Financial Stability Under Heterogeneous Expectations

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1 Abstract

This paper proposes a modest modification to an existing asset pricing model to evaluate how different specifications of heterogeneous expectations affect financial stability. The model is then applied to demonstrate whether financial innovation, in the form of credit insurance, can accentuate instability. Results show that stability outcomes in a CDS economy are dominated by those of a reference economy for different specifications of probability functions defining the beliefs of market participants. Finally, the model is applied to evaluate whether reserve requirements can generate more stable outcomes. Results suggest that this is unlikely to occur at realistically low levels of required reserves.

2 Acknowledgements

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Contents

3 Introduction

The financial crisis of 2008 and the ensuing economic malaise have captured the attention of academics and policymakers alike and have cast a spotlight upon the need to protect financial systems against the adverse effects of destabilizing disturbances. While the magnitude of the crisis has elicited public policy reforms in many countries, debate and disagreement persist. The lack of consensus highlights the need for further inquiry. In fact, to adequately evaluate the merits of proposed solutions, it would be helpful to first identify an ultimate cause of instability. This paper amends an existing theoretical model to explain how technological innovations accentuate heterogeneous beliefs, which, in turn, undermine financial stability in the absence of appropriate regulatory safeguards. Thus, unlike popular debate which emphasizes lurid tales of excess and crony capitalism, this paper identifies a seemingly benign but insidious vehicle of instability - innovation.

Section 2 provides a general overview of important theoretical literature pertaining to equilibrium asset pricing under heterogeneous expectations. Special attention is given to Fostel and Geanakoplos (2012), which outlines a simple multi-period model that compares prices across different levels of financial innovation. Neave (2013) is also discussed in detail as it evaluates how results in the Fostel and Geanakoplos paper can be influenced by changes in the distribution of agents' expectations. Section 3 proposes a modest extension to the Fostel and Geanakoplos model by introducing institutions that accept deposits as a means of financing asset acquisitions. Like the individual agents in the original setup, these banks are also characterized by heterogeneous beliefs. An analytical procedure, first proposed by Neave (2013), is then applied in Section 4 to evaluate how the frequency of bank insolvency varies as the distribution of beliefs is adjusted. Finally, the same approach is used to compare stability under different levels of financial innovation and required reserves. Defaults in a simple leverage economy are shown to be less prevalent compared with an economy where trading of CDS securities is observed. This is primarily due to the fact that credit insurance allows institutions to tailor their payoff profiles to their individual beliefs while individual asset holdings become skewed toward favoured states of the world that vary from institution to institution. The realized state of nature then bestows large payoffs to some firms and large losses to others. Increasingly stringent reserve requirements are shown to exert either a stabilizing or destabilizing influence, depending upon specific starting and final values of enforced reserves. Section 5 offers concluding remarks.

4 Literature Review

The purpose of this section is to document relevant insights gleaned from previous research analyzing how heterogeneous expectations and/or financial innovation influence stability. Many of the ideas presented below are used in subsequent sections to develop a model of the financial sector. For this reason, particular emphasis is given to Fostel and Geanakoplos (2012) as well as Neave (2013) since these studies pertain most closely to the subject matter of Section 3.

Miller (1977)

Edward Miller offers a simple and highly intuitive introduction to the influence of heterogeneous expectations on asset prices and returns. Contrary to what is assumed by orthodox asset pricing models, Miller asserts that probability distributions of future returns are not common knowledge, implying that agents must develop their own subjective estimates of the likelihood of future outcomes.[1] Thus, the real world has a tendency to be characterized by Knightian uncertainty which entails heterogeneity of agents' beliefs. Miller takes such heterogeneity as given and assumes that there exists

[1] Edward Miller, "Risk, Uncertainty and Divergence of Opinion," *The Journal of Finance* 24, no.4 (1977), 1155.

a market for N shares of a given equity asset against which no short-selling is permitted.[2] The model is characterized by arbitrarily many utility-maximizing agents, each of which has sufficient funds to purchase one share of the risky asset.[3] It follows directly that the equilibrium equity price, R, will adjust to ensure that the number of agents that purchase shares will be exactly equal to N, the fixed supply of shares outstanding. So long as the number of shareholders, N, is less than half the total number of prospective investors, the share price will exceed the mean of all appraised values generated by the entire pool of agents.[4] If the mean of all such valuations is an unbiased measure of share value, then it is possible for assets to be overpriced relative to underlying fundamentals. Thus, a firm that wishes to optimize the value of its equity will invest in projects financed exclusively by an optimistic minority of potential investors even when such projects are not economical. Moreover, when actual shareholders represent a minority of all possible investors, any accentuation in the heterogeneity of agents' beliefs will increase the observed equity price implying that the extent of overvaluation is positively related to diverging forecasts of future outcomes.

This model can be applied to explain observed peculiarities of empirical stock return data. For example, it is well documented that average equity returns are negatively related to the corresponding variance of returns, which contradicts what the traditional CAPM model would predict. In Miller's model, however, risk entails uncertainty since agents can only form subjective judgments of state-contingent probabilities, allowing the prices of risky stocks to be determined by the most optimistic class of potential investors.[5] As time passes outcomes are observed attenuating both risk and uncertainty, leading to price depreciation and underperformance.[6] The same

[2]Miller, "Risk, Uncertainty and Divergence of Opinion," 1151-1152.
[3]Miller, "Risk, Uncertainty and Divergence of Opinion," 1151.
[4]Miller, "Risk, Uncertainty and Divergence of Opinion," 1153.
[5]Miller, "Risk, Uncertainty and Divergence of Opinion," 1155.
[6]Miller, "Risk, Uncertainty and Divergence of Opinion," 1155-1156.

logic can also be applied to explain the medium-term underperformance of new equity issues.[7] Divergence of investor sentiment pertaining to a particular stock should be greatest around the initial listing date. As uncertainty diminishes with time, so should the share price, ceteris paribus.

It's worth noting that the model's results continue to hold when amendments are made to its initial assumptions. For example, when investors are assumed to be risk-averse, instead of risk-neutral, they will consider both the expected return and variance of all possible investments and tailor their individual portfolio to fall on the efficient frontier at a point that reaches their highest possible indifference curve.[8] Of course, under heterogeneous expectations, agents no longer evaluate efficiency from an objective perspective, leading each individual to choose a unique portfolio. For example, a given stock may be purchased by optimists but lie inside the efficient frontier of pessimists and be ignored by this latter class of investors.[9] Thus, as before, it is possible that only a small portion of agents will purchase a given security.[10]

The model can also be extended to account for short selling. Since short sales generate additional supply of a particular security, they tend to exert downward pressure on market prices.[11] Given that riskier stocks are likely to be characterized by large numbers of very pessimistic evaluations in addition to many optimistic evaluations, short selling initiated by pessimists should mitigate the upward pressure on market prices originating from the most optimistic agents. In reality, this offsetting effect may only be modest, however, since it is not profitable to short-sell shares that are expected to both underperform and maintain a positive return. This should limit the prevalence of short-selling, a claim which is consistent with empirical data showing that the number of short positions is usually small relative to the number of shares

[7]Miller, "Risk, Uncertainty and Divergence of Opinion," 1156.
[8]Miller, "Risk, Uncertainty and Divergence of Opinion," 1159.
[9]Miller, "Risk, Uncertainty and Divergence of Opinion," 1160.
[10]Miller, "Risk, Uncertainty and Divergence of Opinion," 1159-1160.
[11]Miller, "Risk, Uncertainty and Divergence of Opinion," 1160.

outstanding.[12] Thus, the model's original results remain valid.

Harrison and Kreps (1978)

Results approximating those of Miller (1977) can be obtained when agents' valuations of a given security are determined endogenously. Harrison and Kreps arrive at such results through a model in which risk-neutral, heterogeneous agents value equity shares by considering what other agents are willing to pay for the stock in question. The shares are assumed to generate future dividend payments that are announced one period in advance and are largely determined by the release of information that is inaccessible to agents at time $t = 0$.[13] Given heterogeneous priors, investors can be divided into groups based upon their beliefs regarding the nature of future disclosures of information, even when members within each group share the same beliefs, as assumed by Harrison and Kreps.[14] Different groups think that such revelations will favour different sequences of dividend payments, leading to diverging estimates of the fundamental value of the equity asset. Since investors are assumed to have infinite wealth and are able to purchase arbitrarily large equity stakes, the group of agents that assigns the largest estimated value to the stock will acquire the entire pool of shares. Together, arbitrage and complete markets ensure the equity stake will be valued by each class of investors as the discounted sum of future expected dividend payments.[15] Under incomplete and/or imperfect markets, however, the option of selling shares at a future date also has value. When these circumstances arise, agents' time $t = 0$ valuation of the security in question will be positively related, not only to the discounted stream of future dividends, but also to the discounted value of whatever payment they could receive for selling the security to a different class of

[12] Miller, "Risk, Uncertainty and Divergence of Opinion," 1162.

[13] J. Michael Harrison and David M. Kreps, "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," *The Quarterly Journal of Economics* 92, no. 2 (1978), 325-326.

[14] Harrison and Kreps, "Speculative Investor Behavior," 326.

[15] Harrison and Kreps, "Speculative Investor Behavior," 323.

investors in the future.[16] This can lead to speculative excess where the equilibrium equity price exceeds the appraised value made by any given agent that is unable to access resale markets.[17] Obviously, this can generate bubbles characterized by prices that depart substantially from fundamentals.

Fostel and Geanakoplos (2012)

Fostel and Geanakoplos develop a model that outlines how financial innovation can generate asset price bubbles under heterogeneous expectations. They apply their model to explain important developments relating to the recent financial crisis of 2008 and show that leverage, tranching, and securitization contributed to upward momentum in asset prices while the subsequent proliferation of credit default swaps likely triggered a precipitous fall in asset values.[18]

Under a general equilibrium model with two possible future states of nature, Fostel and Geanakoplos allow agents to trade two assets of different risk classes, each of which yields units of a consumption good at time $t = 1$.[19] The riskless asset, X, is akin to cash and provides a payoff of 1 unit in both states of the world while the risky asset, Y, generates a payoff of 1 unit in the upstate and $R < 1$ units in the downstate. Agents are risk-neutral and have heterogeneous beliefs regarding future outcome probabilities.[20] Each endowed with one unit of X and one unit of Y at $t = 0$, agents subsequently enter the marketplace to exchange assets and adjust their portfolio holdings to suit their individual beliefs.[21] Since beliefs are generated along a continuum, equilibrium is characterized by a marginal agent who is indifferent between holding the portfolio of an optimist and that of a pessimist.[22]

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[16] Harrison and Kreps, "Speculative Investor Behavior," 326.

[17] Harrison and Kreps, "Speculative Investor Behavior," 324.

[18] Ana Fostel and John Geanakoplos, "Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes," *American Economic Journal: Macroeconomics* 4, no. 1 (2012), 190.

[19] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 196.

[20] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 196-197.

[21] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 197.

[22] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 198.

Fostel and Geanakoplos model four separate economies each differentiated by their level of financial innovation. Agents in the first economy, in which there is no leverage, no tranching, and no trading of credit default swaps, are permitted to trade only their asset endowments.[23] In this economy, the marginal buyer is an agent who is indifferent between purchasing and selling the risky asset. Agents who are more optimistic than the marginal buyer will sell their initial endowments to purchase the risky asset exclusively, those who are less optimistic will purchase only the riskless asset. In either case agents will exchange their entire endowment of one asset in exchange for the other.

In the leverage economy, still characterized by an absence of tranching and credit insurance, agents can borrow funds from one another to finance the purchase of the risky asset, which also serves as collateral to secure the loan.[24] As before, optimists and pessimists are identified by comparing their beliefs to those of a marginal buyer who is indifferent between holding the two assets.[25] In equilibrium, optimists sell their endowment of the riskless asset and borrow funds to acquire additional units of the risky asset while pessimists liquidate their holdings of the risky asset, lend to optimists, and use whatever funds remain to purchase units of the riskless asset. Unlike the no-leverage economy, once agents can borrow against collateral the risky asset is absorbed by a small group of agents corresponding to those that are most optimistic.[26] Thus, the marginal buyer of this economy is more optimistic than the marginal buyer of the reference economy, meaning that the risky asset will certainly be more expensive. This seems intuitive since leverage should accentuate differences between upstate and downstate payoffs of equilibrium portfolios. In fact, buying the risky asset with borrowed money allows the purchaser to create an Arrow-Debreu

[23] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 201.
[24] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 202.
[25] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 202-203.
[26] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 203.

security which generates a payoff in the upstate only.[27] This position would be very appealing to an investor who is particularly optimistic. In conjunction with the additional purchasing power granted to agents by the ability to access loanable funds, this ensures that the risky asset issue will be absorbed by a small number of market participants representing the most optimistic contingent.[28]

Fostel and Geanakoplos next consider an economy in which the risky asset can be tranched directly by isolating the upstate and downstate payoff streams.[29] Equilibrium is now characterized by two marginal buyers and three classes of agents. The most optimistic group will purchase all units of the risky asset and sell the downstate tranche, the middle group will sell all their holdings of the risky asset in exchange for the riskless asset, while a third group will sell their entire asset endowment to purchase the downstate tranche exclusively. The tranching economy generates even higher asset prices than the leverage economy. Moreover, it is possible for the price of the risky asset to exceed the appraised value assigned to it by any given investor.[30] This can occur because tranching allows the owner of the risky asset to tailor payoff streams to the unique preferences of heterogeneous buyers. Since tranching of this type effectively creates two Arrow-Debreu securities, one for each state of nature, it allows for more complete market segmentation than was possible under leverage alone.

The fourth economy is characterized by the presence of a credit default swap which insures against all losses on the risky asset.[31] It is created by transferring the downstate payoff of the riskless asset to the purchaser. By acquiring $(1 - R)$ units of this security, the owner is protected against losses on the risky asset since the downstate payoff of the credit insurance, $1 - R$ in conjunction with the downstate payoff

[27]This is true since the downstate payoff of asset Y serves as collateral, payable to the lender

[28]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 203.

[29]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 204.

[30]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 205.

[31]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 206.

of the risky asset, R provides the purchaser with the same total payoff that would be obtained by holding one unit of the riskless asset alone. Moreover, since issuers of the CDS must post collateral in terms of units of the riskless asset, the insurance payment is guaranteed and the new financial innovation is akin to tranching cash.[32] In addition, payoff streams generated by the risky asset can continue to be isolated using the procedure outlined in the previous paragraph.[33] In equilibrium there is one marginal buyer compared to which all agents that are relatively optimistic will purchase all of the original two assets in the economy and sell all downstate tranches (including the credit default swap) while pessimists will liquidate their initial endowments to acquire the isolated downstate payoff streams.[34] The most striking result arising from this equilibrium is that the introduction of the CDS into the economy lowers the price of the risky asset.[35] This is due the fact that agents consider the tranched downstate asset stream and the credit default swap to be perfect substitutes. Thus tranching of the riskless security will increase its value for the same reason that tranching of the risky asset increased the value of Y. It follows that the introduction of the credit default swap will increase the relative value of X, which serves as the numeraire, decreasing the value of the risky asset in equilibrium.[36]

As Fostel and Geanakoplos explain, the findings of their model are entirely consistent with observed data arising from the recent financial crisis in the United States. Securitization, which was first introduced during the 1970s, in conjunction with tranching, allowed financial engineers to customize payoff streams to suit the specific needs of different buyers, raising the value of mortgage loan collateral.[37] These practices, which gained considerable popularity during the 1990s and early 2000s, preceded the adoption of credit default swaps, which first appeared during the 1990s

[32] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 213.
[33] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 206.
[34] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 207.
[35] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 213.
[36] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 213.
[37] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 190-194.

to insure holders of corporate and sovereign bonds.[38] In fact, it was not until the mid-2000s that credit default swaps were widely applied to mortgage securities, due to an incipient appreciation for default risks in the mortgage market.[39] Of course, the sudden collapse of the U.S. real estate market occurred in 2007, after the proliferation of trading in credit insurance. It therefore seems plausible that credit default swaps may have triggered this precipitous fall in asset prices, as the Fostel and Geanakoplos model predicts.[40]

Neave (2013)

Neave uses the framework developed by Fostel and Geanakoplos to evaluate how equilibrium results can be affected by changes in state-contingent payoffs and agents' probability functions. Again, agents are risk-neutral price-takers and are assumed to exhibit heterogeneous expectations.[41] They are each given an endowment of a particular asset which is then sold to procure funds used to purchase Arrow-Debreu securities.[42] As Fostel and Geanakoplos explain, such an Arrow-Debreu economy provides the same equilibrium outcomes as the CDS economy discussed above.[43]

In such a model, every equilibrium outcome is associated with a specific benchmark portfolio value which is used in subsequent comparative statics analysis. This portfolio consists of one unit each of an Arrow-Debreu upstate security and an Arrow-Debreu downstate security.[44] The portfolio's expected return, standard deviation, and Sharpe ratio can usually be computed if it is assumed that all states of nature are characterized by a given objective state-contingent probability. Neave assumes such a probability exists and is equal to 0.5 for each state.[45] Under a sample homogeneous

[38]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 191.

[39]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 191.

[40]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 190.

[41]Edwin Neave, "Heterogeneous Expectations And Financial Equilibrium: Comparative Statics Analyses," *First International Conference on Banking and Finance - Bali*, (2013),6-7.

[42]Edwin Neave, "Heterogeneous Expectations," 7-8.

[43]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 208.

[44]Edwin Neave, "Heterogeneous Expectations," 13.

[45]Edwin Neave, "Heterogeneous Expectations," 13-14.

expectations economy, the price of the upstate and downstate Arrow-Debreu securities (P_{ADU} and P_{ADD} respectively) are equal to one another, the expected return and volatility of the portfolio have a value of zero, and the Sharpe ratio is undefined.[46] Under heterogeneous expectations, P_{ADU} is not always equal to P_{ADD} and one class of investors will own securities characterized by negative expected returns. Any fall in the price of the upstate security when it is already less (more) than 0.5, will tend to increase (decrease) expected return, the Sharpe ratio measure, and volatility. This implies that substantial deviations away from the homogeneous expectations economy characterized by $P_{ADU} = P_{ADD}$ lead to increases in each of the three pertinent statistical measures. These findings are tested repeatedly throughout the paper by way of numerical analysis.

Neave first considers the effects of an upward shift in investors' probability functions.[47] This corresponds to an increase in all agents' level of optimism although estimates of future outcome probabilities are not always adjusted by an equal amount across all individuals.[48]. Moreover, this also implies and is implied by increases in P_{ADU}, in the number of agents that are optimistic relative to the marginal buyer, in expenditures on the upstate AD security relative to the downstate claim, and in aggregate expenditures on all AD securities.[49] Similarly, a numerical analysis shows that a downward shift in the probability function leads to a decrease (increase) in the expected return, standard deviation, and Sharpe ratio of the reference portfolio anytime P_{ADU} is above (below) 0.500.[50] This makes sense given that such a downward shift is shown to imply a decrease in P_{ADU}. A related extension to the Fostel and Geanakoplos model consists of a counterclockwise rotation of a given probability function, representing an accentuation of disagreement.[51] Every rotation is defined to

[46]Edwin Neave, "Heterogeneous Expectations," 14.
[47]Edwin Neave, "Heterogeneous Expectations," 2.
[48]Edwin Neave, "Heterogeneous Expectations," 2,13.
[49]Edwin Neave, "Heterogeneous Expectations," 15.
[50]Edwin Neave, "Heterogeneous Expectations," 17.
[51]Edwin Neave, "Heterogeneous Expectations," 17.

occur around a locus corresponding to a specific point along the original probability function. Whenever the original buyer is at a point characterized by a more (less) pessimistic probability value than that associated with the locus, the counterclockwise rotation generates a decrease (increase) in P_{ADU} as well as in the number of optimists in equilibrium.

Changing the payoff profiles of different assets also elicits changes in equilibrium results. For example, Neave shows how lower payoffs in the upstate generate a corresponding increase in P_{ADU} while the number of optimists in equilibrium decreases.[52] The three portfolio statistics are also affected in accordance with the values of both Arrow-Debreu prices. Another subject of inquiry consists of adjusting the distribution of asset prices so as to magnify the differences between upstate and downstate outcomes while preserving the expected payoff across both states. This change elicits a fall in the price of the upstate security while the number of optimists increases in equilibrium.[53] Since $P_{ADU} > P_{ADD}$ in this particular instance, the decline in P_{ADU} produces a decrease in all three portfolio statistics.

By introducing leverage, Neave considers yet another extension in which agents can borrow funds to finance the purchase of the upstate security only, making it more valuable in equilibrium while reducing the number of optimistic purchasers.[54] Taking the relative value of both securities into consideration, portfolio statistics adjust in the expected direction.

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[52]Edwin Neave, "Heterogeneous Expectations," 18.
[53]Edwin Neave, "Heterogeneous Expectations," 19.
[54]Edwin Neave, "Heterogeneous Expectations," 20.

5 A Model of Financial Stability Under Heterogeneous Expectations

5.1 Financial Stability in the Leverage Economy

This section proposes a simple extension to the standard Fostel and Geanakoplos model to explore the influence upon financial stability of adjusting specifications. Unlike the original model, there are now three distinct time points, $t = -1, 0, 1$. Initially, at $t = -1$, it is assumed that there are N financial institutions characterized by homogeneous expectations, each of which owns two units of the riskless X asset and two units of the risky Y asset (instead of a single unit of each, as assumed in the original model). It seems reasonable to think of X as cash and Y as loans made to agents and firms that exist outside the financial sector. Since risk-preferences are still risk-neutral, these assets can be valued by their expected payoffs. Every institution is assumed to have an initial debt-to-equity ratio of 1.0, meaning that it owes depositors an amount equal to half the starting value of the assets on its balance sheet. Thus, there is the possibility of insolvency should asset values decline below what is owed to creditors. Depositors accept the risk-free rate of interest (assumed to be 0%) since their funds are insured by the government. The zero rate implies that what is owed to depositors remains constant from one point in time to the next provided withdrawals are equal to the inflow of new deposits.

At time $t = 0$, the economy endures a shock which accentuates uncertainty. As an example, assume that a new technology is brought to market. At the time this innovation is first introduced, it may not be obvious exactly how it will affect firms in the nonfinancial sector. As lenders try to revise their assessment of default probabilities, they will be forced to make hasty, premature, and speculative judgments based upon only limited information pertaining to the new innovation. Such uncertainty, as argued by Miller, elicits heterogeneous expectations, which then affect the behaviour of financial institutions.[55] Thus, every firm is now characterized by a function, q_U, defining their estimates of upstate probabilities. The input of this function is a parameter, $h \epsilon (0,1)$ unique to each institution. Since q_U is strictly increasing in h, it makes sense to think of h as being a measure of relative optimism. As a result of heterogeneous expectations, the banks in this model may then decide that they can optimize firm value by exchanging assets. Here, the asset market closely approximates the corresponding setup in the Fostel-Geanakoplos model, though for the sake of simplicity this paper restricts its focus to two sample cases - the leverage economy, and the CDS economy. In the leverage economy, pessimists will use the funds raised through the sale of their initial endowment portfolio, less the amount lent to optimists, to purchase all units of X . Since the value of what is sold must exactly equal the value of what is purchased, transactions initiated by pessimists must satisfy the following cash constraint where h_1 denotes the proportion of pessimists in equilibrium, $1-h_1$ the proportion of optimists, and δ represents one half of aggregate funds borrowed by optimists:

$$2[(P_Y + P_X)(h_1) - \delta] = 2P_X \tag{1}$$

$$2[(P_Y + 1)(h_1) - \delta] = 2 \tag{2}$$

Moreover, notice that since pessimists will only lend up to an amount equal to what can be covered by collateral in the downstate, aggregate lending by institutions of this type will be equal in value to the total downstate payoffs of all risky assets held by optimists. Since optimists hold all units of Y in equilibrium, the following must be true (recall that $R < 1$ is the downstate payoff of one unit of the risky asset):

$$2R = 2\delta \tag{3}$$

[55] Miller, “Risk, Uncertainty and Divergence of Opinion,” 1151.

Optimists, however, will choose not to purchase any units of X but will instead borrow money from pessimists to supplement the proceeds of their endowment sale allowing for the acquisition all units of the risky asset, Y:[56]

$$2[(P_Y + P_X)(1 - h_1) + \delta] = 2P_Y \tag{4}$$

$$2[(P_Y + 1)(1 - h_1) + R] = 2P_Y \tag{5}$$

The price of the riskless asset is normalized to one and exactly equals its expected return, while risk-neutrality ensures that the price of the risky asset must also equal its expected return, as evaluated by the marginal buyer, defined to be indifferent between the positions of optimists and pessimists. This concept is expressed in equation (6) where $q_U^{h_1}$ indicates the probability assigned by the marginal investor to the realization of the upstate:

$$P_Y = q_U^{h_1} 1 + (1 - q_U^{h_1})R \tag{6}$$

Following the exchange of assets and their corresponding payoff streams the economy moves to time $t = 1$. Here there are two possible states of nature that can be realized - an upstate and a downstate. The observed state of the world then determines the payoffs of all assets in the economy. Any institution for which the value of its liabilities is smaller than the value of its assets is then found to be insolvent and defaults on its obligations.

In order to determine the value of assets and liabilities, it is first necessary to compute the total number of different assets owned by a particular firm. Every optimist and pessimist has an initial endowment of two units of the riskless asset, X, and two units of the risky asset, Y. Once the exchange market opens, both optimists and pessimists sell their initial endowments. Pessimists lend to optimists up to the

[56] Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 202-203.

amount that can be fully collateralized. What remains is used to purchase all units of the X asset. Since their are a total of two units of X and a total of h_1 pessimists, each pessimist ends up with $\frac{2}{h_1}$ units of X. Since the downstate payoff of the Y asset, R, is used as collateral to completely secure the loan, and there are two units of Y in the economy, the total amount of funds lent by all pessimists is equal to $2R$. Moreover, each loan provides an amount of funds equal to R, meaning the total number of loans extended by institutions of this type is equal to two. Finally, because the number of pessimists in equilibrium is equal to h_1, each will make $\frac{2}{h_1}$ loans to optimists. Thus, for pessimists in the leverage economy, it follows that:

$$X_P^1 = 2/h_1 \tag{7}$$

$$Y_P^1 = 0 \tag{8}$$

$$B_P^1 = 2/h_1 \tag{9}$$

where each equation represents the number of units of a particular asset owned by a given investor at a specific point in time. Superscripts denote the point in time at which the equality holds, while the subscipt indicates the type of institution under consideration - either O for an optimist or P for a pessimist. The domain of possible assets includes X, Y, and B (which denotes repayment owed to a given creditor institution, where one unit represents a value owed equal to δ). Since repayment is guaranteed by collateral, B is a riskless asset and its price will equal its constant payoff, $R < 1$.

Optimists use the money borrowed from pessimists to supplement what is raised through their endowment sale. The extra money allows them to purchase all units of Y. Since there are two units of Y available and $1 - h_1$ optimists, each optimist acquires $\frac{2}{1-h_1}$ units of the risky asset. Applying the notation developed in the previous paragraph:

$$X_O^1 = 0 \tag{10}$$

$$Y_O^1 = 2/(1 - h_1) \tag{11}$$

$$B_O^1 = 0 \tag{12}$$

Final asset values for each investor of type i at time t are then given as a sum of products, where each product multiplies the number of units held of a given asset by its payoff at time t. For riskless assets, X and B, such payoffs will equal P_X and P_B respectively. For the risky asset, Y, the observed payoff is the realization of a discrete random variable, λ_Y, which can take one of two possible values, each corresponding to a specific state of the world. Thus:

$$A_i^t = X_i^t(P_X) + Y_i^t \lambda_Y^t + B_i^t(P_B) \tag{13}$$

$$A_i^t = X_i^t + Y_i^t \lambda_Y^t + B_i^t(R) \tag{14}$$

Upstate

$$A_O^1 = X_O^1(P_X) + Y_O^1 \lambda_Y^1 + B_O^1(P_B) \tag{15}$$

$$A_O^1 = 0(P_X) + \frac{2}{(1 - h_1)}(1) + 0(P_B) \tag{16}$$

$$A_O^1 = \frac{2}{(1 - h_1)} \tag{17}$$

Downstate

$$A_O^1 = X_O^1(P_X) + Y_O^1 \lambda_Y^t + B_O^1(P_B) \tag{18}$$

$$A_O^1 = 0(P_X) + \frac{2}{(1 - h_1)}(0) + 0(P_B) \tag{19}$$

$$A_O^1 = 0 \tag{20}$$

Upstate and Downstate

$$A_P^1 = X_P^1(P_X) + Y_P^1\lambda_Y^1 + B_P^1(P_B) \tag{21}$$

$$A_P^1 = \frac{2}{h_1}(P_X) + (0)\lambda_Y^1 + \frac{2}{h_1}(P_B) \tag{22}$$

$$A_P^1 = \frac{2}{h_1}(P_X + P_B) \tag{23}$$

$$A_P^1 = \frac{2}{h_1}(1 + R) \tag{24}$$

Balance sheet liabilities can be found using the same procedure. Since pessimists extend a total of two loans to optimists and there are $1 - h_1$ optimists, each optimist must repay a number of loans equal to $\frac{2}{1-h_1}$. Thus, the liabilities held by a given optimist can be expressed as:

$$d_O^1 = D \tag{25}$$

$$b_O^1 = \frac{2}{1 - h_1} \tag{26}$$

where each equation expresses the number of units of a particular liability held by each firm of type i at time t. Liabilities can be of two types - deposits, given as $d_i^t = D$, and debt repayments owed to other institutions, b_i^t, where one unit of b_i^t, represents an amount owing equal to δ.

Pessimists don't have any liabilities outside of what is owed to depositors:

$$d_P^1 = D \tag{27}$$

$$b_P^1 = 0 \tag{28}$$

Total value of liabilities held by a given firm of type i at time j are then given as:

$$L_i^t = D + b_i^t\delta \tag{29}$$

$$L_O^1 = D + b_O^1 \delta \tag{30}$$

$$L_O^1 = D + \frac{2}{1-h_1}\delta \tag{31}$$

$$L_P^1 = D + b_P^1 \delta \tag{32}$$

$$L_P^1 = D \tag{33}$$

Table 1: Asset and Liability Positions of a Representative Optimist and Pessimist

	Optimist		Pessimist	
	Pre-Exchange	Post-Exchange	Pre-Exchange	Post-Exchange
X	2	0	2	$\frac{2}{h_1}$
Y	2	$\frac{2}{(1-h_1)}$	2	0
B	0	0	0	$\frac{2}{h_1}$
d	D	D	D	D
b	0	$\frac{2}{1-h_1}$	0	0

Final equity values for given institutions of type i at time t are then found to be:

$$E_i^t = Max(A_i^t - L_i^t, 0) \tag{34}$$

Thus:

Upstate

$$E_O^1 = Max(A_O^1 - L_O^1, 0) \tag{35}$$

$$E_O^1 = Max(\frac{2}{1-h_1} - D - \frac{2}{1-h_1}\delta, 0) \tag{36}$$

$$E_O^1 = Max(\frac{2}{1-h_1}(1-\delta) - D, 0) \tag{37}$$

Downstate

$$E_O^1 = Max(A_O^1 - L_O^1, 0) \tag{38}$$
$$E_O^1 = Max(0 - D - \frac{2}{1-h_1}\delta, 0) \tag{39}$$
$$E_O^1 = 0 \tag{40}$$

Upstate and Downstate

$$E_P^1 = Max(A_P^1 - L_P^1, 0) \tag{41}$$
$$E_P^1 = Max(\frac{2}{h_1}(1+R) - D, 0) \tag{42}$$

The final step of the analytical procedure involves evaluating financial stability of the economy in question. To this end, expected losses to shareholders and creditors can be computed using either $t = 0$ or $t = -1$ expectations as reference points. Adopting the second of the two possibilities, the aggregate losses to equityholders and creditors can be computed as (where EL and CL denote aggregate losses to equityholders and creditors respectively and η represents the recovery rate on obligations of defaulting firms:

Case I: E_O^1, $E_P^1 = 0$

$$EL = E_O^{-1}(1-h_1) + E_P^{-1}(h_1) \tag{43}$$
$$CL = L_O^1(1-h_1)(1-\eta) + L_P^1(h_1)(1-\eta) \tag{44}$$

Case II: $E_O^1 = 0$, $E_P^1 \neq 0$

$$EL = E_O^{-1}(1-h_1) \tag{45}$$
$$CL = L_O^1(1-h_1)(1-\eta) \tag{46}$$

Case III: $E_O^1 \neq 0,\ E_P^1 = 0$

$$EL = E_P^{-1}(h_1) \tag{47}$$

$$CL = L_P^1(h_1)(1-\eta) \tag{48}$$

Case IV: $E_O^1 \neq 0,\ E_P^1 \neq 0$

$$EL = 0 \tag{49}$$

$$CL = 0 \tag{50}$$

Aggregate economy-wide losses are given as:

$$TL = EL + CL \tag{51}$$

Losses can then be computed as proportions of equity, liability, and asset values respectively (remember that the superscripts are not exponents but instead refer to the point in time under consideration):

$$EL_{prop} = \frac{EL}{E_O^{-1}(1-h_1) + E_P^{-1}(h_1)} \tag{52}$$

$$CL_{prop} = \frac{CL}{L_O^1(1-h_1) + L_P^1(h_1)} \tag{53}$$

$$TL_{prop} = \frac{EL + CL}{A_O^{-1} + A_P^{-1}} \tag{54}$$

5.2 Financial Stability in the CDS Economy

In the CDS economy, optimists acquire all X and Y assets and strip the corresponding downstate payoff streams which they then sell to pessimists as the downtranche and

CDS asset respectively. Thus, they face the following cash constraint:

$$2[(1-h_1)(P_X+P_Y)+P_{DT}+P_{CDS}]=2(P_X+P_Y) \tag{55}$$

$$(1-h_1)(1+P_Y)+P_{DT}+P_{CDS}=(1+P_Y) \tag{56}$$

Pessimists, meanwhile, sell their initial endowment and use the proceeds to purchase the downstate tranche and the CDS.[57] Such transactions are governed by the following relationship:

$$2h_1(P_X+P_Y)=2[P_{DT}+P_{CDS}] \tag{57}$$

$$h_1(1+P_Y)=P_{DT}+P_{CDS} \tag{58}$$

Because the payoffs to the downtranche asset are exactly R times the payoffs of the CDS in every state of the world, the price of the first security must also be R times the price of the latter:

$$P_{DT}=(R)P_{CDS} \tag{59}$$

Moreover, since price and payoff of the first asset are exactly proportional to those of the CDS, the returns to both securities are equivalent. Thus, all participants must be indifferent between tranching the riskless and the risky asset to obtain an Arrow-Debreu upstate claim. This implies that:

$$\frac{1}{P_Y-P_{DT}}=\frac{1}{1-P_{CDS}} \tag{60}$$

$$P_Y-(R)P_{CDS}=1-P_{CDS} \tag{61}$$

$$P_{CDS}=\frac{P_Y-1}{R-1} \tag{62}$$

[57]Fostel and Geanakoplos, "Tranching, CDS, and Asset Prices," 207.

Substituting (62) and (59) into (56) allows for the price of the risky asset to be expressed in terms of h_1. Notice that there is a negative relationship between P_Y and the proportion of pessimists in equilibrium:

$$P_Y = \frac{(1+R)-(1-R)h_1}{(1+R)+(1-R)h_1} \tag{63}$$

Finally, it must also be the case that the marginal investor will be indifferent between acquiring an Arrow-Debreu upstate security (consistent with the behaviour of optimists in equilibrium) and purchasing the credit default swap (consistent with the behaviour of pessimists). Equation (64) ensures that the returns to both positions will be equivalent.

$$\frac{q_U^{h_1}}{P_Y - P_{DT}} = \frac{1-q_U^{h_1}}{P_{CDS}} \tag{64}$$

Substituting (59) and (62) into (64), yields an equation, which, in conjunction with (63) can be used to find equilibrium values.

Time $t = 1$ balance sheets are characterized by a set of equations, analogous to those used to describe the leverage economy. Since optimists acquire all available units of X and Y, and there are $(1-h_1)$ optimists in equilibrium, each institution of this type will own $\frac{2}{1-h_1}$ units of both traditional assets.

$$X_O^1 = 2/(1-h_1) \tag{65}$$

$$Y_O^1 = 2/(1-h_1) \tag{66}$$

$$DT_O^1 = 0 \tag{67}$$

$$CDS_O^1 = 0 \tag{68}$$

Pessimists then sell their endowments of X and Y to purchase the downstate securities. Since there are 2 units of both DT and CDS and h_1 pessimists in equilibrium:

$$X_P^1 = 0 \tag{69}$$

$$Y_P^1 = 0 \tag{70}$$

$$DT_P^1 = 2/h_1 \tag{71}$$

$$CDS_P^1 = 2/h_1 \tag{72}$$

Asset values can then be calculated as:

$$A_i^t = X_i^t P_X + Y_i^t \lambda_Y^1 + DT_i^t \lambda_{DT}^1 + CDS_i^t \lambda_{CDS}^1 \tag{73}$$

Upstate

$$A_O^1 = X_O^1 P_X + Y_O^1 \lambda_Y^1 + DT_O^1 \lambda_{DT}^1 + CDS_O^1 \lambda_{CDS}^1 \tag{74}$$

$$A_O^1 = \frac{2}{1-h_1}(1) + \frac{2}{1-h_1}\lambda_Y^1 \tag{75}$$

$$A_O^1 = \frac{2}{1-h_1}(1) + \frac{2}{1-h_1}(1) \tag{76}$$

$$A_O^1 = \frac{4}{1-h_1} \tag{77}$$

$$A_P^1 = X_P^1 P_X + Y_P^1 \lambda_Y^1 + DT_P^1 \lambda_{DT}^1 + CDS_P^1 \lambda_{CDS}^1 \tag{78}$$

$$A_P^1 = \frac{2}{h_1}(\lambda_{DT}^1) + \frac{2}{h_1}(\lambda_{CDS}^1) \tag{79}$$

$$A_P^1 = \frac{2}{h_1}(0) + \frac{2}{h_1}(0) \tag{80}$$

$$A_P^1 = 0 \tag{81}$$

Downstate

$$A^1_O = X^1_O P_X + Y^1_O \lambda^1_Y + DT^1_O \lambda^1_{DT} + CDS^1_O \lambda^1_{CDS} \tag{82}$$

$$A^1_O = \frac{2}{1-h_1}(1) + \frac{2}{1-h_1}\lambda^1_Y \tag{83}$$

$$A^1_O = \frac{2}{1-h_1}(1) + \frac{2}{1-h_1}(0) \tag{84}$$

$$A^1_O = \frac{2}{1-h_1} \tag{85}$$

$$A^1_P = X^1_P P_X + Y^1_P \lambda^1_Y + DT^1_P \lambda^1_{DT} + CDS^1_P \lambda^1_{CDS} \tag{86}$$

$$A^1_P = \frac{2}{h_1}(\lambda^1_{DT}) + \frac{2}{h_1}(\lambda^1_{CDS}) \tag{87}$$

$$A^1_P = \frac{2}{h_1}(R) + \frac{2}{h_1}(1) \tag{88}$$

$$A^1_P = \frac{2(R+1)}{h_1} \tag{89}$$

Optimists, by selling all CDS and downtranche securities, are obligated to make a promised payment to pessimists in the downstate. Thus, since two units of both assets are created, and given that there exists $1-h_1$ optimists in equilibrium, each investor of this type will carry $\frac{2}{1-h_1}$ units of each security as liabilities in addition to what is owed to depositors:

$$d^1_O = D \tag{90}$$

$$cds^1_O = \frac{2}{1-h_1} \tag{91}$$

$$dt^1_O = \frac{2}{1-h_1} \tag{92}$$

Pessimists do not sell any investment assets that entail promised payments to optimists. Thus, they only owe what is promised to depositors.

$$d_P^1 = D \tag{93}$$

$$cds_P^1 = 0 \tag{94}$$

$$dt_P^1 = 0 \tag{95}$$

$$L_i^t = D + dt_i^t(\lambda_{DT}^1) + cds_i^t(\lambda_{CDS}^1) \tag{96}$$

Upstate

$$L_O^1 = D + dt_O^1(\lambda_{DT}^1) + cds_O^1(\lambda_{CDS}^1) \tag{97}$$

$$L_O^1 = D + \frac{2}{1-h_1}(\lambda_{DT}^1) + \frac{2}{1-h_1}(\lambda_{CDS}^1) \tag{98}$$

$$L_O^1 = D + \frac{2}{1-h_1}(0) + \frac{2}{1-h_1}(0) \tag{99}$$

$$L_O^1 = D \tag{100}$$

Downstate

$$L_O^1 = D + dt_O^1(\lambda_{DT}^1) + cds_O^1(\lambda_{CDS}^1) \tag{101}$$

$$L_O^1 = D + \frac{2}{1-h_1}(\lambda_{DT}^1) + \frac{2}{1-h_1}(\lambda_{CDS}^1) \tag{102}$$

$$L_O^1 = D + \frac{2}{1-h_1}(R) + \frac{2}{1-h_1}(1) \tag{103}$$

$$L_O^1 = D + \frac{2(R+1)}{1-h_1} \tag{104}$$

Upstate and Downstate

$$L_P^1 = D + dt_P^1(\lambda_{DT}^1) + cds_P^1(\lambda_{CDS}^1) \quad (105)$$

$$L_P^1 = D + 0(\lambda_{DT}^1) + 0(\lambda_{CDS}^1) \quad (106)$$

$$L_P^1 = D \quad (107)$$

Table 2: Asset and Liability Positions of a Representative Optimist and Pessimist

	Optimist		Pessimist	
	Pre-Exchange	Post-Exchange	Pre-Exchange	Post-Exchange
X	2	$\frac{2}{1-h_1}$	2	0
Y	2	$\frac{2}{1-h_1}$	2	0
DT	0	0	0	$\frac{2}{h_1}$
CDS	0	0	0	$\frac{2}{h_1}$
d	D	D	D	D
dt	0	$\frac{2}{1-h_1}$	0	0
cds	0	$\frac{2}{1-h_1}$	0	0

Equity values are then:

Upstate

$$E_O^1 = Max(A_O^1 - L_O^1, 0) \quad (108)$$

$$E_O^1 = Max(\frac{4}{1-h_1} - D, 0) \quad (109)$$

$$E_P^1 = Max(A_P^1 - L_P^1, 0) \tag{110}$$

$$E_P^1 = Max(0 - D, 0) \tag{111}$$

$$E_P^1 = 0 \tag{112}$$

Downstate

$$E_O^1 = Max(A_O^1 - L_O^1, 0) \tag{113}$$

$$E_O^1 = Max(\frac{2}{1-h_1} - D - \frac{2(R+1)}{1-h_1}, 0) \tag{114}$$

$$E_O^1 = Max(\frac{-2R}{1-h_1} - D, 0) \tag{115}$$

$$E_O^1 = 0 \tag{116}$$

$$E_P^1 = Max(A_P^1 - L_P^1, 0) \tag{117}$$

$$E_P^1 = Max(\frac{2(R+1)}{h_1} - D, 0) \tag{118}$$

To evaluate financial stability, equations (43)-(54) can be applied exactly as they were in the leverage economy.

6 Analysis

The model developed in Section 3 is now applied to evaluate how financial stability is affected by changes in model specifications. Attention is given to adjustments in the probability functions characterizing each institution, the introduction of specific financial innovations, and to the effects of different regulatory regimes governing reserve requirements. Each of the above elements are discussed in turn. For the remainder of the paper, it is assumed that $\eta = 0$.

6.1 Shifting Probability Functions

Neave (2013) arrives at general analytical results demonstrating how the price of an investment asset, as well as the proportion of optimists in equilibrium can be affected by shifts in investors' probability functions. This section attempts to extend these results by showing how shocks that influence general levels of optimism can also impact the stability of the financial sector.

Consider first the leverage economy. Following the procedure outlined by Neave, observe that in equilibrium the marginal investor must be indifferent between taking the position of an optimist and adopting the profile of a pessimist.[58]. It follows that the returns to both positions, as measured by the indifferent investor, must be the same. Thus:

$$\frac{\frac{2q_U^{h_1}}{1-h_1}+\frac{2R(1-q_U^{h_1})}{1-h_1}-\frac{2R}{1-h_1}}{\frac{2P_Y}{1-h_1}-\frac{2R}{1-h_1}} = \frac{\frac{2}{h_1}+\frac{2R}{h_1}}{\frac{2P_X}{h_1}+\frac{2R}{h_1}} \tag{119}$$

$$\frac{q_U^{h_1}+R(1-q_U^{h_1})-R}{P_Y-R} = \frac{1+R}{1+R} \tag{120}$$

$$\frac{(1-R)q_U^{h_1}}{P_Y-R} = \frac{1+R}{1+R} \tag{121}$$

$$\frac{(1-R)q_U^{h_1}}{1+R} = \frac{P_Y-R}{1+R} \tag{122}$$

Rearranging equation (5) to solve for P_Y yields:

$$P_Y = \frac{1+R-h_1}{h_1} \tag{123}$$

[58] Edwin Neave, "Heterogeneous Expectations," 8,9.

Substituting (123) into (122) gives:

$$\frac{(1-R)q_U^{h_1}}{1+R} = \frac{[\frac{1+R-h_1}{h_1}]-R}{1+R} \tag{124}$$

$$\frac{(1-R)q_U^{h_1}}{1+R} = \frac{(1+R-h_1-Rh_1)}{(1+R)h_1} \tag{125}$$

$$\frac{(1-R)q_U^{h_1}}{1+R} = \frac{1+R-h_1(1+R)}{(1+R)h_1} \tag{126}$$

$$\frac{(1-R)q_U^{h_1}}{1+R} = \frac{(1+R)(1-h_1)}{(1+R)h_1} \tag{127}$$

$$\frac{(1-R)q_U^{h_1}}{1+R} = \frac{1-h_1}{h_1} \tag{128}$$

$$\implies \frac{(1-R)q_U^{h}}{1+R} = \frac{1-h}{h} \quad \forall \text{ possible equilibrium values of h} \tag{129}$$

The function on the left-hand-side of (129), which can be referred to as V(h), is increasing in h, while the function on the right-hand side, W(h), is decreasing in h for all positive values of h. The equality holds when $h = h_1$. By plotting V(h) and W(h) together against h, the point of equilibrium would occur where both curves intersect. Any upward shift in q_U would also act to shift V(h) up and to the left, decreasing the equilibrium value of h_1 and increasing the corresponding number of optimists. Since P_Y is a decreasing function of h_1, as given by (123), this implies that the price of the risky asset will increase following an upward shift in q_U.

These findings should translate into increased instability. When assets are first traded, pessimists acquire all units of the riskless asset, and are owed debt repayments backed by collateral. Both positions offer guaranteed returns. As long as pessimists are solvent immediately following the close of asset markets, which will always be the case since the market value of what they sell exactly equals the market value of what is purchased, they will remain solvent at time $t = 1$. Optimists, meanwhile, will never default in the upstate. To see this, note that the risky asset has a maximum payoff of 1 in the upstate and zero in the downstate. Thus no utility-maximizing agent would ever pay a price greater than 1 for this asset. So long as $P_Y < 1$, the purchasing optimist, which holds only the risky asset in equilibrium, will always profit in the upstate. In the downstate, however, the optimist will always receive a payoff of zero since it must return the downstate payoff of the risky asset as collateral to the lender. Thus, it will never be able to repay depositors at time $t = 1$ and will be forced into insolvency. Therefore, in the upstate, no institutions fail, though in the downstate all optimists must fail. Since increasing levels of general optimism are associated with a higher proportion of optimists in equilibrium, any upward shift in the probability function is guaranteed to increase the number of defaulting institutions in the downstate while leaving the number of failing institutions in the upstate unchanged.

To test the validity of the above logic, numerical simulations are employed using the following nine probability function specifications:

$$q_U^h = 0.05\iota + 0.1h \tag{130}$$

where $\iota \in \{1, 3, 5, ..., 15, 17\}$. Moreover, it is also assumed that $R = 0.2$ and $D = 1.6$. Results are displayed in Table 3:

Table 3: Shocks to General Levels of Optimism - Leverage Economy

Simulation	Probability Function	h_1	P_Y	TL% (Upstate)	TL%(Downstate)
1	$0.05{+}0.1h_1$	0.91386	0.31311	0	0.08614
2	$0.15{+}0.1h_1$	0.86386	0.38911	0	0.13614
3	$0.25{+}0.1h_1$	0.81883	0.46550	0	0.18117
4	$0.35{+}0.1h_1$	0.77809	0.54225	0	0.22191
5	$0.45{+}0.1h_1$	0.74107	0.61929	0	0.25893
6	$0.55{+}0.1h_1$	0.70730	0.69658	0	0.29270
7	$0.65{+}0.1h_1$	0.67639	0.77411	0	0.32361
8	$0.75{+}0.1h_1$	0.64800	0.85184	0	0.35200
9	$0.85{+}0.1h_1$	0.62184	0.92975	0	0.37816

Consistent with the model's predictions, the results of Table 3 demonstrate that greater levels of optimism generate smaller numbers of pessimists in equilibrium, higher prices for the risky asset, and greater instability.

Under the CDS economy the results are somewhat different. Again, in equilibrium the marginal investor should have no preference between behaving as an optimist or as a pessimist. Thus, the expected returns to both positions must be equivalent:

$$\frac{[\frac{2}{1-h_1} + \frac{2}{1-h_1}]q_U^{h_1}}{\frac{2P_X}{1-h_1} + \frac{2P_Y}{1-h_1} - \frac{2P_{DT}}{1-h_1} - \frac{2P_{CDS}}{1-h_1}} = \frac{(1-q_U^{h_1})[\frac{2}{h_1} + \frac{2R}{h_1}]}{\frac{2P_{DT}}{h_1} + \frac{2P_{CDS}}{h_1}} \tag{131}$$

$$\frac{(1+1)q_U^{h_1}}{1+P_Y - P_{DT} - P_{CDS}} = \frac{(1-q_U^{h_1})(1+R)}{P_{DT} + P_{CDS}} \tag{132}$$

Inserting (59) into (132):

$$\frac{2q_U^{h_1}}{1+P_Y-(R)P_{CDS}-P_{CDS}} = \frac{(1-q_U^{h_1})(1+R)}{(R)P_{CDS}+P_{CDS}} \tag{133}$$

$$\frac{2q_U^{h_1}}{1+P_Y-(1+R)P_{CDS}} = \frac{(1-q_U^{h_1})(1+R)}{(1+R)P_{CDS}} \tag{134}$$

Now, substituting (62) for P_{CDS} yields:

$$\frac{2q_U^{h_1}}{1 + P_Y - (1+R)[\frac{P_Y - 1}{R-1}]} = \frac{(1-q_U^{h_1})(1+R)}{(1+R)[\frac{P_Y-1}{R-1}]} \tag{135}$$

$$\frac{2q_U^{h_1}}{1 + P_Y - (1+R)[\frac{P_Y - 1}{R-1}]} = \frac{(1-q_U^{h_1})(R-1)}{P_Y - 1} \tag{136}$$

$$\frac{2(R-1)q_U^{h_1}}{(R-1)(1+P_Y) - (1+R)(P_Y-1)} = \frac{(1-q_U^{h_1})(R-1)}{P_Y - 1} \tag{137}$$

$$\frac{2(R-1)q_U^{h_1}}{R + (R)P_Y - 1 - P_Y - P_Y + 1 - (R)P_Y + R} = \frac{(1-q_U^{h_1})(R-1)}{P_Y - 1} \tag{138}$$

$$\frac{2(R-1)q_U^{h_1}}{-2(P_Y - R)} = \frac{(1-q_U^{h_1})(R-1)}{P_Y - 1} \tag{139}$$

$$\frac{(R-1)q_U^{h_1}}{R - P_Y} = \frac{(1-q_U^{h_1})(R-1)}{P_Y - 1} \tag{140}$$

$$\frac{(R-1)q_U^{h_1}}{(1-q_U^{h_1})(R-1)} = \frac{R - P_Y}{P_Y - 1} \tag{141}$$

$$\frac{q_U^{h_1}}{1-q_U^{h_1}} = \frac{R - P_Y}{P_Y - 1} \tag{142}$$

$$\frac{q_U^{h_1}}{1-q_U^{h_1}} = \frac{P_Y - R}{1 - P_Y} \tag{143}$$

$$\frac{q_U^{h}}{1-q_U^{h}}(1 - P_Y) + R = P_Y \quad \forall \text{ equilibrium values of } h \tag{144}$$

The left-hand-side of equation (144), which can be denoted by V(h), is an increasing function of h, while the right-hand-side of (144), denoted by W(h), is a decreasing function of h. Both observations follow from the fact that P_Y becomes a decreasing function of h once h is substituted for h_1 in equation (63). The point of intersection of both curves, V(h) and W(h) gives h_1. Any upward shift in the probability function would boost V(h) upward while W(h) would remain unchanged. This would act to reduce h_1 in equilibrium, leading to an increase in the proportion of optimists. Given that there exists a negative relationship between the price of the risky asset and the proportion of pessimists in equilibrium (again, this follows from equation (63)), the upward shift in the probability function would be expected to elicit an increase in P_Y.

In the CDS economy, increasing levels of optimism may or may not accentuate instability. In equilibrium, all optimists hold all the risky and riskless assets but sell off the downstate payoffs of each. Essentially they own Arrow-Debreu securities. Obviously, no risk-neutral agent would ever purchase such an asset if its price was greater than one. Of course, paying less than one guarantees a profit and an increase in equity in the upstate. Payoffs in the downstate are equal to zero, implying that all equity will be lost should such a state of nature be observed at time $t = 1$. Thus, in the upstate no optimists will default, while all optimists default in the downstate. This is similar to what is observed in the leverage economy.

Unlike the leverage economy, however, the presence of credit default swaps ensures that pessimists will now purchase Arrow-Debreu assets that generate payoffs exclusively in the downstate. An argument analogous to the one employed in the previous paragraph ensures that in equilibrium no pessimists will default in the downstate while all pessimists will default in the upstate. Thus, an upward shift in the probability function, through a reduction in the proportion of pessimists in equilibrium, will generate fewer defaults in the upstate and more defaults in the downstate. The total effect of such a shock on aggregate stability is ambiguous and depends upon the actual (objective) likelihood of each state being realized.

Table 4 shows the results of a numerical analysis of outcomes associated with different levels of general optimism arising from shocks to institutions' probability functions. Again, $R = 0.2$ and $D = 1.6$, as before. The results are entirely consistent with the preceding analysis. Shocks that increase optimism have a tendency to decrease the proportion of pessimists in equilibrium, h_1, to increase the price of the risky asset, P_Y, and to reduce (increase) the number of defaults in the upstate (downstate).

Table 4: Shocks to General Levels of Optimism - CDS Economy					
Simulation	Probability Function	h_1	P_Y	TL% (Upstate)	TL%(Downstate)
1	$0.05+0.1h_1$	0.80054	0.30404	0.80054	0.19946
2	$0.15+0.1h_1$	0.68246	0.37460	0.68246	0.31754
3	$0.25+0.1h_1$	0.57472	0.44598	0.57472	0.42528
4	$0.35+0.1h_1$	0.47616	0.51809	0.47616	0.52384
5	$0.45+0.1h_1$	0.38577	0.59086	0.38577	0.61423
6	$0.55+0.1h_1$	0.30265	0.66421	0.30265	0.69735
7	$0.65+0.1h_1$	0.22604	0.73808	0.22604	0.77396
8	$0.75+0.1h_1$	0.15525	0.81242	0.15525	0.84475
9	$0.85+0.1h_1$	0.08968	0.88717	0.08968	0.91032

Another important consideration worthy of further analysis is the effect of CDS instruments themselves on financial stability. To gain insight, it makes sense to compare outcomes in the CDS economy with those of the leverage economy. First, notice that when CDS assets are traded, defaults will be observed in both states of the world - insolvency afflicts pessimists in the upstate and optimists in the downstate. By contrast, the leverage economy is only characterized by defaults in the downstate. Thus, to show that CDS securities accentuate instability it is sufficient to show that the proportion of defaulting firms in the downstate (and by extension the number of optimists in equilibrium), is greater under the CDS economy than under the leverage economy.

The starting point of the proof consists of Proposition 5 from Geanakoplos and Fostel, which states that P_Y in the leverage economy is always greater than in the CDS economy for all strictly monotonic and continuous q_U^h as well as all $0 < R < 1$. Taking this as given, it is simple to show that $h_1^{CDS} < h_1^L$ where superscripts are used to distinguish between economies.

From equation (5):

$$P_Y^L = (1 - h_1^L)(1 + P_Y^L) + R \tag{145}$$

$$P_Y^L = 1 + P_Y^L - h_1^L - h_1^L P_Y^L + R \tag{146}$$

$$P_Y^L - P_Y^L + h_1^L P_Y^L = 1 - h_1^L + R \tag{147}$$

$$h_1^L P_Y^L = 1 - h_1^L + R \tag{148}$$

$$P_Y^L = \frac{1 - h_1^L + R}{h_1^L} \tag{149}$$

From equation (63):

$$P_Y^{CDS} = \frac{(R+1) + h_1^{CDS}(R-1)}{(R+1) - h_1^{CDS}(R-1)} \tag{150}$$

Since $P_Y^L > P_Y^{CDS}$:

$$\frac{(R+1)+h_1^{CDS}(R-1)}{(R+1)-h_1^{CDS}(R-1)} < \frac{1-h_1^L+R}{h_1^L} \tag{151}$$

$$\frac{(R+1)+h_1^{CDS}(R-1)}{(R+1)-h_1^{CDS}(R-1)} < \frac{1+R}{h_1^L} - 1 \tag{152}$$

$$\frac{(R+1)+h_1^{CDS}(R-1)}{(R+1)-h_1^{CDS}(R-1)} + 1 < \frac{1+R}{h_1^L} \tag{153}$$

$$\frac{(R+1)+h_1^{CDS}(R-1)}{(R+1)-h_1^{CDS}(R-1)} + \frac{(R+1)-h_1^{CDS}(R-1)}{(R+1)-h_1^{CDS}(R-1)} < \frac{1+R}{h_1^L} \tag{154}$$

$$\frac{2(R+1)}{(R+1)-h_1^{CDS}(R-1)} < \frac{1+R}{h_1^L} \tag{155}$$

$$\frac{2}{(R+1)-h_1^{CDS}(R-1)} < \frac{1}{h_1^L} \tag{156}$$

$$2h_1^L < (R+1) - h_1^{CDS}(R-1) \tag{157}$$

$$\frac{2}{R-1}h_1^L < \frac{R+1}{R-1} - h_1^{CDS} \tag{158}$$

$$\frac{2}{1-R}h_1^L > \frac{R+1}{1-R} + h_1^{CDS} \tag{159}$$

$$h_1^L > \frac{(1-R)(R+1)}{2(1-R)} + \frac{1-R}{2}h_1^{CDS} \tag{160}$$

$$h_1^L > \frac{R+1}{2} + \frac{1-R}{2}h_1^{CDS} \tag{161}$$

If it can be shown that the right-hand-side of (161) is greater than h_1^{CDS}, then $h_1^L > h_1^{CDS}$ holds trivially. In math notation:

$$\frac{R+1}{2} + \frac{1-R}{2}h_1^{CDS} > h_1^{CDS} \implies h_1^L > h_1^{CDS} \tag{162}$$

$$\frac{R+1}{2} > h_1^{CDS} - \frac{1-R}{2}h_1^{CDS} \implies h_1^L > h_1^{CDS} \tag{163}$$

$$\frac{R+1}{2} > h_1^{CDS}[1 - \frac{1-R}{2}] \implies h_1^L > h_1^{CDS} \tag{164}$$

$$\frac{R+1}{2} > h_1^{CDS}[\frac{2-1+R}{2}] \implies h_1^L > h_1^{CDS} \tag{165}$$

$$\frac{R+1}{2} > h_1^{CDS}[\frac{R+1}{2}] \implies h_1^L > h_1^{CDS} \tag{166}$$

Equation (166) holds since $h_1^{CDS} < 1$.

Thus, we have shown that stability outcomes in the leverage economy weakly dominate those in the CDS economy. This follows from the fact that, in the upstate, all pessimists are insolvent in the CDS economy while none are insolvent in the reference economy. In the downstate all optimists default in both economies, but the proportion of optimists is larger in the CDS economy. Thus, default losses in the latter economy are always greater than those in the leverage economy.

Another way of considering how credit default swaps might exert a destabilizing influence upon financial systems is to recognize that their existence implies complete tranching. Thus, institutions can tailor their asset portfolios to match their individual beliefs. When initial asset endowments are not overly skewed toward generating payoffs in one particular state, the trading of asset components to match the heterogeneous beliefs of market participants ensures that individual payoff profiles will be highly asymmetric across states and across participants. Thus, the trading of CDS securities improves the payoffs to winners while exposing losers to more significant losses.

Finally, since the effects of credit default swaps upon stability continue to be hotly debated, it makes sense to consider their introduction as a form of innovation that could accentuate heterogeneous beliefs. Optimists, for example, may be persuaded that credit insurance will improve the performance of financial systems and enhance returns. Pessimists may view such an innovation as a pernicious development. The emergence of such disparate perspectives may, ironically, contribute to the prevalence of CDS trading, given that the exchange of credit insurance depends upon differences of opinion. Thus, the very fact that debate regarding alleged destabilizing effects of financial innovations has not been settled conclusively may actually accentuate their ability to destabilize financial systems.

The above analysis leads to an interpretation of financial crises that may differ somewhat from the discussion presented by Fostel and Geanakoplos. As discussed in Section 2, they assert that leverage and primitive tranching technology led to inflated risky-asset values which then declined following the introduction of the credit default swap innovation. The institutional extension to this model offers a more detailed explanation of how such innovation can precipitate a crisis. While price changes help to explain the mechanics underlying the revised model, they are only one variable of interest. In fact, the primary reason why financial stability is compromised in the extended model following the introduction of the credit default swap is because complete tranching allows heterogeneous institutions to skew their payoff profiles toward one state of the world exclusively. Thus, instead of retaining modestly positive equity values, which would occur under an autarkic economy where all agents were forced to hold their balanced endowment portfolios, institutions either make a large positive payoff in one state of the world or a significant loss in the other state of the world. While observed prices must be consistent with the new equilibrium, they are not necessarily an essential consideration when developing an intuitive understanding of the model's implications.

6.2 Rotating Probability Functions

A discussion of rotating probability functions follows naturally from the preceding subsection. Here we examine financial stability across ten different functions that vary based upon an accentuation of disagreement among institutions while preserving the mean of their upstate probability estimates. More specifically, probability functions are given as:

$$q_U^h = 0.05\iota + (1 - 0.1\iota)h \tag{167}$$

where $\iota \in \{0, 1, 2, ..., 8, 9\}$.

Any decrease in ι will tend to make (167) steeper, implying that institutions' expectations are characterized by greater heterogeneity. In the leverage economy, we know already that V(h) and W(h) are given by the left-hand-side and right-hand-side respectively of equation (129). Substituting (167) into (129) gives:

$$\frac{(1-R)(0.05\iota + (1-0.1\iota)h)}{(1+R)} = \frac{1-h}{h} \quad \forall \text{ possible equilibrium values of h} \qquad (168)$$

As demonstrated by Neave (2013), the effect of this change in equilibrium depends upon the equilibrium value of h relative to the locus of rotation. From (167), for example, the locus of rotation is 0.5. For all values of h such that $h > 0.5$, the new, steeper, probability function lies above the original function. Thus, a counterclockwise rotation will generate equilibrium effects on h_1 and P_Y that mimic those of an upward shift. For values of h such that $h < 0.5$, a rotation accentuating disagreement (i.e. one that is counterclockwise) will move h_1 and P_Y in the same direction as a downward shift in the probability function.

To test this conjecture it makes sense to evaluate a numerical example. The different probability functions defined in equation (167), are used to find a measure of stability for each specification. Again, $R = 0.2$ and $D = 1.6$. Results are provided in Table 5.

Table 5: Accentuation of Heterogeneity - Leverage Economy					
Simulation	Probability Function	h_1	P_Y	TL% (Upstate)	TL%(Downstate)
1	0.45+0.1h_1	0.74107	0.61929	0	0.25893
2	0.40+0.2h_1	0.73293	0.63727	0	0.26707
3	0.35+0.3h_1	0.72546	0.65411	0	0.27454
4	0.30+0.4h_1	0.71859	0.66995	0	0.28141
5	0.25+0.5h_1	0.71221	0.68489	0	0.28779
6	0.20+0.6h_1	0.70629	0.69902	0	0.29371
7	0.15+0.7h_1	0.70076	0.71243	0	0.29924
8	0.10+0.8h_1	0.69558	0.72517	0	0.30442
9	0.05+0.9h_1	0.69072	0.73732	0	0.30928
10	h_1	0.68614	0.74891	0	0.31386

Thus, the results in Table 5 mimic those of Table 3. Consider the probability function associated with the first numerical simulation as a reference function. The value of h_1 associated with this first example, $h_1 = 0.74107$ example, is greater than the value of h associated with the locus of rotation, $h = 0.5$. Thus, a counterclockwise rotation of q_U^h, which has the effect of increasing disagreement between agents, should have the same effect as an upward shift in the probability function. This should lead to more optimism in equilibrium, smaller values of h_1, larger values of P_Y, and greater instability as measured by $TL\%(Downstate)$.

Table 6 presents a similar example using the CDS economy where $R = 0.2$ and $D = 1.6$. Again, the applied rotation is counterclockwise. Unlike the previous example, however, the initial equilibrium value of h_1, 0.38577, actually lies below the locus of rotation, 0.5. Thus, the effects of this change should be the same as would be the case if expectations became more pessimistic. In fact, this is exactly what is observed. As expectations diverge, h_1 rises and P_Y falls. The increased prevalence of pessimists in equilibrium shifts the weight of defaults from the downstate toward the upstate, although the effect of such a shift on total expected defaults is ambiguous and depends on the objective probability of each state being observed. Notice, however, that defaults are once again more prevalent in the CDS economy for any given state of the world when compared to outcomes in the leverage economy. This also reinforces the observation that P_Y is not the only variable of interest, as these results can only be understood as a chain of events in which the change in the price of the risky asset comprises only one component.

Table 6: Accentuation of Heterogeneity - CDS Economy

Simulation	Probability Function	h_1	P_Y	TL% (Upstate)	TL%(Downstate)
1	$0.45+0.1h_1$	0.38577	0.59086	0.38577	0.61423
2	$0.40+0.2h_1$	0.39491	0.58319	0.39491	0.60509
3	$0.35+0.3h_1$	0.40276	0.57666	0.40276	0.59724
4	$0.30+0.4h_1$	0.40955	0.57105	0.40955	0.59045
5	$0.25+0.5h_1$	0.41548	0.56619	0.41548	0.58452
6	$0.20+0.6h_1$	0.42070	0.56193	0.42070	0.57930
7	$0.15+0.7h_1$	0.42532	0.55818	0.42532	0.57468
8	$0.10+0.8h_1$	0.42945	0.55485	0.42945	0.57055
9	$0.05+0.9h_1$	0.43315	0.55187	0.43315	0.56685
10	h_1	0.43649	0.54919	0.43649	0.56351

6.3 Cash Reserve Requirements

Since the riskless asset offers a guaranteed real return of zero percent, it is akin to cash. In the real world, regulators may establish a minimum cash reserve ratio in an effort to limit aggregate leverage. The purpose of this section is to evaluate how this important policy variable affects financial stability.

Consider a modified version of the leverage economy where all agents must retain a specified portion of their initial endowment of the riskless asset, which thus inhibits the complete transfer of asset X from optimists to pessimists. Let ψ denote the required retention ratio of the riskless asset endowment. Optimists will continue to sell their initial endowments and borrow from pessimists to acquire all units of the risky asset and as few units of the riskless asset as regulations allow. Thus, their transfer of funds is governed by equation (169) below (where $\epsilon = 1 - \psi$):

$$2(P_X + P_Y)(1 - h_1) + 2R = 2P_Y + 2(1 - \epsilon)(1 - h_1)P_X \tag{169}$$

$$(P_X + P_Y)(1 - h_1) + R = P_Y + (1 - \epsilon)(1 - h_1)P_X \tag{170}$$

$$P_X(1 - h_1) + P_Y(1 - h_1) + R = P_Y + (1 - \epsilon)(1 - h_1)P_X \tag{171}$$

$$P_X(1 - h_1)\epsilon + R = P_Y h_1 \tag{172}$$

$$P_Y = \frac{\epsilon(1 - h_1) + R}{h_1} \tag{173}$$

Pessimists will still sell their initial endowments and lend to optimists. Whatever funds remain are applied to the purchase of all riskless assets except those held by optimists. Thus:

$$2(P_X + P_Y)h_1 - 2R = [2 - 2(1 - \epsilon)(1 - h_1)]P_X \tag{174}$$

$$(P_X + P_Y)h_1 - R = [1 - (1 - \epsilon)(1 - h_1)]P_X \tag{175}$$

Finally, in equilibrium, the price of the risky asset must be equal to its expected return, as judged by the marginal investor:

$$P_Y = q_U^{h_1} 1 + (1 - q_U^{h_1})R \tag{176}$$

Since optimists start with two units of X (and two units of Y), and are required to hold a proportion, equal to $1-\epsilon$, of their initial riskless asset endowment, each optimist will hold $2(1-\epsilon)$ units of X in equilibrium. Of course, optimists still purchase all units of Y. Since there are two available units and $1-h_1$ optimists, each institution of this type will hold an amount of the risky asset equal to $\frac{2}{1-h_1}$ after exchange markets have closed. Thus, assets held by a given optimist can be expressed as:

$$X_O^1 = 2(1-\epsilon) \tag{177}$$

$$Y_O^1 = 2/(1-h_1) \tag{178}$$

$$B_O^1 = 0 \tag{179}$$

Pessimists, meanwhile, will hold all available units of the risky asset not held by optimists. Since optimists hold a total of $2(1-\epsilon)(1-h_1)$ units of X, and there are only two units available in the economy, pessimists will acquire an aggregate total of $2-2(1-\epsilon)(1-h_1)$ units. Since there are h_1 pessimists in equilibrium, each will hold an amount of X equal to $\frac{2-2(1-\epsilon)(1-h_1)}{h_1}$. Ownership of the B asset is the same as in the original leverage economy. Thus, asset holdings of a given pessimist can be summarized as:

$$X_P^1 = \frac{2-2(1-\epsilon)(1-h_1)}{h_1} \tag{180}$$

$$Y_P^1 = 0 \tag{181}$$

$$B_P^1 = 2/h_1 \tag{182}$$

Final asset values for each investor of type i at time j are then:

Upstate

$$A_O^1 = X_O^1(P_X) + Y_O^1(\lambda_Y^1) + B_O^1(P_B) \tag{183}$$

$$A_O^1 = 2(1-\epsilon)(P_X) + \frac{2}{(1-h_1)}(\lambda_Y^1) + 0(P_B) \tag{184}$$

$$A_O^1 = 2(1-\epsilon)(1) + \frac{2}{(1-h_1)}(1) \tag{185}$$

$$A_O^1 = 2(1-\epsilon) + \frac{2}{(1-h_1)} \tag{186}$$

Downstate

$$A_O^1 = X_O^1(P_X) + Y_O^1(\lambda_Y^1) + B_O^1(P_B) \tag{187}$$

$$A_O^1 = 2(1-\epsilon)(P_X) + \frac{2}{(1-h_1)}(\lambda_Y^1) + 0(P_B) \tag{188}$$

$$A_O^1 = 2(1-\epsilon)(1) + \frac{2P_Y}{(1-h_1)}(0) \tag{189}$$

$$A_O^1 = 2(1-\epsilon) \tag{190}$$

Upstate and Downstate

$$A_P^1 = X_P^1(P_X) + Y_P^1(\lambda_Y^1) + B_P^1(P_B) \tag{191}$$

$$A_P^1 = \frac{2-2(1-\epsilon)(1-h_1)}{h_1}(P_X) + 0(\lambda_Y^1) + \frac{2}{h_1}(P_B) \tag{192}$$

$$A_P^1 = \frac{2}{h_1}(P_X + P_B) - \frac{2(1-\epsilon)(1-h_1)}{h_1}(P_X) \tag{193}$$

$$A_P^1 = \frac{2}{h_1}(1+\delta) - \frac{2(1-\epsilon)(1-h_1)}{h_1} \tag{194}$$

Liabilities are exactly as described in equations (25)-(33).

Table 7: Asset and Liability Positions of a Representative Optimist and Pessimist				
	Optimist		Pessimist	
	Pre-Exchange	Post-Exchange	Pre-Exchange	Post-Exchange
X	2	$2(1-\epsilon)$	2	$\frac{2-2(1-\epsilon)(1-h_1)}{h_1}$
Y	2	$\frac{2}{(1-h_1)}$	2	0
B	0	0	0	$\frac{2}{h_1}$
d	D	D	D	D
b	0	$\frac{2}{1-h_1}$	0	0

Equity values are then:

Upstate

$$E_O^1 = Max(A_O^1 - L_O^1, 0) \tag{195}$$

$$E_O^1 = Max(2(1-\epsilon) + \frac{2}{(1-h_1)} - D - \frac{2}{1-h_1}\delta, 0) \tag{196}$$

$$E_O^1 = Max(2(1-\epsilon) + \frac{2}{1-h_1}(1-\delta) - D, 0) \tag{197}$$

Downstate

$$E_O^1 = Max(A_O^1 - L_O^1, 0) \tag{198}$$

$$E_O^1 = Max(2(1-\epsilon) - D - \frac{2}{1-h_1}\delta, 0) \tag{199}$$

Upstate and Downstate

$$E_P^1 = Max(A_P^1 - L_P^1, 0) \tag{200}$$

$$E_P^1 = Max(\frac{2}{h_1}(1+\delta) - \frac{2(1-\epsilon)(1-h_1)}{h_1} - D, 0) \tag{201}$$

Equations (43)-(54) also apply to describe expected losses to stakeholders.

To interpret how cash reserve requirements influence stability, equation (171) serves as a useful starting point. Reproduced below for convenience, it can be re-arranged in the following way:

$$P_X(1-h_1) + P_Y(1-h_1) + R = P_Y + (1-\epsilon)(1-h_1)P_X \tag{202}$$

$$P_X(1-h_1)(1-(1-\epsilon)) + P_Y(1-h_1) + R = P_Y \tag{203}$$

$$P_X(1-h_1)(\epsilon) + P_Y(1-h_1) + R = P_Y \tag{204}$$

$$(1-h_1)(\epsilon) + P_Y(1-h_1) + R = P_Y \tag{205}$$

$$(1-h_1)(\epsilon + P_Y) + R = P_Y \tag{206}$$

Notice, however, that equation (206) is simply a modified version of equation (5), which is also reproduced below for convenience:

$$(1-h_1)(1+P_Y) + R = P_Y \tag{207}$$

The left-hand-side of both equations (206) and (207) represents the total quantity of funds used to purchase the risky asset. Since $\epsilon < 1$, however, reserve requirements act as a negative shock to demand. Thus, since fewer funds are applied to the purchase of Y, it is expected that the price of the risky asset will decrease as a result. To see how this change in P_Y might affect h_1, the familiar $V(h)$ and $W(h)$ functions are constructed below. Equilibrium implies that the expected returns to the portfolios of optimists and pessimists, as measured by the marginal investor, must exactly equal each other. Thus:

$$\frac{2(1-\epsilon)+q_u^{h_1}\frac{2}{1-h_1}+(1-q_u^{h_1})\frac{2R}{1-h_1}-\frac{2R}{1-h_1}}{2(1-\epsilon)P_X+\frac{2P_Y}{1-h_1}-\frac{2R}{1-h_1}} = \frac{\frac{2-2(1-\epsilon)(1-h_1)}{h_1}+\frac{2R}{h_1}}{[\frac{2-2(1-\epsilon)(1-h_1)}{h_1}]P_X+\frac{2R}{h_1}} \tag{208}$$

$$\frac{2(1-\epsilon)+\frac{1}{1-h_1}[2q_u^{h_1}+2R(1-q_u^{h_1})-2R]}{2(1-\epsilon)+\frac{2P_Y}{1-h_1}-\frac{2R}{1-h_1}} = 1 \tag{209}$$

$$\frac{2(1-\epsilon)+\frac{1}{1-h_1}[(2-2R)q_u^{h_1}]}{2(1-\epsilon)+\frac{2P_Y-2R}{1-h_1}} = 1 \tag{210}$$

$$\frac{(1-\epsilon)+\frac{1}{1-h_1}[(1-R)q_u^{h_1}]}{(1-\epsilon)+\frac{P_Y-R}{1-h_1}} = 1 \tag{211}$$

$$(1-\epsilon)+\frac{P_Y-R}{1-h_1} = (1-\epsilon)+\frac{1}{1-h_1}[(1-R)q_u^{h_1}] \tag{212}$$

$$\frac{P_Y-R}{1-h_1} = \frac{1}{1-h_1}[(1-R)q_u^{h_1}] \tag{213}$$

$$P_Y-R = (1-R)q_u^{h_1} \tag{214}$$

Substituting from equation (173) for P_Y gives:

$$\frac{\epsilon(1-h_1)+R}{h_1}-R = (1-R)q_u^{h_1} \tag{215}$$

$$\frac{\epsilon(1-h)+R}{h}-R = (1-R)q_u^{h} \quad \forall \text{ equilibrium values of h} \tag{216}$$

The left-hand-side of (216), denoted $V(h)$, is a decreasing function of h while the right-hand-side, denoted $W(h)$, is an increasing function of h. Equilibrium occurs at $h = h_1$, where both curves intersect. Any decrease in ϵ, corresponding to increased reserve requirements will shift V(h) downward while leaving W(h) unchanged. These changes should act to decrease h_1, increasing the proportion of optimists in equilibrium. Remember also that reserve requirements decrease the quantity of the risky asset held by a given optimist while increasing their holdings of the riskless asset. The net result is that the equity position of defaulting firms in the downstate should become less negative while the proportion of insolvent firms increases. This continues with progressive decreases in ϵ until eventually optimists will hold sufficient quantities of the riskless asset that they are no longer at risk of default. It may then be useful to evaluate the value of ϵ at which the increase in instability reverses abruptly and no defaults are observed. To this end, another numerical example is applied and the corresponding results listed in Table 8. For this particular example, it is assumed that $q_U^h = h$, $R = 0.2$, and $D = 1.6$.

Table 8: Cash Reserve Requirements - Leverage Economy

Simulation	ϵ	h_1	P_Y	TL% (Upstate)	TL%(Downstate)
1	1.0	0.68614	0.74891	0	0.31386
2	0.9	0.67179	0.73743	0	0.32821
3	0.8	0.65587	0.72470	0	0.34413
4	0.7	0.63809	0.71047	0	0.36191
5	0.6	0.61803	0.69443	0	0.38197
6	0.5	0.59517	0.67614	0	0.40483
7	0.4	0.56873	0.65498	0	0.43127
8	0.3	0.53759	0.63007	0	0.46241
9	0.2	0.50000	0.60000	0	0.50000
10	0.1	0.45293	0.56235	0	0.0000
11	0.0	0.39039	0.51231	0	0.0000

As expected, the results indicate that ϵ, h_1, and P_Y all move in the same direction. Increases in cash reserve requirements decrease the price of the risky asset and decrease the number of pessimists in equilibrium. For small and intermediate values of ϵ stronger cash reserve requirements are associated with more defaults. Of course, once ϵ is sufficiently high, downstate equity values of optimists will become positive due to the buffer offered by increased holdings of riskless assets. At this point, no optimistic firms default, and stability is ensured. Unfortunately, the level of ϵ at which this occurs is very low, implying that reserve requirements would be large enough to restrict credit severely which could have adverse consequences for economic growth. Thus, reserve requirements may not be a promising avenue of reform geared toward preserving stability under heterogeneous expectations. Instead, policymakers may wish to scrutinize the trading of credit default swaps which this paper suggests may exert a destabilizing influence upon financial systems.

Of course, results arising from the analysis of this section are subject to an important caveat. Since all obligations in the model, other than deposits, are secured by sufficient collateral to ensure all promised payments are made to counterparties, the model does not adequately account for credit risk. For this reason, it is unable, and does not purport, to explain defaults arising from contagion, an important component of the most recent crisis. Thus, conclusions about the merits of specific policy reforms geared toward enhancing stability cannot be made exclusively on the basis of the preceding analysis. This does not, however, negate the significance of any of the results in the paper. Rather, this caveat merely suggests that the scope of what the model explains is substantial but not yet all-encompassing. It is therefore an integral piece of a larger puzzle. Policy implications of the model are worthy of consideration but should also be weighed carefully against those arising from other studies that focus upon features of the financial system not considered here.

7 Conclusion

This study presents an extended version of an existing model of asset pricing under heterogeneous expectations and introduces institutions as the market participants of interest. The corresponding analysis shows that under a leverage economy, where no tranching of payoff streams is observed, an upward shift in institutions' probability functions generates more defaults in equilibrium. Under a CDS economy, the same shift elicits fewer defaults in the upstate and more in the downstate, while the aggregate impact on financial stability is ambiguous. The effects of accentuating heterogeneity through a rotation in the probability function depend upon the position of the locus of rotation. Anytime the proportion of pessimists in equilibrium, h_1, is greater (less) than the value of h corresponding to the locus, a counterclockwise rotation will have similar effects to those of an upward (downward) shift in the probability function.

By comparing outcomes in the leverage economy to those in the CDS economy, it can be shown that the former dominate the latter in terms of measures of stability. This is primarily because credit insurance provides market participants with an opportunity to tailor their asset holdings to match their heterogeneous beliefs. This leads to a situation where optimists and pessimists have payoff profiles that are heavily skewed in opposite directions, effectively ensuring that once a given state of nature is realized, one of the two groups will default while the other profits handsomely.

Finally, the model suggests that in the leverage economy, cash reserve requirements may not be an effective mechanism of preserving stability. This may initially seem counterintuitive but makes sense if one considers that more stringent requirements will act to increase the number of optimists in equilibrium. While the absolute value of negative equity attributed to each optimist is smaller, so long as it remains negative the number of defaults in equilibrium will be positively related to required reserves. Only at unrealistically large values of reserve ratios do stricter regulations eliminate negative equity and protect against defaults.

References

[1] Fostel, Ana, and John Geanakoplos. "Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes," *American Economic Journal: Macroeconomics* 4, no. 1 (2012): 190-225.

[2] Harrison, Michael J., and David M. Kreps. "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," *The Quarterly Journal of Economics* 92, no. 2 (1978): 323-336.

[3] Miller, Edward. "Risk, Uncertainty and Divergence of Opinion," *The Journal of Finance* 24, no. 4 (1977): 1151-1168.

[4] Neave, Edwin. "Heterogeneous Expectations and Financial Equilibrium: Comparative Statics Analyses," *First International Conference on Banking and Finance - Bali*, (2013): 1-31.