Realization Utility and its General Equilibrium Implications for IPO Markets

by

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An essay submitted to the Department of Economics

in partial fulfillment of the requirements for

the degree of Master of Arts

Queens University

Kingston, Ontario, Canada

August 2013

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Abstract

In this paper I present both partial equilibrium and general equilibrium analyses of realization utility in the context of a comparison with the expected utility framework. The results of the partial equilibrium section indicate that, unlike expected utility investors, realization utility investors: make discrete changes in their portfolio allocation decisions; place a particularly high requirement on the level of return required before any investment is made; demonstrate the disposition effect at higher levels of returns; and converge to expected utility behaviour at the highest levels of returns. Analyzing the realization utility investor's decision making process, I find that, consistent with Barberis and Xiong [2009], the combination of realization utility and diminishing sensitivity to gains/losses provides a credible explanation of the disposition effect. To capture the important effect of the initial price paid for stock on the realization utility investor's subsequent portfolio rebalancing decisions, the general equilibrium framework is based on the market for initial public offerings. In this setting, expected utility investors are able to attain welfare gains by driving up initial bids for the stock in order to game the realization utility investor's portfolio rebalancing decision. The results indicate that mental accounting and realization utility are potentially capable of explaining 'herd-like' investor behaviour of which more sophisticated expected utility investors are able to exploit.

Acknowledgements

First, I would like to thank my immediate supervisor, Professor Sumon Majumdar, for providing guidance and support during the process of writing this essay. Second, I owe a great many thanks to Frank Milne for thoughtful advice and insight. Most importantly, I would like to thank my family, especially Bob, for providing outstanding motivation.

Contents

I. Introduction

Realization utility [Barberis and Xiong 2009] is a promising new behavioral theory in the ongoing investigation into a number of trading patterns and market behaviours considered inconsistent with the main tenants of rational expectations. The theory drops the assumption that utility is derived from evaluating net asset value at fixed portfolio review dates, and instead posits that individual investors derive utility from changes in wealth levels occurring at the actual moment of transaction. Shefrin and Statman [1985] first suggest the alternative source of utility as one of five key behavioural ingredients (including tax considerations, self-control, regret aversion, and prospect theory) composing a model with the potential to explain the disposition effect i.e. the tendency of individual investors to hold "losing" stocks too long, and sell "winning" stocks too soon. Many authors[1] have subsequently referenced that model when explaining the disposition effect, and, until recently, most have focused on informal arguments based on the prospect theory value function. Barberis and Xiong [2009] solve for the partial equilibrium optimal portfolio allocation decisions of a prospect theory investor in a binomial asset price framework. Examining simulated investor trading strategies, they find that the behavioural biases incorporated into prospect theory alone cannot satisfactorily explain the disposition effect. However, by combining a prospect theory value function with the principles of mental accounting[2], Barberis and Xiong [2009] are able to explain the disposition effect using "Realization Utility".

Applying prospect theory to realized gains and losses represents a substantial deviation from the standard models of investment behavior. Nonetheless, a clear need to address the failure of standard models has advanced the challenges directed at fully rational behaviour that lie at the heart of behavioural economics and finance. Of major importance to this dialogue has been the investigation into the potential

[1] See for example Barber, Odean, and Zhu (2008); Griffin, Nardari, and Stulz (2005); Odean (1998a, 1998b); Weber (1998).
[2] Mental accounting refers to the set of cognitive operations performed by individuals in the organization and evaluation of financial activities. See Thaler (1999) for a good primer on the subject.

causes of the disposition effect. Proponents from the rational and behavioural camps have offered a variety of explanations which seemingly fall short of the realization hypothesis: both in their abilities to explain the disposition effect as well as to provide insight into other empirical evidence and new behaviours.

Perhaps the best reason to consider realization utility comes from a recent breakthrough providing scientific evidence that realization utility indeed plays a role in the financial decision making process. Frydman et al. [2012] are the first in the literature to demonstrate that neural activity provided by functional magnetic resonance imaging (fMRI) can test between behavioural theories. In their experiment they monitor activity in the area of the brain known to encode the value of decisions. By placing subjects in financial situations, the experimenters are able to distinguish between expected utility and realization utility models by examining the temporal correlations between realized gains and losses and brain activity. The results indicate strong evidence for the realization utility hypothesis in that activity in this area of the brain exhibits a sharp spike at exactly the moment the gain is realized. Moreover, the magnitude of the signal at the moment of realization correlates with the strength of the disposition effect.

A number of implications of realization utility in general equilibrium are distilled by Barberis and Xiong [2011]. Because prices are not fixed, but are free to respond to forces of supply and demand, the approach allows the authors to move beyond simply explaining trading biases, to considering how such biases may manifest in aggregate price and volume patters. According to their analysis, realization utility, when combined with a linear or piece-wise linear utility function and a sufficiently high discount rate, can explain both the disposition effect as well as the individual investor preference for volatile stocks observed by Kumar [2009]. Furthermore, it is the presence of these biases at the individual investor level which forms the basis of the explanation for the aggregate market behaviours displayed by their model, such as: the effect of historical highs on the propensity to sell; the low average return of volatile stocks; the higher volume of trade in rising markets; the heavy trading associated with highly valued assets; and stock market momentum.

In order to set the context for interpreting my own findings, it is necessary to provide a brief discussion of Barberis and Xiong's [2011] general equilibrium model. They work in continuous time with an infinite investment horizon, where the prices of dividend paying assets (stocks) follow a Brownian motion, investors are restricted to hold either a unit of stock or a unit of the risk free asset, and trades are subject to transaction costs. In addition to having the realization utility preference specification mentioned in the previous paragraph, investors also discount future expected realized utility and face the possibility that they will be forced out of the asset market by a liquidity shock. The corresponding optimal strategy followed by the investor is to sell the stock if its price rises above a specific liquidation point (always greater than the initial purchase price), and to otherwise maintain the same position unless forced into liquidation. In this sense the model is able to give insight into the disposition effect: on average, most asset sales correspond with gains and few sales correspond with losses. However, I believe that the approach does not offer a complete explanation of the disposition effect because of the trading restriction placed on investors. It seems acceptable that if the investor is not able to purchase more shares of the stock after the initial purchase then his problem is reduced to determining the optimal price at which to sell his single share. It follows naturally then that he would only want to sell that share after experience a gain – and hence always exhibit a disposition unless forced to exit markets – since it would be irrational for him to willingly realize a painful loss only to find himself applying the same stock selection criteria, albeit to the same universe of stocks, that united him with the loosing trade in the first place.

The binomial framework implemented in Barberis and Xiong [2009] does not impose restrictions on investors' trading strategies and therefore allows an examination of the mechanism which causes the realization utility investor to realize gains early over purchasing more shares of the risky asset. Ergo, in addition to contrasting realization utility and expected utility investors in the binomial setting of Barberis and Xiong [2009], I also examine: what drives realization utility investors to exhibit the disposition effect; and the sensitivity of realization utility preferences and trading strategies to diminishing sensitivity, loss aversion, and time discounting. Working with two time periods, I find that the realization utility investor's strategy exhibits discrete jumps as returns of the risky asset increase: up to a given threshold there is no investment made at all; immediately above this threshold stock is purchased and subsequent gains are realized and losses held; as returns increase, the investor reaches another threshold where the proceeds of gains are reinvested and losses are still held; finally, at returns beyond the highest threshold the risky asset is so attractive that it is optimal for the realization utility investor to anticipate realizing losses early so that he may increase the number of shares purchased on the first trading date. The strategy differs considerably from the expected utility investor who is generally always willing to purchase some number of shares of the risky asset and who steadily increases the number of shares purchased after gains and sold after losses as the return of the risky asset increases.

Regarding the source of the disposition effect, I find that realization utility combined with a linear, or piece-wise linear, utility function and time discounting is only able to explain the postponement of loss realization. In order to obtain the full disposition effect it is necessary to include diminishing sensitivity in the function used to evaluate gains and losses. This is because diminishing sensitivity increases the relative marginal positive contribution of realizing gains early against larger expected future gains that may be obtained from purchasing more shares, while simultaneously increasing the relative marginal negative contribution of realizing losses early against holding on to the loosing stock position.

Furthermore, the role of diminishing sensitivity in the context of realization utility is completely flipped from its standard interpretation as being directly related to the expected utility investor's level of risk aversion. Increased diminishing sensitivity results in: a reduced return threshold required for initial investment; an increased range of returns over which a stronger disposition effect holds; and, when combined with time discounting and low levels of loss aversion, can result in a preference for a specific level of volatility.

The partial equilibrium section indicates that the realization utility investor's portfolio rebalancing decision is highly dependent on the initial price paid for the risky asset. In the general equilibrium section of this paper I therefore examine the realization utility and expected utility preference specifications in a model that incorporates the investors' initial purchase decision. This is accomplished by including a securities dealer who auctions shares of the risky asset to investors in a manner analogous to an initial public offering. Furthermore, implications about realization utility in general equilibrium are inferred by comparing general equilibrium outcomes between markets that contain only expected utility investors and markets that contain both expected utility and realization utility investors. The analysis is split into two parts. In the first part, I obtain an analytical solution for general equilibrium by ignoring wealth constraints and diminishing sensitivity to wealth outcomes and gains/losses. In this context, I find that welfare gains to the expected utility investor are increasing in the extent to which the realization utility investor is afflicted by behavioural biases. In the second part of the analysis, I present the results of a numerical calculation capable of handling wealth effects and the complexities of solving for realization utility bid and offer curves when investors' exhibit the quality of diminishing sensitivity. The implications of realization utility in general equilibrium are surprising. Mental accounting causes realization utility investors to demand relatively more shares than expected utility investors even at bids above the realization utility investors' optimal entry point. This tendency provides an opportunity for expected utility investors to drive up the initial stock price in order to increase the discount and volume of shares available for purchase when the realization utility investor realizes gains during the interim portfolio rebalancing date.

The next section of the paper sets the stage by reviewing some of the major empirical inconsistencies of expected utility theory that are accounted for by Kahneman and Tverksy's prospect theory [1979], a primary ingredient in the realization utility specification. Additionally, I discuss the origins and importance of the disposition effect in the trading of financial market participants and motivate the subsequent investigation into realization utility. Section III covers realization utility and expected utility in partial equilibrium. Section IV includes analytical solutions and a description of the numerical methodology of the general equilibrium model of IPOs as well as a discussion of the results. Section V concludes and provides suggestions for future research into realization utility and its potential applications to IPO markets.

II. Literature Review

To understand how individuals make decisions under risk in real world situations, e.g. investment and insurance, economists study choices between gambles, or prospects. Prospects are well defined agreements that yield a set of possible monetary outcomes $(x_1, \ldots, x_n)$ occurring with a corresponding set of probabilities $(p_1, \ldots, p_n)$. In expected utility theory, choices made between prospects are based on the following assumption: (1) outcomes are evaluated under a concave utility function and are weighted according to their objective probabilities; (2) individuals have symmetrical risk attitudes to gains and losses; (3) utility is evaluated based on final wealth. The theory is still accepted as the dominant normative and descriptive model of decision making under risk, and forms the basis of theories in widespread use such as the Capital Asset Pricing Model [Sharpe 1964].

However, the results of laboratory experiments indicate that some individuals behave consistent with expected utility theory, and that others are subject to behavioural biases inconsistent with what the theory deems as rational. One of the most famous violations relates to probabilities and is known as the Allais paradox [Allais, 1953]. The paradox arises from what has been termed the certainty effect, in short: reductions in the desirability of prospects are smaller when the probabilities of risky prospects are reduced than when the probabilities of certain prospects are reduced by the same proportion. On the other end of the probability scale we are concerned with the effect of augmenting probabilities when the outcomes are unlikely and merely possible. MacCrimmon and Larsson [1979] and Kahneman and Tversky [1979] find that when presented with prospects offering slim chances of payouts, individuals demonstrate a preference for the riskier gamble. However, as the probabilities of the payouts increase, the outcomes are no longer seen as unlikely possibilities, but more as probable outcomes and preferences consequently shift to the less risky gamble.

Of course, prospects are defined by the probabilities of outcomes as well as the outcomes themselves, and there is no shortage of evidence indicating that individuals behave irrationally in this domain as well. In fact, it seems as though the human perceptual apparatus is inherently irrational in the context of monetary outcomes when viewed through the lens of expected utility theory. This is because individuals are intrinsically more sensitive to the progression of changes or differences in their environment than they are to the progression of its state [Helson 1964]. Why this may be remains a question for psychologists, or perhaps evolutionists, but the idea that individuals' experiences are informed more by changes in the environment than its actual state is undeniably familiar to us. Just consider the old adage commonly told to those who find themselves subject to an uncomfortable change in their environment: "you'll get used to it".

Consider the following decision problem between financial scenarios owing to Kahneman and Tversky [1979]. In the first scenario, individuals begin with $1,000 and are required to choose between two prospects. The first prospect pays $1,000 with probability ½, and the second prospect pays $500 with probability 1. Therefore, in the first gamble final period wealth outcomes are $2,000 and $1,000 with probabilities of ½, and in the second gamble there is a single outcome of $1,500. Kahneman and Tversky find 84% of their test subjects chose the sure gamble. In the second scenario, individuals begin with $2000 and are required to choose between two prospects. The first of which pays -$1,000 with probability ½, and the second of which pays -$500 with certainty. Similar to the first scenario, the final period wealth outcomes are $2,000 and $1,000 with probabilities ½ for the first prospect, and $1,500 in the second. Here, the authors find that only 31% of individuals chose the sure gamble. The problem not only illustrates that gains and losses should be considered as the primary drivers of the decision making process, but that individuals are risk averse in the domain of gains and risk seeking in the domain of losses.

In addition to having different risk attitudes in the domains of gains and losses, individuals also exhibit differences in the relative impact of identically sized gains and losses on their welfare. The behavioural bias, known as loss aversion, is well known in psychology and economics [Galanter and Pliner 1974] and, now that we have established that the carriers of value should be gains and losses, its effect can be illustrated with an example. Suppose you are offered a gamble which offers \$50 and -\$50 each occurring with a probability of ½. You are then offered a second gamble where the magnitude of the outcomes is increased to \$100 and -\$100 and asked which of the two gambles do you prefer? In most cases, individuals prefer the bet with smaller stakes, indicating that they derive greater disutility from losses than utility from identically sized gains.

The behavioural biases just described are incorporated by Kahneman and Tversky into Prospect Theory[3]. In its most general form the value of a prospect according to their theory is given by

$$V(x,p;y,q) = \pi(p)v(x) + \pi(q)v(y). \qquad 2.1$$

The value function, v, takes the form

$$v(x) = \begin{cases} x^{\alpha} & for \quad x \geq 0 \\ -\lambda(-x)^{\alpha} & \quad x < 0 \end{cases}, \quad 0 < \alpha < 1, \lambda > 1, \qquad 2.2$$

where the argument x corresponds with the gain or loss associated with an outcome defined relative to a fixed reference level. The value function is clearly concave in the region of gains and convex in the region of losses, implying the correct risk attitudes in the positive and negative domains of outcomes. Furthermore, λ is generally greater than 1, so that individuals are more sensitive to losses than they are to gains, consistent with investor loss aversion indicated in the previous example by the preference for smaller stakes.

A particularly important deviation from expected utility theory, consistently explained in the literature using informal arguments based on prospect theory, is the tendency of financial market

[3] Prospect Theory in fact refers to the broader process of the framing of outcomes in addition to their evaluation. For the purpose of this paper I use the term to refer only to the evaluation of outcomes.

participants to ride loosing trades too long and sell winning trades too soon. Shefrin and Statman [1985] are the first in the literature to demonstrate the possible existence of the disposition using stock and mutual fund trading data from individual investors in the United States. The authors look at the proportion of sales corresponding with realized gains and losses for different holding periods and find that a greater proportion of sales coincide with gains. Systematically, the observed pattern of realized gains and losses could be potentially explained by tax considerations combined with a behavioural model incorporating prospect theory, mental accounting, regret aversion, and self-control. Shefrin and Statman note, however, that tax considerations are expected to entice investors to realize gains as early as possible and losses as late as possible in order to take advantage of the reduced short term gains tax. The feature is not strongly presented in the data, and the authors conclude that the pattern of realized gains and losses could be explained by a combination of tax considerations and the disposition to sell winners early and ride losers. Odean [1998] refines the methodology by rigorously defining the disposition effect as a large difference in the proportion of gains realized to the proportion of losses realized[4]. Their data is composed of 10, 000 trading records of individual investor accounts at a large brokerage firm. Odean [1998] finds strong evidence supporting the existence of a preference to sell winners and hold losers for a number of different definitions of the gain/loss reference point[5]. Of particular mention, tax considerations cannot explain the pattern of realizations except in the month of December. Additionally, individuals do not seem motivated by a desire to rebalance their portfolio, a reluctance to incur higher trading costs associated with lower priced stocks, or even subsequent portfolio performance. The authors conclude that the observed pattern of realized gains and losses is consistent with prospect theory or an irrational belief in mean reversion.

The potential implications of the disposition effect for the aggregate market are largely determined by which market participants exhibit the behavior, for example, if only a large number of

[4] To clarify, Odean (1998) defines the proportion of gains(losses) realized as
$Realized\ Gains(losses)/[Realized\ Gains(losses) + Paper\ Gains(losses)]$
[5] These include: initial purchase price, the average purchases price, the highest purchase price, and the most recent purchase price

small investors (so that aggregate wealth in a given asset is still small) demonstrate the disposition effect then the impact might be negligible. However, Locke [2000] looks at professional trading activity in the Chicago Mercantile Exchange's most actively traded currencies and finds strong evidence that trading elites might also suffer from the tendency. More interestingly, he finds implicit support that the disposition effect is indeed the result of behavioural biases or irrational beliefs in that less successful traders are more likely to hold losing trades too long and exit winning trades too soon. In addition to stocks, currencies, and mutual funds, Heath et al. [1999] find empirical evidence of the disposition effect in the exercise of employee stock options when looking at records of over 50,000 employees from seven major corporations. And Case and Shiller [1988] report evidence of the disposition effect in questionnaire surveys of home owners regarding real estate decisions taken during the real estate boom of the mid 80's. To distinguish between the two competing explanations of the disposition effect, Camerer and Weber [1998] construct a hypothetical trading laboratory in which participants are able to buy and sell stocks with known payoffs and probabilities. The authors are able to use the fact that participants are aware of outcome probabilities to eliminate an irrational belief in mean reversion as a possible explanation of the disposition effect. The evidence indicates that the disposition to sell winners too soon and ride losers too long is a tendency pervasive among different market participants and across market types. Other authors such as Yan and Yang [2010], Shumway [2006], and Grinblatt and Han [2000] look at the implications of the disposition effect for the aggregate market and find that it can drive momentum and can explain the positive return-volume correlation observed in stock prices.

The standard explanation of the disposition effect seems to rely on the prospect theory preference specification. According to prospect theory [Kahneman and Tversky 1979]; individuals evaluate monetary outcomes based on the magnitude of the resulting monetary gain or loss defined relative to a specific reference point; demonstrate risk aversion in choices involving sure gains and risk seeking behaviour in choices involving sure losses; and place relatively more weight on losses then they do on gains (i.e. loss aversion). The prospect theory argument goes as follows: after experiencing a gain in his/her investment decision, the investor now faces a choice involving sure gains and risk aversion indicates a tendency to sell the risk of the position and lock in a sure gain; after experiencing a loss, the investor now faces a choice involving sure losses and risk seeking behaviour combined with loss aversion indicates a tendency to hold the risk of the position. The argument is so pervasive it can be found in nearly all of the articles documenting the effect, but does it really offer an explanation?

Hens and Vlcek [2005] examine a simple two period model with a risk free asset and a binomially distributed risky asset in order to investigate the implications of prospect theory on investor trading strategies. In this setting, investors are myopic in that when making initial investment decisions they do not anticipate their optimal subsequent rebalancing decisions, gains and losses are calculated relative to initial wealth, and, consistent with the standard application of prospect theory, utility is derived from holding period gains. The authors are unable to generate the disposition effect for reasonable parameter values of the prospect theory preference parameters. It turns out, if the risky asset has return characteristics that would cause the investor to exhibit the disposition effect in his second period trade decision, then the expected return is not high enough to cause the investor to purchase the stock in the first place. Hens and Vlcek [2005] claim that if we can argue that the investor has purchased the stock in the first period then prospect theory is able to explain a sort of "ex-post" disposition effect. Using the same approach, but with normally distributed returns, Kaustia [2010] obtains similar outcomes.

Other models, which remove the assumption of myopic investors and include additional investment periods and behavioural assumptions, have produced more promising results. Barberis and Xiong [2009] obtain an analytical solution for the fully rational prospect theory investor trading in the same setting as Hens and Vlcek [2005]. Consistent with their results, Barberis and Xiong [2009] find that prospect theory is unable to explain the disposition effect for reasonable parameter values when there are only two time periods and investors have rational expectations. However, they find that increasing the number of time periods does indeed produce the disposition effect, although it is not as pronounced as expected. Unsatisfied with their initial findings, Barberis and Xiong [2009] are able to achieve a great improvement by questioning the standard application of prospect theory to holding period gains and losses and instead, for the first time in a formal model, use realized gains and losses. Still, the idea of using realized gains and losses dates back to Shefrin and Statman's [1985] discussion of the disposition effect in their mention of mental accounting as a possible factor. According to mental accounting, individuals view their financial decisions in separate "mental accounts": when a stock is purchased a new mental account is opened, recording details of the transaction such as investment name and purchase price; when the same stock is sold, the corresponding mental account is closed and the investment decision evaluated. Now, mental accounting, or narrow framing as it is often called, is almost always applied in the context of prospect theory in the sense that utility is derived from the performance of individual stock (or mental accounts) evaluated at the point of final sale. Barberis and Xiong [2009, 2011] propose that, in fact, the most natural time to evaluate the transaction is at the moment of any sale, hence realized gains and losses become carriers of utility. Barberis and Xiong [2011] also argue that realization utility can be explained by the underlying cognitive processes regarding how individuals think about their investing history and how they evaluate financial decisions. The first process has already been mentioned in the context of prospect theory. The second, "evaluation", process proposes that investors use a simple heuristic to,

> ""Guide their trading, one that says: "selling a stock at a gain relative to the purchase price is a good thing--it is what successful investors do." After all, an investor who buys a number of stocks in sequence and manages to realize a gain on all of them *does* end up with more money than he had at the start. The flip side of the same heuristic says: "selling a stock at a loss is a bad thing--it is what unsuccessful investors do." Indeed, an investor who buys a number of stocks in sequence and realizes a loss on all of them *does* end up with less money than he had at the start"

Along similar lines as Barberis and Xiong [2009], Meng [2012] modifies the standard application of prospect theory by introducing a new behavioural assumption concerning the evaluation of outcomes. In the standard application, gains and losses are calculated relative to either the status quo or the risk free rate [Barberis and Xiong 2009], here they are calculated relative to expectations. Meng [2012] removes diminishing sensitivity (concavity) from the prospect theory value function and focuses only on loss aversion. By setting the reference point significantly higher than the risk free rate, but still below expectations, he is able to successfully explain the disposition effect.

Based on the work of Hens and Vlcek [2005], I offer a simple explanation to their finding. Consider a stock with return characteristics such that it is purchased by the prospect theory investor with reference levels set to the status quo or to the level set by Meng [2012]. In the next period the stock increases in value and potential gains from selling shares are calculated for the two reference levels. Both investors examine the trade-off between selling shares early to lock in gains and holding out until the next period. Under the status quo, gains and losses are calculated relative to the initial purchase price, and so the investor may realize a large gain now or wait until the next period which, according to his expectations, will provide him with the chance of an even larger gain and the chance of a small loss. In most cases, as observed by Hens and Vlceck[2005] and Barberis and Xiong[2009], the status quo investor prefers the gamble and often increases his position in the risky asset. However, under the alternative reference point gains are calculated relative to a higher reference level. Meaning the investor chooses between realizing a small gain now or waiting until the next period which provides the chance of a larger (but still small) gain and also the chance of a large loss. Despite the intuitive explanation of the disposition effect offered by Meng's[2012] approach, I decide to focus on realization utility. Mainly because, not only is it Meng's own opinion that "realization utility is a true to life psychological factor", but, I find the magnetic resonance imaging experiments performed by Frydman et al. [2012] compelling enough to indicate that realization utility reflects the way individuals actually view investment decisions.

III. Partial Equilibrium

The partial equilibrium framework is identical to the set-up in Barberis and Xiong [2009]. I consider a portfolio choice problem with three dates: $t = 0$, 1, and 2. On dates $t = 0$ and $t = 1$, investors are able to trade in liquid markets for both risk free and risky assets, and at date $t = 2$ they are forced to liquidate all of their positions. The net return of the risk free asset is equal to zero so that $R_f = 1$. The net return of the risky asset per period is independently binomially distributed according to,

$$R_{t,t+1} = \begin{cases} R_h & with\ probability\ 1/2 \\ R_l & with\ probability\ 1/2 \end{cases}. \qquad 3.1$$

Per period returns of the risky asset are calculated from the two period mean growth rate μ and standard deviation of returns σ according to

$$R_h = \mu^{\frac{1}{2}} + \sqrt{(\mu^2 + \sigma^2)^{\frac{1}{2}} - \mu} \qquad 3.2$$

and

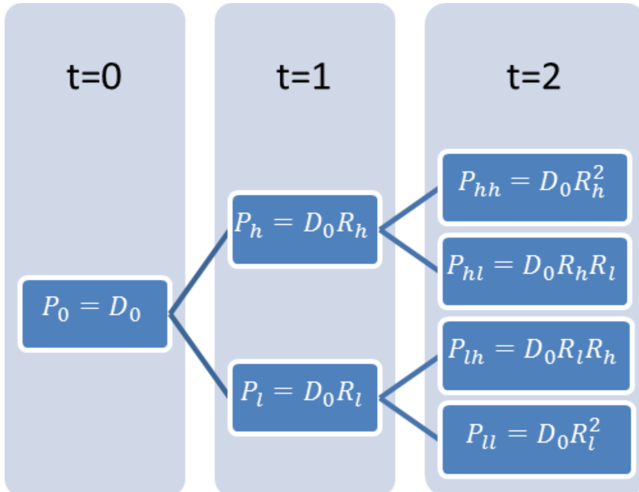
$$R_l = \mu^{\frac{1}{2}} - \sqrt{(\mu^2 + \sigma^2)^{\frac{1}{2}} - \mu}, \qquad 3.3$$

where the values of μ and σ are restricted to satisfy the no arbitrage condition

$$R_l < 1\ and\ R_h > 1. \qquad 3.4$$

The price of the risky asset therefore evolves according to the familiar binomial tree depicted in Figure 3.1.

Figure 3. 1
Binomial Stock Price Evolution



An investor may purchase shares at $t = 0$, rebalance his portfolio at $t = 1$, and must sell all of his holdings at $t = 2$ as a result of a liquidity shock. Therefore, the expected utility investor solves

$$\max_{x_0, x_1} E_0\{W_2^\gamma\} \tag{3.5}$$

subject to the non-negativity of wealth constraint,

$$W_2 = W_0 + x_0 P_0 (R_{0,1} - 1) + x_1 P_1 (R_{1,2} - 1) \geq 0, \tag{3.6}$$

where γ is directly related to the investor's coefficient of relative risk aversion, x_0 and x_1 correspond with the number of shares of the risky asset held at times $t = 0$ and $t = 1$, and W_2 equals the investors wealth level at time $t = 2$. The solution to equations 3.5 and 3.6 is well known to the literature [Cox and Huang 1989], accordingly I solve for the optimal portfolio allocations by working backwards from $t = 2$. The number of shares of the risky asset held by the investor at $t = 1$ is given by

$$x_1 = W_1 \beta_1, \tag{3.7}$$

where

$$W_1 = W_0 + x_0 P_0 (R_{0,1} - 1) \tag{3.8}$$

and

$$\beta_1 = \left(\frac{\left(P_1 - D_0 R_{0,1} R_l\right)^{\frac{1}{\gamma-1}} - \left(D_0 R_{0,1} R_h - P_1\right)^{\frac{1}{\gamma-1}}}{\left(P_1 - D_0 R_{0,1} R_l\right)^{\frac{\gamma}{\gamma-1}} + \left(D_0 R_{0,1} R_h - P_1\right)^{\frac{\gamma}{\gamma-1}}} \right) \qquad 3.9$$

And at $t = 0$, the investor purchases shares in amount equal to

$$x_0 = W_0 \frac{\beta_0 - 1}{(P_h - P_0) + \beta_0 (P_0 - P_l)} \qquad 3.10$$

where

$$\beta_0 = \beta_{hl} \frac{1 + \beta_l (D_0 R_l R_h - P_l)}{1 + \beta_h (D_0 R_l R_h - P_h)} \qquad 3.11$$

and

$$\beta_{hl} = \left(\frac{D_0 R_l R_h - D_0 R_l^2}{P_l - D_0 R_l^2} \frac{D_0 R_h^2 - P_h}{D_0 R_h^2 - D_0 R_l R_h} \frac{P_0 - P_l}{P_h - P_0} \right)^{\frac{1}{\gamma-1}}. \qquad 3.12$$

Contrastingly, the realization utility investor derives utility from realized gains and losses at times $t = 1$ and $t = 2$. Following the approach of Barberis and Xiong [2009], I calculate gains and losses as the difference between the selling price and the average price paid for shares. Accordingly, the realization utility investor's objective is to solve

$$\begin{aligned} \max_{x_0, x_1} E_0 \{ & v[(x_0 - x_1)(P_1 - P_0)] 1_{\{x_1 < x_0\}} \\ & + v[x_1 (P_2 - P_{ave})] 1_{\{x_1 > 0\}} \}, \end{aligned} \qquad 3.13$$

where

$$P_{ave} = \begin{cases} P_0 & \\ \dfrac{x_0 P_0 + (x_1 - x_0) P_1}{x_1} & for \end{cases} \begin{matrix} x_1 \le x_0 \\ \\ x_1 > x_0 \end{matrix}, \qquad 3.14$$

subject to the non-negativity of wealth constraint,

$$W_2 = W_0 + x_0 P_0 \left(R_{0,1} - 1\right) + x_1 P_1 \left(R_{1,2} - 1\right) \ge 0. \qquad 3.15$$

The argument of the first term in equation 3.13 corresponds with the gain/loss on shares voluntarily sold at $t = 1$ i.e. $number\ shares\ sold * change\ in\ price = (x_0 - x_1)(P_1 - P_0)$. The argument of the second term corresponds with the gain/loss on shares forcefully liquidated at $t = 2$, calculated relative to the average price paid for shares. The contribution to overall utility in either case is determined by evaluating realized gains using the prospect theory value function, $v(\cdot)$, defined in equations 2.1 and 2.2. The indicator functions $1_{\{x_1<x_0\}}$ and $1_{\{x_1>0\}}$ ensure that utility from transactions is only realized when shares of the risky asset are sold. Notice that the number of shares of the risk free asset does not enter into equation 3.13. This is because its net return is zero, thereby making it impossible to realize a gain or a loss on this component of the investor's portfolio.

Barberis and Xiong [2009] solve 3.13-15 by working backwards in time starting from $t = 2$. At $t = 2$ the investor's decision is trivial as he is forced to liquidate his entire portfolio. Continuing on to $t = 1$, the optimal value of x_1 conditional on x_0, P_0, and P_1 is obtained by solving

$$\begin{aligned} J(x_0, P_1) = \max_{x_1 \in [0, W_1/(P_1(1-R_l))]} & E_1\{v[(x_0 - x_1)(P_1 - P_0)]1_{\{x_1<x_0\}} \\ & + v[x_1(P_2 - P_1) + x_0(P_1 - P_0)]1_{\{x_1>0\}}\}, \end{aligned} \tag{3.16}$$

where x_1 is constrained by the condition of non-negative wealth in the final period. And at $t = 0$, x_0 is determined by

$$S(x_0) = \max_{x_0 \in [0, W_0/(P_0(1-R_l))]} E_0 J(x_0, P_1), \tag{3.17}$$

where the possible values of x_0 are constrained to ensure that $t = 1$ wealth is non-negative.

This is the appropriate approach to solving the portfolio allocation problem of the expected utility investor in the binomial stock price framework. Applying this method to the problem of the realization utility investor, however, results in a slight oversight of the optimal investment strategy; likely resulting because realization utility clearly allows for violations of path dependence. Simply put, the approach does not account for potential increases in expected realized utility from the perspective of $t = 0$ that can be obtained from deviating from the optimal strategy at $t = 1$ conditional on x_0 and P_0. For example, consider the possible solution to 3.16 and 3.17 where the investor chooses the strategy $x_l = x_0$ in node P_l (note that the following argument holds in general for choices of x_h). This choice of x_l restricts x_0 to the range bound by zero and the maximum value of x_0 which, in turn, is jointly determined by the choice of $x_l = x_0$ and the non-negativity of wealth condition i.e. $x_{0,max} = W_0/P_0\left(1 - R_d^2\right)$. It may, however, be possible to increase initial expected utility by selling shares in node P_l, i.e. deviating from $x_l = x_0$, in order to increase $x_{0,max}$ from $W_0/P_0\left(1 - R_d^2\right)$ up until $W_0/P_0(1 - R_d)$.

Mathematically, the potential trade-off can be captured by including the additional step in the calculation method

$$\max_{x_l \leq x_l^*} \left\{ \frac{dJ(x_0, P_h)}{dx_0} \frac{dx_0}{dx_l} - \left| \frac{dJ(x_0, P_l)}{dx_l} \right| \right\}, \qquad 3.18$$

where $x_l{}^*$ is the value of x_l obtained from solving 3.16. This last step can be interpreted intuitively: before deciding on how many shares to purchase at $t = 0$ the investor considers the benefit of cutting his losses early in order to leverage the size of the initial investment in the risky asset[6]. In summary, the optimal solution to 3.13-15 is obtained by first solving equations 3.16 and 3.17, and then checking if benefits can be obtained from deviating from the strategy in node P_l using equation 3.18.

I begin the analysis by solving for the optimal portfolio allocations of the realization and expected utility investors. Table 1 reports the trading strategies using the prospect theory preference parameters estimated by Kahneman and Tversky (1992), i.e. $\alpha = 0.88$, $\lambda = 2.25$. Additionally, I set $\gamma = 0.88$ so that the expected utility and realization utility investors have identical levels of diminishing sensitivity. The initial stock price and wealth levels of both investors are equal to 100.

[6] I do not mention potential benefits that could be obtained from deviating from the optimal strategy in node P_h because in all the cases examined the maximum value of x_0 is bound by the investor's trading decision in node P_l.

Table 1

Optimal Shares Allocations of Realization and Expected Utility Investors

	σ = 0.20						σ = 0.25						σ = 0.30					
μ	**Realization Utility**			*Expected Utility*			**Realization Utility**			*Expected Utility*			**Realization Utility**			*Expected Utility*		
	x_0	**x_h**	**x_l**	*x_0*	*x_h*	*x_l*	x_0	x_h	x_l	x_0	x_h	x_l	x_0	x_h	x_l	x_0	x_h	x_l
1.06	**0.0**	**0.0**	**0.0**	*8.7*	*18.4*	*0.6*	**0.0**	**0.0**	**0.0**	*6.2*	*11.7*	*0.9*	**0.0**	**0.0**	**0.0**	*4.7*	*7.9*	*1.0*
1.07	**5.2**	**4.0**	**5.2**	*9.5*	*21.2*	*0.4*	**0.0**	**0.0**	**0.0**	*6.8*	*13.5*	*0.6*	**0.0**	**0.0**	**0.0**	*5.2*	*9.3*	*0.8*
1.08	**5.5**	**8.3**	**5.5**	*10.2*	*24.1*	*0.2*	**4.1**	**3.1**	**4.1**	*7.3*	*15.3*	*0.4*	**0.0**	**0.0**	**0.0**	*5.6*	*10.6*	*0.6*
1.09	**5.8**	**9.5**	**5.8**	*10.9*	*27.2*	*0.1*	**4.3**	**3.4**	**4.3**	*7.8*	*17.1*	*0.3*	**3.5**	**2.6**	**3.5**	*6.0*	*11.8*	*0.4*
1.1	**6.1**	**11.0**	**6.1**	*11.6*	*30.8*	*0.0*	**4.5**	**6.6**	**4.5**	*8.3*	*19.0*	*0.2*	**3.6**	**2.8**	**3.6**	*6.3*	*13.0*	*0.3*
1.11	**6.5**	**12.8**	**6.5**	*12.4*	*34.9*	*0.0*	**4.7**	**7.4**	**4.7**	*8.7*	*20.9*	*0.1*	**3.7**	**3.0**	**3.7**	*6.7*	*14.3*	*0.2*
1.12	**13.4**	**28.7**	**0.0**	*13.3*	*39.8*	*0.0*	**4.9**	**8.3**	**4.9**	*9.2*	*23.1*	*0.0*	**3.8**	**5.5**	**3.8**	*7.0*	*15.6*	*0.1*
1.13	**14.4**	**33.8**	**0.0**	*14.4*	*45.8*	*0.0*	**5.2**	**9.3**	**5.2**	*9.7*	*25.6*	*0.0*	**4.0**	**6.0**	**4.0**	*7.3*	*16.9*	*0.1*
1.14	**15.6**	**40.3**	**0.0**	*15.6*	*53.3*	*0.0*	**5.4**	**10.5**	**5.4**	*10.3*	*28.3*	*0.0*	**4.1**	**6.6**	**4.1**	*7.7*	*18.4*	*0.0*
1.15	**16.9**	**48.6**	**0.0**	*16.9*	*62.7*	*0.0*	**10.9**	**22.8**	**0.0**	*10.9*	*31.6*	*0.0*	**4.3**	**7.3**	**4.3**	*8.0*	*20.0*	*0.0*
1.16	**18.6**	**74.9**	**0.0**	*18.6*	*74.9*	*0.0*	**11.6**	**26.1**	**0.0**	*11.6*	*35.4*	*0.0*	**4.5**	**8.1**	**4.5**	*8.4*	*21.8*	*0.0*
1.17	**20.6**	**91.0**	**0.0**	*20.6*	*91.0*	*0.0*	**12.4**	**30.0**	**0.0**	*12.4*	*39.9*	*0.0*	**4.7**	**9.0**	**4.7**	*8.8*	*23.8*	*0.0*
1.18	**23.0**	**113.0**	**0.0**	*23.0*	*113.0*	*0.0*	**13.2**	**34.8**	**0.0**	*13.2*	*45.4*	*0.0*	**9.3**	**18.9**	**0.0**	*9.3*	*26.2*	*0.0*

Table 1 indicates that the choice of $\gamma = 0.88$ practically translates into risk neutral behavior for the expected utility investor: at low expected returns, expected utility is maximized by selling the majority of shares purchased at $t = 0$ in node P_l and using the proceeds from gains to purchase more shares in node P_h; as expected returns increase the number of shares held in node P_l smoothly trends to zero as the investor moves towards purchasing the maximum number of shares possible at $t = 0$ and in node P_h. However, if the level of risk aversion were to increase, the investor would opt for a more conservative strategy: reducing the size of the initial investment and subsequently realizing smaller losses in node P_l and purchasing a smaller number of shares in node P_h. The behavior of the expected utility investor in Table I indicates that the optimal portfolio strategy simplifies to allocating a constant proportion of wealth to the risky asset that depends on the investor's level of risk aversion and the assets return characteristics.

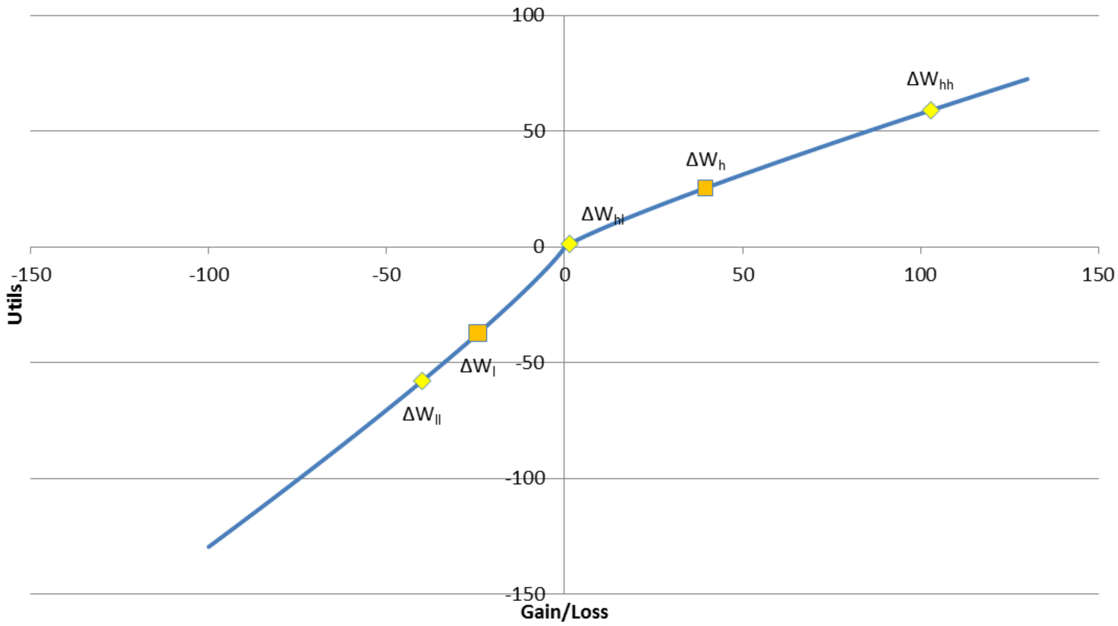
Unlike the expected utility investor, the realization utility investor's strategy does not trend smoothly in the space spanned by returns and volatilities. Instead, the investor uses a decision rule to guide discrete changes in his strategy. Starting with the investor's initial investment: if the stock's Sharpe ratio is above the minimum threshold value of approximately 0.3 then an investment is made; otherwise, if the Sharpe ratio is greater than approximately 0.6, the optimal decision is to take an even larger – loss leveraged – position by liquidating the stock at node P_l. The two different strategies at P_l explain the observed trends and jumps in x_0. For moderately attractive stocks, the investor's strategy is to maintain the same position in node P_l and the value of x_0 is, therefore, fixed by the binding of the non-negativity constraint in node P_{ll}. This explains the gradual increase in the size of x_0 as the expected return of the risky asset increases i.e. holding volatility fixed, equation 3.3 indicates R_l is an increasing function of μ. For more attractive stocks, the investor is willing to sell all of his shares in node P_l so that the value of x_0 is fixed by the binding of the non-negativity constraint in P_l. The trades at node P_h indicate that x_h follows a similar rule: for lower values of μ the investor chooses to realize utility early so that $x_h < x_0$; as μ increases the investor subsequently chooses to increase the size of his position, so that $x_h > x_0$, until he reaches a final cut off point where it is optimal to invest all available wealth in the risky asset. For the preference parameters used, the cut off Sharpe ratios exhibit the following the relationship:

$$Sharpe_{X_l=X_0} < Sharpe_{X_h<X_0} < Sharpe_{X_h>X_0} < Sharpe_{X_l=0} < Sharpe_{X_{h,max}}$$

According to the definition supplied by Odean [1998], the disposition effect holds whenever the assets Sharpe ratio is greater than $Sharpe_{X_l=X_0}$ and less than $Sharpe_{X_h>X_0}$. By calculating the cut off Sharpe ratios, the disposition approximately holds for the preference parameters estimated by Kahneman and Tversky [1992] and the range of expected returns and standard deviations in Table 1 whenever $0.3 < Sharpe < 0.36$.

In order to understand the behaviour of the realization utility investor it is useful to begin by recalling Barberis and Xiong's [2009] explanation of why prospect theory does not result in the early realization of gains and the postponement of losses consistent with the disposition effect. Their results indicate that, similar to the realization utility investor, the prospect theory investor will invest in stocks depending on whether the asset's Sharpe ratio is above a certain threshold. For example, when using the same preference parameters and time periods used to generate the data in Table 1, the minimum expected return required for investment is around 1.1 when volatility equals 0.3. Figure 2 illustrates the gains and losses corresponding with the investor's strategy at the different price outcomes on the horizontal axis, as well as the corresponding utility from each gain or loss calculated using the prospect theory value function on the vertical axis. The high Sharpe ratio indicates that the magnitude of gains is relatively larger than the magnitude of losses, so that the investor is further from the kink in the value function after paper gains than after paper losses at $t = 1$. Consequently, after a gain, the investor increases his share holdings so that the worst possible outcome at $t = 2$ (ΔW_{hl}) lies just in front of the kink in the value function: it is suboptimal to take a larger position because gambling past the kink means that loss aversion would greatly reduce his expected utility. Similarly, after a loss, the investor reduces his share holdings so that the best possible outcome at $t = 2$ (ΔW_{hl}) lies just to the right of the kink.

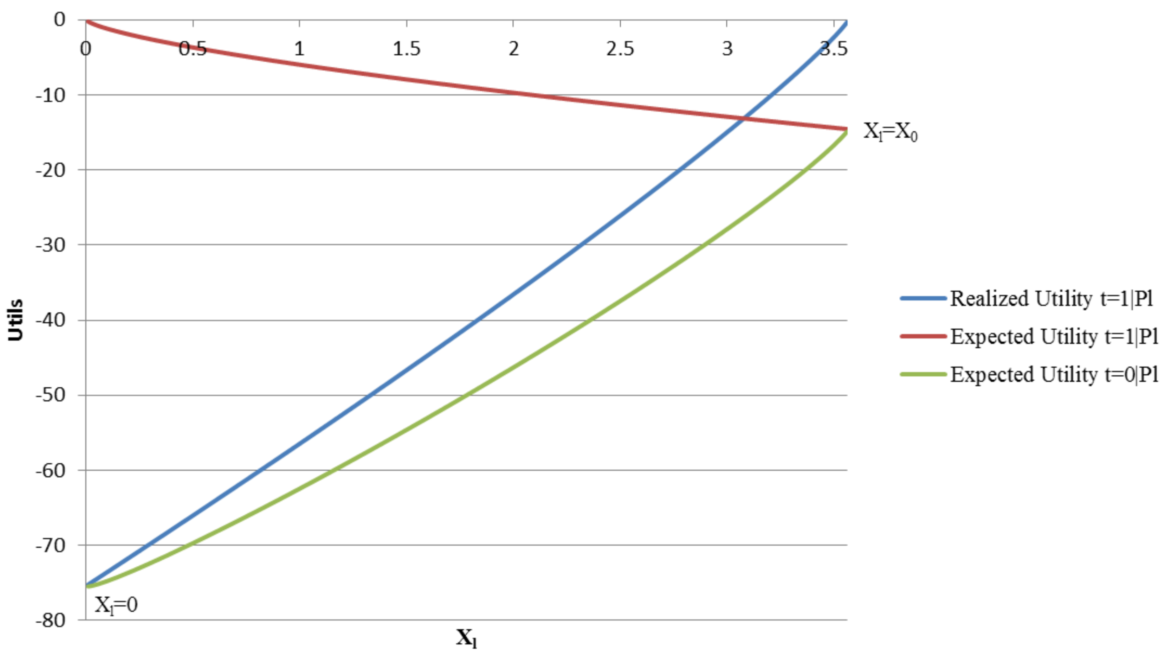
Figure 2
Prospect Theory Investor Portfolio Rebalancing Decision



I now show that, by forcing the investor to derive utility from each realized gain or loss, realization utility produces the disposition effect for the same returns and volatilities used in Figure 2. First, let us inspect the decision in state P_l. The component of equation 3.13 corresponding with $t = 2$ outcomes pushes the realization utility investor towards the prospect theory investor's decision of selling shares to the point that a small gain occurs in node P_{hl}. However, in order to obtain this allocation the investor must realize a loss and experience an associated disutility. It turns out that the cost of realizing this loss early outweighs the benefit from the reduction in expected losses at $t = 2$. The effect of diminishing sensitivity that drives the result can clearly be seen in the plot of expected realized utility from the perspective of $t = 0$ conditional on node P_l and its components (realized utility and expected realized utility at $t = 1$) shown in Figure 3: as x_l decreases from x_0 towards the value of zero, the negative contribution to utility from expected realized utility slowly diminishes while the negative contribution from realized utility rises much more rapidly.

Figure 3

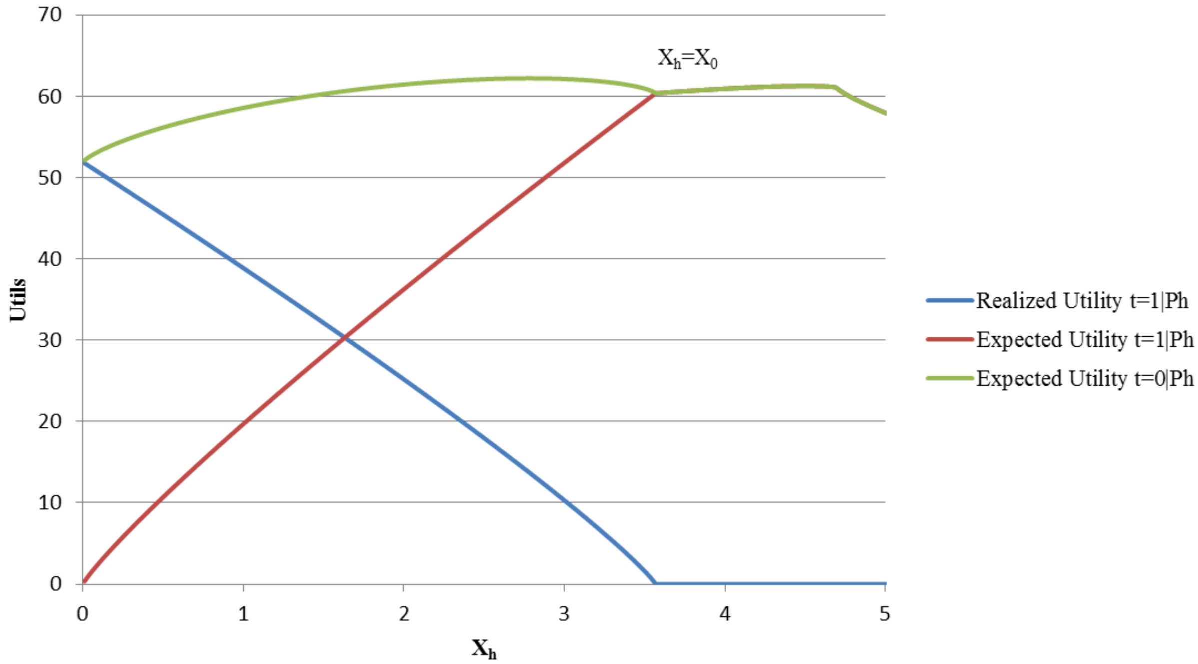
Realized and Expected Utility Decomposition Conditional on State P_l



Similar to the rebalancing decision in node P_l, the $t = 2$ component of the investor`s decision problem in node P_h pushes the realization utility investor toward the prospect theory investor’s strategy of purchasing more shares of the risky asset. However, starting from $x_h = x_0$, Figure 4 shows that the marginal contribution from incremental realized gains (moving to the left) outweighs the marginal contribution of increased expected returns (moving to the right).

Figure 4

Realized and Expected Utility Decomposition Conditional on State P_h



Thus, in the region of gains, realization utility combined with diminishing sensitivity contributes to the disposition effect because the utility from realized gains contributes positively to the investor's overall strategy: providing an incentive to sell winners early. And, conversely, utility from realized losses contributes negatively to the investor's overall strategy: providing an incentive to ride out losing investments. Contrary to Barberis and Xiong [2011], I find that removing diminishing sensitivity, so that gains and losses are evaluated using a linear or piece wise liner utility function, and including time discounting can only explain half the disposition effect i.e. investors are not willing to realize gains early, but tend to hold onto loosing trades. Considering their model set-up, it seems like their explanation of the disposition effect may rely more on the restriction that investors can hold one share of the risky asset in their portfolio. The restriction removes the possibility that investors may be able to increase their exposure to the risky asset, thereby reducing the portfolio allocation problem to determining the optimal price at which to sell the single share of the risky asset. It follows naturally then that the investor would only want to sell that share after experience a gain – and hence always exhibit a disposition – since it would be irrational to realize a painful loss only to find oneself applying the same stock selection criteria, albeit to the same universe of stocks, that united the investor with the loosing stock in the first place. It therefore seems like the combination of diminishing sensitivity and realization utility provides a more solid explanation of the disposition effect in that it holds even when the realization utility investor is able to purchase more shares of the risky asset at future trading dates.

To determine the effects of diminishing sensitivity and loss aversion, Table 2 shows the realization utility investor's optimal portfolio allocation for a range of preference parameters and expected returns when $\sigma = 0.3$. The second panel shows that increasing the degree of diminishing sensitivity augments the strength of the disposition effect when $\lambda = 1$; there is both a greater tendency to realize gains early and to ride loosing trades. From the previous discussion, it should be clear why this is the case: greater diminishing sensitivity decreases the attractiveness of holding out for larger expected gains at $t = 2$ relative to realizing a smaller gain immediately; it also decreases the attractiveness of realizing losses relative to holding on to loosing trades. The first panel shows that the effect of diminishing sensitivity is more dramatic when $\lambda = 2.25$ in that it allows for investment in stocks that would otherwise be unattractive. This is because it not only has an intertemporal effect, but also has a substitution effect that acts across the different branches in the binomial tree by augmenting the contribution of the realized gain in node P_h relative to the contribution of the large loss at node P_{ll}.

Table 2

Preference Parameters and Realization Utility Optimal Share Allocations

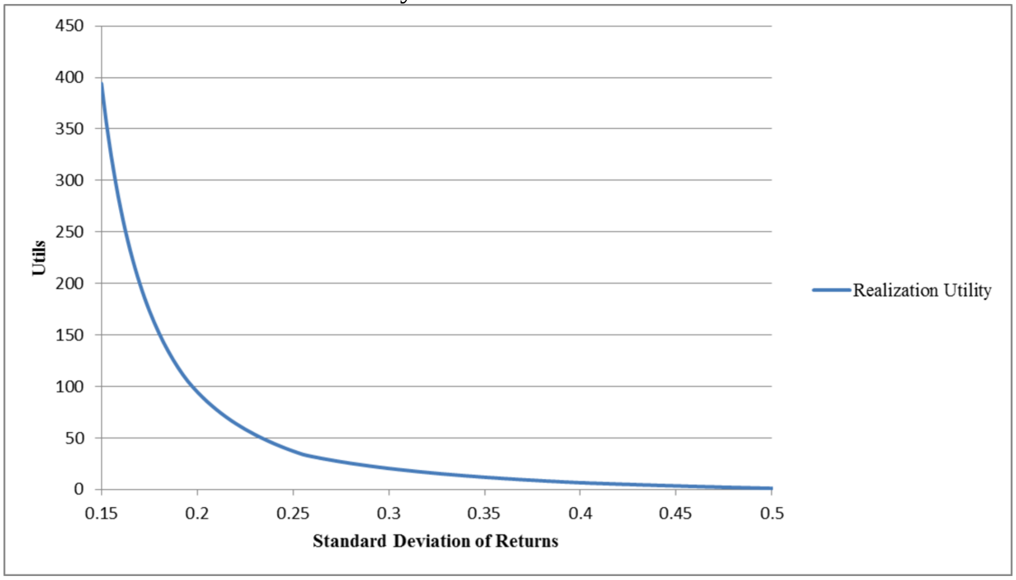
	λ = 2.25									λ = 1								
μ	**α = 0.88**			*α = 0.7*			**α = 0.5**			*α = 0.88*			**α = 0.7**			*α = 0.5*		
	x_0	x_h	x_l	x_0	x_h	x_l	x_0	x_h	x_l	x_0	x_h	x_l	x_0	x_h	x_l	x_0	x_h	x_l
1.04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.05	0.0	0.0	0.0	0.0	0.0	0.0	3.0	1.3	3.0	5.5	3.2	0.0	3.0	1.3	3.0	3.0	1.3	3.0
1.06	0.0	0.0	0.0	0.0	0.0	0.0	3.1	1.4	3.1	5.7	3.6	0.0	3.1	1.5	3.1	3.1	1.4	3.1
1.07	0.0	0.0	0.0	0.0	0.0	0.0	3.2	1.5	3.2	5.9	4.0	0.0	3.2	1.6	3.2	3.2	1.5	3.2
1.08	0.0	0.0	0.0	3.3	1.7	3.3	3.3	1.6	3.3	6.1	4.4	0.0	3.3	1.7	3.3	3.3	1.6	3.3
1.09	3.5	2.6	3.5	3.5	1.8	3.5	3.5	1.7	3.5	6.4	13.1	0.0	3.5	1.8	3.5	3.5	1.7	3.5
1.1	3.6	2.8	3.6	3.6	2.0	3.6	3.6	1.8	3.6	6.6	14.0	0.0	3.6	2.0	3.6	3.6	1.8	3.6
1.11	3.7	3.0	3.7	3.7	2.1	3.7	3.7	1.9	3.7	6.8	14.9	0.0	3.7	2.1	3.7	3.7	1.9	3.7
1.12	3.8	5.5	3.8	3.8	2.2	3.8	3.8	2.0	3.8	7.1	16.0	0.0	3.8	2.2	3.8	3.8	2.0	3.8
1.13	4.0	6.0	4.0	4.0	2.4	4.0	4.0	2.1	4.0	7.4	17.3	0.0	4.0	2.4	4.0	4.0	2.1	4.0
1.14	4.1	6.6	4.1	4.1	2.5	4.1	4.1	2.2	4.1	7.7	18.6	0.0	7.7	4.7	0.0	4.1	2.2	4.1
1.15	4.3	7.3	4.3	4.3	2.7	4.3	4.3	2.3	4.3	8.1	20.2	0.0	8.1	5.0	0.0	4.3	2.3	4.3
1.16	4.5	8.1	4.5	4.5	2.8	4.5	4.5	2.4	4.5	8.5	21.9	0.0	8.5	5.3	0.0	4.5	2.4	4.5
1.17	4.7	9.0	4.7	4.7	3.0	4.7	4.7	2.6	4.7	8.9	23.9	0.0	8.9	5.7	0.0	4.7	2.6	4.7

In addition to preference parameters affecting the cut off Sharpe ratios, it also seems that the level of volatility has some bearing as well. For the parameters used to generate the data in Table 1, the minimum Sharpe ratio required to make an initial investment in the stock ranges from 0.3 when volatility equals 0.3, to 0.32 when volatility equals 0.05. Additionally, the upper limit at which the disposition effect holds ranges from 0.37 when volatility equals 0.3, to 0.35 when volatility equals 0.05. Intuitively, the first observation indicates a slight preference for volatility, or at the least a slight relative preference for volatility, in that the investor places lower demands on the expected return required for an initial investment the higher the volatility. However, by token, the second observation seems to imply the contrary in that the investor places higher demands on the expected return required for reinvestment of capital gains into the risky asset. Given the apparent contradictions regarding preferences inferred from the investor’s trading strategy response to changes in the return distribution, it is necessary to take a closer look at the preferences of the realization utility investor

I start investigating preferences by plotting initial expected realized utility as a function of the standard deviation of returns using the preference parameters estimated by Kahneman and Tversky [1992]. Note, however, that even extreme deviations from the assumed preference parameters, like removing diminishing sensitivity and loss aversion and introducing time discounting, do not significantly alter the relationship shown in Figure 5. The result seems contrary to Barberis and Xiong’s [2011] finding that realization utility investors have a preference for volatility because that requires that initial expected utility be increasing in the level of volatility. I find that the apparent discrepancy can be resolved by considering how the simplifying assumption that investors are restricted to holding one share of the risky asset in their portfolio connects to our framework, as well as to the greater context of financial economic modeling.

Figure 5

Realization Utility vs. Standard Deviation of Returns



The simplifying assumption imposed on investor trading in Barberis and Xiong (2011) has already had an important ramification for this partial equilibrium analysis in that it provides indirect support for why diminishing sensitivity in combination with realization utility is required in order to obtain the disposition effect. The finite time horizon used in this analysis makes it easier, indeed, to consider investor behavior without imposing restrictions on portfolio allocation decisions. It also, however, forces us to consider the investor's wealth constraint when determining the optimal investment strategy. The same consideration need not be made in their model because, in addition to the holding constraint, all stocks trade at a price equal to the investor's wealth level. The non-negativity of wealth constraint is, therefore, trivially satisfied because stock prices cannot go negative. Conversely, by allowing the investor the freedom of choosing the optimal portfolio it is necessary to consider the effect that the assets return distribution has on the value of the investment via the non-negativity of wealth constraint.

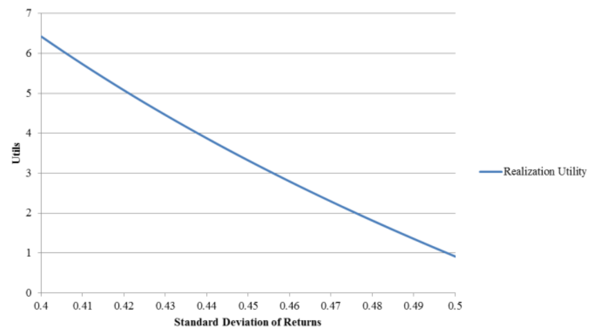
For example, consider the relative value of two different risky assets for which the investor follows the strategy $x_l = x_0$ and $x_h \leq x_0$. Both assets have the same expected return of 1.05, but the first has a volatility of 0.1, whereas the second has a volatility of 0.3. This means that the investor is able to purchase around 22 shares of the first stock and only 3 shares of the second. Thus, looking at the initial expected utility of the investor for the two different prospects captures both a pure preference effect and a leverage effect, the former of which captures the utility obtained from holding a single share and the latter of which corresponds with a multiplicative effect owing to the wealth constraint and return distribution. By looking at the pure preference component it could be possible to observe the purported preference for volatility. This may or may not seem reasonable depending on one's view of how individuals interact in real financial markets. It can be argued that no financial markets have perfect liquidity and that, therefore, there must be a limit to the number of shares that can be purchased before driving up offer prices. If we assume that all markets are equally liquid, so that the investor is not able to purchase more shares of one stock than another, then we should, perhaps, examine only the pure preference component.

Previous work on Prospect Theory Barberis et al. [2001] can provide insight into the pure preferences of an investor who behaves according to the realization utility specification. My earlier finding indicates that the $t = 1$ component of expected realized utility does not affect the investor's optimal portfolio allocation and resulting utility when volatility is below a certain cut-off level. Hence in this region, I would expect the investor to display preferences consistent with those of the prospect theory investor who derives utility from annual gains and losses. Barberis et al. [2001] find that this investor demonstrates preferences that are congruent with the Capital Asset Pricing Model, and hence consistent with the relationship shown in Figure 5. Conversely, the region where realization utility does affect welfare and portfolio allocations can be subdivided by strategy across volatility levels. At the lower end of the spectrum $x_h \geq x_0$ and $x_l = x_0$ and as volatility increases $x_h \leq x_0$ and $x_l = x_0$. I thoroughly analyze the investor's preference in the second region. The analysis is particularly straightforward and is consistent with the motivation for looking at pure preferences: there is no need to include additional controls to account for the effects of wealth levels and return distributions because there are no shares purchased at $t = 1$. The same cannot be said for the lower end of the spectrum because $x_h \geq x_0$. I subsequently leave this region for future research as I expect preferences in this region to be similar to those of the prospect theory investor who derives utility from annual gains and losses.

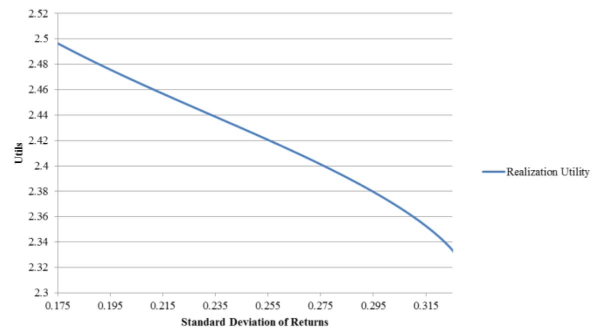
Looking at pure preferences, the extent of risk aversion shown in Panel 1.a seems reduced as the curve is flatter compared to Figure 5. Panel 1.b shows that reducing the loss aversion coefficient, $\lambda = 1$, has a major impact on the investor's level of risk aversion in that expected utility is nearly flat across different levels of volatility. Given that the investor's optimal strategy for all of the data points in the figure is consistent with the disposition effect there is a simple explanation: as volatility increases and loss aversion decreases, the investor is able to obtain greater utility from realizing a larger gain in node P_h while being less sensitive to the proportionally larger loss in node P_{ll}. The lower graphs in Panel 1 take the investigation further by first increasing the level of diminishing sensitivity (left) and then also introducing time discounting (right). The combination of low loss aversion and high diminishing sensitivity produce a preference for a specific level of volatility: as the volatility of returns increases past the local maximum (in terms of investor initial expected utility), the size of the gain realized at node P_h increases to the point where realized utility is no longer more lucrative relative to the expected outcomes at $t = 2$ and initial expected utility eventually begins to decrease. Panel 1.d shows that introducing a positive time discount factor into the investor's decision making process increases both the strength of the preference for volatility and the volatility level at which expected utility is maximized. Intuitively, time discounting results in more emphasis being placed on earlier outcomes and hence augments the underlying effects that produce the disposition effect and the preference for volatility. In the next section, I examine how the differences in realization utility and expected utility investors' trading strategies manifest in general equilibrium.

Panel 1

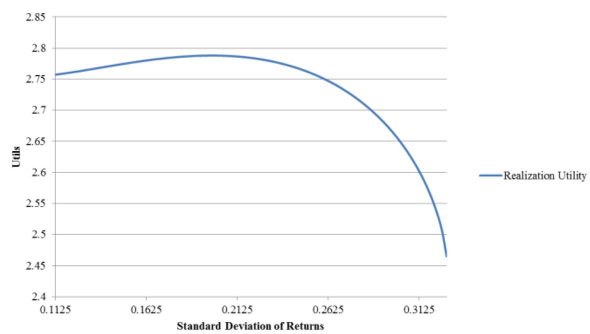
Realization Utility Pure Preferences vs. Standard Deviation of Returns for Various Preferences Parameter Values



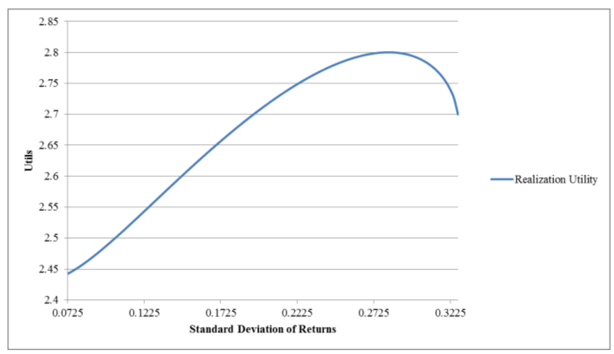
a.) $\alpha = 0.88,\ \lambda = 2.25,\ \delta = 1$



b.) $\alpha = 0.88,\ \lambda = 1,\ \delta = 1$



c.) $\alpha = 0.7,\ \lambda = 1,\ \delta = 1$



d.) $\alpha = 0.7,\ \lambda = 1,\ \delta = 0.9$

IV. General Equilibrium

The purpose of the general equilibrium analysis is to examine the differences between financial markets composed of only expected utility investors (hereafter referred to as expected utility markets) and those composed of both realization utility and expected utility investors (realization utility markets). I take the simplest approach and assume that there are either two expected utility investors or an expected utility and a realization utility investor trading in the market. Furthermore, in order to implement the calculation in the two period binomial framework without relying on specifying initial investor share allocations, I introduce a securities dealer who sells shares to investors at $t = 0$. This serves two purposes: (1) it makes it easier to interpret the effects of investor wealth levels and preferences on equilibrium prices because initial stock allocations are not exogenously specified, but, rather, result from forces of demand that reflect anticipated future outcomes; (2) it creates an economic scenario analogous to the auctioning of an initial public offering (IPO).

The section begins with the definition of general equilibrium followed by an analytical derivation of general equilibrium outcomes when investor preferences do not have the property of diminishing sensitivity and the dealer offers shares according to a linear supply schedule. Examining the analytical model indicates that expected utility investors are able to obtain welfare improvements by setting the equilibrium bid price paid to the securities dealer in order to extract maximum value from the biases affecting the realization utility investor's trading decision at $t = 1$. Incorporating diminishing sensitivity improves the realism of the analysis, but also requires the use of numerical methods. The second half of this section is dedicated to a discussion of the numerical general equilibrium calculation, a more comprehensive model of the dealer's supply schedule, and an analysis of general equilibrium outcomes under a range of wealth distributions and expected utility and realization utility preference parameters.

The general equilibrium framework follows the structure of the partial equilibrium analysis: $R_{\mathrm{f}} = 1$, returns of the risky asset are distributed according to equation 3.1, and all market participants liquidate their portfolios at $t = 2$. The collection formed by prices P_0^* at $t = 0$ and P_h^* and P_l^* at $t = 1$, and portfolio allocation plans $x_{i,0}^*$ at $t = 0$ and $x_{i,h}^*$ and $x_{i,l}^*$ at $t = 1$ constitute a Radner equilibrium if for every market participant i the following conditions are satisfied:

$$\max_{x_{i,0}, x_{i,h}, x_{i,l}} E\{U_i(x_{i,0}, x_{i,h}, x_{i,l})\} \qquad 4.1$$

subject to

$$W_1 = W_0 + x_0 P_0 (R_{0,1} - 1) \geq 0 \qquad 4.2$$

and

$$W_2 = W_0 + x_0 P_0 (R_{0,1} - 1) + x_1 P_1 (R_{1,2} - 1) \geq 0. \qquad 4.3$$

To facilitate obtaining an analytical solution it is useful to use the following form for the dealer's supply schedule

$$f(P_0) = A(P_0 - D_0), \qquad 4.4$$

where D_0 equals the price paid by the dealer for shares held in inventory and the constant A gives the number of shares supplied by the dealer for each dollar received above the minimum offer price D_0. The dealer's profits are then equal to

$$\pi(P_0) = f(P_0) * (P_0 - D_0), \qquad 4.5$$

so that dealer welfare is necessarily increasing in P_0. Furthermore, when selling shares to the market the dealer allocates shares to investors in proportion to their relative levels of demand.

Following the approach used in the partial equilibrium analysis, determining general equilibrium prices and trades begins by working backwards through the binomial tree. I begin by solving for general equilibrium with two risk neutral expected utility investors with equal wealth levels (W_{EU}). At $t = 2$ determining trading behavior is trivial as all participants are forced to exit their position in the risky asset as a result of a liquidity shock. At $t = 1$, the expected utility of either investor conditional on P_0 and the initial stock position is

$$E\{W_2\}_{t=1} = W_{EU} + x_0(P_1 - P_0) + x_1\left(\frac{D_0R_{0,1}R_h + D_0R_{0,1}R_l}{2} - P_1\right). \qquad 4.6$$

Equation 4.6 indicates that investors are indifferent to buying or selling shares at $P_1^* = D_0R_{0,1}(R_h - R_l)/2$: above this price, x_1 is multiplied by a negative term, implying the optimal strategy is to sell; below this price, x_1 is multiplied by a positive term, implying the optimal strategy is to buy. Since both investors are indifferent to buying or selling at the same price, the indifference price corresponds with the equilibrium price. Moreover, in order to avoid arbitrarily designating one investor as the buyer and the other as the seller trading volumes are nil so that equation 4.6 becomes

$$E\{W_2\}_{t=1} = W_{EU} + x_0\left(\frac{D_0R_{0,1}R_h + D_0R_{0,1}R_l}{2} - P_0\right). \qquad 4.7$$

Continuing backwards through the tree, initial expected utility is given by

$$E\{W_2\}_{t=0} = \frac{1}{2}\left\{E\{W_2\}_{t=1|P_h} + E\{W_2\}_{t=1|P_l}\right\} = W_{EU} + \frac{x_0(\mu D_0 - P_0)}{2}. \qquad 4.8$$

Equation 4.8 shows that initial expected utility is increasing in the number of shares that each investor purchases from the dealer when $\mu D_0 > P_0$ and that the marginal benefit of purchasing more shares is the same for both investors. Combined with the fact that both investors have identical wealth levels, this means that the two investors demand shares in equal proportions from the dealer at each value of P_0 so that

$$x_0 = \frac{f(P_0)}{2}. \qquad 4.9$$

Substituting 4.9 into 4.8 gives initial expected utility in terms of P_0,

$$E\{W_2\}_{t=0} = W_{EU} + \frac{f(P_0)(\mu D_0 - P_0)}{2}. \qquad 4.10$$

The optimal price for the expected utility investors to bid is given by the first order condition of equation 4.10 with respect to P_0. Taking the first derivative of equation 4.10, setting to zero, and solving for P_0, the optimal bid price disregarding wealth constraints is

$$P_{0,bid}^{*} = D_0 \frac{\mu + 1}{2}, \tag{4. 11}$$

and the corresponding optimal number of shares purchased by each investor is

$$x_{0,bid}^{*} = \frac{(\mu - 1)AD_0}{4}. \tag{4. 12}$$

Initial demand is, however, limited to the maximum number of shares that can be purchased at $t = 0$. Considering that there are no trades at $t = 1$, the maximum number of shares that each investor can purchase from the dealer is bound by the worst possible return to the risky asset at $t = 2$ so that

$$x_{0,max} = \frac{W_0}{P_0 - D_{ll}}. \tag{4. 13}$$

Setting the left hand side of equation 4.13 equal to $f(P_0)/2$ and solving for the corresponding bid price gives

$$P_{0,max} = \frac{A(D_{ll} + D_0) \mp \sqrt{\left(A(D_{ll} + D_0)\right)^2 + 4a(AD_{ll}D_0 - W_0)}}{2A}. \tag{4. 14}$$

Finally, if $x_{0,bid}^{*} > x_{0,max}$ then

$$P_0^{*} = P_{0,max} \tag{4. 15}$$

otherwise

$$P_0^{*} = P_{0,bid}^{*}. \tag{4. 16}$$

Equations 4.15 and 4.16 show that equilibrium prices in expected utility markets are completely determined by the price at which the dealer begins to offer shares, the return distribution of the risky asset, and investor initial wealth levels. Since wealth constraints have the effect of limiting initial demand, and potentially forcing investors to sell shares, for the remainder of the analytical section I choose to ignore them. This simplifies the analysis by making it possible to focus on the price dynamics that arise entirely from differences in preferences and to ignore complications due to the interaction between preferences and wealth constraints.

Moving on to the realization utility market analogue, I find that in equilibrium the realization utility investor never purchases shares at $t = 1$. Additionally, the realization utility investor sells shares at $t = 1$ depending on the value of P_0 relative to P_{hl}. Starting with the realization utility investor's purchase decision using node P_l as an example, the investor's objective is to maximize (by equation 3.13)

$$E\{RU\}_{P_l}|x_l \geq x_0 = \frac{\delta}{2}\left\{v\big(x_0(P_l - P_0) + x_l(D_{hl} - P_l)\big) + v\big(x_0(P_l - P_0) + x_l(D_{ll} - P_l)\big)\right\}. \quad 4.17$$

To determine the form of $v(\cdot)$, it is easier to examine its arguments by substituting $x_l = x_0 + \Delta x_l$. The gains/loss of the first term in equation 4.17 is then equal to

$$G_{hl} = x_0(P_l - P_0) + (x_0 + \Delta x_l)(D_{hl} - P_l) = x_0(D_{hl} - P_0) + \Delta x_l(D_{hl} - P_l), \qquad 4.18$$

which is positive or negative depending on the value of P_0 relative to D_{hl} and the value of Δx_l relative to x_0. Examining the cases, if $P_0 \leq D_{hl}$ then by the no arbitrage condition ($D_{hl} > P_l$) $G_{hl} \geq 0$ for all choices of Δx_l. Conversely, if $P_0 > D_{hl}$, then $G_{hl} < 0$ when

$$\Delta x_l < \frac{x_0(P_0 - D_{hl})}{D_{hl} - P_l}, \qquad 4.19$$

and $G_{hl} \geq 0$ when

$$\Delta x_l \geq \frac{x_0(P_0 - D_{hl})}{D_{hl} - P_l}. \qquad 4.20$$

Since the argument of the second term is always negative (by the no arbitrage condition $P_{ll} \leq P_l \leq P_0$), equation 4.17 takes two forms depending on the sign of G_{hl}: when $G_{hl} < 0$

$$E\{RU\}_{P_l}|G_{hl} < 0 = \lambda\delta\left\{x_0(P_l - P_0) - \Delta x_l\left(P_l - \frac{D_{hl} + D_{ll}}{2}\right)\right\}, \qquad 4.21$$

and when $G_{hl} \geq 0$

$$E\{RU\}_{P_l}|G_{hl} \geq 0 = -\frac{x_0\delta(1+\lambda)(P_0 - P_l)}{2} + \frac{\Delta x_l\delta\big((D_{hl} + \lambda D_{ll}) - P_l(1+\lambda)\big)}{2}. \qquad 4.22$$

Maximization of realization utility implies that when $G_{hl} < 0$, the realization utility investor is indifferent to demanding shares or maintaining $x_l = x_0$ at the expected utility investor's indifference price, i.e. $P_l = (D_{hl} + D_{ll})/2$. Because the realization utility investor is indifferent to holding or buying and the expected utility investor is indifferent to selling or buying, the welfare outcomes associated with the realization utility investor maintaining the same position or purchasing more shares of the risky asset are equivalent. Additionally, equation 4.22 implies that when $G_{hl} \geq 0$ the realization utility investor buys shares only if

$$P_l < \frac{(D_{hl} + \lambda D_{ll})}{(1+\lambda)}. \qquad 4.\ 23$$

The right hand side of equation 4.23 is always less than or equal to $P_l = (D_{hl} + D_{ll})/2$ for $\lambda \geq 1$ so that the realization utility investor's maximum bid price is never greater than the expected utility investor's minimum offer price in node P_l. The realization utility investor's bid in node P_h follows a similar pattern. When both gain/loss outcomes at $t = 2$ are the same sign, the maximum bid equals the expected utility investor's indifference price. Moreover, when the outcomes are the opposite sign, greater sensitivity to losses reduces the perceived attractiveness of holding more shares and subsequently lowers the maximum bid price. In summary, there is the possibility that the realization utility investor may purchase shares from the expected utility investor at $t = 1$ when the indifference prices of both investors are equal. However, in the following segment it is apparent that these trading strategies are either Pareto inferior to the realization utility investor's sell strategy or are welfare equivalent. In the latter case, the realization utility investor neither purchases nor sells shares because it does not make sense to arbitrarily assign one investor as the seller and the other as the buyer.

It is easier to understand the realization utility investor's decision to supply shares in equilibrium by framing the investor's problem so that it is comparable with risk neutral expected utility maximization. To facilitate obtaining an analytical solution I will assume that the expected utility investor has sufficient wealth to purchase all of the shares supplied by the realization utility investor regardless of current and historical stock prices. Table 4.1 compares the contributions of the two strategies to the realization utility account in node P_h when $P_0 > D_{hl}$.

Table 4. 1

	Sell	**Hold**
Contribution to Utility Account	$x_0(P_h - P_0)$	$\frac{\delta x_0}{2}\{(D_{hh} - P_0) + \lambda(D_{hl} - P_0)\}$

Similarly, Table 4.2 compares the marginal contribution to final period expected wealth of the expected utility investor for the sell and hold strategies in node P_h.

Table 4. 2

	Sell	**Hold**
Contribution to Wealth Account	$x_0(P_h - P_0)$	$\frac{x_0}{2}\{D_{hh} + D_{hl} - 2P_0\}$

The investors' minimum offer prices are determined by the value of P_h which equates the contribution to the account of the sell and hold strategies. Tables 4.1 and 4.2 show that the value of the sell strategy is identical for both investors and is increasing in P_h. Furthermore, greater sensitivity to losses and time discounting decrease the value of the hold strategy to the realization utility investor resulting in this investor having the lowest offer price whenever $\lambda > 1$ or $\delta < 1$. Solving for the realization utility investor's minimum offer price by equating the value of selling and holding shares gives

$$P_{h,RU}^{*}|P_0 > D_{hl} = P_0 + \frac{\delta}{2}\{(D_{hh} - P_0) + \lambda(D_{hl} - P_0)\}. \qquad 4.24$$

Recall from the discussion of the expected utility market that the expected utility investor's maximum bid price corresponds with the investor's indifference price. Since the realization utility investor's minimum offer is below the expected utility investor's maximum bid price, both parties find it beneficial to trade $x_{0,RU}$ shares. I set the equilibrium price equal to the mid-point of the maximum bid and minimum offer so that

$$P_h^*|P_0 > D_{hl} = \frac{D_{hh}(1+\delta) + D_{hl}(1+\delta\lambda) + P_0\big(2-\delta(1+\lambda)\big)}{4}. \qquad 4.25$$

A comparison of the realization utility account under the sell and hold strategies in node P_l when $P_0 \leq D_{hl}$ is presented in Table 4.3. The table shows that in the region of losses, time discounting tends to increase the value of the hold strategy for the realization utility investor by reducing the impact of losses. Since the value of the sell strategy is increasing in P_l, time discounting, therefore, tends to increase the realization utility investor's minimum offer price. However, greater sensitivity to losses tends to decrease the realization utility investor's minimum offer price. Removing time discounting, Table 4.4 shows the expected utility and realization utility account under the two strategies when the contributions of both strategies are multiplied by $1/\lambda$ [7].

Table 4. 3

	Sell	**Hold**
Contribution to Utility Account	$\lambda x_0(P_l - P_0)$	$\frac{\delta x_0}{2}\{(D_{hl} - P_0) + \lambda(D_{ll} - P_0)\}$

[7] Since the minimum offer price is determined by the value of P_l which sets the contributions of the sell and hold strategies equal, multiplying both contributions by a constant does not alter the calculated offer price.

Table 4. 4

	Sell	**Hold**
Contribution to Utility Account	$x_0(P_l - P_0)$	$\frac{x_0}{2}\left\{\frac{1}{\lambda}(D_{hl} - P_0) + (D_{ll} - P_0)\right\}$
Contribution to Wealth Account	$x_0(P_l - P_0)$	$\frac{x_0}{2}\{D_{hl} + D_{ll} - 2P_0\}$

Surprisingly, greater sensitivity to losses causes the realization utility investor to realize losses early in general equilibrium, not because it exacerbates the expected large loss in P_{ll}, but because it decreases the realization utility investor's perceived attractiveness of gains in node D_{hl} relative to the expected utility investor. It is, however, difficult to say who has the lowest offer price when $\delta < 1$ because this depends on the asset's return distribution, the realization utility investor's preference parameters, and the initial stock price. Indeed, when the realization utility investor's offer price is below the expected utility investor's offer price (indifference price), the two investor's trade at the midpoint of the lowest offer and highest bid so that equilibrium occurs at

$$P_h^*|P_0 \leq D_{hl} = \frac{D_{hl}(\lambda + \delta) + D_{hl}\lambda(1 + \delta) + P_0\big(2\lambda - \delta(1 + \lambda)\big)}{4}. \qquad 4.\ 26$$

Furthermore, when the realization utility investor's offer is greater than the expected utility investor's offer, there are no trades and the equilibrium price is equal to the expected utility investor's offer price (indifference price). The realization utility investor's maximum offer price in node P_h when $P_0 < D_{hl}$ and node P_l when $P_0 \geq D_{hl}$ can be obtained similarly.

The established trade prices and volumes at $t = 1$ can now be used to calculate initial expected utility and realized utility, determine the relative strengths of initial investor demand, and to solve for initial equilibrium bid prices at $t = 0$. For example, when $P_0 \leq D_{hl}$ and trades occur in node P_l, expected realized utility equals

$$E\{RU\}_{t=0}|P_0 \leq D_{hl}$$

$$= \frac{\delta x_{0,RU}}{8}\{(D_{hh} - P_0)(1+\delta) + (D_{hl} - P_0)(1 + 2\delta + \lambda) + (D_{ll} - P_0)\lambda(1+\delta)\}, \quad 4.27$$

and expected utility equals

$$E\{U\}_{t=0}|P_0 \leq D_{hl}$$

$$= W_{EU} + x_{0,EU}(\mu D_0 - P_0) + \frac{x_{0,RU}}{8\lambda}\{(D_{hh} - P_0)\lambda(1-\delta) + (D_{hl} - P_0)\big(2\lambda - \delta(1+\lambda)\big) + (D_{ll} - P_0)\lambda(1-\delta)\big\}. \quad 4.28$$

The first two terms in equation 4.28 are identical to the expected utility investor's expected utility when trading in expected utility markets. Moreover, since the term multiplying $x_{0,RU}$ is necessarily positive when $\lambda > 1$ or $\delta < 1$ and $P_0 \leq \mu D_0$, the expected utility investor achieves welfare improvements in realization utility markets relative to expected utility markets. Equations 4.27 also indicates that realization utility demand is equal to the maximum number of shares that can be purchased at P_0 as long as P_0 is below a fixed threshold value required for the term multiplying $x_{0,RU}$ to be positive and, hence, the realization utility investor to enter the market. Obviously, the realization utility can only receive relatively more shares offered by the dealer if the expected utility investor simultaneously demands relatively fewer shares. Denoting the proportion of total shares sold by the dealer that are allocated to the realization utility investor with the variable β and substituting into equation 4.28 gives

$$E\{U\}_{t=0}|P_0 \leq D_{hl}$$

$$= W_{EU} + f(P_0)(1-\beta)(\mu D_0 - P_0) + \frac{f(P_0)\beta}{8\lambda}\{(D_{hh} - P_0)\lambda(1-\delta) + (D_{hl} - P_0)\big(2\lambda - \delta(1+\lambda)\big) + (D_{ll} - P_0)\lambda(1-\delta)\big\}. \quad 4.29$$

Inspecting equation 4.29, if the per share value of purchasing shares at $t = 0$ (second term) exceeds the per share value of purchase shares in at $t = 1$ (third term), then expected utility is necessarily decreasing in β and, like the realization utility investor, the expected utility investor demands as many shares as possible at each value of P_0. When this occurs both investors demand an equal number of shares from the dealer at $t = 0$ because they have equal wealth levels. Otherwise the realization utility investor demands all of the shares offered by the dealer. In order to account for the two cases, it is necessary to calculate candidate equilibria under the assumption that the expected utility investor waits until $t = 1$ to enter the market ($\beta = 1$) as well as under the assumption that expected utility and realization utility investors demand shares equally ($\beta = 1/2$). The equilibrium that provides the greatest utility to the expected utility investor corresponds with the true equilibrium since the initial demand decision is made at the expected utility investor's discretion. Furthermore, because of the discontinuity in equilibrium trades when $P_0 = D_{hl}$ and the fact that trades in node P_l depend on the value of P_0, it is necessary to calculate the optimal bidding behavior of investors in the different price ranges and under different ex-ante assumptions of trades in node P_l. For example, the optimal bid prices of the realization utility and expected investors obtained by the first order conditions of equations 4.27 and 4.28 when $\beta = 1/2$ are

$$P_{0,RU}^{*}|P_0 \leq D_{hl} = \frac{D_0}{2} + \frac{D_{hh}(1+\delta) + D_{hl}(1+2\delta+\lambda) + D_{ll}\lambda(1+\delta)}{4(1+\delta) + 2\big(2\lambda + \delta(1+\lambda)\big)} \qquad 4.30$$

and

$$\begin{aligned} &P_{0,EU}^{*}|P_0 \leq D_{hl} \\ &= \frac{4\lambda D_0(\mu+1)}{\big(12\lambda - \delta(3\lambda+1)\big)} \\ &+ \frac{(D_{hh}+D_0)\lambda(1-\delta) + (D_{hl}+D_0)\big(2\lambda - \delta(1+\lambda)\big) + (D_{ll}+D_0)\lambda(1-\delta)}{2\big(12\lambda - \delta(3\lambda+1)\big)}; \end{aligned} \qquad 4.31$$

Considering that the dealer's profits are increasing in P_0, general equilibrium occurs at the highest bid of the two investors. Furthermore, since this bid price has been calculated under the assumption of trades in node node P_l it is necessary to check that the realization utility investor's offer price is indeed below the expected utility investor's bid price under the highest bid at $t = 0$. If this is not the case, then general equilibrium corresponds with the highest bid under the assumption of no trades in node P_l. Considering equilibrium behavior on either side of the discontinuity, the highest bid in either region of P_0 equals

$$P_0^*|P_0, D_{hl} = \max\{P_{0,RU}^*|P_0, D_{hl}, P_{0,EU}^*|P_0, D_{hl}\}. \tag{4.32}$$

If $E\{RU(P_0^*|P_0, D_{hl})\}_{t=0} < 0$ or $E\{U(P_0^*|P_0, D_{hl})\}_{t=0} < W$, then one of the investors bids so aggressively that the other is not willing to enter the market and we obtain a non-trading equilibrium in the region of P_0. If this occurs when $P_0 < D_{hl}$ then the final equilibrium price is determined by the bidding behaviour of the most aggressive investor when the other investor does not participate in the market. Otherwise, general equilibrium is determined by the value of $P_0^*|P_0, D_{hl}$ that maximizes the expected utility or expected realized utility of the most aggressive bidder in the region $P_0 \geq D_{hl}$.

For the case of $\lambda = 1$ and $\delta = 1$, maximization of realized utility corresponds with maximization of gains which equates to maximization of wealth. Consequently, the realization utility and expected utility market outcomes are identical. Table 4.5 shows a comparison of expected utility and realization utility market outcomes when $\lambda = 1.5$. For each value of μ, I present results starting with the value of σ that sets the risky assets Sharpe ratio equal to 1, followed by intermediate values across which $P_0^* < D_{hl}$ and $P_0^* \geq D_{hl}$, and ending at the highest value that still allows the realization utility investor to enter the market.

Table 4. 5

General Equilibrium Outcomes with no Diminishing Sensitivity or Time Discounting

μ	σ	**Realization Utility Market(λ=1.5,δ=1)**							*Expected Utility Market* *					D_{hl}	*Sharpe*
		P_0\{RU\}	**P_0**	**P_h**	**P_l**	**$E\{U\}_{t=0}$**	**r_{RU}**	**r_{EU}**	*P_0*	*P_h*	*P_l*	*$E\{U\}_{t=0}$*	*r_{EU}*		
1.05	0.05	**102.13**	**102.50**	**108.53**	**101.27**	**3.2**	**2.3**	**2.6**	*102.50*	*108.53*	*101.47*	*3.1*	*2.4*	**104.88**	*1.00*
1.05	0.17	**101.27**	**102.47**	**116.98**	**92.92**	**3.2**	**2.4**	**2.6**	*102.50*	*116.98*	*93.02*	*3.1*	*2.4*	**103.63**	*0.29*
1.1	0.1	**104.27**	**104.99**	**117.06**	**102.56**	**13.0**	**4.6**	**5.1**	*105.00*	*117.06*	*102.94*	*12.5*	*4.8*	**109.55**	*1.00*
1.1	0.33	**102.70**	**104.90**	**133.08**	**86.90**	**12.5**	**4.8**	**4.9**	*105.00*	*133.08*	*86.92*	*12.5*	*4.8*	**105.16**	*0.30*
1.1	0.34	**102.61**	**105.17**	**133.73**	**86.23**	**12.5**	**4.6**	**4.6**	*105.00*	*133.77*	*86.23*	*12.5*	*4.8*	**104.87**	*0.29*
1.15	0.15	**106.41**	**107.48**	**125.58**	**103.87**	**29.1**	**6.7**	**7.5**	*107.50*	*125.58*	*104.42*	*28.1*	*7.0*	**114.03**	*1.00*
1.15	0.42	**104.62**	**107.35**	**144.23**	**85.75**	**28.1**	**7.1**	**7.1**	*107.50*	*144.23*	*85.77*	*28.1*	*7.0*	**107.57**	*0.36*
1.15	0.43	**104.47**	**107.76**	**144.84**	**85.10**	**28.2**	**6.7**	**6.8**	*107.50*	*144.90*	*85.10*	*28.1*	*7.0*	**107.22**	*0.35*
1.15	0.5	**103.96**	**107.85**	**149.18**	**80.42**	**28.9**	**6.4**	**6.9**	*107.50*	*149.58*	*80.42*	*28.1*	*7.0*	**104.60**	*0.30*
1.2	0.2	**108.56**	**109.97**	**134.09**	**105.21**	**51.7**	**8.8**	**9.8**	*110.00*	*134.09*	*105.91*	*50.0*	*9.1*	**118.34**	*1.00*
1.2	0.49	**106.68**	**109.81**	**153.97**	**85.98**	**50.1**	**9.3**	**9.3**	*110.00*	*153.97*	*86.03*	*50.0*	*9.1*	**110.38**	*0.41*
1.2	0.5	**106.46**	**110.33**	**154.60**	**85.36**	**50.1**	**8.7**	**8.8**	*110.00*	*154.64*	*85.36*	*50.0*	*9.1*	**110.00**	*0.40*
1.2	0.66	**105.30**	**110.57**	**164.16**	**74.90**	**52.3**	**8.1**	**9.1**	*110.00*	*165.10*	*74.90*	*50.0*	*9.1*	**103.05**	*0.30*

Because there are no trades at $t = 1$ in the expected utility market, optimal bid prices are determined entirely by the trade-off between increases in expected wealth obtained from purchasing the risky asset and the cost of purchasing shares. However, realization utility market outcomes for $\lambda = 1.5$ are more complex. The data indicate that the expected utility investor is the driver of equilibrium prices since P_0^* is always greater than the optimal bid price of the realization utility investor ($P_{0,RU}^*$). Additionally, the expected utility investor's optimal bid price in realization utility markets differs from expected utility markets. Recalling that the expected utility investor experiences welfare gains from trading with the realization utility investor at $t = 1$, the expected utility investor, therefore, alters the initial bid in order to extract maximum value from the behavioural biases incorporated into realization utility.

The data show that the expected utility investor either decreases or increases the initial bid depending on how the optimal bid in expected utility markets compare to the critical price outcome affecting the realization utility investor's strategy, i.e. P_{hl}. When the optimal bid in expected utility markets is below D_{hl}, welfare gains in realization utility markets can be obtained by lowering initial bids. The optimal degree of adjustment is determined by the trade-off between the discount received on shares purchased in node P_l, which is decreasing in P_0, and the number of shares purchased directly from the dealer as well as from the realization utility investor, which are increasing in P_0. Conversely, when the optimal bid in expected utility markets is greater than D_{hl}, welfare gains are obtained by increasing initial bids. The optimal degree of adjustment is determined by the trade-off between the greater discount and number of shares purchased in node P_h with the cost of purchasing shares directly from the dealer.

For the data used in Table 4.5, the maximum Sharpe Ratio that just allows the realization utility investor to participate in the market is on average approximately equal to 0.3. At low levels of μ the absolute level of volatility is never high enough to alter the skew of the distribution. However, at higher levels of μ, the absolute level of volatility is quite high and large upward moves in the stock price (D_{hh}) need to be accompanied not only by large downward moves in P_{ll}, but also, because the stock price is bound by zero from below, large downward moves in P_{hl}. The tendency for P_{hl} to decrease in the level of volatility combined with the expected utility investor's bidding strategy explains the correlation between the initial stock price and the level of volatility observed at higher levels of μ.

Removing diminishing sensitivity to losses and introducing time discounting means that the expected utility investor can only improve his welfare by gaming the realization utility investor's decision to realize gains early. Table 4.6 shows that as a result the initial stock price in realization utility markets is always greater than it is in expected utility markets. Lastly, the greatest welfare improvements are obtained by including loss aversion and time discounting as shown in Table 4.7.

Table 4. 6

General Equilibrium Outcomes with no Diminishing Sensitivity or loss aversion

μ	σ	Realization Utility Market(λ=1,δ=0.9)							*Expected Utility Market* *					D_{hl}	Sharpe
		$P_0\{RU\}$	P_0	P_h	P_l	$E\{U\}_{t=0}$	r_{RU}	r_{EU}	P_0	P_h	P_l	$E\{U\}_{t=0}$	r_{EU}		
1.05	0.05	**102.55**	**102.55**	**108.24**	**101.47**	**3.3**	**2.2**	**2.7**	*102.50*	*108.53*	*101.47*	*3.1*	*2.4*	**104.88**	*1.00*
1.05	0.22	**102.71**	**102.71**	**119.58**	**89.53**	**3.7**	**1.8**	**3.0**	*102.50*	*120.47*	*89.53*	*3.1*	*2.4*	**102.72**	*0.23*
1.05	0.23	**102.72**	**102.72**	**120.25**	**88.83**	**3.7**	**1.8**	**3.0**	*102.50*	*121.17*	*88.83*	*3.1*	*2.4*	**102.51**	*0.22*
1.1	0.1	**105.10**	**105.10**	**116.47**	**102.94**	**13.3**	**4.4**	**5.2**	*105.00*	*117.06*	*102.94*	*12.5*	*4.8*	**109.55**	*1.00*
1.1	0.32	**105.30**	**105.30**	**131.04**	**87.60**	**14.3**	**3.8**	**5.5**	*105.00*	*132.40*	*87.60*	*12.5*	*4.8*	**105.44**	*0.31*
1.1	0.33	**105.31**	**105.31**	**131.69**	**86.92**	**14.3**	**3.8**	**5.5**	*105.00*	*133.08*	*86.92*	*12.5*	*4.8*	**105.16**	*0.30*
1.15	0.15	**107.64**	**107.64**	**124.69**	**104.42**	**29.8**	**6.4**	**7.6**	*107.50*	*125.58*	*104.42*	*28.1*	*7.0*	**114.03**	*1.00*
1.15	0.41	**107.89**	**107.89**	**141.77**	**86.45**	**31.6**	**5.8**	**7.9**	*107.50*	*143.55*	*86.45*	*28.1*	*7.0*	**107.91**	*0.37*
1.15	0.42	**107.90**	**107.90**	**142.41**	**85.77**	**31.6**	**5.7**	**7.9**	*107.50*	*144.23*	*85.77*	*28.1*	*7.0*	**107.57**	*0.36*
1.2	0.2	**110.19**	**110.19**	**132.90**	**105.91**	**53.0**	**8.4**	**9.8**	*110.00*	*134.09*	*105.91*	*50.0*	*9.1*	**118.34**	*1.00*
1.2	0.48	**110.45**	**110.45**	**151.16**	**86.69**	**55.5**	**7.7**	**10.1**	*110.00*	*153.31*	*86.69*	*50.0*	*9.1*	**110.76**	*0.42*
1.2	0.49	**110.46**	**110.46**	**151.80**	**86.03**	**55.6**	**7.7**	**10.1**	*110.00*	*153.97*	*86.03*	*50.0*	*9.1*	**110.38**	*0.41*
1.2	0.5	**110.47**	**110.47**	**152.43**	**85.36**	**55.7**	**7.6**	**10.1**	*110.00*	*154.64*	*85.36*	*50.0*	*9.1*	**110.00**	*0.40*

Table 4. 7

General Equilibrium Outcomes with no Diminishing Sensitivity

μ	σ	Realization Utility Market(λ=1.5,δ=0.9)							*Expected Utility Market* *					D_{hl}	Sharpe
		$P_0\{RU\}$	P_0	P_h	P_l	$E\{U\}_{t=0}$	r_{RU}	r_{EU}	P_0	P_h	P_l	$E\{U\}_{t=0}$	r_{EU}		
1.05	0.05	**102.13**	**102.50**	**108.23**	**101.34**	**3.4**	**2.2**	**2.8**	*102.50*	*108.53*	*101.47*	*3.1*	*2.4*	**104.88**	*1.00*
1.05	0.07	**101.99**	**102.50**	**109.57**	**100.00**	**3.4**	**2.2**	**2.8**	*102.50*	*109.95*	*100.05*	*3.1*	*2.4*	**104.77**	*0.71*
1.05	0.08	**101.97**	**102.57**	**110.25**	**99.35**	**3.4**	**2.2**	**2.7**	*102.50*	*110.65*	*99.35*	*3.1*	*2.4*	**104.70**	*0.63*
1.05	0.17	**101.16**	**102.77**	**119.58**	**89.53**	**3.7**	**1.7**	**2.9**	*102.50*	*120.47*	*89.53*	*3.1*	*2.4*	**102.72**	*0.29*
1.1	0.1	**104.27**	**104.99**	**116.46**	**102.70**	**13.6**	**4.4**	**5.5**	*105.00*	*117.06*	*102.94*	*12.5*	*4.8*	**109.55**	*1.00*
1.1	0.15	**103.91**	**104.98**	**119.80**	**99.40**	**13.5**	**4.4**	**5.5**	*105.00*	*120.58*	*99.42*	*12.5*	*4.8*	**108.98**	*0.67*
1.1	0.16	**103.98**	**105.14**	**120.48**	**98.72**	**13.5**	**4.2**	**5.3**	*105.00*	*121.28*	*98.72*	*12.5*	*4.8*	**108.84**	*0.63*
1.1	0.32	**103.09**	**105.27**	**131.04**	**87.60**	**14.3**	**3.8**	**5.5**	*105.00*	*132.40*	*87.60*	*12.5*	*4.8*	**105.44**	*0.31*
1.1	0.33	**102.97**	**105.44**	**131.67**	**86.92**	**14.3**	**3.7**	**5.4**	*105.00*	*133.08*	*86.92*	*12.5*	*4.8*	**105.16**	*0.30*
1.15	0.15	**106.41**	**107.48**	**124.68**	**104.08**	**30.4**	**6.4**	**8.0**	*107.50*	*125.58*	*104.42*	*28.1*	*7.0*	**114.03**	*1.00*
1.15	0.21	**105.99**	**107.47**	**128.67**	**100.15**	**30.3**	**6.5**	**7.9**	*107.50*	*129.79*	*100.21*	*28.1*	*7.0*	**113.10**	*0.71*
1.15	0.22	**106.12**	**107.69**	**129.35**	**99.51**	**30.3**	**6.3**	**7.7**	*107.50*	*130.49*	*99.51*	*28.1*	*7.0*	**112.91**	*0.68*
1.15	0.4	**105.18**	**107.84**	**141.13**	**87.12**	**31.5**	**5.8**	**7.9**	*107.50*	*142.88*	*87.12*	*28.1*	*7.0*	**108.24**	*0.38*
1.15	0.41	**104.97**	**108.07**	**141.76**	**86.45**	**31.6**	**5.6**	**7.7**	*107.50*	*143.55*	*86.45*	*28.1*	*7.0*	**107.91**	*0.37*
1.2	0.2	**108.56**	**109.97**	**132.89**	**105.48**	**54.1**	**8.4**	**10.4**	*110.00*	*134.09*	*105.91*	*50.0*	*9.1*	**118.34**	*1.00*
1.2	0.27	**108.08**	**109.95**	**137.52**	**100.94**	**53.8**	**8.4**	**10.3**	*110.00*	*138.97*	*101.03*	*50.0*	*9.1*	**117.00**	*0.74*
1.2	0.28	**108.28**	**110.24**	**138.20**	**100.33**	**53.7**	**8.2**	**9.9**	*110.00*	*139.67*	*100.33*	*50.0*	*9.1*	**116.78**	*0.71*
1.2	0.48	**107.29**	**110.41**	**151.16**	**86.69**	**55.5**	**7.7**	**10.1**	*110.00*	*153.31*	*86.69*	*50.0*	*9.1*	**110.76**	*0.42*
1.2	0.49	**106.96**	**110.72**	**151.77**	**86.03**	**55.6**	**7.4**	**9.8**	*110.00*	*153.97*	*86.03*	*50.0*	*9.1*	**110.38**	*0.41*

The two types of bidding behaviour displayed by expected utility investors correspond with discrete changes in the realization utility investors trading strategy that depend on the initial stock price and the mode of the terminal stock price at $t = 2$. For the greater range of volatility in which the realization utility investor is willing to enter the market, volatility is relatively low and the realization utility investor's offer in node P_l is below the expected utility investor's bid. The expected utility investor responds by gaming the realization utility investor's sell decision by decreasing initial bid prices because this tends to lower the price at which the realization utility investor sells shares in node P_l. At higher levels of volatility, the realization utility investor's offer in node P_h is below the expected utility investor's bid. The optimal bidding strategy is to drive up the initial stock price in order to increase the number of shares sold by the realization utility investor in node P_h as well as to increase the discount received on those shares. In the following section, I show that including wealth considerations and diminishing sensitivity removes the realization utility investor's decision to sell shares in node P_l. Consequently the model predicts that IPO prices in realization utility markets are almost always greater than in expected utility market controls.

In order to calculate general equilibrium outcomes when investor preferences exhibit diminishing sensitivity it is necessary to use numerical methods. Furthermore, diminishing sensitivity implies that the expected utility investor's coefficient of relative risk aversion and initial wealth level must be identical if we are to compare welfare across different choices of the dealer supply schedule, realization utility investor preferences, and wealth distributions. This is because the expected utility investor's welfare is determined by the different price outcomes at $t = 2$ evaluated under a concave utility function and is, therefore, necessarily dependent on the investor's initial wealth level. Consequently, to examine the impact of trading with realization utility investors on expected utility welfare I begin by calculating general equilibrium outcomes when there are only expected utility investors in the market; considering both the cases of homogeneous and heterogeneous preferences. I then calculate general equilibrium when one of the expected utility investors is replaced by a realization utility investor and compare the results with the expected utility market outcomes. To provide further insight into the differences between the two types of markets I perform the same comparison for various wealth distributions, levels of dealer liquidity, and risky asset volatilities.

Calculating general equilibrium numerically allows for a more complete specification of the dealer's supply function. Stoll [1978a] posits that the ownership structure of dealerships, e.g. proprietorships, partnerships, and closely held corporations, indicates that their bid and offer curves can potentially be explained by assuming that they act passively to maximize expected utility. In this sense, the dealer sets his bid-ask spread, or the cost of his services, by equating pre-trade and post-trade expected utilities. Stoll [1978b] finds empirical evidences supporting the bid/ask modeling assumption via econometric analysis of spreads on the NASDAQ. I view Stoll's work [1978a and 1978b] as a test of the hypothesis that the behaviour of a securities dealer can be modeled using the expected utility framework. Stoll examines the determinants of bid-ask spreads under the assumption that the main market function of the dealer is to supply liquidity for sequential trades separated by short time horizons for a number of risky assets held in the dealer's inventory. Given the short time horizons and the nature of market making as a means of maintaining client relationships while potentially earning the bid-ask spread, it makes sense to calculate bid-ask spreads by equating pre-trade and post-trade expected utilities. However, in the context of an IPO, the dealer purchases shares from the issuing corporation with the goal of clearing the acquired inventory at a higher price. Were the sequence of transaction not to improve the expected utility of the dealer there would be no reason to enter the IPO market. I therefore obtain the dealer's supply decision by calculating the number of shares to offer that maximizes expected utility at a given bid price.

I model the dealer's preferences with the same CRRA utility specification used for the expected utility investor. The choice of CRRA preferences implies that the dealer's wealth level and proportion of wealth allocated to the risky asset impacts the dealer's supply decision. In order to make comparisons of general equilibrium outcomes for different levels of dealer wealth and risky asset return distributions I, therefore, include the dealer's inventory acquisition decision as a preliminary step to the general equilibrium calculation. To simplify the calculation I assume that the dealer has no information regarding potential bids at $t = 0$. Additionally, the dealer is not allowed to participate in trading at $t = 1$. Under these assumptions, the dealer's optimal level of inventory is obtained from solving the following portfolio allocation problem

$$\max_{x_0} E_0\{W_2^{\varepsilon}\} \tag{4.33}$$

subject to

$$W_2 = W_0 + x_D\big(D_0 R_{0,1} R_{1,2} - P_D\big) \geq 0. \tag{4.34}$$

where ε is related to the dealer's level of risk aversion (in the same way that the expected utility investor's level of risk aversion is related to γ), and P_D and x_D are the price and volume of shares purchased from the issuing corporation. The number of shares that the dealer supplies at $t = 0$, (Δx_0) is then given by

$$\max_{\Delta x_0} E_0\{W_2^{\varepsilon}\}\,, \tag{4.35}$$

subject to

$$W_2 = W_D + (x_D - \Delta x_0) D_0 R_{0,1} R_{1,2} - x_D P_D + \Delta x_0 P_0 \geq 0. \tag{4.36}$$

Figure 4.1 shows that the number of shares offered by the dealer is increasing in P_0 and also scales proportionally with the level of dealer wealth. Furthermore, Figure 4.2 shows that the dealer's initial expected utility is always increasing in P_0. In a more complete model of IPO markets it would be ideal that the dealer have knowledge of investor demand at $t = 0$ as well as to be able to participate in trades at $t = 1$. The purpose of including the dealer as an expected utility investor at this stage, however, is only to provide an economically consistent mechanism for comparing bidding behavior in expected utility and realization utility markets under different risky asset return distributions and dealer wealth levels. Numerical calculations show that general equilibrium outcomes are independent of absolute wealth levels and depend only on the relative wealth levels of market participants. This is probably a result of the CRRA utility specification as well the market clearing mechanism that the dealer offers shares to market participants in proportion to their relative levels of demand. I leave the formal proof for future research.

Figure 4. 1

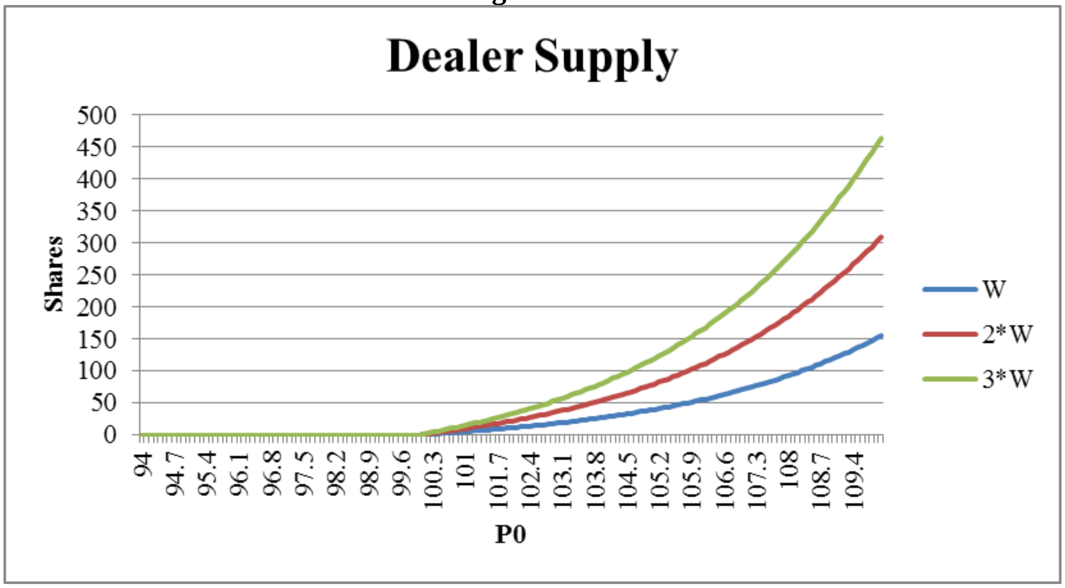
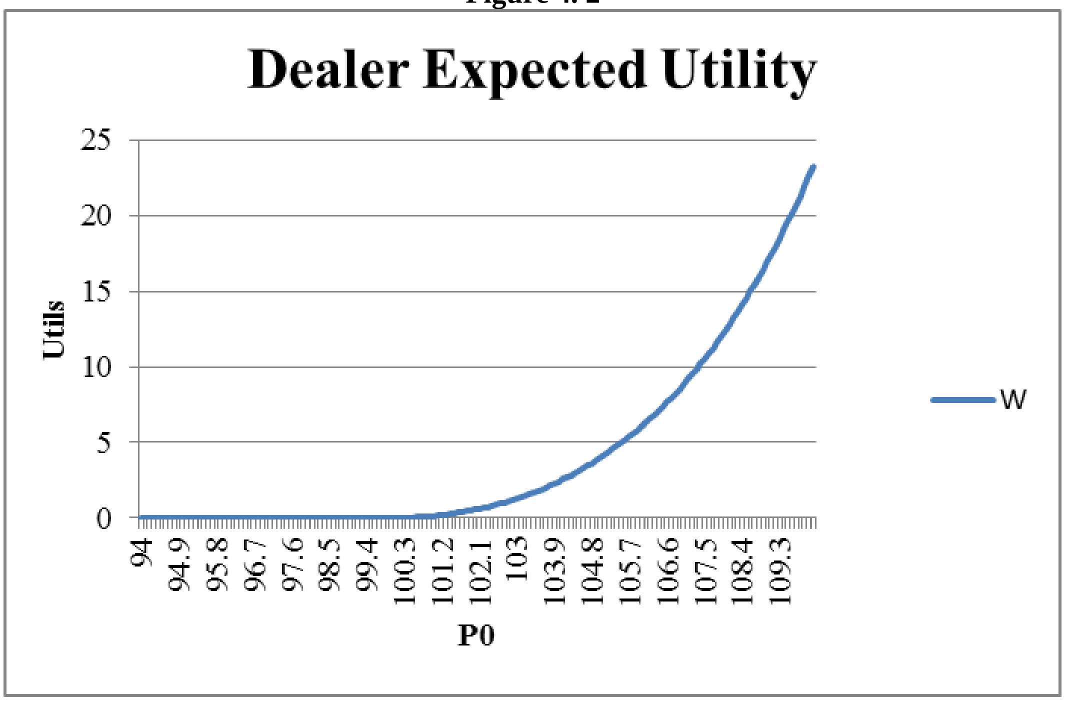


Figure 4. 2

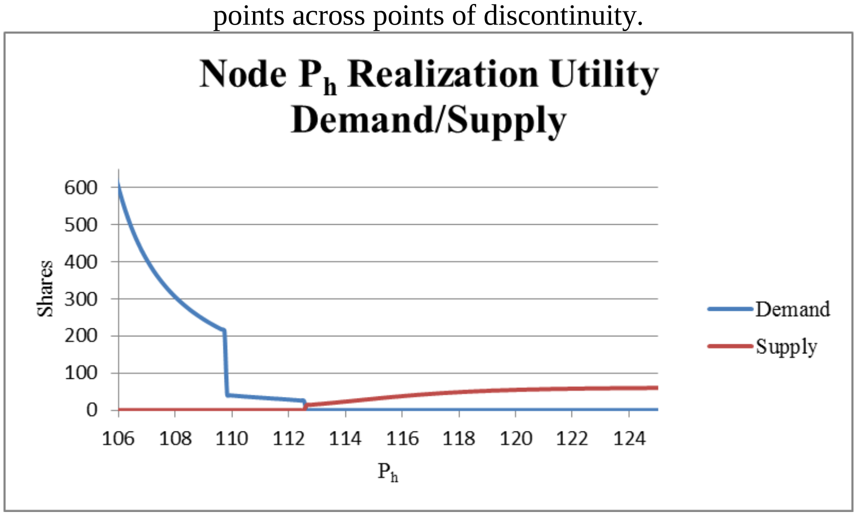


The numerical general equilibrium calculation starts by working backwards through the binomial tree. In order to calculate the investor demand and supply schedules at $t = 1$, it is necessary to condition on the initial stock price, the number of shares supplied by the dealer, and the proportion of shares supplied by the dealer that are demanded by each investor. Moreover, rational expectations requires that investor initial demand is fully informed regarding the ensuing equilibrium trades and prices at $t = 1$. Initiating the search for general equilibrium therefore requires a starting value of P_0 and an assumption regarding the relative levels of investor demand at $t = 0$, the former of which is chosen randomly and the latter of which is set equal to the proportion of relative investor wealth levels. Since dealer's expected utility is increasing in P_0, the calculation subsequently searches for the highest value of P_0 which maximizes the expected utility or expected realized utility of the most aggressive bidder conditional on equilibrium at $t = 1$ and on the dealer allocating shares to investors in proportion to relative levels of demand.

The discrete changes in the realization utility investor's optimal portfolio allocation observed in the partial equilibrium section translate into the general equilibrium framework. Figure 4.3 shows that both realization utility demand and supply curves at $t = 1$ exhibit discontinuous jumps. It is, therefore, necessary to include a method for handling cases where the expected utility investor's supply or demand curve passes through, or "intersects", at the point of discontinuity. Over the infinitesimally small price range over which the demand/supply of the realization utility investor abruptly changes, the supply/demand of the expected utility investor is effectively flat. Equilibrium trading volume is therefore determined by which demand/supply decision makes the realization utility investor best off. Using discontinuities in realization utility demand as an example, this implies that equilibrium occurs immediately to the left of the discontinuity if the realization is better off having a larger purchase order partially filled than a smaller order completely filled; otherwise equilibrium occurs immediately to the right of the discontinuity. Furthermore, the functional form of calculated realized utility changes depending on whether the investor demands or supplies shares. Combined with the possibility of partially filled orders, this fact, means that it is necessary to calculate distinct candidate sub-equilibria at each $t = 1$ price node and choose the corresponding strategy that provides the greatest utility to the realization utility investor. The approach, therefore, ensures that the realization utility investor bases his decision to supply or demand shares on strictly feasible strategies. Lastly, since trades at $t = 1$ are calculated conditional on P_0 and $t = 0$ demand, it is necessary to check if investors can obtain welfare improvements by deviating from these strategies in order to alter $t = 0$ demand. This is similar in spirit to the adjustment I made to Barberis and Xiong's [2009] calculation of the realization utility investor's optimal portfolio allocation in the partial equilibrium section. In that case, improvements were obtained by deviating from the optimal ex-post initial stock purchase strategy of holding onto losses to the optimal ex-ante initial stock purchase strategy of realizing losses early because the latter strategy allows for a greater number of shares to be purchased at $t = 0$.

Figure 4. 3

Jumps in Realization Utility Demand/Supply: jumps are discontinuous, vertical lines merely connect points across points of discontinuity.



The data indicates that general equilibrium results depend on the relative wealth levels of investors. In order to facilitate an investigation into expected utility welfare under different wealth distributions and across market types it is necessary to fix the absolute wealth level of one of the expected utility investors, I subsequently set the wealth level of one of the expected utility investors to the arbitrary value of $10,000.

Implications of realization utility for general equilibrium are obtained by comparing outcomes between expected utility and realization utility markets. Furthermore, explanations of the expected utility investor's strategy in realization utility markets and expected utility markets with heterogeneous preferences are inferred by cross-examining prices, trades, and investor welfare with expected utility markets where investors have homogeneous preferences. Consequently expected utility markets with homogenous preferences form the basis of comparison in the following analysis; results are presented in Table 4.8 for a range of dealer and investor wealth levels and choices of γ. Intuitively, more risk tolerant investors (higher γ) are willing to pay higher bid prices in order to receive more shares of the risky asset. Moreover, increasing the dealer's wealth level (thereby increasing the number of shares that the dealer offers at each price by a proportional amount) allows investors to attain their desired level of risk exposure at lower bids. Lastly, an important consequence of homogeneous preferences is that there are never trades at $t = 1$ regardless of the distribution of wealth. This final point makes the case of homogeneous preferences particularly useful in explaining the more complex heterogeneous preference outcomes because it implies that the bid prices in Table 4.8 are based purely on the investors' desired levels of risk exposure at $t = 0$ and do not reflect anticipated trades at $t = 1$.

Table 4. 8

Homogeneous Preference Expected Utility Market Outcomes

W_D^*	W_{EU}^*	**$\Gamma = 0.7$**					*$\Gamma = 0.8$*					**$\Gamma = 0.95$**				
		r	**P_0**	**V_0**	**P_h**	**P_l**	*r*	*P_0*	*V_0*	*P_h*	*P_l*	**r**	**P_0**	**V_0**	**P_h**	**P_l**
2	15	**5.34**	**109.17**	**31.7**	**135.67**	**93.77**	*5.21*	*109.30*	*32.5*	*135.78*	*93.83*	**5.02**	**109.51**	**33.8**	**135.97**	**93.93**
	20	**5.25**	**109.27**	**32.3**	**135.75**	**93.82**	*5.15*	*109.37*	*32.9*	*135.84*	*93.86*	**5.00**	**109.52**	**33.9**	**135.99**	**93.94**
	30	**5.15**	**109.36**	**32.9**	**135.84**	**93.86**	*5.09*	*109.43*	*33.4*	*135.90*	*93.90*	**4.98**	**109.54**	**34.0**	**136.00**	**93.95**
10	15	**6.46**	**108.02**	**126.5**	**134.76**	**93.11**	*6.05*	*108.44*	*137.8*	*135.12*	*93.33*	**5.27**	**109.25**	**161.0**	**135.78**	**93.77**
	20	**6.16**	**108.33**	**134.6**	**134.98**	**93.30**	*5.81*	*108.68*	*144.4*	*135.29*	*93.48*	**5.19**	**109.32**	**163.4**	**135.83**	**93.82**
	30	**5.82**	**108.67**	**144.2**	**135.25**	**93.50**	*5.56*	*108.94*	*152.0*	*135.48*	*93.64*	**5.11**	**109.40**	**165.9**	**135.89**	**93.87**
40	15	**8.69**	**105.81**	**307.1**	**133.61**	**91.39**	*8.05*	*106.44*	*357.5*	*134.23*	*91.68*	**6.98**	**107.50**	**453.5**	**135.51**	**92.78**
	20	**8.12**	**106.36**	**351.6**	**133.83**	**91.87**	*7.48*	*107.00*	*406.2*	*134.40*	*92.19*	**6.03**	**108.46**	**552.9**	**135.53**	**93.04**
	30	**7.41**	**107.07**	**412.9**	**134.18**	**92.43**	*6.83*	*107.65*	*467.9*	*134.67*	*92.74*	**5.59**	**108.91**	**604.5**	**135.62**	**93.50**

Table 4.9 shows the general equilibrium outcomes for expected utility markets with heterogeneous preferences for varying degrees of difference in the investors' coefficients of relative risk aversion. Included in the table are the expected utility of each investor in the homogeneous control

markets as well as the optimal bid price and the more risk tolerant investor's (higher γ) proportion of total initial demand. For all of the data presented in the table the more risk tolerant investor is always the highest bidder at $t = 0$ and always the purchaser of shares in node P_h. Gains to welfare and returns (compare returns with Table 4.8) are achieved by the more risk tolerant investor. Additionally, the gains are increasing in the difference between the investors' relative levels of risk aversion and wealth levels. The key to understanding the more risk tolerant investor's strategy lies in the fact that he/she is always the highest bidder and therefore drives bids beyond the less risk tolerant investor's optimal entry point. The equilibrium response of the less risk tolerant investor is to demand a smaller proportion of shares which, in turn, allows the more tolerant investor to obtain the desired level of risk exposure at a lower bid price relative to the corresponding homogeneous market outcome. Furthermore, the correlation in the increase in trading volume in node P_h, the less risk tolerant investors wealth level, and welfare gains indicates that the more risk tolerant investor obtains further improvements to welfare from purchasing shares from the less tolerant investor in node P_h. Therefore the purchase of shares in node P_h may provide an additional incentive for the more risk tolerant investor to reduce his initial bid price because doing so increases the number of shares demanded by the less risk tolerant investor and hence also likely increases his/her willingness to sell shares in node P_h.

Table 4. 9

Heterogeneous Preference Expected Utility Market Outcomes

W_D^*	W_{EU}^*	$\Gamma_1 = 0.8, \Gamma_2 = 0.7$														
		r_1	r_2	$E\{U_1\}_{t=0}$	$E\{U_1\}_{t=0}^{\dagger}$	$E\{U_2\}_{t=0}$	$E\{U_2\}_{t=0}^{\dagger}$	P_0	$P_0^{\ddagger}$	V_0	β_0	$\beta_0^{\dagger}$	P_h	V_h	P_l	V_l
2	15	5.24	5.22	1601.1	1600.0	391.1	391.6	109.28	109.30	32.4	0.28	0.33	135.75	1.2	93.79	0.0
	20	5.18	5.16	1597.7	1596.3	634.4	634.9	109.34	109.37	32.8	0.44	0.50	135.80	1.4	93.83	0.0
	30	5.12	5.10	1593.9	1592.6	1029.0	1029.3	109.41	109.43	33.2	0.61	0.67	135.87	1.4	93.87	0.0
10	15	6.13	6.02	1655.5	1651.9	400.5	401.9	108.39	108.44	136.3	0.29	0.33	135.02	6.5	93.16	0.0
	20	5.93	5.83	1641.6	1636.9	646.6	648.1	108.61	108.68	142.4	0.45	0.50	135.15	7.9	93.33	0.0
	30	5.68	5.61	1625.8	1620.9	1043.3	1044.6	108.87	108.94	149.7	0.62	0.67	135.34	7.6	93.52	0.0
20	15	6.96	6.77	1708.9	1704.0	409.6	411.5	107.57	107.65	230.4	0.30	0.33	134.52	12.6	92.42	0.0
	20	6.63	6.47	1686.3	1679.4	659.0	661.2	107.92	108.02	247.8	0.46	0.50	134.65	15.9	92.75	0.0
	30	6.24	6.11	1659.7	1651.9	1058.5	1060.5	108.33	108.44	269.4	0.62	0.67	134.88	15.7	93.11	0.0
W_D^*	W_{EU}	$\Gamma_1 = 0.95, \Gamma_2 = 0.7$														
2	15	5.03	5.00	6392.5	6383.1	390.8	391.6	109.50	109.51	33.8	0.25	0.33	135.95	5.8	93.81	0.0
	20	5.02	4.98	6376.4	6364.8	634.1	634.9	109.52	109.52	33.9	0.39	0.50	135.95	8.6	93.84	0.0
	30	5.01	4.97	6357.8	6346.5	1028.7	1029.3	109.54	109.54	34.0	0.56	0.67	135.96	10.8	93.88	0.0
10	15	5.34	5.18	6703.7	6664.8	399.3	401.9	109.22	109.25	160.1	0.26	0.33	135.68	29.5	93.12	0.0
	20	5.29	5.11	6629.0	6578.7	645.3	648.1	109.30	109.32	162.5	0.40	0.50	135.68	43.4	93.31	0.0
	30	5.24	5.04	6543.3	6490.9	1042.0	1044.6	109.40	109.40	165.6	0.57	0.67	135.70	54.1	93.51	0.0
20	15	5.72	5.46	7054.0	6992.1	407.4	411.5	108.85	108.91	298.5	0.27	0.33	135.46	58.1	92.02	0.0
	20	5.60	5.29	6917.3	6831.8	656.5	661.2	109.03	109.09	309.1	0.41	0.50	135.44	85.4	92.52	0.0
	30	5.49	5.13	6760.7	6664.8	1056.0	1060.5	109.23	109.25	321.2	0.58	0.67	135.45	106.4	93.00	0.0

Table 4.10 shows general equilibrium outcomes for a range of realization utility and expected utility investor preference parameters. Where trading volume is greater than zero at $t = 1$, the realization utility investor is always the seller of shares in node P_h, and the purchaser of shares in node P_l. Starting with the prospect theory preference parameters estimated by kahneman and tversky, the high level of loss aversion means that the realization utility investor is highly sensitive to the initial bid price. Consequently, in order to prevent the expected utility investor from driving up bids to homogeneous market levels, the realization utility investor generally demands fewer shares from the dealer. Moreover, in the case of trading with an extremely risk tolerant investor (i.e. $\gamma = 0.95$), initial bids are so high that the realization utility investor is not even willing to enter the market. The impact of loss aversion on realization utility welfare can be reduced by including time discounting. This increases the realization utility investor's initial demand by reducing the impact of losses at $t = 2$ and also increases the tendency to realize gains early. The expected utility investor responds to increased demand at $t = 0$ by increasing initial bids in order to obtain the desired level of risk exposure at $t = 0$. Welfare gains are therefore the greatest for more risk tolerant expected utility investors because they are able to attain their desired level

of risk exposure at relatively lower bid prices and are better able to take advantage of the realization utility investors decision to realize gains in node P_h. Lastly, removing loss aversion altogether greatly increases the realization utility investors risk tolerance to the point of being the highest bidder. The data show that equilibrium bid prices are substantially higher than expected utility market outcomes. Furthermore, bid prices are increasing in trading volumes in node P_h: implying that the realization utility investor's bid is driven by how many shares he/she anticipates on selling in node P_h.

Table 4. 10
Realization Utility Market Outcomes

α	λ	δ	γ	r_{RU}	r_{EU}	$r_{EU}^{\dagger}$	$E\{RU\}_{t=0}$	$E\{U_2\}_{t=0}$	$E\{U_2\}_{t=0}^{\dagger}$	P_0	$P_0^{\dagger}$	V_0	β_0	$\beta_0^{\dagger}$	P_h	V_h	P_l	V_l
0.88	**2.25**	**1**	**0.7**	*4.02*	*4.45*	*4.35*	*35.2*	*673.8*	*673.9*	*110.25*	*110.20*	*494.3*	*0.51*	*0.50*	*124.5*	*110.9*	*103.6*	*0.0*
0.88	**2.25**	**1**	**0.8**	*4.12*	*4.40*	*4.00*	*17.2*	*1738.4*	*1717.3*	*110.43*	*110.58*	*523.0*	*0.41*	*0.50*	*124.8*	*101.9*	*103.7*	*0.0*
0.88	**2.25**	**1**	**0.95**	*0.00*	*0.00*	*3.29*	*0.0*	*0.0*	*7028.0*	*0.00*	*111.34*	*0.0*	*0.00*	*0.50*	*0.0*	*0.0*	*0.0*	*0.0*
0.88	**2.25**	**0.9**	**0.7**	*3.82*	*4.55*	*4.35*	*63.4*	*674.2*	*673.9*	*110.29*	*110.20*	*500.3*	*0.51*	*0.50*	*124.4*	*161.6*	*103.6*	*0.0*
0.88	**2.25**	**0.9**	**0.8**	*3.90*	*4.34*	*4.00*	*48.4*	*1739.0*	*1717.3*	*110.45*	*110.58*	*525.9*	*0.41*	*0.50*	*124.7*	*145.2*	*103.7*	*0.0*
0.88	**2.25**	**0.9**	**0.95**	*3.31*	*3.41*	*3.29*	*6.7*	*7536.3*	*7028.0*	*110.94*	*111.34*	*612.1*	*0.11*	*0.50*	*125.4*	*48.3*	*103.9*	*0.0*
0.88	**1**	**0.9**	**0.7**	*2.71*	*3.71*	*4.35*	*557.7*	*666.4*	*673.9*	*111.50*	*110.20*	*726.7*	*0.58*	*0.50*	*124.0*	*247.0*	*103.3*	*0.0*
0.88	**1**	**0.9**	**0.8**	*2.64*	*3.37*	*4.00*	*538.6*	*1703.7*	*1717.3*	*111.68*	*110.58*	*765.5*	*0.53*	*0.50*	*124.4*	*263.8*	*103.5*	*0.0*
0.88	**1**	**0.9**	**0.95**	*2.63*	*2.79*	*3.29*	*543.6*	*7057.1*	*7028.0*	*111.96*	*111.34*	*834.8*	*0.47*	*0.50*	*125.2*	*293.6*	*104.2*	*117.4*

General equilibrium dynamics for different wealth distributions and risky asset return distributions are generally unchanged when the realization utility investor has zero loss aversion. On the other end of the spectrum, the high level of loss aversion estimated by Kahneman and Tversky means that the realization utility investor is unwilling to participate in markets with more aggressive expected utility investors, especially at higher levels of volatility and relatively greater levels of expected utility investor wealth. To continue investigating general equilibrium dynamics I have, therefore, decreased the coefficient of loss aversion from 2.25 to 1.5. Furthermore, to simplify the analysis I have removed time discounting; the data is shown in Table 4.11. The outcomes are governed by a common dynamic that emerges from the effects of mental accounting and loss aversion on realization utility investor initial demand as well as the interaction between diminishing sensitivity and realization utility on the investor's tendency to realize gains early. Mental accounting means that demand for the risky asset at $t = 0$ is not based on the risky assets contribution to future expected wealth like it is for the expected utility investor. Rather, as explained in the partial equilibrium section, the realization utility investor tries to gain as much exposure as possible to an asset that can be traded to produce realized gains and losses that result in a net positive contribution to the realization utility account. The choice of $\lambda = 1.5$ reduces the optimal bid price of the realization utility investor so that the expected utility investor drives equilibrium bid prices, however, it is not so high that the realization utility investor significantly reduces demand for the risky asset at higher bids set by the expected utility investor. Consequently, the expected utility investor responds by increasing bids relative to the homogeneous market control. Whether or not this improves expected utility welfare depends mainly on the expected utility investors preferences. At lower levels of risk tolerance, the relative demand of the realization utility investor is so great that the cost to the expected utility investor of raising bid prices has a negative overall impact on investor welfare. However, at higher levels of risk tolerance, the cost of pumping up the initial stock price is minimal and welfare gains are obtained from the increased number of shares the realization utility sells in node P_h. Examining expected utility welfare and bidding behavior for different realization utility investor and dealer wealth levels solidifies the rationale. For low levels of risk tolerance, the expected utility investor is able to obtain welfare improvements for the lowest levels of realization utility wealth. However, at higher realization utility wealth levels, the proportionately greater increase in realization demand relative to expected utility demand in the homogeneous control market means that the cost of pumping up the initial share price eliminates gains from shares purchased in node P_h. Conversely, for high levels of risk tolerance, realization utility demand increases in wealth commensurately with expected utility demand in the control market. Consequently, the cost of pumping up the initial stock price is practically fixed, and the investor is able to benefit from the greater number of shares purchased in node P_h that result from the increased realization utility investor initial demand. In all cases, expected utility welfare in realization utility markets is improving in the relative size of the dealer. Intuitively, the corresponding increase in dealer supply makes it relatively cheaper to take advantage of the realization utility investor’s tendency to realize gains early by pumping up the stock price. This is reflected by much higher bid prices relative to the homogeneous market control.

Table 4. 11

Realization Utility Market Outcomes

W_D^*	W_{RU}^*	r_{RU}	r_{EU}	$r_{EU}^\dagger$	$E\{RU\}_{t=0}$	$E\{U_2\}_{t=0}$	$E\{U_2\}_{t=0}^\dagger$	P_0	$P_0^\dagger$	V_0	β_0	$\beta_0^\dagger$	P_h	V_h	P_l	V_l
		$\alpha = 0.88$ & $\lambda = 1.5$, $\Gamma = 0.7$														
	5	5.14	5.36	5.34	18.8	635.5	636.1	109.25	109.17	32.2	0.43	0.33	135.57	8.2	93.79	0.0
2	10	5.00	5.31	5.25	24.6	634.2	652.8	109.40	109.27	33.1	0.60	0.50	135.59	11.9	93.84	0.0
	20	4.79	5.28	5.15	28.2	633.0	668.5	109.62	109.36	34.5	0.75	0.67	135.60	15.7	93.89	0.0
	5	5.94	6.57	6.46	81.1	651.1	634.9	108.17	108.02	130.4	0.40	0.33	134.43	26.4	93.18	0.0
10	10	5.51	6.42	6.16	105.3	646.0	648.1	108.60	108.33	142.2	0.58	0.50	134.40	41.8	93.38	0.0
	20	4.85	6.41	5.82	111.4	641.1	661.2	109.27	108.67	161.7	0.75	0.67	134.32	61.1	93.57	0.0
	5	6.77	7.56	7.41	150.1	666.5	633.6	107.22	105.81	213.4	0.39	0.33	133.72	37.0	92.52	0.0
20	10	6.20	7.35	6.97	200.9	658.6	643.0	107.78	106.36	240.6	0.57	0.50	133.62	60.6	92.86	0.0
	20	5.37	7.34	6.46	222.0	650.4	652.8	108.59	107.07	284.0	0.73	0.67	133.43	91.0	93.20	0.0
W_D^*	W_{EU}	**$\alpha = 0.88$ & $\lambda = 1.5$, $\Gamma = 0.8$**														
	5	5.10	5.24	5.21	16.6	1599.2	1600.0	109.33	109.30	32.7	0.37	0.33	135.71	7.4	93.84	0.0
2	10	5.01	5.20	5.15	22.8	1595.4	1596.3	109.42	109.37	33.3	0.54	0.50	135.73	11.0	93.87	0.0
	20	4.89	5.17	5.09	27.6	1591.8	1592.6	109.55	109.43	34.1	0.71	0.67	135.74	14.7	93.90	0.0
	5	5.70	6.15	6.05	72.9	1649.7	1651.9	108.50	108.44	139.5	0.36	0.33	134.86	27.1	93.35	0.0
10	10	5.39	6.02	5.81	97.6	1634.3	1636.9	108.81	108.68	148.2	0.53	0.50	134.86	42.8	93.51	0.0
	20	4.94	5.96	5.56	111.1	1618.6	1620.9	109.27	108.94	161.8	0.70	0.67	134.82	61.8	93.65	0.0
	5	6.37	6.99	6.83	138.8	1701.7	1704.0	107.71	106.44	237.1	0.35	0.33	134.30	41.3	92.77	0.0
20	10	5.91	6.78	6.46	187.6	1676.5	1679.4	108.16	107.00	260.4	0.53	0.50	134.22	67.3	93.06	0.0
	20	5.25	6.69	6.05	213.0	1649.7	1651.9	108.81	107.65	296.3	0.70	0.67	134.08	100.3	93.34	0.0
W_D^*	W_{EU}	**$\alpha = 0.88$ & $\lambda = 1.5$, $\Gamma = 0.95$**														
	5	4.99	5.03	5.02	14.8	6383.1	6383.1	109.51	109.51	33.8	0.33	0.33	135.95	7.0	93.93	0.0
2	10	4.97	5.02	5.00	21.0	6364.9	6364.8	109.53	109.52	34.0	0.50	0.50	135.96	10.6	93.94	0.0
	20	4.94	5.01	4.98	26.8	6346.6	6346.5	109.56	109.54	34.2	0.67	0.67	135.96	14.2	93.94	0.0
	5	5.16	5.31	5.27	63.3	6665.2	6664.8	109.25	109.25	161.3	0.33	0.33	135.69	32.4	93.77	0.0
10	10	5.07	5.26	5.19	88.2	6579.7	6578.7	109.35	109.32	164.3	0.50	0.50	135.70	49.7	93.82	0.0
	20	4.94	5.22	5.11	109.5	6492.8	6490.9	109.49	109.40	168.6	0.67	0.67	135.69	68.2	93.87	0.0
	5	5.43	5.65	5.59	122.4	6993.3	6992.1	108.93	107.50	303.0	0.33	0.33	135.48	59.1	93.49	0.0
20	10	5.22	5.53	5.42	167.2	6834.9	6831.8	109.13	108.46	315.2	0.50	0.50	135.46	92.4	93.64	0.0
	20	4.95	5.45	5.27	200.7	6671.4	6664.8	109.40	108.91	331.5	0.67	0.67	135.42	129.8	93.76	0.0

Tables 4.12 and 4.13 show general equilibrium outcomes in heterogeneous expected utility markets and realization utility markets as a function of volatility and dealer wealth. In expected utility markets the more risk tolerant investor achieves greater welfare gains at higher levels of volatility. This is because the less risk tolerant investor is more sensitive to the level of volatility. Consequently, the difference in the investors' desired level of risk exposure increases with volatility. Welfare gains are then achieved because the more risk tolerant investor is able to attain his desired level of risk exposure at lower bid prices. Similarly, in realization utility markets, the performance of expected utility investors is improving in the level of volatility. However, only the most risk tolerant investors are able to achieve

welfare gains. The data show that at high levels of volatility, the bids of expected utility investors are high enough that realization utility investors are unwilling to enter the market. The result indicates that, even with a moderate sensitivity to losses, realization utility investors are more sensitive to volatility. However, unlike in the expected utility market, this does not lead the realization utility investor to significantly reduce demand and the expected utility investor responds by increasing bid prices relative to the homogeneous market control. Furthermore, at the highest levels of volatility the realization utility investor exhibits an even stronger disposition effect by purchasing more shares of the risky asset after losses. Considering that the expected utility investor's trading decisions are driven by the desire to hold a roughly constant proportion of wealth in the risky asset, the desire to sell shares in node P_l is increasing in the level of volatility because larger losses have a greater impact on the investor's wealth level.

Table 4. 12

Heterogeneous Preference Expected Utility Market Outcomes and Volatility

σ	W_D^*	r_1	r_2	$E\{U_1\}_{t=0}$	$E\{U_1\}_{t=0}^\dagger$	$E\{U_2\}_{t=0}$	$E\{U_2\}_{t=0}^\dagger$	P_0	$P_0^\ddagger$	V_0	β_0	$\beta_0^\dagger$	P_h	V_h	P_l	V_l
		$\Gamma_1 = 0.8, \Gamma_2 = 0.7$														
	2	*3.28*	*3.26*	*1618.6*	*1616.0*	*640.7*	*641.6*	*111.36*	*111.38*	*141.3*	*0.54*	*0.50*	*125.38*	*8.5*	*104.23*	*0.0*
0.15	10	*4.09*	*4.01*	*1724.4*	*1717.3*	*671.5*	*673.9*	*110.52*	*110.58*	*538.0*	*0.53*	*0.50*	*124.96*	*40.2*	*103.57*	*0.0*
	20	*4.85*	*4.72*	*1825.6*	*1817.0*	*701.1*	*704.1*	*109.75*	*109.81*	*847.3*	*0.52*	*0.50*	*124.75*	*69.2*	*102.87*	*0.0*
	2	*5.18*	*5.16*	*1597.7*	*1596.3*	*634.4*	*634.9*	*109.34*	*109.37*	*32.8*	*0.56*	*0.50*	*135.80*	*1.4*	*93.83*	*0.0*
0.3	10	*5.93*	*5.83*	*1641.6*	*1636.9*	*646.6*	*648.1*	*108.61*	*108.68*	*142.4*	*0.55*	*0.50*	*135.15*	*7.9*	*93.33*	*0.0*
	20	*6.63*	*6.47*	*1686.3*	*1679.4*	*659.0*	*661.2*	*107.92*	*108.02*	*247.8*	*0.54*	*0.50*	*134.65*	*15.9*	*92.75*	*0.0*
	2	*6.25*	*6.23*	*1593.6*	*1592.5*	*633.2*	*633.6*	*108.24*	*108.27*	*18.2*	*0.57*	*0.50*	*145.91*	*0.5*	*83.63*	*0.0*
0.45	10	*7.05*	*6.94*	*1623.2*	*1619.2*	*641.0*	*642.2*	*107.47*	*107.57*	*80.3*	*0.57*	*0.50*	*144.93*	*3.3*	*83.16*	*0.0*
	20	*7.81*	*7.61*	*1652.6*	*1646.7*	*648.7*	*650.5*	*106.76*	*106.90*	*141.3*	*0.56*	*0.50*	*144.13*	*7.1*	*82.61*	*0.0*
σ	**W_D^***	***$\Gamma_1 = 0.95, \Gamma_2 = 0.7$***														
	2	*3.07*	*3.03*	*6485.0*	*6462.4*	*640.1*	*641.6*	*111.60*	*111.60*	*151.8*	*0.57*	*0.50*	*125.51*	*43.3*	*104.23*	*0.0*
0.15	5	*3.17*	*3.08*	*6730.1*	*6679.2*	*651.9*	*673.9*	*111.51*	*111.52*	*364.0*	*0.57*	*0.50*	*125.42*	*106.0*	*103.92*	*0.0*
	7.5	*3.25*	*3.13*	*6887.7*	*6856.1*	*658.8*	*704.1*	*111.43*	*111.43*	*498.0*	*0.57*	*0.50*	*125.37*	*146.6*	*103.70*	*0.0*
	2	*5.02*	*4.98*	*6376.4*	*6364.8*	*634.1*	*634.9*	*109.52*	*109.52*	*33.9*	*0.61*	*0.50*	*135.95*	*8.6*	*93.84*	*0.0*
0.3	10	*5.29*	*5.11*	*6629.0*	*6578.7*	*645.3*	*648.1*	*109.30*	*109.32*	*162.5*	*0.60*	*0.50*	*135.68*	*43.4*	*93.31*	*0.0*
	20	*5.60*	*5.29*	*6917.3*	*6831.8*	*656.5*	*661.2*	*109.03*	*109.09*	*309.1*	*0.59*	*0.50*	*135.44*	*85.4*	*92.52*	*0.0*
	2	*6.09*	*6.03*	*6356.8*	*6346.3*	*632.8*	*633.6*	*108.42*	*108.43*	*18.7*	*0.64*	*0.50*	*146.13*	*4.0*	*83.65*	*0.0*
0.45	10	*6.39*	*6.15*	*6533.9*	*6488.1*	*639.7*	*642.2*	*108.19*	*108.23*	*90.3*	*0.63*	*0.50*	*145.73*	*20.9*	*83.19*	*0.0*
	20	*6.72*	*6.29*	*6733.9*	*6655.6*	*646.8*	*650.5*	*107.92*	*108.00*	*173.1*	*0.62*	*0.50*	*145.36*	*42.4*	*82.52*	*0.0*

Table 4. 13
Realization Utility Market Outcomes and Volatility

σ	W_D^*	r_{RU}	r_{EU}	$r_{EU}^\dagger$	$E\{RU\}_{t=0}$	$E\{U_2\}_{t=0}$	$E\{U_2\}_{t=0}^\dagger$	P_0	$P_0^\dagger$	V_0	β_0	$\beta_0^\dagger$	P_h	V_h	P_1	V_1
		$\alpha = 0.88$ & $\lambda = 1.5$, $\Gamma = 0.7$														
0.15	2	3.16	3.37	3.37	74.2	640.2	641.6	111.38	111.25	140.0	0.57	0.50	125.20	42.7	104.24	0.0
	10	4.02	4.48	4.35	319.9	671.5	673.9	110.33	110.20	506.7	0.54	0.50	124.52	120.8	103.65	0.0
	20	4.82	5.35	5.17	580.5	701.9	704.1	109.45	109.35	773.8	0.53	0.50	124.20	164.0	103.09	0.0
0.3	2	5.00	5.31	5.25	24.6	634.2	634.9	109.40	109.27	33.1	0.60	0.50	135.59	11.9	93.84	0.0
	10	5.51	6.42	6.16	105.3	646.0	648.1	108.60	108.33	142.2	0.58	0.50	134.40	41.8	93.38	0.0
	20	6.20	7.35	6.97	200.9	658.6	661.2	107.78	107.50	240.6	0.57	0.50	133.62	60.6	92.86	0.0
0.45	10	5.96	7.47	7.28	38.6	641.9	642.2	107.45	107.20	80.0	0.53	0.50	143.64	25.0	83.20	5.0
	20	6.64	8.82	8.14	83.1	648.3	650.5	106.93	106.35	146.0	0.60	0.50	142.24	43.4	82.88	10.4
σ	W_D^*	**$\alpha = 0.88$ & $\lambda = 1.5$, $\Gamma = 0.8$**														
0.15	2	3.11	3.25	3.25	69.2	1614.2	1616.0	111.46	111.38	143.2	0.53	0.50	125.32	41.7	104.28	0.0
	10	3.77	4.13	4.00	304.2	1715.3	1717.3	110.64	110.58	557.1	0.51	0.50	124.79	136.7	103.79	0.0
	20	4.47	4.90	4.73	570.5	1816.2	1817.0	109.86	109.81	876.1	0.51	0.50	124.53	196.8	103.26	0.0
0.3	2	5.01	5.20	5.15	22.8	1595.4	1596.3	109.42	109.37	33.3	0.54	0.50	135.73	11.0	93.87	0.0
	10	5.39	6.02	5.81	97.6	1634.3	1636.9	108.81	108.68	148.2	0.53	0.50	134.86	42.8	93.51	0.0
	20	5.91	6.78	6.46	187.6	1676.5	1679.4	108.16	108.02	260.4	0.53	0.50	134.22	67.3	93.06	0.0
0.45	10	6.43	7.32	6.91	33.9	1623.7	1619.2	107.55	107.57	81.4	0.43	0.50	144.39	22.0	83.27	5.2
	20	6.50	8.11	7.58	76.4	1644.2	1646.7	107.21	106.90	153.4	0.54	0.50	143.25	46.6	83.00	10.1
σ	W_D^*	**$\alpha = 0.88$ & $\lambda = 1.5$, $\Gamma = 0.95$**														
0.15	2	2.99	3.03	3.04	64.8	6460.3	6462.4	111.64	111.60	151.4	0.50	0.50	125.51	44.1	104.38	0.0
	10	3.20	3.33	3.29	279.4	7030.8	7028.0	111.36	111.34	695.8	0.50	0.50	125.33	193.1	104.19	0.0
	20	3.99	4.15	4.07	574.1	7658.4	7652.8	110.50	110.50	1068.3	0.50	0.50	125.25	285.9	103.94	0.0
0.3	2	4.97	5.02	5.00	21.0	6364.9	6364.8	109.53	109.52	34.0	0.50	0.50	135.96	10.6	93.94	0.0
	10	5.07	5.26	5.19	88.2	6579.7	6578.7	109.35	109.32	164.3	0.50	0.50	135.70	49.7	93.82	0.0
	20	5.22	5.53	5.42	167.2	6834.9	6831.8	109.13	109.09	315.2	0.50	0.50	135.46	92.4	93.64	0.0
0.45	20	6.24	6.72	6.48	59.9	6692.2	6655.6	108.03	108.00	176.1	0.45	0.50	145.32	53.9	83.46	12.5

Analysis of the analytical and numerical models indicate that general equilibrium outcomes in realization utility markets are mainly driven by the expected utility investor's capacity to trade against the realization utility investor's behavioural biases. The analytical model removes diminishing sensitivity and controls for the effects of wealth on investor trading decisions. In this setting, the expected utility investor always experiences welfare gains when trading with the realization utility investor when $\lambda > 1$ or $\delta < 1$. Furthermore, because of the realization utility investor's sensitivity to losses, the expected utility investor's demand is the primary driver of the IPO opening price. The numerical model provides a richer picture of general equilibrium dynamics by incorporating wealth effects and diminishing sensitivity. Whether or not the expected utility investor experiences welfare gains depends mainly on his/her level of risk tolerance, the dealer's supply schedule, and the relative wealth level of the realization utility investor.

Mental accounting means that the realization utility investor's initial investment decision is based on obtaining as much exposure to an asset that the can be traded in order to produce gains and losses with a net positive contribution to realized utility. This can be contrasted with the expected utility investor who tries to obtain a specific level of exposure to the risky asset as a share of total wealth. Consequently, in order for the expected utility investor to purchase the desired number of shares of the risky asset from the dealer, IPO prices must rise so that the dealer sells a greater total number of shares. Obtaining the desired level of risk exposure to the IPO is costly, especially for less risk tolerant expected utility investors. However, the realization utility investor's strong tendency to realize gains and hold losses provides an opportunity for the expected utility investor: by increasing the IPO price, the expected utility investor is able to put more shares into the realization utility investor's portfolio that can then be purchased at a considerable discount when the realization utility investor decides to realize gains early. The data show that the trading mechanism can lead to large improvements in expected utility welfare as well to the expected utility investor driving up IPO prices well beyond levels observed in the expected utility market control. The general equilibrium result provides the basis for an interesting interpretation of mental accounting in the context of realization utility if we consider how the model's prediction of IPO behaviour relates to real financial markets. In real financial markets there are many bidders at $t = 0$ and shares are continually sold by the dealer as new orders come in. The result that the realization utility investor demands a large number of shares at $t = 0$ even as bid prices keep rising indicates that the model may be capturing a form of 'momentum chasing' or 'irrational exuberance'. In this vein, the expected utility investor takes advantage of the 'momentum chasing' behavior by bidding up the initial stock price in order to benefit from the massive sell off in shares corresponding with positive future news about the stock.

V. Conclusion

In this paper I examine the realization utility specification in both partial equilibrium and general equilibrium settings in the context of a comparison with the expected utility framework. In partial equilibrium, I examine the drivers of realization utility trades over a range of preference parameter values when there are no restrictions placed on the investor's strategy. The discussion indicates that Barberis and Xiong's (2009) explanation of the disposition effect is superior to that of Barberis and Xiong (2011) because the latter does not consider the impact of trade restrictions inherent to that model when interpreting investor behavior. Furthermore, I uncover an error in Barberis and Xiong(2009) methodology for calculating the realization utility investor's optimal portfolio allocation in partial equilibrium. Examining expected utility and realization utility portfolio allocations, I find that correcting for this error leads to a more realistic picture of realization utility investor portfolio rebalancing: at higher returns the investor stops exhibiting the disposition effect and begins to trade more like an expected utility investor by realizing losses early. Additionally, I find surprising results regarding the interpretation of prospect theory preference parameters when implementing realization utility. Diminishing sensitivity, which tends to lead to risk aversion in expected utility specification, actually leads to greater levels of risk tolerance. In fact, when combined with time discounting and very low sensitivity to losses, it is possible that the realization utility investors prefer to invest in riskier assets. The result arises because diminishing sensitivity decreases the investor's sensitivity to large losses, while time discounting and diminishing sensitivity both tend to increase the relative value of gains realized at $t = 1$. It may therefore be useful to estimate prospect theory preference parameters in the context of realization utility by examining individuals' choices using prospects that simulate asset price behavior by evolving through time.

In the general equilibrium section I develop a model that allows me to examine dynamics that arise from differences in expected utility and realization portfolio rebalancing as well as initial stock purchase decisions. This is accomplished by introducing a securities dealer into the setting and has the added benefit of making the general equilibrium setting analogous to an initial public offering. The analysis indicates that realization utility investors indeed exhibit the disposition effect when trading with expected utility investors and that this plays a central role in the models dynamics. The results of the model indicate that mental accounting implies that realization utility investors purchase shares of the risky asset, not with the intention of reaching some specified level of risk exposure, but with the intention of obtaining as much exposure as possible to an asset that can be traded in order to produce realized gains and losses with a net positive contribution to realized utility. The response of the expected utility investor is to then drive up the initial stock price in order to increase the number of shares that the realization utility investor will sell at a discount after positive news releases about the stock. This is the key result of the general equilibrium section, and it has implications for the way we view realization utility. Intuitively, the model seems to capture herd like, or momentum chasing, behavior often associated with less sophisticated investors. In future research I would like to examine the general equilibrium dynamics when the dealer is capable of trading in markets at $t = 1$ and is fully informed regarding investor trading behavior. This approach will likely result in stock price behavior that is even more consistent with the observed trends of IPO prices in that, since the dealer's profits are increasing in the initial stock price, the dealer will be able to identify a mechanism that could encourage the expected utility investor to drive initial stock prices even higher.

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