Queen's University
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Optimal Management of an Invasive Species

An essay submitted in
partial fulfillment
of the requirements for
the Degree of
Master of Arts in Economics
at
Queen's University
Kingston, Ontario, Canada
by

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August 2013

Abstract

Most invasive species management models use either preventive or removal measures, without considering adaptive measures. In particular, models are concerned mostly with the population level and not the damage susceptibility the population evokes. This paper will use the Tulkens model for optimal pollution management in terms of an invasive species. It will do this by strategically balancing removal, damages and adaptation in a static context. As well, a focus will be made on the Asian Carp that is currently threatening the Great Lakes.

Acknowledgements

First and foremost, I am deeply indebted to my supervisor, Devon Garvie, whose guidance, support and putting up with relentless emails and phone calls, made this paper possible. Additionally, and just as importantly, without the support of my family, friends and girlfriend, Brittany, this also would not have been possible. All of which listened to my thoughts and rambles, and knew when to give advice and when to listen, despite likely finding the topic utterly dull. I would also like to thank my fellow colleagues and friends that I have met over the year, who were a big part in making Kingston special. Finally, I would like to thank the Faculty of Economics at Queen's University for providing me with the wealth of knowledge that I will carry throughout my economic career.

Contents

1 Introduction

This paper will design a policy rule such that a governing body can optimally manage an invasive species through balancing of investments in removal and adaptive measures. The environmental sector is one of the biggest areas in economics that requires closer examination to better allocate funds and improve the economy. It is here that billions of dollars can be potentially lost due to damaged ecosystems caused by invasive species. Invasive species make an impact on economic stability through lost biodiversity, decreased production, or even hospitalization. The introduction of these intruders may be entirely unintentional or misguided policy decisions, however, they can happen at any time, making an optimal management strategy imperative. Underestimating the impact of a new species in an environment can lead to great expenditures which are only recently being understood. The introduction of biodiversity economic evaluations has provided insight to the impact of a new species in an environment. For example, in the past, it was very difficult to put a price tag to the number of native birds in a country, however, the value of such a thing can be great with evaluations including use value, existence value and bequest value. As well, if these intruders decimate the population of another species through competition for food or predatory instinct, a ripple effect will occur, impacting industry and other habitats, among others. Thus, the effect of a careless introduction of an invasive species can lead to billions in lost revenue for governments [1].

Recessions, limited funds and high unemployment are all at the forefront of citizen's minds during struggling economic times. This emphasizes the need for an optimal management strategy of a *controllable* problem. Misguided solutions or lack of understanding can lead to economic disaster. An example of this is the introduction of the Asian Carp in the Lakes of Mississippi. This was seen as a potential solution to another invasive species, and now has rapidly progressed to an international emergency [2]. Typically, there is a strong stigma attached to allocating funds to environmental protection. The effects are difficult to observe and is thus given less priority. Fortunately, this perception has been changing recently due to the introduction of economic evaluations indicating major misallocations of funds and underestimated damages. The economic impact of these non-native beings is enormous and the proper management is crucial to a long run, sustainable future.

The model developed in this paper will be introduced in a static context, with removal as the only option initially for a social planner to decrease total costs of invasion. Removal measures will indicate any input or expenditure that will decrease the population of the species. The model will then introduce the option of adaptation, which are any inputs that serve to decrease the *effects* of the invading body on damages. The introduction of adaptation will complicate the decision process for the social planner, and achieving a balance between the two variables will be a priority. Past models have attempted to solve the problem of an invasive species, however there are many issues with certain strategies that are crucial to optimal management. A review of past studies and progress made by recent studies follows in the Literature Review section.

This model will also emphasize the fact that every species will affect the economy *differently.* Each species will have varying effects on damages, which can be attributed to differing starting populations, for instance. The way in which one removes and adapts will vary from species to species. These varying levels of impact on an ecosystem will be observed through an ecosystem change proxy, encompassing all facets of how a species will impact nature and the economy.

In order to display the model more clearly, a simulation of the Asian Carp invasion into the Great Lakes will be performed. This case is of interest due to the current status of the issue being contested in the United States and Canada with potential damage estimates nearing billions of dollars. This is just one particular example of the importance of an optimal strategy for dealing with an invasion. This also emphasises the attention required for policymakers in the world today, to focus on mitigating potential environmental disasters, and the resulting economic repercussions.

2 Asian Carp

Before delving into the details of the model, it is of interest to explore the inspiration for this paper; the Asian Carp. In the 1970's, people of the aquaculture sector in Arkansas and Mississippi were becoming increasingly irritated by parasites in their ponds which lead to protests for action [2]. The protests were heard, and the Asian carp was suggested as a potential solution.

In order to introduce a new species, the farmers were required to obtain approval from the organization that controlled species introduction, the United States Fish and Wildlife Service (USFW). An entire evaluation was done using undisclosed techniques and the species was approved for introduction, at the satisfaction of the aquaculturists. However, Thomas and Hansen (2000) [3] ran their own evaluation, and found that the Carp should have indeed been blacklisted by the USFW and a mistake was made in the approval process. Thomas and Hansen evaluated the potential damages of an invasion on many different criteria including: the potential of a large social loss occurring, identifying the parties affected, and the financial coverage of the worst case social loss. After the evaluation, they deemed that the loss was so great that in no way should the Carp have been introduced. Despite the approval, at the time, the Asian Carp did indeed help remove pesticides from ponds and the program was seen as a success. This success can be attributed to the consuming nature of the Asian Carp, which can eat up to 40 percent of its body weight in a day. As well, their diets consist of eating nearly anything, and they have very few natural predators, so populations can flourish [2]. Despite doing their job of removing harmful pesticides from the ponds, populations of the Carp grew quickly and in large numbers, but were not seen as a major problem until major rainfalls caused flooding to the ponds.

Flooding in the 1970's lead to the Asian Carp escaping into the Mississippi River. Over the next 20 years, the Carp made their way up the river (over 3000 km), eating everything in their path and growing their numbers greatly. This up-river journey ended at the Chicago Area Waterway System, which is a canal system that connects the river with the Great Lakes. Damages have been great during the travel up the United States; from injured boaters, to sharp drops in populations of other species in the river with strong market value [2]. This is due to the competition for food and altering of habitats for native species such as phytoplankton, snails and mussels. Analysts soon realized this potential threat and decided to attempt to delay the invasion into the Great Lakes. This is because an invasion into the Great Lakes would be extremely detrimental to the economy and the ecosystem in the surrounding area. The Great Lakes fishing industry alone contributes approximately 7 billion dollars to the economy annually, making avoidance of any damage to the industry imperative, especially in a struggling economy.

Attempts to prevent the invasion have been ongoing for more than a decade. In the year 2000, an electric barrier was introduced between the Great Lakes and the Chicago canal system as an attempt to prevent invasion. This barrier, while temporarily holding the fish outside the great lakes, is susceptible to power outages, and this failure of the barrier can lead to an invasion at any time. In 2010, federal agencies in the United States secured funding of nearly 80 million dollars for improved monitoring of the fish around the barrier, as well as environmental DNA (eDNA) tests and other research and development practices to prevent an invasion. Despite these investments there is one other possible solution that has been receiving attention recently; the closure of the canal entirely. This solution is met with strong opposition due to the amount of daily trade that would be lost through the canal in addition to the lack of substantial evidence of the Carp being actually present in the Lakes. The closure of the Canal is seen as an attractive preventive measure, as it would delay the invasion substantially. Despite this, fish can perhaps find another way to the Great Lakes, either through tiny river offshoots or even by careless fishermen inadvertently using the wrong bait. Thus, despite all of these investments, an invasion can still be seen as inevitable and policymakers must be prepared to handle an invasion and all of its consequences.

If an economic model is split into a pre/post invasion context, the current timing of this model would be during the pre-invasion stage. However, the post-invasion is of interest due to the inevitability of the invasion. In this case, if studies are done to obtain estimates on the growth rate of the species, planners can have a good idea how fast the species will grow in the environment of the Great Lakes. Adaptive techniques need to be developed immediately in order to prepare for the invasion. And most importantly, a careful evaluation of potential damages and costs of removal and adaptation should be completed. This is the most crucial step, as the results of the model will not be nearly as cost-effective if estimates are grossly underestimated. Despite this very interesting case of the Asian Carp, this paper develops a general model of optimal management. This is done in order for this model to be applicable to other cases of invasions, where inflicted damages can vary.

3 Literature Review

The study of invasive species is a relatively new area of interest to the economic world, as it has historically been reserved for ecologists and biological scientists. Despite the large association between economic damages and infiltrations, economists have not been involved in the study of invasive species until fairly recently. Optimal management lends itself very well to a cost-benefit analysis, and is thus well suited for economists. The problem of invasive species differs from a traditional cost-benefit analysis due to the biological nature of the problem. Population growth rates, nutritional habits and carrying capacities of a particular species were all seen as biological issues and hence were not studied by economists. However, these new species have enormous economic implications, and an invasive species can lead to very significant economic damage at the local, national and even international level. A recent estimate by the U.S. Office of Technology Assessment (OTA) figured that almost 100 billion dollars of monetary losses are due to alien species [1]. Thus, the need for economic modeling to minimize damage is a necessity for the long run sustainability of industries and societies that depend on the equilibrium of nature, which is disturbed when non-native species are introduced. Even after including economics into models, there have been varying opinions on the optimal management of species invasion. This section will be an overview of these past studies, and what sets models apart from one another.

The need for economic analysis of invasive species led to an explosion in the field in the early 1980's with a wide variety of models attempting to optimally manage a nonindigenous species infiltration. Some models focused on the study of optimal prevention, whereas others focused on the optimal removal of a species that has already disturbed the equilibrium of nature. As well, some even studied an optimal combination of both prevention and control to varying degrees of success. The differing views of economists are due to the varying effects of the rate and location of population introduction, the changing damage costs, as well as their respective fluctuating growth rates. All of these differ from species to species, and thus, some assumptions in models that work well for some infiltrations, will not work well for others. Hence, the goal of the study of these alien species is to find and develop a model that is much more general such that it can cover the unique characteristics of each species.

Before examining the varying models more closely, there must be a discussion of the important problems that are common to each model. Firstly, there is the issue of understanding the arrival of the invasive species and its rate of introduction. If one model makes a very lax assumption about the number of arrivals, the model will not be effective with removal measures and a similar case can be made if the assumption is too strong. This is easier said than done, as predicting the arrival of new species requires modeling with a large amount of uncertainty and associated probabilities. Secondly, a proper assessment of damages is critical in obtaining an appropriate strategy. A suitable damage evaluation can help a governing body (who has limited funding) prioritize particular threats with a cost-effective policy. Finally, growth rates of species are crucial to optimal management. A clear understanding of species minimum carrying capacity is vital to obtaining the best management policy. If a species grows very slowly, for example, then removal costs may not be a very high priority. The appropriate solution to these problems is critical to the development of an optimal management strategy for invasive species.

There are a multitude of papers developing models for optimal preventive techniques. One of the first papers using this idea is an examination of biological pollution under ignorance by Horan et al. (2002) [4]. This paper specifies that invasive species are difficult to deal with in a risk-management framework and that a different approach must be taken. They argue that assigning a probability to a one-time event without any historical evidence is invalid. Furthermore, decisions regarding establishment of species should be made under conditions of incomplete information about probability (Williamson 1996 [5]).The main contribution of this paper is this emphasis on the high amount of uncertainty with regards to species arrival, which would be incorporated into future models. However, the problem with this method is that it completely ignores the idea of control. For a particular species, such as the airborne Asian Soybean Rust [6], prevention may be impossible and costly, but appropriate control methods can be much cheaper and more effective.

Another preventive model is one developed in Margolis et al. (2005) [7], in which the effects of trade policy on invasive species are discussed. Trade is the main mode of transportation for an alien species, as they can easily go undetected in inspection on large cargo ships or airplanes. Restricting trade to zero would eliminate a high percentage of these invasions, despite hundreds of years of economic trade policy indicating the large benefits from trade. However, when it comes to dealing with particularly dangerous areas, a policymaker may want to deter trade through the use of tariffs. A dangerous area can be defined as a country that may have a potentially dangerous species that would cause copious amounts of damage if brought into the receiving country. This logic is counter to the benefits of free trade which enable countries to specialize in the products they produce most efficiently. Thus, the problem becomes not one of optimal policy but one of a political variety. This is because private interest groups may be hesitant to instill tariffs to decrease chances of a new species being introduced into a foreign environment due to a loss in profits. Again this model is logical in that it underlines the importance of trade in the transportation of invasive species as well as tariffs being a possible policy tool for prevention. However, it does not incorporate an optimal strategy for when the pest actually does invade the area where it will cause damage. Nor is there mention of an adaptive measure for when removal becomes very costly after invasion. Hence a mix of removal strategy, such as a tariff combined with a control strategy seems much more attractive than a strict preventative strategy as it has no safety net for when the invasion does eventually occur.

Finally, Horan and Lupi (2005) [8] examine other incentive based strategies to reduce the chance of a new species being introduced into the Great Lakes. The two methods suggested are a performance proxy-based incentive and a technology-based incentive. Similar to Margolis et al. (2005), this model again states that the best policy option for mitigating invasions is the restriction of trade but realizes this to be unrealistic. There is thus an examination of a second best solution, which can take the form of uniform technology standard for example, which in this case is a ballast control for ships. Ballast acts as a speedway for invasive species to enter new environments and the controlling of ballast can decrease the chance of an alien species arriving unexpectedly. The problem with this idea is the determination of said standard. This problem is very similar to the one found with emission control, in that a third party should not be determining the standard, as it is difficult to establish the appropriate level of technology. In face of a strict technology standard, there is no incentive to innovate nor is cost efficiency achieved because this third party does not have full information of the best production techniques for a firm. Therefore, this preventive measure for an invasive species may not be optimal and another direction should be followed.

In contrast to the optimal study of prevention, the study of control techniques were also explored by many economists over the world. One particular paper of interest is Eisworth and Johnson (2002) [9], who developed a dynamic optimal control model for the management of new species. This model states that optimal management is dependent on ecological factors such as the carrying capacity of the invaded species as well as its growth rate. Eisworth and Johnson note that pollution models and invasive species models are very closely related but can differ in some important respects. One such example is that the natural rate of stock change for the nonindigenous species is almost always positive due to its lack of predators. However, the rate of growth for a pollutant stock is negative, as countries become more environmentally conscious over time or their production techniques become more efficient. The authors note that invasive species models take into account biological factors such as carrying capacity, growth rates and growth limits, which are not present in pollution models. Despite these drawbacks, the connection between the models is of interest to this author and will be explored in more detail with Tulkens and Steenberghe's (2009) [10] model of emissions later on in this paper. The Eisworth and Johnson model acknowledges adaptation as an interesting idea in managing invaders, yet chooses to ignore it for a larger focus on removal, which can be seen as a potential weakness of the paper.

One of the biggest contributors to the control of invasive species literature is Jason Shogren. In one of his many contributions to the literature, Shogren (2000) [11] identifies that decision makers protect themselves from the risk of invasion in two ways: mitigation and adaptation. Mitigation is the conventional idea of removal and control of species through capital investments that can decrease the current population of an invaded species. Adaptation, on the other hand, deals with the change in attitude and behaviour of decision makers which can decrease ones susceptibility to damages from the species. Adaptation can also involve capital investments as a form of changed attitudes. Together, mitigation and adaptation determine the overall risk of an invasion as well as the cost of said invasion. However, the model is examined from a risk of damages perspective, and does not present a clear balancing between removal and adaptation.

Another paper of interest is one of Olson and Roy (2007) [12], which is a model of control, and differs from their more famous paper on the combination of prevention and control, Olson and Roy (2005) [13]. In the 2007 paper, there is a development of a control model which is described as an optimal capital accumulation problem. One particular point of interest in their model is the determination of when it is optimal to control a population, to eradicate a population and when to do nothing. The determinants of one of these options are the growth rate of the invasion, the control cost of the invasion and the damage cost. In all cases the initial size of the invasion is crucial to determining the optimal policy choice and this importance is stressed in Burnett et al. (2011) [14], which will be discussed further in the mixed model portion of this review. As well, a lack of information in any determinant can have dire consequences on the optimal control level choice. The paper makes mention of prevention as a determinant of the probability of invasion, however it is not explored in order to focus on control methods. A drawback of this model is that it is much more complex than other control models where simplicity is emphasized. As well, besides the removal of species to decrease damages, there is no other option mentioned for a social planner to decrease damages after the invasion has occurred.

The final portion of this Literature Review is a discussion of the combination of differing strategies involving prevention, control and/or adaptation for the optimal management of an alien species. The assessment of damages is critical to any study of invasive species, not only from a society point of view, but also for an appropriate determination of one of these levels of prevention, removal or adaptation. In order to properly assess damages, one must take into account the strategy utilized, and this is the motivation behind following a mixed strategy of managing an infiltration. In particular, a mixed strategy offers much more freedom for a social planner, and thus levels can be more easily adjusted to changes in damage assessments. Recently, papers have followed a mixed strategy due to this allowed freedom for social planners to devise an efficient and cost effective strategy.

One of the first mixed models is found in Olsen and Roy (2005), who developed a combined model of prevention and removal for examining an uncertain biological invasion. The paper is a good foundation for mixed modeling, in that it stresses the interdependences between prevention and control; however there are some issues that require attention. One strong assumption within the model is that it requires growth of the non-indigenous species to begin immediately after the first arrival. This is difficult to accept, as one would presume there is a minimum required population such that the species can reproduce and sustain its numbers. As well there is no mention of adaptive measures that can be explored once the invasion does inevitably occur.

Another paper that inspired further studies on optimal mixed strategies is Leung et al. (2005) [15]. This paper solves for an optimal combination of the two strategies of prevention and removal. In order to make the model more manageable, it assumes that the post-invasion prevention level is zero (where the model is split into a pre/post invasion context). This is similarly done by Burnett et al. (2007) [16], with the logic being that once invasion takes place, spending on prevention should be zero and all remaining funds dedicated to removal. More specifically, once the damage of invasion is done, there is no practical use in trying to reduce the damages through prevention and a focus should be made on removal. However, as will be shown in a later paper (Burnett et al. 2011), a proposed solution to the prevention and control strategy is keeping the population at a particular level, post-invasion. This paper inspired the Burnett et all. 2011 paper, however, it does not address any other measure of decreasing damages after the invasion, which is seen as a drawback to the model.

Burnett et al. (2011) uses a dynamic control model for an optimal management strategy in a pre and post invasion context. This extended Olson and Roy (2005) as well as the Leung et al.(2005) by solving a two-stage dynamic optimization problem. The two stages are pre-invasion and post-invasion, with post-invasion determining the optimal path of species removal along with the associated damage costs and post-invasion removal costs. These paths were determined based on a calculated steady state for the population. This framework was inspired by a catastrophe model by Tsur and Zemel (2006) [17], who used a similar idea, but applied it to a broader sense of environmental disasters. The motivation behind this stability level is greater certainty of the current population. This would give better estimates of damages and costs and thus a more appropriate allocation of funds. In order for this to be possible, prevention must be maintained even after an invasion occurs, in addition to spending on removal. This idea of an optimal population level is interesting, however, there may be a few drawbacks associated with a steady state level. Keeping a population at a particular level may be very expensive in the face of high growth rates and unknown locations of pests. Thus, perhaps a higher focus should be spent on adapting to the growing population, in addition to control. If there is a very large focus on a steady state, and an unexpected event occurs that increases the population greatly, there are no adaptive measures in place, which could lead to uncontrollable damage costs. Using the post-invasion results, an optimal path of removal can be solved in the pre-invasion stage using a hazard rate arrival function as a basis for predicting the arrival of a new species. This hazard rate is another issue that can be found with this paper. The hazard rate is extremely reliant on surveys from scientists making their best assumption for when the invasion will take place. Unexpectedly, probabilities become quite complicated and non-intuitive and the model becomes very difficult to comprehend. Despite the idea of a pre/post invasion split being very interesting, it involves far too many strong assumptions for it to be seen as a very convincing argument to include itself in a mixed model.

Finally, Tulkens and Steenberghe (2009) use a different mixed model by including adaptation in their analysis (in addition to removal) but in a climate change context. The Tulkens paper is an interesting idea because of its inclusion of adaptation in the damage function, which already includes the temperature change. The temperature change is what causes the damages, and can be affected by emission removal levels. Increased adaptation levels will shift down and flatten the marginal damage curve, and hence reduce the damage felt by the increased temperature change. From this, an overall environmental cost function can be found which can determine optimal levels of removal, adaptation and damages. Additionally, the emphasis on adaptation costs, especially in a dynamic context, can be a very important addition to the invasive species literature. This is because it can serve as reference guide for deciding on investment projects that will decrease susceptibly to damages caused by species invasion. This is due to the benefits of adaptation in earlier periods, being felt long into the future. Another caveat of the general emissions model is that it is not tied down to a particular species and is open to be interpreted in a general context. As has been stated throughout this Literature Review, the study of invasive species is very case dependent, and it is hard to come up with a general model for all cases. This gives the Tulkens model an advantage over the narrow view of the Burnett model.

Despite the literature predominately featuring studies by ecologists and biological scientists, the study of invasive species has not been fully explored until fairly recently. However, even with economics being introduced to the literature, it seems that is has been difficult for economists to completely agree upon a model that applies to a more general framework (as is this case with most economic problems). This can be attributed to the varying nature of a species arrival rate, growth rate, or damage function as key reasons for the differences observed. A mixed policy of removal and adaptation appears to be a very attractive strategy as the two individual policy options are included, and thus even if removal or adaptation is optimally zero, it will be revealed in the mixed model. Armed with this knowledge, there appears to be a superior strategy to deal with pesky invasions that will greatly decrease the harm felt by victims of these infiltrations.

4 Model

4.1 Overview

The goal of this model is find an optimal solution for policymakers to minimize costs in the face of an invasive species. In order to achieve this, a mixed model of removal and adaptation is recommended. Past models have used pure strategies such as strict prevention or strict removal; however, these models do not offer the required versatility a social planner needs in order to optimally manage an invasion. In addition, some previous models have split the problem into a pre-invasion/ post-invasion context, where an optimal prevention level is found in the pre-invasion stage. The merits of such a model can serve to delay an imminent invasion. However, the probabilities and assumptions involved in deciding the best prevention level are very difficult to grasp. Location and timing of the invasion are unknown, and can be simply caused by fishermen using certain bait by accident and introducing a harmful species to the environment. Thus, for simplicity and concreteness, the following model assumes that the invasion has already occurred, and an optimal balancing of removal and adaptation is required to minimize total costs.

Due to increasing removal costs, a model of only removal will eventually lead to one of two scenarios: Removal costs eventually becoming so great, that the associated budget is exhausted or alternatively, removal spending stops due to high costs, and the population of the invasive species grows so high that damage costs are monumental. Thus, there are needs for an additional tool which social planners can use to avoid situations that are presented in a removal only model.

Adaptation can lower the damages caused by a given population. However, it does not decrease the population of a species, which is a drawback of using an adaptation-only model. As one can imagine, a model of only adaptation would also lead to very high costs. This is due to increasing adaptation costs, which are caused by technological or feasibility constraints. This strong focus on adaptation would lead to the population becoming overwhelming high due to a lack of removal. The sheer size of this population would lead to very high damages, despite a society being highly adaptive, as the prospect of invulnerability to damages seems highly unlikely.

With a mixed model, total costs can be minimized, and the welfare of anyone affected by the invasion will be improved. The model presented uses a simple and intuitive process that can be easy for policymakers to comprehend and implement. This is a caveat that is not afforded by other, pure strategy invasion models.

In order to simplify the model, a static approach is taken. Firstly, the model is solved without adaptation, where the only tool a social planner possesses for minimizing total costs is removal, y. Removal, in this context, reduces damages, $D(\cdot)$, through the reduction of the invasive species population, n. Next, the problem will be made more complicated with the addition of adaptation, α, which is any input or expenditure that reduces one's susceptibility to damages caused by the species. This static model will be solved backwards, where adaptation is determined before the level of removal is chosen.

4.2 Static model without adaptation

The main impact of an invasive species will be measured through an ecosystem quality change proxy, ΔQ, which is a composite asset comprised of a bundle of services including: life support, resource supplier, waste sink and amenity services. This ecosystem change depends on the initial population of the invasive species, $\bar{n}$, as well as the removal level, y. Let

$$\Delta Q = \Delta Q(\bar{n}, y)$$

be the functional form of the ecosystem change. An increase in the ecosystem change will harm the environment and the economy, and can be observed through the function:

$$D = D(\Delta Q),$$

where an increase in ΔQ will worsen damages: $\frac{\partial D}{\partial \Delta Q} > 0$ at an increasing rate: $\frac{\partial^2 D}{\partial \Delta Q^2} > 0$. As the starting population increases, ΔQ grows due to the larger amount of the invasive species causing changes. Alternatively, as the removal level increases and the population decreases, ΔQ declines. Formally, this can be seen in the derivatives: $\frac{\partial \Delta Q}{\partial \bar{n}} > 0$ and similarly with the removal level[1], $\frac{\partial \Delta Q}{\partial y} < 0$.

[1] Additionally, the second derivatives are assumed to be positive and negative respectively, i.e. $\frac{\partial^2 \Delta Q}{\partial y^2} > 0$ (the removal becoming less effective over course of invasion) and $\frac{\partial^2 \Delta Q}{\partial \bar{n}^2} < 0$ (the starting population becoming less important as the population becomes higher).

It follows that as $\bar{n}$ increases, damages are worsened, $\frac{\partial D}{\partial \bar{n}} > 0$ and using similar logic, $\frac{\partial D}{\partial y} < 0$.

To start, a solution to the invasion problem will be derived with respect to the use of removal measures in order to decrease the population of the given species. The cost function is given by:

$$J = c(y) + D(\Delta Q)$$

where $c(y)$ is the cost of removal. Removal cost is increasing with removal ($c' > 0$), at an increasing rate ($c'' > 0$) due to the fact that the more one removes, the lower the population gets, and the harder it is to find and remove said species. This logic makes sense with a simple example of the costs associated with finding one snake (and removing it) in a forest of no other snakes as opposed to finding 50 in a forest of thousands of snakes.

The only choice variable in ΔQ is the removal level, and thus the minimization problem follows:

$$\min_{y} J = c(y) + D(\Delta Q) \qquad \text{s.t. } \Delta Q = \Delta Q(\bar{n}, y)$$

$$\frac{\partial J}{\partial y} = c'(y) + \frac{\partial D}{\partial \Delta Q}\frac{\partial \Delta Q}{\partial y} = 0$$

thus,

$$y^* : c'(y) = -\frac{\partial D(\Delta Q)}{\partial \Delta Q}\frac{\partial \Delta Q}{\partial y} \tag{1}$$

The optimal removal level is one which equalizes the marginal cost of removal with the marginal benefit received from avoiding damages through removal. This can be seen in Figure 1, where the intersection occurs at y^*. Notice if the removal level is below the optimal value, $y^1 < y^*$ this implies that the marginal benefit of one more unit of removal is greater than marginal cost of that same unit, and thus a social planner can decrease total costs more with an increase in removal. Similarly, if the removal level is above the optimal value, $y^2 > y^*$, the opposite is true, where marginal cost is greater than marginal benefit and a social planner can decrease costs by decreasing the removal level. Thus, y^* is the equilibrium removal level.[2]

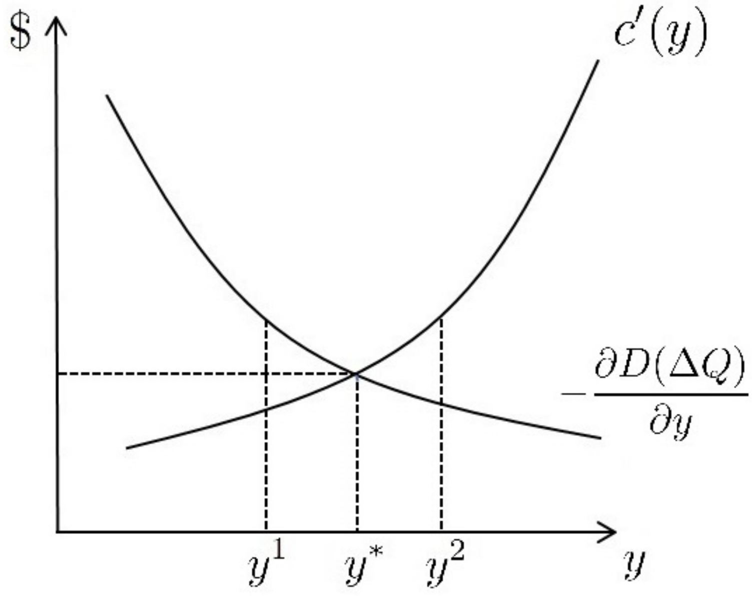

Figure 1: Optimal choice of removal in a static model without adaptation

4.3 Static model with adaptation

Adaptation provides an additional tool that can decrease damages and further minimize the total cost of species invasion. Unlike removal, which reduces damage cost by reducing population size (movement along the damage curve), adaptation acts to decrease the adverse effects of the population. Hence, a reduction in damage costs, for any given population size, can be seen as a downward shift of the damage curve. An example of adaptation can be protective equipment on boats to decrease the damage felt by jumping Asian Carp, whereas a toxin in the water would be an act of removal.

[2]The second order condition is given for future use, which will be referred to as SOC_y : $c'' + \frac{\partial^2 D}{\partial \Delta Q^2}\left(\frac{\partial \Delta Q}{\partial y}\right)^2 + \frac{\partial^2 \Delta Q}{\partial y^2}\frac{\partial D}{\partial \Delta Q} > 0$, ensuring cost minimization.

The total cost function is given by:

$$J = c(y) + a(\alpha) + D(\Delta Q, \alpha)$$

Notice that adaptation enters the total cost function twice. Firstly, the direct cost of adaptation, $a(\alpha)$, includes investments in adaptive techniques or any expenditure involved with adaptation. As the adaptation level increases, as does the cost, $\frac{\partial a}{\partial \alpha} > 0$ at an increasing rate $\frac{\partial^2 a}{\partial \alpha^2} > 0$. Clearly, an increase in the expenditure cost will increase total cost. Additionally, the benefit of adaptation is included in the damage function. That is, for every ΔQ (which is fixed when choosing the adaptation level), an increase in adaptation will lead to a decrease in damage costs, $\frac{\partial D}{\partial \alpha} < 0$ with the rate of change decreasing, $\frac{\partial^2 D}{\partial \alpha^2} > 0$. As well, ΔQ and adaptation affect damages in opposite directions, thus $\frac{\partial^2 D}{\partial \Delta Q \partial \alpha} < 0$ and similarly $\frac{\partial^2 D}{\partial \alpha \partial \Delta Q} < 0$.

The problem will be solved backwards in two stages. In the first stage, the social planner chooses α, to minimize the adaptive cost function (K), treating ΔQ as given:

$$\min_{\alpha} K = a(\alpha) + D(\Delta Q, \alpha)$$

$$\frac{\partial K}{\partial \alpha} = a'(\alpha) + \frac{\partial D(\Delta Q, \alpha)}{\partial \alpha} = 0$$

And after isolating α,

$$\alpha^*(\Delta Q) : a'(\alpha) = -\frac{\partial D(\Delta Q, \alpha)}{\partial \alpha} \tag{2}$$

This first order condition states that the optimal choice of adaptation is one that balances the marginal cost of adaptation with the marginal benefit of adaptation (the reduction in damages from implementation of adaptive measures).[3] Notice as well that the optimal choice of adaptation is a function of the ecosystem change

[3] Additionally, the second order condition, $SOC_\alpha : a'' + \frac{\partial^2 D(\Delta Q, \alpha)}{\partial \alpha^2}$, is assumed to be positive to ensure a cost minimum exists.

alone. Once this level of adaptation is substituted into the total cost function, as will be done in order to solve for the optimal removal level, a cost minimizing level of ΔQ will be found. This can then be substituted back into $\alpha^*(\Delta Q)$ in order to obtain an optimal level of adaptation that is strictly a function of exogenous parameters. The effects of a changing ΔQ (as well as additional exogenous parameters) on adaptation will be explored in the Comparative Statics section.

In order to examine the short and long run effects of adaptation on damages, we must first define the *optimally adapted cost function*:

$$f^*(\Delta Q) = a(\alpha^*(\Delta Q)) + D(\Delta Q, \alpha^*(\Delta Q))$$

which is the result of substituting $\alpha^*(\Delta Q)$ into the adaptive cost function, K. Notice that at any point along the function, adaptation is optimal.

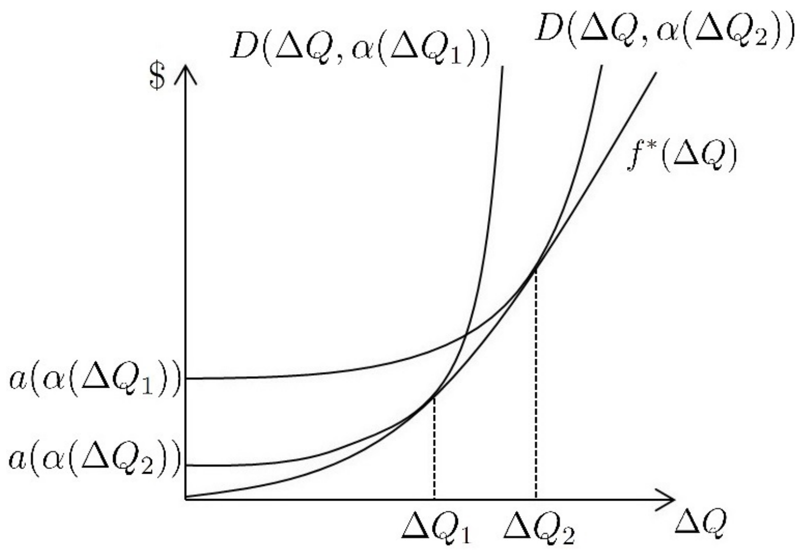

Figure 2: Suffered damage costs, enveloped by the optimally adapted cost function

Figure 2 shows multiple *suffered damage costs* functions, which are the damage portion of f^*, i.e. $D(\Delta Q, \alpha(\Delta Q))$. Notice that f^* appears as the outer envelope of all suffered damage costs functions. For a fixed level of ecosystem change, ΔQ, the suffered damage cost functions differ based on the amount of adaptation expenditures, $a(\alpha)$.[4]

Formally, the difference between the outer envelope f^* and the suffered damage cost functions is due to varying levels of adaptation expenditures being fixed with respect to ΔQ over time. As is the case with total cost curves in classical microeconomics, certain short run input costs are fixed.[5] However, in the long run, these fixed costs become variable, and a social planner can adjust adaptation to optimally manage the invasive species. In every period, the social planner will chose the minimum of the short run cost function and thus, the optimally adaptive cost function envelopes the individual suffered damage cost functions.

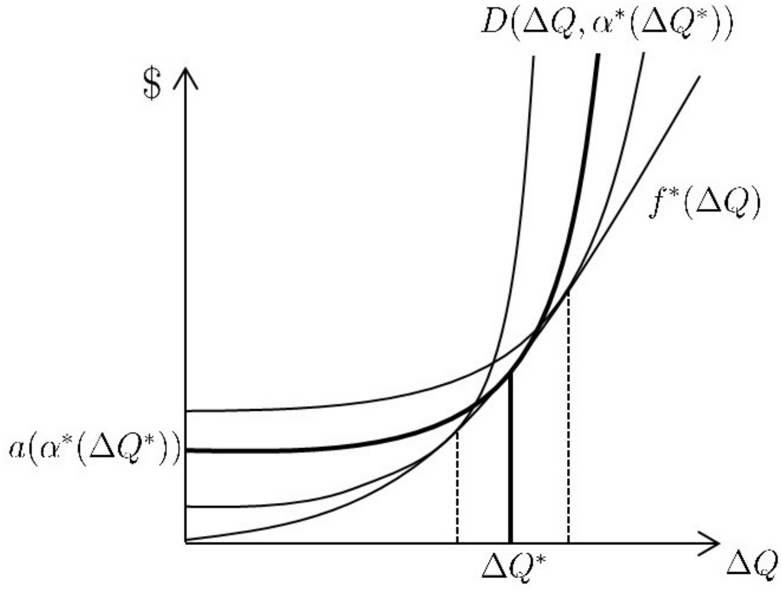

Figure 3: Optimal choice of adaptation

For any given level of ecosystem change, ΔQ^*, the optimal adaptation expenditure, $a(\alpha^*(\Delta Q^*))$, is one in which its associated suffered damage cost function, $D(\Delta Q, \alpha^*(\Delta Q^*))$, is tangent to the optimally adapted cost function, $f^*(\Delta Q)$ at the given ecosystem level, ΔQ^*. In particular, the optimal adaptation level is one in which the slope of its associated suffered damage cost at ΔQ^* equals the slope of the optimally adapted cost function, $f^*(\Delta Q^*)$ as shown in Figure 3.

[4] $a(\alpha(\Delta Q_1))$ and $a(\alpha(\Delta Q_2))$ in Figure 2.

[5] Adaptation expenditures ($a(\alpha)$) are fixed in the short run, and thus α enters $D(\Delta Q, \alpha)$ as a parameter.

Formally,

$$\frac{df^*}{d\Delta Q} = a' \frac{d\alpha^*}{d\Delta Q} + \frac{\partial D}{\partial \Delta Q} + \frac{\partial D}{\partial \alpha^*}\frac{d\alpha^*}{d\Delta Q}$$

Re-arranging:

$$= \left[a' + \frac{\partial D}{\partial \alpha^*}\right]\frac{d\alpha^*}{d\Delta Q} + \frac{\partial D}{\partial \Delta Q}$$

And after making use of the first order condition being zero for α, we are left with:

$$\left.\frac{df^*(\Delta Q)}{d\Delta Q}\right|_{\Delta Q=\Delta Q^*} = \left.\frac{\partial D(\Delta Q, \alpha^*)}{\partial \Delta Q}\right|_{\Delta Q=\Delta Q^*} \tag{3}$$

The optimal adaptation expenditure level is one in which the marginally adapted damage cost incurred by ecosystem change is equal to the marginal suffering damage cost only and does not include adaptation expenditures. The absence of the cost of adaption $a(\cdot)$ is significant because this emphasizes the connection between the suffered damage cost function and the optimally adapted cost function. It implies that the only aspect of importance is the effect of adaptation on equalizing the slopes of both functions and not the physical cost of adaptation.

We have shown that adaptation can decrease damages and provide the social planner with an additional tool in optimally managing an invasive species. However, the social planner must adapt *efficiently*, otherwise, cost minimization is not achieved. Misappropriating adaptation, or maladaptation, can be seen in two different ways in Figure 4.

If there is a target ecosystem change level, say ΔQ^* (as in Figure 3), we can use two suffered cost functions to examine the implications of maladaptation. Firstly, take the adaptation expenditure level, $a(\alpha(\Delta Q_1))$ which has an associated suffered damage cost function, $D(\Delta Q, \alpha(\Delta Q_1))$ tangent to f^* at ΔQ_1. It is clear from Figure 4 that at the target ΔQ^*, adaptation is too little, which is represented by the vertical distance AC. The evidence of this lower amount of adaptation is that the slope of $D(\Delta Q, \alpha(\Delta Q_1))$ at ΔQ^* is higher than the slope of $f^*(\Delta Q^*)$,

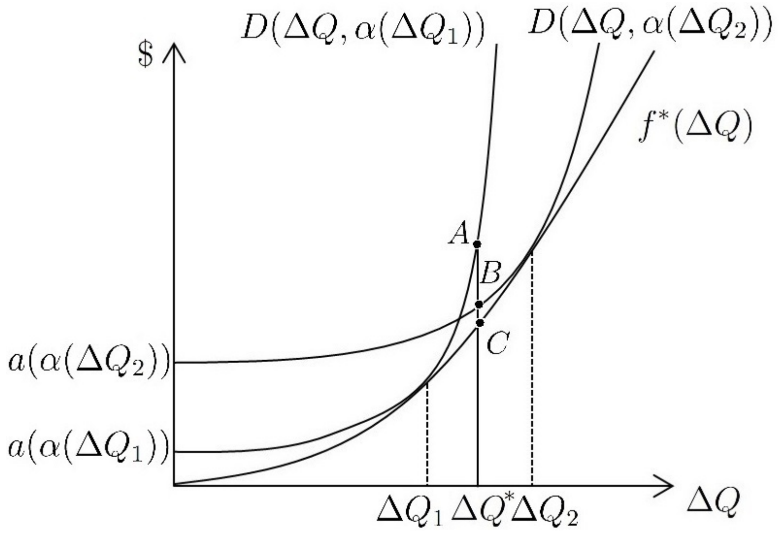

Figure 4: Example of Maladaptation

violating the optimal rule of tangency. The savings achieved from the decreased damage cost (as a result of adaptation) is more than the cost of adaptation and hence the social planner should invest in more adaptive measures.

Alternatively, with associated adaptation level ΔQ_2, the suffered damage cost function, $D(\Delta Q, \alpha(\Delta Q_2))$ has a flatter slope than f^*, at the point ΔQ^*. This non-tangency is an indication of a non-optimal level of adaptation. The flatter suffered damage cost curve indicates excessive adaptation, which can be represented by the vertical distance between the two curves, BC. Thus, cost savings will occur if the social planner invests less in adaptive measures. In particular, the higher costs from an increase in damages (due to the decrease in adaptation), is in fact cheaper than the necessary costs associated with maintaining a high level of adaptation.

Upon finding the optimal adaptation level, the social planner will then decide on an optimal removal level, y^*. This will be achieved by choosing y in order to minimize J^*, the optimally adapted total cost function:

$$\min_y J^* = c(y) + a(\alpha^*(\Delta Q)) + D(\Delta Q, \alpha^*(\Delta Q)) \qquad \text{s.t. } \Delta Q = \Delta Q(\bar{n}, y)$$

$$\frac{\partial J^*}{\partial y} = c'(y) + \frac{\partial D}{\partial \Delta Q}\frac{\partial \Delta Q}{\partial y} + \left[a'(\alpha^*) + \frac{\partial D}{\partial \alpha^*}\right]\frac{d\alpha^*}{d\Delta Q}\frac{d\Delta Q}{dy} = 0$$

The term in the square brackets is zero due to adaptation being chosen optimally

$$y^* : c'(y) = -\frac{\partial D(\Delta Q(\bar{n}, y), \alpha(\Delta Q))}{\partial \Delta Q}\frac{\partial \Delta Q(\bar{n}, y)}{\partial y} \tag{4}$$

Notice that the optimal removal level again balances the marginal cost of removal with the marginal benefit received from reducing damages through removal and subsequently, adaptation. However, the optimal removal level under adaptation is less than the removal level found under the no adaptation case, as seen in equation 1.

To see this, recall the first order conditions for removal under adaptation (y^A) and no adaptation (y^N):

$$y^A : c'(y) + \frac{\partial D(\Delta Q, \alpha^*)}{\partial \Delta Q}\frac{\partial \Delta Q}{\partial y} = 0$$

and

$$y^N : c'(y) + \frac{\partial D(\Delta Q)}{\partial \Delta Q}\frac{\partial \Delta Q}{\partial y} = 0$$

Substituting y^N into y^A yields:

$$\left[\frac{\partial D(\Delta Q, \alpha^*)}{\partial \Delta Q} - \frac{\partial D(\Delta Q)}{\partial \Delta Q}\right]\frac{\partial \Delta Q}{\partial y} > 0$$

As the term in the brackets is negative due to the assumption of $\frac{\partial^2 D}{\partial \Delta Q \partial \alpha} < 0$.

Thus, $y^N > y^A$ because the marginal cost is greater than the marginal benefit evaluated at the no adaptation removal level ($c' > \frac{\partial D(\Delta Q, \alpha^*)}{\partial \Delta Q}$) and can be seen in Figure 5.

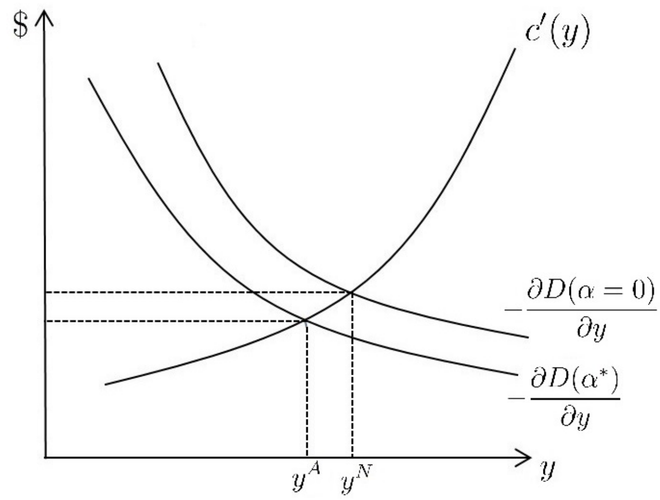

Figure 5: Comparing removal levels under adaptation and no adaptation

4.4 Comparative Analysis for Static model

In order to observe some interesting comparative statics, a few additional parameters are required. An expanded definition of the derived first order conditions[6] will begin with the addition of slope and/or shift parameters to the adaptation cost function (k) and the damage function (v). These parameters (both greater than zero) will help determine the shape and location of each respective curve, and ultimately, the optimal adaptation level, α^*. With a better understanding of what determines α^*, a more appropriate understanding of the optimal removal level y^* will follow. As well, the budget allocation between removal and adaptation can be understood more clearly from comparative statics and will be explored more thoroughly in the simulation experiment following this section.

The more specific first order condition for adaptation is now given by:

$$\alpha^*(\Delta Q; k, v) : a'(\alpha; k) = -\frac{\partial D(\Delta Q, \alpha; v)}{\partial \alpha}$$

Total differentiation yields:

$$\left[a'' + \frac{\partial^2 D}{\partial \alpha^2}\right] d\alpha + \frac{\partial^2 D}{\partial \alpha \partial \Delta Q} d\Delta Q + \frac{\partial^2 a}{\partial \alpha \partial k} dk + \frac{\partial^2 D}{\partial \alpha \partial v} dv = 0 \tag{5}$$

[6] See equations 2 and 4.

It is assumed that the slope of the adaptive cost curve increases with k, $\frac{\partial a}{\partial k} > 0$ and the damage cost function increases with v, $\frac{\partial D}{\partial v} > 0$. The overall impact of k and v on adaptation will be determined in the removal stage of the problem following the adaptation analysis. This is because adaptation is determined in the first stage as a function of the ecosystem change, ΔQ, which is not fully determined until the second stage when the optimal removal level is found.

The impact of an increase in k on adaptation is given by:

$$\frac{d\alpha}{dk} = -\frac{\left[\frac{\partial^2 a}{\partial \alpha \partial k}\right]}{SOC_\alpha} < 0$$

An increasing k, $(k_2 > k_1)$, moves the marginal cost curve for adaptation up and to the left of the original marginal cost curve[7] which results in a decrease in adaptation, $\alpha_2^*(\Delta Q) < \alpha_1^*(\Delta Q)$. Recall this optimal adaptation level is for any ΔQ, and the actual effect of k on α^* will not be fully realized until *after* the optimal removal level is determined in the second stage of the cost minimization problem. This is because the second stage will determine the actual value of ΔQ that is then substituted into $\alpha^*(\Delta Q)$, which is done in order to obtain α^* as a function of only exogenous parameters. Thus, we will analyze the effects of the various parameters on the removal level before discussing the overall effects of k on the final value of α^*.

Similarly, for an increase in v $(v_2 > v_1)$:

$$\frac{d\alpha}{dv} = -\frac{\left[\frac{\partial^2 D}{\partial \alpha \partial v}\right]}{SOC_\alpha} > 0$$

The numerator is assumed to be negative due to the effect of adaptation on damages being negative, and the effect of v on damages being positive. Thus, an

[7]See left diagram of Figure 7 where $a'(k_2)$ is above $a'(k_1)$.

increase in v shifts the marginal benefit of adaptation curve outwards,[8] increasing the value of adaptation, $\alpha_2^*(\Delta Q) > \alpha_1^*(\Delta Q)$, for any value of ΔQ. Again, the overall effect of v on α^* will be determined after the second stage, when the optimal removal is determined, and substituted back into $\alpha^*(\Delta Q)$.

The effect of an increase in the ecosystem change on adaptation will be given by:[9]

$$\frac{d\alpha}{d\Delta Q} = -\frac{\frac{\partial^2 D}{\partial\alpha\partial\Delta Q}}{SOC_\alpha} > 0$$

Thus, as ΔQ increases, optimality requires higher spending on adaptive measures. As damages increase, the social planner is faced with the problem of decreasing these damages. Removal can only go so far, due to the increasing nature of removal costs. Hence, the social planner can decrease the susceptibility of the ecosystem to these damages through more investments in adaptation as an alternative to continuous investments in only removal.

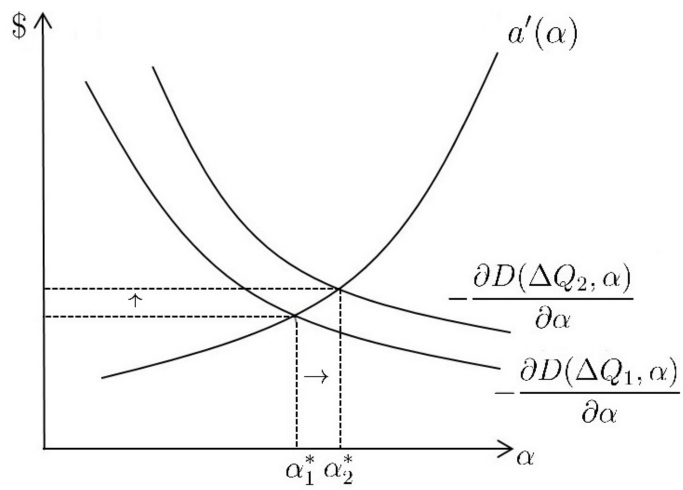

Figure 6: Comparative statics for adaptation and ecosystem change

This can be observed graphically in Figure 6, where $\Delta Q_2 > \Delta Q_1$, shows that the increase in ΔQ (indicated by the shift outwards of the damage function), requires a higher level of adaptation for optimality ($\alpha_2^* > \alpha_1^*$), for a fixed level of the marginal adaptation cost, a'.

[8] See left diagrams of Figure 9 and Figure 10.

[9] An increase in ΔQ can be caused by an increase in the starting population, $\bar{n}$ or a decrease in the removal level, y.

The addition of slope and/or shift parameters to the removal analysis can enhance the study of the optimal removal level. The first order condition will now include the parameters s, k and v which will act as slope/shift parameters for the removal cost, the adaptation cost and the damage cost respectively. The first order condition for removal is now given by:

$$y^* : c'(y; s) = -\frac{\partial D(\Delta Q(\bar{n}, y), \alpha(\Delta Q; k, v); v)}{\partial \Delta Q} \frac{\partial \Delta Q(\bar{n}, y)}{\partial y}$$

Totally differentiating this first order condition for removal yields:

$$\left[\left(\frac{\partial^2 D}{\partial \Delta Q^2} + \frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial \Delta Q}\right)\frac{\partial \Delta Q}{\partial \bar{n}}\frac{\partial \Delta Q}{\partial y} + \frac{\partial^2 \Delta Q}{\partial y \partial \bar{n}}\frac{\partial D}{\partial \Delta Q}\right] d\bar{n} + \frac{\partial^2 c}{\partial y \partial s} ds +$$

$$\left[\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial k}\frac{\partial \Delta Q}{\partial y}\right] dk + \left[\left(\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial v} + \frac{\partial^2 D}{\partial \Delta Q \partial v}\right)\frac{\partial \Delta Q}{\partial y}\right] dv +$$

$$\left[c'' + \frac{\partial^2 D}{\partial \Delta Q^2}\left(\frac{\partial \Delta Q}{\partial y}\right)^2 + \frac{\partial^2 \Delta Q}{\partial y^2}\frac{\partial D}{\partial \Delta Q} + \frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial \Delta Q}\left(\frac{\partial \Delta Q}{\partial y}\right)^2\right] dy = 0 \quad (6)$$

Notice that the term associated with dy is in fact the second order condition for removal with adaptation, $SOC_{y\alpha}$. In order for cost minimization to occur, this second order condition must be positive, which will require additional assumptions.[10] Simplifying this second order condition:

$$SOC_y^* + \left[\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial \Delta Q}\left(\frac{\partial \Delta Q}{\partial y}\right)^2\right]$$

Where SOC_y^* possesses the same form as SOC_y in the static model without adaptation, however SOC_y^* includes adaptation, and is not equivalent SOC_y but is used for notational purposes. The term SOC_y^*, like its counterpart SOC_y, remains positive, despite the damage functions being different. The portion,

$$\left[\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial \Delta Q}\left(\frac{\partial \Delta Q}{\partial y}\right)^2\right]$$

[10]This is not the same second order condition as in the static model without adaptation, as damages are now a function of ΔQ *and* α.

is negative, as the term $\frac{\partial \alpha}{\partial \Delta Q}$ was shown to be positive, and the other two partial derivatives are negative. This summation of a positive term and a negative term leaves the sign of $SOC_{y\alpha}$ ambiguous. However, if we assume the absolute value of this negative portion is never larger than SOC_y^*, the expression $SOC_{y\alpha}$ will be positive. This can be accomplished if we assume the derivative, $\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}$ is sufficiently small. This assumption can be justified if adaptation is very effective at decreasing damages. For example, we if differentiate the damage function $D(\Delta Q, \alpha) = \frac{\Delta Q^2}{\alpha^{10}}$:

$$\frac{\partial^2 D}{\partial \Delta Q \partial \alpha} = -\frac{20 \Delta Q}{\alpha^{11}},$$

where even with an increase in damages caused by a higher ΔQ, a level of adaptation greater than 1 will offset this increase in damages greatly. If the costs of removal and adaptation are the same, a social planner would have incentive to invest in more adaptation, which would result in a sufficiently small derivative and thus, making $SOC_{y\alpha}$ positive. This assumption will follow throughout the rest of the analysis.

In order to complete the analysis of the effect that a higher k value has on removal and adaptation, the comparative static is derived:

$$\frac{dy}{dk} = -\frac{\left[\frac{\partial^2 D}{\partial \Delta Q \partial \alpha} \frac{\partial \alpha}{\partial k} \frac{\partial \Delta Q}{\partial y}\right]}{SOC_{y\alpha}} > 0$$

The components of the numerator are all negative due to prior assumptions and the denominator is assumed to be positive as was discussed above. Given these assumptions, the removal level will increase with the larger value of k. This can be seen in the right diagram of Figure 7, where removal increases from y_1^* to y_2^* as a result of the marginal benefit of removal curve shifting outwards. Additionally, an increasing removal level will decrease the ecosystem change from ΔQ_1 to ΔQ_2. Recall that $\frac{d\alpha}{d\Delta Q} > 0$ and thus the optimal adaptation level will also decrease. This overall decrease in α^* can be seen in the left diagram of Figure 7, where the marginal benefit of adaptation curve has shifted down as a result of the lower ΔQ, and now intersects the lower marginal cost curve, $a'(k_2)$. Intuitively, an increase in adaptive costs, while holding all other parameters and costs constant, will result in a shifting away from expensive adaptive measures, and a relatively cheaper method, such as removal, is seen as a more attractive option for minimizing total costs.

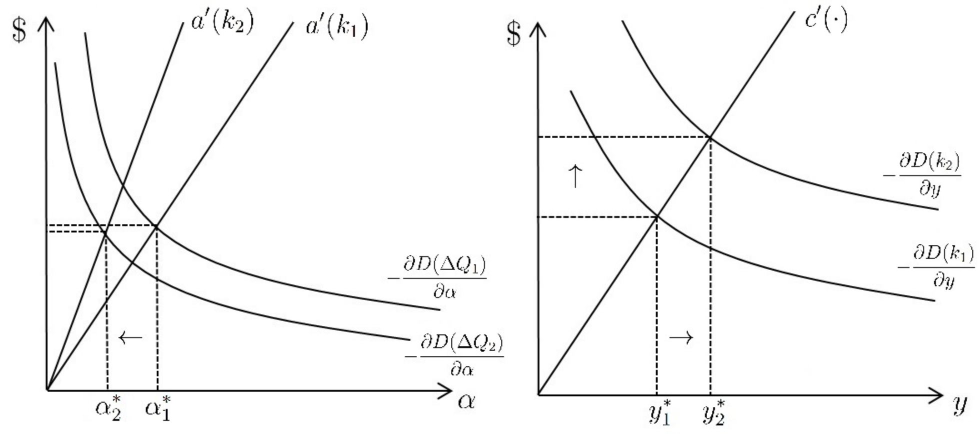

Figure 7: Varying the exogenous parameter k

Similar analysis can be done for the removal cost parameter s, where $\frac{\partial c}{\partial s} > 0$. In the first stage of the problem, adaptation remains unchanged from its optimal value, $\alpha_1^*(\Delta Q)$. This value is then substituted into the total cost function, in order to find the optimal removal value. However, the marginal removal cost function has been shifted up $\big(c'(s_2) > c'(s_1)\big)$, and thus the optimal removal level is lower than its original value. This can be seen in the right diagram of Figure 8, with the lower removal level, $y_2^* < y_1^*$. This lower removal level will increase the ecosystem change from ΔQ_1 to ΔQ_2, resulting in an increased adaption level, α_2^* (using the same analysis as in Figure 6). As was the case with the increased adaptation cost, the social planner will move away from the method that has become relatively more costly, which in this scenario is removal.

Formally,

$$\frac{dy}{ds} = -\frac{\left[\frac{\partial^2 c}{\partial y \partial s}\right]}{SOC_{y\alpha}} < 0$$

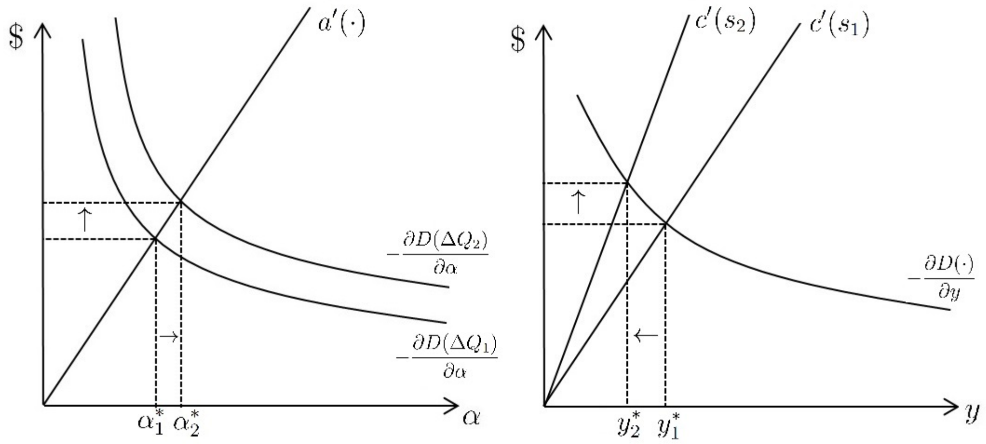

Figure 8: Varying the exogenous parameter s

In order to observe the overall result of the effect of an increase in v on adaptation, we must examine the effects of v on the removal level:

$$\frac{dy}{dv} = -\frac{\left[\left(\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial v} + \frac{\partial^2 D}{\partial \Delta Q \partial v}\right)\frac{\partial \Delta Q}{\partial y}\right]}{SOC_{y\alpha}} \lesseqgtr 0$$

The parameter v is assumed to increase with the damage cost function and thus increases the marginal benefit of removal curve ($\frac{\partial^2 D}{\partial \Delta Q \partial v} > 0$), making the second portion of the term in square brackets of the numerator negative. However, the first portion of this numerator,

$$\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial v}\frac{\partial \Delta Q}{\partial y}$$

is positive.[11] The opposing nature of v on the numerator is caused by the substitution of a higher $\alpha^*(\Delta Q)$ into $D(\Delta Q, \alpha^*(\Delta Q))$, which diminishes the marginal benefit of removal.[12] Thus, the social planner must examine these opposing effects of the parameter v on the numerator of $\frac{dy}{dv}$ in order to conclude any results. If the numerator is positive, $\frac{dy}{dv}$ is negative and the optimal removal level will decrease from its original value. Implying that the effect of $\alpha^*(\Delta Q)$ decreasing the marginal benefit of removal is stronger than the increase in the marginal benefit received

[11] As $\frac{\partial \alpha}{\partial v}$ was shown to be greater than zero.

[12] Recall that the higher v increased $\alpha^*(\Delta Q)$.

from the higher v. This lower optimal removal level would increase ΔQ and in turn, α^* would increase further, as v has already shifted the marginal benefit of adaptation outwards in the first stage (which lead to $\alpha_2^*(\Delta Q) > \alpha_1^*(\Delta Q)$). This can be seen in Figure 9 below:

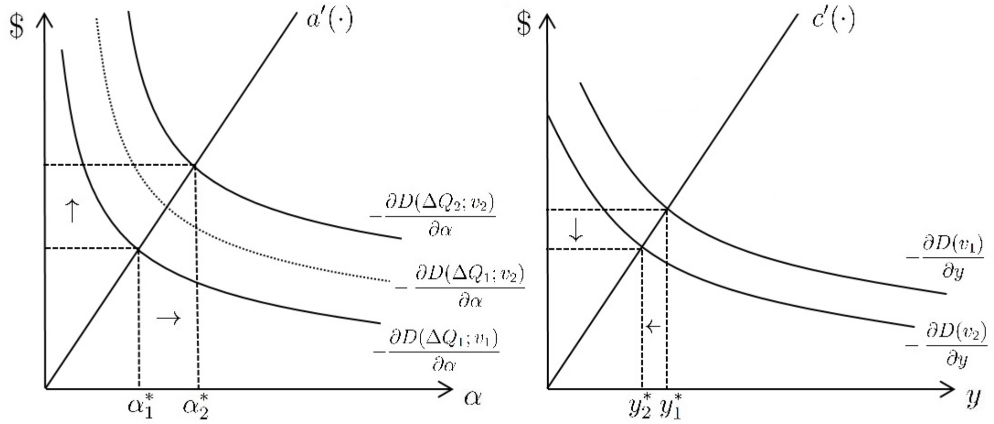

Figure 9: Varying the exogenous parameter v: Decreasing removal level

Alternatively, if the numerator is negative, a higher v will increase the optimal removal level. This increased y^*, will decrease ΔQ, and in turn, lower the optimal adaptation level. However, it is unclear if the overall effect on adaptation is positive or negative. This overall effect on adaptation depends on what has the stronger effect on the marginal benefit of adaptation curve. If the initial increase in the marginal benefit curve caused by the higher v is large enough to offset the decrease in the curve that occurs when ΔQ decreases, then the overall effect on adaptation will be positive, i.e. $\alpha_2^* > \alpha_1^*$. On the other hand, if the decrease in ΔQ is so great, it can offset the initial increase of the marginal benefit from the higher v and lower adaptation i.e. $\alpha_1^* > \alpha_2^*$.[13] The case of $\frac{dy}{dv} > 0$ with an overall positive effect on adaptation is presented in Figure 10.

An intriguing exogenous parameter involves seeing the effect of a change in the starting population on the removal level. The starting population statistic is interesting to policymakers as it acts as an indicator for how quickly populations

[13] A similar case will be examined graphically in Figure 11 for the comparative analysis of the starting population, $\frac{dy}{d\bar{n}}$, as a similar ambiguity in regards to the overall effect on adaptation arises.

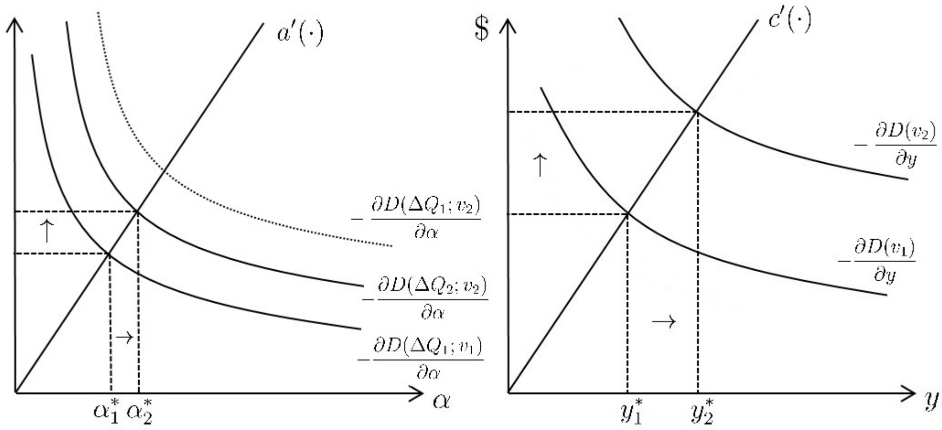

Figure 10: Varying the exogenous parameter v: Increasing removal level

will grow. In the static context, a starting population is simply associated with higher damage, however, it is interesting to policymakers to see how removal should change when the starting population is increased.

$$\frac{dy}{d\bar{n}} = -\frac{\left[\left(\frac{\partial^2 D}{\partial \Delta Q^2} + \frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial \Delta Q}\right)\frac{\partial \Delta Q}{\partial \bar{n}}\frac{\partial \Delta Q}{\partial y} + \frac{\partial^2 \Delta Q}{\partial y \partial \bar{n}}\frac{\partial D}{\partial \Delta Q}\right]}{SOC_{y\alpha}} \lesseqgtr 0$$

The only derivative in the numerator whose sign has yet to be defined is $\frac{\partial^2 \Delta Q}{\partial y \partial \bar{n}}$. Initially, the increase in removal, improves the quality of the ecosystem with a smaller ΔQ. However, once the starting population increases, it will have the opposite effect on the ecosystem change. These two opposing effects on ΔQ reveal the derivative, $\frac{\partial^2 \Delta Q}{\partial y \partial \bar{n}}$, to be negative. With this assumption in mind, the numerator can be split into two parts:

$$A = \left[\frac{\partial^2 D}{\partial \Delta Q^2}\frac{\partial \Delta Q}{\partial \bar{n}}\frac{\partial \Delta Q}{\partial y} + \frac{\partial^2 \Delta Q}{\partial y \partial \bar{n}}\frac{\partial D}{\partial \Delta Q}\right] < 0$$

and

$$B = \left[\frac{\partial^2 D}{\partial \Delta Q \partial \alpha}\frac{\partial \alpha}{\partial \Delta Q}\frac{\partial \Delta Q}{\partial \bar{n}}\frac{\partial \Delta Q}{\partial y}\right] > 0.$$

This makes the sign of the numerator unclear, as was the case with an increase in v. Recall that the increase in the starting population will increase ΔQ_1 to ΔQ_2, increasing the optimal adaptation level in the first stage (due to $\frac{d\alpha}{d\Delta Q} > 0$).

This increase in $\alpha^*(\Delta Q)$ will have the effect of decreasing the marginal benefit of removal curve. If the numerator of $\frac{dy}{d\bar{n}}$ is positive (i.e. $|B| > |A|$), the removal level will decrease with the starting population, as the marginal benefit of removal will shift down with the higher level of $\bar{n}$. This diminished removal level will increase ΔQ, and in turn, increase α^* further.[14] This is due to adaptation being seen as a more attractive option for the social planner to decrease damages caused by a higher $\bar{n}$.

The opposite case, ($|A| > |B|$), gives a negative numerator which makes $\frac{dy}{d\bar{n}}$ positive. In particular, the social planner will invest in more removal as a result of the higher starting population. The increase in removal is attributed to the shifting outwards of the marginal benefit of removal curve caused by the effect of a higher $\bar{n}$ being greater than the decrease in the marginal benefit caused by the higher level of adaptation ($\alpha^*(\Delta Q_2) > \alpha^*(\Delta Q_1)$). An increased emphasis on removal, decreases ΔQ, which will diminish the optimal adaptation level. However, as was the case with an increase in v, the overall effect on adaptation is unclear. The case of $\frac{dy}{d\bar{n}} > 0$ is presented in Figure 11, where the overall effect on adaptation is negative, i.e. $\alpha_1^* > \alpha_2^*$.[15]

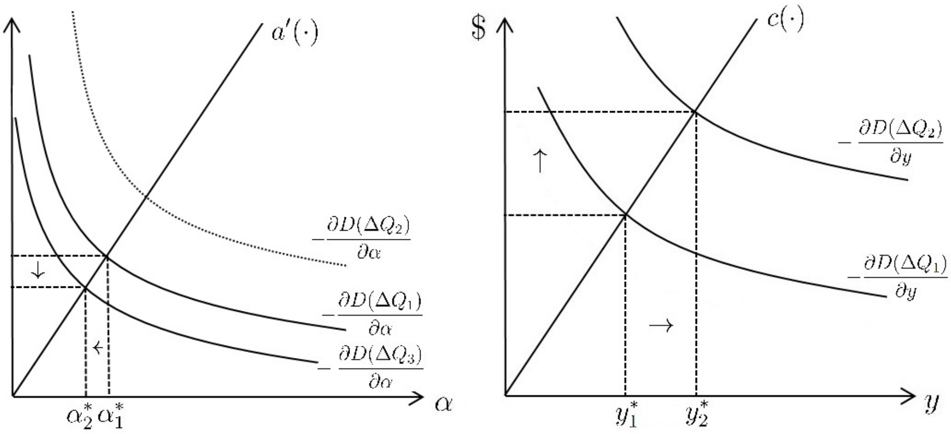

Figure 11: Varying the exogenous parameter $\bar{n}$

[14]Same analysis as in Figure 9.

[15]The case where the overall adaptation level increases despite removal increasing is very similar to Figure 10.

4.5 Asian Carp Simulation Experiment

In order to observe the impacts of exogenous parameters on removal and adaptation levels, such as s, k, v and $\bar{n}$, a simulation experiment is undertaken. This simulation will be in the context of the Asian Carp invasion that is currently on the verge of invading the Great Lakes. With extremely high damages looming, this simulation will serve as example of how to optimally manage the invasion through removal and adaptive measures.

The simulation assumes, as in the model, that the invasion has already taken place, and the species is already present in the Great Lakes, though damages have yet to take form. Costs of removal are given by $c(y; s) = sy^2$, where s is again a slope parameter that is greater than zero. Similarly, the cost of adaptation is given by $a(\alpha; k) = k\alpha^2$, where k is also a slope parameter greater than zero. The functional form of the ecosystem change is given by:[16]

$$\Delta Q = \sqrt{\bar{n}}(10 - \sqrt{y}),$$

and affects damages positively:

$$D(\Delta Q, \alpha; v) = \frac{v \Delta Q^{3/2}}{\alpha},$$

where v is a positive slope parameter for the damage function. The total cost function is given by:

$$J = sy^2 + k\alpha^2 + \frac{v\Delta Q^{3/2}}{\alpha}$$

In the first stage of the problem, the social planner minimizes the total cost function:

$$\min_{\alpha} J = 2k\alpha - \frac{v\Delta Q^{3/2}}{\alpha^2} = 0$$

[16]In order for ΔQ to be non-negative, the removal level must be less than 100. Otherwise, removal will be higher than the population, and will be a waste of funds, as one cannot remove more than the population.

which gives the optimal level of adaptation, for a given ecosystem change level ΔQ:

$$\alpha^*(\Delta Q; v, k) = \left[\frac{v}{2k}\right]^{1/3} \sqrt{\Delta Q}$$

In the second stage of the problem, the social planner chooses the optimal removal level, given the adaptation level solved in the first stage:

$$J(\alpha^*) = sy^2 + k(\alpha^*)^2 + \frac{v\Delta Q^{3/2}}{\alpha^*}$$

$$J(\alpha^*) = sy^2 + k\left[\frac{v}{2k}\right]^{2/3} \Delta Q + \frac{v\Delta Q^{3/2}(2k)^{1/3}}{v^{1/3}\sqrt{\Delta Q}}$$

After simplification:

$$J(\alpha^*) = sy^2 + k^{1/3}v^{2/3}\Delta Q\left[2^{-2/3} + 2^{1/3}\right]$$

$$J(\alpha^*) = sy^2 + 1.89k^{1/3}v^{2/3}\sqrt{\bar{n}}(10 - \sqrt{y})$$

and minimization of the adapted total cost function:

$$\min_y J(\alpha^*) = 2sy - \frac{1.89k^{1/3}v^{2/3}\sqrt{\bar{n}}}{2\sqrt{y}} = 0$$

gives:

$$y^*(k, v, \bar{n}, s) = \left[\frac{0.47k^{1/3}v^{2/3}\sqrt{\bar{n}}}{s}\right]^{2/3}$$

And subbing this optimal value of removal, back into $\alpha^*(\Delta Q; v, k)$ reveals:[17]

$$\alpha^*(k, v, \bar{n}, s) = \left[\frac{v}{2k}\right]^{1/3} \bar{n}^{1/4}\sqrt{(10 - \sqrt{y^*})}$$

The effects of the parameters $k, s, v, \bar{n}$ (all of which are greater than zero) on the optimal levels of removal and adaptation can be found from comparative statics. We will begin with the effects of these parameters on the removal level

[17]Notice the optimal values of removal and adaptation are strictly functions of exogenous parameters.

(where the exponent has been carried through):

$$y^* = 0.61k^{2/9}v^{4/9}\bar{n}^{1/3}s^{-2/3}$$

The effect of an increase in k:

$$\frac{dy}{dk} = 0.14k^{-7/9}v^{4/9}\bar{n}^{1/3}s^{-2/3} > 0$$

The effect of an increase in s:

$$\frac{dy}{ds} = -0.41k^{2/9}v^{4/9}\bar{n}^{1/3}s^{-5/3} < 0$$

The effect of an increase in v:

$$\frac{dy}{dv} = 0.27k^{2/9}v^{-5/9}\bar{n}^{1/3}s^{-2/3} > 0$$

The effect of an increase in $\bar{n}$:

$$\frac{dy}{d\bar{n}} = 0.20k^{2/9}v^{4/9}\bar{n}^{-2/3}s^{-2/3} > 0$$

Notice that there is no ambiguity as to the effect of any of the parameters on removal. From the model, there was ambiguity in the derivatives: $\frac{dy}{dv}$ and $\frac{dy}{d\bar{n}}$, however this is not the case in this simulation.

Comparative statics are also done for adaptation:

$$\alpha^* = 0.79k^{-1/3}v^{1/3}\bar{n}^{1/4}\left(10 - 0.78k^{1/9}v^{2/9}\bar{n}^{1/6}s^{-1/3}\right)^{1/2}$$

The effect of an increase in k:

$$\frac{d\alpha}{dk} = -0.26k^{-4/3}v^{1/3}\bar{n}^{1/4}\left(10 - \sqrt{y^*}\right)^{1/2} - 0.03k^{-11/9}v^{5/9}\bar{n}^{5/12}s^{-1/3}\left(10 - \sqrt{y^*}\right)^{-1/2} < 0$$

The effect of an increase in s:

$$\frac{d\alpha}{ds} = 0.10k^{-2/9}v^{5/9}\bar{n}^{5/12}s^{-4/3}\left(10-\sqrt{y^*}\right)^{-1/2} > 0$$

The effect of an increase in v:

$$\frac{d\alpha}{dv} = 0.26k^{-1/3}v^{-2/3}\bar{n}^{1/4}\left(10-\sqrt{y^*}\right)^{1/2} - 0.07k^{-2/9}v^{-4/9}\bar{n}^{5/12}s^{-1/3}\left(10-\sqrt{y^*}\right)^{-1/2} \lesseqgtr 0$$

The effect of an increase in $\bar{n}$:

$$\frac{d\alpha}{d\bar{n}} = 0.20k^{-1/3}v^{1/3}\bar{n}^{-3/4}\left(10-\sqrt{y^*}\right)^{1/2} - 0.05k^{-2/9}v^{5/9}\bar{n}^{-7/12}s^{-1/3}\left(10-\sqrt{y^*}\right)^{-1/2} \lesseqgtr 0$$

Notice the ambiguity of the effect on adaptation with regards to the derivatives $\frac{d\alpha}{dv}$ and $\frac{d\alpha}{d\bar{n}}$. Recall in the first stage of the problem, both v and $\bar{n}$ increase $\alpha^*(\Delta Q)$. However, as was shown above, both v and $\bar{n}$ positively affect the removal level. An increase in the removal level, will decrease ΔQ, which is then substituted back into $\alpha^*(\Delta Q)$ to obtain an optimal level of adaptation that is a function of only exogenous parameters. The effect of subbing in a lower value of ΔQ into $\alpha^*(\Delta Q)$ is a lower optimal level of adaptation, α^*. Thus, the social planner must examine the opposing effects on adaptation, in order to see the overall effect of an increase in v or $\bar{n}$. We can derive rules for when these comparative statics are positive and when they are negative. The parameter v will increase adaptation if:

$$0.26k^{-1/3}v^{-2/3}\bar{n}^{1/4}\left(10-\sqrt{y^*}\right)^{1/2} > 0.07k^{-2/9}v^{-4/9}\bar{n}^{5/12}s^{-1/3}\left(10-\sqrt{y^*}\right)^{-1/2}$$

$$3.71k^{-1/9}v^{-2/9}\bar{n}^{-1/6}s^{1/3}(10-\sqrt{y}) > 1$$

Note that:

$$\sqrt{y} = 0.78k^{1/9}v^{2/9}\bar{n}^{1/6}s^{-1/3}$$

If we multiply both sides by 0.78:

$$2.89(10 - \sqrt{y}) > \sqrt{y}$$

Solving for the removal level reveals that $\frac{d\alpha}{dv}$ will be positive whenever y is less than 55.00.

Similarly, a condition for whenever $\frac{d\alpha}{d\bar{n}}$ is positive is found using the exact same method as above. This method finds that $\frac{d\alpha}{d\bar{n}}$ is positive whenever y is less than 57.40.

An actual simulation of the model is of interest, as it will serve to show the budget allocation between removal and adaptation. This information is of high importance to a social planner, in order to better allocate funding and research into the method that is more prominent. Each parameter will varied one at a time in order to observe their effects on removal and adaptation. The status quo will be referred to as the Base Case, and can be seen in Table 1.[18] This will be the reference for all following comparisons.[19]

Table 1: Base Case displaying balance between adaptation and removal ($k = 1, s = 1, v = 1$ and $\bar{n} = 1000$)

Expenditure	Level	Spending($)	% of Total Spending	Δ Base Case
Removal	6.05	36.54	19.57	0
Adaptation	12.26	150.23	80.43	0

Notice that the budget allocation highly emphasizes adaptation as a means for minimizing total costs. Due to adaptation and removal having the same costs functions, adaptation is seen as a much more attractive option for decreasing damages than removal.

[18] Any combination of these parameters must ensure that the removal is less than 100, otherwise, funds will be wasted as the social planner will be removing more than the damage caused by the population. Removal is also bounded by 0 to ensure a non-negative level.

[19] Units are omitted in order for a more general model that emphasizes the relative levels of adaptation and removal.

Increasing adaptation costs possess no ambiguity with respect to its effect on removal and adaptation, as was shown in the model and in the comparative statics part of this simulation. As a result, the social planner moves to a cheaper method for decreasing damages caused by the Asian Carp. This can be observed in the increased emphasis on removal in the budget allocation of Table 2, where removal increases almost 15%. Despite this, investments are still higher for adaptation. However, if the cost of adaption increases greatly, the level of adaptation approaches zero and policymakers focus their attention on the relatively cheaper method, removal.

Table 2: Balancing between adaptation and removal with increased adaptive costs ($k = 100, s = 1, v = 1$ and $\bar{n} = 1000$)

Expenditure	Level	Spending($)	% of Total Spending	Δ Base Case
Removal	16.82	282.94	34.16	+14.59
Adaptation	2.34	545.43	65.84	−14.59

Increasing removal costs act in a very similar way to increasing adaptation costs. The budget allocation, as can be seen in Table 3, shows an increase in adaptation of over 15%. The social planner focuses more on adaptation, and shifts spending from the expensive removal methods of catching Asian Carp, to relatively cheaper adaptive methods such as protective gear for boats to decrease damages.

Table 3: Balancing between adaptation and removal with increased removal costs ($k = 1, s = 100, v = 1$ and $\bar{n} = 1000$)

Expenditure	Level	Spending($)	% of Total Spending	Δ Base Case
Removal	0.28	7.87	4.01	−15.56
Adaptation	13.74	188.66	95.99	+15.56

The effect of an increase in v on removal is unambiguous in this simulation, and can be observed in Table 4. Thus, with every increase in v, removal increases. As well, adaptation will also increase with v, but to a point. This is due to the opposing effects that determine $\frac{d\alpha}{dv}$. While v is increasing, the removal level is increasing, which gives more weight to the negative portion of $\frac{d\alpha}{dv}$. As long as y is less than 55.00, $\frac{d\alpha}{dv}$ will be positive. However, once this level of removal is reached, an increase in v will cause a decrease in adaptation, $\frac{\partial \alpha}{\partial v} < 0$. This result is also the cause for the shift in budget allocation. The removal level increases with v at a steady rate, and adaptation also increases but at a slowing rate, and this rate eventually becomes negative once the removal level of 55.00 is reached. This explains why, despite the adaptation level increasing from the Base Case (12.26 to 36.82), the allocation of removal and adaptation has changed considerably. The 'switchover' level of v in this particular simulation is 140.87, upon which, any further increase in v will have an overall negative effect on adaptation.[20]

Table 4: Balancing between adaptation and removal with increased damage costs from v ($k = 1, s = 1, v = 100$ and $\bar{n} = 1000$)

Expenditure	Level	Spending($)	% of Total Spending	Δ Base Case
Removal	46.80	2190.66	61.77	+42.20
Adaptation	36.82	1355.64	38.23	−42.20

Using the same analysis as the case with v, an increase in the starting population will unambiguously increase the removal level. As well, the adaptation level will increase with $\bar{n}$ to a point, upon which any further increase in $\bar{n}$ will decrease adaptation. The switchover level of $\bar{n}$ that makes y greater than 57.40 is a population of 833,194. Again, notice that the overall adaptation level has increased from the Base Case, however, the budget allocation has become more focused on removal.

Table 5: Balancing between adaptation and removal with increased starting population ($k = 1, s = 1, v = 1$ and $\bar{n} = 100000$)

Expenditure	Level	Spending($)	% of Total Spending	Δ Base Case
Removal	28.06	787.28	45.66	+26.09
Adaptation	30.61	936.88	54.34	−26.09

A summary of the total spending on removal and adaptation, the damage costs and the total cost of the invasion are presented below in Table 6:

[20] This switchover level is found by solving for the level of v that gives a removal level 55, with the parameters given in Table 4

Table 6: Total costs associated with invasion

Parameter	Total Spending($)	Damage Cost($)	Total Cost($)
Base case	186.77	300.46	487.24
Increasing k	828.37	1090.85	1919.21
Increasing s	196.53	377.32	573.85
Increasing v	3546.30	2711.28	6257.58
Increasing $\bar{n}$	1724.16	1873.77	3597.93

5 Conclusion

The intent of this paper is to emphasize the importance in developing an optimal strategy to endure an imminent invasion, such as the Asian Carp. The model presented is not limited to the Asian Carp, rather, is designed to effectively model the optimal coping mechanism for the invasion of any species. The environmental damages created by invasive species are reflected in the differing geographic regions, species and introduction levels. Developing a general model is ideal because it can be applied to all situations. Implementing this model as regulatory practice in various levels of government may be useful to social planners because of the models flexibility. The proposed model does not impose strict removal or prevention, and avoids complicated assumptions regarding the location and timing of a species intrusion. Government policy must include removal and adaptive methods and adopt a balanced approach in formulating a strategic model to mitigate the effects of invasive species. This paper has outlined the benefit of balancing removal and adaptive methods, namely, an increased level of power for a social planner, where the removal will act to lower the population to a more manageable level, while adaptation will serve to alleviate damages to the ecosystem caused by elevated population levels. Without adaptation integrated into the model, the task of removal becomes daunting and may not be feasible to a government with limited funds. An integrative model balancing removal and adaptation is necessary for managing an invasion; a model that includes only a single element will not suffice to minimize costs. The future health of an ecosystem has great repercussions for the environment, other native species, the economy and its citizens.

Thus, an optimal management strategy of an invasive species is essential to the sustainability of the planet and the economy.

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