Asset and Option Price Dynamics with Private Information:
Theory and Evidence from S&P 500 Index Options

by

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Abstract

This paper is divided up into two sections; theoretical and empirical. An extension to Glosten and Milgrom's (1985) model is developed to account for changes in options prices when presented with private information. Put-call parity arguments are used to relate the changes in asset prices through the changes in options prices. In a world where the only actions are buying and selling assets or options, the extended model suggests that the spread between call and put options decreases when the conditional expectation of a down state increases, as well as the converse. S&P 500 options data and the CBOE VIX, both from the year 1993, are used to test this hypothesis. It is concluded that not only is the spread most significant when considering "in-the-money" options, but also that call options provide an overall better signal when faced with conditions of uncertainty. It is also found that deviations from put-call parity are negatively correlated with the slope parameter on the call-put difference. This suggests that changes in the VIX respond to optimism or pessimism as measured by the call-put spread.

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Contents

1 Introduction

The financial crisis of 2007-2008 has brought into question the accuracy of many of the standard models used in finance and economics. Classic assumptions governing these models have proven to derail financial markets in many countries, and theories are thus forced to go back into development. To correct for the misspecification of these models, they must be rebuilt to reflect the realities apparent in the financial market; ie, non-constant volatility and changing expectations among others. Creating a model that can accurately reflect these realities is one of the toughest challenges for financial economists. The approach often rests upon the frontier of scientific technique; even at that point there may still exist misspecification. Thus, there is a major margin for error as the course of events in the financial market bears some uncertainty. Since there are high costs in constructing an exact replica of reality, making close approximations is justified. The point I am trying to make is not to say that market makers are doing a poor job at creating models, but that it is difficult to create a model that is even close to reflecting the dynamics of real markets. The most recent example is that of the financial crisis. Even the most state of the art models of 2006 were not able to capture what would proceed to be the biggest recession since the great depression.

My goal in proceeding sections is to develop a model of private information to see whether the behavior of insiders is apparent in the market. The simple idea I shall be exploring is how private information is channeled through an asset price. What I hope to draw out of this is a relation between asset prices and the dynamics of financial speculation. These ideas shall then be tested empirically against S&P 500 and VIX data from 1993.

I use the term private information to refer to information that can possibly influence an individual's perception of how an asset will behave over some period. The other more important term is the so-called pessimistic financial speculation. This refers to actions that you would take if you had acquired some negative information. There are ultimately three strategies that an individual can take when given private negative information[1], however, we shall however be focusing primarily on two of them. The two strategies are buying a put option and selling the asset. This is contrasted to optimistic financial strategies, which would consist of the purchase of a call option and/or buying the asset.[2] By choosing to look at the effect of options in the market, we can analyze the degree of expectation that evolves throughout, which will ultimately help us understand the behavior of investors over any period.

[1] I use the terms negative and pessimistic information interchangeably.

[2] This ultimately assumes that the individual initially holds the asset.

2 Financial Speculation

Before heading on to the literature review, I'd like to first introduce the mechanisms by which investors can let their views be known to the market. An investor is an individual who, conditional on information, takes positions in an asset. She can channel this information by selling an asset short, purchasing options, or simply trading the asset. Including short selling in the model gets very complex, so I've chosen to abstain from incorporating it. However, I find it valuable to discuss the mechanics of short selling and hence why I am leaving it out.

Short selling is the process by which an investor borrows securities that she does not own and proceeds to sell them at the market price. The short seller anticipates that the price of the asset will decrease, so when she is required to retrieve the shares to return them to the lender, there will (hopefully) be a profit of the amount equal to the original market price of the asset minus the market value of the asset upon retrieval. Such a position is risky so far as the asset price remains equal to or lower than the price upon selling, and also that the short seller will generally have to put up some form a collateral. One of the major concerns with short selling is that there is no limit to how much the short seller can lose, it is potentially infinite; this is because the price of the asset can theoretically rise with no upper bound. In contrast to a put option, there is an upper bound of the amount of profit that can be made. The maximum profit the short seller can make is the amount equal to the market value of the asset upon selling; this is because the lowest price that an asset can take is zero, also implying a lower bound of profits. Although we cannot observe what type of information is acquired by short sellers, we generally like to believe that they are looking at market fundamentals such as the earnings and book values of the companies at hand, as proposed by Deshow, Hutton, Muelbroek, and Sloan (2000), although this is not always the case. Short sellers are generally limited to the amount of shares that are issued and are generally required to pay a fee when the shares are settled. Because the acquisition of shares is sometimes difficult even when dealing with hard to acquire assets, the fees charged by brokers vary across firms. It is therefore difficult to obtain an exact pricing formula for the asset-lending fee. Performing studies on the the lending fee would be quite useful, but the idea is out of the scope of this paper. Other limitations on short selling include the Uptick rule. The rule was originally enacted in the Securities Exchange Act of 1934, and was removed on July 6, 2007, until a modified version of it was adopted on February 24, 2010. This current version states that the original uptick rule applies when the price of a security has fallen by ten percent or more in the previous trading day, otherwise the uptick rule need not be accounted for. Regulation T is another regulation on short selling such that the borrower must post 50% collateral upon borrowing the stock.

With the nature of the costs associated with and the regulations, it makes modeling the act of short selling very difficult. We shall therefore abstain from discussing the theoretical implications of short selling throughout this paper as the analysis of options is an alternative way to acquire the same result.

An option is a financial contract that gives the owner the right, but not the obligation, to buy or sell an asset at a given strike price determined during the writing of the contact. This type of contract requires a premium up front and is only profitable if the absolute difference between the asset price and the strike price is both positive and greater than the initial premium paid. The contract allows an individual to exploit information about the asset value and proceeding accordingly to purchase it. The contract can also be thought of as insurance against the rise or fall of the asset. The other important aspect of this contract is how the premium is determined. Options are generally priced using the Black-Scholes formula Black (1973). Rather than trading at the Black Scholes (BS) price, they typically trade at prices close to it. Traders often find it useful to examine the BS price compared to the traded price as well as the implied volatility of the option. Investors use implied volatility as a measure of future expectations about the market. It is generally more useful for this purpose than historic volatility. The implied volatility is generally calculated through the Black-Scholes formula given either the bid or ask prices. This interpretation will be useful for the empirical study.

The strategy of trading options as a speculation device will be discussed extensively throughout the rest of this paper. The main question at hand is whether we can use option prices to understand the behavior of investors in the market.

The rest of the paper is organized as follows. Section three describes the current literature and the direction I intend to take in order to tackle the problem at hand. Section four represents the model and basic logic of the environment. We discuss the model and its implications in section five. Empirical evidence is presented in section six. Section seven bases a ground for future work, and section eight concludes.

3 Direction and Current Literature

3.1 Theoretical Review

How does private information affect asset prices? The answer to this question has been studied for decades. The question is of interest because traditional pricing theories almost always assume the classical assumptions of perfect information, market completeness, and agent homogeneity. Early theories about market uncertainty go back to 1964, where Sharpe developed the theory behind capital asset equilibrium prices in the presence of risk. The standard assumptions used by Sharpe date back to Arrow and Debreu (1954), whose ideas were formally stated by Lucas (1972) as the rational expectation hypothesis. While these theories have influenced our beliefs and foundations of other modern pricing theories, they do not express environments consisting of asymmetric information and agent heterogeneity; these are common realities in markets.

Theories of information implications date back to Grossman and Stiglitz (1975) and Grossman (1981), where they challenge the rational expectations hypothesis by asserting that the existence of costs associated with looking for information demonstrates that asset prices may not reflect full information in the market. They also represent a more limited view that information is strictly channeled through either buying or selling an asset; this can not be the case when options are available. Harrison and Kreps (1978) show how equilibrium with differing expectations leads to prices that would be more than what any investor would be willing to pay for. Ideas representing the logic of how negative information impacts asset prices dates back to Edward Miller (1977). Miller claims that the absence of short sellers will put upward pressure on prices, as they will only reflect the expectations of optimistic agents. Miller's ideas are of interest in the absence of short selling. When this is the case, prices cannot reflect information from pessimistic investors, which biases the price. Therefore, arbitrage will be limited. This inherently contradicts the rational expectations hypothesis. Specific models of asset prices and heterogeneous beliefs discussed in Williams (1977), who revises the capital asset pricing model to include heterogeneous beliefs through known variances and covariance s of different investor portfolios, and Wang (1993), who devises an environment of capital asset prices where there is heterogeneous beliefs about future dividends. Detemple and Murthy (1994) present similar logic to that of Wang, where beliefs are updated and weighted by an agent's amount of wealth. The idea of updating beliefs is interesting because it provides tractability in the theories of price adjustment. This idea is especially apparent in Glosten and Milgrom (1984), where changing beliefs based on private information has a direct impact on the bid and ask price setting of a market specialist. Their model also establishes a martingale property that allows for a deterministic probability; hence, we can identify the expected asset price.

To elaborate on theories capturing private information through short selling, we turn to Amihud and Mendelson (1986) who show the relation between the big-ask spread and asset prices is represented in a continuous dynamic setting where there are multiple holding periods. They find strong evidence that is consistent with their hypothesis that asset prices are positively related to the bid-ask spread. Diamond and Verrecchia (1987) apply Glosten and Milgrom to assess asset price adjustment to private information when there are constraints on short sales. The model is good so far as it is very tractable and simple to adjust. The model focuses specifically on short selling rather than looking at specific angles through options. They conclude that short sale constraints don't necessarily lead to an upward bias in prices. The idea of relating options to short selling and asset pricing dates back to Figelwski (1981) where he shows empirically that put options can be substitutes for short selling. Neilson (1989) proves the existence of a capital market equilibrium where there is allowed short selling. More recently, there has been much more literature on the constraints that short sales have on option and asset prices. Ofek, Richardson, and Whitelaw (1996) look at how violations of put-call parity are impacted by short sale constraints. Dechow, Hutton, Muelbroak, and Sloan (2001) argue that short sellers are strictly looking at market fundamentals such as company EPS (earnings per share) and other bookkeeping fundamentals. They also proceed to show that high EPS his positively related to positive excess returns. Abreu and Brunnermeier (2002) develop a model of private information where arbitrage opportunities are delayed in the market even if investors hold private information. They go on to show that the presence of transaction costs and regulations hinder an investor from eliminating the arbitrage immediately. Thus, there is a delay in the channelling of information.

3.2 Empirical Review

Since there will be focus on elements of the S&P 500 Index, it is important to review some of the literature on the empirical evidence of the relations between option prices and asset prices. Figelwski and Webb (1993) show that the spread between option prices are a result from the ability to short sale. Short sale data from the S&P 500 was used. The sample consisted of 342 firms as of April 1985. Data from Call options were acquired from the Chicago Board of Options Exchange (CBOE), which were first introduced in 1979. The panel-time series regression used poses problems because there is evidence of selection bias. This results because the short interest is strongly correlated with excess stock returns. Although the biases may skew the results, it is still a good proxy for determining negative information about asset prices. This paper at hand is important as it underlies the basic logic that will be evident within the following sections. Skinner (1988) uses option data from the CBOE and the American Stock Exchange (AMEX). Of the options data, 301 companies are used who have list options since 1977; thus, the interval period was from 1977-1980. His paper proclaims that stock return volatility decreases after options are listed with the stock. They also show that this variance is in no way related to the historic variance of the underlying stock. Stein and Lamont (2003) plot 3 series' over a monthly basis from 1995-2002; the NASDAQ total index, 60 day moving average from CBOE's daily put call ratio, and the value weighted short interest ratio for all NASDAQ companies. They show that aggregate short interest is in general countercyclical during recessionary periods, especially during the dot-com bubble. As a result of the same countercyclical fashion of the put-call parity, they suggest that short selling has limited impact in stabilizing the entire market. Earlier empirical works of put-call parity are done by Klemkosky and Resnick (1979). They test the hypothesis of the put-call parity to see if the identity is useful in creating efficient markets. They use options data from the CBOE and a period between july 1977 and June 1978. Their major results are that the data is consistent with the put-call parity and that deviations from this are due to over prices of call options. Finucane (1991) tests the hypothesis of whether the put call parity identity can be a good predictor of the S&P 100 Index. They use bid-ask option quotes from the CBOE from a period of December 2, 1985 and November 30, 1988. They conclude that the deviation from put-call parity leads the market by 15 minutes. Cremers and Weinbaum (2010) dictate that abnormal stock returns are associated by abnormal call prices (if the return is positive) and put prices (if the return is negative). They use the spread between implied volatility data from puts and calls acquired from OptionsMetrics. They use a sample period of January 1996 to December 2005.

We have presented some of the vast literature on the theoretical and empirical aspects of options, spreads, and stock returns. Since the main goal is to see how options prices reflect private information, it makes much sense to focus on those instruments. From a theoretical sense there has been no model that explicitly captures an asset price as a function of options, especially in the context of the put-call parity, which we will discuss. The intent will ultimately be to develop a model to account for investor behavior through the acquisition of options. Only then will I be able to fully capture the desired results.

4 The Model

The model to be developed at hand is an extension of the basic model of Glosten and Milgrom (1985). I develop the model such that it captures not only the bid-ask spread of the stock price, but also have it include the bid-ask price of a call and put. This approach is straightforward to develop and can provide us with easy to understand intuition that will naturally lead to empirical implications.

The logic of the model is very straightforward. It is a pure dealership model where there is one specialist carrying out the transactions of each investor. The idea is that the specialist in the model sets the bid and the ask price of the asset such that he is willing to sell at the ask price and buy at the bid price. The fundamental difference between Glosten et al (1985) and ours is that we assume the specialist will only be able to set the bid-ask price of the asset through the bid-ask of the options. This draws us to the first assumption of the model.

The specialist sets the asset's bid and ask prices, V_b and V_a, according to the following identity.

$$V_b = k(1 + r)^{-t} + (c_b - p_b) \tag{1}$$

and

$$V_a = k(1 + r)^{-t} + (c_a - p_a) \tag{2}$$

where $k(1 + r)^{-t}$ is the present value of the at the money strike price given the time to expiration t, the risk free rate r, and $(c_b - p_b)$ and $(c_a - p_a)$ is the spread between the call price and the put price. This is identity is known as the put call parity, and it demonstrates the relation between the value of the stock, the strike price of the options, and the put-call spread. The identity follows from an equilibrium reached under no transactions costs, no arbitrage, and not other external events. We can now represent the bid and the ask price of a stock as a function of the spread between the puts and the calls to the bad-ask price of stock price. The choice to alter the original model to include this identity serves two purposes. The first is that it allows easy analysis of how the asset price is changed through the call and put prices. Secondly, while the put-call parity holds in theory, it doesn't usually hold in practice. Therefore, the study of these deviations will be of key interest.

I am using the logic from Figlewski and Webb (1993) to justify the nature of the particular spread. They focus on negative information and claim that in the acquisition of it people will begin to purchase more put options and buy less calls. Thus, it is logical to assume that the prices should decline in p and increase in c. Another key aspect of the original model is that the bid-ask prices are endogenously determined. We shall be honoring this endogeneity for the remainder of the model.

Upon the arrival of investors, the specialist has chosen the bid-ask prices given his private information S; he may only change these prices once after a sale is made. To make the model even more tractable, we also assume that only unit trades take place. Relaxing this assumption would mean that we would have to account for an optimal investor strategy that accounts for the sale of certain quantities. To avoid adding complexity, this approach will be ignored.

A key concept in this paper will be that of the informational characteristics. The market consists of two types of investors, informed and uninformed traders. Informed investors have information about the true value of the stock price F. I assume also that at time T_T no investor has an informational advantage, and let this also be the time where F is realized. This is essentially the time where everyone agrees on the price of the asset as they all have the same information set.

The arrival process by investors to the specialist is straightforward. Upon arrival to the specialist, the informed trader will either sell assets or buy options. Uninformed investors will do either depending on the information set. It is also not known to the specialist whether the investor is informed or uninformed. Also suppose that each investor arrives one by one and that there can only be one arrival every instant.

We also assume that each investor is risk neutral and has a utility function. The utility function is in place to give motivation for uninformed investors to trade. In general, we have the following utility function: $U(.) = \rho x F + C$, where ρ is a preference parameter, x is the quantity of shares held, and C is consumption. For low values of ρ, we say that the investor has a strong desire to consume today. High values suggest a strong desire to invest for the future. We also assume that ρ is unknown to the specialist as it is a preference parameter, therefore we can say that it is independent of F.

Investors arrive to the specialist knowing pieces of information. The uninformed investors know the current bid-ask prices of options and the asset, historic prices or the asset, and all other public information available. The informed investors know the same as the uninformed plus additional information that would be considered private.

Denote P_t as the public information set available at time t. This information includes the bid-ask spread and historical prices. Thus, this is the information of the uninformed investor upon arrival at time t. An informed investor has access to more information, as some of it is private. Thus, his information upon arrival is P_t, and J_t, where J_t is the private information taken to be negative.

We can now specify decision making of the investor. An investor will take positive positions in the asset if,

$$Z_t > V_a$$

and take negative positions if,

$$Z_t < V_b$$

where the V terms have already been defined. I'd like to stress the meaning of positive and negative positions. Negative positions consist of buying a put or selling the asset. Positive positions consist of buying or holding a call.

Z_t is given by

$$Z_t = \rho_t((1 - u_t)E[F|P_t, J_t] + u_t E[F|P_t])$$

where u_t is the probability that the investor is uninformed.

Given this information about the behavior of the investors, the specialist will rationally choose the bid-ask of the puts and calls after each order, implying a change in the bid-ask of the asset. Similar to the investors problem, the specialist also has an information set S_t, where he doesn't know whether the investor is informed or not. One may challenge our assumption by saying that since are assuming that private information is negative, won't the specialist know whether this investor is informed by his actions? The answer to this question is no. Suppose that an uninformed investor chooses a strategy by purchasing a put or by short selling based on his information set P_t, then there is no reason for the specialist to infer that this investor is either uninformed or informed, since the investor is acting on public information.

The expected profits to the specialist are very similar to that of Glosten et al (1985).

This condition is :

$$E[(V_a - F)I_{z_t > V_a} + (F - V_b)I_{Z_t < V_b}|S_t] \tag{3}$$

where $I_{z_t > V_a}$ and $I_{Z_t < V_b}$ are indicator functions of the two events $Z_t < V_b$ and $Z_t > V_a$ happening.

If we carry through the expectation, the profit function can be rewritten:

$$(V_a - E[F|S_t, Z_t > V_a])P(Z_t > V_a|S_t) - (V_b - E[F|S_t, Z_t < V_b])P(Z_t < V_b|S_t) \tag{4}$$

where $E[.|.]$ is the conditional expectation operator which holds only if there are zero transaction costs or short selling costs. Since equation (4) is difficult to analyze on its own, I require the specialist to have zero expected profits. This will prove useful in the following development.

To illustrate the the zero profit condition, suppose that a specialist sets the ask such that,

$V_{a1} > E[F|S_t, Z_t > V_a]$.

Naturally, speculators will enter the market and undercut the the specialist by setting $V_{a2} < V_{a1}$. This will happen until the desired result is obtained.

Another core assumption to avoid arbitrage opportunities is that $V_a > E[F] > V_b$. While this is more of a heuristic proof, Glosten et al (1985) follow similar logic to us in their original paper and do not follow through with a formal proof .[3] This assumption is also necessary to allow for changes in the expected asset price based on the spreads; which we will discuss below.

Another important aspect with respect to intuition is the understanding of the specialists revision based on the strategies involved with by the investors. Suppose an investor engages in a pessimistic strategy, then the specialist will revise the bid-ask prices of the options such that the asset price is forced downward. This is a natural market reaction and the reverse can be said about an optimist strategy. For example, suppose a trade takes place at time t such that the investor purchases a put option. This would imply that $Z_t < V_b$. The bid price will be then updated such that the new V_{b1} is less than V_b.

Using the logic above, of any trade h, the transaction price will be:

$$V_{a_h} P(Z_h > V_a | S_h) + V_{b_h} P(Z_h < V_b | S_h) \tag{5}$$

which is equivalent to,

$$E[F|S_h, Z_h > V_{a_h}] P(Z_h > V_{a_h}|S_h) + E[F|S_h, Z_h > V_{a_h}] P(Z_h < V_{b_h}|S_h) \tag{6}$$

Where the time at which trade k is executed is before the terminal time T_T, hence $T_h < T_T$.

Looking at the equation above, this is just $E[F|S_h]$. We can this impose that the transaction price at trade h is $p_h = E[F|S_h]$, which is a martingale.[4] Since the value

[3]These ideas are presented in Glosten and Milgrom (1985) page 81, where the reason for not proving the result is discussed further.

[4]The proof of this is simple and presentd on page 81 of Glosten and Milgrom (1985).

of the asset is known the instant the trade is made, and given all information S_h, we have that $E[F|p_h] = E[F|S_h]$.

The assumption of no transaction costs plays a major role in the derivation of these results. Including transaction costs borne by the lender yields a more complicated result that I will not attempt to discuss in our simple analysis. Further detail in this market can be found in Ohara and Oldfield (1982), especially when the main transaction cost at hand are inventory costs.

5 Discussion

We now proceed to show the model in practice by looking at the dynamics. Up to this point we have discussed nothing about options in general. The importance of the previous section was simply to develop the ideas and key points of interest when looking at the decision making of each investor. Next, we show how the options will play a major role in the equilibrium asset price.

To use the simplest model possible, suppose there are two states of the world, and the fundamental value of the asset, F, is either 3 or 10. Where 10 is a good state of the world, and 3 is not. Thus, the expected value of this asset is:

$E[F] = 3\pi + 10(1-\pi)$, where π is the specialists probability of a downstate conditional on his information set. Let it also be the case that $\rho = 1$ for informed investors. Since they have private information about the state of the world, they will always trade. Since trading by uninformed investors depends on the preference parameter ρ, we need to understand the probability distribution of the parameter. To make the arguments as simple as possible, I shall take the distribution of ρ to be 1/2. This such that with probability 1/2, ρ for uninformed investors can be either 0 or $M < \infty$, where M is some finite constant. At the ask price, the investor will take a positive position if $10 - V_a > 0$, and thus the probability that this will occur is $\pi(1-u_t)(10-V_a)$. The uninformed investor will buy at V_a if $0.5u_t(V_a - E[F])$. So for the break even condition of the specialist to hold, we must have that:

$$\pi(1-u_t)(10-V_a) = 0.5u_t(V_a - E[F]) \tag{7}$$

The same logic presented presented here can be applied to the bid price as well, where we have:

$$(1-u_t)(1-\pi)(V_b - 3) = 0.5u_t(E[F] - V_b) \tag{8}$$

Since the specialist makes zero profits, he must set the bid-ask prices such that when an informed investor buys the asset with expected loss to the specialist $\pi(1-u_t)(10-V_a)$, the specialist must sell to the uninformed investor an equal amount giving him an expected return $\pi(1-u_t)(10-V_a)$.

Since we have already defined V_a and V_b in the previous section, we can adjust the equations to reflect our desired information. Hence we have:

$$\pi(1-u_t)(10-k(1+r)^{-t}-(c_a-p_a)) = 0.5u_t(k(1+r)^{-t}+(c_a-p_a)-E[F]) \quad (9)$$

and also,

$$(1-u_t)(1-\pi)(k(1+r)^{-t}+(c_b-p_b)-3) = 0.5u_t(E[F]-k(1+r)^{-t}-(c_b-p_b)) \quad (10)$$

where $V_b = k(1+r)^{-t}+(c_b-p_b)$, and $V_a = k(1+r)^{-t}+(c_a-p_a)$

We now have two equations and can therefore solve for the bid spreads and the ask spreads separately.

Define $B_b = (c_b-p_b)$ and $B_a = (c_a-p_a)$, then, solving the equations we have the following:

$$B_a = [-\pi((k(1+r)^{-t}-10)(1-u_t)-3.5u_t)-2.5u_t]/[\pi(1-u_t)+0.5u_t] \quad (11)$$

Suppose the probability of the next investor being informed is 0.5, and that the strike price is between 10 and 3, then we have that:

$$B_a = [-\pi(k(1+r)^{-t}-10)0.5-\pi 1.75-2.5u_t]/[\pi 0.5+0.25]. \quad (12)$$

The above expression shows that when the conditional probability of a downstate is increasing, i.e. π increases which is given by the beliefs of the specialist as his information set is updating, then the term B_a decreases. In other words, call ask prices are decreasing and put-ask prices are increasing.

Solving for B_b yields, the following:

$$B_b = [\pi(3u_t0.5 + (1-u_t)(k(1+r)^{-t}-1) - u_t5) - u_t0.5(10-k) - (1-u_t)(k(1+r)^{-t}-1)]/[(1-u_t)(1-\pi) + u_t0.5] \quad (13)$$

Lets suppose there is a some private negative information that investors know, then we may wish to study these equations more carefully. Assuming that the specialists conditional probability π changes downward as investors with negative private information are trading, we can see how the bid-ask of options change with respect to π.

We are interested in seeing what happens when the following events take place. i) The market is flooded with private information and the strike price is low, ii) The market is flooded with private information and the strike price is high, iii) The market is not flooded with private information and the strike price is low, and finally iv) The market is not flooded with private information and the strike price is high. Cases i), iii), and iv) yield results that imply a convergence of call bid-ask spreads, and a larger spread of put bid-ask spreads. Case ii) we find that the spreads are moving together. We notice also that this is the case where there is a lot of private information and the stock price is high. This case presents a general decrease in call prices and and increase in put prices. This is the general result I expected would happen given negative private information.

Cases i), ii), and iv) demonstrate that even if there is any private information where the strike price is high or low, the call bid-ask looks to converge and the put bid-ask prices will be more spread out. To understand why this is the case, consider a specialist where high expectations are updating with respect to actions towards him through the investors. As more negative information comes into play, the specialist has the incentive to profit by setting a low put bid price and a high put ask price, since the information reflects such an instance. Call prices will converge because optimist information is already fully represented by all public information, and there is no incentive to deviate from creating a large spread. Of course, to satisfy the no-profit condition of the specialist, we must have some other have some other reaction to not only make that condition hold, but to also create stability in the equilibrium price.

Since we have mixed results of the interrelation, it is more useful to look at the total change in the options as π increases.

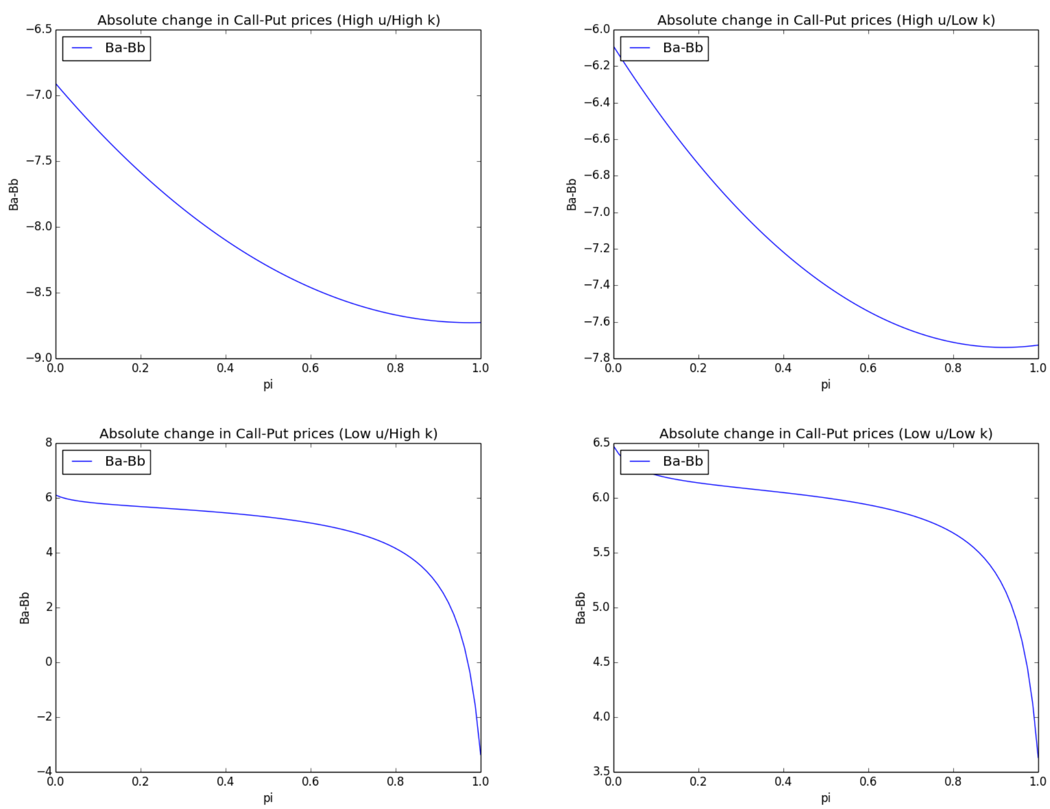
Consider a new variable

$$B_a - B_b = (c_a - p_a) - (c_b - p_b) = (c_a - c_b) - (p_a - p_b) \tag{14}$$

Constructing this measure more useful since we can see the total change in the options when there is a change in the conditional probability π.

Figure 1 shows the measure for cases i)-iv):

Figure 1: Call-Put spreads as π increases



It can be observed that the new quantity defined decreases as π increases, demonstrating our main theoretical result to be tested. To observe the dynamic of how information is translated, we simple look at a decreasing π. This hypothesis is consistent with the theoretical intuition and empirical results of Figlewski et al (1993). We now move onto the empirical section of the paper to determine if the relation holds.

6 Empirical Evidence From The S&P 500

The sections before us presented a model of how private information is translated into asset prices through the sale of options. In particular, we focused on a specialist market and demonstrated how call and put bid ask prices are determined through the realization of private information. As a result, we show that this translates into asset prices. The key result is that upon the acquisition of private information, the specialist reacts by setting the bid-ask prices. We use the put-call parity relation to demonstrate how the price setting of the specialist translates into the asset prices. The decision to utilize the put call parity in the theoretical model makes it easy for us to construct empirical tests that are easy to draw from the theory. We also don't hinder any of the main theoretical features as a result.

Our primary interest is to understand the actions of investors with private information. We have shown from the theory that the conditional expectation of the specialist, π, will change as investors, both informed and uninformed, make orders. As π decreases, call and put prices change such that call orders would decrease and put orders will increase. The reverse if true for when π increases. Another way to look at this is to think about changes in π as a change in uncertainty. We shall maintain this notion throughout the reset of the section.

6.1 Data

To begin with the empirical analysis, we shall use the bid-ask prices of call and put options. This data has been acquired from Wulin Suo in the Queen's School of Business and the frequency is in minutes. The range of this data is from January 4, 1993 to September 15, 1993 and there are 60,000 observations in total for puts and calls.

The data is transformed such that calls and puts match the date, day to maturity less than 30 days, and an equal strike price. Roughly 97% of the put-call data have, on average, maturities of 30 days or less; the rationale for this is further explained in the proceeding paragraphs. Another problem at hand is that the VIX data, to be presented later, is in days and not minutes. Since we have already matched put and calls of the same day and strike together, and are only considering maturities of 30 days or less, I found it logical to transform the data such that the options are recorded on a daily basis. To do this, we simply take the daily average of each variable. While we do lose information when managing the data in this way, we are now able to compare it with the VIX and draw other conclusions.

I also find it important to discuss money-ness. The theory presented above makes no inferences based on this, but we will perform analysis within this section that highlights some of its implications. To determine money-ness, we consider the following measure:

$$k_t/S_t(1+r)^t \approx 1 \tag{15}$$

where the equality indicates that the option is issued and traded near the money. For the measure of the interest rate, I've chosen to use the annual federal reserve bank rate for the year 1993. This rate was constant at about 3%.[5] Since this is a yearly average, we must convert it to a monthly average. This move is justified since, as I will explain throughout, the rolling regression window will be 30 days. Hence, for discounting purposes, we take the above formula to be:

$$k_t/S_t(1.03)^{1/12} \tag{16}$$

Luckily, as the data presents itself, we are fortunate to have the money-ness within the following range:

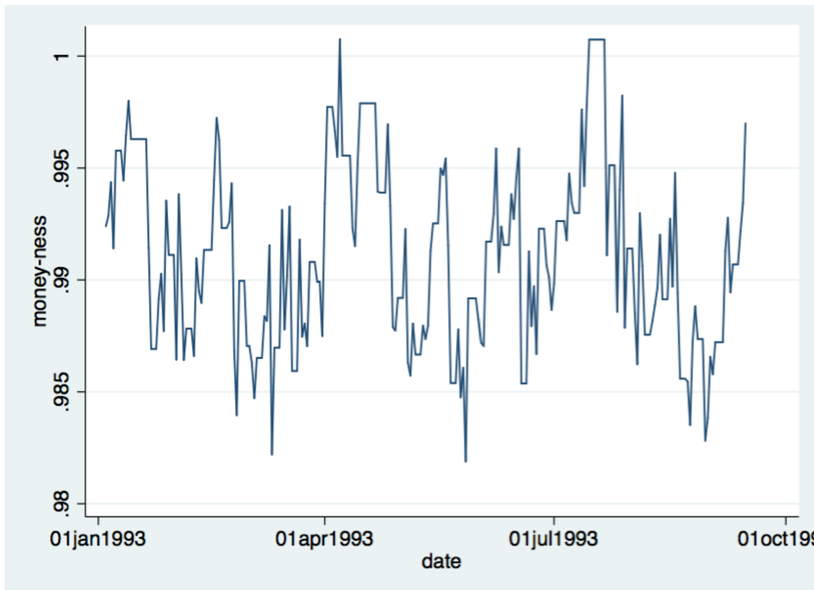
$$0.98 < k_t/S_t(1.03)^{1/12} < 1.002 \tag{17}$$

It is quite often the case that options do not trade when they are out of the money; this is the reason as to why the interval is very tight around the money. The money-ness

[5] The constant interest rate has been acquired from the website http://tradingeconomics.com

is calculated by discounting the time series of strikes by the one month interest rate and further dividing that series by the S&P 500 Index. Graphically below, we can see the money-ness of options over time.

Figure 2: Moneyness of Options



Smaller intervals will also be considered throughout the section to determine if money-ness demonstrates results favorable to our theory.

Critical for the analysis is to find a measure for uncertainty. A standard proxy for market uncertainty is the CBOE Volatility Index (VIX). This measure is calculated using a generalized formula The foluma is rather complex so we shall not go into the details.[6] The VIX has been acquired from the Wharton Research and Database Center, a web-based business data research service from The Wharton School of the University of Pennsylvania.[7] I shall be using daily data from January 4, 1993 to September 15, 1993 to be consistent with the option data.

We have acquired the bid and the asks of the options since this is the main variable in the theory. The problem is, however, that it is not always possible to trade at the bid and the ask, so I shall abstain from using these options directly. Instead of using equation (14), we shall use the spread between the average of the call and put bid/ask

[6] The actual method can be found through the CBEO website. http://www.cboe.com/micro/VIX/vixintro.aspx

[7] The WRDS login interface. https://wrds-web.wharton.upenn.edu/wrds/

prices, equation (18) below. Calculating the option price this way is commonly used by the CBOE to calculate the put and call prices to be used to calculate the VIX. Options prices in general are also easier to obtain. By the previous arguments, we have the following:

$$cps = (c_a + c_b)/2 - (p_a + p_b)/2 \tag{18}$$

The abbreviation *cps* stands for call-put spread, and will be used extensively throughout.

To justify our choice, consider the following figure.

Figure 3: Spearman's Rho of Ba-Bb and cps

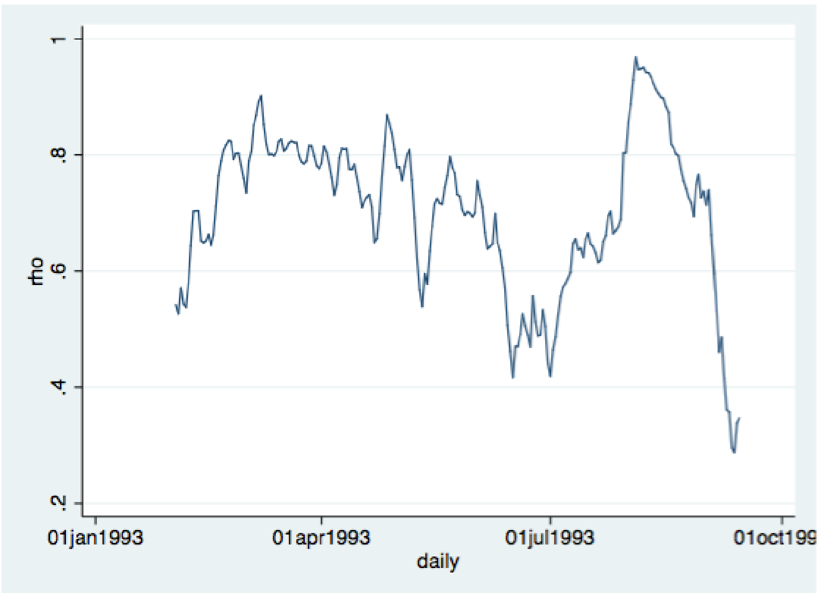


Figure 3 shows spearman's rho over sub-samples of 30 to look at the correlations between *cps* and $B_a - B_b$ over time. The correlations maintain positive for the entire sample, and 80% of the observations are between 0.6 and 0.8. This approach is of key importance because we employ the same methodology with the other parameters to be estimated.

6.2 Rolling Regressions

To begin, we obtain an estimate using the first 30 observations in the sample. We then store the estimate. We then proceed to obtain the second estimate by using the 2nd observation to the 31st. This estimate is stored. This iteration proceeds until the sample has been completed, and what we end up with is a graph like figure 3 showing the evolution of the correlation over the sample period. The estimates stored are commonly known as ”rolling estimates” and are obtained by running a rolling regression with the specified period. I chose 30 days to be the window as that is the average expiration period of the options. It is also the maximum estimation period of options used to calculate the VIX.[8] We will be applying rolling regressions within the next subsections.

6.3 Empirical Results

6.3.1 Considering *cps*

Now that we have a proxy for uncertainty and a key parameter of interest, we are in a position to test our hypothesis. Like before, I will abstain from running a single regression on the entire dataset as it will not deliver enough information. I shall therefore be focusing on rolling regressions to look at the estimates over time. We should also note that for this section and the remainder of the empirical section, each put and call observation are taken to have the same strike price.

We construct the following rolling regression:

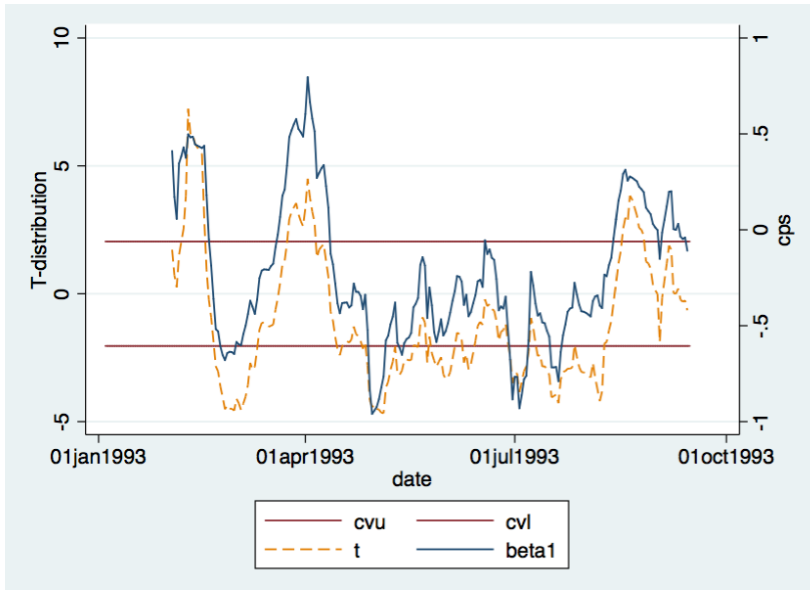
$$VIX = \alpha_{i:M} + \beta_{i:M} cps + \mu_{i:M} \tag{19}$$

where $i : M$ is the rolling interval, $\beta_{i:M}$ is the slope parameter on cps, $\alpha_{i:M}$ is a constant, and $\mu_{i:M}$ is the residual.

[8] The average maturity date in the original sample was 35 days. To exploit the same methodology as the VIX, we abstained from using option that have maturities of over 5 and a half months. This decreased our observations amount by a mere 1.8%, while providing us with an average maturity date of 30.085 days.

Figure (4) shows exactly this.

Figure 4: Evolution of Beta over time



We see the estimate *beta* over time as well as the significance level. The area between the two red bars is the region of insignificance which are the values ±2.042 obtained from a traditional t-table. Our simple hypothesis is the following:

$$H_0 : Beta = 0 \tag{20}$$
$$H_1 : Beta \neq 0 \tag{21}$$

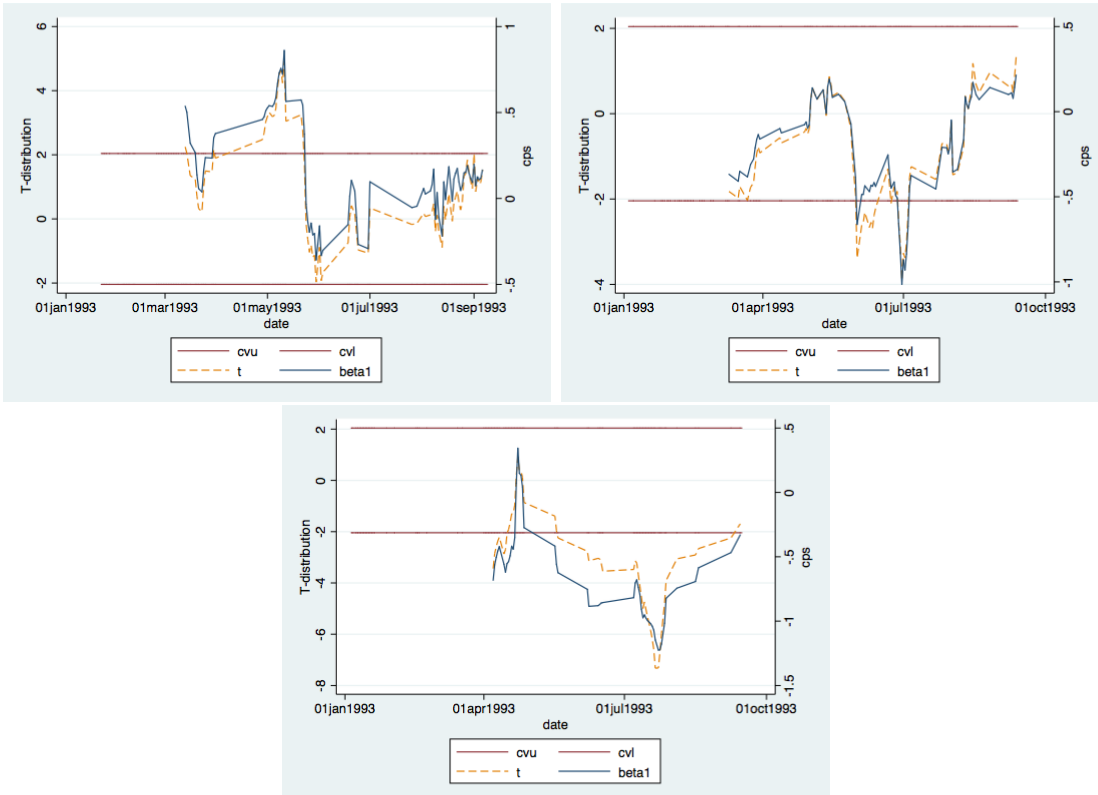
The estimates are compared against a two tailed t-distribution at the $\alpha = 0.05\%$ level. It can be seen that most of the extreme high positive and low negative observations are significant over time, as we would have expected based on the theory. Rather than to have put the statistics for each regression into a table at the appendix, I thought it would be easier to plot the areas of significance based on the critical t-values where the t-values are calculated in the traditional way.[9]

Notice that these estimates were created by using money-ness where $0.98 < k_t/S_t(1+r)^t < 1.002$. We shall proceed by looking at other intervals of money-ness, focusing primarily on the intervals $0.98 : 0.99, 0.99 : 0.995$ and $0.995 : 1.002$, as the data is

[9]The critical value to be used to compare to the t-distribution is: $t^* = \beta/se(\beta)$.

distributed between them. The graphs below show the estimates, and are in order from the intervals we have just presented.

Figure 5: Beta and its significance with different moneyness intervals



I chose to leave out other intervals simply because there weren't enough observations to follow though.

Figure (5) show the results over time with different intervals of money-ness. The results are quite similar to figure (4) in that most of the observations take on negative values; this is especially the case when we are considering an interval further and further from the money. Although we see both positive and negative signs, not all of them show to be significant. We see more significant parameters by considering the tighter money-ness interval 0.995-1.002. These results are not only significant, but the parameters are what we were expecting from theory. Although we lose more information by considering more and more in the money observations, more trading occurs near the money. We would therefore expect these observations to better adhere to our theory.

From these observations, it looks as though in the money options do a consistent job at demonstrating negative and positive relations between the *cps* and the VIX. The results demonstrate how increasing uncertainty through the VIX can be positively related to *cps* if there is positive private information, and negatively related to *cps* if there is negative private information. This is consistent with our theoretical results.

6.3.2 Considering c and p

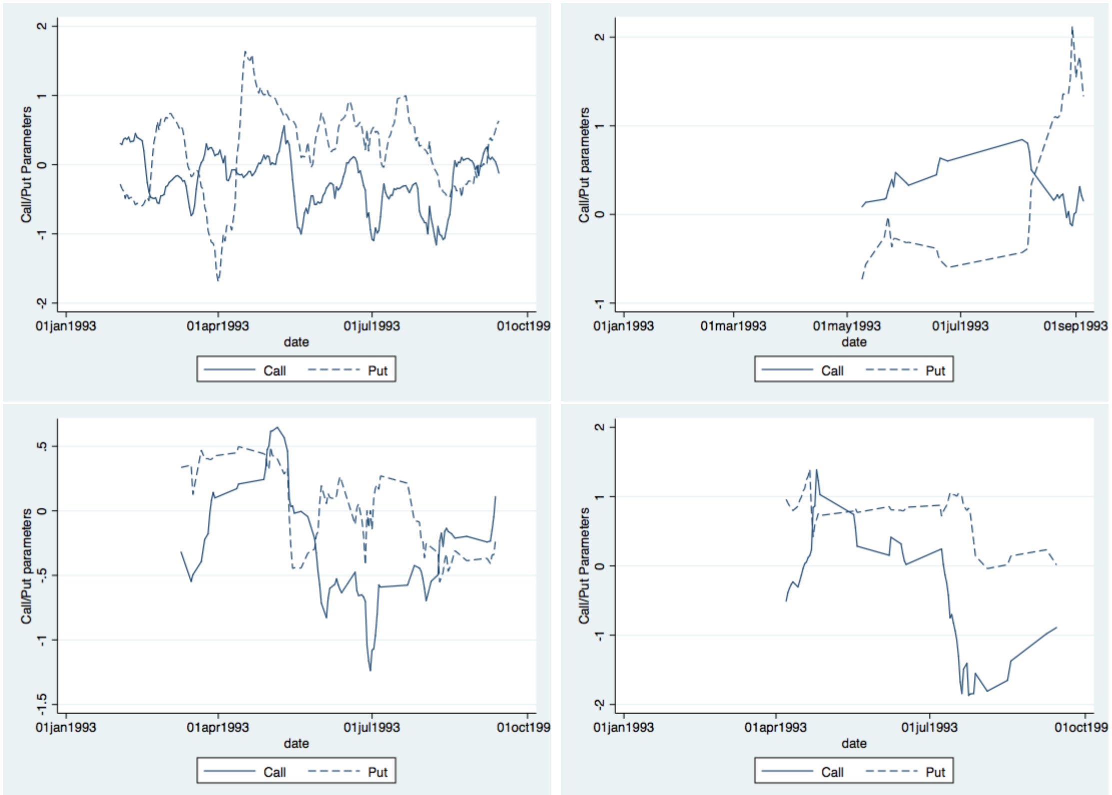
Now that we have shown periods of our variable changes as uncertainty arises, it may also be useful to consider the individual contributions of the puts and the calls.

We now look at the following modified regression:

$$VIX = \alpha_{i:M} + \beta_{1i:M}c + \beta_{2i:M}p + \epsilon_{i:M} \tag{22}$$

The benefit of breaking down the regression in this way is that we can check the source of the uncertainty, whether it was through the calls or the puts. In the absence of short selling, it would be expected that a decrease in call prices should put upward pressure on put prices.

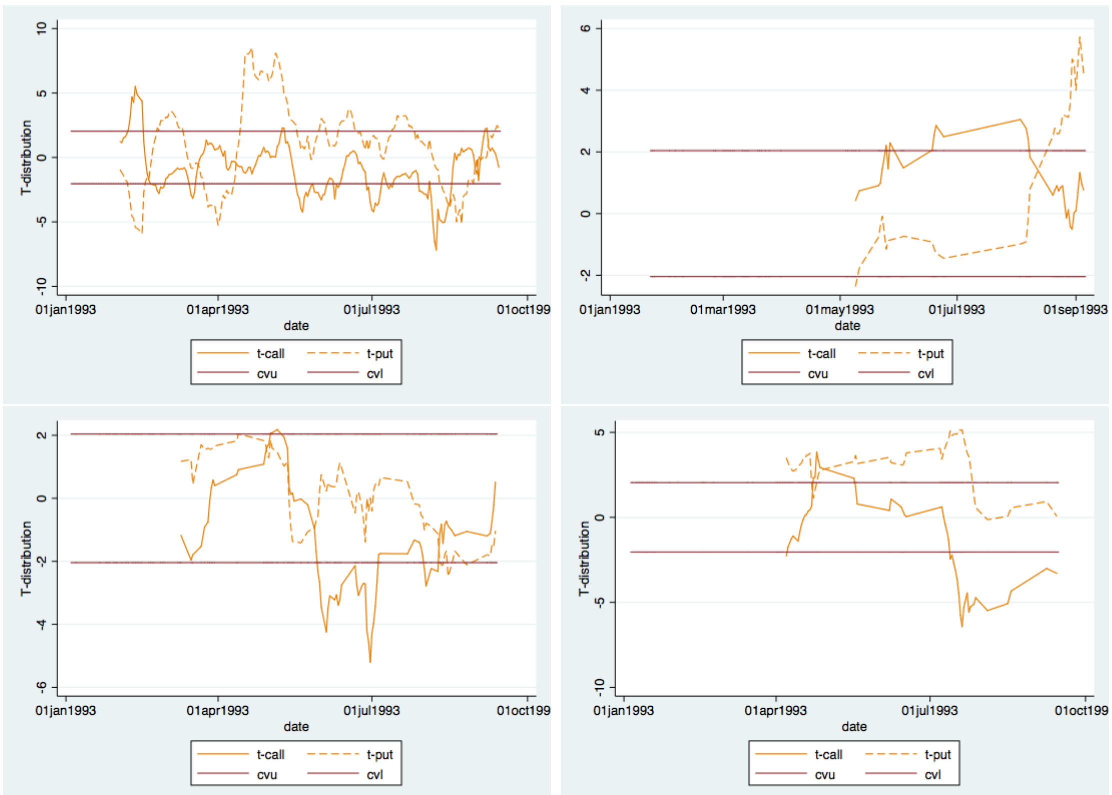
Figure 6: Put and Call Beta's with different moneyness intervals



The figure above shows the evolution of parameters on the call and put options in the following order: $0.98 : 1.002, 0.98 : 0.99, 0.99 : 0.995$, and $0.995 : 1.002$. We see what looks to be consistent results from the model. For much of the data, it is noticed that the estimates almost always yield the opposite sign for any given period, this is consistent with the theory. In keeping that in mind, it is also observed that the options, either call or put, have different signs at different times. This is also consistent with the theoretical results. Suppose for example that the parameter of the call option is negative, then as the price of the call increases the corresponding effect should be a decrease in the Volatility index. We see consistency of this result by looking at figure (6). It is the same story for the put options. Increasing put option prices correspond to an increase in the Volatility index, and can also be observed by looking at most of the parameter estimates in the figure. The reverse is also true such that the volatility index can increase when the call option parameter has a positive coefficient, and the put has a negative. This result allows the determination of whether there is optimistic or pessimistic information afloat.

It will also be useful to look at the significance of these parameters as well. To avoid a cluttered figure 6, I created figure (7) below to compliment it. The figure shows the significant of the parameter estimates above in the order of money-ness that we have described before.

Figure 7: Significance of Option parameters of difference intervals



Since one of our central questions at hand is determining which option contributes more to changes in the VIX, we can look at the significance of the parameters represented in figure 5.

Overall, the put options had more areas of significance. Call options became more transparent when the sample was tighter around the money. There are no consistent patterns of call and put parameter signs as we move closer to the money as far as analyzing investor behavior. While being consistent with the theory, the only purpose money-ness serves in this exercise is look at different classes of investors who may purchase options that are out of the money. For example, consider the second quadrant of figure 7. Given investors who purchase at this interval, we can say that there was a spread of private positive information from may until august, where private negative information dominated. Reverse effects are seen when considering investors trading at the tight around the money options. Since options rarely trade far from the money, I would conclude that the tight around the money interval would be the appropriate figure to look at to capture the informational existence.

We shall not, however, ignore what we have seen in the second quadrant. Our analysis concerns private information, so of course from may to august and investor with positive private information is going to trade below the money.

6.4 Analysis

There are a few important things to take away from the previous exercise. Even given the small sample of data, especially when dealing with the money-ness intervals, some pretty interesting results have arisen.

While looking at the effects present in figure (4), we notice a major trough from mid-february to mid march, which cannot be explained theoretically. In February 26th, 1993, a bomb went off in the basement of the World Trade Center. Perhaps investors became unconfident in the options markets and therefore turned to short selling. This is a possibility. Other than this abnormal spike in the data, we now turn to what else the results suggest.

When considering below the money intervals, we notice both positive and negative coefficients. While most of the coefficients are insignificant, it is difficult to obtain a solid relation when speaking of this interval. Moving closer and closer to an in-the-money interval, we get positive and negative parameters, but most are still insignificant. Moving towards a very tight around the money interval, we see parameters that are consistent with the theory and are significant. A noteworthy pattern is noticed. It almost seems that during the first half of the sample, the put call spread positively contributes to the VIX, while it negatively contributes during the second half. This would have happened throughout the entire sample had there not been that major spike. This result suggests that there may have been an instance of widespread optimism in the first 6 months, and pessimism in the second. While we cannot explain the nature of the change in expectations, we can at least infer that there is information managing the expectations.

We now turn to observe the contributions of the puts and the calls to the analysis. In considering the sample as a whole, it seems as if the put's provide a better signal of changes in the VIX. As with before, the spike from mid-february to mid march cannot be explained theoretically and we shall use the same arguments as before with respect to the bombing in early 1993.

By considering intervals of money-ness that are more out of the money, we notice that the calls exhibited positive signs and are significant. Moving towards a closer to the money interval, the only significant contribution to the VIX are the call prices. Finally, considering a tight interval around the money, we notice a combination of both call and put contributions to the VIX where the puts were towards the end of the sample.

We notice a few patterns here. When considering our created variable *cps*, we can say that this parameter is useful in explaining changes in the VIX so long as the options are closer to the money; at least this is where more significant options are observed. When considering each options contribution separately, the puts seem to dominate overall contribution to the VIX when considering the entire sample. When looking at out of the money intervals, calls seem to dominate at least as much as the put options. From the individual contributions, money-ness seems to favor the contribution of call options over put options. Other intervals >1 can bring to light a pattern consistent with measuring option prices of different intervals and how they relate to uncertainty. Unfortunately, lack of data prohibited me testing the interval.

The fact that we have determined that call options play more of a role in contributing to the VIX rather than puts is rather intuitive. The simple answer is that when faced with private information, pessimists can do more than just sell the asset or buy put options, they can sell short as well. As far as optimists are concerned, they only have the option to buy the asset or buy call options. Clearly, having alternatives to channel information should show inconsistency in some of those alternatives when relating them to uncertainty.

To this point we have also left out the notion of transactions costs. The theoretical results from our model section relied heavily upon the assumption of zero transactions costs. In reality, positive transaction costs can limit arbitrage opportunities since there may be no net profit by a trade once the transactions costs are paid. The arbitrage opportunity is therefore slow to diminish. This reality will therefore be attributed to many of the observed areas of insignificance.

6.5 Link between Theory and Practice

Up to this point we have not established a direct link from our theory to the empirical results presented. The main results of from the theory section are derived because we used the put-call parity condition. Table 1 below shows the correlations between the VIX and the Deviation from Put-call Parity, denoted $DPCP$. There looks to be a positive correlation across all intervals and the significance is better with intervals closest to and around the money. These results show that positive deviations from put-call parity demonstrate greater uncertainty or volatility in the market. The reverse would be true for a negative deviation.

Table 1: Correlation Results

Spearman's Rho: VIX, DPCP				
Interval	0.98:0.99	0.99:0.995	0.995:1.002	0.98:1.002
Rho	0.2435	0.1955	0.4390	0.1052
	(0.1652)	(0.0067)	(0.000)	(0.0937)
Observations	64	191	83	255
The () represents the P-Value of Spearman's Rho				

From these results, we can already infer the result we would obtain by running correlations between the cps coefficient and DPCP. However, table 2 shows this result formally.

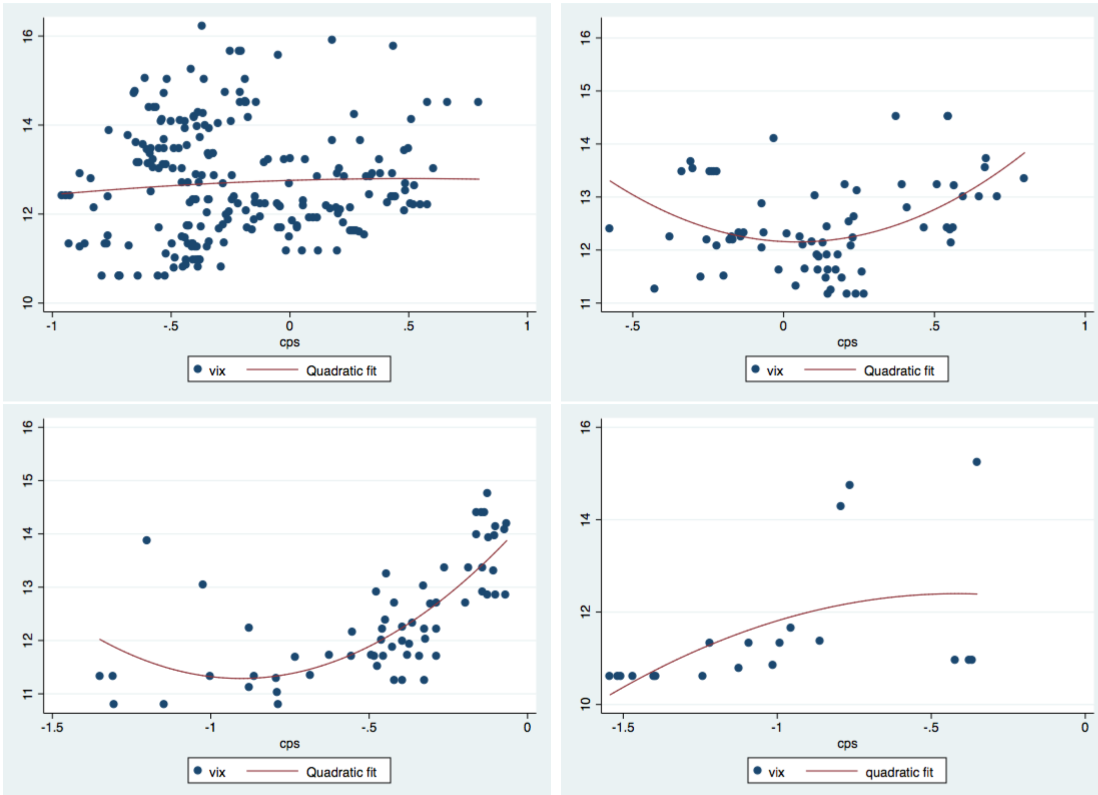
Table 2: Correlation Results

Spearman's Rho: $Beta_{cps}$, DPCP				
Interval	0.985:0.99	0.99:0.995	0.995:1.002	0.98:1.002
Rho	-0.0644	-0.0918	-0.64731	-0.0574
	(0.3453)	(0.0242)	(0.002)	(0.3912)
Observations	217	114	20	225
The () represents the P-Value of Spearman's Rho				

As we would have expected, the correlation between the coefficient and the put-call parity deviation is negative across the board where the significance varies. When there is a positive deviation from put-call parity, we can infer that the index is overvalued. This argument is justified since market prices generally reflect the net market response to a mix of opinions. Speculators then act accordingly.

This result is important because it suggests that the VIX isn't necessarily an indicator of pessimistic information in the market. While increases in the VIX are associated, for the most part, in significant decreases in *cps*, the reverse is also true in some instances. Because we observed in instances where the index is undervalued the slope parameter is positively related the VIX, we can say that the VIX is also capturing optimistic uncertainty, or positive private information. The result here is that general asymmetric information, positive or negative, about the direction of the index is captured by the VIX, but the direction itself can be understood by looking options prices. Figure (9) demonstrates exactly this result.

Figure 8: Volatility Smiles of the VIX and Slope Parameter of CPS



The order of the quadrants are the money-ness intervals we have explained before. Since many of the observations are insignificant, we obtain mix results. As with before, this lack of significance is attributed to transactions costs, forecasting errors, and perhaps other exogenous events. This is why we notice that the smile is the opposite for some intervals. Our desired result is noticed in the second and third quadrant, and is what we would otherwise expect for the other quadrants if there were no limits to arbitrage or other market frictions.

6.6 VIX growth rate speculation

The same tests are run on the 30 day growth rate of the VIX. The idea here is that speculators may not be betting against a movement in the VIX tomorrow, but rather its end of month growth. The results from this approach are demonstrated in appendix B. Very similar behavior is noticed with analyzing the *cps* slope parameter, but when considering smaller intervals of money-ness, the result becomes more insignificant.

When looking at the put and call contributions separately, mixed results are noticed. When relating these results to the theory, there doesn't seem to be any particular pattern as we move more in the money.

While we do not notice a particular pattern from puts and calls with respect to the growth of the VIX, it is noticed that the volatility smiles are more in line with the theory as we move more in the money. This can be seen in Figure 13 of appendix B.

This result ultimately suggests that puts and calls are not great at consistently explaining the growth rate of the VIX. However, the slope parameter on *cps* appears to do a reasonable job at explaining the increase in private information, which is reflected in the growth rate, so long as the options are very tight around the money.

7 Future Research

The link between theory and practice is ever so important. From the theoretical section presented in the beginning, many of the analytical results hold only due to the assumptions that we have imposed. To accurately link our theory with data, may of those assumptions must be relaxed. Such assumptions include relaxing the no-transaction costs assumption as well as the no frictions aside from the private information. Such a procedure could provide not only reasoning that is in line with the data, but some new insights.

While we discussed in extensive detail the action of short selling, we have left it out of the theory and the empirical section. The issue was that it created complications when extending the Glosten et al (1985) model. Moving forward, it should be noted that short selling can play a major role in the relation that is being discovered. Such an analysis can provide alternative insights either complimenting or challenging the results presented.

Much of the motivation in understanding how investors operate is to understand what alternatives they have available. We stressed earlier on the strategies available to a pessimistic investor, but neglected to thoroughly discuss short selling. Although short selling is very common in practice, it was chosen to be left out of the theory and empirical sections due to its complexity.

One of the major challenges of this paper was transforming the data into something that could be better analyzed. The raw data at hand was in per-minute frequency and the variables of interest were put and call bid and ask prices, strike prices, time to maturity, the CBOE volatility index (VIX), and the risk free interest rate. Per-minute data is often difficult to analyze and is therefore left to the school of high frequency thought. The data is thus transformed into daily data to be made easier to analyze. Daily data is also the smallest frequency obtainable from the CBOE database from WRDS. The problem with shrinking the data is that we lose lots of information and effectively decrease the sample size. For further insight into the general theory presented, it would be desirable to acquire minute data of the VIX, so our sample size can be largely represented while avoiding small sample biases.

The choice to consider the money-ness of options was costly. Narrowing down our already short observation list to tight money-ness intervals reduced our observation size significantly for some intervals. While desired results were still obtained, not enough data points could be extracted to obtain a clearer picture of what was actually going on. With respect to this, it would still be desirable to have had minute data for the VIX and then to proceed with the same tests.

A common problem for researchers when given a data set is how the data should be used for testing purposes. Vector autoregressive models (VAR) or vector error correction models (VECM) are often used to deal with time series data. Time series is a very complex and misunderstood field which makes the approach very difficult to grasp for many researchers who lack knowledge thereof. Results in time series are often difficult to understand and articulate, making a study in the field a piece of work on its own. We therefore stick to multivariate OLS models to test our hypothesis of interest. While desired results were obtained, there are still several issues with this approach. Firstly, since our original sample was consolidated into daily observations, performing a 30 day rolling regression used a sample of 30, as opposed to the well over two thousand we would have used had we used minute data. Obviously when we have such as small sample, our parameter estimates become biased and/or inconsistent. Secondly, by using the VIX as a proxy for uncertainty, our regression attempts to explain this from the dynamics of options prices; hence, regressing the VIX on the put and call prices. The results obtained are what was expected from the theory, but it is quite possible that the model is misspecified. Assuming that only puts and calls explain the VIX is a bold assumption from an empirical standpoint, but it need not hinder the result we are trying to put fourth. I would have otherwise suggested that short interest be used in the regression as well, since this can also reflect an investors response to private information.

The span of the data is only nine months beginning in 1993. Taking a similar approach while using recent data, especially if it is attainable from the crisis, could prove to provide alternate insights about the theoretical results derived. Just based on what was observed during that time, I would expect there to be similar results, but perhaps a lot more insignificance. The reason being is that there was widespread illiquidity among assets, and therefore very high transaction costs, which was argued played a role in the insignificance of certain parameters. This issue will have to be investigated further.

The paper at hand is only suggesting that private information can be signaled through options prices by looking at the spread. higher spreads indicating increased private positive information, and lower spreads indicating private negative information. Of course, one may question how we compare the spreads for tomorrows observations. The answer to this would be to compare them against the empirical distribution from the previously observed data points. Adding this feature to this paper would be interesting, but again, it could indeed be a large contribution on its own. Therefore, I would suggest the application of this idea for future research in this field.

8 Conclusion

In conclusion, our primary question at hand is to understand how private information can be detected in the market. A basic extension of Glosten and Milgrom (1985) is developed to understand the dynamics of investor behavior. The environment is a specialist market where investors come to this specialist either to buy assets or options. The argument of put-call parity is used such that the specialist will set the bid-ask prices of options, which then translate to the asset price. Given orders from investors, the specialist updates his conditional expectation about the asset price. This change is apparent in the options prices. Using a zero profit condition for the specialist, it is found that the spread between call and put prices increases when conditional expectations about an downstate increases, and the reverse happens otherwise.

Since it cannot be observed directly what the conditional expectation of the specialist is, a proxy must be used. Therefore, to test the hypothesis at hand, we use the publicly available CBOE Volatility index (VIX) as a measure of uncertain expectations. Rather than capturing a single company to generate the options relation, the S&P 500 Index is used. The options data acquired is from January 4, 1993 to September 15, 1993, as is converted from minute to daily frequency while only considering maturities of at most a month. Rolling OLS regressions are used over thirty day periods, and the strike is discounted by the one month risk-free interest rate at 3%. The purpose of this is because we are also interested to see if money-ness plays a role in determining the result we seek.

Rather than to use the difference between the bid-ask spreads of the options as was presented in the theoretical section, it was more intuitive to use the spread between the bid-ask of calls and puts. It is shown using Spearman's Rho that this new parameter *cps* is positively correlated over time with the theoretical variable, in addition, the average of the bid-ask prices are commonly used especially when calculating the implied volatility; it is also easier to get the raw options prices in practice, than the bid-ask options prices.

Overall, the regression output showed mixed results, both positive and negatively significant slope parameters over time. This is what would be expected given the theory and model specification. While we do see areas of insignificance, they tend to decrease as the observations go closer in the money. While the parameter *cps* is the key interest, it is also useful to look at the individual contributions of the put and calls. For the most part, the correct signs are on the slope parameters. More specifically, the puts parameters positive while the calls are negative. While we sometimes see the reverse, it isn't typically an issue since our primary interest is to look at private information, both optimistic and pessimistic. While we see areas of insignificance less and less as the observations are closer to the money and depending where in the sample it is, this shall be associated to significant world events, transactions costs, and finite sample biases/misspecification.

The same tests are performed to see if investors speculate against the 30 day growth rate of the VIX, without the contribution of the puts and calls. It is found that much of the observations are statistically insignificant, becoming worse as we go closer into the money.

To draw inferences from the theory, deviations from put-call parity are compared against the VIX. Treating the two as separate series', it is found using Spearman's Rho that the deviations are positively correlated with the VIX, while only significant within tight intervals around the money. The deviations are also negatively correlated with the slope parameter on *cps*. The correlation becomes more significant the more tightly around the money. This result suggests that positive deviations from put-call parity is a signal that the index is overvalued, and therefore investors will buy more puts and less calls, hence $Beta_{cps} < 0$. The reverse is true for a negative deviation. Without examining explicitly the deviations from put-call parity, the result could potentially be inferred by looking at the call-put spread as it acts as a signal of over/undervaluation.

This ultimately leads us to the final result, and the most relevant to the theory. Since the VIX has been used as a proxy for uncertainty, it is expected that the slope parameters on *cps* both increase and decrease with a high VIX. The result is a smile like curve. This can actually be seen by looking at different money-ness intervals.

The theoretical result suggests that changes in *cps* are associated with changing expectations. From the smiles, this leads us to believe that the VIX captures not only private pessimistic information, as is commonly the case, but also private optimistic information. The same result is seen when looking at the 30 day growth of the VIX, where the smiles are more apparent as we go closer to the money.

A Appendix

A.1 Data Maniplulation

This section explains more deeply how the data was altered for testing purposes and how the tests were run.

The data was originally acquired as two separate files; one for calls, one for puts. The files were also originally in excel. I ran into problems using excel, so I decided to transfer the data into a format that is understood by the program ”Stata”. Doing it this way is easier since I am more proficient in it, and it is also programmable to carry out all of the desired statistical results.

After both files were ready to be used in Stata, the put data had to be merged with the call data. Since both data sets are within the same range, minimal problems are encountered. Luckily, state has a nice command called ”merge” which allows you to take one variable from a data set and combine it with another set based on other key variables. I've chosen to merge the calls with the puts according to the date and the strike price. The exact state command is:

merge 1:1 varlist using filename

where ”varlist” consists of ”puts date strike” and ”filename is the name I've given to the put file.

Once matched, it was time to collapse the data from minute to daily. The following state command does exactly this:

collapse (mean) index bid ask cbid cask strike, by(date)

where ”bid ask cbid cask” are the put-bids, put-asks, call-bids, and call-asks respectively.

The command collapses the data based on the date which was a new variable generated to reflect the days present in the sample; this command is as follows:

gen daily = date(date, ”DMY”, 1993)

which, again, creates a variable that represents each day that data is available.

Overall, state was easier to use in manipulating the data.

A.2 Statistical Methodology

Stata was used to perform the rolling regressions. The use of rolling regressions rather than typical regressions is that it required the use of programming within stata. The following is a sample of the code that was used on the data:

```
(1)gen beta1=.
(2)gen se3=.
(3)local begin=1
(4)local begin1=30
(5)local end=255
(6)while ‘begin1’<‘end’ {
(7)reg vix cps in ‘begin’/‘begin1’
(8)matrix se = vecdiag(cholesky(diag(vecdiag(e(V)))))
(9)matrix betamat = e(b)
(10)svmat se
(11)replace se3 = se1[1] in ‘begin’/‘begin1’
(12)replace beta1 = el(betamat,1,1) in ‘begin’/‘begin1’
(13)drop se1
(14)drop se2
(15)local begin=‘begin’+1
(16)local begin1=‘begin1’+1}
```

Lines (1) and (2) are dummy series' that were created so we could eventually store the values. Lines (2) and (3) shows the initial window that the program will run the first regression. At the end of the code, lines (15) and (16), you can see Local start = 'start"+1 and Local start4 = 'start4'+1, which suggests that the program is now starting from 2 to 31. This happens until the sample ends; we see where it ends from line (5). Line (7) is the regression comment itself. Lines (8) and (9) extract the matrix of the beta estimates and the standard error estimates. Lines (11) and (12) take the first elements in those matrices, the standard error and the slope parameter, and places them in the corresponding position in lines (1) and (2).

This is an explanation of the code in a nutshell, but it is quite straight forward. Once we have the series' of the betas and the standard errors, (beta1 and se3), we produce t-values but dividing the two. A graph is then created through the following command:

twoway line t cvu cvl date || line beta1, date, yaxis(2)

Which is exactly figure 4. Similar codes are used for the remainder of the graphs.

B Appendix: Statistical Results from Growth Rate Analysis

Figure 9: CPS parameter and significance against 30 day growth of VIX

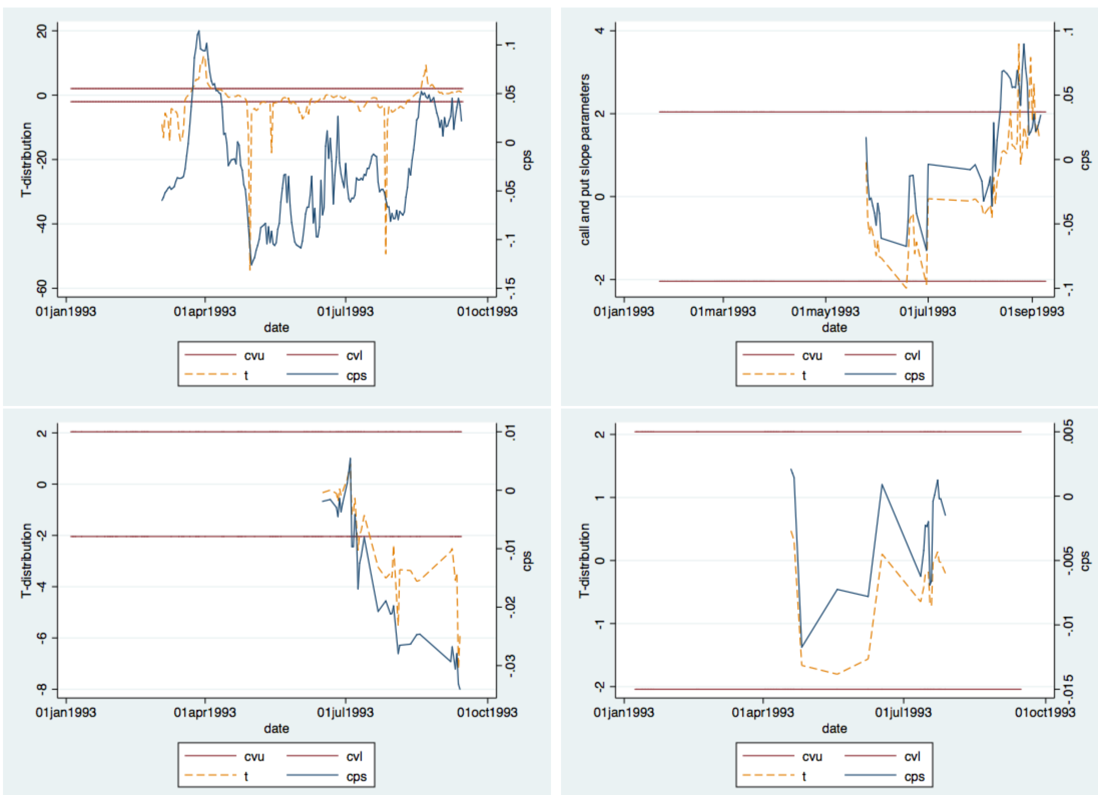


Figure 10: Call and Put parameters against 30 day growth of VIX

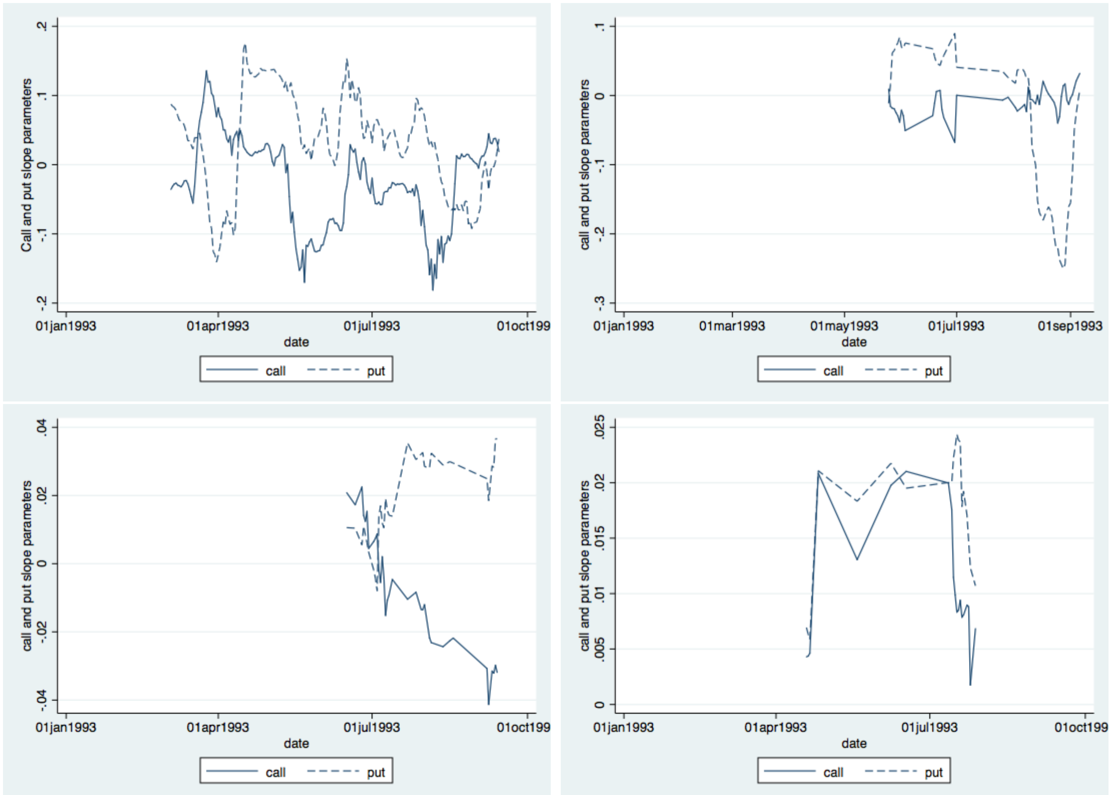


Figure 11: significance of Call and Put parameters against 30 day growth of VIX

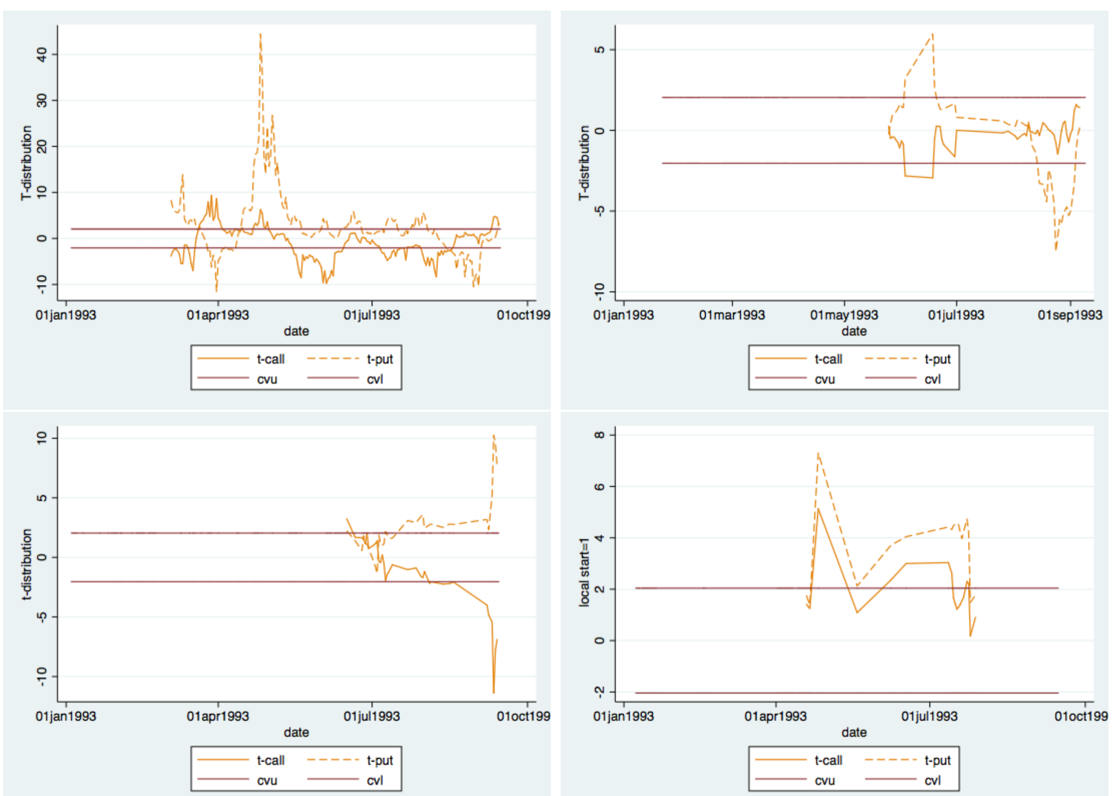
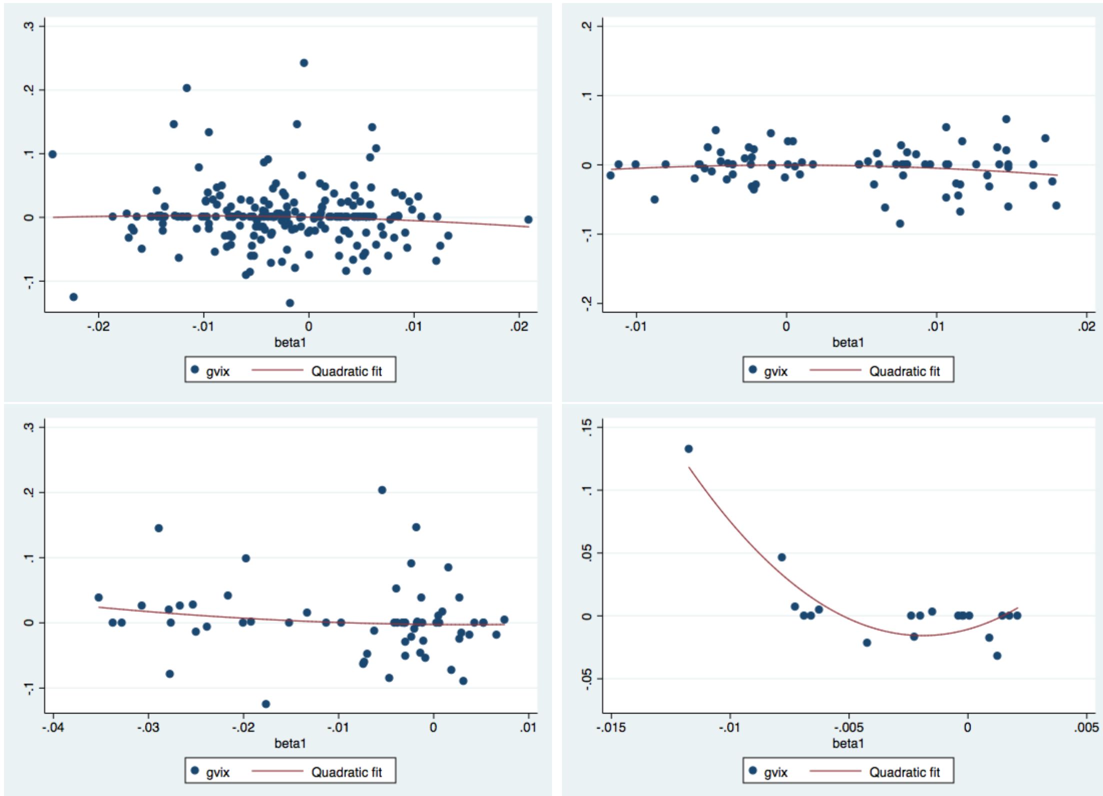


Figure 12: Volatility Smiles of the 30 day VIX growth rate and Slope Parameter of CPS



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