WHEN THERE'S NOWHERE TO HIDE: A SPILLOVER ANALYSIS IN THE S&P 500 USING FORECAST ERROR VARIANCE DECOMPOSITION

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An essay submitted to the Department of Economics in partial fulfilment of the requirements for the degree of Master of Arts

> Queen's University Kingston, Ontario, Canada July 2014

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Abstract

In this essay, we investigate the S&P 500 index daily intra-market linkages in volatility and liquidity. We document and analyse patterns in spillover for both variables in the time and cross-sectional dimensions. We carry our investigation using a generalized vector autoregressive framework, in which forecast-error variance decompositions are invariant to variable ordering, and the novel spillover index developed by Diebold and Yilmaz in 2012. Their spillover index, recently coined connectedness index gives an aggregated estimation of the market resilience to shocks. Measuring the level of interdependence among stocks enables researchers and practitioners to identify periods during which markets are more vulnerable in terms of risk transmission. In parallel, it provides a quick graphical tool to identify systemic risk variation and monitor risk spreading across the market. We contribute to the existing literature with the addition of two major aspects. First, we introduce a methodology allowing researchers to handle larger sets of variables, inspired by the network theory literature and recent developments in statistics. Also, we are the first to analyse liquidity spillover. Our results highlight that the choice of variables under investigation can lead to bias in the spillover results, especially in the crisis build-up detection. Also, propagation mechanisms of liquidity and volatility can evolve in a substantial different fashion illustrating that liquidity dry-out are not necessarily linked to market risk increase.

Aknowledgements

First and foremost, I would like to thank my supervisor Allan Gregory for his guidance and advice along the way. I thank my beautiful wife Marianne for her wisdom and the tremendous amount of help she offered me throughout my entire academic life. A special thanks to my family that provides me with all the possible support. Finally, thanks to my friends, Pierre, Jean-Franois, James, Marc, Phil, Valois, Boivin and many others for the exceptional year we had together. I cherish the memories we have, I look forward to meet with you in the future and, most of all, I wish you the best success in all your enterprises. This essay was written using LATEX. The programming was done using R version 3.1.1.

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1 Introduction

A sound understanding of market linkages is of crucial interest with regard to financial activities such as risk management, asset allocation and regulatory framework implementation. Recently, researchers provided evidence supporting the fact that crises share common patterns (e.g., Reinhart & Rogoff 2008, Corsetti et al. 2001). The natural next step is to investigate such relationships in order to improve our understanding of crisis dynamics and develop tools enabling market participants to detect the symptoms and evolution of crises.

Bolstered by the recent financial events, a large body of literature was published in order to deepen our understanding of connectedness patterns in returns and volatility. Research on the topic can be broken down into two broad categories: the analysis of financial contagion (e.g., Forbes & Rigobon 1999, 2002), which assesses whether a crisis in a given market can contaminate other markets, and the analysis of market spillovers and their evolution through time (e.g., Engle et al. 1990; Diebold & Yilmaz 2009, 2012, 2014). The latter category does not discriminate between contagion and interdependence, but investigates the magnitude of the spillovers among the studied variables. The interest for spillover analysis was largely nurtured by the dynamics of the 2008 crisis, specifically because the crisis, which broke out in a single market, finally spread to the entire financial system. From a risk management perspective, spillover monitoring enables the tracking of crises evolutions and ultimately search for pattern similarities in the process. Studies in this area previously shed light on comovements in the mean and volatility of equity returns and were thus extended to all classes of assets and regions (e.g., Barunik et Al. 2013).

The spillover index of Diebold and Yilmaz (2009) can also be adapted to the emerging literature on market connectedness and network modelling. Although it does not specify the spillover determinants, it gives an aggregate measure of the interdependence level among the studied variables. In this sense, it provides researchers with insightful information about the stability of the propagation channel through time, as well as the ability to distinguish between variables that are more likely to transmit disturbances and those more likely to receive disturbances. Finally, their spillover index is in line with other systematic measures such as the CoVaR tools of Adrian and Brunnermeier (2011) and the marginal expected shortfall developed by Acharya et al. (2010).

Furthermore, research on topics related to market microstructures (e.g., liquidity) attracted a considerable amount of attention over the last decade. We are now better able to characterize liquidity linkages with other market indicators, such as the risk level. Broadly defined, the liquidity level of a stock may be considered as a proxy to characterize the informative quality level conveyed by the stock price. In other words, the more liquid a stock is, the better its price reflects the market participants expectations about future events. These assertions have been under careful scrutiny in many studies (e.g. Vayamos 2004, Cespa & Foucault 2014), where the researchers found a significant negative relationship between the level of risk in the market and its inherent liquidity level. These researches pointed out that liquidity provides insightful information on the risk level in the market.

Following the lead of Diebold and Yilmaz (2012), we investigate the evolution of intra-market spillovers in the S&P 500 focusing on two measures related to risk, volatility and liquidity in the ten sectors of the Global Industry Classification Standard (GICS): Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Telecommunication Services and Utilities. First, we adapt Diebold and Yilmazs spillover index to measure the intra-market spillovers of volatility and liquidity. Second, using the adapted index, we measure the intra-market liquidity and volatility spillovers among stocks listed on the S&P 500 index. Such methodology enables us to quantify the joint evolution of volatility and liquidity across time. To the best of our knowledge, this is the first attempt to model spillover over such a large set of data. Lastly, the topic of liquidity spillover has received much less attention than volatility in the current literature. Our paper fills this gap by providing a first investigation of the dynamics of liquidity spillover patterns as well as a comparative analysis between volatility and liquidity spillovers.

The ability to conduct spillover analysis in a data rich environment (i.e., large dataset) is the most crucial contribution of this essay. Selecting a small subsample of a studied population brings about biases that have mostly been

ignored from the current literature. Diebold and Yilmaz (2014) briefly discuss the population selection bias, stating that the results obtained with their methodology were highly sensitive to the selected variables under investigation. In this regard, we provide a way to handle large and high dimensional systems while performing a spillover analysis.

The rest of the paper will be structured as follows. Section 2 reviews the literature on spillover. In Section 3 describes the proxy used in this study and provides summary statistics of the measures. Section 4 introduces the methodology we applied to work with an high dimensional data set. In section 5, we explain the methodology used for the variance decomposition. Section 6 and 7 discuss the spillover results. Finally, section 8 is the conclusion with recommendations for further researches.

2 Literature Review

2.1 Volatility Spillover

The analysis of volatility gained considerable attention during the last decade, mostly since it became clear that a throughout understanding of its dynamics was of prime necessity for most economic agents. In this regard, a rich body of literature was created on various topics pertaining to volatility and its linkages in financial markets and asset classes.

Masson (1998) characterized three distinct types of shocks based on their propagation mechanisms: (1) aggregate shocks, (2) country-specific shocks, and (3) shocks which cannot be explained by economic fundamentals. Aggregate shocks may be caused by a common set of factors, such as a change in the economic fundamentals affecting uniformly several economies. Countryspecific shocks concerns shocks in a set of countries that may affect the economic fundamentals of other countries. The intuition here is that there exists some sort of directionality in the transmission mechanism. For example, a change in trade agreement and policy coordination among countries may be view as country-specific as they will potentially cause repercussions on competitiveness in other countries. Finally, he defined the pure contagion propagation mechanism (the third type of shock) as an increase in co-movement which cannot be explained by the first two economics channels.

Most of the early studies on spillover analysis address the identification of the transmission mechanism at play in various situations. Engle et al. (1990) carried an analysis on volatility spillover and examined if countryspecific and world-wide (aggregate) news arrival had distinguishable effects on volatility. More specifically, they conducted their analysis on intra-daily volatility in foreign exchange markets in order to explain the causes of clustering in exchange rate. They carried their analysis using a GARCH model on the intra-daily yen/dollar exchange rate to test whether the impact of news were country-specific. Furthermore, many researchers have investigated market comovements with the objective to distinguish between market interdependence, attributable to Masson's first two propagation mechanisms, and contagion shocks, the third propagation channel. Forbes and Rigobon (1999; 2002) looked at cross-country correlations of asset returns in tranquil and crisis periods, defining contagion as a structural break in the parameters of the underlying data-generating process.

More recently, Diebold and Yilmaz (2009) introduced a spillover index based on the forecast error variance decomposition of a N-variable vector autoregression. Their spillover methodology enabled researchers to measure the contribution of each variables exogenous shocks to the other variables. Their suggested approach distinguishes itself from the core literature as it does not directly aim at differentiating contagion from interdependence, but grants the ability to assess the proportion of spillover specific to each study variable. Their methodology does not require a formal contagion test in order to produce results, empowering researchers with information on systemic risk in the market at every observation. Their work relates to systemic risk researches, as it produces information on the market interdependence level and its transmission channels. It enriches the systemic risk measurement literature and brings a new dimension to risk management techniques, such as the value-atrisk (VaR), by allowing the identification of patterns in the time and crosssectional dimensions. Their methodology recently gained significant attention and was adapted to a broader set of analyses. Nevertheless, the examination of intra-market spillover received less coverage and is still limited to a restrained subset of the studied population.

In their first version, Diebold and Yilmaz (2009) applied the Cholesky variance decomposition to obtain the forecast error variance decomposition used in their index. Rapidly, the method faced a large amount of criticism from the academic community with respect to the lack of theoretical background regarding the ordering of the variables in the vector autoregression. Given that the variable ordering could not be anchored in the economic theory, two avenues were proposed. First, Löbner and Wagner (2012) suggested that the cholesky decomposition be performed over the N! possible combinations. Their methodology has the benefit of producing orthogonalized errors; however, it no longer produces a single version of the spillover measure. Rather, they choose a median version of the index, selected among all the N! potential outcomes. The package FastSom, developed for the Cran project in R, efficiently computes N! potential orders. Second, Diebold and Yilmaz (2012) revisited the work of Pesaran and Shin (1998) on generalized variance decomposition to solve the ordering issue. The generalized variance decomposition is robust to variable order in the VAR, under the hypothesis that variables are normally distributed. Under Wolds theorem, it is possible to invert a covariant stationary process into an infinite moving average, yielding the empirical covariance of the errors. Then, the error decomposition is achieved through the filtration of the shocks, in our case the standard deviation of the errors (σ_{ii}) , using their empirical distribution. This approach has the advantage of producing an exact measure of the spillovers.

2.2 Liquidity measure

Concurrently with the increased interest for volatility analysis and its implications for risk assessment, the analysis of market microstructures gained considerable momentum during the last decade. Market liquidity is closely related to stock price informativeness; liquid assets potentially convey more precise information on market participants expectations. In this sense, liquidity is determinant to the price discovery process as it allows stocks to incorporate new information at a minimal cost, frictionlessly. Many theoretical studies (e.g. Vayamos 2004; Brunnermeier & Perdersen 2009) revealed that liquidity fluctuations are intrinsically related to surges in market risk level. In this sense, we expect liquidity levels to covary with volatility.

Of the first researchers to investigate spillover patterns of liquidity, Vayamos (2004) found that large fund managers are among the potential causes of this spillover phenomenon. These fund managers impact the liquidity channels through the composition of their investment portfolio. Under the assumption that the liquidity premium is a convex function of the volatility he stated that managers are less willing to hold illiquid assets during period of high volatility. Intuitively, when volatility is low, managers are not concerned with withdrawals because a movement of several standard deviations is required for their fund performance to fall below a targeted return threshold. Thus, the perceived value of liquidity, the liquidity premium, is very small and almost insensitive to volatility. However, when volatility increases, the probability of investors withdrawals grows rapidly, since the funds are more likely to underperform, and so does liquidity premium. The dynamics between the level of risk in the market and the need for liquidity of fund managers, embodied by the liquidity premium, shed evidence of potential comovements between liquidity spillover and changes in the market risk regime.

Cespa and Foucault (2012; 2014), among others, suggested potential mechanisms to explain the liquidity spillover phenomenon across assets of the same class. Researchers carrying out empirical studies on liquidity spillover linked its dynamics to market volatility. Cespa and Foucault (2014) emphasized the implication of cross-asset learning, a spillover transmission mechanism for liquidity dry-ups and comovements. The cross-asset learning implies that as the liquidity of one asset decreases, its price becomes less informative to liquidity providers in other assets. Therefore, the liquidity of the other assets drops as liquidity providers withdraw from these assets. They investigated the consequences of volatility shocks on the concentration of liquidity across assets in one arbitrary market constituted by cross-market arbitrageurs. Such liquidity spillover mechanism highlights dependence patterns in liquidity linkages and can be understood as a risk propagation mechanism as well as a source of fragility in the market. This feedback loop provides a new explanation for comovements in liquidity and liquidity dry-ups.

Although these researches findings showed a strong relationship between volatility and liquidity, there is evidence suggesting that these variable fluctuations are potentially linked to differing sets of observable phenomena (Dionne &

Chun 2013). Additionally, researchers have long acknowledged that the liquidity risk provides complementary information on volatility in the risk management business, which is attested by the importance of the liquidity-adjusted value-at-risk of Bangia et al. (1998) and the liquidity beta of Amihud and Mendelson (1991). However, the literature on liquidity spillover has received far less attention and there has been no attempt at modelling its evolution through time.

3 Data

We studied the volatility and liquidity spillovers of stocks listed on the S&P 500 index for the period ranging from 12/30/2002 to 06/20/2014, for a total of 2889 daily observations. We removed from our data sample all stocks that did not span the entire time range. By doing so, we missed the new entries, among which are found potentially large new players such as Facebook, Alibaba and Netflix. However, this is common practice in all the studies that we surveyed in our literature review.

For the construction of the realized volatility measure and the Amihud"s illiquidity measure, we obtained our data from $PiTrading¹$, a high-frequency data provider. Prior to computing the realized volatility measure, we used the data cleaning methodology suggested by Barndorff-Nielsen (2008), whenever applicable. The cleaning routine related to the order book was conducted by

¹http://pitrading.com/, Historical Stock Data Package

our data provider, as we only received the trade data. As part of our cleaning routine, we deleted all entries occurring before 9h30 and after 16h00. In the rare situation where we had an entry with a missing or zero value, we replaced it by the previous observation in that given day. If the first value of the day was missing, we used the following observation to avoid overnight price variation.

Our analysis necessitated the transformation of the raw data into two proxies: one for volatility, named realized volatility, and another one for liquidity, named Amihud's illiquidity measure. The next two subsections provide a brief overview of these measures and their fundamentals.

3.1 Volatility Measurement

Volatility modelling has been one of the most active fields of research in financial time series econometrics during the last decade. Academics made considerable progress in understanding stochastic volatility and, subsequently, a wide variety of models emerged to capture the process dynamics.

With the increased availability of intraday high-frequency data on financial assets, the econometrics researchers considered applying novel techniques from the stochastic process theory to volatility modelling. These opportunities led to the development of a measure coined realized volatility, derived from the quadratic variation process. In contrast with the well-known ARCH/GARCH estimation methodology, which essentially treats volatility as a latent variable, the realized volatility (RV) treats it as an observed variable based on past observations, yielding an ex-post measure.

3.1.1 Truncated Realized Volatility

It is widely recognized that modern financial markets operate in an approximated continuous fashion throughout the trading time. It is then reasonable to consider a continuous-time process as the underlying data generating process of these discrete observations. Andersen and Associates (2003) developed a formal argument to support the link between the realized volatility process and the quadratic variation process. To set forth the notation, let $p_{i,t}$, the logarithmic price, denote the time $0 \leq i$ at day $[0 \leq t \leq T]$ defined on a complete probability space (Ω, \mathcal{F}, P) . Ω is the fundamental set, \mathcal{F}_t is the information set, and P is a probability measure defined over Ω . The process can be expressed in its simplified form as a Brownian semimartingale with a pure jump process:

$$
P_t = \int_0^t \alpha_s ds + \int_0^t \sigma_s dW_s + J_t \tag{1}
$$

- P_t denotes the logarithmic price
- $-\alpha$ is a continuous locally bounded variation process
- $-\sigma$ is a strictly positive and cadlag (right-continuous with left limits) stochastic volatility process

- W_t ∼ (0,dt) is a standard Weiner process

 $-J_t$ is a pure jump process

Then the Quadratic Variation process (QV) can be expressed as:

$$
[P_t, P_t] = \int_0^t \sigma_s^2 ds + \sum_{s \le t} (\Delta J_s)^2 \tag{2}
$$

And the stochastic differential equation becomes:

$$
d[P_t] = \sigma_t^2 dt + (\Delta J_t)^2 \tag{3}
$$

Based on equation (3), it is clear that the stochastic differential equation estimates the spot volatility plus the jump component of the stock diffusion process. The realized volatility measure can be generally understood as a discrete time approximation of the stochastic differential equation presented in (3). Capitalizing on high frequency data, the realized volatility allows researchers to approximate the spot volatility through the summation of the returns computed at a small interval (i.e. $\Delta t \to 0$).

Since the first measure of realized volatility introduced in Andersen and Bollerslev (1998a), hundreds of competing methodologies have been advanced to improve the initial RV and cope with financial data stylized facts. Most of the recent development aimed at better capturing the jump component in the stock process and solving market microstructure limitations. Market microstructures, such as the liquidity risk, create distortions in the return observations. These effects tend to generate spurious correlations, rendering inconsistent the estimation via the conventional realized volatility measures.

We based our volatility measure choice on a recommendation from Liu, Patton and Sheppard (2012), who published a survey conducted over 400 realized measures at various time aggregations. They tested the effectiveness of measures for different applications in time series econometrics. Since our research involves the implementation of forecasting techniques and the treatment of stocks ranging from more to less liquid, we were concerned about finding the preferred measure specific to our data. Based on these considerations, we chose the best realized volatility estimator in forecasting individual equity, namely the 15-minute truncated realized volatility.

The 15-minute truncated realized volatility measure was first introduced by Mancini (2009) in order to provide researchers with a nonparametric jumprobust estimator of the volatility. The justification for the use of jump-robust estimators was found in Andersen et al. (2007): disentangling the continuous path component from the jump component in the stock return process enables a more reliable volatility forecasting, since jumps are directly associated with specific macroeconomic news announcements. Consequently, most of the predictability in return volatility results from the non-jump component (Andersen et al. 2007). The truncated realized volatility separates the two processes in the following fashion:

TR-RV_t =
$$
\sum_{i=1}^{n} (y_{t,i})^2 I\{y_{t,i}^2 \le r(\Delta i)\}
$$
 (4)

For $y_{t,i} = P_{t,i} - P_{t,i-1}$ being the stock return on day "t" at time "i"={15,30,...,375,390}, since we sum the returns at every 15 minutes during a given trading day.

In the original article, Mancini (2009) defined $r(\Delta i)$ as being the nonparametric threshold at which we potentially discard the time "i" squared return from the truncated realized volatility daily summation. She explained that the absolute value of the increment in the Brownian motion path tends asymptotically to zero at the same speed as the deterministic function $\sqrt{2 \cdot \Delta i \ln(\frac{1}{\Delta i})}$. Therefore, for small Δi , the instant squared returns $(y_{t,i}^2)$ should not be larger. In a case where $r(\Delta i) \geq 2 \cdot \Delta i \ln(\frac{1}{\Delta i})$, it is likely that a jump occurred. We defined $r(\Delta i)_t$ based on the methodology used by Liu, Patton and Sheppard (2012).

Then
$$
r(\Delta i)_t = 4\sqrt{\frac{BVP_{t-1}}{n}} \; ; \; n = 26 \tag{5}
$$

Where "n" is the number of sampled intraday returns at a 15-minute sampling frequency (26), and BPV_{t-1} is the previous day's Bipower estimator using a one-minute sampling frequency. We computed the Bipower estimator using one-minute frequency since Liu, Patton and Sheppard specified that it performed better at this sampling frequency.

Such that
$$
BVP_{t-1} = \sum_{i=2}^{n} |y_{t,i-1}| |y_{t,i}|
$$
; n =390 (6)

For $i = \{2, 3, \ldots, 388, 289, 390\}$, since we sum the returns at every 1 minute during a given trading day. The use of the Bipower estimator as a threshold is motivated by the fact that it is *robust* to jumps. Thus, when we divide it by the number of sampled daily observations, we obtain an intuitive jump-robust nonparametric threshold.

3.2 Liquidity Measure

In the theory of market microstructure, it is generally accepted that the liquidity cannot be represented as a single variable. A thorough definition of market liquidity encompasses the four axes identified in Harris (2003): tightness, depth, resilience and immediacy. From these four aspects have emerged two broad classes of liquidity proxies: trade-based and order book-based measures. The two classes of proxies show a limited level of correlation, suggesting that the choice of classes decisively impacts the quality of any empirical analysis. Furthermore, Goyenko et al. (2009) showed that both classes capture different aspects of liquidity. Thus, researchers should base their choice on the research focus and the underlying assumption of the study. Irvine and Kandel (2000) provided a framework for choosing which type of measure best suits a research goal. They provided an intuitive model based on the investment horizon of financial agents. Broadly defined, we can segment investors into two groups: patient or impatient. Impatient investors, such as arbitrageurs, trade at relatively high frequency and require immediate execution of their orders.

Conversely, patient investors, such as institutional investors, face less pressure to immediately execute their trades allowing them to look for occasional liquidity pools in order to execute their orders inexpensively. For the latter group of investors the lower the uncertainty around the execution price is, the more liquid is the market.

3.2.1 Amihud's Illiquidity Measure

We investigated the relationship between volatility and liquidity from the point of view of these patient investors. In this sense, Goyenko et al. (2009) advised the use of Amihuds illiquidity measure (IL) as the preferred proxy in the tradebased category. The measure can be defined as:

$$
IL_{t,n} \equiv \frac{|r_{t,n}|}{P_{t,n}VO_{t,n}} 10^6 \tag{7}
$$

For the stock "n" on day "t":

 $P_{t,n}$ = price of stock

 $VO_{t,n}$ Daily share trading volume (in one share unit)

It is important to understand that the smaller this ratio is for a given observation, the more liquid the stock is considered for that specific day. Like any proxy, this measure is not a panacea and it is important to bear in mind that Amihuds illiquidity measure fails to capture the immediacy dimension of liquidity. However, it captures the most relevant aspects of the assessment of the execution price uncertainty level, namely uncommitted liquidity and market resilience (see Moyaert 2012).

3.3 Data treatment and stylized fact

The distribution of realized volatility is serially correlated, strongly right skewed and clearly leptokurtic. The serial correlation issue is handled using the vector autoregression model. However, the other stylized facts on our variable distribution raise concerns for the variance decomposition process, illustrated in section 4.1, as it relies on normality assumptions. Andersen and associates (2003) advocate that the distribution of the realized logarithmic volatility, in our case $\ln(\text{TR-RV}_{15})$, was approximately normal. Thus, we used the natural logarithm of our volatility measure to perform the variance decomposition process. Concurrently, we applied the same transformation to our liquidity measures as they exhibit major similarities with the volatility.

4 Vector Autoregression in Data Rich environment

When modelling spillover in a given market, there is no theoretical guidance for selecting the stocks. Most of the earlier studies handpicked 10 to 20 stocks based on the largest capitalization at a given date in the studied time interval. However, there is no theoretical ground to justify such selection, which potentially leads to biased results, especially in spillover analysis as pointed out by Diebold and Yilmaz (2014).

To mitigate the difficulties linked to the selection bias, we relied on recent developments in high dimensional data analysis, which encompasses the least absolute shrinkage and selection operator (LASSO) family. More precisely, we used an extension called the adaptive elastic net regression. These types of regression were developed to manage systems characterized by matrices with large numbers of coefficients or high dimensional systems, which are doomed by the curse of dimensionality. High dimensional analysis is applied in situations where the number of coefficients to estimate is larger than the number of observations, which renders traditional estimation methods inadequate or simply impossible to use. The literature on the topic evolved in two major directions: data/coefficient shrinkage techniques and principal component analysis. Given the nature of our work, we decided to keep our data in level. The coefficient shrinkage technique thus seemed to provide a natural extension to handle the challenges associated with large dataset analysis. Elastic net regression represents an effective way to control the trade-off between bias and variance in the estimated coefficient matrix. This type of regression is valid under the hypothesis that the coefficient matrix has a sparse representation (i.e., only a subset of the matrix coefficients are nonzero). Another motivation for introducing coefficient shrinkage techniques to our analysis comes from the inherent biased nature of the population selection. As Diebold and Yilmaz (2014) briefly discussed, handpicking a subset of data from the market is likely to bias the results of the spillover analysis. On the other hand, coefficient shrinkage techniques may induce bias in the estimation of the coefficients by wrongly assigning a zero value to nonzero coefficients. In spite of this, our strategy determines the most relevant relationship among the variables in a statistical fashion, whereas the former strategy has no theoretical justification little empirical validity.

We based the choice of our shrinkage technique on data properties highlighted in the network analysis literature. Indeed, we know from financial network analysis (Barigozzi $\&$ Brownlees 2014) that the stock market has central stocks, also called star nodes, which are linked to a large number of additional stocks. These central stocks indirectly connect with other stocks creating a neighbourhood (i.e., companies sharing common characteristic with a tendency to move together) (Chudik & Pesaran 2013). In this sense, the adaptive elastic net is well suited for financial data analysis and allows the estimation of larger VAR models containing several correlated variables. In these VAR models where many of the variables move jointly, our selected estimation procedure leaves out irrelevant variables but does not exclude correlated variables that may be relevant as part of a group (Furman 2014). The adaptive elastic net benefits from the properties attributed to the adaptive LASSO, namely the selection and asymptotic oracle properties, as well as from the finite sample grouping effect inherited from the ridge penalty. In this branch of literature, the oracle effect is understood as the ability to correctly select the sparsity pattern in the VAR coefficient matrix. In a situation where there are groups of variables among which the correlations are very high, the lasso tends to select only one variable from the group and set the other coefficients to zero (Zou & Hastie 2005; Kock & Callot 2012). We used the package GcdNet produced by Hui Zou and associates (2014). The technique we used can be defined in the following way:

$$
\beta^{Anet} = (1 + \frac{\lambda}{NT}) \operatorname{argmin}_{\beta} \{ ||y - X\beta||_2^2 + \lambda [\alpha \sum_{k=1}^K \beta_k^2 + (1 - \alpha) \sum_{k=1}^K \omega_k |\beta_k|| \} (8)
$$

α: Trade-off between the adaptive LASSO and the ridge regression, for $\alpha=1$ represents the ridge regression and $\alpha=0$ represents the adaptive LASSO. λ : the shrinkage, or penalizing, parameter.

N: Number of variables.

T: Number of daily observations.

 $K:$ Number of subintervals in which our sample is partitioned to perform the cross-validation procedure.

We chose λ and α by minimizing the cross-validated mean-square-error, using 10 folds $(k=10)$. Cross-validation estimates the expected level of fit for a model independently from the data used. In other words, it attempts to optimize the predictive power of the regression using the tuning parameters, λ and α , as the model selection criteria. The original sample is randomly partitioned into K equal-sized subsamples. Then, from the K subsamples, one is randomly selected as the validation data segment for testing the model. We then used the remaining K-1 subsamples to estimate the model.

Finally, it has been demonstrated that the adaptive elastic net produces excellent impulse response results and sometimes even outperforms the oracle estimates (Furman 2014).

5 Methodology - intra-market Spillover in the S&P 500

Once we estimated the vector autoregression process using the adaptive elastic net methodology, our estimated results can simply be expressed as a vector autoregression with "P" lag coefficient:

$$
Y_t = \Phi_1 Y_{t-1} + \dots + \Phi_p Y_{t-p} + \epsilon_t \tag{9}
$$

where $Y_t = [rv_{t,1}, rv_{t,2}, ..., rv_{t,k}]$ or $[il_{t,1}, il_{t,2}, ..., il_{t,k}], rv_{t,i}$ are the truncated realized volatility variables, $il_{t,i}$ are the Amihud's illiquidity measures, $\Phi_1, ..., \Phi_p$ are matrices of coefficients of the lagged values of Y_t , and $\epsilon_t \sim (0, \sum)$ is a vector of identically and independently distributed disturbances. In order to produce the forecast error variance decomposition, we are require to work with the infinite moving average of our estimated system. Under the Wold theorem, it is possible to invert our process and represent it as an infinite moving average using the following methodology

$$
[I_m - \Phi_1 L - \dots - \Phi_p L^p] Y_t = \epsilon_t \tag{10}
$$

$$
Y_t = [I_m - \Phi_1 L - \dots - \Phi_p L^p]^{-1} \epsilon_t \tag{11}
$$

$$
Y_t = \Phi(L)^{-1} \epsilon_t = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i} \tag{12}
$$

where L is the lagged operator and ψ_i can be identified using the recursive substitution $\psi_i = \Phi_1 \psi_{i-1} + \Phi_1 \psi_{i-1} + \Phi_2 \psi_{i-2} + ... + \Phi_p \psi_{i-p}, \psi_0 = I_m$ and $\psi_i = 0$ for any $i < 0$.

5.1 Pesaran & Shin Variance decomposition

Variance decomposition is the key of Diebold and Yilmazs methodology, which allows us to discern the fraction of each shock attributable to a variable x_j on each of the x_i forecast. However, the residuals of the infinite moving average process are often correlated, which renders any direct analysis of the errors meaningless since it is necessary to work with uncorrelated residuals to perform the Diebold and Yilmazs spillover index. As a replacement technique to the well-known Cholesky decomposition, the authors used the generalized forecast variance decomposition developed by Pesaran and Shin (1998). This technique also produces uncorrelated impulse responses and has the additional advantage of being order-invariant. For an invertible process, we can derive the generalized impulse response function at horizon h with a shock on the j^{th} element in the following way:

$$
GI_{Y}(h, \delta_{j}, \Omega_{t-1}) = E(Y_{t+h}|\epsilon_{j,t} = \delta_{j}, \Omega_{t-1}) - E(Y_{t+h}|\Omega_{t-1})
$$
\n(13)

where Ω_{t-1} is the information set in the economy at time "t-1" and δ is a m x 1 vector of shocks hitting our system. Equation (13) can be understood as an shock impulse on the jth only. The contribution of this shock to the other variables is obtained using the historical distribution of the errors. Hence, under the hypothesis that the errors follow a multivariate normal distribution, the conditional effect on the errors is defined given a shock on the jth equation at time "t" for the forecast horizon "h" while setting $\delta_j = \sqrt{\sigma_{jj}}$ the standard deviation of ϵ_{it} as:

$$
E(\epsilon_t | \epsilon_{j,t} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, ..., \sigma_{Kj})' \sigma_{jj}^{-1} \delta_j = \sum e_j \sigma_{jj}^{-1}
$$
(14)

where e_j is an m by 1 selection vector with unity as its j_{th} element with zeros elsewhere and \sum is the covariance matrix.

Borrowing this methodology, Diebold and Yilmaz (2009) defined the own variance shares of every stock as fractions of the H-step-ahead error variances in forecasting the variable x_i given a shock to x_i , for $i = 1, 2, ..., N$. Intuitively, spillovers represent the contribution attributable to shocks x_j , for $j = 1, 2, ..., N$ such that $i \neq j$, to the error variance forecast of x_i .

Henceforth, Diebold and Yilmazs H-step-ahead forecast error variance decompositions $(\theta_{ij}(H))$ can be expressed as:

$$
\theta_{ij}(H) = \frac{\sigma_{ii}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \sum e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \sum A_h' e_i)}; \text{ For } H = 1, ..., 10
$$
\n(15)

The sum of each row of the generalized method is not equal to one. The authors normalized each entry of the matrix by the row sum²:

$$
\tilde{\theta}_{ij}(H) = \frac{\theta_{ij}(H)}{\sum_{j=1} \theta_{ij}(H)}\tag{16}
$$

Note that Σ $\sum\limits_j {{\tilde \theta _{ij}}(H)} = \frac{{{\theta _{ij}}(H)}}{{\sum\limits_{j = 1}^{} {{\theta _{ij}}(H)}}}$ $\frac{\theta_{ij}(H)}{\sum\limits_{j=1}^{n} \theta_{ij}(H)} = 1$ and that $\sum\limits_{j,i} \tilde{\theta}_{ij}(H) = \frac{\theta_{ij}(H)}{\sum\limits_{j=1}^{n} \theta_{ij}(H)}$ $\frac{\theta_{ij}(H)}{\sum\limits_{j=1}^{n}\theta_{ij}(H)}=N$

Depending on the sign, positive or negative, and the magnitude of \sum of A_h , it is possible that $\tilde{\theta}_{ij}(H)$ be positive of negative. However, our identification technique does not grant us the ability to distinguish between positive and negative spillover shocks.

In order to identify spillovers between sectors, we ordered the variables by sector in the VAR. An illustration of the forecast error variance decompositions with two sectors, utilities (U) and finance (F) can be represented in the matrix Γ:

²Conversely, it is possible to normalize with respect to the columns

$$
\Gamma(H) = \begin{pmatrix}\n\tilde{\theta}_{U_1,U_1} & \tilde{\theta}_{U_1,U_2} & \cdots & \tilde{\theta}_{F_1,U_K} \\
\tilde{\theta}_{U_2,U_1} & \tilde{\theta}_{U_2,U_2} & \cdots & \tilde{\theta}_{F_2,U_K} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{\theta}_{F_K,U_1} & \tilde{\theta}_{F_K,U_2} & \cdots & \tilde{\theta}_{F_K,F_K}\n\end{pmatrix}
$$

sectioned into four quadrants:

Utilities Spillover	$U \leftrightarrow F$
$F \leftrightarrow U$	Finance Spillover

From this matrix, we were interested in obtaining four distinct measures: total spillover, directional spillover, net directional spillover and pairwise spillover.

5.1.1 Total Spillover

We adapted the spillover measure from Diebold and Yilmaz (2012) to aggregate the $\tilde{\theta}_{ij}(H)$ of every stock listed in the S&P500 into their respective GICS sectors.

The total spillover index measures the contribution of shock spillovers to the forecast error variance across the selected stocks (Diebold & Yilmaz 2012). It can be expressed as follows:

$$
S(H) = \frac{\sum_{i,j=S_l}^{S_u} \tilde{\theta}_{ij}(H)}{\sum_{i,j=S_l}^{S_u} \tilde{\theta}_{ij}(H)} \bullet 100
$$
\n(17)

where S_l and S_u define the initial and ending index of each sector stacked in the VAR. For example, the first 10 stocks stacked in the VAR are from the Consumer Discretionary sector. Their *initial* and *ending index* are 1 and 10. Then, by operating within these indexes, we include the spillover effect specific to this sector only.

5.1.2 Directional Spillover

The directional spillover provides general information on the strength of the propagation channel between one sector and the other sectors of the market. We defined sectors transmitted spillover as:

$$
S_{\bullet S}(H) = \frac{\sum_{i=S_l}^{S_u} \sum_{j=1}^{N} \tilde{\theta}_{ij}(H) - \sum_{i,j=S_l}^{S_u} \tilde{\theta}_{ij}(H)}{\sum_{i=S_l}^{S_u} \sum_{j=1}^{N} \tilde{\theta}_{ij}(H)} \bullet 100
$$
(18)

And sectors received spillover as:

$$
S_{S\bullet}(H) = \frac{\sum_{i=1}^{N} \sum_{j=S_l}^{S_u} \tilde{\theta}_{ij}(H) - \sum_{i,j=S_l}^{S_u} \tilde{\theta}_{ij}(H)}{\sum_{i=1}^{N} \sum_{j=S_l}^{S_u} \tilde{\theta}_{ij}(H)} \bullet 100
$$
(19)

Then, each sectors net transmitted spillover is defined as: $S_{net} = S_{\bullet S}(H)$ – $S_{S\bullet}(H)$

5.1.3 Pairwise Spillover

The pairwise spillover allows us to determine the specific direction of the spillover between each sector, which enables us to identify specific contributors to the market disturbance and quantify the strength of linkages between sectors. The pairwise spillover from sectors \boldsymbol{A} to \boldsymbol{B} can be defined as:

$$
S_{AB}(H) = \begin{pmatrix} \sum_{i,j=S_l}^{S_u} \tilde{\theta}_{ij}(H) & \sum_{i,j=S_l}^{S_u} \tilde{\theta}_{ji}(H) \\ \frac{\sum_{i=S_l}^{S_u} N}{\sum_{i=S_l}^{N} \sum_{k=1}^{N} \tilde{\theta}_{ik}(H)} & \sum_{j=S_l}^{S_u} \sum_{k=1}^{N} \tilde{\theta}_{jk}(H) \end{pmatrix} \bullet 100
$$
 (20)

This measure can be understood as the difference between the gross transmission from sectors \boldsymbol{A} to \boldsymbol{B} , and sectors \boldsymbol{B} to \boldsymbol{A} .

5.2 Identification of the Vector Autoregression

Spillover analysis is highly dependent on the variable set under which the investigation is carried. Recent researches on intra-market (e.g., Barunik et al. 2012) investigated solely a subset of the market. Under this consideration and to depict the most relevant view of the intra-market linkages, we decided to conduct our analysis over all stocks listed on the S&P 500.

To produce our results, we performed a rolling forecast using a window size of 10% of the total 2889 daily observations, included between 12/30/2002 and $06/20/2014$, for our 30 (or 93) variables. In this sense, the results at time "t" are obtained by estimating the system for the period ranging from $[t - 10\%$ 2889, t]. For every iteration, we estimate the model.

However, working with such a large set of variables poses an important challenge: the consistent identification of the VAR(P) coefficients. In order to mitigate the curse of dimensionally related to vector autoregression, we decided to follow the literature procedure and fix the number of lags in our VAR to one, VAR(1) (e.g., Diebold & Yilmaz 2009; Barunik et al. 2012; Urbina Calero 2013). Fixing the number of lags in the VAR is not generally accepted as a good practice. However, endogenizing the lag length in VARs is done at the expense of our degree of freedom and, in our largely dimensional experiment, this technique can potentially result in a greater number of estimated variables than observations. Thus, under this consideration and the fact that every author in the field used a fixed number of lags, we maintained our decision to fix it at one, $VAR(1)$.

5.3 Confidence Interval for the Spillover Index

We constructed a bootstrap confidence interval to estimate to robustness of our spillover results. We used the methodology for the variance decomposition confidence interval detailed by Lütkepohl (2004). Given our methodology, two issues needed to be addressed for the construction of the confidence interval. First, since our vector autoregression does not contain an intercept, the mean of our residuals was potentially not equal to zero. To be consistent with the methodology of Lütkepohl (2004) we subtracted the mean of the residual matrix. Second, although we tested the robustness of our results for different lag specifications and obtained similar results for 1 and 2 lags, our model may exhibit serially correlated residuals. We spot checked different segments of our experiment and tested for the presence of serial correlation in our residuals. Most segments tested positive and, after examination of the partial autocorrelalogram, we observed that the persistence pattern was weak and quickly converge to zero for the section observed. These results indicate that our residuals were potentially serially correlated, thus we decided to use block bootstrap to account for their dependence dynamics. The block bootstrap is a simple fully nonparametric methodology that accounts for dependence in the bootstrapped variables (see Mackinnon 2002).

We proceeded according to the following steps to construct our set of bootstrapped variance decomposition forecast.

1- Estimate the VAR(1)

2- Collect the residuals and center them (Lütkepohl 2004) by substracting their mean. Then, resample the residuals using blocks size of 10 observations in order to construct the bootstrapped vector of residuals (ϵ_t^*) .

3- Generate a set of bootstrapped variables y_t^* using the bootstrapped vector of residuals in a recursive fashion to conserve the time dependence of the y_t such that:
$$
y_1^* = \Phi_1 y_0 + \epsilon_1^*
$$

$$
y_2^* = \Phi_1 y_1^* + \epsilon_2^*
$$

$$
\dots
$$

$$
y_t^* = \Phi_1 y_{t-1}^* + \epsilon_t^*
$$

4- Using the set of bootstrapped observations y_t^* , reestimate the VAR(1), compute the spillover index, store the values obtained from the bootstrap experiment and repeat steps 1 to 4, 999 times $(B = 999)$.

5- Finally, select the 2,5% and 97,5% quantiles from our results to construct the 95% confidence interval.

For every iteration of our rolling spillover index, we followed the methodology stated above. The construction of the bootstrap confidence interval is a computationally intensive approach. Since our work already involved 2546 iterations in our rolling process to estimate a VAR(1) of 30 or 93 financial stocks, we faced definitive technical limitations in term of computing power. Under these considerations, we decided to limit the number of bootstrap trials to 999. The total computational time required to run the entire code was approximatively 91 hours. We used R version 3.1.1. for the entire analysis and the vector autoregression was estimated using the adaptive elastic net package gcdnet and the parallelism package FOREACH. We ran the experiment using 8 processors on an Intel Core i7-3630QM with 2.40GHz.

6 Results

6.1 Robustness Analysis of the Results

Since there is no theoretical ground for the construction of our spillover index, we tested the sensitivity of our results for various specification of the model against a benchmark result. We set the benchmark result based on the $VAR(1)$ including the 30 stocks listed on the S&P 500 with the largest market capitalization as of $06/20/2014$.

The first aspect that we stressed in our analysis refers to the size of the rolling window. We considered three potential scenarios with window sizes of 5%, 7.5% and 10%, respectively equivalent to 145, 216 and 289 daily observations of each stocks (appendix 9.4.3). Most of the studies on the topic were conducted using a window size varying between 200 and 300 daily observations per selected stock. On one hand, having more observations for estimating the system at every iteration should produce more efficient coefficient results. On the other hand, one may argue that having a smaller window size allows researchers to put more weight on more recent data and thus better capture upcoming changes in the spillover patterns. We found that the size of the rolling window influenced our result in the most striking fashion. Specifically, the 5% rolling window size produced more erratic results than the 7.5% and 10% ones. However, all three sizes captured the same specific surge events in spillover. The main difference resulting from the choice of the rolling size was the persistence of the specific surge in the spillover index. As the window size

increased, the spillover index took longer to subside to pre-crisis levels. Since most of the literature on spillover analysis was created using a window size between 200 and 300 daily observations, we decided to maintain our rolling forecast window at 10%. Moreover, this approach offered more degrees of freedom in the estimation of our regression.

Secondly, we investigated the effects of using multiple lags for our VAR equation (appendix 9.4.3). Our benchmark model was fixed at one lag, following the methodology used by most researchers. However, in order to assess the potential information content provided by more persistent models, we also considered two potential lags. Our robustness test indicated that there were few differences between the $VAR(1)$ and the $VAR(2)$.

Finally, we tested the sensitivity of the forecast horizon length. From a theoretical perspective, the choice of our forecast horizon is intrinsically linked with the investigation objective; it serves the same purpose as the studied horizon in value-at-risk. However, this study focused on the phenomenal analysis of spillover patterns among stocks; therefore there is no direct guidance for selecting the optimal number of steps with respect to the forecast horizon. Thus, we performed the analysis at different time horizons, and more precisely two, six and ten steps ahead in order to assess the robustness of our results. This approach allowed us to control for the VAR stability and simultaneously account for the time persistence of the spillovers in our analysis. It is important to clarify one aspect relative to the horizon forecast size. By construction,

the spillover index is a cumulative sum of the forecast error variance decomposition. Thus, we are likely to observe an increase in the spillover results as the forecast horizon under investigation increases. By looking at the spillover index in figure (2), we noticed that there was an important discrepancy between the two- and six-day ahead spillover results. However, the gap between six- and ten-day ahead results was negligible. In this perspective, our results showed that for forecast horizons of ten days and more, the spillover results were not likely to exhibit important differences with the six-day ahead horizon forecast results.

The 95% confidence interval (Appendix 9.4.1) was constructed for the volatility and liquidity spillover at the two-, six-, and ten-day ahead forecast horizons. First, for both sets of variables, the confidence interval results were narrowed indicating that spillover distribution density was relatively tight. This suggests that the reported results were reliable, all things equal. The confidence interval of the volatility spillover attracted our attention, as it was not symmetrical and was skewed toward larger values of the spillover. This result raised potential concerns on the distribution of the errors. Indeed, the generalized error forecast decomposition requires to have normally distributed data in order to be consistent. We acknowledge the limitation that this finding put of the validity of our error forecast decomposition. However, Urbina Calero (2013) compared the results obtained from the N! reordering technique using the Cholesky decomposition and the generalized forecast decomposition; both set of results were within the same confidence interval.

Additionally, our work investigated the spillover patterns between the 30 largest corporations (3 per sector) listed on the S&P 500. We tested the robustness of the results with a different dataset size. Therefore, we subsequently increased our selection to the 93 largest companies (10 per sector and 3 for telecommunication services) (Appendix 9.3). Our results indicated that the selection of the sample leads to major differences in the spillover magnitude. In appendix 9.4.4, it is clear that both sample sizes captured the market dynamics, but the results for the larger sample reached a higher level of spillover, up to 20% higher (Appendix 9.4.4). Based on financial network studies, these different results clearly corroborate our initial intuition. By performing the spillover analysis on a subset of hand-picked stocks, researchers do not account for the dynamics of existing linkages in the market. Then, the ability to correctly identify and measure linkages are subject to the researchers sample selection. Our argument in favour of working with larger datasets when the research objective is to detect crises in the entire market could be strengthened by the investigation of the inter-market spillovers, such as the link between the fix income, the foreign exchange, commodities and stock market. However, this new research avenue is beyond the scope of this essay.

6.2 Static analysis

Total Spillover

The total liquidity spillover measure obtained for the entire studied period, that is from $12/30/2002$ to $06/20/2014$, was 18% (Appendix 9.5.1). The own share of variance of each sector, the table diagonal, was relatively homogeneous among all sectors, ranging from 77.5 to 87.3. The financial and industrial sector stood apart in term of spillover transmission, with 33.5 and 35.7 respectively, which were 10 points over the third largest transmitter. The total volatility spillover measure was 39.8%, and the own share of variance of every sector was again quite homogeneous. Here also, the financial and industrial sectors were the major volatility spillover transmitters. Interestingly, the telecommunication services and utility sectors were the biggest shock total receivers.

6.2.1 Pairwise Spillover

For the pairwise spillover examination, both volatility and liquidity analyses identified the telecommunication services, materials, and utilities sectors as being the most prominent net pairwise transmitters. The pairwise spillover between these three sectors and both the financial and industrial sectors particularly stood out in term of magnitude. Finally, the major net pairwise receivers were the financial and materials sectors.

Figure 1: Liquidity: Static Full Sample Pairwise Spillover

Figure 2: Volatility: Static Full Sample Pairwise Spillover

$\mathbf{0}$	$\mathbf{0}$	-1	$\mathbf{1}$	$\mathbf{0}$	\overline{c}	$\overline{1}$	$\overline{3}$	6	θ	Mat.
$\overline{0}$	$\pmb{0}$	-2	$\pmb{0}$	$\overline{0}$	$\mathbf{1}$	$\mathbf{0}$	$\overline{3}$	$\overline{4}$	$\bf 8$	Uti.
$\mathbf{1}$	$\overline{2}$	$\mathbf{0}$	\overline{c}	$\mathbf{1}$	\overline{c}	$\mathbf{1}$	$\sqrt{3}$	$\overline{5}$	$\overline{7}$	Telco
-1	$\pmb{0}$	-2	$\pmb{0}$	$\mathbf 0$	1	$\mathbf{0}$	$\mathbf{1}$	$\overline{3}$	$\overline{5}$	Info
$\bf{0}$	$\mathbf{0}$	-1	$\mathbf{0}$	$\mathbf{0}$	$\mathbf 0$	$\mathbf{0}$	$\mathbf{1}$	$\overline{2}$	$\overline{3}$	C.S.
-2	-1	-2	-1	$\mathbf 0$	$\bf{0}$	$\mathbf 0$	$\mathbf{1}$	$\mathbf{3}$	$\overline{4}$	C.D.
-1	$\mathbf{0}$	-1	$\pmb{0}$	$\mathbf{0}$	$\pmb{0}$	$\mathbf{0}$	$\boldsymbol{0}$	$\mathbf{1}$	$\overline{3}$	H.C.
-3	-3	-3	-1	-1	-1	$\mathbf{0}$	$\mathbf 0$	\overline{c}	$\overline{4}$	Ener.
-6	-4	-5	-3	-2	-3	-1	-2	$\mathbf 0$	$\sqrt{2}$	Ind.
-8	-8		-5	-3	-4	-3	-4	-2	$\pmb{0}$	Fin.
Materials	Utilities	Telep	Into, Teori	Constitution of the land	Const Osc	Health Cape	Enemel	Industries	Friedricks	

The list of abbreviations can be read as follows: Telco = Telecommunication services, Mat = Materials, Uti = Utilities, C.S. = Consumer Staples, Ener = Energy, H.C. = Health Care, Info = Information services, C.D. = Consumer Discretionary, Ind = Industrial and Fin = Financial.

In the above table, the blue tiles represent negative pairwise spillover between two sections and the red ones are positive pairwise spillover. The pairwise spillover can be understood as the gross spillover transmission of the sector on the horizontal axis minus the gross spillover transmission of its corresponding sector on the vertical axis. The numbers presented in the tables

are normalized on a scale of 100 (-100) and large absolute value of pairwise spillover coefficients imply that one sector contributed substantially to the level of risk of the other sector. From a cross-sectional perspective, the static analysis revealed that both studied variables had the same dominant sectors in terms of spillover. However, volatility spillover was significantly higher than liquidity spillover. It was interesting to note that, based on the full sample static analysis, both volatility and liquidity linkages shared similarities with respect to their dominant sectors.

6.3 Dynamic Spillover

A visual inspection of the total liquidity and volatility spillovers revealed six interesting segments in the series, identified in the graphics presented hereunder with Roman numerals I to VI. For the purpose of our analysis, we included the S&P 500 VIX index, a general benchmark of risk for market participants 3 .

³The VIX index is an aggregated measure of risk averaged from in-the-money call and put options on the S&P 500

Figure 3: Total Market Spillover

In figure (a) and (b), the 2-, 6- and 10-day ahead forecast horizons are respectively represented by the blue zone, the grey line and the pink zone.

Liquidity and volatility total spillovers exhibited approximately the same regime changes in the first four segments. In section II, which coincided with the beginning of the subprime crisis in mid-2007, we observed an increase in volatility (indicated by VIX). In segment III, the market crash of September 2008 plummeted the S&P 500. Conversely, the market risk level measured by VIX increased to an all-time high accompanied by increases in liquidity and volatility total spillovers. We recorded the most pronounced market spillover increase in both variables during this period. To confirm our visual inspection of the results, we ran a bai-perron break test using the methodology introduce in Appendix 9.2 on the total volatility spillover results. We then identified the different regime periods found for the total volatility spillover on the four

graphics presented. Interestingly, we observed that the volatility and liquidity spillover with the VIX tends to experience breaks at the same periods. All three rose sharply during the crash of September 2008, but the liquidity spillover barely reacts when the crisis first declared at the end of 2007. Also, total liquidity spillover did not revert to its pre-crisis level while total volatility spillover subsided to its pre-crisis level in two marked drops concomitant with the two most important purchase of US securities by the United States federal reserve (Appendix 9.7.5).

As we mentioned before, spillovers are an aggregate measure of connectedness and encompass without distinction all the potential transmission mechanisms for a given type of variables: liquidity or volatility. They can be understood as a measure of systemic risk, in comparison to an absolute measure of risk represented by the VIX. It is interesting to observe that most spillover surges coincided with spikes in the VIX, although total spillovers evolved in plateaus without revolving to their initial level as quickly as the VIX.

6.3.1 Total Spillover by Sector

Liquidity

We observed a homogeneous response for all total liquidity spillovers by sector, which match that obtained for the total market spillover (Appendix 9.6.1). From a general perspective, the spillover in each sector was not reduced to its pre-crisis level and was significantly more volatile than it was before the events of 2008. Moreover, two salient facts attracted our attention. The financial,

energy and industrial sectors suffered the most significant surges in spillover, reaching in the worst of the crisis 12%, 20% and 12% of intra-sector interdependence, respectively. From a liquidity risk management perspective, sector diversification did not seem to pose a significant challenge as the intra-market connectedness did not reach dramatic levels. Furthermore, it was interesting to note that significant variation in liquidity spillover was only observed in the worst of the crisis. This observation supports the intuition developed in Vayamos (2004), on the convexity of the liquidity spillover as a function of the risk level in the market. Finally, the telecommunication sectors spillover regime was less persistent and substantially lower than any other sector.

Volatility

Sectorial volatility spillover patterns (Appendix 9.6.1) presented more diverse patterns of change throughout the studied interval. The energy, information technologies and industrial sectors suffered the most marked and long-lasting increase in volatility spillover of all the studied sectors, reaching 80% of sector connectedness. However, the most interesting change of regimes was recorded in the consumer staples, information technology, health care, and consumer discretionary sectors. The 2007 and 2008 market events marginally impacted the spillover level in these sectors, with a minor increase of 10 to 15 points, but during the period of 2010 to 2012, all these sectors experienced a sudden rise of up to 60 points to rapidly abate at the end of that same segment.

6.4 Directional, Net Pairwise and Net Spillover

Liquidity

The examination of directional spillover (Appendix 9.6.2) revealed that all sectors were almost equally receiver and transmitter of directional spillover. Acknowledged by the net spillover, we observed that all sectors were null net liquidity spillover transmitters for most of the studied period, with the only exception of the period encompassed by the 2008 crisis. However, the magnitude of transmission and reception channels intensified after the events that declared in 2007. The financial and energy sectors experienced the highest increase in both transmitted and received spillover from the rest of the market. The telecommunication services and utilities sectors exhibited the lowest level of connectedness with the other sectors over the whole period. The consumer staples, consumer discretionary, health care, information technologies and material sectors experienced marginal variation in terms of directional spillover, potentially indicating that the propagation mechanisms of these sectors were more robust to changes in market conditions.

Moreover, we observed that net pairwise spillover (Appendix 9.6.3) increased in 2007, and based on our results, there was no clear indication that the situation will revert to pre-crisis levels for any of the sectors. This was the most pronounced result of the liquidity spillover section, which indicated that the gross transmission of liquidity among sectors reached a new paradigm, stronger and more intricate. It was interesting to note that the net pairwise spillover between all sectors and the financial and energy sectors was negative during almost the entire sample. At the same time, the telecommunication services sector displayed the highest level of net pairwise spillover. The remaining sectors did not exhibit clear patterns in net pairwise spillover, although the sign of the relationship (i.e., positive/negative net pairwise spillover) appeared to remain constant during the studied interval.

Volatility

First, the size of the directional and net spillovers for the volatility was generally larger than that observed for the liquidity. In concert with liquidity directional spillover, volatility directional spillover (Appendix 9.7.2) revealed that sectors were likely to remain net transmitters or net receivers during most of the studied period. The magnitude of the transmission and reception channels intensified after the events that declared in 2007. The financial and energy sectors distinguished themselves from other sectors by reaching the highest level of directional spillover. Interestingly, the directional spillover surge observed in the energy sector occurred before the event of 2007 and slowly reverted to its initial level throughout the studied period. Again, the telecommunication services sector exhibited the lowest level of connectedness with the other sectors over the entire period. Moreover, during period IV of our sample, the financial, consumer staples, information technologies and material sectors suffered a sudden shift in their net directional spillover, which coincided with the surge recorded in the sector-level total spillover. This drastic change of regime corresponded with two marked increases of VIX. Aside from this drastic event, the other sectors did not display any significant variation during the studied period. Again, this potentially indicates that the connection of these sectors within the market was more robust to market conditions.

We observed a sharp increase in the net pairwise spillover (Appendix 9.7.3) of the financial sector toward all sectors, starting in 2007 and lasting until mid-2009, which was consistent with the financial crisis. The consequential breaks that we noticed earlier in most of the sector-level total spillover were evident. No clear pattern emerged from our results, which prevented us from determining unequivocally which sectors were central to the net pairwise spillover during that period. Finally, we found particularly interesting the fact that, aside from the large break of 2010, most of the net pairwise spillover remained stable across the studied horizon.

7 Discussion

The liquidity and volatility total spillovers evolved in a similar fashion: the significant rises of the VIX index correspond to major increases in both variables. However, the spillover index for the volatility was substantially stronger, suggesting that volatility propagation mechanisms are predominant. The examination of total spillovers at the sector level revealed that spillover significantly differs among sectors and studied variables. The phenomena are generally more erratic and sector-level discrepancies more noticeable in the volatility results.

The major drops in the S&P 500 level occur simultaneously with increases in the total market spillover index. As we explained before, the spillover index can be understood as the systematic risk level in the market.In this sense, we expect the market price level to drop in reaction to a sudden surge in the volatility spillover index as market participants now require a larger return for the risk associated with their market positions. Additionally, our index does not differentiate between good and bad shocks of volatility, limiting our ability to infer on the impact of these two different types of volatility shocks. The most simplistic model such as the CAPM can help derive this intuition. However, once the market prices the new level of systemic risk, we observe that the S&P 500 has a tendency to recover, although the spillover level remains substantially high. A potential explanation comes from the risk habituation of the major market participants (Weber et al. 2012). In time of great turmoil, Weber and associates found that market participants become less sensitive to the stream of negative news in the market. Even if the level of systematic risk remains high, market participants required returns adjust to the new paradigm (appendix 9.7.4). The patterns we find in our research are in line with their findings.

Liquidity

The dynamic section of our experiment reveals that the total market spillover reached its highest level (80%) during the 2008 crisis. This result indicates

that a generally high level of dependence among stocks is exacerbated in crisis period. At the disaggregated level, although sector-specific total spillovers experience simultaneous surges, the increased magnitude in spillover differs considerably among sectors. The financial, energy and industrial sectors experience a significantly higher rise in total spillover during the worst part of the crisis while the rest of the studied sectors increased moderately, barely reaching 40-50%. These dynamics can be attributed to the fact that expectations of increased liquidity risk are transferred to the most interdependent sectors in the index. Additionally, the dynamic and static analyses of the directional and net pairwise spillovers suggest that spillover among sectors represents a limited part on the shock transmission within the market.

Strikingly, the transmitted and received spillovers for all sectors evolve in a similar fashion to the total market liquidity spillover. We have two potential hypotheses to reconcile the joint evolution of all sectors. First, the systematic directional spillover evolution potentially defines liquidity spillover as a market-wide phenomenon rather than a sectorial one. Second, it is possible that liquidity spillover in the stock market are influenced by external factors that not included in our study.

Finally, total and net pairwise liquidity spillovers for every sector do not subside to their pre-crisis level. This new paradigm indicates that, all things equal, liquidity shocks are more likely to propagate inside the market. Thus, the risk of liquidity crunch in the stock market is considerably higher than it

was before the crisis. At this point, we can only suggest potential causes for this persistent increase in liquidity interdependence among our stocks. First, the increased use of algorithmic trading for cross-asset learning is a potential explanation for this new liquidity spillover regime. At the same time, liquidity risk has gained considerable attention since the last crisis and it is possible that market participants then started to be increasingly concerned with liquidityinduced risks.

Volatility

In terms of volatility spillover, our analysis clearly identified the energy and financial sectors as predominant players in the transmission of shocks. One may argue that these sectors presumably act as the prominent transmission mechanisms of shocks coming from financial/economics and energy news, through portfolio rebalancing mechanisms. Until the major break in 2008, the sectorlevel total and net pairwise spillovers remained stable in most of the sectors, except in the financial and energy sectors. To some extent, these dynamics suggest that market systemic risk is mostly dictated by financial and energy news, unsurprisingly. Finally, most sectors do not subside to their pre-crisis level in terms of net pairwise total market and sector-level spillovers. These findings, despite the all-time low level of the VIX, underscore the market feverishness toward potential news. We highlight that the perceived level of risk is low, but risk transmission is likely to propagate among stocks with more velocity than before the events of 2007. Also, the telecommunication services and utilities sectors exhibit the lowest level of directional and pairwise spillovers. These two industries are commonly referred to as anti-cyclical. They offer a hedge over the market fluctuations. In this sense, it is natural for the directional spillover received and transmitted by and from these two sectors to be low.

We interpret the high level of intra-sector interdependence combined with a low net pairwise and directional spillover among sectors as an indication that sector-specific shocks are more likely to be contained within themselves. This points out that some sectors are more likely to suffer from risk propagation from within. Then, a sector-specific liquidity shock in one these sectors is likely to be contained within the sector, which increases the risk of experiencing a vicious feed-back loop in subsets of the market that have limited propagation potential with other sectors. This high level of intra-sector interdependence coupled with low transmission channels among sectors may be explained by portfolio managers allocation restriction between sectors, which forces managers to keep positions in risky sectors.

8 Conclusion

We applied Diebold and Yilmazs spillover framework, covering a large set of variables listed on the S&P 500, and extended our analysis to liquidity variables. Our empirical experiment on daily stock data, divided based on the Global Industry Classification Standard (GICS), provides ample evidence in support of the market interdependence at the disaggregate sectorial level. We confirmed with the general patterns observed for both liquidity and volatility that the theoretical framework arguing that liquidity spillovers are dependent upon the market risk level. However, our results demonstrate that sector-level total and directional spillovers differed dramatically between volatility and liquidity, which suggests that tracking both variables for risk monitoring would provide additional information in terms of risk management. Also, we notice that both variables experience a substantial increase in their net pairwise spillover level, an argument in favour of a more interdependent market within and among sectors. Finally, we introduce a new technique enabling researchers to handle large sets of data in the analysis of the market spillover. This new methodology could be extended to an even more diversified set of data. We believe that research including data on the fix income, commodity and foreign exchange market would benefit from that approach.

For future research, we have two potential developments. First, it might be meaningful to distinguish between liquidity positive (inflows) and negative (outflows) shocks between the stock market and the fix income market to capture the effect of the central bank action on stock market liquidity. During the last financial crisis, a major part of the problem was caused by liquidity dry-ups in specific subsets of the market that ultimately spread. While this research demonstrates that transmission channels among sectors show no clear patterns, it is of major interest to investigate the asymmetry of liquidity spillover in term of inflows/outflows. Including other types of assets in the

analysis, especially the debt market, may allow researchers to target the liquidity transmission effect of central bank action of the stock market. These findings can improve our understanding of liquidity propagation mechanisms and help fund managers to better track risky assets.

Finally, potential research developments may enable researchers to simultaneously examine inter-market and intra-market spillover. In this sense, studying the macro and micro relationship between the stocks market with the commodities, the fix income market and the foreign exchange market could be of great interest. Our work highlights that sectors share similarities in their general evolution, although there also exist key divergences in regime changes. Investigating these relationships in a richer context may help depict a better portray of the spillover dynamics.

9 Appendix

9.1 Data Cleaning

A diligent cleaning routine is of prime necessity for volatility estimators using high frequency data. When applicable, we used the procedure developed by Barndorff-Nielsen et al. (2008) to clean the high frequency data.

9.2 Break Identification

We used Bai-Perrons break test to segment our total volatility spillover results of the S&P 500 into different periods. Bai-Perron performed an endogenous break detection to identify an unknown number of breaks in the studied series. We tested for changes in the mean of the index over the entire sample assuming a naive model with only one constant: a regressor. At the 1%, 5% and 10% confidence level, using the sequential optimization method, we found that both liquidity and volatility total market spillover series experienced breaks concordant with the events of September 2008. We performed robustness checks using window sizes of 5%, 10% and 15%.

The following section provides the intuition behind the test. We can consider m endogenous structural breaks in the return series, which can be written as the following set of equations:

$$
r_t = \alpha_1 + \epsilon_t, t = 1, ..., T_1
$$

$$
r_t = \alpha_2 + \epsilon_t, t = T_1, ..., T_2
$$

$$
r_t = \alpha_m + \epsilon_t, t = T_{m-1}, \dots, T_m
$$

...

where $\alpha_1,\,\alpha_2,...,\,\alpha_m$ are the means of the index for each regime. Bai-Perron (2003) returned the estimated break dates resulting from the minimization process of the sum of squared residuals by choosing the optimal number of breaks,"m". For the return series, it can be expressed as:

$$
(\hat{T}_1, \hat{T}_2, ..., \hat{T}_m) = argmin_{T_1, ..., T_m} SSR_T(T_1, T_2, ..., T_m)
$$
\n(21)

Where
$$
SSR_T(T_1, T_2, ..., T_m) = \sum_{i=1}^{m} \sum_{t=T_{i-1}+1}^{T_i} (r_t - \alpha_i)^2
$$
 (22)

9.3 10 largest market capitalization of each GICS Sectors in the S&P 500

Table 1: Stocks included in the small Spillover experiment (30 largest S&P 500 companies) are in bold.

9.4 Robustness Test Results

9.4.1 95% Confidence Interval of the Total Market Volatility Spillover, for $VAR(1)$ and 10% rolling window size

9.4.2 95% Confidence Interval of the Total Market Liquidity Spillover, for $VAR(1)$ and 10% rolling window size

9.4.3 Rolling Window Size and Lag sensitivity

Figure 10: $VAR(1)$ and $VAR(2)$ with 5% rolling window size

Figure 13: Blue = $VAR(1)$, Black = $VAR(2)$

9.4.4 Sample Size sensitivity

Figure 16: Blue = large sample (93 stocks), Black = small sample (30 stocks)

9.5 Result figures

9.5.1 Full Sample, static Spillover

GICS sector	Cons. Disc.	Cons. Stap.	Energy	Fin.	Health	Indus.	Info. Tech.	Mat.	Teleco.	Uti.	Received
Cons. Disc.	79.8	1.3	2.4	4.6	1.7	4.7	2.5	1.6	0.1	1.2	20.1
Cons. Stap.	2.0	85.3	1.5	2.7	1.4	3.0	$1.6\,$	1.0	0.1	1.4	14.7
Energy	2.8	$1.1\,$	79.8	4.4	1.6	4.8	2.5	$1.6\,$	0.1	1.2	20.1
Fin.	3.2	1.2	2.4	81.8	1.7	4.3	2.3	$1.6\,$	0.1	1.4	18.2
Health	2.3	1.1	1.7	3.2	83.7	3.6	2.0	1.1	0.1	1.2	16.3
Indus.	3.6	1.3	2.7	4.6	1.9	79.9	2.8	1.7	0.1	1.4	20.1
Info. Tech.	2.6	1.2	2.2	3.7	1.6	4.6	81.6	1.3	0.1	1.0	18.3
Mat.	3.2	1.2	2.7	4.7	1.8	4.9	2.5	77.5	0.2	1.3	22.5
Teleco.	2.3	1.3	2.0	3.2	1.4	3.1	$1.6\,$	1.0	82.8	1.4	17.3
Uti.	1.5	1.1	$1.5\,$	2.4	1.2	2.7	1.3	0.8	0.1	87.3	12.6
Transmitted	23.5	10.8	19.1	33.5	14.3	35.7	19.1	11.7	$1.0\,$	11.5	Total: 18%

Table 3: Volatility Spillover Table (%), Full Sample

9.6 Liquidity Spillover

9.6.1 Total Spillover

9.6.2 Directional and Net Spillover

Figure 19: Each graphic, lettered (a) to (j), represent the net pairwise spillover between all the sectors and the sector in the subtitle.

9.7 Volatility Spillover

9.7.1 Total Spillover

60

 $\overline{40}$

20 \circ

 20

 $\frac{1}{2}$ $\overline{\varphi}$

 20

 $\frac{10}{2}$ \ddot{a} ω \circ ϕ \ddot{a}

 \overline{a}

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 \circ

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 \supseteq

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 \sim

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 $\ddot{\Omega}$

9.7.3 Pairwise Spillover

Figure 22: Each graphic, lettered (a) to (j), represent the net pairwise spillover between all the sectors and the sector in the subtitle.

9.7.4 S&P 500 Index returns

9.7.5 Quantitative Easing

U.S. Federal Reserve: Treasury and Mortgage-Backed Securities Held
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