1437	0.4103	1.0000		
Montreal	0.3369	0.1087	0.1903	0.2784	0.3769	1.0000	
Halifax	0.2078	0.1529	0.1851	0.0762	0.2541	0.3567	1.0000

Given the absence of any formal test, it is difficult to say whether or not the common factor is likely to be biased in this regard. There is a bit of a pattern in Western Canada with Calgary sharing a high contemporaneous correlation with both Edmonton and Vancouver, however the correlation between Edmonton and Vancouver is not so high.

Given the empirical results obtained in Section 2.4 it does not appear as though the model is significantly biased in this regard.

2.3 Diagnostic Testing

While Teranet provides house price indexes for 11 major urban centres in Canada, the final model only incorporates 7 indexes. Initially, the dynamic factor model was estimated using all 11 indexes however the resulting model produced poor diagnostic results. This indicates bias arising out of the fact that the identifying assumptions made at the beginning of Section 2.1 are not satisfied by the empirical model. Poor diagnostic results indicate that the AR process for c_t and $u_{i,t}$ have not been correctly specified within the model[11]. The solution to this is to increase the order of the AR process that each of the variables is specified to follow. However, with 11 indexes, increasing the order of each $u_{i,t}$ from $q = 1$ to $q = 2$ requires the estimation of an additional 11 parameters. This becomes unwieldy very quickly and the estimation of any 11 index model with $q > 1$ failed. In order to make the model more parsimonious Victoria, Winnipeg, Hamilton, and Quebec City were excluded from the model on the basis that they were the smallest markets. Limiting the model to just seven indexes allowed the estimation of a model of order $p = 3$ and $q = 3$ which produced diagnostic results that were markedly better. As the 7 cities chosen to remain in the model represent a substantial part of the Canadian housing market and are geographically heterogeneous, the 7 city dynamic factor model was selected as the best candidate.

Two tests were conducted to assess the diagnostic fit of the model.

Test of Serial Correlation in the Observable Disturbances

The test of serial correlation in the observable disturbances is similar to the one carried out by Stock and Watson (1988). Recall that in Section 2.1 the identifying assumption that the co-movements in the growth rates of the city-specific indexes are captured entirely by c_t. This implied that the covariance matrix of the vector of idiosyncratic errors, $u_{i,t}$ was diagonal and also that the n+1 disturbances in Equations (2) and (3) were uncorrelated. The purpose of this test is to determine whether or not these conditions hold for the empirical model.

The test works as follows. We define $e_{i,t} = y_t - y_{t|t-1}$. This is the one-step ahead forecast error in period t for city-specific index i (for $i = 1, ...7$) estimated using the Kalman Filter with $p = 3$ and $q = 3$.

[11] Recall from equations (2) and (3) that the orders of those processes are given by p and q respectively.

The following regression is then estimated

$$e_{i,t} = \phi_1 e_{j,t-1} + ... + \phi_p e_{j,t-p} + \epsilon_t$$

where $(j = 1, ...7)$ and the number of lags is set to six. An F-test of the joint significance of the coefficients is conducted and the p-values for each of these tests was obtained. Table 3 reports the p-values obtained from this test.

Table 3: Test of Serial Correlation in the Observable Disturbances

Dep. Variable	e_{Hal}	e_{Mon}	e_{Ott}	e_{Tor}	e_{Edm}	e_{Cal}	e_{Van}
e_{Hal}	0.7791	0.8340	0.0362	0.1549	0.7761	0.7565	0.3434
e_{Mon}	0.3935	0.5398	0.5423	0.6143	0.0178	0.2104	0.9741
e_{Ott}	0.2560	0.5881	0.0369	0.6058	0.0707	0.4383	0.2222
e_{Tor}	0.1985	0.1542	0.3463	0.0176	0.0506	0.3903	0.0409
e_{Edm}	0.1894	0.5891	0.1282	0.0041	0.8429	0.3258	0.1827
e_{Cal}	0.3980	0.0272	0.1454	0.0270	0.5701	0.2795	0.5411
e_{Van}	0.1970	0.0740	0.0195	0.4018	0.1875	0.0242	0.3763

Note: Yellow denotes joint significance of the regressors at the 5% level. Red denotes joint significance at the 1% level.

Test of White Noise in the Observable Disturbances

The purpose of the second test is to assess the fit of the dynamic factor model. If the dynamic factor model fits the data then the one-step ahead forecast errors should be white noise. To this end a Portmanteau test of white noise is conducted on the each of the observable disturbances described in the last section. Table 4 presents the results from this procedure.

Table 4: Portmanteau Test of White Noise with 9 Lags

Innovation	Q-statistic	p-value
Vancouver	11.20	0.2625
Calgary	10.19	0.3356
Edmonton	8.72	0.4639
Toronto	25.16	0.0028***
Ottawa	15.26	0.0840*
Montreal	13.17	0.1549
Halifax	4.18	0.8988

*** significant at the 1% critical value
** significant at the 5% critical value
* significant at the 1% critical value

While the results of the two diagnostic tests conducted in this section are far from ideal, they represent a degree of progress over previous specifications. Table 5 reports the results from the first diagnostic test of serial correlation in a more condensed form alongside the best results achieved with the 11 index model when $p = 2$ and $q = 1$. Despite having a higher percentage of regressions that returned coefficients that had joint significance at the 5% level, the 7-City Index model displayed substantially fewer problematic tests that were significant at the 1% level. Overall the performance of the 7-City Index model was better with 26.4% of its test regressions being significant at the 10% level or lower compared with 31.3% for the 11-City index model.

Table 5: Comparison of Models

Significance Level	7 City Index	11 City Index
10%	6.1%	7.4%
5%	18.3%	14.0%
1%	2.0%	9.9%
Total	26.4%	31.3%

Note: The numbers underneath the 7 City Index and 11 City Index headings denote the percentage of regressions from the first diagnostic test that yielded coefficients that were jointly significant at the indicated level.

The performance of the 7-City index model on the second test also greatly outstripped that of its counterpart. 7 of the 11 series of forecast errors for the 11-City index model were not white noise at the 1% level of significance compared with 1 problematic series at the same level of confidence for the 7-City index model.

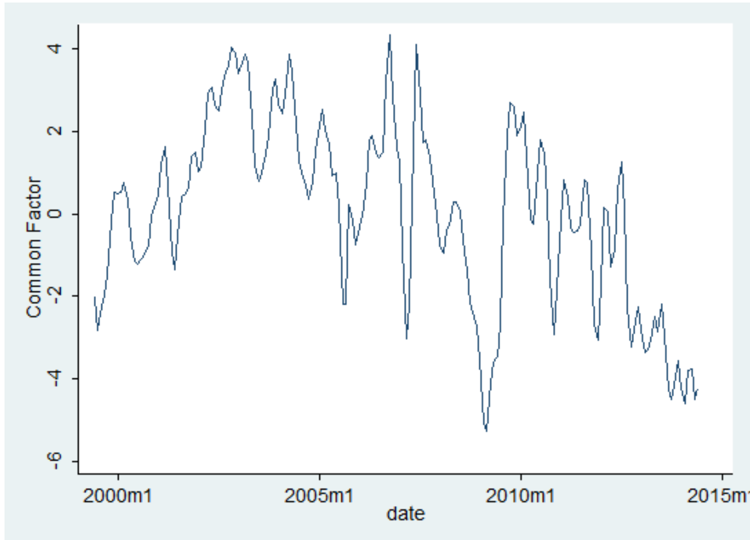
While diagnostic testing reveals that there is some degree of bias present in the final model, correcting for this appeared to be beyond the scope of this paper. Whatever bias remained was thus accepted and analysis continued. The estimation results of the final 7-city index model are reported in Section A of the Appendix.

2.4 Empirical Results

In this section the dynamic factor model is employed to yield two results: the average 2-year rolling correlation between the common factor and the seven city-specific indexes and a variance decomposition. The estimate of the common factor is depicted in Figure 4.

This figure implies that common movements in growth were generally positive during the first part of the sample, prior to the recession in 2008. Unsurprisingly the common factor exhibits

Figure 4: The Estimated Common Factor



a precipitous dip in response to the recession before recovering. Another observation is the fact that common growth in housing prices across the nation seems to have been stronger prior to the recession compared with the post recession period of 2010 onwards. Common growth after 2013 appears particularly weak, with prices largely declining. Referring to Figure 1, declining in prices have characterized the housing markets in Ottawa, Montreal, and Halifax over this recent period.

An attempt was made at explicitly characterizing the nature of the common by factor by regressing the growth rates of nation-wide data related to the average mortgage lending rate, non-shelter inflation, per-capita GDP, and labour force growth[12]. Persistence in the common factor was also accounted for by including 2 lags of the common factor. This procedure is similar to the one found in Fadiga and Wang (2009). Their results showed a relation between the common factor and the movements of various macroeconomic fundamentals, particularly the fed funds rate. Unfortunately the results obtained from utilizing that approach within the context of this study indicated that movements in the common factor were entirely associated with its own persistence (its lagged values) and not with the fundamentals included in the regression.

While this result is admittedly disappointing, it does not indicate that the common factor is being determined entirely by persistence but likely reflects a common problem inherent with this type of approach. Achen (2000) notes that regressions like the one described above can often

[12] 12 lags of each of the independent variables were also included in a separate specification to capture their delayed effect but similar results were obtained.

produce misleading results. He uses statistical theory to demonstrate that when autoregressive terms are incorporated into regressions involving independent variables, the lagged terms terms will often bias the results by dominating the effect of otherwise relevant independent variables. This has often produced non-nonsensical results that contradict otherwise sound economic theory and has often led practitioners to draw dubious conclusions.

Yet another alternative approach is to utilize a structural VAR alongside a forecast error variance decomposition (as is done in Section 3 to conduct analysis at the local level) to link fundamentals to the common factor. This is problematic however as the Choleski decomposition implies that the ordering of the variables is significant to the results[13] and economic theory does not readily provide a potential ordering. The use of sign restrictions is an alternative way to identify the structural shocks and has the benefit of being a more agnostic method in that it requires practitioners to make fewer assumptions with regard to the model. Such an approach was adopted by Del Negro and Otrok (2007) within the same context as this paper to successfully link GDP, the fed funds rate, the average 30-year mortgage rate, and inflation to their estimate of the nationwide common factor for the American housing market. Unfortunately, the implementation of sign restrictions to identify the structural shocks in the model is technically advanced and appears to be beyond the scope of this essay.

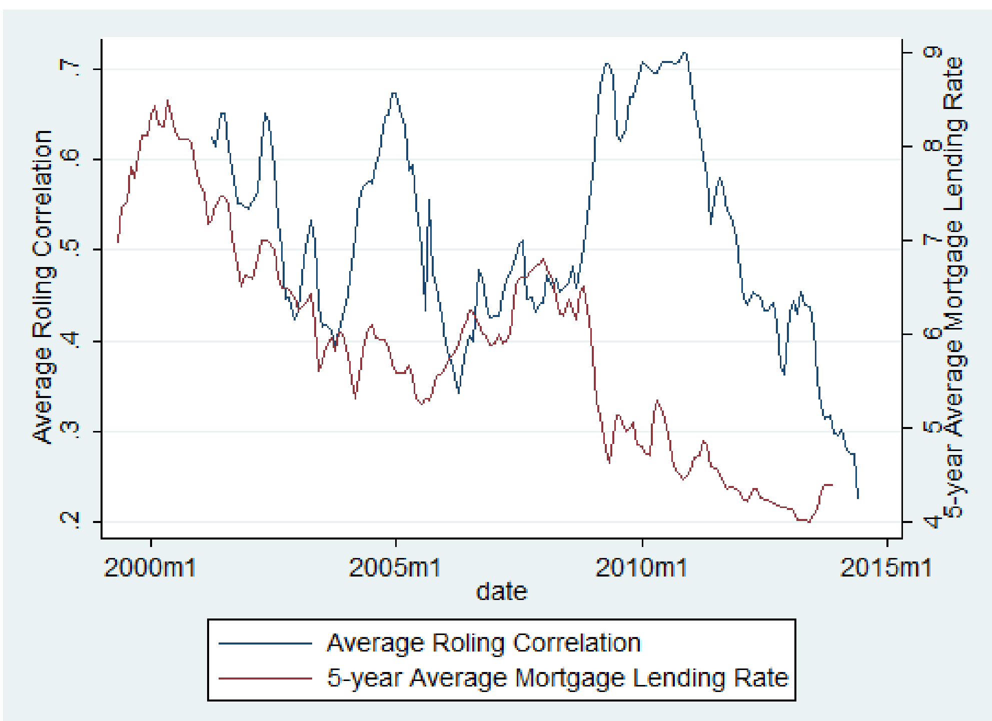
Rather than explicitly characterizing the common factor, I instead make the assumption that it is driven by some unidentified set of macroeconomic fundamentals as well as similarities in supply and demand conditions across local markets. I also note that while national factors may have a direct affect on housing prices through the mechanisms explained in Section 1, they also likely exert an indirect affect on prices in local markets by influencing the similarity of supply and demand conditions across markets. For instance both Adrian and Shin (2008) and Musso et al. (2011) find that monetary policy affects investment within the housing sector which means that supply conditions at the local level can potentially be altered by changes in monetary policy. Nationwide shocks to GDP such as the world-wide economic recession can also influence per-capita income across the country to a great degree. Nationwide inflationary pressures may also determine growth in construction costs by influencing the cost of inputs.

[13]For an explanation of this see Section 3.1.

Rolling Correlation

With an estimate of the common factor obtained it is possible to calculate the average rolling correlation between it and the individual house price indexes. Computing the rolling correlation between each city-specific indexes and the common factor and then taking an average of the correlations provides a simple and intuitive measure of how the importance of the common factor changes over time. An attempt to establish a link between monetary policy and the importance of the common factor is also made by graphing the 5-year average residential mortgage rate alongside the average rolling correlation. The idea behind this being that changes in the average mortgage lending rate should cause a higher degree of co-movement in markets across the country. Thus either positive or negative changes in monetary policy should lead to a spike in the average correlation between the cities in the sample and the common factor. Figure 5 plots the result of this.

Figure 5: Average 2-Year Rolling Correlation



The average rolling correlation fluctuates quite a lot over the sample period, peaking briefly at 0.7 at the beginning of 2005 and returning to this same level for a sustained period of time during the financial crisis and subsequent recession. At best, the relationship between the rolling correlation and the mortgage rate can be described as tenuous. However, at the beginning of the sample period some upward spikes in correlation do coincide with increases (but not decreases) in the average mortgage lending rate. What is perhaps most striking is that the sudden rise in the importance of the common factor at the end of 2008 coincides nearly perfectly with a substantial drop in average lending rates. The causality of this is however quite dubious and is probably less related to monetary policy and more so to do with a generalized panic that caused housing prices to fall across the country. Moreover, a fall in lending rates should imply that prices increase, yet Figure 4 exhibits highly negative rates of growth for this same period. Low mortgage rates may however be able to explain why the importance of the common factor remained high until midway 2011 well after the recession was ended. At the beginning of the crisis it seems given that consumers would be reluctant to purchase a home: despite record low costs of financing, Figure 1 demonstrates falling prices during this time. The quick recovery of housing prices may be due to a delayed adjustment to lower rates of financing and may explain the quick and sustained recovery of prices in markets across the country.

The final interesting thing to note about Figure 5 is that today, the average correlation of the common factor with local prices is at its lowest within the sample period. This suggests that as of late housing prices in local markets have been moving on increasingly divergent paths.

The Variance Decomposition

It is possible to utilize the estimation results reported in Section A of the appendix to obtain a measure of the contribution of the common factor to price fluctuations in each city-specific index. This is of interest because if the common factor were to explain a significant degree of price variations within each of the markets, monetary policy would likely be behind the growth in housing prices across the country[14].

The variance decomposition adopted here was first used by by Gregory et al. (1997) in a paper studying the impact of world business cycles on output, consumption, and investment for a number of countries. In a similar study, Kose et al. (2003) also use a variance decomposition to measure the contribution of worldwide and regional factors to the variance of aggregate variables for a number of countries.

[14] This idea is taken from Del Negro and Otrok (2007) and provides motivation for their study of the American housing market which is similar in nature to this essay.

In order to carry out this variance decomposition, it is necessary to assume that the common and series-specific factors are orthogonal. It is then possible to to express the variance of each index city-specific index i as

$$\sigma_i^2 = \gamma_c^{i^2} \sigma_c^2 + \sigma_{i,u}^2 \tag{12}$$

As described in Section 2.1 During the estimation process the variance of the common component σ_c^2 is normalized to unity in order to identify the scale of the common factor. The estimate of the variance of each index explained by the common factor may then be expressed as

$$\hat{R}_c^{2^i} = \frac{\hat{\gamma}_c^{i^2} \hat{\sigma}_c^2}{\hat{\gamma}_c^{i^2} \hat{\sigma}_c^2 + \hat{\sigma}_{i,\epsilon}^2 \hat{\sigma}_{i,u}^2} \tag{13}$$

where $\hat{\sigma}_{i,\epsilon}^2$ is the estimated variance of the innovation to the series specific component. $\hat{\sigma}_c^2$ and $\hat{\sigma}_c^2$ are computed using the estimated coefficients for the common and idiosyncratic processes respectively. Recall that the orders of both these processes are both equal to 3. Computing the variance for an $AR(3)$ process is slightly involved and as such a brief outline of the procedure is included in Section B of the appendix. The results of the variance decomposition are reported in Table 6.

Table 6: Variance Decomposition Results

City	R-squared
Vancouver	36%
Calgary	41%
Edmonton	13%
Toronto	8%
Ottawa	27%
Montreal	32%
Halifax	17%

Note: The percentages denote the proportion of variance in each index that is attributable to the common factor.

The results indicate that the common factor explains the price dynamics of the seven housing markets to varying degrees. For Vancouver, Calgary, Ottawa, and Montreal the common factor explains a good deal of local price variations while for the remaining 3 cities, it is quite low. This is evidence against the idea that expansionary monetary policy has been driving the growth of house prices across the nation and may indicate that unique supply and demand conditions at the local level can overwhelm the affect of fundamentals at the national level.

It is however interesting to note that Edmonton and Calgary have very different R^2's which is contrary to what one may expect given that supply and demand conditions in these cities are likely to be similar, and likely differ greatly from the rest of the country. Alberta has enjoyed a prolonged boom over the past decade that has brought vast numbers of new workers to its cities and raised average incomes to levels well in excess of the national average, yet housing prices in Calgary are readily explained by the common factor while prices in Edmonton are not.

A potential explanation for this phenomenon is that different markets may react differently to changes in monetary policy. This could be the case if the supply of housing is constrained either by geography, civil boundaries, or if the supply-side of the market is sluggish to adjust to new demand conditions. A decrease in the cost of financing a home caused by loose monetary policy will shift housing demand outwards by increasing its affordability. If the supply of housing readily expands to meet this new demand then we would expect prices to remain relatively stable. However, for a city like Vancouver which is walled in by the ocean on one side and the mountains on the other, it may be difficult to expand the supply of housing. The obvious solution would be to build upwards but this may take longer and certain households may decidedly prefer to live in the suburbs. Prices therefore have nowhere to go but up. Montreal may face the same type of geographical constraint given that much of it is situated on an island. With regards to Calgary it does seem as though land-use restrictions implemented at the beginning of the last decade have impeded housing construction. In a report published in April of 2014, Wendell Cox argues that "with its strong urban containment policies, the Calgary area could be at risk of repeating the even-more severe cost escalation that has occurred in metropolitan areas with longer histories of urban containment policy, such as Vancouver and Sydney"[15]. To determine if this idea indeed has any weight, data from Section 3 is used to calculate the total number of housing completions per 1000 from 1999:5 to 2014:4. The results are reported in Table 7.

Far from substantiating the idea that housing construction in Calgary has been sluggish, that particular market boasts the highest number of housing completions per thousand people over the sample period of any of the other cities. It therefore does not appear as though the above represents an adequate explanation as to why Calgary and Edmonton have such different R^2's.

[15] Pp. 5, Cox (2014)

Table 7: Total Housing Completions Per 1000 People

City	Completions
Vancouver	119
Calgary	195
Edmonton	173
Toronto	122
Ottawa	131
Montreal	97
Halifax	114

Alternatively, the fact that the common factor is not of importance in certain markets may be taken as evidence that prices are moving out of step with fundamentals. This would imply the existence of a speculative bubble within these markets and may be a situation in which prices are being determined more by their own history than by fundamentals, either local or nationwide.

At any rate, the results in Table 5 are difficult to interpret due to the fact that they only represent part of the picture. Local factors are most certainly of importance too and thus a closer look at their affect on housing markets can potentially provide additional insight. This is the focus of Section 3.

3 The Importance of Local Factors

The results of the variance decomposition in the previous section motivate the analysis continued in this section. The fact that less than half of the price fluctuations in each local market are attributable to the common factor leaves a potentially significant role for local supply and demand factors. This is consistent with the conclusions of Allen et al. (2009) and Maclean (1994) discussed in Section 1 who study the Canadian housing market as well and who utilize approaches and time periods which differ from that of this paper.

While the *potential* role for local factors is significant, it may be the case that the persistence in housing prices explains a great degree of their behaviour. If this were to be the case and prices were moving outside of fundamentals then we would have the Himmelberg et al. (2005) definition of a bubble. In order to determine the importance of persistence in each urban market, it is necessary to control for all of the fundamentals both nationwide and local that are thought to be relevant. Although earlier analysis failed to establish *which* macroeconomic variables are important, it is possible to control for their influence by utilizing the idiosyncratic errors (the portion of each price index which is not explained by the common factor) estimated by the dynamic factor model for each city-specific house price index.

In the following analysis the idiosyncratic errors, $u_{i,t}$, are incorporated into a structural VAR model with several other local factors. As stated in the previous section, the dynamic factor model is assumed to capture the effects of macroeconomic fundamentals as well as the similarity of supply and demand conditions across markets. Under this assumption, if a particular market experiences large deviations in, say, the growth of per capita income from the national average, that market will be less related to the common factor. For this reason all local factors are entered into the model as deviations from the national average. This will be expanded upon in Section 3.2.

Using a Choleski decomposition to identify the structural shocks of the model, a forecast error variance decomposition is performed to determine the relative importance of local fundamentals and persistence.

3.1 The Theoretical Model

The method selected to conduct analysis for this portion of the paper is a structural vector auto regression (SVAR). While the use of either a vector error correction model or a fully modified OLS were considered, these models require the data to be I(1) which was not the case for two of the five variables included in the model. The SVAR approach controls for endogeneity in the model which is a useful property given that the dataset contains information related to the price of housing as well as demand and supply-side factors. With an SVAR it is also possible to perform a forecast error variance decomposition (FEVD) which is employed do determine the relative importance of each the variables in describing local variations in housing prices. An overview of the technical aspects of the model borrowed from Enders (2009) will now be given.

Consider the following first-order VAR with two variables

$$y_t = b_{10} - b_{12}z_t + c_{11}y_{t-1} + c_{12}z_{t-1} + \epsilon_{yt} \tag{14}$$

$$z_t = b_{20} - b_{21}y_t + c_{21}y_{t-1} + c_{22}z_{t-1} + \epsilon_{zt} \tag{15}$$

where

$$\epsilon_{it} \sim \; iid(0, \sigma^2_{\epsilon i}) \qquad\qquad Cov(\epsilon_y, \epsilon_z) = 0$$

It is also generally assumed that y_t and z_t are stationary. The structure of the above model is such that y_t and z_t are allowed to contemporaneously affect one another. ϵ_{yt} and ϵ_{zt} are pure innovations and and are interpreted as exogenous shocks to y_t and z_t respectively. Equations (14) and (15) can be expressed in matrix notation as

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{yt} \\ \epsilon_{zt} \end{bmatrix} \tag{16}$$

In more compact form the system is given by

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \epsilon_t \tag{17}$$

Equations (16) and (17) represent the SVAR. Multiplying both sides of the equation by B^{-1} gives

$$x_t = A_0 + A_1 x_{t-1} + e_t \tag{18}$$

It will also be useful for sake of clarity to express Equation (18) in a less condensed form

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{19}$$

Equations (18) and (19) represent the unstructured form of the original model. This system is estimable by OLS due to the fact that it contains only predetermined variables and the error terms can be shown to be white noise with constant variance. The error terms are composites of the original innovations from Equations (16) and (17)

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{1 - b_{21}b_{12}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{yt} \\ \epsilon_{zt} \end{bmatrix} \tag{20}$$

It thus follows that $Cov(e_{1t}, e_{2t}) \neq 0$. Correlation between the two errors is problematic since this implies that a contemporaneous shock to one innovation has a contemporaneous affect on another. Clearly this is undesirable if we are interested in ascertaining the *pure* affect of a shock to one variable on the other. Note the SVAR has the useful property that the structural innovations do not exhibit any correlation. The problem arises in that it is not directly estimable by OLS since the key assumption that y_t and z_t be uncorrelated with the innovations is violated. The unstructured VAR given in Equation (19) proves useful in this instance because the parameters obtained from its estimation (6 coefficients, 2 variances, 1 covariance) can be used to indirectly obtain estimates of the parameters for the SVAR. The reader will however note that while we obtain 9 parameters from estimating Equation (19), Equation (16) requires the estimation of 10 parameters (8 coefficients, 2 variances). The system is thus under-identified.

The Recursive VAR and Choleski Decomposition

One possible way to identify the SVAR is suggested by (Sims, 1980). This method involves imposing a recursive structure on the model which requires us to restrict certain parameters in the VAR. For instance we can assume that y_t is contemporaneously affected by z_t but not the other way around. This is what is known as a Choleski Decomposition and is characterized by the triangular matrix in the leftmost element of Equation (21). The structural VAR in Equation (16) now becomes

$$\begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{yt} \\ \epsilon_{zt} \end{bmatrix} \tag{21}$$

Again multiplying by B^{-1} the system is given by

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} - b_{12}b_{20} \\ b_{20} \end{bmatrix} + \begin{bmatrix} (c_{11} - b_{12}c_{21}) & c_{12} - b_{12}c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{yt} - b_{12}\epsilon_{zt} \\ \epsilon_{zt} \end{bmatrix} \tag{22}$$

Comparing the above to Equation (19) makes it clear that the system is now identified with

$$a_{10} = b_{10} - b_{12}b_{20} \qquad a_{20} = b_{20} \qquad e_1 = \epsilon - b_{12}e_z$$

$$a_{11} = c_{11} - b_{12}c_{21} \qquad a_{21} = c_{21} \qquad e_2 = e_z$$

$$a_{12} = c_{12} - b_{12}c_{22} \qquad a_{22} = c_{22} \qquad Cov_{12} = -b_{12}\sigma_z^2$$

While this method of identifying the structural shocks within the model is widely used in the literature, it does suffer from the criticism that the ordering of the variables matters. When we identify the model in this manner we are in fact making the assumption that the first variable in the model has a contemporaneous affect on all of the other variables, but not vice versa; that the second variable in the model has a contemporaneous affect on all the variables (except the first variable) but not vice versa and so on. Results obtained when we estimate impulse response response functions or FEVDs are often not robust to different orderings. This is clearly problematic given that economic intuition is not always helpful in determining what the correct way to order the variables is. For the purposes of this study, the recursive approach was judged to be appropriate given that it is utilized by Head et al. (2014) to identify a model which uses a data set similar to the one found in this paper. Their intuition is described Section 3.3 of this paper.

Forecast Error Variance Decomposition

The forecast error variance decomposition quantifies how much of the change in a variable is due to its own shock and how much is due to shocks in other variables. Typically at shorter horizons the change in a given variable can mainly be explained by its own shock but as the horizon increases the importance of the other variables grows. To outline the theory behind the FEVD it is first necessary to re-write the unrestricted VAR in vector moving autoregressive (VMA) form. We begin by rewriting Equation (18) as

$$X_t = \frac{A_0}{I - A_1 L} + \frac{e_t}{I - A_1 L} \tag{23}$$

where L denotes the lag operator. It can be shown that

$$\frac{A_0}{I - A_1 L} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} \tag{24}$$

where $\bar{z}$ and $\bar{y}$ denote the means of z_t and y_t respectively. If we assume that the model is stationary, or that the roots of $I - A_1 L$ lie outside of the unit circle then it is possible to express the second component of Equation (23) as

$$\frac{e_t}{I - A_1 L} = \Sigma_{i=0}^{\infty} A_1^i e_{t-i} = \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} e_{1,t-i} \\ e_{2,t-i} \end{bmatrix} \tag{25}$$

We utilize Equation (19) to replace the composite errors in the above equation with the structural innovations ϵ_t. Using Equations (19),(24), and (25), Equation (23) can thus be re-expressed as

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \underbrace{\frac{A^i}{1 - b_{12} b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}}_{\phi_i} \begin{bmatrix} \epsilon_{y,t-i} \\ \epsilon_{z,t-i} \end{bmatrix} \tag{26}$$

ϕ_i is known as the impact multiplier. This traces the effect on either y_t or z_t of a one unit shock to a particular structural innovation. For example the immediate effects of a shock to $\epsilon_{z,t}$ on y_t and z_t are given by

$$\frac{dy_t}{d\epsilon_{z,t}} = \phi_{12}(0) \qquad \frac{dz_t}{d\epsilon_{z,t}} = \phi_{22}(0)$$

Similarly the the one-period ahead shocks are given by

$$\frac{dy_{t+1}}{d\epsilon_{z,t}} = \phi_{12}(1) \qquad \frac{dzt+1}{d\epsilon_{z,t}} = \phi_{22}(1)$$

Equation (24) written with the impact multipliers becomes

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix}^i \begin{bmatrix} \epsilon_{y,t-i} \\ \epsilon_{z,t-i} \end{bmatrix} \tag{27}$$

Or, more compactly

$$x_t = \bar{X} + \sum_{i=0}^{\infty} \phi_i \epsilon_{t-i} \tag{28}$$

Once the system is put in proper VMA form we can calculate the n-period forecast errors of x. Starting from period 1

$$x_{t+1} = \bar{X} + \phi_0 \epsilon_{t+1} + \phi_1 \epsilon t + \phi_2 \epsilon t - 1 + ...$$

$$E_t[x_{t+1}] = \bar{X} + \phi_1\epsilon_t + \phi_2\epsilon t - 1 + ...$$

Which gives the 1-period forecast error

$$x_{t+1} - E_t(x_{t+1}) = \phi_0\epsilon_{t+1}$$

Utilizing the same type of method we can obtain the 2-period forecast error

$$x_{t+2} - E_t(x_{t+2}) = \phi_0\epsilon_{t+2} + \phi_1\epsilon_{t+1}$$

This process easily generalizes to any horizon. The n-period forecast error is given by

$$\begin{aligned} x_{t+n} - E_t(x_{t+n}) &= \phi_0\epsilon_{t+n} + \phi_1\epsilon_{t+n-1} + \phi_2\epsilon_{t+n-2} + ... + \phi_{n-1}\epsilon_{t+1} \\ &= \sum_{i=0}^{n-1} \phi_i\epsilon_{t+n-i} \end{aligned}$$

Recall that x_t is a vector of y_t and z_t. The n-step-ahead forecast error of y_t is given by

$$\begin{aligned} y_{t+n} - E_t(y_{t+n}) = (&\phi_{11,0}\epsilon_{y,t+n} + \phi_{11,1}\epsilon_{y,t+n-1} + ... + \phi_{11,n-1}\epsilon_{y,t+1}) \\ &+ (\phi_{21,0}\epsilon_{z,t+n} + \phi_{21,1}\epsilon_{z,t+n-1} + ... + \phi_{21,n-1}\epsilon_{z,t+1}) \end{aligned} \tag{29}$$

The first component in Equation (29) gives the proportion of variance in y_t that is explained by a shock it its own innovation. The second component in the equation gives the proportion of variance that can be explained by the innovation associated with z_t. As the horizon increases the size of the first component decreases relative to that of the second.

3.2 Local Data

The variables chosen for the model are intended to reflect supply and demand factors for housing at the local level and are similar to those chosen by Allen et al. (2009) in their study of the American housing market.

Measures of income and the number of households are chosen to characterize the demand for housing. Obtaining publicly available data for income at the monthly frequency for specific urban markets in Canada proved to be infeasible. Instead, seasonally adjusted data at the monthly frequency was obtained for the total compensation of employees at the *provincial* level[16] and divided by the total provincial population. To measure the number of households in each market, monthly data for the size of each city's labour force was obtained. I choose to view this as a proxy for household formation occurring within a city, either through increased immigration or demographic changes.

The cost of building homes and the number of completions are chosen to characterize the supply side of the local market of each city. The Apartment Construction Index (APCI) is used to proxy the cost of building houses[17]. The APCI measures changes in contractors' selling price of a representative apartment building, although notably excludes the cost of land. While it may be more appropriate to follow Allen et al. (2009) in using building permits and a union wage index to measure the cost of constructing housing, it was necessary to keep the number of variables in the model to a minimum in order to keep the Choleski ordering manageable. The APCI is only available at quarterly frequency, it was therefore necessary to expand it to monthly frequency by using interpolation. Seasonally adjusted data for the total number of housing completions in each city was also obtained at the monthly frequency. In order for this data to be comparable across cities, the number of completions was divided by the population of each city in 1000's. A more detailed description of the data and where it was obtained from is included in section C of the Appendix. Figures 6-7 plot the data for each city as well as the national averages in levels.

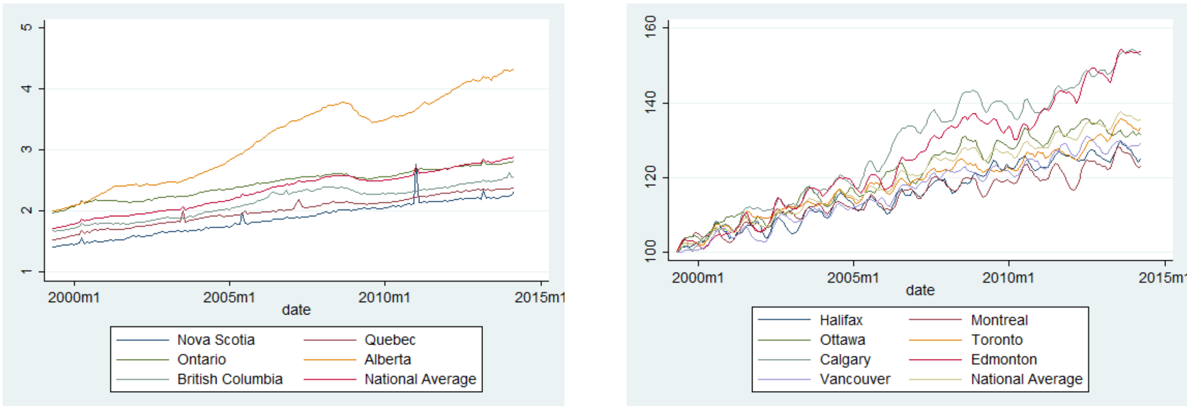
As Figure 6 demonstrates, income growth across the country has displayed a similar pattern with the notable exception of Alberta which has grown well in excess of the national average. The labour forces of Edmonton and Vancouver have also experienced substantial growth well in excess of the other cities in the model. On the supply side, several cities displayed a substantial run-up in the cost of constructing housing beginning in 2006 then experienced a decline just prior to the start of 2009.

The idiosyncratic fluctuations in the city-specific indexes obtained from the dynamic factor model are also included in the VAR. This represents the variance in prices that is not accounted for by nationwide factors and similarities in local supply and demand conditions across markets.

[16]This is computed by Statistics Canada and represents an important part of their calculation of GDP.

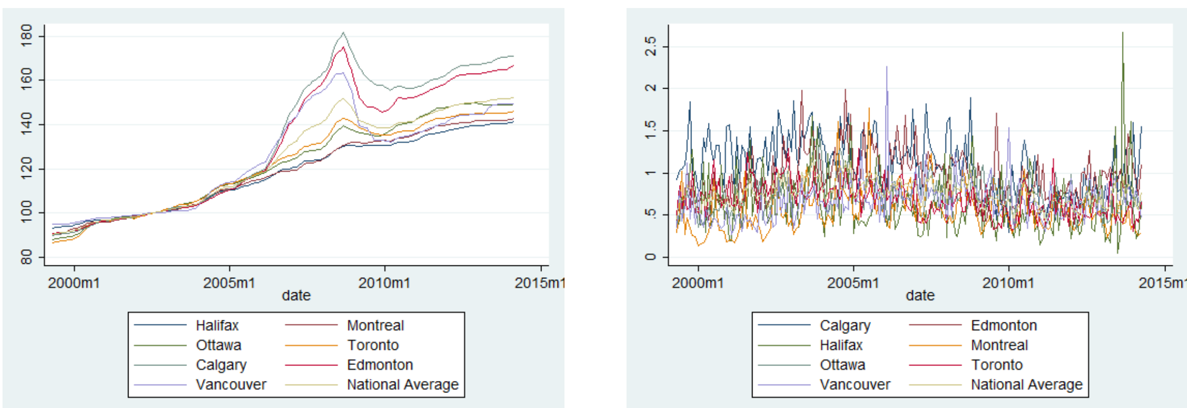
[17]A technical report published in May 2013 by the Alberta Treasury Board indicates that that this is an appropriate proxy for the general cost of constructing housing

Figure 6: Local Demand Factors



(a) Monthly Per Capita Income (1000's of Dollars) (b) Labour Force, 1999:5 as base year

Figure 7: Local Supply Factors



(a) Apartment Construction Index (b) Housing Completions per 1000 people

It is thus appropriate to include the local supply and demand factors in the VAR as deviations from their respective national averages. With the exception of housing completions, all the local factors were non-stationary. The logarithmic growth rates for each city-specific measure of income, labour force, and the APCI, as well as their respective national averages were thus calculated using Equation (11). The deviations from the national average for each city specific data set were then calculated by subtracting the national average from each of the city-specific variables. All variables were again tested for stationarity and the null hypothesis of a unit root was rejected in all instances.

With the exception of Edmonton, the labour force data in deviations from the national average displayed seasonality which was corrected for using the FWL procedure described in Section 2.2. Housing completions for Montreal were also corrected for seasonality. All the data included in the model were then normalized in the manner described in Section 2.2.

3.3 Estimation and Diagnostic Results

The following model was estimated for each of the seven cities in the sample

$$By_{i,t} = \Gamma_0 + \Gamma_1 y_{i,t-1} + ... + \Gamma_p y_{i,t-p} + \epsilon_t \tag{30}$$

where $y_{i,t} = (\Delta Inc_{i,t}, u_{i,t}, \Delta APCI_{i,t}, Comp_{i,t}, \Delta LF_{i,t})$[18] in that order and $\epsilon_{i,t}$ is a vector of structural shocks. As explained in Section 3.1 the ordering of the variables in the vector $y_{i,t}$ affects the results of the FEVD. The ordering of the variables is similar to that of Head et al. (2014). They argue that when incomes rise in a particular city relative to the national average, new workers relocate to that city thereby increasing the size of its labour force. A larger labour force and higher incomes increases the demand for housing, which puts upward pressure on prices. Higher prices induce more investment within the housing sector and the number of housing completions rises. Unfortunately the authors do not incorporate a measure of the cost of constructing housing $\Delta APCI_{i,t}$ which is included in this model. Several different specifications were run and results were generally robust when $\Delta APCI_{i,t}$ was ordered after $u_{i,t}$[19].

A standard (reduced form VAR) was estimated for each of the seven cities. Before performing the FEVD, it is necessary to perform two diagnostic tests to determine the fit of the model. The modelling process is now outlined:

1. The final prediction error (FPE) was obtained to determine an initial lag length for which to estimate the VAR model.
2. The VAR was estimated using the lag length indicated by the FPE.
3. Two diagnostic tests were run:
   (a) Verifying the stability condition of the VAR that all the eigenvalues of the system lie inside the unit circle.
   (b) An LM test of order 12 to determine the presence of autocorrelation in the residuals.

[18]Denoting respectively per capita income, idiosyncratic prices, the Apartment Construction Index, completions/1000 people, and labour force as they were obtained in Section 3.2

[19]For the sake of brevity, only the results from the given ordering are reported.

4. If unsatisfactory diagnostic results were obtained the order of the model was increased by one and re-estimated.

5. Steps 3 and 4 repeated until satisfactory diagnostic results had been obtained.

Generally, good diagnostic results were obtained for each of the seven models estimated with the notable exception of Ottawa which was slightly problematic at some of the higher order lags. Unfortunately increasing the order of the model did not remedy these results and the model reported reflects the best possible specification that could be obtained. Diagnostic results are reported in Section D of the Appendix.

3.4 Forecast Error Variance Decomposition Results

With the seven city-specific VARs properly constructed, it is possible to perform a forecast error variance decomposition. The FEVD tells how much of a change in a variable is due to its own shock and how much is due to shocks to other variables. The variable of interest is the idiosyncratic growth rate of prices $u_{i,t}$. Thus the results presented in Table 8 indicate the importance of shocks to the local variables as well as shocks to $u_{i,t}$ in determining movements in $u_{i,t}$. Local factors that are highly correlated with the national average are not expected to have a large degree of explanatory power. With this in mind, the contemporaneous correlation of each of the 4 supply and demand factors with their respective national averages are also reported in Table 8 which gives a frame of reference.

Table 8 presents several notable results. What is perhaps most notable is the fact that persistence in the local price fluctuations accounts for over half of their variation in all markets except for Edmonton. This finding is consistent with the result obtained by Case and Shiller (1989) that price movements in housing markets are mainly determined by their own history. Persistence in prices also seem to be less important in Western Canada where more of the changes in prices are attributable to fundamentals.

However, price fluctuations in housing markets do appear to be determined to a large extent by local demand-side factors as well. Changes in income at the local level represent the second most important factor in determining housing prices, with the exception of Calgary where shocks to the labour force variable are most important. Growth in the labour force in Western Canada in

Table 8: Forecast Error Decomposition Results

Forecast Horizon (Months)	Δinc_t	Δu_t	$\Delta APCI_t$	$Comp_t$	ΔLF_t
Vancouver					
3	0.00	0.96	0.00	0.00	0.01
6	0.18	0.62	0.03	0.06	0.11
9	0.19	0.54	0.03	0.07	0.16
12	0.19	0.53	0.04	0.09	0.15
Correlation w/ National Average	0.5372	–	0.9077	0.3618	0.7422
Edmonton					
3	0.26	0.50	0.00	0.04	0.23
6	0.37	0.33	0.01	0.06	0.22
9	0.36	0.29	0.02	0.08	0.26
12	0.35	0.28	0.02	0.08	0.28
Correlation w/ National Average	0.4595	–	0.9534	0.6602	0.7914
Calgary					
3	0.03	0.97	0.00	0.01	0.17
6	0.05	0.65	0.03	0.06	0.21
9	0.15	0.55	0.03	0.09	0.19
12	0.16	0.51	0.03	0.09	0.21
Correlation w/ National Average	0.4595	–	0.9369	0.6437	0.7524
Toronto					
3	0.09	0.87	0.02	0.00	0.01
6	0.21	0.69	0.03	0.05	0.02
9	0.26	0.64	0.03	0.05	0.03
12	0.29	0.62	0.03	0.05	0.03
Correlation w/ National Average	0.5960	–	0.8559	0.4388	0.7812
Ottawa					
3	0.13	0.86	0.00	0.00	0.00
6	0.22	0.74	0.01	0.02	0.01
9	0.29	0.64	0.01	0.04	0.01
12	0.31	0.61	0.01	0.05	0.02
Correlation w/ National Average	0.5960	–	0.7986	0.5670	0.6624
Montreal					
3	0.06	0.92	0.00	0.01	0.01
6	0.06	0.83	0.00	0.01	0.01
9	0.14	0.83	0.00	0.01	0.03
12	0.15	0.80	0.01	0.02	0.03
Correlation w/ National Average	0.4962	–	0.5565	0.5496	0.8261
Halifax					
3	0.08	0.90	0.01	0.01	0.00
6	0.13	0.81	0.01	0.02	0.02
9	0.14	0.74	0.03	0.07	0.02
12	0.14	0.73	0.03	0.08	0.02
Correlation w/ National Average	0.8152	–	0.7768	0.5346	0.7760

excess of the national average has also been of importance in Western Canada. In Eastern Canada changes in the labour force are basically irrelevant.

Interestingly supply-side factors fail to explain much of any variation in prices at the local level for all the markets considered. This result is somewhat perplexing and could be due to the fact that the Apartment Construction Index is an inadequate measure of the cost of construction due to the fact that it does not account for the price of land; on the other it bears noting that growth in the APCI in most markets tends to be highly correlated with the national average.

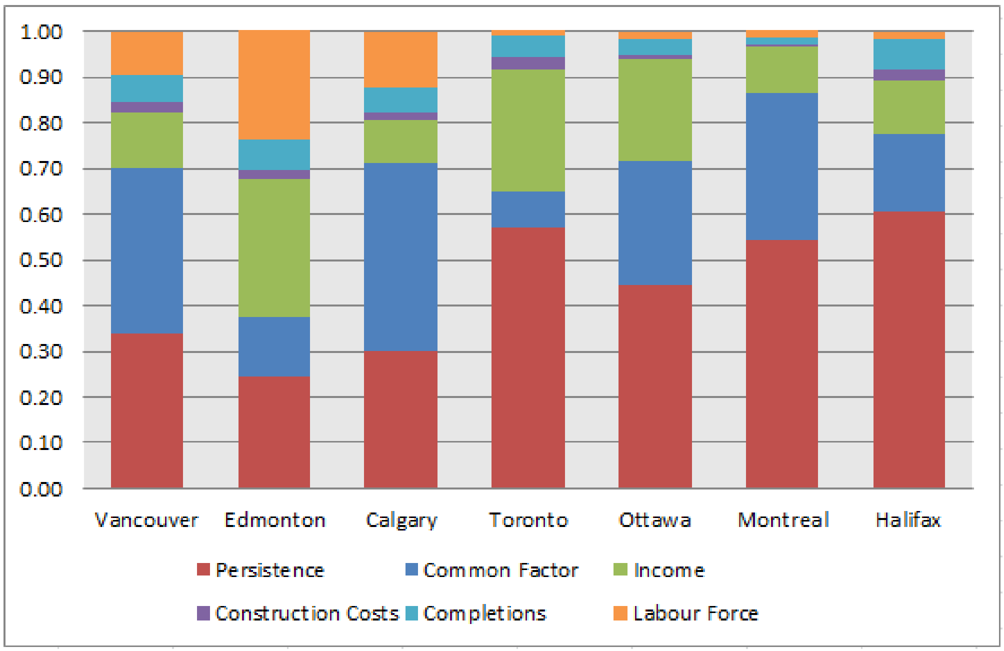
The inability of completions/1000 people to explain much of the variation in local prices across all seven markets may be evidence that the demand for housing is elastic. Somewhat surprisingly, academic research on this topic is limited and the literature that does exist seems quite archaic given the tendency of many economic phenomena to change over time. Hanushek and Quigley (1980) describe the difficulties in estimating the price elasticity of demand for housing. Firstly, the sheer heterogeneity of housing makes the direct observation of prices impossible. Secondly, there exist significant search, transactions, and moving costs associated with changing dwellings which implies that a given household's consumption may be very different from its utility maximizing level in a static equilibrium. The authors use data obtained from an experiment that provided housing subsidies to low income households in Phoenix and Pittsburgh. Their results indicate that the demand for housing is quite elastic, with $\epsilon = -0.359$ for Pittsburgh and $\epsilon = -0.409$ for Phoenix. However this evidence cannot be said to hold substantial weight given that it is a study involving low-income households in two American cities, close to 35 years ago. It may however make sense that housing demand would be elastic, given that purchasing a house represents the single largest expenditure that the vast majority of households make and it may not be terribly difficult for the majority of households to delay this expenditure, at least in the short to medium-run.

Section E of the Appendix also reports the FEVD results graphically. These figures indicate the confidence intervals for the results. They depict wide confidence intervals for Toronto, Ottawa, Montreal, and Halifax suggesting that the FEVD results for these cities are imprecise and that persistence may explain as much as all of the variation in prices. Much more precise estimates were obtained for cities in Western Canada.

4 Conclusion

The goal of this paper has been to quantify the relative importance of macroeconomic fundamentals, local factors, and persistence in determining house prices in urban markets across Canada. The purpose of this exercise was to reconcile two conflicting bodies of literature and to find evidence for localized housing bubbles in Canada. Figure 8 combines the results of Sections 2 and 3 and depicts the overall findings of this paper.

Figure 8: Macroeconomic Fundamentals, Local Factors, and Persistence



Given the assumptions made throughout this paper, the effect of nationwide fundamentals on local markets is captured by the common factor estimated in Section 2. The importance of the common factor varies greatly from city to city however a reasonable explanation of this phenomenon proves to be elusive and the problem is compounded by the fact that the determinates of the common factor are unknown. Unfortunately this presents a major shortcoming of my study. Perhaps a more desirable approach would have been to include regional factors within the model for Western and Eastern Canada respectively alongside the national factor. It is possible that such an approach would have produced a more characterizable common factor and reduced the bias present in the final model.

With regard to local factors, per-capita income is of importance to markets across the country.

Changes in the size of the labour force in Western markets is also of importance while in the East it is largely irrelevant. Overall, supply-side factors are also relatively unimportant. The fact that the supply of housing does not affect prices at the local level may be evidence that the demand for housing is elastic.

Persistence explains a large proportion of housing price fluctuations across the nation, particularly in Eastern Canada. The fact that house prices are predominately determined by their own history rather than by fundamentals may be evidence of a speculative bubble within Eastern Canada, particularly in Toronto and Halifax where neither macroeconomic nor local fundamentals appear to explain price movements in those markets.

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A Estimation results of the Dynamic Factor Model

	ϕ_1	ϕ_2	ϕ_3	γ_1	d_1	d_2	d_3
Common	1.327 (0.000)***	-0.737 (0.009)***	0.303 (0.047)**				
Vancouver				0.253 (0.000)***	0.695 (0.000)***	0.113 (0.302)	0.695 (0.000)***
Calgary				0.324 (0.000)***	0.570 (0.000)***	0.452 (0.005)***	-0.190 (0.036)**
Edmonton				0.137 (0.005)***	0.606 (0.000)***	0.428 (0.000)***	-0.192 (0.012)**
Toronto				0.116 (0.029)**	0.577 (0.000)***	0.249 (0.002)***	-0.332 (0.000)***
Ottawa				0.212 (0.000)***	0.159 (0.039)**	0.343 (0.000)***	-0.298 (0.000)***
Montreal				0.236 (0.000)***	-0.091 (0.330)	-0.062 (0.489)	-0.295 (0.001)***
Halifax				0.167 (0.000)***	-0.027 (0.721)	-0.093 (0.211)	-0.319 (0.000)***

*** significant at the 1% critical value
*** significant at the 5% critical value
** significant at the 1% critical value

City	$\hat{\sigma}^2_\epsilon$
Vancouver	0.289 (0.000)***
Calgary	0.296 (0.000)***
Edmonton	0.238 (0.000)***
Toronto	0.501 (0.000)***
Ottawa	0.605 (0.000)***
Montreal	0.640 (0.000)***
Halifax	0.747 (0.000)***

*** significant at the 1% critical value
*** significant at the 5% critical value
** significant at the 1% critical value

B Computation of the variance for an AR(3) process

Consider the following $AR(3)$ process

$$z_t = c + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 z_{t-3} + a_t$$

Given that $E[z_t] = \mu$ this becomes

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} \phi_3 \tilde{z}_{t-3} + a_t$$

Where $\tilde{z}_t = z_t - \mu$. This yields the autocorrelation formula

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho k - 2 + \phi_3 \rho_{k-3}$$

Note that for a stationary process $\rho_i = \rho_{-i}$. Thus for $k = 1$ the autocorrelation formula is

$$\rho_1 = \phi_1 + \phi_2 \rho_1 + \phi_3 \rho_2$$

Solving for ρ_1 yields

$$\rho_1 = \frac{\phi_1 + \phi_3 \rho_2}{1 - \phi_2}$$

Similarly k=2 yields

$$\rho_2 = \rho_1(\phi_1 + \phi_3) + \phi_2$$

and k=3 yields

$$\rho_3 = \phi_1 \rho_2 + \phi_2 \rho_1 + \phi_3$$

This gives a system of 3 equations in 3 unknowns. ρ_1, ρ_2, ρ_3 can therefore be solved for as functions of the coefficients ϕ_1, ϕ_2, ϕ_3. Finally, the variance of an $AR(P)$ process can be shown to be

$$\gamma_0 = \frac{\sigma_\epsilon^2}{1 - \phi_1 \rho_1 - .... - \phi_p \rho_p}$$

So the estimated coefficients $\hat{\phi_1}, \hat{\phi_2}, \hat{\phi_3}$ are plugged into the above equation to yield the estimated variance of the process, $\hat{\gamma}_0$.

C Data Description

Per Capita Income

- Wages, Salaries, and Employer's Social Contributions
  - Source: Statistics Canada, Table 382-0006
  - Province-wide
  - Frequency: Monthly
  - Seasonally Adjusted
- Population
  - Source: Statistics Canada, Table 282-0116
  - Province-wide
  - Frequency: Monthly
  - Seasonally Adjusted

Labour Force

- Source: Statistics Canada, Table 282-0111
- City-specific
- Frequency: Monthly
- Not seasonally adjusted

Apartment Construction Index

- Source: Statistics Canada, Table 327-0044
- City-Specific
- Frequency: Quarterly
- Not seasonally adjusted

Housing Completions

- Source: Statistics Canada, Table 027-0048
- City-Specific
- Frequency: Monthly
- Not seasonally adjusted
- City Population
  - Source: Statistics Canada, Table 282-0116
  - City Specific
  - Frequency: Monthly
  - Seasonally adjusted

D Diagnostic results of the seven VAR Models

Vancouver

FPE Selection Criteria		3
Stability		yes
Optimal Lags		7
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.3219
2	25	0.2329
3	25	0.1388
4	25	0.5411
5	25	0.1063
6	25	0.4548
7	25	0.3522
8	25	0.1224
9	25	0.0980
10	25	0.0898
11	25	0.8713
12	25	0.0615

Edmonton

FPE Selection Criteria		1
Stability		yes
Optimal Lags		7
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.7214
2	25	0.8292
3	25	0.2882
4	25	0.7190
5	25	0.8225
6	25	0.3376
7	25	0.8521
8	25	0.7358
9	25	0.0726
10	25	0.4026
11	25	0.6872
12	25	0.3538

Calgary

FPE Selection Criteria		8
Stability		yes
Optimal Lags		8
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.4330
2	25	0.7109
3	25	0.4021
4	25	0.3282
5	25	0.5832
6	25	0.1461
7	25	0.0910
8	25	0.9151
9	25	0.0574
10	25	0.1612
11	25	0.4063
12	25	0.3140

Toronto

FPE Selection Criteria		4
Stability		yes
Optimal Lags		4
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.5279
2	25	0.1808
3	25	0.5861
4	25	0.3203
5	25	0.6375
6	25	0.0874
7	25	0.8818
8	25	0.0448
9	25	0.1997
10	25	0.8540
11	25	0.6917
12	25	0.3743

Ottawa

FPE Selection Criteria		3
Stability		yes
Optimal Lags		7
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.7776
2	25	0.6185
3	25	0.2355
4	25	0.6846
5	25	0.4289
6	25	0.0571
7	25	0.9293
8	25	0.1279
9	25	0.0.006
10	25	0.9954
11	25	0.0586
12	25	0.0258

Montreal

FPE Selection Criteria		3
Stability		yes
Optimal Lags		8
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.9016
2	25	0.8207
3	25	0.1221
4	25	0.3357
5	25	0.4485
6	25	0.1375
7	25	0.9704
8	25	0.4322
9	25	0.6275
10	25	0.8705
11	25	0.4675
12	25	0.4639

Halifax

FPE Selection Criteria		3
Stability		yes
Optimal Lags		7
Residual Autocorrelation		
Lag #	df	Prob $>\chi^2$
1	25	0.9862
2	25	0.8841
3	25	0.1260
4	25	0.2120
5	25	0.6790
6	25	0.1080
7	25	0.7764
8	25	0.7358
9	25	0.0290
10	25	0.2653
11	25	0.8842
12	25	0.6591

E FEVD Confidence Intervals

Vancouver, Edmonton, and Calgary

Toronto, Ottawa, Montreal, and Halifax