Competing Media of Exchange in a Frictional Monetary Model

Author:

Ralph Oliquino

Supervisor:

Dr. Marco Cozzi

An essay submitted to the Department of Economics

in partial fulfillment of the requirements for the degree of Master of Arts

Queen's University

Kingston, Ontario, Canada

September 2013

Acknowledgements

I am grateful to my supervisor Dr. Marco Cozzi, for his guidance throughout the process of writing this paper. I would also like to thank Dr. Allen Head for his helpful comments, and Dr. Amy Sun for sparking my interest in Monetary Economics. Thank you as well to my colleague Stephen Snudden for the many helpful discussions.

I would like to acknowledge Queen's University for its financial support.

Lastly, thank you to Dr. Dirk Everaert for his continued support.

Contents

1 Introduction

The purpose of this paper is to study the concurrent use of fiat money and real assets in exchange. We begin by posing the question: why hold money in the first place? On the basis of fundamental value, it is difficult to justify holding intrinsically worthless fiat money in lieu of equity shares or capital goods. The obvious response to this question is that money facilitates exchange: it is the most *liquid* asset. But then, which market frictions give rise to money's liquidity?

Answering our first question has preoccupied economists since as early as Hicks (1935), who identified the coexistence of money and real assets as "the central issue in the pure theory of money." Hicks also recognized that answering the coexistence problem required answering our second question: understanding why money is valued necessitates understanding the frictions that are best overcome by money. In other words, Hicks recognized that money is essential foremost for its role as a medium of exchange. Much of the literature following Hicks, however, neglects frictions entirely. These papers employ reduced-form assumptions that money enters the utility function as if it were a consumption good, or that money is the only asset that satisfies a cash-in-advance constraint to ensure that money is held. Money is assumed to be valued.

In the words of Wallace (1998), "money should not be a primitive in monetary theory"; the state of monetary macroeconomic models was dissatisfactory for Wallace and other scholars in the field. Only recently has a literature coalesced that addresses the issues raised by Hicks. Surveys in Williamson and Wright (2011, 2012) identify this group under the banner of *New Monetarism*. What these theories have in common is an insistence on the use of micro-foundations—employing techniques from Search Theory such as bargaining and random matching—to make explicit the act of exchange and the frictions that impede those processes. Indeed, Wallace (1998) sets forth a dictum that is illustrative of aims of this group: Wallace prescribes specifying the physical environment, while refraining from dictating agents' actions. The intent is that it is a result that money is valued—emerging endogenously in the model—rather than having an assumption enforce the result. By better understanding money's essential role in the process of exchange, we can better understand the monetary transmission mechanism.

In this paper, we adhere to Wallace's dictum and employ a version of the model in Lagos and

Wright (2005) (henceforth LW) to address the coexistence of fiat money and real assets. Borrowing from Bryant and Wallace (1979)—who suggest that a real cost must be incurred to trade with large-denomination nominal bonds—we argue that trading with real assets also imposes a real cost. Since real assets are immobile, one who wishes to use them as a medium of exchange must resort to trading with claims to their ownership. This raises commitment issues that hinder such trades: appraisal of value, verification of claims and enforcing the transfer of ownership. Here, we take the extreme position and say that assets claims cannot be traded at all without appealing to the aid of financial intermediaries. Being specialized institutions, intermediaries can solve the problems associated with appraisal, verification, and transfer, allowing agents to trade with their asset holdings at a cost: the intermediary is a commitment mechanism. We will show that this allows money to have an explicit welfare-improving role as a less costly medium of exchange.

The rest of the paper is organized as follows. The second section provides a brief overview of the monetary search literature, focusing on the developments that have led to the model employed here. The third section sets forth the environment of the model. The fourth works through the agent's decision rules. The fifth discusses equilibrium and variations on the model. The sixth considers a version of the model with a discrete stochastic process. The seventh concludes.

2 The Literature

A precursor to New Monetarist models is that developed in Kiyotaki and Wright (1989, 1993) (henceforth KW). This model places agents in an anonymous environment characterized by a lack of record-keeping and enforcement. Agents consume and produce only a certain subset of consumption goods, and cannot consume the good that they themselves produce. Since trading partners are encountered at random, a double coincidence of wants problem emerges whenever a match is formed where one or both parties are not interested in the production of the trading partner. A lack of record-keeping and enforcement preclude the use of pure credit such that fiat money has an essential role as a medium of exchange. Like other early papers, KW encountered the technical problem of dealing with the distribution of money across agents. Since agents trade with each other, they care about the distribution of money, a potential trading partner's money holdings influencing their potential gains from trade. The problem arises with repeated rounds of trading: idiosyncrasies in trading opportunities over time make the distribution of money holdings intractable. Certainly, computational approaches could provide insights, but an analytical solution would require dealing with the distribution problem.[1] KW overcame this issue by enforcing strong restrictions on either the divisibility of money, or the amount of it that agents can hold. While not desirable, these restrictions allowed for a tractable environment in which money is valued for its use in exchange.

One of the first tractable models to relax these restrictions is the model in Shi (1997) (henceforth SS), where perfectly divisible fiat money circulated freely among agents. The model uses *large households* comprised of agents with different roles in the marketplace, who maximize the utility of the entire household. Agents enter the marketplace carrying a certain amount of money, and encounter trading partners at random. Once the marketplace closes, agents of the same household pool their money together. Since agents within a household exist in a continuum, pooling of money eliminates uncertainty and creates a degenerate distribution of money across households. Degeneracy dispenses of the need to track the distribution of money as a state variable, affording a tractable model without the strong restrictions of previous work.

More recently developed is the model in LW, which also allows a degenerate distribution to exist in equilibrium. As opposed to the large household framework, degeneracy is achieved using quasi-linear preferences, along with alternating markets with distinct characteristics. Agents begin each period in a decentralized market similar in structure to that in KW: an anonymous environment with randomized matching creates a double coincidence of wants problem that can be solved only with money. Following decentralized trade, agents enter a centralized Walrasian market where the value of money clears the market. As we will see in the model developed in this paper, quasi-linearity of utility eliminates the income effect of money in the centralized market: with homogenous preferences, all agents leave the centralized market with the same quantity of money, irrespective of each agent's money holdings coming into the market. While using very different structures to achieve a degenerate distribution of money, LW and SS arrive at similar equilibrium decision rules;

[1]Green and Zhou (2002) and Chiu and Molico (2010), amongst others, use computational methods in monetary models with search frictions.

indeed, the two models can be interchanged in many contexts. A particular advantage to LW, however, is that the value of money is determined competitively in the centralized market. In the large household framework, the value of money is specific to the individual household, leading to complications absent in LW. In this paper we opt for the LW framework, introducing real assets that can compete with money as a medium of exchange.

Extensions to LW which allow for capital to act as a medium of exchange include Lagos and Rocheteau (2008) and Geromichalos et al. (2007).[2] Lagos and Rocheteau (2008) introduce a storage technology that allows agents to create capital from the centralized market good. This capital competes with money as a medium of exchange in decentralized trading. It emerges that fiat money has an essential (Pareto-improving) role so long as the storage technology that creates capital is insufficiently productive. Geromichalos et al. (2007) introduce an asset ressembling the trees in Lucas (1978). Trees exist in a fixed quantity and produce a dividend of fruit every period in the centralized market. Geromichalos et al. find that money has an essential role when the fixed stock of trees is too small to satisfy the liquidity needs of the economy. Note how the result in this modelling approach provides a similar real world interpretation to the result in Lagos and Rocheteau: a shortage of liquid assets allows money to be valued. Indeed, one can see the fixed stock of assets in Geromichalos et al. as a special case of the storage technology in Lagos and Rocheteau. In this paper, we opt for the interpretation provided by a fixed stock of assets.

3 The Environment

The environment is similar to Geromichalos et al. (2007). Time is discrete. There is a $[0, 1]$ continuum of infinitely-lived agents. Each period is divided into two subperiods: a decentralized market (DM) in the day, followed by a centralized market (CM) at night. The discount factor is $\beta \in (0, 1)$; agents discount between periods but not between subperiods. There are two perfectly divisible, non-storable consumption goods x and X. In the day, the special good x can be produced one-for-one with labor h. At night, the general good X (the *numéraire*) is consumed by all agents

[2] Here we focus on models where capital is not completely illiquid. See Aruoba et al (2011) and Waller (2011) for an environment where illiquid capital is incorporated into LW, proposing a version of the neoclassical growth model with search frictions.

and can be produced one-for-one with labor H. Preferences are represented by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[u(x_{t+s}) - c(h_{t+s}) + U(X_{t+s}) - H_{t+s} \right],$$

where $U' > 0,\ U'' < 0,\ u' > 0,\ u'' < 0,\ c' > 0,\ c'' \geqslant 0,\ u(0) = c(0) = 0$. The Inada conditions hold: $\lim_{x\to\infty} u'(x) = 0$ and $\lim_{x\to 0} u'(x) = \infty$. There exist $X^* \in (0,\infty) : U'(X^*) = 1$, where $U(X^*) > X^*$, and a quantity $q^* \in (0,\infty) : u'(q^*) = c'(q^*)$. Unless otherwise noted, we will omit time indexes and denote variables corresponding to the next period with subscript $+1$.

Agents begin the day in the DM, a search environment characterized by random matching, anonymity and limited enforcement: potential trading partners are encountered at random, credit is infeasible and trades are *quid pro quo*. The good consumed in this market, x, comes in many varieties, of which an agent consumes and produces only a subset. An agent cannot consume the x that he himself produces. Each round of decentralized trading, an agent has a probability α of encountering someone drawn at random from the population. The probably that agent i consumes the good produced by agent j, but not vice-versa (a single coincidence of wants), is γ. By symmetry, the probability that agent j consumes the good produced by agent i, but not vice-versa, is also γ. There are no double coincidence of wants. We call an agent that consumes in the DM a *buyer* and an agent that produces a *seller*. An agent that meets someone with whom there is a single-coincidence of wants is said to be *matched*. Thus, an agent has an equal probability $\sigma = \alpha\gamma$ of being matched as either a buyer or a seller. Note that since there are no double-coincidence of wants $\sigma \leqslant 1/2$, where $\sigma = 1/2$ implies that the agent will always be matched as either a buyer or a seller.

Once an agent is matched with a potential trading partner, the pair must bargain over the *terms of trade*. Since there is a double coincidence of wants problem, and since anonymity precludes payment with credit, we do not yet know what the buyer can provide the seller in exchange for x. For now, we set forth the trading protocol: a buyer decides the terms of trade and makes a take-it-or-leave-it offer to the seller. In other words, the buyer has all the bargaining power. We assume that an indifferent seller accepts the terms of trade—already, we can foresee that the buyer will choose an allocation such that he captures all potential trade surplus.[3]

Following decentralized trade, agents enter the Walrasian CM. In this frictionless market, an agent can costlessly trade with anyone else, allowing him to purchase the exact quantity of the general good X that he wishes to consume. The frictionless setting also allows the agent to adjust his portfolio of assets for the next period. We proceed by discussing what this portfolio can consist of.

Agents have access to an intrinsically worthless object we call *money.* Money is durable, perfectly divisible, portable, and non-counterfeitable. Money cannot be consumed and promises only its purchasing power in the CM ϕ. Notice how, despite having no intrinsic value, money's physical properties make it an attractive medium of exchange in decentralized trade; in particular, since money cannot be counterfeited, a seller need not fear accepting fake fiat even in the anonymous environment of the DM. We denote an agent's money holdings as m and the aggregate money stock as M. The money supply grows at the rate μ such that $M_{+1} = (1 + \mu)M$. Monetary policy is conducted by the government through lump-sum transfers (taxes) at night in the amount of $\tau = \mu M$, where $\mu < 0$ implies a lump-sum tax.

In addition to money, there is a fixed stock of real assets we can think of as *trees.* These assets are durable and perfectly divisible. No one can produce trees and the economy is endowed with a fixed stock A. Trees are also immobile and the entire stock is located in the CM. Agents can own a proportion of the total stock of trees; we denote an agent's holdings with a. At the beginning of each CM, the economy's trees produce a random quantity δ of perishable fruit—these dividend shocks have the natural interpretation of aggregate productivity shocks. Dividends follow the stochastic process $F(\delta)$ with support $[0, \delta_{max}]$. Dividend shocks are independent across periods such that $\{\delta_t\}$ is a sequence of i.i.d. random variables. A costless technology exists that informs all agents at the beginning of each DM of the productivity of trees—the level of δ—that will be observed in that period's CM. Finally, agents at night can costlessly exchange claims on trees at the price ρ; however, in the day agents are unable to trade with assets unless they appeal to *intermediaries.*

[3] While the trading protocol used here is simple, it nonetheless captures the essential inefficiencies that can arise with periodic bargaining. The original LW model used Nash bargaining, which leads to a hold-up inefficiency so long as the buyer does not possess complete bargaining power—the bargaining protocol used here is the limiting case when the buyer *does* have all the bargaining power.

This final point is elaborated below.

An agent must appeal to specialized institutions, intermediaries, to facilitate the exchange of assets in the DM: this is rationalized by the environment of the DM and the physical characteristics of trees. First, trees are immobile and can only be accessed while in the CM; while this does not preclude the exchange of asset claims, it does imply that the seller cannot observe the buyer's stock of trees. Note how this issue of information asymmetry is only aggravated by the presence of anonymity and its corollary, lack of record keeping, in the DM—the seller can neither assess the value of the buyer's trees, nor assess if the buyer fairly conducted past trades. To see the implications of this setting, we can consider the scenario where the buyer issues counterfeit asset claims: the seller has no means to authenticate the buyer's claims. Further, the seller cannot observe the trade history of the buyer, the seller cannot infer the true value of the buyer's tree holdings, nor the probability that the claims are valid. In particular, if counterfeit claims can be costlessly produced, and there is no means to punish a counterfeiter, then the seller has no incentive to accept asset claims at all. While costless counterfeiting and lack of enforcement are somewhat strong assumptions, they rationalize the complete illiquidity of assets in decentralized trade.

We can consider a second scenario that arises due to the immobility of the asset. Since trees can only be accessed in the CM, transfer of ownership can only be affected at night. Thus, we can think of the buyer's payment of asset claims as a form of credit: in the day, the buyer purchases x and promises repayment by conferring ownership of his trees to the seller at night. This credit arrangement, however, is infeasible when there is a lack of enforcement. If there is no mechanism to enjoin the buyer to repay his debt, or there is no penalty for a buyer that reneges, then a buyer's promise of repayment is not credible—a seller would reject any offer which promises to provide assets in the future. Thus, the decentralized trading environment leads to illiquidity of the asset.

The two scenarios presented here suggest that assets are illiquid in the day market. Both arguments reduce to a commitment issue between the buyer and seller arising from information asymmetries. Either, or both scenarios can be used to justify our stance that assets cannot be traded in the DM without appealing to intermediaries. As specialized institutions, intermediaries can help overcome the frictional search environment: the intermediary can validate the buyer's claims to asset holdings and can ensure the safe transfer of ownership. In other words, our intermediaries act as a commitment mechanism. We assume that the industry of financial intermediaries is perfectly competitive such that the price for the homogenous good (i.e. the service), is ξ. The fee ξ is a lump-sum payment measure in terms of utility that a buyer may choose to pay once he is matched. If the buyer pays ξ, then the intermediary facilitates the exchange of all asset claims for the buyer in that period. Buyers must pay ξ for each period in which they wish to trade with their tree holdings.[4]

Before proceeding to the agents' value functions in the next section, we summarize the timeline of events in each period:

1. Agents enter the DM from the previous period's CM with their money and asset holdings. They are informed of the quantity of fruit, δ, that trees will yield that night.

2. Each agent is matched as a buyer, or as a seller with equal probability σ. A matched buyer decides whether he will incur the the fixed utility cost ξ—if he does, he can use both money and assets as payment in the DM, otherwise his only means of payment is money.

3. Buyer and seller pairs bargain over the terms of trade, where the buyer makes a take-it-or-leave-it offer. The seller chooses to accept or reject the offer. If rejected, the match ends and both agents proceed to the CM. If the terms are accepted, the match ends with the seller producing the specified quantity of the special good and the buyer handing over his payment of money, and possibly assets.

4. Agents enter the CM bringing with them their stocks of money and assets. The economy's stock of trees bears fruit and agents receive a quantity proportional to their share of the total stock.

5. Finally, agents choose the quantity of the general good to consume and the amount of labor to supply. Agents can also adjust their portfolios, optimizing the quantity of money and assets

[4]The model developed here is related to that in Lester et al. (2012). They argue that due to anonymity and the ability to counterfeit asset claims, an information technology is necessary for the exchange of assets. In their model, however, agents must invest *ex ante* into the technology—equilibrium emerges from strategic complementarities. In this paper, since the agent pays for the information technology (the intermediary) at the time of trade, the illiquidity of assets arises from the uncertainty in dividends.

to carry into the next period.

4 Decision Rules

In this section, we discuss an agent's decision rules as he navigates the decentralized and centralized markets in each period. Let the agent's value function in the DM be V and in the value function CM be W.

4.1 Centralized Trade

We begin with the CM. Following decentralized trade, agents enter the night market carrying a quantity m of money and owning a stock a of trees. The CM opens: the economy's stock of trees bears fruit and each agent receives a share $a\delta$. Since fruit is perishable, agents must consume their share within the current CM. Given this, the agent decides the quantity of the general good X to consume, the amount of labor H to supply, and the quantity of money $\hat{m}$ and assets $\hat{a}$ to bring into the next period. Note also that monetary injections (contractions) also occur in the CM: agents receive a lump-sum transfer (tax) in the amount of $\tau = \phi\mu M$, $\mu < 0$ implying a lump-sum tax.[5] Subject to his budget constraint, the agent's value function satisfies the Bellman equation

$$\begin{aligned} W(m,a,\delta) = & \max_{X,\,H,\,m_{+1},\,a_{+1}} \left\{U(X) - H + \beta\,\mathbb{E}\left[V(m_{+1},a_{+1},\delta_{+1})\right]\right\} \\ & \text{st. } X = H + \phi(m - m_{+1}) + (\rho+\delta)a - \rho a_{+1} + \tau, \end{aligned} \tag{1}$$

where there are nonnegativity constraints on X, m_{+1} and a_{+1}. Since utility is linear in labor, we can substitute for H from the budget constraint and find that the solution to X is always X^*. Using this fact, we rewrite (1) as

$$\begin{aligned} W(m,a,\delta) = & \; U(X^*) - X^* + \phi m + (\rho+\delta)a + \tau \\ & + \max_{m_{+1}\geqslant 0,\,a_{+1}\geqslant 0} \left\{-\phi m_{+1} - \rho a_{+1} + \beta\,\mathbb{E}\left[V(m_{+1},a_{+1},\delta_{+1})\right]\right\}. \end{aligned} \tag{2}$$

[5] To be precise, transfers are received at the beginning of the CM, while taxes are levied at the end of the CM after the money has been used in transactions. This timing ensures that transfers affect the value of money alone.

The value function (2) has several implications. First, W is linear in m and and in a with slopes ϕ and $(\rho + \delta)$, respectively. Second, the agent's CM problem reduces to choosing the quantities of money and assets to bring into the next period's DM. We call this decision—the maximization problem in (2)—the agent's *portfolio problem*. Third, the state variables m and a do not figure into the portfolio problem; that is, the choice of next period's money and assets holdings is independent of the holdings that the agent brought into the current CM. This result emerges from the quasi-linearity of the utility function; we will see that it allows for the existence, in equilibrium, of a degenerate distribution of money and assets across agents.

4.2 Decentralized Bargaining

Before stating the agent's value function in the DM, we wish to derive the results of the bargaining problems that an agent may encounter. Agents enter the DM from the previous period's CM and are immediately informed of the current period's aggregate productivity shock δ, i.e., the quantity of fruit that trees will yield later that night. Each agent is then matched as a buyer, or a seller, with equal probability σ. The matched buyer decides whether he wishes to incur the cost ξ in order to be able to trade with his asset holdings. For convenience, we refer to a match in which asset claims exchange hands as a *type–a* trade and a match in which only money exchanges hands as a type *type–m* trade. Once the buyer has decided on the type of trade, the buyer and seller pair engages in the bargaining game: the buyer decides the terms of trades and makes a take-it-or-leave-it offer to the seller; if the seller accepts, then the specified terms of trade are exchanged and the match ends; if the seller rejects, then the match ends immediately. We will analyze each type of trade from the perspective of the buyer who must decide on the terms of trade to offer to the seller.

4.2.1 Trades with Money

We first consider a buyer's problem in a type–m trade, where the buyer chooses not to pay ξ. For this type of match, denote with q_m the amount of special good produced by the seller and with d_m the buyer's payment in terms of money. Then, the buyer's problem is to choose the terms of trade (q_m, d_m) that maximize his trade surplus. Note that the buyer's best response is to set the terms of trade such that seller accepts the offer. Since we assume that an indifferent seller always accepts the offer, the buyer will choose q_m and d_m such that the seller's trade surplus is nonnegative; we call this the seller's participation constraint. Let m and a be the buyer's holdings of money and assets at the time of the trade, and $\tilde{m}$ and $\tilde{a}$ be those of the seller. The productivity shock in the current period is denoted δ. We emphasize that the costless information technology informs all agents of the level of the amount of fruit that trees will yield in the proceeding CM—this allows us to treat δ as a state variable even in the DM. The type–m bargaining problem from the perspective of the buyer is

$$\begin{aligned} \max_{q_m,\, d_m} \quad & \{u(q_m) + W(m - d_m, a, \delta) - W(m, a, \delta)\} \\ \text{st.} \quad & -c(q_m) + W(\tilde{m} + d_m, \tilde{a}, \delta) - W(\tilde{m}, \tilde{a}, \delta) \geqslant 0, \\ & 0 \leqslant d_m \leqslant m, \\ & q_m \geqslant 0. \end{aligned} \tag{3}$$

The objective function in (3) is exactly the buyer's trade surplus. The first constraint is the seller's participation constraint. The two remaining constraints are feasibility constraints; $d_m \leqslant m$ states that the buyer's payment in money cannot exceed the amount of his money holdings, i.e., credit is ruled out.

Using the results on the linearity of W, (5) simplifies to

$$\begin{aligned} \max_{q_m,\, d_m} \quad & \{u(q_m) - \phi d_m\} \\ \text{st.} \quad & -c(q_m) + \phi d_m \geqslant 0, \\ & 0 \leqslant d_m \leqslant m, \\ & q_m \geqslant 0. \end{aligned} \tag{4}$$

Note how the seller's money and assets do not figure into the buyer's bargaining problem—only the buyer's portfolio affects the terms of trade. To solve the bargaining problem, note that in maximizing his own trade surplus, the buyer will leave the seller with zero trade surplus, i.e., the participation constraint holds at equality. We proceed by substituting the participation constraint into the objective and considering the unconstrained optimum quantity—the buyer's choice of q_m if he were not constrained by $d_m \leqslant m$. That quantity is q^*, the efficient allocation in decentralized trade. Given this result, we can enforce the constraint and find terms of trade

$$q_m(m) = \begin{cases} q^* & \text{if } \phi m \geqslant c(q^*), \\ c^{-1}(\phi m) & \text{if } \phi m < c(q^*), \end{cases} \tag{5}$$

$$d_m(m) = \begin{cases} \frac{c(q^*)}{\phi} & \text{if } \phi m \geqslant c(q^*), \\ m & \text{if } \phi m < c(q^*). \end{cases} \tag{6}$$

As we identified above, the terms of trade are a function of the buyer's portfolio alone—specifically, only the buyer's money holdings affect the type–m terms of trade. In words, (5) and (6) state that the buyer spends all of his money up until he can afford the efficient quantity of the special good. If the buyer can afford the efficient quantity, then he offers just enough to such that the seller is willing to produce q^*.

Before considering the type–a bargaining problem, we note a result that emerges from the terms of trade (5) and (6). If $\phi m \geqslant c(q^*)$, the buyer can purchase the efficient quantity with money alone: the buyer has no incentive to pay with his tree holdings and incur the cost ξ. In this model, real assets have a role as a medium of exchange so long as agents carry insufficient money holdings to purchase the efficient level of DM output. This leads to the following lemma

Lemma 1. *If a matched buyer's money holdings, m, are such that $\phi m \geqslant c(q^*)$, then the buyer does not pay ξ and uses only money as a medium of exchange.*

4.2.2 Trades with Money and Assets

If $\phi m < c(q^*)$, then it may be possible for the buyer to derive more trade surplus in a type–a trade. The precise conditions under which type–a trades are preferred over type–m trades will be derived further on; for now, we determine the terms of trade assuming that the buyer has found it optimal to pay ξ. For a type–a trade, let q_a be the quantity of the special good produced, d_m^ξ be the amount of money exchanged, and d_a be the amount of assets exchanged. The buyer makes the offer such that the seller is willing to accept; as in the type–m trade, this implies the presence of the seller's participation constraint. Once again, let m and a represent the buyer's portfolio, $\tilde{m}$ and $\tilde{a}$ the seller's portfolio, and δ the productivity shock. Then the buyer's type–a bargaining problem is

$$\begin{aligned} \max_{q_a,\, d_m^\xi,\, d_a} \quad & \Big\{ u(q_a) + W(m - d_m^\xi, a - d_a, \delta) - W(m, a, \delta) - \xi \Big\} \\ \text{st.} \quad & -c(q_a) + W(\tilde{m} + d_m^\xi, \tilde{a} + d_a, \delta) - W(\tilde{m}, \tilde{a}, \delta) \geqslant 0 \\ & 0 \leqslant d_m^\xi \leqslant m, \\ & 0 \leqslant d_a \leqslant a \\ & q_a \geqslant 0. \end{aligned} \tag{7}$$

The objective function in (7) is the buyer's trade surplus; notice that it is decreased by the fixed cost ξ. The first constraint is the seller's participation constraint which states that the seller's trade surplus must be nonnegative—to maximize the buyer's trade surplus, the participation constraint will hold with equality. The three remaining constraints are feasibility constraints. $d_m^\xi \leqslant m$ and $d_a \leqslant a$ rule out any form of credit. Using the linearity of W, (7) simplifies to

$$\begin{aligned} \max_{q_a,\, d_m^\xi,\, d_a} \quad & \Big\{ u(q_a) - \phi d_m^\xi - (\rho + \delta) d_a - \xi \Big\} \\ \text{st.} \quad & -c(q_a) + \phi d_m^\xi + (\rho + \delta) d_a \geqslant 0 \\ & 0 \leqslant d_m^\xi \leqslant m, \\ & 0 \leqslant d_a \leqslant a, \\ & q_a \geqslant 0. \end{aligned} \tag{8}$$

We can solve (7) in the same manner as the type–m bargaining problem. Without loss of generality, we assume that the buyer expends all of his money before resorting to payment with his assets. Since a type–a trade only occurs when the buyer has insufficient money holdings to purchase q^*, this assumption implies that

$$d_m^\xi = m. \tag{9}$$

That is, the buyer always expends his money holdings in a type–a trade. The remaining terms of trade are

$$q_a(m, a, \delta) = \begin{cases} q^* & \text{if } \phi m + (\rho + \delta)a \geqslant c(q^*), \\ c^{-1}\left[\phi m + (\rho + \delta)a\right] & \text{if } \phi m + (\rho + \delta)a < c(q^*), \end{cases} \tag{10}$$

$$d_a(m, a, \delta) = \begin{cases} \frac{c(q^*) - \phi d_m^\xi}{(\rho + \delta)} & \text{if } \phi m + (\rho + \delta)a \geqslant c(q^*), \\ a & \text{if } \phi m + (\rho + \delta)a < c(q^*). \end{cases} \tag{11}$$

In words, if the buyer's money and asset holdings are insufficient to purchase q^*, then the buyer gives up his entire portfolio to purchase the quantity $c^{-1}\left[\phi m + (\rho + \delta)a\right]$. If the buyer can afford q^*, then he expends his money and pays for the remainder with his assets. As in type–m trades, only the buyer's portfolio figures into the terms of trade.

4.2.3 The Choice to Trade with Assets

Now that we have solved the bargaining problem for each type of trade, we can determine when the buyer chooses to pay the intermediaries' fee ξ. By Lemma 1, when $\phi m \geqslant c(q^*)$, the buyer will not pay ξ, opting instead for type–m trades; when $\phi m < c(q^*)$, we will show that the aggregate productivity shock determines which type of trade is chosen. First notice that $q_a \geqslant q_m$; however, the buyer is willing to incur ξ only if the trade surplus in a type–a trade exceeds the trade surplus in a type–m trade, i.e., the benefit of being able to trade with asset claims compensates for the fixed cost ξ. Thus, the buyer's trade surplus can be defined as

$$\mathcal{S}(m, a, \delta) = \max\left\{ u(q_m) - \phi d_m, \quad u(q_a) - \phi d_m^\xi - (\rho + \delta)d_a - \xi \right\}. \tag{12}$$

The first argument in the maximum operator is the buyer's surplus in a type–m trade, the second argument is the surplus in a type–a trade. Notice that the surplus in a type–a trade is increasing in

δ, while surplus in a type–m trade is constant in δ.[6] For any level of m and a for which $\phi m < c(q^*)$, we can define a threshold level of productivity δ_a that equates the surpluses in type–m and type–a trades. That is, δ_a is defined implicitly by

$$u\left[c^{-1}(\phi m)\right] - \phi m \;=\; u\left[c^{-1}\left[\phi m + (\rho + \delta_a)a\right]\right] - \phi m - (\rho + \delta_a)a - \xi. \tag{13}$$

We assume that if the current period's productivity shock is exactly δ_a, then buyers opt for type–a trades. This leads us to a decision rule that dictates when the buyer pays ξ; this is formalized in the following Proposition.

Proposition 1. *If $\phi m < c(q^*)$ and $\delta \geqslant \delta_a$, then a matched buyer pays the fee ξ and consumes $q_a = c^{-1}[\phi m + (\rho + \delta)a]$, where q_a reaches its maximum, q^*, if and only if $\phi m + (\rho + \delta)a \geqslant c(q^*)$. If $\phi m \geqslant c(q^*)$ or $\delta < \delta_a$, then the matched buyer does not pay ξ and consumes $q_m = c^{-1}(\phi m)$, where $q_m = q^*$ if and only if $\phi m \geqslant c(q^*)$.*

Using the results of the two bargaining problems, we now define V; the agent's value function in the DM satisfies the Bellman equation

$$V(m, a, \delta) = \sigma\, \mathcal{S}(m, a, \delta) + W(m, a, \delta). \tag{14}$$

Looking at (14), the first term is the agent's trade surplus if he is matched as a buyer, which happens with probability σ. The agent can also be matched as a seller with probability σ; however, this disappears from (14) since the seller is always left with zero trade surplus irrespective of the type of trade. The second term in (14) is the continuation value of the CM which follows the current DM.

4.3 The Portfolio Problem

We have characterized the agent's value functions V and W; both are deterministic functions of m, a and the i.i.d. shock δ. Thus, we can reduce the agent's problem in each period to the choosing

[6] For the second argument in $\mathcal{S}$, $\frac{\partial \mathcal{S}}{\partial \delta_a} = a\left[\frac{u'\left[c^{-1}[\phi m + (\rho+\delta_a)a]\right]}{c'\left[c^{-1}[\phi m + (\rho+\delta_a)a]\right]} - 1\right] \geqslant 0$, and " $=$ " iff. $c^{-1}\left[\phi m + (\rho + \delta_a)a\right] \geqslant q^*$.

the quantities of money and assets to carry into the next period: the agent's portfolio problem. To find a recursive solution, notice how we can substitute (2) into (14) and rewrite (14) as

$$V(m, a, \delta) = \sigma\, \mathcal{S}(m, a, \delta) + \phi m + (\rho + \delta) + \nu, \tag{15}$$

where ν is the constant continuation value of the agent's portfolio choice problem. Substituting (15) into the maximization problem in (2) and dropping the constant ν, we characterize the agent's portfolio choice problem as

$$\max_{m_{+1} \geqslant 0,\, a_{+1} \geqslant 0} \Big\{ -(\phi - \beta\phi_{+1})\, m_{+1} \;\; - [\rho - \beta\, \mathbb{E}(\rho_{+1} + \delta_{+1})]\, a_{+1} \;\; + \beta\sigma\, \mathbb{E}\, [\mathcal{S}(m_{+1}, a_{+1}, \delta_{+1})] \Big\}. \tag{16}$$

From (16), $\phi - \beta\phi_{+1}$ is the net cost of carrying an additional dollar into the next period: an agent gives up ϕm_{+1} units of consumption in the current period for money which carries the discounted value $\beta\phi_{+1} m_{+1}$ the next period. Similarly, $\rho - \beta\mathbb{E}(\rho_{+1} + \delta_{+1})$ is the expected net cost of investing in a unit of asset: an agent gives up ρa_{+1} units of consumption for assets which provide the expected, discounted *cum dividend* value $\beta\, \mathbb{E}(\rho_{+1} + \delta_{+1})\, a_{+1}$ the next period. The remaining expression in (16), $\beta\sigma\, \mathbb{E}\, [\mathcal{S}(m_{+1}, a_{+1}, \delta_{+1})]$ is the agent's expected discounted decentralized trade surplus. If he is matched as a buyer in the next period, it is the agent's choice of m_{+1} and a_{+1} that determines the quantity of special good that he can purchase. Note that this trade surplus is increasing in m_{+1} and in a_{+1}, but it is necessarily bounded by the seller's participation constraint—the maximum quantity that the seller is willing to produce is q^*. Thus, a solution exists if the following two conditions hold: $\phi - \beta\phi_{+1} \geqslant 0$ and $\rho - \beta\, \mathbb{E}(\rho_{+1} + \delta_{+1}) \geqslant 0$. Note how these conditions imply that the agent has no incentive to carry more money and assets than what is necessary to purchase the efficient DM quantity. Further, when these conditions hold at equality, then carrying money or purchasing assets is costless. We will discuss what this implies in the next section.

Given nonnegativity conditions on the costs of m_{+1} and a_{+1}, a solution exists for the portfolio problem. What we require, however, is a unique solution: ensuring that $\mathcal{S}$ is jointly concave implies a degenerate distribution of money and assets leaving the CM. See Appendix A for a discussion of the conditions necessary for concavity. Here, we will assume that the conditions for concavity are satisfied. The first order conditions are then

$$-(\phi - \beta\phi_{+1}) + \beta\sigma \frac{\partial}{\partial m}\mathbb{E}\big[\mathcal{S}(m_{+1}, a_{+1}, \delta_{+1})\big] \leqslant 0 \qquad \text{and `` = '' iff. } m > 0, \tag{17}$$

$$-\left[\rho - \beta\,\mathbb{E}\,(\rho_{+1} + \delta_{+1})\right] + \beta\sigma \frac{\partial}{\partial a}\mathbb{E}\big[\mathcal{S}(m_{+1}, a_{+1}, \delta_{+1})\big] \leqslant 0 \qquad \text{and `` = '' iff. } a > 0. \tag{18}$$

To interpret these first order conditions, we proceed to the next section where we characterize the tationary equilibrium.

5 Equilibrium

To summarize the results thus far, we have reduced the agent's choice problems over the entire period to the single portfolio problem in the CM (16). This decision, choosing the money and assets holdings to bring into the next period, is independent of the portfolio which the agent brought with him into the CM—this is a result of the quasi-linear specification of the utility function. Assuming that the portfolio problem is jointly concave in its arguments, all agents leave the CM with the same quantities of money and assets. Indeed, given degenerate distributions of money and assets, market clearing conditions for the CM can be stated as: $m_{+1} = M_{+1}$ and $a_{+1} = A$. These conditions dictate the "supply curve" for money and assets at the end of the period. Demand is captured by the first order conditions, which will be discussed in the next section.

Here, we specify the stationary equilibrium. We focus on a particular type of steady state in which real prices are constant over time: the aggregate money supply maintains a constant value each period such that $\phi M = \phi_{+1} M_{+1}$, and the asset price is constant across periods such that $\rho = \rho_{+1}$. See Appendix B for a discussion of these stationarity conditions. In addition to stationarity, we further impose that the *real interest rate* r is constant and pinned down by the discount factor, i.e., $\beta = (1 + r)^{-1}$.

Given this stationary setting, we can introduce two intuitive expressions to interpret the first order conditions. First, we can characterize the *nominal interest rate* i. Suppose that the government issues risk-free nominal bonds that are completely illiquid—they cannot be traded in the DM. These securities are sold for a dollar in the CM and must be redeemed for $1 + i$ dollars in the next period's CM. Despite their illiquidity, these securities can be priced through the Fisher equation such that $1 + i = (1 + \mu)(1 + r)^{-1}$. Second, we can define the *fundamental price* of the asset as ρ^*. This is the asset price that would prevail if trees were completely illiquid and valued only for their discounted stream of dividends. Since dividend realizations δ are i.i.d., the unconditional mean of dividends over time can be defined as $\overline{\delta} = \int_0^{\delta_{max}} \delta \, \mathrm{d}F(\delta)$. Thus, the fundamental price $\rho^* = \overline{\delta}/r$.

5.1 Monetary Policy and Asset Demand

We are looking for interior solutions to the portfolio problem. Given i and ρ^*, we invoke stationarity and re-express (17) and (18) as [7]

$$i = \sigma \int_0^{\delta_a} \left[\frac{u'\left[c^{-1}(\phi m)\right]}{c'\left[c^{-1}(\phi m)\right]} - 1 \right] \mathrm{d}F(\delta) \; + \sigma \int_{\delta_a}^{\delta_{max}} \left[\frac{u'\left[c^{-1}\left[\phi m + (\rho + \delta)a\right]\right]}{c'\left[c^{-1}\left[\phi m + (\rho + \delta)a\right]\right]} - 1 \right] \mathrm{d}F(\delta), \tag{19}$$

$$\rho = \rho^* \; + \frac{\sigma}{r} \int_{\delta_a}^{\delta_{max}} \left[\frac{u'\left[c^{-1}\left[\phi m + (\rho + \delta)a\right]\right]}{c'\left[c^{-1}\left[\phi m + (\rho + \delta)a\right]\right]} - 1 \right] (\rho + \delta) \, \mathrm{d}F(\delta). \tag{20}$$

The expression (19) makes explicit the opportunity cost of holding money for decentralized trade. Rather than investing a dollar into a government bond for a nominal return i, the agent carries the dollar into the DM—the benefit of an additional dollar in decentralized trade is exactly the right-hand-side of (19). Since money is used in both type–m and type–a trades, utility from the marginal dollar can be decomposed into two expressions. We call the first integral in (19) the *liquidity premium* of money in type–m trades; it captures the utility for the marginal dollar spent in the a type–m trade. Similarly, the second integral in (19) is the liquidity premium of money in type–a trades. Since holding money benefits the agent only if he is matched as buyer, both liquidity premia are weighted by the probability of being matched as a buyer σ. Note that $q_a \geqslant q_m$ implies that the marginal dollar earns more utility in a type–m trade compared to a type–a trade.

We now turn to (20), which expresses the asset price ρ in terms of its deviation from its

[7] Note how we drop time subscripts altogether. Prices, and choice variables m and a all correspond to the following period.

fundamental value. For sufficiently high realizations of tree productivity δ, the agent's assets can be used as a medium of exchange in the DM. The benefit for using assets in decentralized trade is captured by the integral in (20)—the greater this benefit, the greater the demand for assets, and the greater ρ deviates from its fundamental price. Notice how the asset is priced fundamentally only if the the integral in (20) vanishes; this can be guaranteed if the decentralized shuts down, such that $\sigma = 0$.

To see the effect of monetary policy on the choice of money holdings, we can consider an economy with a lower rate of inflation (or a higher rate of deflation) μ. In stationary equilibrium, this implies a smaller i, i.e., there is a less of an opportunity cost to holding money. It follows that agents are willing to carry more money into the DM each period, increasing the quantity that the a matched buyer is able to purchase in both type–m and type–a trades. We can see this in (19). A decrease in the nominal interest rate i must be met by a decrease in the liquidity premia of money in both types of trades. Since the $\frac{u'(q)}{c'(q)} - 1$ is decreasing in q, we must have that the agent chooses to hold more money for decentralized trade. Note how there is a secondary effect to an increase in the agent's money holdings: the more money a matched buyer holds, the less likely he is to resort to trading with assets. Recall that the dividend threshold δ_a is defined implicitly, for some level of m and a, by (13). Differentiation gives

$$\frac{\partial \delta_a}{\partial m} = \frac{\phi}{a} \left[\frac{u'(q_a)}{c'(q_a)} - 1 \right]^{-1} \left[\frac{u'(q_m)}{c'(q_m)} - \frac{u'(q_a)}{c'(q_a)} \right] \Bigg|_{\delta = \delta_a}, \tag{21}$$

where $q_m = c^{-1}(\phi m)$ and $q_a = c^{-1}\left[\phi m + (\rho + \delta)a\right]$. The derivative (21) is positive for all $q_a > q_m$; thus, less inflationary monetary policy increases money balances held by agents and decreases reliance on assets as a medium of exchange.

We can now discuss optimal monetary policy. The results of the bargaining problems suggest that the there is a level of real balances that allows a buyer to purchase the efficient DM quantity with money alone—the monetary policy that allows this is the Friedman rule, which specifies deflating at the rate of impatience such that $\phi = \beta\phi_{+1}$. From the portfolio problem (16), the Friedman rule implies that the net cost of holding an additional dollar is zero—equivalently, the policy implies a zero nominal interest rate. Since $\frac{u'(q)}{c'(q)} - 1 = 0$ if and only if $q = q^*$, from (19), the Friedman rule implies that $q_m = q_a = q^*$. With no opportunity cost to holding money, the agent will carry as much as is necessary to purchase the efficient quantity in the DM. Note that while the Friedman rule suggests that in a type–a trade $q_a = q^*$, Lemma 1 tells us that the buyer need not pay ξ: the buyer can purchase q^* with money alone. Allowing the agent unlimited money balances eliminates the use of costly assets in exchange.

Thus far, we have assumed that both money and assets are valued in equilibrium—we have implicitly assumed that inflation is sufficiently low for money to be valued. Notice that, while the buyer's trade surplus is bounded by the seller's participation constraint, the opportunity cost of holding money i is not. For high levels of inflation, the agent will choose not to carry money altogether. Further, while we have found the Friedman rule to be optimal, it is not a policy often observed in practise. We would like to know the degree to which monetary policy can deviate from the Friedman rule when real assets are present. To address these questions, we proceed to the next section, where we specify the stochastic process for the aggregate shock δ.

6 An Example

In this section, we consider the case of a discrete stochastic process for δ: with probability π_H trees produce high dividends δ_H, and with probability $\pi_L = 1 - \pi_H$ trees produce low dividends δ_L. The rest of the environment follows what we have set forth above. We first assume that[8]

$$\delta_L < \delta_a \leqslant \delta_H. \tag{22}$$

In other words, for some level of m and a, buyers choose type–a trades if dividends are high and type–m trades when dividends are low. We will identify below under what conditions this assumption holds.

[8]The "$\leqslant$" sign is due to our assumption (see Proposition 1) that a buyer is indifferent between a type–m and a type–a trade opts for the latter.

Notice that we can invoke stationarity to rewrite the portfolio problem (16) as

$$\max_{m\geqslant 0,\, a\geqslant 0} \left\{ -i\phi m \, - r(\rho-\rho^*)a \, + \, \sigma \int_0^{\delta_{max}} \mathcal{S}(m,a,\delta)\, \mathrm{d}F(\delta) \right\}, \tag{23}$$

where the fundamental asset price $\rho^* = \overline{\delta}/r = (\pi_L\delta_L + \pi_H\delta_H)/r$. Applying the stationarity conditions in this way allows us to see immediately the costs and benefits of holding money and assets, as was discussed in the previous section. Given our assumption on the use of assets in the DM, we can substitute for the expected trade surplus such that (23) becomes

$$\max_{m\geqslant 0,\, a\geqslant 0} \Big\{ -i\phi m \, - r(\rho-\rho^*)a \, + \sigma\, \pi_L \Big[u\big[c^{-1}(\phi m)\big] - \phi m \Big] \\ + \sigma\, \pi_H \Big[u\big[c^{-1}\left[\phi m + (\rho+\delta_H)a\right]\big] - \phi m - (\rho+\delta_H)a - \xi \Big] \Big\}. \tag{24}$$

In words, the expected trade surplus in (24) is the probability weighted average of the trade surplus in both types of trades. For low dividend realizations, which happens with probability π_L, trees are not sufficiently productive to justify paying the fee ξ—a matched buyer uses only money to purchase the quantity $c^{-1}(\phi m)$. For high dividend realizations, occurring with probability π_H, the additional benefit of trading with assets is no less than ξ, allowing the buyer to also trade with assets and purchase the quantity $c^{-1}\left[\phi m + (\rho+\delta_H)a\right]$.

6.1 Monetary Equilibrium

In this case of a binomial distribution for δ, the concavity conditions discussed in Appendix A are always satisfied. Solving for the first order conditions of (24), and after some manipulation, we have the following expressions:

$$i = \sigma\, \pi_L \left[\frac{u'\big[c^{-1}(\phi m)\big]}{c'\big[c^{-1}(\phi m)\big]} - 1 \right] + \sigma\, \pi_H \left[\frac{u'\big[c^{-1}\left[\phi m + (\rho+\delta_H)a\right]\big]}{c'\big[c^{-1}\left[\phi m + (\rho+\delta_H)a\right]\big]} - 1 \right], \tag{25}$$

$$\rho = \frac{\overline{\delta} + \sigma\, \pi_H\, \delta_H \left[\frac{u'\left[c^{-1}[\phi m + (\rho+\delta_H)a]\right]}{c'\left[c^{-1}[\phi m + (\rho+\delta_H)a]\right]} - 1 \right]}{r - \sigma\, \pi_H \left[\frac{u'[c^{-1}[\phi m + (\rho+\delta_H)a]]}{c'[c^{-1}[\phi m + (\rho+\delta_H)a]]} - 1 \right]}. \tag{26}$$

The expression (25) is the analogue of (19), the right-hand-side representing the sum of money's liquidity premia. The second expression (26) is likewise similar to (20); however, specifying the simple stochastic process allows us to find a more explicit expression for the asset price ρ. We see in (26) that ρ is a contraction and that it is decreasing in a, thus yielding a downward sloping demand curve for assets. Notice that the asset price can only equal its fundamental value if: (a) the DM shuts down and $\sigma = 0$, or (b) the quantity in a type–a trade $c^{-1}\left[\phi m + (\rho + \delta_H)a\right]$ is equal to the efficient allocation in the DM q^*. The market clearing conditions for the CM imply that (b) is satisfied when

$$c^{-1}\left[\phi M + (\rho + \delta_H)A\right] \geqslant q^*. \tag{27}$$

We know from (25) that the Friedman rule implies $c^{-1}(\phi M) \geqslant q^*$, ensuring that (27) holds.[9] If we are away from the Friedman rule, then (27) is more likely to hold with larger high dividend realizations δ_H or with a larger aggregate asset stock A—intuitively, if assets are sufficiently productive to purchase q^* when they *are* used as a medium of exchange, then the marginal unit of the asset can no longer benefit the buyer.

We now address the coexistence question when we are away from the Friedman rule. Substituting (25) into (26) finds

$$\rho = \frac{\overline{\delta} + \sigma\,\pi_H\,\delta_H \left[\frac{u'\left[c^{-1}[\phi m + (\rho+\delta_H)a]\right]}{c'\left[c^{-1}[\phi m + (\rho+\delta_H)a]\right]} - 1\right]}{r - i + \sigma\,\pi_L \left[\frac{u'\left[c^{-1}(\phi m)\right]}{c'\left[c^{-1}(\phi m)\right]} - 1\right]}. \tag{28}$$

From the denominator in (28), a positive ρ requires

$$i < r + \sigma\,\pi_L \left[\frac{u'\left[c^{-1}(\phi m)\right]}{c'\left[c^{-1}(\phi m)\right]} - 1\right]. \tag{29}$$

In words, the opportunity cost of holding money i must be less than the sum of the real interest

[9] As before, we know by Lemma 1 that the buyer need not engage in a type–a trade.

rate r and the liquidity premium of money in a type-m match. We can also express (29) as

$$\mu < \beta \, \sigma \, \pi_L \left[\frac{u'\big[c^{-1}(\phi m)\big]}{c'\big[c^{-1}(\phi m)\big]} - 1 \right], \tag{30}$$

which makes explicit a necessary condition for inflation: for money and assets to coexist, the rate of inflation μ must be less than the discounted value of money's liquidity premium in type–m trades. In this model, money fulfills a unique role as a medium of exchange due to ξ. For periods where dividends are low, the cost of intermediaries prohibits the use of assets such that only money can yield trade surplus for the buyer: that trade surplus is captured by the liquidity premium in (30). If the liquidity premium in type–m trades increases—for example, due to an increase in the probability of low dividends π_L—then a monetary equilibrium can allow for higher levels of inflation μ: a greater deviation from the Friedman rule. The less productive is the asset, the greater the scope for money in exchange.

6.2 Liquid Assets

What if money does not possess a unique role? That is, what if buyers are willing to trade with assets despite low dividend yields? This would be the case when our assumption (22) *does not* hold; instead we would have that

$$\delta_a \leqslant \delta_L < \delta_H. \tag{31}$$

Following the analysis above, the asset price in stationary monetary equilibrium with (31) would be

$$\rho = \frac{\overline{\delta} + \sigma \, \pi_L \, \delta_L \left[\frac{u'\big[c^{-1}(\phi m)\big]}{c'\big[c^{-1}(\phi m)\big]} - 1 \right] + \sigma \, \pi_H \, \delta_H \left[\frac{u'\big[c^{-1}[\phi m + (\rho+\delta_H)a]\big]}{c'\big[c^{-1}[\phi m + (\rho+\delta_H)a]\big]} - 1 \right]}{r - i}. \tag{32}$$

Notice that the denominator implies $\mu < 0$. Thus, so long as assets are liquid—in the sense that agents are always willing to incur the cost ξ to trade with assets—then money can be valued only if there is deflation.

Now we want provide the conditions under which buyers are always willing to trade with assets—that is, the conditions under which money has a unique role as a medium of exchange for low dividend shocks. Equivalently, we want to see when our assumption (22) *does* hold. For this, we turn to the market clearing conditions. Enforcing that $m = M$ and $a = A$, the assumption (22) implies that DM trade surpluses are such that

$$u\big[c^{-1}\left[\phi M + (\rho + \delta_L)a\right]\big] - \phi M - (\rho + \delta_L)A - \xi \ \ < \ \ u\big[c^{-1}\left(\phi M\right)\big] - \phi M, \tag{33}$$

and

$$u\big[c^{-1}\left(\phi M\right)\big] - \phi M \ \ \leqslant \ \ u\big[c^{-1}\left[\phi M + (\rho + \delta_H)A\right]\big] - \phi M - (\rho + \delta_H)A - \xi. \tag{34}$$

Since type–a trades only occur away from the Friedman rule, we assume $\phi M < c(q^*)$. Thus, we can ignore real money balances ϕM and focus our attention on the aggregate tree stock A. Together (33) and (34) imply

$$u\big[c^{-1}\left[\phi M + (\rho + \delta_H)A\right]\big] - u\big[c^{-1}\left[\phi M + (\rho + \delta_L)A\right]\big] \ \ > \ \ (\delta_H - \delta_L)A. \tag{35}$$

We can note that the right-hand-side of the inequality (35) is linear in A. Since $u(\cdot)$ is concave, the inequality is less likely to hold for large A, i.e., a larger aggregate asset stock makes the buyer more likely to pay ξ despite that the dividend shock is low. This is a result of the intermediaries' fee being a fixed cost—once ξ is paid, a buyer can pay with any quantity of asset claims he wishes. Since CM market clearing implies that the amount of the agent's assets is A, a larger stock A decreases the relative cost of ξ. Thus, a larger asset stock makes it less likely that money fulfills a unique role in decentralized trade.

7 Conclusion

This paper has proposed an answer to the classic problem of the coexistence of fiat money and real assets. It has argued that the physical properties of assets make them an unsuitable medium of exchange. In particular, when agents are forced into environments where informational frictions raise moral hazard problems, a commitment mechanism is necessary to facilitate the exchange assets. While specialized institutions can solve the commitment problem, these institutions impose a real cost that renders trades in assets inefficient.

Fiat money has a welfare improving role by allowing the agent to engage in trades that he would otherwise turn down—money has a unique role as a medium of exchange. At the Friedman rule, there is no opportunity cost to holding money: the agent can satiate his liquidity needs with money, eliminating the need to trade with assets altogether. If there is a unique role for money, then money and assets can coexist with a positive rate of inflation. If we take away that unique role, then money is valued only if it offers a return, i.e., when there is deflation. A larger aggregate asset stock makes it less likely that this unique role exists.

The model presented here has attempted to follow the dictum set forth by Wallace (1995). A worthless object called money is valued due to its ability to overcome frictions in exchange. While the Friedman role remains the optimal monetary policy, this model can allow for the more realistic prescription of positive inflation, a result emerging directly from money's role as a medium of exchange.

Appendix

Appendix A

For convenience, let

$$\Gamma(x) \equiv \left[\frac{u'(c^{-1}(x))}{c'(c^{-1}(x))} - 1\right]$$

Note that

$$\Gamma'(x) = \frac{c'(c^{-1}(x))u''(c^{-1}(x)) \, - \, u'(c^{-1}(x))c''(c^{-1}(x))}{[c'(c^{-1}(x))]^2},$$

which is negative for all $c^{-1}(x) \geqslant 0$.

Let the objective of the portfolio problem (16) be Ψ. Since we are focused on stationary equilibrium, we can invoke stationarity (see section 5) from (16) and find the first order conditions

$$\Psi_m = -(\phi - \beta\phi) \; + \sigma\beta\phi \int_0^{\delta_{max}} \Gamma(\phi m) \, \mathrm{d}F(\delta) \; + \sigma\beta\phi \int_{\delta_a}^{\delta_{max}} \Gamma[\phi m + (\rho + \delta)a] \, \mathrm{d}F(\delta), \tag{36}$$

$$\Psi_a = -[\rho - \beta(\rho + \delta)] \; + \sigma\beta \int_{\delta_a}^{\delta_{max}} \Gamma[\phi m + (\rho + \delta)a](\rho + \delta) \, \mathrm{d}F(\delta). \tag{37}$$

Note how these can be used to find (19) and (20).

From (36) and (37), it follows that

$$\begin{aligned}\Psi_{mm} = {} & \sigma\beta\phi^2 \int_0^{\delta_{max}} \Gamma'(\phi m) \, \mathrm{d}F(\delta) \; + \sigma\beta\phi^2 \int_{\delta_a}^{\delta_{max}} \Gamma'[\phi m + (\rho + \delta)a] \, \mathrm{d}F(\delta) \\ & + \sigma\beta\phi \; \frac{\partial \delta_a}{\partial m} \left[\Gamma(\phi m) - \Gamma[\phi m + (\rho + \delta)a]\right]\Bigg|_{\delta = \delta_a},\end{aligned} \tag{38}$$

$$\begin{aligned}\Psi_{aa} = {} & \sigma\beta \int_{\delta_a}^{\delta_{max}} \Gamma'[\phi m + (\rho + \delta)a](\rho + \delta)^2 \, \mathrm{d}F(\delta) \\ & + \sigma\beta \; \frac{\partial \delta_a}{\partial a} \, \Gamma[\phi m + (\rho + \delta)a](\rho + \delta)\Bigg|_{\delta = \delta_a},\end{aligned} \tag{39}$$

$$\Psi_{ma} = \sigma\beta\phi \int_{\delta_a}^{\delta_{max}} \Gamma'[\phi m + (\rho+\delta)a](\rho+\delta)\,\mathrm{d}F(\delta) \tag{40}$$
$$+\sigma\beta\phi\ \frac{\partial\delta_a}{\partial a}\left[\Gamma(\phi m) - \Gamma[\phi m + (\rho+\delta)a]\right]\Big|_{\delta=\delta_a}.$$

Recall that the threshold δ_a is defined implicitly by (13), which we reproduce here:

$$u\left[c^{-1}(\phi m)\right] - \phi m \ = \ u\left[c^{-1}\left[\phi m + (\rho+\delta_a)a\right]\right] - \phi m - (\rho+\delta_a)a - \xi.$$

This implies

$$\frac{\partial\delta_a}{\partial m} = \frac{\phi}{a}\,\frac{\Gamma(\phi m) - \Gamma[\phi m + (\rho+\delta)a]}{\Gamma[\phi m + (\rho+\delta)a]}, \tag{41}$$

$$\frac{\partial\delta_a}{\partial a} = -\frac{\rho+\delta_a}{a}. \tag{42}$$

Using (41) and (42), (38) - (40) become

$$\Psi_{mm} = \sigma\beta\phi^2 \int_0^{\delta_{max}} \Gamma'(\phi m)\,\mathrm{d}F(\delta)\ + \sigma\beta\phi^2 \int_{\delta_a}^{\delta_{max}} \Gamma'[\phi m + (\rho+\delta)a]\,\mathrm{d}F(\delta) \tag{43}$$
$$+\sigma\beta\,\frac{\phi}{a}\ \frac{\left[\Gamma(\phi m) - \Gamma[\phi m + (\rho+\delta)a]\right]^2}{\Gamma[\phi m + (\rho+\delta)a]}\Bigg|_{\delta=\delta_a},$$

$$\Psi_{aa} = \sigma\beta \int_{\delta_a}^{\delta_{max}} \Gamma'[\phi m + (\rho+\delta)a](\rho+\delta)^2\,\mathrm{d}F(\delta) \tag{44}$$
$$+\sigma\beta\ \frac{(\rho+\delta)^2}{a}\,\Gamma[\phi m + (\rho+\delta)a]\Big|_{\delta=\delta_a},$$

$$\Psi_{ma} = \sigma\beta\phi \int_{\delta_a}^{\delta_{max}} \Gamma'[\phi m + (\rho+\delta)a](\rho+\delta)\,\mathrm{d}F(\delta) \tag{45}$$
$$-\sigma\beta\phi\ \frac{(\rho+\delta)}{a}\left[\Gamma(\phi m) - \Gamma[\phi m + (\rho+\delta)a]\right]\Big|_{\delta=\delta_a}.$$

The cross partial derivative (45) is always negative. Thus, conditions that guarantee joint concavity in m and a of the stationary portfolio problem are

$$\Psi_{mm} < 0,\ \Psi_{aa} < 0.$$

Note that a discrete stochastic process for δ—such as that studied in section 6—always satisfies these concavity conditions, ensuring degeneracy for the distributions of money and assets.

Appendix B

Despite uncertainty in asset dividends, a stationary equilibrium with a constant asset price can be justified in the environment of the model. To illustrate, suppose the first order condition for a_{+1} (16) holds with equality. We can reintroduce explicit time subscripts and solve for the asset price:

$$\rho_t = \beta\, \mathbb{E}_t\,(\rho_{t+1} + \delta_{t+1}) + \beta\sigma\, \frac{\partial}{\partial a}\mathbb{E}_t\big[\mathcal{S}(m_{t+1}, a_{t+1}, \delta_{t+1})\big]. \tag{B1}$$

Since dividend shocks δ are i.i.d., we can define the unconditional mean of δ_t as $\overline{\delta}$ so that

$$\rho_t = \beta\ \left[\mathbb{E}_t(\rho_{t+1}) + \overline{\delta}\right] + \beta\sigma\, \frac{\partial}{\partial a}\mathbb{E}_t\big[\mathcal{S}(m_{t+1}, a_{t+1}, \delta_{t+1})\big].$$

Repeatedly iterating ρ_t forward and substituting, we can express the asset price as

$$\rho_t = \frac{\beta}{1-\beta}\overline{\delta}\ + \lim_{s\to\infty}\beta^s\mathbb{E}_t(\rho_{t+s+1})\ + \sigma\sum_{s=1}^{\infty}\ \beta^s\frac{\partial}{\partial a}\mathbb{E}_t\big[\mathcal{S}(m_{t+s}, a_{t+s}, \delta_{t+s})\big]. \tag{B2}$$

Note that for $\beta = (1+r)^{-1}$,

$$\frac{\beta}{1-\beta}\overline{\delta} = \frac{\overline{\delta}}{r} = \rho^*,$$

where ρ^* is the fundamental price of the asset—the price that prevails for a completely illiquid asset that is held only for its flow of dividends. Also, given an appropriate transversality condition, the limit in (B2) vanishes. Thus

$$\rho_t = \rho^*\ + \sigma\sum_{s=1}^{\infty}\ \beta^s\frac{\partial}{\partial a}\mathbb{E}_t\big[\mathcal{S}(m_{t+s}, a_{t+s}, \delta_{t+s})\big]. \tag{B3}$$

We can note a few things in the expression (B3). First, the asset price can be decomposed into its fundamental price ρ^*, plus a term which captures the asset's expected benefit in decentralized trade. Second, while not explicit, the period t trade surplus $\mathcal{S}_t$ is a function of the asset price ρ_t. Third, notice that the period t realization of the asset shock does not figure into the current period's asset price. Since the choice of a_{t+1} is independent of a_t, so the price of a_{t+1} is independent of the productivity of a_t. This independence suggests that multiple price paths can satisfy (B1). Notice that a constant asset price can exist in equilibrium. Specifically, there is a stationary equilibrium where: the choice of $(m_{t+1},\, a_{t+1})$ is the same across periods; the change in ϕ_t is defined by $\phi_t M_t = \phi_{t+1} M_{t+1}$; and $\rho_t = \rho_{t+1}$. Imposing this stationary equilibrium and dispensing with time subscripts, (B3) can be expressed as

$$\rho = \rho^* \; + \frac{\sigma}{r}\, \frac{\partial}{\partial a} \int_0^{\delta_{max}} \mathcal{S}(m, a, \delta)\, \mathrm{d}F(\delta). \tag{B4}$$

Notice that (B4) yields our expression for the stationary asset price (18).

It is interesting to compare the asset price in the LW framework with the asset price of a standard Lucas tree in the DSGE framework. Consider an infinitely-lived, representative agent maximizing his expected lifetime utility over discrete time. Utility $U(\cdot)$ has the typical assumptions and consumption is c_t. The agent receives exogenous income w_t each period. A unit of the Lucas tree is a_t which carries the price ρ_t and provides a random dividend δ_t each period. Dividend shocks are i.i.d. This problem can be represented by

$$\begin{aligned} \max_{\{c_{t+s},\, a_{t+s+1}\}_{s=0}^{\infty}} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \\ & st.\;\; c_{t+s} + \rho_{t+s} a_{t+s+1} = w_{t+s} + (\rho_{t+s} + \delta_{t+s}) a_{t+s}. \end{aligned}$$

A solution is characterized by the Euler equation

$$U'(c_t)\, \rho_t = \beta\, \mathbb{E}_t \big[U'(c_{t+1})\, (\rho_{t+1} + \delta_{t+1}). \big]$$

Using repeated iteration and substitution, and enforcing the appropriate transversality condition, we can express the asset price as

$$\rho_t = \frac{1}{U'(c_t)} \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \, U'(c_{t+s}) \, \delta_{t+s}.$$

Comparing with the asset price from the LW framework, the asset price here is a function of the current period shock δ_t through the marginal utility $U'(c_t)$. Despite i.i.d. dividend shocks, the asset price here does not depend on future realizations alone.

References

[1] Aruoba, Borağan C, Christopher J Waller and Randall Wright (2011). "Money and Capital." *Journal of Monetary Economics*, 58(2): 98-116.

[2] Bryant, John and Neil Wallace (1979). "The Inefficiency of Interest-bearing National Debt." *Journal of Political Economy*, 87(2): 365-381.

[3] Chiu, Jonathan and Miguel Molico (2010). "Liquidity, Redistribution, and the Welfare Cost of Inflation.? *Journal of Monetary Economics*, 57(4): 428-438.

[4] Geromichalos, Athanasios, Juan Manuel Licari, Jos é Suárez–Lledó (2007). "Monetary Policy and Asset Prices." *Review of Economic Dynamics*, 10: 761-779.

[5] Green, Edward J and Ruilin Zhou (2002). "Dynamic Monetary Equilibrium in a Random Matching Economy." *Econometrica*, 70(3): 929-969.

[6] Hicks, John R (1935). "A Suggestion for Simplifying the Theory of Money." *Economica*, 2(5): 1-19.

[7] Kiyotaki, Nobuhiro and Randall Wright (1989). "On Money as a Medium of Exchange." *Journal of Political Economy*, 97(4): 927-954.

[8] Kiyotaki, Nobuhiro and Randall Wright (1993). "A Search-theroretic Approach to Monetary Economics." *American Economic Review*, 83(1): 63-77

[9] Lagos, Ricardo and Guillaume Rocheteau (2008). "Money and Capital as Competing Media of Exchange." *Journal of Economic Theory*, 142: 247-258.

[10] Lagos, Ricardo and Randall Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113(3): 463-484.

[11] Lester, Benjamin, Andrew Postlewaite and Randall Wright (2012). "Information, Liquidity, Asset Prices, and Monetary Policy." *Review of Economic Studies*, 79: 1209-1238.

[12] Lucas, Robert E (1978). "Asset Prices in an Exchange Economy." *Econometrica*, 46(6): 1429-1445.

[13] Shi, Shouyong (1997). "A Divisible Model of Fiat Money." *Econometrica*. 65(1): 75-102.

[14] Wallace, Neil (1998). "A Dictum for Monetary Theory." Federal Reserve Bank of Minneapolis *Quarterly Review*, 22(1): 20-26.

[15] Waller, Christopher J (2011). "Random Matching and Money in the Neoclassical Growth Model: Some Analytical Results." *Macroeconomic Dynamics*, 15(s2): 293-312.

[16] Williamson, Stephen D and Randall Wright (2010). "New Monetarist Economics: Methods." Federal Reserve Bank of St. Louis *Review*, 92(4): 265-302.

[17] Williamson, Stephen D and Randall Wright (2011). "New Monetarist Economic: Models." In B. Friedman and M. Woodford (Eds.), Handbook of Monetary Economics.