A Study of Counterparty Credit Risk and Credit Value Adjustment

by

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Abstract

In this study, I examine the importance of Counterparty Credit Risk (CCR) in financial risk management, and the role played by Credit Value Adjustment (CVA) when pricing and hedging CCR. Regulatory frameworks and ways of mitigating for CCR are introduced. I derive and explain different types of CVA formula including generalized unilateral CVA formula, with/without wrong-way risk, with collateral and netting, and Bilateral CVA formula. I discuss the key components of CVA for the purpose of hedging CCR. Static hedging and dynamic hedging are explained using examples. The credit default swap (CDS) is introduced as the key product in the hedging of CCR. I then discuss the CDS risks which are specific to CCR.

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Contents

1 Introduction

Over the last two decades, financial risk management has experienced revolutionary changes. The changes were mostly triggered by the collapse of some large financial institutions that had inadequate risk management, for example, a few well-known ones, Lehman Brothers (2008), WorldCom (2002), Enron (2001), Long Term Capital Management (1998), and Barings (1995). The losses from underestimating financial risks are huge, and are very likely not only to destroy the financial institution itself, but to create a chain effect to harm the whole well-being of the host country's financial system. Without the government rescue, there would be more disasters of huge losses raised from insufficient risk management. American International Group Inc. is an example, who required over US$100 billion from the US government to cover its losses.

Financial risks are normally recognized as market risk, liquidity risk, operational risk, and credit risk etc. More recently, Counterparty Credit Risk (CCR) is considered as one of the key financial risk factors. CCR is defined as the risk that the counterparty to a financial contract will default prior to the expiration of a trade and will not therefore make the current and future payments required by the contract. It is one of the most complex risks to deal with in risk management. To fully understand CCR, it requires a good knowledge of financial risks since it is driven by the combination of all possible risks. The large and growing over-the-counter (OTC) derivatives market is subject to CCR. As a common knowledge in financial field, OTC derivatives is a powerful instrument. However, dealing with derivatives potentially can cause huge losses without caution. This also makes CCR the key component of risk management.

Now CCR is one of the hottest topics within the financial markets with much interest around Credit Value Adjustment (CVA)as the market value of CCR. Credit Value Adjustment (CVA) is defined as the difference between the risk-free portfolio value and the true portfolio value that takes into account the possibility of a counterparty's default. CVA is one of the most important parts in the Basel Accords. The Basel Accords refer to the banking supervision recommendations on banking regulations (Basel I, Basel II and Basel III) issued by the Basel Committee on Banking Supervision (BCBS). Under the current Basel II standards, banks are subject to a capital charge primarily to cover the losses arising from the actual default of a counterparty of an over-the-counter (OTC) derivative contract. Under the proposed Basel III framework, the capital charge will be enhanced by a new charge, called the Credit Valuation Adjustment Risk Capital Charge.

Many years before 2007, most market participants had underestimated the magnitude of CCR because of the implicit of "too big to fail" assumption. The consequence of the disastrous derivatives and financial risk management is the credit crisis in 2007 and its onwards negative waves throughout the global financial markets. A typical example is the collapse of Lehman Brothers (2008). Many banks and other financial institutions use CVA as the measurement of the market value of CCR. This leads to another important aspect of current risk management - how to correctly implement CVA system. Based on Basel Committee on Banking Supervision (2009), roughly two-thirds of CCR losses were due to CVA losses and only one-third were due to actual defaults. Now many banks start to price and actively hedge CVA.

The plan for the remainder of this paper is as follows. In Section 2, I provide some reviews of recent literature concerning CCR and CVA. In Section 3, I introduce in detail of the background knowledge needed to understand, quantify, and mange CCR. Section 4 analyzes different types of CVA formula including no wrong-way risk, with wrong-way risk, with collateral, with netting, and Bilateral CVA formula. Section 5 discusses the main components to be hedged in the CVA including default probability, recovery rate, exposure, cross-dependencies, and term structure. In Section 6, I conclude and summarize some of the key areas for future development and improvement.

2 Literature Review

There has been no shortage of relevant previous literature on CCR and CVA. Pykhtin and Zhu (2006) presented the treatment of CCR of OTC derivatives under Basel II. They showed a framework for calculating the minimum capital requirements for CCR. They also provided a modelling framework for calculating expected exposure (EE) profiles. Their paper discussed a general approach to capturing collateralized exposure. This approach can be used to compute the collateral at a future time as a function of uncollateralized exposure at another date[1]. Pykhtin and Zhu introduced two methods to compute collateral. One is a straightforward approach using Monte Carlo simulation to determine the amount of transferred collateral. However, this approach requires more computation time. Thus, they suggested another method which is simple and fast by avoiding the simulation of exposure at the secondary dates. In 2007 Pykhtin and Zhu provided a guide to modelling credit exposure and CCR. They defined CVA as the price of CCR and discussed approaches to its calculation. Pykhtin (2011) developed a general framework for CCR capital requirement according to both Basel II and Basel III. The paper showed the importance for banks to calculate a CVA capital charge. Two applications of this framework were introduced under the market risk approach and the credit risk approach. All the works done by Pykhtin and Zhu motivate my study on the concept of CVA risk capital charge for CCR under Basel II and Basel III.

Arora, Gandhi, and Longstaff (2012) argued that CCR had become one of the highest-profile risks facing by participants in the financial market. Instead of developing a framework like in the case of Pykhtin and Zhu, their paper examined CCR pricing using an extensive proprietary data set of contemporaneous credit default swap (CDS) transaction prices and quotes on the same underlying firm. The result identified directly how CCR affected the prices of the credit derivatives. Under the assumption of CDS liabilities are unsecured, they found that the price of CCR seems

[1]Collateral agreement is discussed in section 3.

to be too small to be explained by models, but, seems consistent with the standard market practice of requiring full collateralization. Strong evidence were found that more and more firms began to price CCR after 2007 crisis. They also analyzed different behaviours of CDS dealers in the US, Europe, and Asia. The result showed that CDS dealers adjusted pricing of CCR based on the industry to which the underlying firm belongs. In sum, they showed the importance of correctly pricing CCR and discussed the changes in CCR pricing before and after 2007 financial crisis. This also motivates me to examine about how CCR is actually priced. In section 4, I will discuss CCR pricing in details.

Saunders (2010) focused in detail on CVA calculation by demonstrating some examples. The unilateral and bilateral CVA calculation were introduced as the market value of CCR. He also explained the role played by CVA in risk management. Numerical examples and analytic approximations were provided in computing CVA. Saunders used example from Pykhtin and Rosen (2010) to demonstrate a general result that exposure correlations only matter if netting and collateral are considered when calculating the contribution of each instrument in a counterparty portfolio to the CVA. CVA contributions (can be positive or negative) are increasing in mean exposure, exposure volatility and correlation. Saunders also discussed the hedging of CVA from the credit risk perspective and market risk perspective. He used example from Gregory (2009) to present CVA as spread and hedging. Issues of hedging CVA were listed in his study. Some key points are multiple sources of risk, bilateral CVA property, and uncertainty in recoveries, etc. This leads to my study of main components of CVA to be hedged such as recovery rate, exposure, and term structure.

Kjaer (2011) extended some results from past papers and books such as Pykhtin and Zhu (2007), Brigo (2008), and Gregory (2009) to propose a generalized CVA formula and extended it to derive some commonly used cases such as the regular unilateral CVA and the regular bilateral CVA. The result could be used for unified calculation of different types of CVAs. Kjaer used an example to show that the CVA could vary widely depending on different types of agreement between counterparties. The result showed that the generalized CVA could be hedged in the fully independent credit model. It depended on the fact that the hedging of the counterparty risk-free value can be funded at the risk-free interest rate. In addition, under certain situations, CCR could be accounted by discounting with the risky curve of the counterparty. This result made it relatively easier to calculate the CVA of portfolios with positive cash flows.

Not like all works mentioned above, Alavian, Ding, Laudicina, and Whitehead (2010) used another approach to demonstrate a basic and introductory review of the components to the CVA which were derived by decomposing a single portfolio's value into a set of binary states. These states were a set of market values of the portfolio (can be positive or negative), default states (default or no default) and recoveries (recover the recovery amount or not). As they mentioned in the article there were some issues for the application of the CVA formula such as strong assumptions on the goodness of the input values, the interdependence of the processes, and the fact that in reality there are many portfolios instead of a single portfolio. However, the purpose of their paper is to encourage and motivate the readers for future work and to represent opportunities for further developments.

3 Counterparty Credit Risk

3.1 Defining CCR

CCR is defined as the risk that the counterparty to a financial contract will default prior to the expiration of a trade and will not therefore make the current and future payments required by the contract. Compare to other types of financial risks, such as market risk, credit risk, operational risk, and liquidity risk, CCR is commonly considered as the most complicated one.

Sometimes, people treat CCR as a special form of credit risk, since for both risks the cause of loss is due to the obligor's default. However, there are two main features that make CCR different from common forms of credit risk. First, the counterparty credit exposure is uncertain. Counterparty credit exposure is the cost of replacing the contract if the counterparty defaults, which is the maximum of the contract's market value and zero (assuming zero recovery value). The unpredictable changes in the contract market value over time as the market moves make the current credit exposure known with certainty, but future credit exposure is uncertain. Second, CCR has bilateral nature, which is due to the fact that the contract market value can change sign and either counterparty can default. To better understand CCR, it requires knowledge of all financial risks, since it is important and necessary to identify the nature of CCR by examining the interaction of different types of financial risks. Therefore, it is crucial to understand and manage CCR for the future health and growth of derivatives products and worldwide financial markets.

3.2 CCR and Credit Derivatives

Not all the derivatives products are subject to CCR. Typically, exchange-traded derivatives are not affected by CCR, because one function of the exchange is to guarantee the contract payment by the derivative to the counterparties[2]. CCR affects financial products whose contacts are privately negotiated between counterparties. There are two main categories of such products over-the-counter (OTC) derivatives and security financing transactions (SFT).

The market for OTC derivatives has grown dramatically in the last decade. Table 1 shows the notional amounts outstanding in global OTC derivatives market during the period of 2002 to 2012. According to the semi-annual OTC derivatives statistics release from the Bank for International Settlements (BIS)[3], OTC derivatives notional amounts outstanding totalled US$633 trillion at end-December 2012 compare to US$128 trillion at end-June 2002, and it reached the peak at US$707 trillion

[2] After 2007 financial crisis, there are more topics about what if the exchange itself fails to cover its obligations. There are more regulations on exchange now, for example, it is required to hold sufficient collateral in hand to cover all its obligations.

[3] The publication is available on the BIS website (www.bis.org).

in 2011. By examining three-year period of growth in OTC derivatives, the rate of growth of notional amounts outstanding was 32% annually in the period 2004-2007. During post-crisis period 2007-2010, the rate of growth was 5% annually. The notional amounts outstanding of OTC derivatives market at end-December 2010 was US$583 trillion, 15% higher than the level recorded in the 2007 survey.

Global OTC derivatives market[1]
Notional amounts outstanding, in billions of US dollars

	Foreign Exchange Contracts	Interest rate contracts	Equity linked contracts	Commodity contracts	Credit default swaps[2]	Others[3]	Grand Total
2012/H2	67,358	489,703	6,251	2,587	25,069	41,611	632,579
2012/H1	66,645	494,427	6,313	2,994	26,931	42,057	639,366
2011/H2	63,349	504,117	5,982	3,091	28,626	42,610	647,777
2011/H1	64,698	553,240	6,841	3,197	32,409	46,498	706,884
2010/H2	57,796	465,260	5,635	2,922	29,898	39,536	601,046
2010/H1	53,153	451,831	6,260	2,852	30,261	38,329	582,685
2009/H2	49,181	449,875	5,937	2,944	32,693	63,270	603,900
2009/H1	48,732	437,228	6,584	3,619	36,098	62,291	594,553
2008/H2	50,042	432,657	6,471	4,427	41,883	62,667	598,147
2008/H1	62,983	458,304	10,177	13,229	57,403	81,719	683,814
2007/H2	56,238	393,138	8,469	8,455	58,244	71,194	595,738
2007/H1	48,645	347,312	8,590	7,567	42,581	61,713	516,407
2006/H2	40,271	291,582	7,488	7,115	28,650	39,740	414,845
2006/H1	38,127	262,526	6,782	6,394	20,352	35,997	370,178
2005/H2	31,364	211,970	5,793	5,434	13,908	29,199	297,670
2005/H1	31,081	204,795	4,551	2,940	10,211	27,915	281,493
2004/H2	29,289	190,502	4,385	1,443	6,396	25,879	257,894
2004/H1	26,997	164,626	4,521	1,270		22,644	220,058
2003/H2	24,475	141,991	3,787	1,406		25,508	197,167
2003/H1	22,071	121,799	2,799	1,040		21,949	169,658
2002/H2	18,448	101,658	2,309	923		18,328	141,665
2002/H1	18,068	89,955	2,214	777		16,496	127,509

[1] Source: Bank for International Settlements OTC derivatives statistics.
[2] By the request from the Committee on the Global Financial System (CGFS), the BIS was initiating the publication of statistics on the market for credit default swaps (CDS) in 2004.
[3] Estimated positions of non-regular reporting institutions.

Table 1: Global OTC derivatives market: Notional amounts outstanding

The notional amounts outstanding provides a measure of market size and a useful information on the structure of the OTC derivatives market but should not be interpreted as a measure of CCR, rather it is a useful measure of the aggregate level of activity. On the other hand, gross market values are defined as the sums of the absolute values of all open contracts with either positive or negative replacement values evaluated at market prices prevailing on the reporting date. Thus, if a dealer's outstanding contracts were settled immediately, the gross market values would represent claims on counterparties. Therefore, gross market values provide a more accurate measure of the scale of financial risk transfer taking place in OTC derivatives market. Table 2 shows the BIS OTC derivatives statistics for gross market values during the period of 2002-2012.

Global OTC derivatives market[1] Gross market value, in billions of US dollars							
	Foreign Exchange Contracts	Interest rate contracts	Equity linked contracts	Commodity contracts	Credit default swaps[2]	Others[3]	Grand Total
2012/H2	2,304	18,833	605	358	848	1,792	24,740
2012/H1	2,217	19,113	645	390	1,187	1,840	25,392
2011/H2	2,555	20,001	679	481	1,586	1,976	27,278
2011/H1	2,336	13,244	708	471	1,345	1,414	19,518
2010/H2	2,482	14,746	648	526	1,351	1,543	21,296
2010/H1	2,544	17,533	706	458	1,666	1,789	24,697
2009/H2	2,070	14,020	708	545	1,801	2,398	21,542
2009/H1	2,470	15,478	879	682	2,973	2,816	25,298
2008/H2	4,084	20,087	1,112	955	5,116	3,927	35,281
2008/H1	2,262	9,263	1,146	2,209	3,192	2,303	20,375
2007/H2	1,807	7,177	1,142	1,898	2,020	1,790	15,834
2007/H1	1,345	6,063	1,116	636	721	1,259	11,140
2006/H2	1,266	4,826	853	667	470	1,609	9,691
2006/H1	1,136	5,445	671	718	294	1,685	9,949
2005/H2	997	5,397	582	871	243	1,659	9,749
2005/H1	1,141	6,699	382	376	188	1,818	10,605
2004/H2	1,546	5,417	498	169	133	1,613	9,377
2004/H1	867	3,951	294	166		1,116	6,395
2003/H2	1,301	4,328	274	128		957	6,987
2003/H1	996	5,459	260	100		1,081	7,896
2002/H2	881	4,266	255	86		871	6,360
2002/H1	1,052	2,467	243	79		609	4,450

[1] Source: Bank for International Settlements OTC derivatives statistics.
[2] By the request from the Committee on the Global Financial System (CGFS), the BIS was initiating the publication of statistics on the market for credit default swaps (CDS) in 2004.
[3] Estimated positions of non-regular reporting institutions.

Table 2: Global OTC derivatives market: Gross market value

Despite the crisis related declines, the recent survey from BIS and the International Swap and Derivative Association (ISDA) shows that the size of OTC derivatives market is unlikely to fall dramatically and would still have a trend to grow. The financial entities do need a solid understanding and proper measuring and managing of CCR.

3.3 CCR and Regulation

The Basel Accords (Basel I 1988, II 2004, and III 2010) refer to the banking supervision Accords (recommendations on banking regulations) issued by the Basel Committee on Banking Supervision (BCBS). BCBS was established as the Committee on Banking Regulations and Supervisory Practices by the central-bank Governors of the group of ten countries at end of 1974, now it has 27 member countries. The Committee does not possess any formal supranational supervisory authority. Rather, it formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement them through detailed arrangements.

The most recent Basel III proposals were developed in response to the deficiencies in financial regulation revealed by the 2007 financial crisis[4]. Compare to Basel I and II (Basel II is the one that implemented by most central banks at current time), there are some changes focused on CCR such as promoting more integrated management of CCR, adding the CVA-risk due to deterioration in counterparty's credit rating, strengthening the capital requirements and risk management of counterparty credit exposures, providing additional incentives to move OTC derivative contracts to central counterparties, and raising CCR management standards by including wrong-way risk. The Basel III proposals for counterparty credit risk contain significant enhancements related to CVA and in particular the needs to account for variation in CVA with a regulatory CVA Value at Risk (VaR) computation.

As I mentioned early, the Basel Committee does not possess any formal supervisory authority, and its conclusions do not, and were never intended to, have legal force. There are difficulties to implement Basel III as an international agreement such as facing different cultures, different structural models, complexities of public policy, and existing regulations. In fact BCBS has received a lot of interpretation questions since the publication of Basel III regulatory frameworks especially for the CCR section.

[4]The Basel III document is available at www.bis.org/publ/bcbs189.pdf.

Following this, the Committee has released four sets of document of frequently asked questions that related to the CCR sections of Basel III rules text. There are also some critics, for example, the American Banker's Association argues that the Basel III proposals, if implemented, would hurt small banks by increasing their capital holdings dramatically and then would hurt economic growth. As a result of those implementing difficulties and critics, the BCBS extended Basel III full implementation schedule to 2019. But those facts just reflect the caution of financial entities when they are dealing with CCR. All the discussion above shows that banks and financial institutions demand a solid understanding, proper measuring and managing CCR. In the following, I will discuss some key points regarding to mitigating CCR.

3.4 Mitigating CCR

Due to the complexities of CCR, mitigating of counterparty risk is not an easy task. The following discussions are in general sense, and we make no particular assumptions on the type of financial institution and the type of financial instrument facing CCR.

Mitigating CCR is a task based on dealing with the key components of CCR including credit exposure, default probability, and expected loss given default (or equivalently to deal with recovery rate). These three key components can be treated separately. However, two or more components are needed to be combined under certain consideration. For example, a CCR with the combination of a small exposure and a large default probability might be considered preferable to the one with a larger exposure and a smaller underlying default probability. The main issue is focused on reducing current and potential future credit exposures. For a long time before 2007 crisis, the most obvious way to mitigate CCR was to trade only with higher credit quality counterparties. With the failure of institutions used to be considered as "too big to fail", this area has proved to be a deadly weakness in spite of overall strength, that can actually or potentially lead to downfall. Here I will introduce two commonly used methods to reduce credit exposure: netting agreements and margin agreements (collateralization).

A netting agreement is a legally binding contract between two counterparties that allows aggregation of transactions between these counterparties in the event of default. The purpose of a netting agreement is to control the exposure to a counterparty across two or more transactions. It is specific to transactions that may take both positive and negative mark-to-market (MtM) values (such as in the case of derivatives). The maximum loss for the surviving counterparty is equal to the sum of the contract-level credit exposures. With netting the credit exposure from all transactions is the maximum of the net portfolio or zero. The following example illustrates the impact of simple bilateral netting on CCR. Suppose there are two trades between counterparties A and B. Institution A has MtM values (+5) and (-4), so B has MtM values (-5) and (+4). If there is default (assume zero recovery rate), the loss will be (+5) without netting and (+1) with netting for A. For B, the loss will be (+4) with no netting and zero with netting. It is clear that netting can reduce CCR for both parties. Note that the example here is the simplest bilateral form of netting for illustration purpose. In practice, there are technical issues such as multilateral netting, negative or positive initial MtM, and correlation between the MtM values. Also, it is necessary to consider other legal and operational risks created by netting.

I just showed that netting can significantly reduce CCR exposure but still limit trading activities with certain counterparties, for example, maybe no institutions want to trade with less credit worthy counterparties. The use of margining (sometimes called collateral) provides the further mitigation of CCR and allowing the market to include less credit sounds counterparties. A margin agreement is a legal collateral support document signed between counterparties which contains the terms and conditions under which they will operate. For instance, there are two counterparties A and B. Under a bilateral collateral agreement, base on margin call frequency (daily margining is becoming a market standard) both counterparties mark all positions to market and check their overall netting value. If counterparty A's netting portfolio value is positive and exceeds B's threshold[5], A will check the terms and conditions of the agreement to calculate the incremental exposure will be collateralized, and B is required to post the collateral (either cash or other securities). Thus, the collateral can be used to reduce the CCR exposure in the event of B's default, since it is not needed to return the collateral in this case. As the market changes, if the excess part of uncollateralized exposure over the threshold decreases, some amount of posted collateral will be returned by the agreement. According to each counterparty's credit rating, there is a specified minimum transfer amount defined as the smallest amount of collateral to be transferred with margin call. This is used to reduce the frequency of insignificant collateral transfers.

The example described above shows that how collateralization can mitigate further CCR exposure beyond netting. Meanwhile, it can potentially create other risks such as operational risk, market risk, and legal issues. It requires implementing with cautions. But there is a market trend that many counterparties will not trade on an uncollateralized basis. Margining agreement still is the most widely used method on CCR mitigation. The use of collateral has increased dramatically since 2003, and it reached a peak in 2008 at almost US$4.0 trillion with a growth rate of 86 percent. According to ISDA Margin Survey 2013, the amount of collateral in circulation in the non-cleared OTC derivatives market was US$3.70 trillion by the end of 2012, and 73.7 percent of all OTC derivatives trades were subject to collateral agreements.

Besides the two CCR mitigating methods just mentioned, there are other ways can be used to mitigate CCR such as dealing with central counterparties and hedging. All methods can reduce CCR, but with additional operational cost and following by other financial risks such as liquidity risk and operational risk. Thus, mitigating CCR can potentially be counterproductive without caution. This point was explained in detail in Gregory (2010).

This section introduced some key aspects of CCR including definition, CCR with

[5] A threshold is a level of exposure below which collateral will not be called, therefore represents an amount of uncollateralized exposure.

OTC derivative market, and ways to mitigate CCR. Those points can be helpful to better understand CCR. The next plausible question will be how to accurately price CCR. In the following section, I will analyze some generalized formulas for pricing CCR. This involves another important concept in risk management - Credit Value Adjustment (CVA) which is the market value of CCR.

4 Pricing CCR

The main idea of correctly pricing CCR with a given counterparty is to calculate the market value of the risk of all outstanding positions. It requires combining credit exposure and default probability with a given counterparty. Credit Value Adjustment (CVA) is defined as the difference between the risk-free portfolio value and the true portfolio value that takes into account the possibility of a counterparty's default. Thus, CVA is the market value of CCR, and calculating CVA is the key for pricing CCR.

Prior to 2007 crisis, some banks had started to price and hedge CVA. However, the standard practice for many financial institutions was to mark the portfolios to market without considering CVA. The cash flows were simply discounted by LIBOR curve as risk-free values. Since the crisis, treatment of CVA has changed dramatically in financial market. Banks currently calculate and actively hedge CVA using different models for pricing CCR. Recently, more and more banks either set up central CVA desk or implement CVA desk in their main business units. Managing CVA is already a part of their trading books including daily MtM, active hedging and enforce market risk limits.

In section 3, I mentioned the bilateral nature of CCR. This makes the calculation of CVA more difficult. There are many models dealing with pricing CCR involved CVA calculation. For example, Pykhtin and Zhu (2007) define CVA as the price of CCR and discuss approaches to its calculation. Alavian, Ding, Whitehead and Laudicina (2010) provide an overview of CVA within the context of collateralized and uncollateralized trading relationships. Saunders (2010) introduces calculating CVA, CVA and wrong-way risk, and hedging CVA. Kjaer (2011) derives a generalized standard bilateral CVA as well as non-standard CVAs. In section 4.1, I will derive a standard unilateral CVA formula where CVA is calculated as an adjustment to the risk-free value of derivatives positions within the netting set to account for CCR.

4.1 A general unilateral CVA formula

In this section, I will derive an equation for a netted set of derivatives positions as an example of general formula for CVA by using the following notations from Gregory (2010). Let $V(t,T)$ be the risk-free MtM value of the netted portfolio at time t with maturity date T. For notational simplicity, we assume that the MtM value is already including discounting. Thus, for any s such that $t < s \leqslant T$, the future uncertain MtM value of the portfolio is $V(s,T)$. Denote $V_r(t,T)$ as the associated risky value with counterparty's default time as τ. Based on these notations, the associated CVA term we are looking for can be expressed as:

$$\mathrm{CVA}(t,T) = V(t,T) - V_r(t,T) \tag{1}$$

The idea is to find the risky value $V_r(t,T)$ as an expression of risk-free value $V(t,T)$ using risk-neutral measure which is the commonly used pricing method for derivatives. Using the indicator function:

$$\mathbb{I}(\theta) \equiv \begin{cases} 1, & if\ \theta = true \\ 0, & if\ \theta = false \end{cases} \tag{2}$$

the risky value is:

$$\begin{aligned} V_r(t,T) = \mathbb{E}^Q \big[& \mathbb{I}(\tau > T)V(t,T) \\ & + \mathbb{I}(\tau \leq T)V(t,\tau) + \mathbb{I}(\tau \leq T)\left(RV(\tau,T)^+ + V(\tau,T)^-\right)\big] \end{aligned} \tag{3}$$

where $V(\tau,T)^+ = max\{V(\tau,T),0\}$, $V(\tau,T)^- = min\{V(\tau,T),0\}$, and $\mathbb{E}^Q[\cdot]$ denotes the risk-neutral expectation.

The first term of equation (3) captures the risky value without counterparty's default. It is just the risk-free value, $V(t,T)$, since $\mathbb{I}(\tau > T) = 1$ when the counterparty does not default before T. The last two terms count the portfolio value when the counterparty does default at time τ ($t < \tau \leq T$). The second term is the value of the portfolio that would be paid by the counterparty before the default time τ. The last term is the payoff at default. It involves two possibilities. If the MtM portfolio value at time $\tau, V(\tau,T)$, is positive then $RV(\tau,T)^+$ is the recovery fraction of the risk-free value will be received by the surviving counterparty, where R is the recovery rate defined as the proportion of a bad debt that can be recovered. If the MtM value is negative then $V(\tau,T)^-$ is the amount need to be paid from the default counterparty. By using the relationship $V(\tau,T) = V(\tau,T)^+ + V(\tau,T)^-$ we have:

$$\begin{aligned} V_r(t,T) = \mathbb{E}^Q [&\mathbb{I}(\tau > T)V(t,T) \\ &+ \mathbb{I}(\tau \leq T)V(t,\tau) + \mathbb{I}(\tau \leq T)\left((R-1)V(\tau,T)^+ + V(\tau,T)\right)\big] \end{aligned} \tag{4}$$

Now, using the fact that $V(t,T) \equiv V(t,\tau) + V(\tau,T)$ and re-arranging equation (4) we obtain:

$$\begin{aligned} V_r(t,T) = \mathbb{E}^Q [&\mathbb{I}(\tau > T)V(t,T) \\ &+ \mathbb{I}(\tau \leq T)V(t,T) + \mathbb{I}(\tau \leq T)\left((R-1)V(\tau,T)^+\right)\big] \end{aligned} \tag{5}$$

Notice that the first two terms of equation (5) can be combined together, since $V(t,T) \equiv \mathbb{I}(\tau > T)V(t,T) + \mathbb{I}(\tau \leq T)V(t,T)$, finally we have the general formula for CVA:

$$\begin{aligned} \text{CVA}(t,T) &= V(t,T) - V_r(t,T) \\ &= \mathbb{E}^Q \left[(1-R)\mathbb{I}(\tau \leq T)V(\tau,T)^+\right] \end{aligned} \tag{6}$$

As I mentioned at the beginning of this section, here we assume that the future uncertain MtM value includes discounting. Without this assumption, equation (6) becomes:

$$\text{CVA}(t,T) = \mathbb{E}^Q \left[(1-R)\mathbb{I}(\tau \leq T)V(\tau,T)^+ \frac{B(t)}{B(\tau)}\right] \tag{7}$$

where $B(t)$ is the value of money-market account at time t.

Equation (6) is a simple unilateral CVA formula given by the risk-neutral expectation of the discounted loss. It is a general framework for calculating CVA for pricing

CCR. However, the complex features of many financial instruments make it difficult to calculate CVA for pricing CCR in practice. In the following parts of this section, I will introduce some practical CVA formulas that can be used for pricing CCR.

4.2 Without/with wrong-way risk

The equations (6) and (7) in section 4.1 can be used for valuing derivatives transactions when we make a simplifying assumption of no dependence between exposures and default events, this also refers to no wrong-way risk assumption. I will show that under this assumption how the CVA calculation could be expressed as the expected exposure (EE) multiplied by default probability, where the EE is defined as the average of only the positive MtM values in the future. But with the presence of wrong-way risk, it is not possible to use the multiplication to calculate CVA since the co-dependence between exposures and default probability. Wrong-way risk refers to the tendency for exposures to be high when default probability is high and vice versa. As an unfavourable dependence, wrong-way risk can cause a substantial increase in CCR. It is also possible to have "right-way risk" where exposures tend to be low when default probability is high. Compare to wrong-way risk, right-way risk is considered as a favourable one since it will reduce CCR. In the following, I will extend equation (6) to derive a new one that can be used in the case of no wrong-way risk.

In order to derive CVA formula without wrong-way risk, I use the approach of survival probability and default probability which were developed by Jarrow and Turnbull (1992, 1995). Let $\mathrm{SP}(t,T)$ be the risk-neutral survival probability in the time interval between t and T, where $\mathrm{SP}(t,T) = \mathbb{E}\left[\mathbb{I}(\tau > T)\right]$ with a negative slope. The negative slope means that the survival probability will decrease as time increases. Then, $1 - \mathrm{SP}(t,T)$ is the risk-neutral default probability. With a constant recovery rate R, the equation (6) can be rewritten as expression (8) as below:

$$\mathrm{CVA}(t,T) = (1-R)\mathbb{E}^Q\left[\mathbb{I}(\tau \leq T)V(\tau,T)^+\right] \tag{8}$$

Since the default time τ can be any point in the interval (t,T), we can calculate the term inside the expectation by using integration over (t, T). We have:

$$\text{CVA}(t,T) = -(1-R)\mathbb{E}^Q\left[\int_t^T B(t,s)\tilde{V}(s,T)^+ d(\text{SP}(t,s))\right] \tag{9}$$

where $B(t,s)$ is the risk-free discount factor and $\tilde{V}(s,T) = V(s,T)|\tau = s$ denotes the future exposure, $V(s,T)$, knowing that default of the counterparty has happened at $\tau = s$. The negative sign in the front is because of the negative slope of $\text{SP}(t,s)$. Under assumption of independence between exposure and default, knowing counterparty's default has no effect on the expected value of the underlying positions. Therefore, without wrong-way risk we have $\tilde{V}(s,T) = V(s,T)$. In the case of $\tilde{V}(s,T) \neq V(s,T)$, usually referred as with wrong-way risk, will be discussed later. Now, equations (9) can be rewritten as:

$$\begin{aligned}\text{CVA}(t,T) &= -(1-R)\mathbb{E}^Q\left[\int_t^T B(t,s)V(s,T)^+ d(\text{SP}(t,s))\right] \\ &= -(1-R)\int_t^T B(t,s)\mathbb{E}^Q\left[V(s,T)^+\right] d(\text{SP}(t,s))\end{aligned} \tag{10}$$

The second line of the above equation is due to the assumption of no wrong-way risk (no co-dependence between exposures and default probability), also the discount factor and survival probabilities are deterministic, so we can take them out of the expectation operator. Thus, we only have one term inside the expectation which is $\mathbb{E}^Q[V(s,T)^+]$. This is just the EE under risk-neutral measure, which can be denoted as $\text{EE}(s,T) = \mathbb{E}^Q[V(s,T)^+]$. Finally, we have the formula for CVA without wrong-way risk:

$$\text{CVA}(t,T) = -(1-R)\int_t^T B(t,s)\text{EE}(s,T)d(\text{SP}(t,s)) \tag{11}$$

Note here that, if we want to show in more general form and assuming without wrong-way risk, equation (11) should be like:

$$\text{CVA}(t,T) = -(1-R)\mathbb{E}^Q\left[\int_t^T B(t,s)\text{EE}(s,T)d(\text{SP}(t,s))\right] \tag{12}$$

However, the exact discounted EE is very difficult to calculate in practice. In stead, there is a commonly used practical framework to simulate the exposure at a fixed set of simulation dates such as:

$$\mathrm{CVA}(t,T) \approx (1-R)\sum_{j=1}^{N} B(t,t_j)\mathrm{EE}(t,t_j)\mathrm{DP}(t_{j-1},t_j) \tag{13}$$

where $\mathrm{DP}(t_{j-1},t_j) = \mathrm{SP}(t,t_{j-1}) - \mathrm{SP}(t,t_j)$ is the marginal default probability in the interval between date t_{j+1} and t_j. The idea is to divide the time interval (t,T) into N periods denoted by $(t_0, t_1, ..., t_N)$ such that $t_0 = t, ..., t_N = T$. Note that there is no negative sign in the front of equation (13) since it has default probability instead of survival probability. This gives a good approximation for CVA calculation. It requires reasonably large N, typically $N = 12$ per year.

With the presence of wrong-way risk, the unfavourable dependence between exposure and counterparty's default event will increase CVA. However, how to adjust the equation (13) to calculate CVA under wrong-way risk is not a easy job. One approach is to adjust the EE or default probability upwards to reflect the wrong-way risk. It is hard to quantify the magnitude of the adjustment in CVA formula (13). Gregory (2010) gives some examples of measuring wrong-way risk including forward trade, foreign exchange (FX), and CDS examples. There is another theoretical approach for calculating CVA with wrong-way risk. It requires to examine the economic relationship between exposure and counterparty's default event. However, it is extremely hard to define and quantify CVA in this method.

4.3 With collateral and netting

Equation (13) can be considered as stand-alone CVA formula without wrong-way risk for a given transaction. As discussed in section 3.4, mitigating CCR is an important part of risk management. Therefore, for any practical CVA calculation, we need to take risk mitigation into account such as collateral and netting. In the following, I will discuss how to modify equation (13) when considering the impact of collateral and netting[6].

[6] The rest of this section is still following the assumption of no wrong-way risk.

First, consider equation (13) for the case of CVA formula with collateral. There is no influence of collateral on the default probability of the counterparty. The only part needed to be adjusted is the expected exposure. Gregory (2010) gives a good and detailed example to compute the CVA where various different collateral assumptions are considered. Four cases of CVA are calculated by using same formula with different EE profiles. The cases are no collateral, collateralization with a 10-day remargin period, the addition of a minimum transfer amount, and the addition of a threshold. Four different CVA calculations are compared. The results show that the impact of collateral reduces the CVA by over five times in the case of collateralization with 10-day remargin period compared to the case without collateral. The cases of minimum transfer amount and threshold also can reduce the CVA by over half. The result also shows that increasing in minimum transfer amount and threshold will increase the CVA towards the case of no collateral.

In the case of CVA formula with netting, incremental CVA is needed to be calculated. Incremental CVA is defined as the difference between the CVA of a portfolio or netting set with and without a given trade. This implies that pricing the new trade is the key to consider the influence of netting on the CVA before and after a new trade, that is:

$$\Delta \text{CVA} = \text{CVA}(NS, trade) - \text{CVA}(NS) = V(trade) \tag{14}$$

where NS (netting set) denotes the set of netted trades with a counterparty and $V(trade)$ is the risk-free value of the new trade, then $\text{CVA}(NS)$ is the CVA for current trades within the netting set and $\text{CVA}(NS, trade)$ is the CVA including the new trade in the netting set. The first equality in the above equation is just the change in CVA. The second equality shows that the change can be represented by the pricing of the new trade with considering the netting impact. This is because any increase in CCR should be charged for by netting, i.e. there should be no change in the risky value of all trades when adding a new trade. Therefore, we need to have:

$$V_r(NS, trade) = V_r(NS) \tag{15}$$

By equation (1), we can rewrite this as:

$$V(NS, trade) - \text{CVA}(NS, trade) = V(NS) - \text{CVA}(NS) \tag{16}$$

Using linearity of the risk-free values of the netted values, the above equation becomes:

$$V(NS) + V(trade) - \text{CVA}(NS, trade) = V(NS) - \text{CVA}(NS) \tag{17}$$

cancelling the terms, we have the second equality in equation (14):

$$V(trade) = \text{CVA}(NS, trade) - \text{CVA}(NS) \tag{18}$$

The result shows that the price (the risk-free value) of a new trade should at least offside the change in CVA due to the CCR of the trade. Now, we can modify the general formula (13) to get the formula for incremental CVA:

$$\Delta\text{CVA} \approx (1 - R) \sum_{j=1}^{N} B(t, t_j) \Delta\text{EE}(t, t_j) \text{DP}(t_{j-1}, t_j) \tag{19}$$

where $\Delta\text{EE}(t, t_j)$ is the only difference between equations (13) and (19) which is the term for incremental change in EE within each time interval caused by the new trade. For any financial instrument, there are some facts when comparing stand-alone and incremental CVA. First, with netting the incremental CVA cannot be higher than the stand-alone CVA. The reason is that netting could not increase exposure as we discussed in section 3.4. Secondly, the incremental CVA can be negative because of hedging effect. For example, if there is a strong negative correlation between the existing trade and the new trade, then the new trade may lead to a loss from the reduction in CVA. Finally, if a new trade has stronger correlation with the existing portfolio (netting set), then it will have a higher incremental CVA. In other words, if a new trade is strongly correlated with existing exposures, then there will be a small change in incremental CVA.

4.4 Bilateral CVA formula

So far we talked about unilateral CVA formula in general. We discussed what is the difference between without and with wrong-way risk. We also showed how to modify the generalized formula with consideration of collateral and netting. Those formulas are most commonly used before 2007 crisis. CVA as pricing for CCR is the charge for a transaction received by the better credit quality counterparty. In most cases, CVA is charged by the bank that trading with corporate counterparties according to the credit quality of the corporate and the expected exposures in the transaction. Post 2007 crisis, "too big to fail" does not hold any more. When pricing CCR, one need to consider the bilateral feature of CCR. CVA calculation needs to be adjusted to consider the possibility that the bank itself may default. For the rest of this section, I will discuss how to derive an expression for Bilateral CVA (BCVA).

Consider the case of trading between a bank and a corporate counterparty, I will use subscript "B" and "C" to stand for the bank and the corporate counterparty. Same as previous section, let $V(t,T)$ be the risk-free MtM value of the netted portfolio at time t with maturity date T. Denote $V_r(t,T)$ as the associated risky value with counterparty's default time as τ_C and the default time of the bank as τ_B. Following these notations, the actual default time τ_d is equal to $min\{\tau_C, \tau_B\}$, i.e. the default time of either the bank or the counterparty that defaults first. R_B and R_C will be the recovery rates for the bank and counterparty respectively. Using the same risk-neutral measure and notations as in section 4.1, the risky value is:

$$\begin{aligned} V_r(t,T) =& \mathbb{E}^Q \left[\mathbb{I}(\tau_d > T) V(t,T) \right. \\ & + \mathbb{I}(\tau_d \le T) V(t,\tau_d) \\ & + \mathbb{I}(\tau_d \le T)\mathbb{I}(\tau_d = \tau_C) \left(R_C V(\tau_d,T)^+ + V(\tau_d,T)^- \right) \\ & \left. + \mathbb{I}(\tau_d \le T)\mathbb{I}(\tau_d = \tau_B) \left(R_B V(\tau_d,T)^- + V(\tau_d,T)^+ \right) \right] \end{aligned} \tag{20}$$

The above expression follows the same logic as in section 4.1. The first line gives the risky value such that neither the bank nor the counterparty will default. The second term is the portfolio value would be paid before any default event. The third line captures the payoff if the counterparty defaults first before the maturity time T. All those terms are similar to the unilateral CVA formula structure. The last line of equation (20) is the extra term compared with unilateral CVA case. It gives the payoff when the bank defaults first before the maturity time T. Notice that the bank and the counterparty have exactly opposite payoff structure with their own default time and recovery rate. Our goal is to find the BCVA formula as:

$$\text{BCVA}(t,T) = V(t,T) - V_r(t,T) \tag{21}$$

Same as unilateral CVA case, using relationship $V(\tau_d,T) = V(\tau_d,T)^+ + V(\tau_d,T)^-$ to replace $V(\tau_d,T)^-$ and $V(\tau_d,T)^+$ in the third and fourth line of equation (20), we have:

$$\begin{aligned} V_r(t,T) =& \mathbb{E}^Q \left[\mathbb{I}(\tau_d > T) V(t,T) \right. \\ & + \mathbb{I}(\tau_d \le T) V(t,\tau_d) \\ & + \mathbb{I}(\tau_d \le T) \left[\mathbb{I}(\tau_d = \tau_C) V(\tau_d,T) + \mathbb{I}(\tau_d = \tau_B) V(\tau_d,T) \right] \\ & + \mathbb{I}(\tau_d \le T) \mathbb{I}(\tau_d = \tau_C) \left(R_C V(\tau_d,T)^+ - V(\tau_d,T)^+ \right) \\ & \left. + \mathbb{I}(\tau_d \le T) \mathbb{I}(\tau_d = \tau_B) \left(R_B V(\tau_d,T)^- - V(\tau_d,T)^- \right) \right] \end{aligned} \tag{22}$$

Again, as before using the fact that $V(t,T) = V(t,\tau_d) + V(\tau_d,T)$. Then the second and third terms can be combined to get $\mathbb{I}(\tau_d \le T)V(t,T)$. Thus the above equation can be expressed as:

$$\begin{aligned} V_r(t,T) =& \mathbb{E}^Q \left[\mathbb{I}(\tau_d > T) V(t,T) + \mathbb{I}(\tau_d \le T) V(t,T) \right. \\ & + \mathbb{I}(\tau_d \le T) \mathbb{I}(\tau_d = \tau_C) \left(R_C V(\tau_d,T)^+ - V(\tau_d,T)^+ \right) \\ & \left. + \mathbb{I}(\tau_d \le T) \mathbb{I}(\tau_d = \tau_B) \left(R_B V(\tau_d,T)^- - V(\tau_d,T)^- \right) \right] \end{aligned} \tag{23}$$

Notice that $\mathbb{I}(\tau_d > T)V(t,T) + \mathbb{I}(\tau_d \le T)V(t,T) = V(t,T)$ and rearrange terms, we finally have BCVA formula as:

$$\begin{aligned} \text{BCVA}(t,T) &= V(t,T) - V_r(t,T) \\ &= \mathbb{E}^Q \left[\mathbb{I}(\tau_d \le T) \mathbb{I}(\tau_d = \tau_C)(1 - R_C) V(\tau_d,T)^+ \right. \\ & \qquad \left. + \mathbb{I}(\tau_d \le T) \mathbb{I}(\tau_d = \tau_B)(1 - R_B) V(\tau_d,T)^- \right] \end{aligned} \tag{24}$$

If we follow the same methodology with no wrong-way risk assumption as in section 4.2, furthermore, under assumption of no default at the same time between the bank and the counterparty, we will have similar practical BCVA formula as unilateral case. It can be approximated as:

$$\begin{aligned} \text{BCVA}(t,T) \approx (1-R_C)\sum_{j=1}^{N} B(t,t_j)\text{EE}(t,t_j)S_B(\text{DP}_C(t_{j-1},t_j)) \\ -(1-R_B)\sum_{j=1}^{N} B(t,t_j)\text{NEE}(t,t_j)S_C(\text{DP}_B(t_{j-1},t_j)) \end{aligned} \tag{25}$$

where S_B and S_C denote the survival probabilities of the bank and counterparty respectively; DP_B and DP_C are the default probabilities of the bank and counterparty. Other terms follow the same meaning of equation (13) (unilateral case), the term $\text{NEE}(\cdot)$ denotes the negative EE from the point of view of the counterparty. The first term in equation (25) is quiet similar with unilateral case. The difference comes from the multiplicative factor which is the bank's survival probability times the counterparty's default probability. It means that if the bank defaults before the counterparty, there is no need for the bank to concern about any loss from counterparty's default. The second term in above equation works like a mirror image of the first one which is from the point of view of the counterparty.

From the above argument, we can say that the BCVA equation (25) is a more generalized case from unilateral case with the possibility that the bank itself may default. We can find some facts and implications of the BCVA formula. First, equation (25) is symmetric. It implies that if two counterparties agree on the BCVA formula, the total amount of CCR in the market would be zero, i.e. the prices of CCR from two counterparties should have same absolute value with opposite sign. Second, the BCVA can be negative which implies that it is possible to have a higher risky value of a portfolio than the risk-free value.

5 Hedging CCR

CVA as the pricing of CCR is introduced in section 4. There is an obvious question followed previous discussion: what is the hedging of CCR associated with CVA. In this section, I will discuss some topics dealing with the hedging of CVA.

5.1 Components of CVA

Recently, more banks start to actively hedge CVA. There are some motivations for financial institutions to hedge CVA. When trading with counterparties, banks want to be as flexible as possible in the types and sizes of transactions. In order to increase earnings, financial institutions need a highly competitive pricing. An institution always tries to avoid potential huge losses from highly volatile CVA. Those goals can be achieved by actively and efficiently hedge CVA.

Same as the case of CVA calculation, hedging CVA involves multiple market variables such as recovery rate, expected exposure, default probability, and term structure. Sometimes the correlation between some variables is needed to be considered for hedging CVA. For example, the correlation between expected exposure and default probability is a key component of hedging CVA with wrong-way risk. For a netted portfolio, there may be a large number of CVA terms to be hedged. Thus, the primary task of hedging CVA is to locate which components of CVA are the ones should be hedged and which ones can be ignored. In practice, this is not a easy task to do. In fact hedging of CVA probably can never be perfect. This is not only because many different market variables get involved, but also some variables simply just cannot be hedged. There are two main reasons make the inability to hedge some variables. First, there may be no such financial instrument available in the market can be used to hedge. Second, the hedging costs are too high to implement. So far we learned the importance of identifying key components of CVA that should be hedged and some issues facing by hedging CVA. Next, I will give a brief introduction of the key components of CVA to be hedged.

For the purpose of hedging, the main components of CVA to be hedged are recovery rate, expected exposure, default probability, cross-dependencies, and term structure. Default event such as default probability and recovery rate usually can be hedged by using credit default swaps (CDSs). A credit default swap is a financial swap agreement that the seller of the CDS will compensate the buyer in the event of a loan default or other credit event. CDS is the mostly taken form of credit derivatives which transfers the default risk from one party to another. For example, in a single-name CDS, the protection buyer pays a premium to the protection seller for a certain notional amount of debt of a reference entity[7]. For the exposure term, one should realize that all the variables involved exposure are needed to be hedged. For example, both foreign exchange risk and foreign exchange volatility should be considered for a foreign exchange forward contract. Cross-dependencies and term structure are the components normally being ignored in practice. However, if they have a significant influence on the sensitivity of CVA, then it is necessary to consider these terms in the hedging of CVA.

All discussions above are in general sense. There are more hedging strategies need to be taken into account. Gregory (2010) examines all of the possible components in detail from the hedging perspective. Next sections will discuss two hedging strategies: static hedging and dynamic hedging.

5.2 Static hedging

Static hedging by buying CDS protection is considered as a reasonable hedging strategy for traditional debt securities such as bonds and loans. Generally speaking, it is an efficient hedging for the credit risk of fixed rate bonds. The idea of this kind of strategy is quiet straightforward. For example in the hedging of a bond, the protection buyer holds the bond with certain face value and buys the same notional of CDS protection referencing the bond issuer. If there is any credit event from the bond

[7] The reference entity may be a corporate, a sovereign or any other form of legal entity which has incurred debt.

issuer, the default loss is hedged by the CDS protection payoff . The CDS payoff will be the bond face value less recovery.

As we mentioned before, there is almost no perfect hedge in the real world. There are issues of the above example of hedging the default risk in a bond with a CDS. An obvious one is the effect from the movement of interest rate in the market. With a CDS protection settlement on a fixed notional value, the CDS hedge notional is the par value of the bond. Thus, the hedge will lead to a loss if the bond is trading above par prior to bond issuer's default. A bond trading below par will have the opposite effect. However, the static hedging of a fixed rate bond still be considered as a reasonable one, since fixed rate bond is most unlikely to trade more than 5-10% away from its par value.

The static hedging also can be used to hedge CCR on a derivative. This is more complicated than the simple standard CDS case due to the uncertainty of the expected exposure at default time. In addition, the hedge is based on the worst case exposure. These points make the static hedging sometimes inefficient and costly. To hedge the uncertain credit exposure at default time from the perspective of hedging CCR, a contingent credit default swap (CCDS) is developed. A CCDS works exactly as standard CDS with one difference that the notional amount of protection is referenced to the MtM value of a specific transaction. Theoretically, a CCDS can be used to perfectly hedge the CCR on a derivative since the notional amount of protection can be linked to the exposure. However, there are practical issues make it not a popular use in hedging CCR. First, it involves a complex documentation. All the details of the transaction must be specified including maturity date, underlying, payment frequency, etc. Second, a CCDS deals with a single transaction. Netting is not available to reduce exposure. Thus, a CCDS is most likely to overhedge the trade. Finally, to hedge CCDS itself involves same amount of work as hedging CCR, since a CCDS simply just transfer the CCR from one party to another.

Let us reconsider the simple standard CDS protection on a bond as a static hedge.

If we want to take interest rate effect into account, the hedge position should be adjusted as the bond price moves with interest rate change, i.e. moves away from the par value. It involves another hedging strategy known as dynamic hedging. It is explained in succeeding section.

5.3 Dynamic hedging

We learned that static hedging can be used under certain conditions. Now, let us discuss some aspects of dynamic hedging. One must realize that there will be other risks followed by dynamic hedging. Think about a dynamic hedging with CDS protection example, there will be another risk need to be considered which is annuity risk. In the past, CDS contracts normally traded with a running spread and no upfront payments. This will lead to a mismatch between the duration of the bond and CDS contract. In other words, there is a mismatch between the delta hedge and default hedge[8]. Therefore, the notional amount of CDS protection used to hedge the bond issuer's CDS premium will not hedge the default risk of the bond. Nowadays, trade with fixed premium and upfront payments in CDS contracts are used to solve the above problem. But this kind of dynamic hedging still does not work perfectly unless the bond spread and the fixed CDS premium are the same.

The above example is relatively simple comparing to other exotics risks associated with more complex derivatives which should be hedged dynamically. For the hedging of exotics risks in CVA, it must be achieved by implementing dynamic hedging carefully. For example, to hedge the credit spread of a 5-year interest rate swap, it cannot be achieved simply by a 5-year CDS protection. Even with fixed CDS premium, a movement in interest rate requires to adjust the credit hedge significantly. The constant readjustment makes the hedging difficult and costly in practice.

Another consideration for the hedging of the CVA is the jump-to-default risk. The jump-to-default risk refers to the risk associated with potential severe credit

[8] A delta hedge is a simple type of hedge to reduce (hedge) the risk associated with the price movements in the underlying asset.

deterioration and /or a sudden credit event. The hedging requires at least two different CDS protection contracts to hedge both the credit spread and jump-to-default risk. Jump-to-default risk can potentially cause a large loss if it is not properly hedged.

All the cases discussed so far are using CDS as the hedging instrument. This is because CDS was designed and used as a key product in the hedging of CCR and credit risk in general. However, CDSs themselves can also have significant CCR due to wrong-way risk and along with other risks. They should be implemented with caution for the hedging of CCR. Next section will briefly discuss some CDS risks that are worth mentioning.

5.4 CDS risks in hedging CCR

The credit derivatives market has grown dramatically due to the highly demand by financial institutions for a means of hedging and diversifying credit risks. Most credit derivatives take the form of the credit default swap (CDS). CDS is considered as the key product in the hedging of CCR and credit risk in general. Since it is widely used in financial market, it is important to be aware of the risks associated with CDS contracts.

As an instrument for the hedging of CCR, CDS itself is subject to CCR as well. Consider the case in which a CDS protection buyer buys credit protection on a reference firm from a protection seller. If the reference firm underlying the CDS contract has an unanticipated default, then the protection seller could suddenly face a huge loss. This could potentially drive the protection seller into distress. The protection buyer may not receive the promised payment specified in the CDS contract. The problem caused by the default of protection seller is one of the sources of CCR in the CDS market.

Another CDS associated risk is recovery risk which is specific to CCR. Consider a case of hedge of a risky bond with a CDS contract. With physical settlement[9], the protection buyer has the option to find the cheapest bond to deliver to the protection seller. The cheaper bond has a lower recovery on the hedge than on the contract. This will lead to a gain for the institution hedging CCR. This "cheapest-to-deliver option" problem is a concern that the protection buyer may push the reference firm into bankruptcy in order to make a gain. Another problem caused by recovery risk is called "delivery squeeze". Again with physical settlement, CDS protection buyers need to buy bonds and deliver them to protection sellers for the purpose of hedging CCR. Without well defined deliverable obligations, the strong demand of the bond will push up the price. The higher bond price will most likely cause a loss for the institution hedging CCR. Finally, there is a potential mismatch between settled recovery and actual recovery[10]. Almost any hedging of CCR is facing this kind of problem. The mismatch comes from the immediate settlement of the CDS contract and much slower recovery claim process. The more complex of a recovery process, the more uncertain of the actual recovery. For example, if the reference firm goes into bankruptcy, the process may take a significant time.

There are more CDS related risks than the issues we discussed. The point here is that using CDS as an instrument for the hedging of CCR must be implemented with careful considerations. Any mistake may lead to a large economic loss and result in a systemic problem in financial market.

6 Conclusion and future works

During the last twenty years, the collapse of some large financial institutions brought market participants' attention on financial risk management. Especially after the 2007

[9]In physical settlement, the protection buyer will deliver to the protection seller defaulted securities of the reference entity with a par value equal to the notional amount of the CDS contract. In return, the protection seller must make a payment of par in cash.

[10]Settle recovery is achieved from a CDS auction cash settlement or from trading the debt security in the market. Actual recovery is the final recovery paid on the debt following a bankruptcy or similar process.

crisis, counterparty credit risk (CCR) is commonly thought as the key financial risk. To fully understand and manage CCR, it requires the knowledge of all other financial risks such as market risk, liquidity risk, operational risk, and credit risk. In this paper, I introduced some regulatory frameworks and different aspects of mitigating CCR. The aim is to help market participants to better understand the importance of CCR and methods can be used to deal with CCR in practice.

Correctly pricing and hedging CCR become the key component of the risk management. There have been many models proposed for pricing CCR. I first introduced the credit value adjustment (CVA) as the measure (pricing) of CCR. Then, briefly discussed some past works such as Pykhtin and Zhu (2007) and Kjaer (2011). I used risk-neutral pricing CCR. A generalized unilateral CVA formula is derived and then be modified to derive other types of CVA formula such as with/without wrong-way risk, with collateral and netting, and Bilateral CVA formula. The formula is generalized meaning that it is not specified to certain financial instrument. It helps to give a general idea about how to price CCR in risk-neutral measurement.

For the hedging of CCR, I discussed some key components of CVA should be hedged and the importance of identifying those components. The credit default swap (CDS) was introduced as the key product in the hedging of CCR. I used a CDS protection contract on a risky bond example to demonstrate two hedging strategies: static hedging and dynamic hedging. Some practical hedging issues were explained along with the example. I discussed some of the CDS risks which are specific to the hedging of CCR. The purpose is to help market participants to better understand the hedging of CCR.

Finally, future research is required to better understand and manage CCR. Financial risk management has experienced revolutionary changes over the last two decades. It is most likely that the CCR also will experience a similar revolution. In financial market, the CCR is still facing lots of challenges such as implementing CVA system, controlling credit exposure, and the need of well designed regulatory frameworks.

There is much work to do to build and maintain a healthy financial market.

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