# Punishing Prejudice or Increasing Intolerance: The Effect of International Trade on Gender Wage Gaps

by

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**Abstract**

In this paper, I develop a monopolistically competitive trade model that combines the "taste for discrimination" model of Becker (1957) and the trade model developed in Krugman (1980) to explain the conflicting empirical results concerning trade's impact on gender wage gaps found in Black and Brainerd (2004), Menon and Meulen Rodgers (2009), and Ben Yahmed (2011). I demonstrate that, under certain conditions, the gender wage gap will rise when there is an increase in trade due to a fall in fixed export costs, while the gender wage gap will fall when there is an increase in trade due to a fall in iceberg trade costs. I then explore the various competitive effects that lead to these results, and show that small trade costs are sufficient, but not necessary, for there to be aggregate welfare gains when consumers move from autarky to trade.

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# Contents

# 1 Introduction

In this paper I develop a monopolitiscally competitive model that combines the discrimination model of Becker (1957) and the trade model of Krugman (1980) to examine the effect of trade on gender wage gaps. I show that movements from autarky to trade always decrease the gender wage gap, while increased trade due to a fall in trade costs will have an ambiguous effect on the gender wage gap. Specifically, a fall in per-unit iceberg trade costs will either decrease or have no effect on the gender wage gap, while a fall in fixed export costs will increase the gender wage gap. It is then shown that women are always better off in trade compared to autarky, while men are better off in trade only when trade costs are sufficiently small. This means that small trade costs are sufficient, but not necessary, for there to be aggregate welfare gains from trade.

The ambiguous effect of increased trade on gender wage gaps is an important result, since recent empirical work on the effect of trade on gender wage gaps, such as Black and Brainerd (2004), Menon and Meulen Rodgers (2009), and Ben Yahmed (2011), has found that increased trade can be associated with both an increase or a decrease in the gender wage gap. Since the model developed in this paper shows that the direction of the gender wage gap change depends on whether per-unit costs or fixed costs fall when trade increases, it is my hope that some of the insights gleamed from this model can be used to guide future empirical research into this topic and decrease some of the ambiguity in the current empirical literature.

Much of the recent empirical literature on trade and gender wage gaps is based on the "taste for discrimination" model of wage gaps developed in Becker (1957). While Becker's monograph primarily deals with racial discrimination in the United States, the underlying mechanism that leads to the racial wage gap in Becker's model can apply to a gender wage gap just as easily as a black-white wage gap. In the basic model, there are two groups of equally productive people that firms can choose to hire; say, men and women. If firms have a taste for discrimination against women, which means that the owners of the firms have discriminatory preferences, then the owners of these firms bear a disutility cost whenever they hire a woman. As a result, women are perceived to be "more costly" than men, which means that if any women are to be hired by discriminatory firms, the female wage will have to lie below the male wage, so that the perceived cost of women, net of their disutility cost, equals the direct monetary costs of hiring male workers.

An interesting implication of Becker's model is that if there are also firms that do not have a taste for discrimination that directly compete with discriminatory firms in a perfectly competitive environment, then wage gaps will have to disappear in the long run.[1] The reason for this is that non-discriminatory firms will produce more efficiently than discriminatory firms, since they will be willing to hire under-priced female labour, which is just as productive as male labour. Thus, non-discriminatory firms will earn higher profits than discriminatory firms[2] and therefore be able to buy out discriminatory firms in the long-run, which will lead to the disappearance of a gender wage gap.

The above argument, however, does not apply to imperfectly competitive markets. As a result, one would still expect a gender wage gap to exist in the long-run, whenever firms have some degree of market power, since discriminatory firms could use this market power to indulge their taste for discrimination. Building on this idea, Bhagwati (2004) and Black and Brainerd (2004) argue that if the gender wage gap is due to the existence of firms with a taste for discrimination, then increased trade can work to decrease the gender wage gap by increasing the degree of market competition within a country. Black and Brainerd (2004) empirically test this hypothesis using U.S. census data, finding that increased imports within the United States were associated with decreases in the gender wage gap for more concentrated industries, i.e. industries where firms had more market power.

However, subsequent empirical work by Menon and Meulen Rodgers (2009) and Ben Yahmed (2011) using data from Indian and Uruguayan household surveys, respectively, has shown that gender wage gaps tend to increase as trade shares increase. As a result, the simple story told by Bhagwaiti (2004) and Black and Brainerd (2004) does not appear to hold generally. This is perhaps to be expected, since these authors predictions were based on intuitive extensions to Becker's model, rather than explicit models of trade and gender wage gaps. This suggests that a more concrete model of discrimination and trade may be needed to fully evaluate the effect that trade can have on the gender wage gap.

With this in mind, Menon and Meulen Rodgers (2009) attempt to explain their results by incorporating a modified version of Becker's discrimination model into a two-sector trade model developed by Borjas and Ramey (1995), where wages are determined by Nash bargaining between firms and workers. In the model, one sector is competitive, while the other is imperfectly competitive. Increased trade is then modelled as an exogenous increase in the number of foreign firms that compete in both markets. Since increased trade results in increased foreign competition, trade then reduces the surplus available to the imperfectly competitive sector, which drives down the relative wages of their workers. Menon and Meulen Rodgers (2009) modify this model by assuming that the firms in the imperfectly competitive

[1] For a formal demonstration of this, see Goldberg (1982).

[2] Siegel et al. (2011) find evidence consistent with this prediction by studying South Korean firm level data, where they find that multinational firms that hire more female labour than domestic firms tend to earn higher profits.

industry have a taste for discrimination against women. As a result, women earn an even smaller portion of the rents earned by the imperfectly competitive industry. The authors then argue that when these rents fall due to increased trade, the female wage should fall more in relative terms than the male wage, since discriminatory firms prefer to cut female wages before they cut male wages. Unfortunately, this is not an actual result of their model, but rather, a story told to justify their assumption that each firm's degree of discrimination increases with trade. As a result, their paper primarily offers an intuitive story for why females wages should fall with trade, rather than a complete model.

Ben Yahmed (2011), on the other hand, explains her empirical findings by developing a partial equilibrium Cournot competition model that incorporates Becker's taste for discrimination. She is then able to show that trade has an ambiguous effect on gender wage gaps, which is consistent with her empirical results. This ambiguity occurs in her model because trade does not only increase the degree of foreign competition faced by domestic firms, but also provides domestic firms with access to new consumers. This allows discriminatory firms to access a larger base of consumers, which means that these firms can potentially gain more market power, thereby making it easier for firms to discriminate against women, and consequently leading to an increase in the gender wage gap. These are important results, since the model is consistent with empirical evidence and increases our understanding of why trade has an ambiguous effect on gender wage gaps. However, since the model is based on Cournot competition, the number of discriminatory firms is exogenous, which means that one important channel through which trade can affect the gender wage gap, i.e. discriminatory firm exit due to increased competition, cannot be examined.[3]

Therefore, to complement this approach, I develop a trade model based on the monopolistic competition framework of Dixit and Stiglitz (1977), which is similar to the trade models developed in Krugman (1979), Krugman (1980) and Melitz (2003). In these models, each firm is the only producer of a single variety of some product, which allows each firm to set prices "almost" like a monopolist. However, every variety is assumed to be easily substitutable for one another, which means that firms still take the pricing behaviour of their rivals into account. Consumers are then assumed to have a "love of variety," which means that each consumer tends to prefer an increase in the number of varieties available to purchase, over an increase in the quantity consumed of a given variety. Gains from trade arise

[3]On the other hand, Ben Yahmed (2011) does consider *selection effects*, i.e. different cost thresholds necessary for different types of firms to produce positive levels of output. Since discriminatory firms and non-discriminatory firms have different cost thresholds, it is then shown that for a certain subset of possible parameter values, discriminatory firms never produce output. While this is similar to an exit effect, note that discriminatory firms do not actually "exit" the market; rather, these are simply cases where discriminatory firms never chose to produce output. If discriminatory firms are able to enter the market and produce positive output, their numbers are entirely exogenous, and thus exit effects are ruled out by construction.

in these models through an increase in the number of varieties available to each consumer, as each consumer gains access to the varieties produced abroad in addition to the varieties produced at home. One important feature of these models is that the number of varieties is endogenous, which means that the number of firms is also endogenous, and therefore firm entry and exit can be modelled formally.

As I will show in this paper, using this monopolistic competition framework to build a trade model that incorporates firms with a taste for discrimination can lead to straightforward and simple results that shed light on the empirically observed effect of trade on gender wage gaps. Most importantly, this model will be able to show under what conditions increased trade decreases the gender wage gap, and vice versa. This is because trade costs are the key mechanism that change the gender wage gap in trade relative to autarky. As a result, different types of trade costs are shown affect the gender wage gap in different ways.

For example, fixed export costs increase the degree of competition between discriminatory firms for male labour. This increased competition can force some discriminatory firms to exit the market. This means that increasing the fixed costs of trade can decrease the degree of effective discrimination in the market, and therefore lead to a decrease in the gender wage gap. On the other hand, since decreasing the fixed costs of trade will increase the amount of trade in equilibrium, this means that increased trade will be associated with an *increase* in the gender wage gap. However, if the fixed costs to trade are particularly large, discriminatory firms may never actually enter the export market, which results in a relative increase in the monopoly power of non-discriminatory firms compared to discriminatory firms, since only the non-discriminatory firms gain access to new consumers. This means that a decrease in iceberg trade costs whenever only fair firms export will increase the monopoly power of non-discriminatory firms, relative to discriminatory firms. This allows these non-discriminatory firms, who primarily hire women, to charge higher prices. These larger revenues are then partly passed on to women in the form of higher wages, which decreases the gender wage gap. As a result, increased trade due to a fall in iceberg trade costs will be associated with a *decrease* in the gender wage gap. Altogether, then, increased trade will be associated with either an increase or decrease in the gender wage gap, *depending on which type of trade costs fall.*

While different trade costs have different effects on the gender wage gap, I also show that whenever a country moves from autarky to trade, the equilibrium gender wage gap will never rise. In fact, for a certain subset of possible equilibria, the gender wage gap will always fall relative to autarky, since trade either causes some discriminatory firms to exit the market, or trade results in a relative increase in the monopoly power of fair firms. These results are consistent with those found in Melitz (2003), which considers the effect of trade on industries with heterogeneous production technologies. In his model, moving from autarky to trade increases the average productivity of entire market by increasing the degree of competition for scarce labour, which forces the least productive firms to exit. It turns out that this sort of competition effect is quite similar to the mechanism that drives down the gender wage gap when a country opens its borders to trade. This is because firms that have a taste for discrimination are similar to firms that have a less efficient production technology. As a result, since Melitz (2003) is able to show that trade forces firms that use less efficient technologies to exit the market, in this paper I am able to show that trade also forces firms that have, in essence, "less efficient preferences," to exit the market.

This paper is organized as follows. Section 2 outlines the model environment. In Section 3, I solve the model in autarkic equilibrium and demonstrate the conditions necessary for the emergence of an endogenous gender wage gap. In Section 4, I modify the model to account for trade between two identical countries. I then demonstrate that movement from autarky to trade will decrease the equilibrium gender wage gap under certain conditions, and then show that trade due to a fall in fixed costs will increase the gender wage gap, while a fall in iceberg trade costs decreases the gender wage gap. In Section 5, I consider the welfare effects of trade and demonstrate that trade always makes women better off, while trade increases male welfare only if trade costs are small. Section 6 concludes and considers possible extensions to the model and future empirical work.

# 2 Model Environment

The basic model environment is similar to that described in Krugman (1980);[4] i.e. firms are monopolistically competitive as in Dixit and Stiglitz (1977), and consumers have a "taste for variety" with constant elasticity of substitution between varieties. The main difference between my model and Krugman's model is the introduction of firm heterogeneity and different types of workers (male and female). Firms will differ in their attitudes towards hiring women, which will lead to the endogenous determination of a male-female wage gap.

## 2.1 Demand

Consider an economy with $L_m$ men and $L_f$ women. Each man or woman is endowed with one unit of male or female labour, respectively, which they supply inelastically. While for production purposes male and female labour is identical, since some producers prefer to discriminate (i.e. use male labour), I assume that each consumer's "type" of labour

[4] With the minor modification that there is a continuum of varieties, as opposed to a discrete number.

endowment is public knowledge.

All men and women have identical preferences of the "taste for variety" form, as in Dixit and Stiglitz (1977), Krugman (1980), and Melitz (2003):

$$U_i = \left( \int_0^N q_i(\upsilon)^\rho \, d\upsilon \right)^{\frac{1}{\rho}} \tag{1}$$

where $q_i(\upsilon)$ denotes consumer $i$'s consumption of variety $\upsilon$, $i \in (1, 2, ..., L_m + L_f)$, $N$ is the measure of varieties available to the consumer, and $0 < \rho < 1$. As was shown in Dixit and Stiglitz (1977) the preferences defined by (1) imply constant elasticity of demand for each variety, call this $\epsilon_d$, given by:

$$\epsilon_d = \frac{1}{1-\rho} \tag{2}$$

where $\epsilon_d > 1$ because $\rho < 1$.

Since all men and women have identical preferences, I use (1) to find any consumer's demand for variety $\upsilon$ by solving an arbitrary consumer's utility maximization problem, i.e.:

$$\underset{q_i(\upsilon)|\upsilon \in [0,N]}{\text{Max}} \left( \int_0^N q_i(\upsilon)^\rho \, d\upsilon \right)^{\frac{1}{\rho}} - \lambda \left( \int_0^N p(\upsilon) q_i(\upsilon) \, d\upsilon - I_i \right) \tag{3}$$

where $I_i$ denotes consumer $i$'s income, and $p(\upsilon)$ is the price of variety $\upsilon$. Solving (3) yields consumer $i$'s Marshallian demand for variety $\upsilon$, which I denote by $d_i^\upsilon$:

$$d_i^\upsilon = \frac{I_i}{p(\upsilon)^{\frac{1}{1-\rho}} \int_0^N p(\psi)^{\frac{-\rho}{1-\rho}} \, d\psi} \tag{4}$$

Note that $p(\upsilon)$ is the price charged for a single variety $\upsilon$ in the above expression. So that there is no confusion, I now let $p^\upsilon \equiv p(\psi)|_{\psi=\upsilon}$, so that $p^\upsilon$ denotes the price of a particular variety $\upsilon$, and let $\mathbf{p} \equiv p(\psi)$ where $\psi \in (0, N)$, so that $\mathbf{p}$ denotes the entire set of prices.

I find the aggregate demand for variety $\upsilon$, denoted by $D^\upsilon(\mathbf{p})$, by up summing each individual consumer's demand for variety $\upsilon$. It follows that:

$$\begin{aligned} D^\upsilon(\mathbf{p}) &= \sum_{i=1}^{L_m+L_f} d_i^\upsilon \\ &= \sum_{i=1}^{L_m+L_f} \frac{I_i}{(p^\upsilon)^{\frac{1}{1-\rho}} \int_0^N p(\psi)^{\frac{-\rho}{1-\rho}} \, d\psi} \end{aligned}$$

$$D^{\upsilon}\left(\mathbf{p}\right) = \frac{I}{\left(p^{\upsilon}\right)^{\frac{1}{1-\rho}} \int_0^N p(\psi)^{\frac{-\rho}{1-\rho}}\, d\psi} \tag{5}$$

Note that the expression for aggregate demand for variety $\upsilon$ is similar in structure to an individual consumer's demand for variety $\upsilon$, except individual income has been replaced by aggregate income. The aggregate demand functions have this useful property because all consumers have identical and homothetic preferences, which means that one could have also found the aggregate demand functions by considering the Marshallian demand of a single consumer who received the entire aggregate income of the economy.[5] As a result, it follows that the elasticity of *aggregate* demand, $\epsilon_D$ is also:

$$\epsilon_D = \frac{1}{1-\rho} \tag{6}$$

I now derive an equation that shall be useful later. Using (5), I find the ratio of the aggregate demand for two different varieties, say, variety $\upsilon$ and variety $\psi$, where $\upsilon \neq \psi$:

$$\frac{D^{\upsilon}}{D^{\psi}} = \left(\frac{p^{\psi}}{p^{\upsilon}}\right)^{\frac{1}{1-\rho}} \tag{7}$$

Intuitively, the above shows that each variety is a substitute for one another, since an increase in $p^{\psi}$ will lead to an increase in the ratio of aggregate demand for variety $\upsilon$ over variety $\psi$. This means that each firm will have to account for the pricing behaviour of their rivals, since their rivals pricing will affect the overall demand for their variety.

## 2.2 Production technology and Becker's "taste for discrimination" model

There is an endogenous measure of $N$ firms, each of which produces its own variety of a product as in the standard monopolistic competition model of Dixit and Stiglitz (1977). All firms have access to the same production technology, which uses a single factor of production called labour. Male and female labour is equally productive, in that all units of labour have a constant marginal product equal to unity. To capture the importance of economies of scale, firms must first build a factory by using $\alpha$ units of labour before they may start producing output. Mathematically:

$$l_m^i + l_f^i = y^i + \alpha \tag{8}$$

[5] This is due to the fact that identical and homothetic preferences are sufficient conditions for the existence of aggregate preferences. See Gorman (1953).

where $y^i$ denotes the total output of variety $i$ produced by firm $i$, $l_m^i$ denotes the total amount of male labour used by firm $i$, and $l_f^i$ denotes the total quantity of female labour used by firm $i$. Letting $w_m$ and $w_f$ denote the wage rate per unit of male and female labour, respectively, it should be clear from (8) that the marginal cost of male labour is $w_m$, while the marginal cost of female labour is $w_f$.

I shall often refer to the wage gap in this model, which mathematically I shall define as the ratio of the male wage to the female wage, so that:

$$\omega \equiv \frac{w_m}{w_f} \tag{9}$$

where $\omega$ is the gender wage gap. Note that whenever $\omega > 1$, women are paid less than men. I call this a positive gender wage gap.

While each firm has access to the same production technology, firms differ in the their *preferences* regarding the use of different types of labour. Following Becker (1957), I assume that there are a number *discriminatory* firms, the owners of which would prefer to not hire women, since each woman hired requires that the firm (owner) bear some disutility cost. This "taste for discrimination" is captured through the use of a *discrimination coefficient*, which I shall denote by $\beta$, and which enters a firm's profit function[6] so that the effective marginal cost for female labour is $\beta w_f$ as opposed to $w_f$, where $\beta > 1$. [7]

For simplicity, I shall assume that all discriminatory firms have the same discrimination coefficient. However, discriminatory firms must also compete with "fair" firms, who *do not* have a taste for discrimination. Thus, the main difference between fair firms and discriminatory firms is that fair firms act according to the true marginal cost of women, $w_f$, while discriminatory firms act as if the marginal cost of women is $\beta w_f$.

## 2.3 Labour market clearing and profits

Since the production technology described by (8) is linear is both inputs, it follows that whenever $1 \leq \omega < \beta$, there will be complete *segregation* in the labour market, as fair

[6]This means that discriminatory firms technically maximize *utility*, of which profits are an argument, rather than profits *per se*. Becker does not worry too much about this distinction, but the formal model environment for some taste for discrimination models that build on his work, such as Arrow (1972) and Goldberg (1982), do account for this difference. Similar to Becker, I shall simply speak of discriminatory firms "profits", where it should be understood that these profits are technically *perceived* profits.

[7]In Becker's original formulation, the discrimination coefficient was of the following form: suppose that some factor of production has a marginal cost equal to $w$. Then discriminatory firms act as if their actual marginal cost is $w(1+d)$, or $w + dw$, where $d$ is what Becker called the "discrimination coefficient," and $dw$ acts as "the exact monetary equivalent of the non-monetary costs." (p. 15). Thus, even though a female labourer's "true" marginal cost is simply $w_f$, discriminatory firms act as if their marginal cost is $(1+d)w_f$. For simplicity, I prefer to let $\beta \equiv 1 + d$, and therefore $\beta > 1$

firms will demand only female labour since $w_f \leq w_m$,[8] while discriminatory firms will hire only male labour, since $w_m < \beta w_f$. [9] However, if $\omega = \beta$, then there will be incomplete segregation, since fair firms will prefer to only hire women ( $\omega = \beta$ implies that $w_f < w_m$), while discriminatory firms will be indifferent to hiring either men or women, since $w_m = \beta w_f$. As a result, discriminatory firms will hire both types of labour, while fair firms will continue to only hire women.

As long as the equilibrium wage gap satisfies $1 \leq \omega \leq \beta$, I can write each type of firm's profits as follows. Let $N^d$ denote the measure of discriminatory firms in equilibrium, and let $N^f$ denote the measure of fair firms, where $N^d + N^f = N$. I then let $y^j$ denote the output of discriminatory firm $j$, where $j \in (0, N^d)$ and $y^k$ denote the output of fair firm $k$, where $k \in (N^d, N^d + N^f)$. By (8) and the pattern of segregation just discussed, a fair firm $k$'s demand for female labour is given by $\alpha + y^k$, while discriminatory firm $j$'s demand for male labour (if $1 \leq \omega < \beta$), or male *and* female labour (if $\omega = \beta$), is given by $\alpha + y^j$. Thus, letting $\pi^k$ denote fair firm $k$'s profit, and $\pi^j$ denote discriminatory firm $j$'s profit, I can write:

$$\pi^j = p^j y^j - w_m(\alpha + y^j) \qquad \text{For } j \in (0, N^d) \tag{10}$$

$$\pi^k = p^k y^k - w_f(\alpha + y^k) \qquad \text{For } k \in (N^d, N^d + N^f) \tag{11}$$

The above profit functions are also valid when $\omega = \beta$, since this means that $w_m = \beta w_f$. Since both factors of production have the same perceived cost to a discriminatory firm, $\alpha + y^j$ is discriminatory firm $j$'s demand for both male and female labour, and therefore $w_m(\alpha + y^j)$ is the total perceived payment made to *both* types of labourers. Thus, $\pi^j = p^j y^j - w_m(\alpha + y^j) = p^j y^j - \beta w_f(\alpha + y^j)$ when $\omega = \beta$, and there is no need to write a different profit expression for the case where there is incomplete segregation.

I now formally state a key assumption of the model:

**Assumption 1.** *In equilibrium, either labour markets clear, or, if there is excess supply in some labour market in equilibrium, the price of that labour input is zero.*

[8] Technically, fair firms would be willing to hire both types of labour in this case. However, since discriminatory firms only hire male labour and there is an unbounded pool of these discriminatory firms (See Section 2.4), I assume that discriminatory firms simply absorb the entire male labour force, so that only female workers are left available to work for fair firms.

[9] For this reason, Welch (1975) has argued that the taste based discrimination model of Becker is primarily a model of labour market segregation, rather than a model of gender wage gaps. However, this is not a completely fair criticism, since the segregation result is entirely due to the linearity of the production function, which is itself only a simplifying assumption for tractability purposes. If one were to use a production function with diminishing returns to both labour types (which could perhaps be due to declining quality of workers as one takes subsequent workers of each type from the labour pool) both types of firms would demand both types of labour.

It follows from this assumption that the equilibrium gender wage gap will has to satisfy $1 \leq \omega \leq \beta$. I prove this in a couple of steps:

**Proposition 1.** *Both labour markets will clear only if* $1 \leq \omega \leq \beta$.

*Proof.* Suppose not, so that either $\omega > \beta$ or $\omega < 1$. If $\omega = \frac{w_m}{w_f} < 1$, then all firms will only want to hire men, since this implies that $w_m < w_f$ and $w_m < \beta w_f$. Thus, there will be no demand for female labour, and therefore the female labour market will not clear, and be in excess supply. Likewise, if $\omega = \frac{w_m}{w_f} > \beta$ , then all firms will only want to hire female labour, since this inequality implies that $w_m > \beta w_f$ and $w_m > w_f$. Therefore, there will be no demand for male labour, which means that the male labour market will not clear, and be in excess supply. Since either $\omega > \beta$ or $\omega < 1$ implies that at least one labour market will not clear, the proof is complete. □

**Corollary 1.** *If* $\omega > \beta$, *the male labour market is in excess supply, and if* $\omega < 1$ *the female labour market is in excess supply.*

**Proposition 2.** *The equilibrium gender wage gap must satisfy* $1 \leq \omega \leq \beta$.

*Proof.* Suppose not, so that either $\omega > \beta$ or $\omega < 1$. First, suppose that $\omega > \beta$. By Corollary 1, if $\omega > \beta$, then the male labour market is in excess supply. Thus, by Assumption 1, $w_m = 0$. This means that $\omega = 0 < \beta$, which contradicts the assumption that $\omega > \beta$. Next suppose that $\omega < 1$. From Corollary 1, this means that the female labour market is in excess supply, and therefore, by Assumption 1, $w_f = 0$. This means that $\omega = \frac{w_m}{w_f} \approx \infty > 1$, contradicting the assumption that $\omega < 1$, which then completes the proof. □

Proposition 2 means that I need only consider two possible cases for labour market equilibrium. Either $1 \leq \omega < \beta$, in which case there is complete labour market segregation as fair firms only hire women and discriminatory firms only hire men, or $\omega = \beta$, in which case there is incomplete segregation as fair firms only hire women and discriminatory firms hire both men and women.

Formally, if $1 \leq \omega < \beta$, then each individual fair firm $k$ demands $\alpha + y^k$ units of female labour. Likewise, each discriminatory firm $j$ demands $\alpha + y^j$ units of labour. Thus, by Assumption 1, the following conditions must hold:

$$L_m = \int_0^{N^d} (\alpha + y^j)\, dj \tag{12}$$

$$L_f = \int_{N^d}^{N^d+N^f} (\alpha + y^k)\, dk \tag{13}$$

However, when $\omega = \beta$, each fair firm $k$ demands $\alpha + y^k$ women as before, but each discriminatory firm $j$ demands $\alpha + y^j$ units of male *or* female labour. Since female workers also work for discriminatory firms in this case, I have to account for the percentage of the female labour force that works for either type of firm. Let $x$ denote the percentage of female workers that work for fair firms. Thi means that $1 - x$ percent of female workers work for discriminatory firms. Thus, the labour market clearing conditions when $\omega = \beta$ are:

$$L_m + (1 - x)L_f = \int_0^{N^d} (\alpha + y^j)\, dj \tag{14}$$

$$xL_f = \int_{N^d}^{N^d + N^f} (\alpha + y^k)\, dk \tag{15}$$

Since the form of the labour market clearing conditions depend on $\omega$, which is an endogenous variable, I have to solve the model for each case separately.

## 2.4 Free entry

I assume that there exists an unbounded (i.e. infinitely large) pool of discriminatory firms who may choose to enter the market. On the other hand, the upper bound on the measure of possible entrants to this market with "fair" preferences is *finite*. I make this assumption concerning the relative size of the pools of possible entrants to capture the *distribution* of discrimination in the economy, as opposed to the "level" or "intensity" of discrimination in an economy, which is captured by $\beta$. Intuitively, this assumption simply means that the measure of discriminatory firms in the economy is *large*, relative to the measure of fair, or non-discriminatory, firms. As a result, this model environment only considers an economy where "most" firms are discriminatory. Moreoever, as shall be proven momentarily, this is a necessary assumption for there to be an endogenous wage gap in the model.[10]

Formally, let $N^{f*}$ denote the upper bound on the measure of possible fair entrants. Whenever all possible fair entrants enter the market in equilibrium, $N^f = N^{f*}$. I make the following assumption concerning entry:

**Assumption 2.** *Discriminatory and fair firms will enter the market until either* (a) *a representative firm of each type earns zero profits, or* (b) *a representative firm's profits are greater than zero but there are no more possible entrants available to enter the market.*

Since the number of possible discriminatory firms is unbounded, and all discriminatory firms are otherwise identical, it follows that discriminatory firms will enter the market until

[10] See Proposition 13, below.

a representative discriminatory firm (and thus, all discriminatory firms) earn zero profits. Fair firms, on the other hand, *may* earn positive profits, if the given number of possible fair entrants $N^{f*}$ is sufficiently small. The idea of "small" is formalized below, where I show under what conditions $N^f = N^{f*}$ or $N^f < N^{f*}$.

# 3 Autarky

## 3.1 Pricing

I now consider each firm's profit maximizing behaviour. As is standard in monopolistic competition models, each firm $i$ chooses their level of output to maximize their profits, given the inverse demand curve for their particular variety, denoted by $p^i\left[\mathbf{p}, D^i(\mathbf{p})\right]$. Since each firm is the only producer of a particular variety $i$, each type of firm's profits can be written as:

$$\pi^j = p^j\left[\mathbf{p}, D^j(\mathbf{p})\right] D^j(\mathbf{p}) - w_m(\alpha + D^j(\mathbf{p})) \qquad \text{For } j \in (0, N^d) \tag{16}$$

$$\pi^k = p^k\left[\mathbf{p}, D^k(\mathbf{p})\right] D^k(\mathbf{p}) - w_f(\alpha + D^k(\mathbf{p})) \qquad \text{For } k \in (N^d, N^d + N^f) \tag{17}$$

First, consider the profit maximization problem of an arbitrary discriminatory firm. Since each firm knows the aggregate demand function for their particular variety, they choose $D^j$ to maximize their profits. Differentiating (16) with respect to $D^j$ and setting that partial derivative equal to zero yields:[11]

$$\begin{aligned}
\frac{\partial p^j}{\partial D^j} D^j(\mathbf{p}) + p^j - w_m &= 0 \\
p^j\left(\frac{\partial p^j}{\partial D^j}\frac{D^j(\mathbf{p})}{p^j} + 1\right) &= w_m \qquad \text{-since } \frac{\partial p^j}{\partial D^j}\frac{D^j(\mathbf{p})}{p^j} = \frac{-1}{\epsilon_D} \\
p^j\left(\frac{\epsilon_D - 1}{\epsilon_D}\right) &= w_m \\
\frac{p^j}{w_m} &= \frac{\epsilon_D}{\epsilon_D - 1} \qquad (18)
\end{aligned}$$

Since (18) will hold for all discriminatory firms, let $p^d$ denote the price charged by all discriminatory firms in equilibrium, so that $p^j = p^d$ for $\forall j \in (0, N^d)$.

[11]The arguments of $\mathbf{p}[.]$ are suppressed for notational simplicity

Recall from (6), that $\epsilon_D = \frac{1}{1-\rho}$. Substituting (6) into (18) yields:

$$\frac{p^d}{w_m} = \frac{1}{\rho} \tag{19}$$

The above result will hold for all possible values of the gender wage gap, since the profit expression given by (16) holds for any pattern of labour demand, including the case where $\omega = \beta$ and discriminatory firms demand female labour as well.

Solving for the profit maximizing quantity of output produced by an arbitrary fair firm $k \in (N^d, N^d + N^f)$ follows the exact same steps outlined above, and yields:.

$$\frac{p^f}{w_f} = \frac{1}{\rho} \tag{20}$$

Where $p^f$ denotes the price charged by all fair firms, i.e. $p^k = p^f$ for $\forall k \in (N^d, N^d + N^f)$.

The equilibrium prices given by (19) and (20) show that each firm charges the same markups over marginal cost, and are standard results in the trade literature involving monopolistic competition with constant elasticity of substitution.[12] While each type of firm may charge the same markup over marginal cost, each firm will not have same marginal costs if $w_m \neq w_f$. This means that different types of firms can still earn different levels of profits due to the fixed cost, $\alpha$, they must pay to start production.

Using (19) and (20), I now prove an important result:

**Proposition 3.** *Discriminatory firms charge more for their varieties if and only if the male wage rate is greater than the female wage rate, i.e. $\omega > 1$.*

*Proof.* Divide (19) by (20):

$$\frac{\frac{p^d}{w_m}}{\frac{p^f}{w_f}} = \frac{\frac{1}{\rho}}{\frac{1}{\rho}}$$

Which is equivalent to:

$$\frac{p^d}{p^f} = \frac{w_m}{w_f} = \omega \tag{21}$$

Proposition 3 follows directly from (21). □

I end up with this result because each firm charges the same markup over marginal cost. Thus, if men are paid more in equilibrium, then for discriminatory firms to earn the same

[12] See, for example, Krugman (1979), Krugman (1980), and Melitz (2003).

markup per unit sold, they will have to price higher than fair firms.[13] More importantly, Proposition 3 means that the gender wage gap in this model will primarily be determined by the *goods pricing* behaviour of the two different types of firms. Thus, competition in the goods market can have an effect on the gender wage gap, which, as we shall soon see, is one of the mechanisms through which trade affects the gender wage gap.

## 3.2 Firm scale and equilibrium measure of firms

Since there is an unbounded pool of possible discriminatory firms, by Assumption 2 it must be that each discriminatory firm earns zero profits in equilibrium. I now impose a zero profit condition on each discriminatory firm to solve for the quantity of output produced by each discriminatory firm in equilibrium, call this $y^d$:

$$\begin{aligned}
\pi^d = p^d y^d - w_m(\alpha + y^d) &= 0 && \text{-Rearranging} \\
\frac{p^d}{w_m} y^d - y^d &= \alpha && \text{-Substitute in (19) for } \frac{p^d}{w_m} \\
\frac{1}{\rho} y^d - y^d &= \alpha && \text{-Rearranging} \\
y^d &= \frac{\alpha\rho}{1-\rho} && (22)
\end{aligned}$$

While discriminatory firms must earn zero profits in equilibrium, since I have assumed that the measure of potential fair firms is finite, fair firms may earn positive profits in equilibrium. Assuming that all fair firms earn the same profits, it follows that if a given representative fair firm earns positive profits in equilibrium, then $N^f = N^{f*}$, i.e. all possible fair firms enter the market. However, whenever $N^{f*}$ firms operating in the market causes a representative fair firm to earn negative profits, each potential fair firms will be unable to enter the market by Assumption 2. Thus there will be "exit" by fair firms until a representative fair firm earns zero profits, which means that in equilibrium $N^f < N^{f*}$ . Thus, I have to determine whether a zero profit condition also holds for fair firms, or whether they earn positive profits in equilibrium, to close the model.

However, I cannot determine the profitability of a representative fair firm until I find their scale, i.e. $y^f$. To accomplish this, I now turn to fully solving the model for each possible case of labour market segregation, i.e. when $\omega = \beta$, and when $1 \leq \omega < \beta$.

[13]This sort of relationship between pricing and wages is fairly standard in simple monopolistic competition models with constant elasticity of substitution between varieties and different varieties of labour. See, for example, Krugman (1991), p. 489, equation (6).

### 3.2.1 Case 1: Incomplete labour market segregation ($\omega = \beta$)

Since the wage gap is already given in this case, then by (21), the difference in pricing between fair firms and discriminatory firms is also given, i.e.:

$$\frac{p^d}{p^f} = \omega = \beta$$

I can then use (7), i.e. the equation relating the ratio of consumer demand between varieties to the ratio of their prices, to find a fair firm's output. Let $D^d$ denote the aggregate demand for a single discriminatory firm's variety, and let $D^f$ denote the aggregate demand for a single fair firm's variety. By (7):

$$\frac{D^f}{D^d} = \left(\frac{p^d}{p^f}\right)^{\frac{1}{1-\rho}} = \beta^{\frac{1}{1-\rho}}$$

Imposing goods market clearing so that $D^f = y^f$ and $D^d = y^d$ yields:

$$\frac{y^f}{y^d} = \beta^{\frac{1}{1-\rho}}$$

Substituting the equilibrium output of discriminatory firms given by (22) and rearranging yields:

$$y^f = \frac{\beta^{\frac{1}{1-\rho}}\alpha\rho}{1-\rho} \tag{23}$$

Note that (23) implies that $y^f > y^d$, since $y^d = \frac{\alpha\rho}{1-\rho}$, and $\beta > 1$. Thus, fair firms produce a greater quantity of output compared to discriminatory firms. From this, I obtain the following result:

**Proposition 4.** *Fair firms earn positive profits if* $\omega = \beta$

*Proof.* Suppose not, so that fair firms earn non-positive profits and $\omega = \beta$. It follows that:

$$\begin{aligned}
\pi^f = p^f y^f - w_f(\alpha + y^f) &\leq 0 && \text{-Rearranging} \\
\frac{p^f}{w_f} y^f - y^f &\leq \alpha && \text{-Substituting (20) in for } \frac{p^f}{w_f} \text{ yields after rearranging} \\
\frac{(1-\rho)}{\rho} y^f &\leq \alpha && \text{-Substitute in (23) and then rearrange, yielding} \\
\beta^{\frac{1}{1-\rho}} &\leq 1
\end{aligned}$$

Which contradicts $\beta > 1$ □

The intuition for this result is fairly straightforward. Whenever $\omega = \beta$, there is a relatively large gender wage gap. Since fair firms hire more female labour than discriminatory firms, which is under-priced relative to male labour, fair firms earn positive profits. Moreover, fair firms are able to produce at a higher scale than discriminatory firms, and are therefore able to take greater advantage of economies of scale than discriminatory firms, which also allows them earn higher profits.

Proposition 4 can also be used to prove the following result:

**Proposition 5.** *All fair firms enter the market if $\omega = \beta$,, i.e. $N^f = N^{f*}$.*

*Proof.* Suppose that $\omega = \beta$. By Assumption 2, either fair firms earn zero profits and $N^f < N^{f*}$, or $N^f = N^{f*}$ and a fair firm earns profits greater than zero. However, by Proposition 4, fair firms earn positive profits if $\omega = \beta$, which rules out the first case, and means that it must be that $N^f = N^{f*}$. □

Since I know the equilibrium number of fair firms as well as the scale of each fair firm, I can now solve for $x$, the percentage of the female labour force hired by fair firms in equilibrium. Since $y^k = y^f$, for $\forall k \in (N^d, N^d + N^f)$, then the labour market clearing condition for the female labour market given by (15) becomes:

$$xL_f = \int_{N^d}^{N^d+N^f} (\alpha + y^f)\, dk = N^f(\alpha + y^f) \tag{24}$$

Substituting $N^f = N^{f*}$ and the equilibrium level of fair firm output given by (23) into (24) and then solving for $x$ yields:

$$x = \left(\frac{N^{f*}}{L^f}\right)\left(\frac{\alpha\rho(\beta^{\frac{1}{1-\rho}} - 1) + \alpha}{1-\rho}\right) \tag{25}$$

Unsurprisingly, the above shows that as $N^{f*}$ increases, the proportion of the female labour force hired by fair firms increases. This is because an increase in the number of fair firms increases the overall demand for female labour by fair firms, and thus fair firms hire a larger portion of the female labour force.

Since $x$ is uniquely determined by (25), I can then implicitly characterize the equilibrium number of discriminatory firms as a function of $x$. Since $y^j = y^d$, for $\forall j \in (0, N^d)$, the male labour market clearing condition given by (14) becomes:

$$L_m + (1-x)L_f = \int_0^{N^d} (\alpha + y^d)\, dj = N^d(\alpha + y^d) \tag{26}$$

Substituting the output of discriminatory firms given by (22) into the above, and solving for $N^d$ yields:

$$N^d = \frac{(L_m + (1-x)L_f)\,(1-\rho)}{\alpha} \tag{27}$$

Recalling that $x$ increases whenever $N^{f*}$ increases, (27) shows that the equilibrium measure of discriminatory firms will *fall* as the measure of potential fair firms increase. This is simply because an increase in the equilibrium measure of fair firms will increase the degree competition for female labour. Since discriminatory firms also hire female labour whenever $\omega = \beta$, the increased competitive pressure hurts discriminatory firms more than fair firms, since fair firms earn positive profits while discriminatory firms earn zero profits. As a result, increasing $N^{f*}$ forces some discriminatory firms exit the market while all the previously established fair firms can continue to remain in the market.

Before engaging in a more detailed analysis of this particular equilibrium, I now turn to the case of complete segregation.

### 3.2.2 Case 2: Complete segregation ($1 \leq \omega < \beta$)

Since equations (20) and (19) imply symmetry in the level of output chosen by each firm type, i.e.$y^j = y^d$, for $\forall j \in (0, N^d)$ and $y^k = y^f$, for $\forall k \in (N^d, N^d + N^f)$, then aggregate demand for male labour is simply $N^d(\alpha + y^d)$ while aggregate demand for male labour is $N^f(\alpha + y^f)$. Thus, labour market clearing conditions (12) and (13) become:

$$L_m = N^d(\alpha + y^d) \tag{28}$$

$$L_f = N^f(\alpha + y^f) \tag{29}$$

The scale of each discriminatory firm is given by equation (22). Therefore, I can substitute this equation into (28) and then solve for the equilibrium number of discriminatory firms, yielding:

$$N^d = \frac{L_m(1-\rho)}{\alpha} \tag{30}$$

It should be clear from (30) that increasing the number of male workers ($L_m$) increases the number of discriminatory firms in equilibrium, while increasing fixed costs of production ($\alpha$) or the elasticity of demand[14] decreases the number of discriminatory firms in equilibrium.

[14] Note that $1 - \rho = \frac{1}{\epsilon_D}$. Therefore, I can rewrite (30) as $N^d = \frac{L_m}{\epsilon_D\,\alpha}$.

Turning to the labour market clearing conditions for fair firms, equation (29) is still an equation in two unknowns, $N^f$ and $y^f$. Rearranging (29) to get the scale of a fair firm as a function of the number of fair firms yields:

$$y^f(N^f) = \frac{L^f}{N^f} - \alpha \tag{31}$$

The following propositions then follow:

**Proposition 6.** *Fair firms earn positive profits if* $\frac{L_f}{N^f} > \frac{\alpha}{1-\rho}$

*Proof.* Suppose not, so that fair firms earn non-positive profits and $\frac{L_f}{N^f} > \frac{\alpha}{1-\rho}$. If fair firms earn non-positive profits, then by Proposition 4, this means that $\omega \neq \beta$. Therefore, $1 \leq \omega < \beta$, and (31) defines a fair firm's output. It follows that fair firms earn non-positive profits whenever:

$$\begin{aligned}
\pi^f = p^f y^f - w_f(\alpha + y^f) &\leq 0 && \text{-Rearranging} \\
\frac{p^f}{w_f} y^f - y^f &\leq \alpha && \text{-Plug in (20)} \\
\frac{1}{\rho} y^f - y^f &\leq \alpha && \text{-Rearranging} \\
(1-\rho) y^f &\leq \alpha\rho && \text{-Plug in (31)} \\
\left(\frac{L^f}{N^f} - \alpha\right)(1-\rho) &\leq \alpha\rho && \text{-Rearranging} \\
\frac{L_f}{N^f} &\leq \frac{\alpha}{1-\rho}
\end{aligned}$$

Which is a contradiction. □

**Corollary 2.** *Fair firms earn zero profits if* $\frac{L_f}{N^f} = \frac{\alpha}{1-\rho}$*, and earn negative profits if* $\frac{L_f}{N^f} < \frac{\alpha}{1-\rho}$*.*

*Proof.* Work through the derivation for (32) backwards. Making the inequality hold with strict equality yields the first part of the corollary. Making the inequality a strict inequality yields the second part of the corollary. □

The intuition underlying Proposition 6 and Corollary 2 is fairly straightforward. Whenever $\frac{L_f}{N^f}$ is "large" i.e. greater than $\frac{\alpha}{1-\rho}$, the measure of fair firms in equilibrium is relatively small compared to the number of women in the market. Since only fair firms hire women whenever $1 \leq \frac{w_m}{w_f} < \beta$, this means that there are relatively few firms competing for access to this factor of production. This smaller degree of competition in the labour market allows fair firms to earn positive profits since there is less upward pressure on female wages. However, whenever $\frac{L_f}{N^f}$ is "small," this means that the number of fair firms competing for female labour is relatively large, and therefore competitive pressure in the labour market drives fair firms profits down to zero by raising the female wage.

This sort of story suggests that the relative profitability of fair firms should be related to the gender wage gap in equilibrium, since the degree of competition for female labour appears to determine the profitability of fair firms. It turns out that this is indeed the case, since the degree of competition in the labour market as defined by $\frac{L_f}{N^{f*}}$, is the primary mechanism for determining the size of the gender wage gap in equilibrium. However, there are a couple of intermediate results that must be proven before this can be formally stated.

**Proposition 7.** *If $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$, all possible fair firms enter the market, i.e. $N^f = N^{f*}$, and each fair firm earns positive profits.*

*Proof.* Suppose that $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$. By Assumption 2, either fair firms earn zero profits and $N^f < N^{f*}$, or fair firms earn profits greater than or equal to zero and $N^f = N^{f*}$. The second case is possible if $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$, since if $N^f = N^{f*}$, then by Proposition 6 each fair firm earns positive profits. I rule out the second case by noting that if $N^f < N^{f*}$ then $\frac{L_f}{N^f} > \frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$, which means that fair firms earn positive profits. □

Proposition 8 means that whenever $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$, I can simply substitute $N^f = N^{f*}$ into (31) to get the equilibrium scale of each fair firm, i.e. $y^f = \frac{L_f}{N^{f*}} - \alpha$. To find the scale of each fair firm when $\frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho}$, I prove that this inequality implies that fair firms earn zero profits, and thus, I can find the scale of fair firms using a zero profit condition.

**Proposition 8.** *If $\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}$ all possible fair entrants will not be able to enter the market in equilibrium, i.e. $N^f < N^{f*}$, and each fair firm earns zero profits.*

*Proof.* Suppose that $\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}$. By Assumption 2, $N^f \leq N^{f*}$. First suppose that $N^f = N^{f*}$. Then by Corollary 2, fair firms earn negative profits since $\frac{L_f}{N^{f*}} = \frac{L_f}{N^f} < \frac{\alpha}{1-\rho}$. Since fair firms cannot earn negative profits by Assumption 2, it follows that $N^f < N^{f*}$. By Assumption 2, whenever $N^f < N^{f*}$, fair firms must earn zero profits, and the proposition follows. □

Assumption 2, combined with Propositions 7 and 8 imply the following Corollary:

**Corollary 3.** *Whenever $\frac{L_f}{N^{f*}} = \frac{\alpha}{1-\rho}$ all fair firms earn zero profits and $N^f = N^{f*}$.*

The intuition for Proposition 8 and Corollary 3 is fairly straightforward. If there are many potential fair entrants, i.e. $\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}$, not every fair firm will be able to enter the market in equilibrium since that would cause each firm to earn negative profits. Therefore, the market only supports a smaller measure of firms in equilibrium, i.e. $N^f < N^{f*}$, with each fair firm earning zero profits. $\frac{L_f}{N^{f*}} = \frac{\alpha}{1-\rho}$ then corresponds to the case where there are "just enough" potential entrants so that all of the fair firms enter the market in equilibrium and each fair firm earns zero profits.

This means that whenever $\frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho}$, I can use a zero profit condition to solve for the scale of each fair firm. Setting fair firms profits equal to zero and solving for $y^f$ yields:

$$\begin{aligned}
\pi^f = p^f y^f - w_f(\alpha + y^f) &= 0 && \text{-Rearranging} \\
\frac{p^f}{w_f} y^f - y^f &= \alpha && \text{-Substitute } \frac{p^f}{w_f} \text{ given by (20)} \\
\frac{1}{\rho} y^f - y^f &= \alpha && \text{-Rearranging} \\
y^f_{\text{ZP}} &= \frac{\alpha\rho}{1-\rho} && \qquad (32)
\end{aligned}$$

I then plug (32) into the female labour market conditions given by (29) to obtain the equilibrium number of fair firms whenever fair firms earn zero profits

$$N^f_{\text{ZP}} = \frac{L_f(1-\rho)}{\alpha} \qquad (33)$$

Note that (32) implies that $y^f_{\text{ZP}} = y^d = \frac{\alpha\rho}{1-\rho}$. This should not be surprising since discriminatory firms also earn zero profits and both types of firms charge the same markups over marginal cost.

I now characterize the equilibrium measure of fair firms and the scale of each fair firm by the following functions of $\frac{L_f}{N^{f*}}$:

$$N^f\left(\frac{L_f}{N^{f*}}\right) = \begin{cases} \frac{L_f(1-\rho)}{\alpha} & \text{if } \frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho} \\ N^{f*} & \text{if } \frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho} \end{cases} \qquad (34)$$

$$y^f\left(\frac{L_f}{N^{f*}}\right) = \begin{cases} \frac{\alpha\rho}{1-\rho} & \text{if } \frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho} \\ \frac{L^f}{N^{f*}} - \alpha & \text{if } \frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho} \end{cases} \qquad (35)$$

The above two equations, combined with (22) and (30) which define $y^d$ and $N^d$, respectively, fully characterize the equilibrium measure of firms, and the scale of each firm when $1 \leq \omega < \beta$. Thus, having now solved for all the aggregate equilibrium quantities in the autarkic equilibrium, I now turn to the gender wage gap.

### 3.3 Autarkic gender wage gap

I now find the gender wage gap as a function of each type of firm's output. By (7):

$$\begin{aligned}
\left(\frac{p^d}{p^f}\right)^{\frac{1}{1-\rho}} &= \frac{D^f}{D^d} && \text{-Impose goods market clearing} \\
\left(\frac{p^d}{p^f}\right)^{\frac{1}{1-\rho}} &= \frac{y^f}{y^d} && \text{-By (21), } \frac{p^d}{p^f} = \frac{w_m}{w_f} = \omega \\
(\omega)^{\frac{1}{1-\rho}} &= \frac{y^f}{y^d} && \text{-Rearranging}
\end{aligned}$$

$$\omega = \left(\frac{y^f}{y^d}\right)^{1-\rho} \tag{36}$$

I can use the above expression to find the equilibrium gender wage gap when $1 < \omega < \beta$. Suppose that $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$ so that by (35), $y^f = \frac{L^f}{N^{f*}} - \alpha$. Moreover, by (22), $y^d = \frac{\alpha\rho}{1-\rho}$. Then by (36):

$$\begin{aligned}
\omega &= \left(\frac{y^f}{y^d}\right)^{1-\rho} && \text{-Substitute in equilibrium outputs} \\
\omega &= \left(\frac{\frac{L^f}{N^{f*}} - \alpha}{\frac{\alpha\rho}{1-\rho}}\right)^{1-\rho} && \text{-Rearranging}
\end{aligned}$$

$$\omega = \left[\left(\frac{L_f}{N^{f*}} - \alpha\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} \tag{37}$$

The reader may notice one curious feature of the gender wage gap defined by (37); it does not depend on $\beta$. This means that the "level" of prejudice a discriminatory firm feels towards women does not determine whether women are underpaid relative to men. This is a surprising result since it could imply that a firm's taste for discrimination is an irrelevant feature of the model. Recall, however, that (37) was derived under some restrictions on the model's parameters. If one relaxes these restrictions, the gender wage gap will depend on $\beta$, but only as special case. I demonstrate this by defining the wage gap more generally. First, I define the following variables:

$$\phi_L \equiv \frac{\alpha}{1-\rho} \tag{38}$$

$$\phi_H \equiv \frac{\alpha}{1-\rho}\left(1 + \rho(\beta^{\frac{1}{1-\rho}} - 1)\right) \tag{39}$$

The following proposition then defines the bounds for the equilibrium wage gap as a function of $\frac{L_f}{N^{f*}}$. Proof is omitted from the main text for the sake of brevity, but can be found in the Mathematical Appendix, Appendix A.

**Proposition 9.** *There will be no wage gap, i.e.* $\omega = 1$*, whenever* $\frac{L_f}{N^{f*}} \leq \phi_L$*, while the wage gap satisfies* $1 < \omega < \beta$ *whenever* $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$*, and satisfies* $\omega = \beta$ *whenever* $\frac{L_f}{N^{f*}} \geq \phi_H$*.*

By combining Proposition 9 and equation (37), I define the wage gap as a function of exogenous parameters for all possible values of $\frac{L_f}{N^{f*}}$.

$$\omega = \begin{cases} 1 & \text{if } \frac{L_f}{N^{f*}} \leq \phi_L \\ \left[\left(\frac{L_f}{N^{f*}} - \alpha\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} & \text{if } \phi_L < \frac{L_f}{N^{f*}} < \phi_H \\ \beta & \text{if } \frac{L_f}{N^{f*}} \geq \phi_H \end{cases} \tag{40}$$

The above shows that the wage gap does depend on $\beta$, but only when $\frac{L_f}{N^{f*}} \geq \phi_H$, or in other words *when there are very few fair firms relative to the number of female workers.*

The intuition for this result is that whenever $\frac{L_f}{N^{f*}} \geq \phi_H$ the measure of potential fair firms is very small, relative to the number of women in the labour force. Consequently, the female labour market will only clear if discriminatory firms absorb some of the female labour force. Since discriminatory firms hire female labour, their taste for discrimination "binds" in the labour market, and the equilibrium wage gap is entirely determined by how much distaste discriminatory firms have for hiring women.

However, when $\frac{L_f}{N^{f*}} < \phi_H$, there are now "enough" fair firms in the economy, in the sense that the aggregate demand for female labour by fair firms entirely absorbs the female labour force. Therefore, a discriminatory firm's "taste for discrimination" no longer binds, and the wage gap no longer depends on $\beta$. Thus, according to this model *the degree to which firms are discriminatory towards women only determines the size of the gender wage gap if there are very few non-discriminatory firms competing with them*

However, if each discriminatory firms' "taste for discrimination" no longer matters when $\frac{L_f}{N^{f*}} < \phi_H$, it may seem curious that there is still a positive gender wage gap when $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$. It turns out that a gender wage gap still exists in this case because there is a smaller supply of fair firms relative to discriminatory firms, which means that fair firms face less competitive pressure in the labour market compared to discriminatory firms *since discriminatory firms do not actually want to hire female labour.* As a result, fair firms do not have to pay female workers as much as men because there is less competition for female labour.

This insight is formalized by the following propositions.

**Proposition 10.** *If* $\frac{L_f}{N^{f*}} \leq \phi_L$*, then in equilibrium* $\frac{N^f}{L_f} = \frac{N^d}{L_m}$*, and there is no gender wage gap, i.e.* $\omega = 1$*.*

*Proof.* By Proposition 9, $\omega = 1$ if $\frac{L_f}{N^{f*}} \leq \phi_L = \frac{\alpha}{1-\rho}$. Thus, the proposition follows once it has been proven that $\frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho}$ implies that $\frac{N^f}{L_f} = \frac{N^d}{L_m}$.

Suppose that $\frac{L_f}{N^{f*}} \leq \phi_L = \frac{\alpha}{1-\rho}$. Then equation (30) describes the equilibrium measure of discriminatory firms, $N^d = \frac{L_m(1-\rho)}{\alpha}$. It follows that:

$$\frac{N^d}{L_m} = \frac{\frac{L_m(1-\rho)}{\alpha}}{L_m} = \frac{1-\rho}{\alpha}$$

Moreover, by (34), $N^f = \frac{L_f(1-\rho)}{\alpha}$, which means that:

$$\frac{N^f}{L_f} = \frac{\frac{L_f(1-\rho)}{\alpha}}{L_f} = \frac{1-\rho}{\alpha}$$

It follows that whenever $\frac{L_f}{N^{f*}} \leq \phi_L = \frac{\alpha}{1-\rho}$

$$\frac{N^f}{L_f} = \frac{N^d}{L_m} = \frac{\alpha}{1-\rho}$$

□

The primary mechanism that leads to this result can be found in Proposition 8. Whenever $\frac{L_f}{N^{f*}} < \phi_L = \frac{\alpha}{1-\rho}$, the measure of potential fair entrants is large. As a result, the market cannot support all potential fair entrants in equilibrium. Thus, fair firms only enter the market until each fair firm earns zero profits. Since discriminatory firms also earn zero profits, this leads to an equilibrium measure of fair firms per unit of female labour equal to the measure of discriminatory firms per unit of male labour. This means there is an equal degree of competition for both male and female labour, which eliminates the gender wage gap.

The next proposition states the effect of $\frac{L_f}{N^{f*}}$ rising above the threshold value of $\phi_L = \frac{\alpha}{1-\rho}$.

**Proposition 11.** *If* $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$*, then the gender wage gap satisfies* $1 < \omega < \beta$*, each fair firm earns positive profits, and* $\frac{N^f}{L_f} < \frac{N^d}{L_m}$*.*

*Proof.* By Proposition 9, whenever $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$, the wage gap satisfies $1 < \omega < \beta$. Moreover, Proposition 7 establishes that whenever $\phi_L < \frac{L_f}{N^{f*}}$, each fair firm earns positive profits. Thus, the proposition follows once it has been proven that $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$ implies that $\frac{N^f}{L_f} < \frac{N^d}{L_m}$.

By Proposition 9, I can use (30) and (34) whenever $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$ since these equations were derived under the condition that $1 \leq \omega < \beta$.

By (30), $N^d = \frac{L_m(1-\rho)}{\alpha}$. It follows that whenever $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$:

$$\frac{N^d}{L_m} = \frac{\frac{L_m(1-\rho)}{\alpha}}{L_m} = \frac{1-\rho}{\alpha}$$

Moreover, by (34), $N^f = N^{f*}$ if $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho} = \phi_L$. It follows that when $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$:

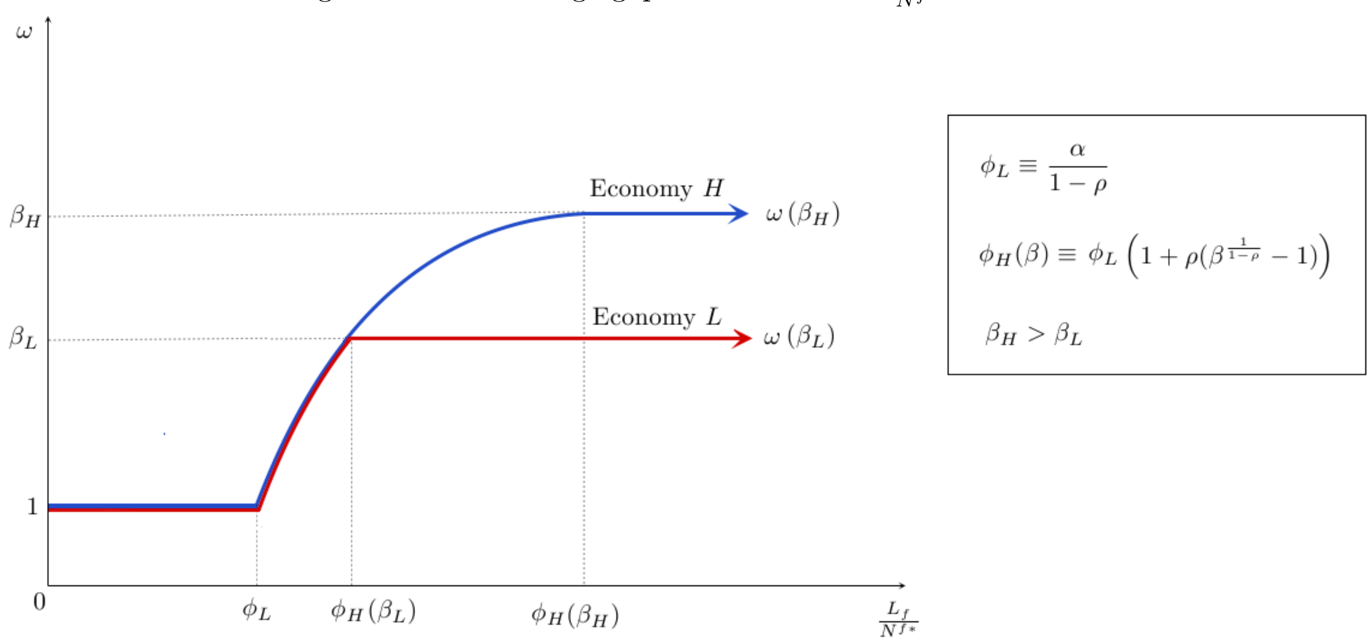
$$\frac{N^{f*}}{L_f} = \frac{N^f}{L_f} < \frac{N^d}{L_m} = \frac{\alpha}{1-\rho}$$

□

Proposition 11 makes the mechanism that leads to a gender wage gap clear. Whenever the measure of potential fair entrants per female labourer is small (but not so small that discriminatory firms need to hire female workers), then there will be less fair firms competing for access to female labour than there are discriminatory firms competing for access to male labour. This means that fair firms do not need to pay their workers as much as discriminatory firms, which leads to a gender wage gap.

In Figure 1, below, I graph the gender wage gap defined by (40) as a function of $\frac{L_f}{N^{f*}}$, to summarize these results.[15]

Figure 1: Gender wage gap as a function of $\frac{L_f}{N^{f*}}$




[15]The shape of the slope and continuity of $\omega$ is proven in the mathematical appendix.

Each line shows the gender wage gap for all possible distributions of prejudice, which are implicitly defined by $\frac{L_f}{N^{f*}}$. Movement along any given line from left to right can then be thought of as showing the effect that an *increasingly prejudiced distribution of discrimination* has on the gender wage gap. Unsurprisingly, as economies develop more prejudiced distributions, the gender wage gap increases, eventually reaching an upper bound where each discriminatory firm's taste for discrimination "binds" and the gender wage gap is determined by the level of prejudice, or the amount of distaste, discriminatory firms feel towards women.

Figure 1 also shows how changing the level of prejudice exhibited by each firm changes the equilibrium gender gap. The blue line, which partially lies above the red line, shows the gender wage gap for an economy, call it economy $H$, where each discriminatory firm has a discriminatory coefficient of $\beta_H$. Likewise, the red line shows the gender wage gap for an economy, call it economy $L$, where discriminatory firms have discriminatory coefficients of $\beta_L$, where $\beta_L < \beta_H$. Note that when $\frac{L_f}{N^{f*}} < \phi_H(\beta_L)$, the gender wage gap is the same for both economies. However, as the distribution of prejudice increases to the point where $\frac{L_f}{N^{f*}} = \phi_H(\beta_H)$, economy $H$ has a larger wage gap. This is because discriminatory firms in economy $L$ are willing to hire women at a higher relative wage rate[16] than are discriminatory firms in economy $H$, because firms in economy $L$ have a smaller discrimination coefficient, i.e. they do not receive as much dis-utility from hiring women. Therefore the gender wage gap continues to rise in economy $H$ as $\frac{L_f}{N^{f*}}$ increases, since there continues to be less competition for female labour than there is demand for male labour, which drives down women's wages relative to men's wages.

One might very well wonder why a discriminatory firm's taste for discrimination binds with *equality* whenever $\frac{L_f}{N^{f*}} \geq \phi_H$, since it could also rise above $\beta$ due to a decreased demand for female labour as the number of fair firms decreases. Note, however, that once $\omega = \beta$, all discriminatory firms are willing to hire female workers, even though this requires each firm to bear a disutility cost. Therefore, if $\frac{L_f}{N^{f*}}$ falls because a number of fair firms decide to leave the market, new discriminatory firms will be willing to enter the market. Once they do, their new demand for female labour "makes up" for the loss of the fair firms demand for female labour. Therefore, $\omega$ will not rise above $\beta$ since the aggregate demand for female labour effectively remains constant. I show this formally with the following proposition:

**Proposition 12.** *If $\frac{L_f}{N^{f*}} \geq \phi_H$, then further increasing $\frac{L_f}{N^{f*}}$ increases the portion of the female labour force hired by discriminatory firms, i.e. $(1 - x)$.*

*Proof.* By Proposition 9, $\omega = \beta$ whenever $\frac{L_f}{N^{f*}} \geq \phi_H$. This means that I can use (25) to find the portion of the female labour force hired by discriminatory firms, which is denoted by

[16] Note that a smaller $\omega$ implies a higher relative wage rate for women.

$(1-x)$. It follows that:

$$1-x=1-\frac{N^{f*}}{L^{f}}\frac{\alpha\rho(\beta^{\frac{1}{1-\rho}}-1)+\alpha}{1-\rho}$$

Partially differentiating the above with respect to $\frac{L_f}{N^{f*}}$ yields:

$$\frac{\partial(1-x)}{\partial\frac{L_f}{N^{f*}}}=-(-1)\left(\frac{N^{f*}}{L^{f}}\right)^{2}\frac{\alpha\rho(\beta^{\frac{1}{1-\rho}}-1)+\alpha}{1-\rho}>0 \tag{41}$$

□

As should be clear from the above discussion, the size of the measure of potential fair firms, relative to the measure of discriminatory firms, is one the key variables that determines the size of the autarkic gender wage gap. However, I began my analysis by assuming that the measure of potential discriminatory firms was always "large" relative to the measure of fair firms, i.e. the measure of potential fair firms was bounded, while the measure of discriminatory firms was not. It is then worth considering what would occur in this economy if these relative size assumptions were reversed.[17]

It turns out, however, that if the measure of fair firms is "large" i.e. unbounded, while the measure of discriminatory firms is finite, there will *never* be a gender wage gap in equilibrium. This can easily be proven formally:

**Proposition 13.** *If the number of prospective fair firms is unbounded (infinite), while the number of discriminatory firms is finite, there will not be a gender wage gap in equilibrium.*

*Proof.* If the number of prospective fair firms is infinite, then fair firms will earn zero profits. Therefore, by (32), $y^f=\frac{\alpha\rho}{1-\rho}$. By male labour market clearing, $y^d=\frac{L_m}{N^d}-\alpha$. Using (36), then, the wage gap can be written as:

$$\omega=\left(\frac{y^f}{y^d}\right)^{1-\rho}=\left[\frac{\alpha\rho}{(1-\rho)\left(\frac{L_m}{N^d}-\alpha\right)}\right]^{1-\rho} \tag{42}$$

Next suppose that the above proposition were not true. Then either $\omega>1$ or $\omega<1$. By Proposition 2, $\omega<1$ is impossible in equilibrium, so it must be that $\omega>1$. Therefore, by (42):

$$\left[\frac{\alpha\rho}{(1-\rho)\left(\frac{L_m}{N^d}-\alpha\right)}\right]^{1-\rho}>1$$

[17] I do not consider the case where the measure of both types of firms is bounded, since the number of varieties will not be endogenous in that case.

Or,

$$\frac{\alpha\rho}{1-\rho} > \frac{L_m}{N^d} - \alpha \tag{43}$$

Next, I determine if discriminatory firms earn positive profits. This is only possible when:

$$\pi^d = p^d y^d - w_m(\alpha + y^d) > 0$$

Substituting $y^d = \frac{L_m}{N^d} - \alpha$ into this expression and rearranging yields:

$$\frac{L_m}{N_m} - \alpha > \frac{\alpha\rho}{1-\rho} \tag{44}$$

However the above cannot be true by (43). Since discriminatory firms cannot earn negative profits, it must be that discriminatory firms earn zero profits. As a result, I can make use of (22) which shows that in equilibrium $y^d = \frac{\alpha\rho}{1-\rho}$. By (36):

$$\omega = \left(\frac{y^f}{y^d}\right)^{1-\rho} = \frac{\frac{\alpha\rho}{1-\rho}}{\frac{\alpha\rho}{1-\rho}} = 1 \tag{45}$$

Which contradicts the assumption that $\omega > 1$. □

I have now demonstrated the key assumptions that lead a gender wage gap in this model. First, there must be some firms that have discriminatory preferences. Moreover, the measure of possible discriminatory firms must be "large" relative to the measure of fair firms. Finally, even if both of the preceding conditions are satisfied, a gender wage gap will only appear if the measure of fair firms is "small" relative to the supply of female labour, so that the degree of competition for female labour is less than the degree of competition for male labour. Altogether, these conditions lead to market equilibria where the presence of discriminatory firms results in less demand for female labour than the demand for male labour, thereby leading to an endogenous gender wage gap. I emphasize these points since understanding how these factors lead to a gender wage gap in autarky shall prove essential to understanding why opening a country to trade decreases gender wage gaps.

# 4 Trade

## 4.1 Overview of trade environment

Suppose that an economy as described in Sections 3 and 4 trades with another country with identical preferences, identical numbers of men and women, and an identical distribution of prospective fair firms and prospective discriminatory firms. Assume that each firm still produces a unique variety, so that if there are $N$ varieties produced in each country, there are $2N$ varieties available to each consumer.

As Krugman (1980) and Melitz (2003) point out, if there are no trade costs in this sort of trade environment characterized by increasing returns to scale and constant marginal costs, then there will be gains from trade due to the increased number of varieties available to each consumer. However, because (6) implies constant elasticity of aggregate demand, trade will not affect the scale, or profitability of each firm. Since $\omega = \left(\frac{y^f}{y^d}\right)^{1-\rho}$, this means that the wage gap will not be affected by trade if there are no trade costs.

Melitz (2003), however, argues that there is empirical evidence suggesting that firms face significant trade costs which factor into their export decisions. These include per-unit trade costs such as transportation costs, as well as various fixed costs, including the costs of learning foreign regulatory requirements or marketing to foreign consumers. To account for these features of a firm's export decision, I assume that there are per-unit trade costs of the iceberg variety, as well as a fixed cost that must be paid to be enter the export market.

Formally, let $\tau > 1$ denote the per-unit iceberg trade costs. As is standard in the literature, iceberg trade costs mean that $\tau$ units of a good must be shipped for 1 unit of the good to arrive in the foreign economy. Let $f_x > 0$ denote the fixed costs associated with entering the export market. For simplicity, I shall model $f_x$ as the number of workers a firm needs to hire to build a ship, which they then use to transport goods to the other country.[18] Thus, the value of the fixed cost is either $w_f f_x$ or $w_m f_x$, depending on whether the firm in question hires men or women.

While demand conditions do not change when moving from autarky to trade, I now distinguish between demand for variety $\upsilon$ that originates domestically and demand due to foreign consumers. To account for this distinction, let $D^{\upsilon H}$ denote aggregate demand for variety $\upsilon$ in the home economy of firm $\upsilon$, and let $D^{\upsilon X}$ denote aggregate demand for variety $\upsilon$ in the foreign country.

Proposition 2 continues to hold in the trade environment, so the equilibrium wage must still satisfy $1 \leq \omega \leq \beta$. Since the pattern of labour demand depends on whether $1 \leq \omega < \beta$ or $\omega = \beta$, I would have to consider both cases in turn to fully solve the model. However, the intuition underlying how trade affects the gender wage gap can easily be understood if one only considers the "middle" case where $1 < \omega < \beta$. Therefore, for expositional clarity *I assume that $1 < \omega < \beta$ for the remainder of this section, unless otherwise stated.*[19] This

[18]I could have also chosen to model $f_x$ as a cost in terms of output. However, since many of the fixed costs of trade likely involve hiring people, such as advertisers, or lawyers to research foreign regulatory requirements, modelling the fixed cost as a labour input requirement, as opposed to a cost in terms of output, makes the most intuitive sense.

[19]However, the model's behaviour when $\omega = 1$ and $\omega = \beta$ is briefly discussed in the Mathematical

means that as I consider trade equilibrium, fair firms will only hire female labour, while discriminatory firms will only hire male labour.

To make the export entry decision as simple as possible, it shall prove useful to make a distinction between "short-run" decisions in this model, and "long-run" equilibrium. I assume that export market entry is a short-run decision that occurs after an autarkic equilibrium is established. Thus, the quantities and prices of other firms that were established in autarkic equilibrium will be taken as given by individual firms in the short-run. However, the final equilibrium I shall be considering is a long-run equilibrium in the sense that I shall allow these quantities and prices to vary, *given* each firm's export status decision that they made in the short-run. I make this distinction so that the short-run equilibrium will then *determine* the long-run equilibrium.

Note that one could also solve this model by simply assuming a pattern of export statuses between firms (all fair firms export, discriminatory firms do not, etc), and from there one could solve for the type of long-run equilibrium implied by each possible case. Unfortunately one would be unable to decide which of the four possible candidates for long-run equilibrium[20] is most likely to occur using this approach. This is because *a firm never has an incentive to start only producing for the domestic market if they export in long-run equilibrium, since their fixed export costs are already sunk*. This means that no firm will ever want to deviate from a long-run equilibrium where they export, and therefore every equilibrium where a given firm type exports is a Nash equilibrium.

This means that a firm's export decision can only be modelled in a meaningful way if one considers a firm's export choice *before* long-run equilibrium. This is why I chose to consider a firm's export decision from the short-run vantage point; it can then be shown that whenever trade costs are high relative to a firm's factory costs, some firms will not gain by unilaterally entering the export market. Thus, I can *rule out* some possible candidate long-run equilibria, by allowing a short-run export decision to determine each firm's long-run export status.

## 4.2 Firm pricing with trade

Before I consider a firm's export decision, I determine what their pricing behaviour would be *if* they were to enter the export market. Therefore, suppose that all types of firms export. Of the total output produced by each firm $i$, call this $y^{iT}$, some portion of that output will be sold domestically, call this $y^{iH}$, and another portion will be shipped abroad, call this $y^{iX}$.

Appendix, where I show the bounds on $\frac{L_f}{N^{f*}}$ consistent with the result that $1 < \omega < \beta$ in trade equilibrium. See Appendix D and E.

[20] These four candidates are: 1) both types of firms export, 2) only fair firms export, 3) only discriminatory firms export, and 4) neither type of firm exports.

It follows that:

$$y^{iT} = y^{iH} + y^{iX} \tag{46}$$

However, only $\frac{y^{iX}}{\tau}$ of the shipped goods actually reach foreign consumers due to iceberg trade costs. Denote this portion of output by $\hat{y}^{iX}$. It follows that:

$$y^{iX} = \tau \hat{y}^{iX} \tag{47}$$

With the above in mind, let $j$ index discriminatory firms as before, $j \in (0, 2N^{dT})$, where $N^{dT}$ is the measure of discriminatory firms operating in a single country in trade equilibrium.[21] Likewise, let $k$ index fair firms, $k \in (2N^{dT}, 2N^{dT} + 2N^{fT})$, where $N^{fT}$ is the number of fair firms operating in a single country in equilibrium . Further let $p^{iH}$ denote the price charged for variety $i$ in the home market, and $p^{iX}$ denote the charged for variety $i$ in the foreign market. I can then write the profits of an arbitrary fair firm and discriminatory firm, respectively, as:

$$\pi^{kT} = p^{kH} y^{kH} + p^{kX} \hat{y}^{kX} - w_f(\alpha + f_x + y^{kT}) \tag{48}$$

$$\pi^{jT} = p^{jH} y^{jH} + p^{jX} \hat{y}^{jX} - w_m(\alpha + f_x + y^{jT}) \tag{49}$$

Substituting (46) and (47) into (48) and (49), yields:

$$\pi^{kT} = p^{kH} y^{kH} + p^{kX} \hat{y}^{kX} - w_f(\alpha + f_x + y^{kH} + \tau y^{kX}) \tag{50}$$

$$\pi^{jT} = p^{jH} y^{jH} + p^{jX} \hat{y}^{jX} - w_m(\alpha + f_x + y^{jH} + \tau y^{jX}) \tag{51}$$

Since no firm will export without also producing for the domestic economy,[22] I also can split the profits for each type of firm into two components; profits due to domestic sales, $\pi^{iH}$ and profits due to exports $\pi^{iX}$, as in Melitz (2003):

$$\pi^{kH} = p^{kH} y^{kH} - w_f(\alpha + y^{kH}) \tag{52}$$

$$\pi^{jH} = p^{jH} y^{jH} - w_m(\alpha + y^{jH}) \tag{53}$$

[21] Since both countries are identical, it follows that each country will have $N^{dT}$ firms operating in equilibrium, and therefore the number of discriminatory firms in the world economy is $2N^{dT}$.

[22] Once $\alpha$ is paid to start producing goods, it always makes sense to produce some output for the domestic economy since producing goods at home has a smaller marginal cost.

$$\pi^{kX} = p^{kX}\hat{y}^{kX} - w_f(f_x + \tau y^{kX}) \tag{54}$$

$$\pi^{jX} = p^{jX}\hat{y}^{jX} - w_m(f_x + \tau y^{kX}) \tag{55}$$

Firms will then choose quantities to maximize each profit expression. By following the same profit maximization procedure described in Section 3.1,[23] one obtains:

$$\frac{p^{fH}}{w_f} = \frac{1}{\rho} \tag{56}$$

$$\frac{p^{dH}}{w_m} = \frac{1}{\rho} \tag{57}$$

$$\frac{p^{fX}}{w_f} = \frac{\tau}{\rho} \tag{58}$$

$$\frac{p^{dX}}{w_m} = \frac{\tau}{\rho} \tag{59}$$

Dividing (56) by (57) and (58) by (59), yields:

$$\frac{w_m}{w_f} = \frac{p^{dH}}{p^{fH}} = \frac{p^{dX}}{p^{fX}} \tag{60}$$

Which means that Proposition 3 still continues to hold with trade, i.e. there is a wage gap if and only if fair firms charge more for their varieties than discriminatory firms. Moreover, dividing (56) by (58) and (57) by (59) yields:

$$\frac{p^{fH}}{p^{fX}} = \frac{p^{dH}}{p^{dX}} = \frac{1}{\tau} \tag{61}$$

From which it follows that each firm type prices relatively lower in their home country, relative to the foreign market.

## 4.3 Short-run export market entry

Since I assume that an autarkic equilibrium is established before a country opens its borders to trade, firms will take the quantities and prices that were previously established in autarkic

[23] I.e. replace each output by their demand schedule, then differentiate these profit functions as in (18). Afterwards, substitute $\epsilon_D = \frac{1}{1-\rho}$ into these expressions, and then impose symmetry so that $j = d$ for $\forall j \in (0, 2N^{dT})$ and $k = f$ for $\forall k \in (2N^{dT}, 2N^{dT} + 2N^{fT})$.

equilibrium as *given* when they make their export market entry decision.

I make the following assumption concerning export market entry:

**Assumption 3.** *A firm will enter the export market if and only if they earn non-negative profits by unilaterally entering the export market from the autarkic equilibrium.*

I make the above assumption to rule out collusion between firms, and to make the export decision problem as simple as possible. Note that the above is not an equilibrium decision *per se*, but rather, a decision "off the equilibrium path" that will determine which equilibrium this model reaches in the long-run.[24]

Let the subscript $o$ denote output decisions of "other" firms. Likewise, let the subscript $u$ denote outputs for the firm unilaterally deciding whether to export. Since I am only considering the case where $1 < \omega < \beta$, equations (22) and (35) describe each type of firm's autarkic output, which the firms considering entering the export market take as given, i.e.:

$$y_o^{dH} = \frac{\alpha\rho}{1-\rho} \tag{62}$$

$$y_o^{fH} = \frac{L_f}{N^{f*}} - \alpha \tag{63}$$

I assume that firms will only choose quantities that clear the goods market, which means that firms must "know" the aggregate demand function for their variety, and act accordingly. It follows that firms will not unilaterally change the amount of output they are producing for the domestic market when they have the opportunity to export.[25] However, one can determine the quantity of output firms will choose to produce for the export market using (7) and goods market clearing, which imply that $\frac{\hat{y}_u^{dx}}{y_o^{dh}} = \left(\frac{p^{dH}}{p^{dX}}\right)^{\frac{1}{1-\rho}}$ must hold. Substituting (62) and (61) into this equation yields:

$$\hat{y}_u^{dX} = \tau^{\frac{-1}{1-\rho}} \frac{\alpha\rho}{1-\rho} \tag{64}$$

Also by (7) and good market clearing, $\frac{\hat{y}_u^{fX}}{y_o^{fH}} = \left(\frac{p^{fH}}{p^{fX}}\right)^{\frac{1}{1-\rho}}$. Substituting (63) and (61) into

[24] The decision to enter the export market could also have been modelled more formally as a game, with each firm considering the possible export choices of other firms, given their expectations concerning long-run equilibrium. This is certainly an approach worth pursuing, and it would be quite interesting to see if this modelling strategy changed the long-run predictions of my model in any major way. However, I leave this problem for future research.

[25] The proof for this is quite simple. First, consider a discriminatory firm's production decision. Goods market clearing requires that (36) still hold, which means that $\omega = \left(\frac{y_o^{fH}}{y_u^{dH}}\right)^{1-\rho}$. However, $\omega$ and $y_o^{fH}$ have not changed relative to their autarkic equilibrium values. As a result, $y_u^{dH}$ has to remain the same as in autarky. A similar proof follows for fair firms.

this equation yields:

$$\hat{y}_u^{fX} = \tau^{\frac{-1}{1-\rho}} \left( \frac{L_f}{N^{f*}} - \alpha \right) \tag{65}$$

I now use the above equations to determine when each type of firm will choose to enter the export market.

**Proposition 14.** *Discriminatory firms enter the export market if and only if* $f_x \tau^{\frac{\rho}{1-\rho}} \leq \alpha$

*Proof.* By Assumption 3, each discriminatory firms will export if and only if they earn non-negative profits by unilaterally entering the export market. Therefore, the following must hold:

$$\pi_u^{dX} = p^{dX} \hat{y}_u^{dX} - w_m(f_x + \tau \hat{y}_u^{dX}) \geq 0$$

Rearranging yields:

$$\frac{p^{dX}}{w_m} \hat{y}_u^{dX} - \tau \hat{y}_u^{dX} \geq f_x$$

Substitute the expression for $\frac{p^{dX}}{w_m}$ given by (59) into the above and rearrange, yielding:

$$\tau \hat{y}_u^{dX} \frac{1-\rho}{\rho} \geq f_x$$

Substituting $\hat{y}_u^{dX}$ given by (64) and rearranging yields:

$$\tau \left( \tau^{\frac{-1}{1-\rho}} \frac{\alpha\rho}{1-\rho} \right) \frac{1-\rho}{\rho} = \tau^{\frac{-\rho}{1-\rho}} \alpha \geq f_x \qquad \text{-Or}$$

$$f_x \tau^{\frac{\rho}{1-\rho}} \leq \alpha$$

□

Proposition 14 is not particularly surprising, since it shows that discriminatory firms will export only when trade costs are relatively small compared to their initial startup costs in the home market.[26] In other words, if trade costs are too high, discriminatory firms do not benefit by unilaterally entering the export market since that would result in them earning negative profits.

I now consider a fair firm's export decision.

**Proposition 15.** *Fair firms enter the export market if and only if* $\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x \tau^{\frac{\rho}{1-\rho}} - \alpha)}{1-\rho} + \phi_L$

[26] Note that since a discriminatory firm's scale does not depend on the equilibrium wage rate, the above proof also applies to the case where $\omega = \beta$.

*Proof.* By Assumption 3, each fair firm will export if and only if such a decision results in them earning non-negative profits in the export market. Therefore, the following must hold:

$$\pi_u^{fX} = p^{fX}\hat{y}_u^{fX} - w_f(f_x + \tau\hat{y}_u^{fX}) \geq 0$$

Rearranging yields:

$$\frac{p^{dX}}{w_f}\hat{y}_u^{fX} - \tau\hat{y}_u^{fX} \geq f_x$$

Substituting in the expression for $\frac{p^{dX}}{w_f}$ given by (58) yields, after some rearranging:

$$\tau\hat{y}_u^{fX}\frac{1-\rho}{\rho} \geq f_x$$

Substituting the expression for $\hat{y}_u^{fX}$ given by (65) yields:

$$\tau^{\frac{-\rho}{1-\rho}}\left(\frac{L_f}{N^{f*}} - \alpha\right)\frac{1-\rho}{\rho} \geq f_x \tag{66}$$

Rearranging yields:

$$\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}} - \alpha)}{1-\rho} + \frac{\alpha}{1-\rho}$$

Recall that $\phi_L \equiv \frac{\alpha}{1-\rho}$. Therefore:

$$\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}} - \alpha)}{1-\rho} + \phi_L$$

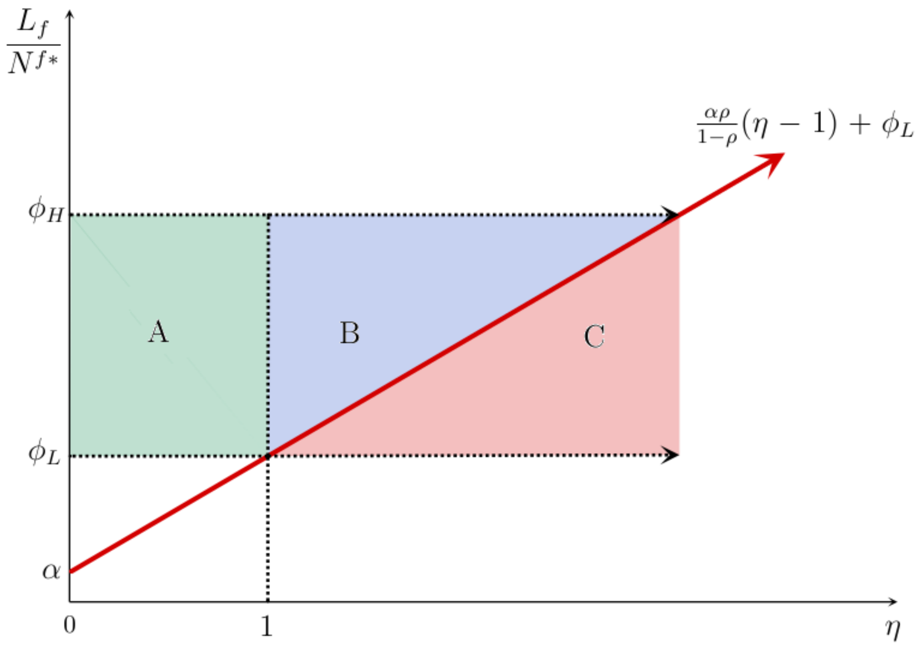
□

The inequality in Proposition 15 is much harder to give an intuitive interpretation. However, it can be used to show that as long as one firm chooses to export, then either *both* types of firms will choose to unilaterally export, or *only fair firms* will choose to export in equilibrium.

**Proposition 16.** *If* $1 < \omega < \beta$ *in autarky, then both types of firms will enter the export market if* $f_x\tau^{\frac{\rho}{1-\rho}} \leq \alpha$*, while only fair firms will enter the export market if* $f_x\tau^{\frac{\rho}{1-\rho}} > \alpha$ *and* $\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \phi_L$.

*Proof.* I prove this proposition with the aid of Figure 2. The vertical axis corresponds to $\frac{L_f}{N^{f*}}$, while the horizontal axis corresponds to $\frac{f_x\tau^{\frac{\rho}{1-\rho}}}{\alpha} \equiv \eta$.

Figure 2: Export Status Cases




If $1 < \omega < \beta$ in autarky, then by (40), it must be that $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$. This corresponds to regions A, B, and C in Figure 2.

By Proposition 14, discriminatory firms will only export when $f_x\tau^{\frac{\rho}{1-\rho}} \leq \alpha$. Rewriting this inequality in terms of $\eta$ yields the *discriminatory firm export inequality*, $\eta \leq 1$. Since I am only considering cases where $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$, this means that discriminatory firms export only when the parameters lie in region A.

By Proposition 15, fair firms will unilaterally decide to export if and only if $\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho}+\phi_L$. Rewriting this inequality in terms of $\eta$ yields the *fair firm export inequality*, $\frac{L_f}{N^{f*}} \geq \frac{\alpha\rho}{1-\rho}(\eta-1)+\phi_L$.

In Figure 2, $\frac{\alpha\rho}{1-\rho}(\eta-1)+\phi_L$ is graphed as a function of $\eta$. It follows from the fair firm export inequality that fair firms export whenever the parameters lie above this line. Since I am only considering cases where $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$, this corresponds to regions A and B.

The above implies that neither firm exports in region C. Since both types of firms export in region A, while only fair firms export in region B, the proposition follows.

□

Proposition 16 implies that a) if discriminatory firms export, fair firms will also export, and b) there exists a range of values of $\frac{L_f}{N^{f*}}$ for which fair firms are still willing to export, even when trade costs are high enough to discourage discriminatory firms from exporting. The reason for this is fairly straightforward; since I am only considering cases where there is a wage gap in autarky, fair firms have lower marginal costs than discriminatory firms because they only purchase under-priced female labour. This means that fair firms will be able to pay for higher trade costs than discriminatory firms.

It follows from Proposition 16 that there are two types of trade equilibria to consider; one where both firms export, and one where only fair firms export. Since the case where neither firm exports is uninteresting, I assume for the remainder of this section that $\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \phi_L$.

## 4.4 Long-run equilibrium with both types of firms exporting

Let $w_m^T$ denote the male wage rate in trade equilibrium, and let $w_f^T$ denote the female wage in trade equilibrium. Further let $\omega^T \equiv \frac{w_m^T}{w_f^T}$, so $\omega^T$ is the gender wage gap in trade equilibrium. As per Proposition 16, I assume that $f_x\tau^{\frac{\rho}{1-\rho}} \leq \alpha$ so that both types of firms enter the export market. Since there is an infinitely large pool of possible discriminatory entrants, it follows by Assumption 2 that discriminatory firms must still earn zero profits in equilibrium. I now solve for the level of total output that results in zero profits for a representative discriminatory firm:[27]

$$\pi^{dT} = p^{dH}y^{dH} + p^{dX}\hat{y}^{dX} - w_m^T(\alpha + f_x + y^{dH} + \tau y^{dX}) = 0$$

Rearranging:

$$\frac{p^{dH}}{w_m^T}y^{dH} + \frac{p^{dX}}{w_m^T}\hat{y}^{dX} - y^{dH} - \tau y^{dX} = \alpha + f_x$$

Substitute in optimal pricing given by (57) and (59)

$$\frac{y^{dH} + \tau\hat{y}^{dX}}{\rho} - y^{dH} - \tau y^{dX} = \alpha + f_x$$

Rearranging:

$$\frac{1-\rho}{\rho}\left(y^{dH} + \tau\hat{y}^{dX}\right) = \alpha + f_x$$

[27]Note that the relevant profits are overall profits, i.e. profits from the export market *and* home market. This may seem like an odd assumption since it means that a firm can, for example, earn positive profits in the home market according to (53), and then earn negative profits in the export market according to (55), which then both sum up to zero according to the discriminatory firm's zero profit condition. However, even if this happens a discriminatory firm will not wish to exit the export market since $f_x$ was *sunk* when the firm decided to enter the export market. This means that no firm would gain by deciding to only serve the home market, since they simply lose customers, without earning back their investment cost of $f_x$. Note, however, that earnings negative profits *overall*, i.e. in both markets together, is inconsistent with long-run equilibrium

Since $y^{dT} = y^{dH} + \tau \hat{y}^{dX}$:

$$\frac{1-\rho}{\rho} y^{dT} = \alpha + f_x$$

Rearranging the above, yields:

$$y^{dT} = \frac{(\alpha + f_x)\,\rho}{1-\rho} \tag{67}$$

Note that (67) shows that there is a *scale effect* for discriminatory firms in trade equilibrium, since $y^{dT} = \frac{(\alpha+f_x)\rho}{1-\rho} > \frac{\alpha\rho}{1-\rho} = y^d$. This is a result of the fixed cost that must be paid to enter the export market; since discriminatory firms costs have increased, they have to scale up their output to cover their increased costs.

While (67) describes the total amount of output produced by each discriminatory firm in equilibrium, to find the wage gap one must find the amount of output sold in the home market, since (36) still has to hold in equilibrium, i.e. $\omega^T = \frac{w_m^T}{w_f^T} = \left(\frac{y^{fH}}{y^{dH}}\right)^{1-\rho}$. [28] Substituting (67) into $y^{dT} = y^{dH} + y^{dX}$ and rearranging yields:

$$y^{dX} = \frac{(\alpha + f_x)\,\rho}{1-\rho} - y^{dH}$$

Since $y^{dX} = \tau \hat{y}^{dX}$ by (47), then:

$$\hat{y}^{dX} = \frac{1}{\tau}\left[\frac{(\alpha + f_x)\,\rho}{1-\rho} - y^{dH}\right] \tag{68}$$

Recall that by (7), $\frac{D^\upsilon}{D^\psi} = \left(\frac{p^\psi}{p^\upsilon}\right)^{\frac{1}{1-\rho}}$. Imposing goods market clearing in the home and foreign market for discriminatory firms yields:

$$\left(\frac{p^{dX}}{p^{dH}}\right)^{\frac{1}{1-\rho}} = \frac{y^{dH}}{\hat{y}^{dX}}$$

Substituting $\frac{p^{dX}}{p^{dH}} = \tau$ and (68) into the above, yields:

$$\tau^{\frac{1}{1-\rho}} = \frac{y^{dH}}{\frac{1}{\tau}\left[\frac{(\alpha+f_x)\rho}{1-\rho} - y^{dH}\right]}$$

Solving for $y^{dH}$:

[28] One could also use the amount of output sold in the foreign market to find the wage gap, since (36) can also be written as $\omega^T = \frac{w_m^T}{w_f^T} = \left(\frac{y^{fX}}{y^{dX}}\right)^{1-\rho}$ in the trade environment.

$$y^{dH} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \frac{(\alpha + f_x)\,\rho}{1-\rho} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} y^{dT} \tag{69}$$

where the last equality follows from (67).

Note that (69) shows that if there were no iceberg trade costs, half of the total output produced by a discriminatory firm would go to the home market.[29] Since it is easily shown that $\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}$ is strictly increasing in $\tau$ and bounded by 1, this means that discriminatory firms increase the portion of total output produced that is sold at home, the higher are the iceberg trade costs, which is an intuitive result.

Next, I solve for the relevant quantities produced by fair firms in equilibrium. Accounting for the workers who are now used to build ships for transporting goods abroad, the female labour market clearing condition becomes:

$$L_f = N^{fT}(\alpha + f_x + y^{fT}) \tag{70}$$

Assuming for the time being that all fair firms enter the market in equilibrium when $1 < \omega^T < \beta$,[30], I solve the above for $y^{fT}$, yielding;

$$y^{fT} = \frac{L_f}{N^{f*}} - \alpha - f_x \tag{71}$$

The above implies that there is a *reverse* scale effect for fair firms, since $y^{fT} = \frac{L_f}{N^{f*}} - \alpha - f_x < \frac{L_f}{N^{f*}} - \alpha = y^f$. Since fair firms continue to earn positive profits whenever both types of firms export and $1 < \omega^T < \beta$,[31] they do not need to scale up their output to remain in the market, as was the case for discriminatory firms. However, since some female workers are now used to build ships to service the export market, less women are directly engaged in the production of output for fair firms, which leads a fall in output.

I now solve for the level of output sold to the domestic market. Substituting the above into (46) and rearranging yields:

$$y^{fX} = \frac{L_f}{N^{f*}} - \alpha - f_x - y^{fH} \tag{72}$$

Since $y^{fT} = y^{fH} + \tau\hat{y}^{fX}$:

$$\hat{y}^{fX} = \frac{1}{\tau}\left(\frac{L_f}{N^{f*}} - \alpha - f_x - y^{fH}\right) \tag{73}$$

[29]If $\tau = 1$, then $y^{dH} = \frac{1}{2}\frac{\rho(\alpha+f_x)}{1-\rho} = \frac{1}{2}y^{dT}$

[30]This is proven in the Mathematical Appendix, Appendix D.

[31]See the Mathematical Appendix, Appendix D.

Using (7) once again, it follows that:

$$\left(\frac{p^{fX}}{p^{fH}}\right)^{\frac{1}{1-\rho}} = \frac{y^{fH}}{\hat{y}^{fX}}$$

Substituting $\frac{p^{fX}}{p^{fH}} = \tau$ and (73) into the above yields:

$$\tau^{\frac{1}{1-\rho}} = \frac{y^{fH}}{\frac{1}{\tau}\left(\frac{L_f}{N^{f*}} - \alpha - f_x - y^{fH}\right)}$$

Solving for $y^{fH}$:

$$y^{fH} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right) = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} y^{fT} \tag{74}$$

Where the last equality follows from (71). The following proposition then follows:

**Proposition 17.** *If $1 < \omega^T < \beta$ in the trade equilibrium, $1 < \omega < \beta$ in autarky, and both types of firms export in equilibrium, then the gender wage gap falls relative to autarky.*

*Proof.* Note that $\omega$ shall continue to denote the autarkic wage gap, while $\omega^T$ denotes the gender wage gap in trade equilibrium. Equation (36) still has to hold in trade equilibrium, i.e. $\omega^T = \left(\frac{y^{fH}}{y^{dH}}\right)^{1-\rho}$. Assuming that $1 < \omega^T < \beta$, one can then substitute (69) and (74) into this expression, yielding.

$$\frac{w_m^T}{w_f^T} = \left[\frac{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)}{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\rho}{1-\rho}}\right]^{1-\rho} = \left[\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha+f_x)\rho}\right]^{1-\rho} \tag{75}$$

Next, suppose that Proposition 17 did not hold. Then both firms export and $\frac{w_m^T}{w_f^T} \geq \frac{w_m}{w_f}$. Substituting (75) and the autarkic wage gap given by (37) into this expression yields:

$$\left[\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha+f_x)\rho}\right]^{1-\rho} \geq \left[\left(\frac{L_f}{N^{f*}} - \alpha\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho}$$

Rearranging yields:

$$\left(\frac{L_f}{N^{f*}}\right) f_x \leq 0$$

Which is a contradiction. □

Note that the expression for $\omega^T$ given by (75) does not depend on $\tau$. The reason for this should be clear from its derivation; each type of firm faces the same iceberg trade costs, and therefore each firm ships the same portion of their total output abroad. As a result, the increase in the domestic price of each firm's variety due to increased shipping abroad[32] is the same for each firm. Therefore, since iceberg trade costs do not affect the ratio of final goods prices between firm types, they also do not affect the size of the gender wage gap.

On the other hand, $f_x$ affects the equilibrium gender wage gap. This is related to the scale effect I mentioned earlier, where discriminatory firms scale up their output due to increased trade costs in the export market.[33] The positive scale effect leads to a fall in the gender wage gap by forcing some discriminatory firms to exit both the home and export market.

To see this formally, consider the male labour clearing condition when discriminatory firms export:

$$L_m = N^{dT}(\alpha + f_x + y^{dT}) \tag{76}$$

Rearranging the above yields:

$$N^{dT} = \frac{L_m}{\alpha + f_x + y^{dT}}$$

Substituting $y^{dT}$ from (67) into the above yields:

$$N^{dT} = \frac{L_m(1-\rho)}{\alpha + f_x} \tag{77}$$

The following proposition then follows:

**Proposition 18.** *When both types of firms enter the export market and $1 < \omega^T < \beta$, then there is an exit effect due to trade for discriminatory firms, i.e. $N^{dT} < N^d$.*

*Proof.* Suppose not. Then both types of firms enter the export market and $N^{dT} \geq N^d$. By (77) above and $N^d = \frac{L_m(1-\rho)}{\alpha}$ this means that:

$$\frac{L_m(1-\rho)}{\alpha + f_x} \geq \frac{L_m(1-\rho)}{\alpha}$$

Which, when rearranged, yields:

$$f_x \leq 0$$

[32] Since demand curves are downward sloping, if any output is shipped abroad thus decreasing the domestic supply of that variety, then goods market clearing requires that the equilibrium price of that variety rise.

[33] The reverse scale effect that applied to fair firms also decreases the gender wage gap, since decreasing the quantity of output sold by fair firms puts upward pressure of the price of each fair variety. Since all firms charge the same markups over marginal cost, this causes the female wage rate to rise relative to the male wage rate. However, this is not a particularly important result, since I could have also chosen to model the fixed cost of entry as a cost in terms of *output*, in which case there would be no reverse scale effect, while there would *still* be a positive scale effect for discriminatory firms. Since the discriminatory firm scale effect is more robust, I focus on its effect on the gender wage gap over the reverse scale effect.

Which contradicts $f_x > 0$ $\square$

Note that the proof of Proposition 18 makes it clear that the fixed costs to enter the export market are necessary for the exit effect of trade to occur in this model. Intuitively, this is because the extra costs associated with trade lead to more labour market competition within a country, which causes discriminatory firms to exit, and a resulting decline in the gender wage gap.

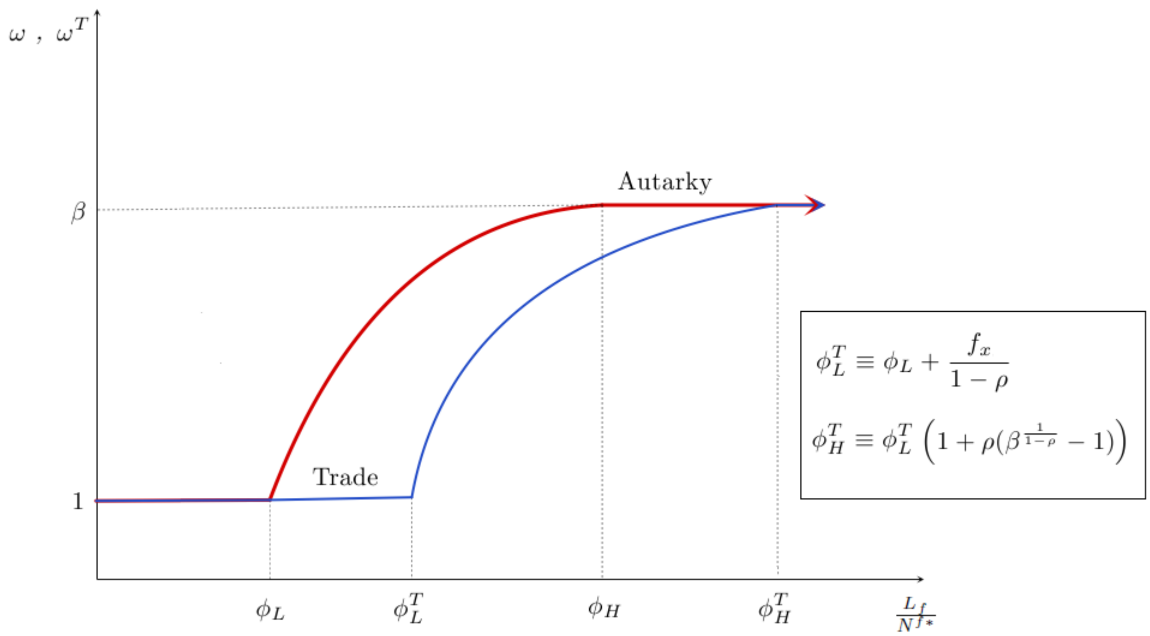
One can think of the process as follows; when a country opens up its borders to trade, discriminatory firms all have an incentive to enter the export market due to increased profit opportunities. However, as fair firms start scaling up their overall output to cover the increased costs of trade, some discriminatory firms as forced to exit the industry, since there no longer enough male workers to support the same number of firms *and* increased scale of production by each discriminatory firm. As a result, competitive pressure forces some discriminatory firms to exit both the home *and* export markets.

As a result of this exit effect, there will be relatively *less* competition for male labour in trade than there was in autarky, i.e. $\frac{N^{dT}}{L_m} < \frac{N^d}{L_m}$. Recalling Propositions 10 and 11 from Section 3, which showed that a gender wage gaps only exists when $\frac{N^d}{L_m} > \frac{N^f}{L_f}$, this means that relatively less fair firms will be necessary to eliminate the gender wage gap. Consequently, if all the fair firms that produced in autarky remain in the market once a country has opened its border to trade, then the gender wage gap will fall, since there is less competitive pressure in the male labour market than the female labour market.

While the above discussion has only focused on the "middle case" where $1 < \omega^T < \beta$, to fully appreciate the effect trade has on the gender wage gap, one should still consider an economy's behaviour around the corners where $\omega^T = 1$ and $\omega^T = \beta$. It turns out, however, that the only appreciable difference between autarky and trade for these cases is that the values of $\frac{L_f}{N^{f*}}$ that result in corners shift. Figure 3, below, shows these shifts, as I graph $\omega^T$ as a function of $\frac{L_f}{N^{f*}}$ alongside the function for $\omega$ in autarky. The interested reader may find a more formal derivation of these results in the Mathematical Appendix, Appendix D.

Figure 3 shows that even if the wage gap is at its upper bound in autarky, then with trade the wage gap may still fall so that $\omega^T < \beta$ as long as $\frac{L_f}{N^{f*}}$ is not overly large. Furthermore, there are now values for $\frac{L_f}{N^{f*}}$ which would have resulted in positive gender wage gap in autarky, that with trade result in the disappearance of the gender wage gap, i.e. $\omega^T = 1$. This means that *trade has lowered the necessary number of fair firms that will lead to the disappearance of a gender wage gap.* This is due to the discriminatory firm exit effect; since there are less discriminatory firms in equilibrium, a smaller number of fair firms are necessary to match the degree of competition in the male labour market.

Figure 3: Gender wage gaps as a function of $\frac{L_f}{N^{f*}}$: $\left(f_x \tau^{\frac{\rho}{1-\rho}} \leq \alpha\right)$




## 4.5 Long-run equilibrium with only fair firms exporting

As was shown in Section 5.3, fair firms are the only exporting firms when $f_x \tau^{\frac{\rho}{1-\rho}} > \alpha$ and $\frac{L_f}{N^{f*}} \geq \frac{\rho(f_x \tau^{\frac{\rho}{1-\rho}} - \alpha)}{1-\rho} + \phi_L$. This means that discriminatory firms only produce for the domestic market, and therefore the profit expression for discriminatory firms is identical to their profit expression in autarky. Moreover, since discriminatory firms still need to earn zero profits by Assumption 2, this means that discriminatory firms produce the same quantity of output for their home market as they did in autarky. Therefore:

$$y^{dH} = \frac{\alpha\rho}{1-\rho} \tag{78}$$

For now, I simply assume that all potential fair firms enter the market and export in equilibrium, so that $N^{fT} = N^{f*}$.[34] I may then make use of (74) which I derived in the previous section, i.e. $y^{fH} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)$. The following proposition then follows:

**Proposition 19.** *If* $1 < \omega^T < \beta$ *in the trade equilibrium,* $1 < \omega < \beta$ *in autarky,* $N^{fT} = N^{f*}$*, and only fair firms export in trade equilibrium, then the gender wage gap falls relative to autarky.*

[34] Unlike the case where both firms export, $1 < \omega^T < \beta$ is not sufficient to guarantee that all fair firms enter the export market. This is not a crucial point, but is proven in the Mathematical Appendix, Appendix E.

*Proof.* Since $\omega^T = \left(\frac{y^{fH}}{y^{dH}}\right)^{1-\rho}$, substituting (78) and (74) into this expression yields:

$$\omega^T = \left( \frac{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)}{\frac{\alpha\rho}{1-\rho}} \right)^{1-\rho} = \left[ \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \left(\frac{L_f}{N^{f*}} - \alpha - f_x\right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} \tag{79}$$

Next, suppose that the proposition did not hold, so that $\omega^T \geq \omega$. Since $1 < \omega < \beta$ and $1 < \omega^T < \beta$ by assumption, the autarkic wage gap is defined (37), while the gender wage gap in trade equilibrium is defined by the above expression. It follows that:

$$\left[ \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \left(\frac{L_f}{N^{f*}} - \alpha - f_x\right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} \geq \left[ \left(\frac{L_f}{N^{f*}} - \alpha\right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho}$$

Which, when rearranged, yields:

$$\frac{\frac{L_f}{N^{f*}} - \alpha}{1+\tau^{\frac{\rho}{1-\rho}}} + \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} f_x \leq 0$$

Which is a contradiction, since $\left(\frac{L_f}{N^{f*}} - \alpha\right) > 0$ if $1 < \omega < \beta$. □

While Proposition 19 shows that the gender wage gap falls relative to autarky when only fair firms export, this cannot be due to an exit effect as before, since discriminatory firms do not enter the export market. As a result, there is no scale effect that leads to discriminatory firm exit.[35]

Instead, wage gaps fall in this case because access to foreign markets *increases* the relative market power of fair firms compared to discriminatory firms, which I will call the *consumer base effect* of trade. While discriminatory firms only have access to $L_m + L_f$ consumers in their home country, fair firms now have access to $2(L_m + L_f)$ consumers. Thus, fair firms can offer smaller quantities of their varieties in each market, raising the price of their varieties compared to the corresponding prices of discriminatory firms. Moreover, since all firms charge the same markups, the increased price of fair varieties relative to discriminatory prices causes the female wage rate to rise relative to the male wage rate. Thus, gender wage gaps fall.

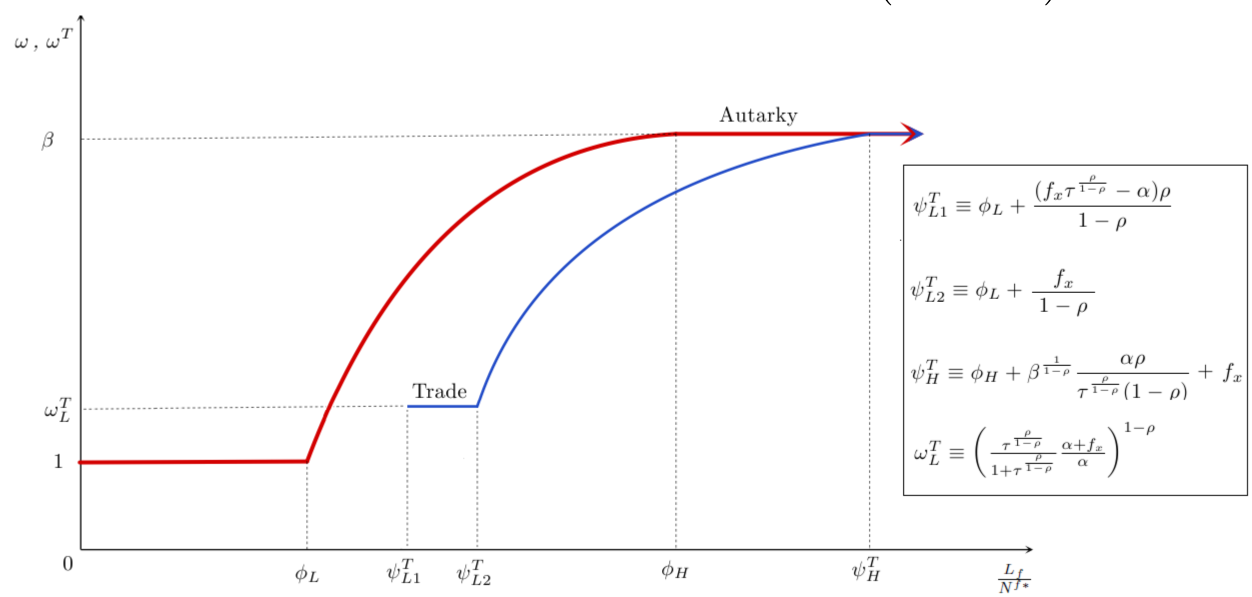
It is worth noting, however, that the consumer base effect also occurs when both firms export. On the other hand, since both firms decrease the supply of output in their home

[35] Note that there is still a reverse scale effect for fair firms, which also works to decrease the gender wage gap. However, since result is not robust to differing specifications for how the fixed cost is to be paid (see footnote 33), I do not discuss this result in detail.

countries by the same proportions in this case,[36] the consumer base effect does not favour one firm over the other, and thus, has no effect on the gender wage gap.

I now plot $\omega^T$ as a function of $\frac{L_f}{N^{f*}}$ alongside the function for $\omega$ in autarky for the case where $f_x\tau^{\frac{\rho}{1-\rho}} > \alpha$ in Figure 4, to further illustrate this result.[37] Note that whenever $\frac{L_f}{N^{f*}} < \frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \phi_L$, fair firms do not chose to export, so there is no trade equilibrium along this interval.

Figure 4: Gender wage gaps as a function of $\frac{L_f}{N^{f*}}$: $\left(f_x\tau^{\frac{\rho}{1-\rho}} > \alpha\right)$




As in the case where both firms export, the values of $\frac{L_f}{N^{f*}}$ that result in the highest possible wage gap of $\beta$ have shifted to the right. This is simply because the consumer base effect puts upward pressure on female wages, which means that discriminatory firms who were previously willing to hire women in autarky no longer hire women. As a result, fair firms absorb the entire female labour force, and each discriminatory firm's taste for discrimination no longer binds in equilibrium.

Note, however, that the wage gap never disappears when only fair firms export.[38] The reason for this is that when there are many fair firms in autarky, i.e. $\frac{L_f}{N^{f*}}$ is small, the wage gap is not particularly large. As a result, fair firms do not have a significant cost advantage compared to discriminatory firms, and therefore do not earn very large profits. Since fair firms earn relatively small profits, some fair firms may be forced to exit the market in the trade equilibrium, since the increased upward pressure on the female wage rate, combined

[36]See the derivation for (75).

[37]Refer to the Mathematical Appendix, Appendix E for the derivation of these results.

[38]This is proven in the Mathematical Appendix, Appendix E.

with the export costs, increases their overall costs beyond their zero profit threshold. Therefore, the market can no longer support all potential entrants in the trade equilibrium, and $N^{fT} < N^{f*}$.

However, the remaining fair firms will produce at a higher scale than discriminatory firms, since fair firms pay higher fixed costs by entering the export market, and thus need to produce more output to break even. Since fair firms produce at a higher scale when they earn zero profits, then it must be that $\frac{N^{dT}}{L_m} > \frac{N^{fT}}{L_f}$, which means that there will be less competition for female workers, relative to male workers, and therefore there is still a positive gender wage gap.[39] However, even though a positive gender wage gap still exists, as Figure 4 makes clear, this wage gap will still be below the wage gap that existed in autarky, which means that trade still decreases the gender wage gap for this special case as well.

Putting this complication aside, note the similarities between Figure 3 and 4. Both involve gender wage gap functions that lie below the autarky gender wage gap function, and show that the values of $\frac{L_f}{N^{f*}}$ that lead to the maximum possible wage gap must rise compared to autarky. However, the mechanisms that lead to these similar looking pictures are quite different. When both types of firms export, the discriminatory firm exit effect primarily drives the gender wage gap down. When only fair firms export, the consumer base effect is primary responsible for lowering the wage gap. This difference between the exit effect and the consumer base effect turns out to matter when one considers the relationship between the gender wage gap and increased trade due to falling trade costs.

## 4.6 Trade and gender wage gap changes: Movement from autarky to trade vs. falling trade costs

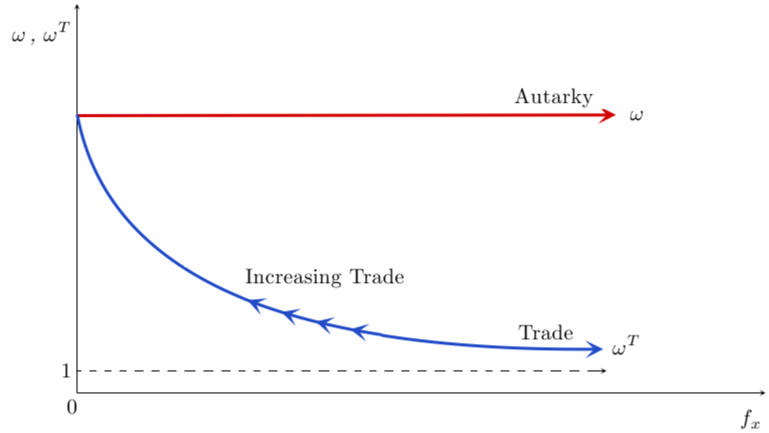
To demonstrate the distinction between movement from autarky to trade, and an increase in trade due to falling trade costs, I now plot $\omega^T$ and $\omega$ as a function of $f_x$ when both firms export in trade equilibrium. Since this requires that $f_x$ satisfy $f_x \tau^{\frac{\rho}{1-\rho}} \leq \alpha$, Figure 5 is only plotted for relatively "small" values of $f_x$.

Note that Figure 5, below, is still consistent with the story told in Section 4.4, since it shows that any movement from autarky to trade, i.e. a jump from the red autarky line to the blue trade line, leads a fall in the gender wage gap. However, the diagram also shows that decreasing $f_x$ *within* the trade equilibrium, which is equivalent to moving along the blue trade line, increases the gender wage gap. This is because decreasing $f_x$ dampens the discriminatory firm exit effect, since discriminatory firms no longer have to scale up their output as much to cover their costs. This means that the market can support a greater

[39] This is proven formally in the Mathematical Appendix, Appendix E.

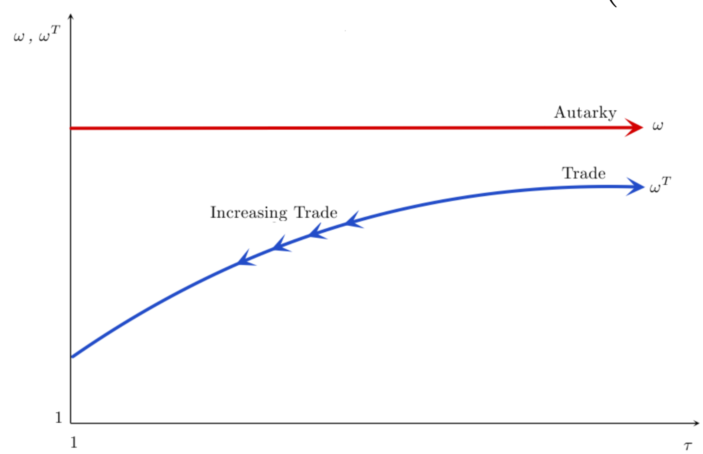
number of discriminatory firms in equilibrium, which increases the demand for male labour, relative to the demand for female labour. As a result the male wage rate appreciates relative to the female wage.

Figure 5: Gender wage gaps as a function of $f_x$: $\left(f_x\tau^{\frac{\rho}{1-\rho}} \leq \alpha\right)$




On the other hand, consider the effect of falling iceberg trade costs. Since iceberg trade costs do not affect the gender wage gap when both firms export, I only consider the case where only fair firms export, so $f_x\tau^{\frac{\rho}{1-\rho}} > \alpha$.

Figure 6: Gender wage gaps as a function of $\tau$: $\left(f_x\tau^{\frac{\rho}{1-\rho}} > \alpha\right)$




As Figure 6 shows, both a movement from autarky to trade and an increase in trade due to a decrease in iceberg trade costs will drive down the equilibrium gender wage gap. This is because decreased iceberg trade costs amplify the consumer base effect for fair firms. As iceberg trade costs fall, less goods are lost in transit. This allows fair firms to gain relatively more monopoly power compared to discriminatory firms, leading to higher equilibrium female wages as fair firms increase their relative prices.

As a result, the story obtained by Figure 5 and 6 is that increased trade between two countries due to a decrease in trade costs will have an *ambiguous* effect on gender wage gaps. Trade will decrease the gender wage gap within trade equilibrium only if increased trade "rewards" fair firms over discriminatory firms. This occurs when trade costs are sufficiently high to keep discriminatory firms from entering the export market, and therefore, the consumer base effect only benefits fair firms.

On the other hand, if trade costs are low so that discriminatory firms enter the export market, the consumer base effect from decreasing iceberg trade costs has no effect on gender wage gaps since both types of firms gain equal degrees of monopoly power. However, decreasing the fixed costs of entry will primarily benefit discriminatory firms, since these decreased costs will allow more discriminatory firms to enter the market. This will then increase the gender wage gap, since discriminatory firms only hire men.

This is an important result, since it sheds some light on the basis for the conflicting empirical findings on the effect of trade on gender wage gaps. Since an observed increase in trade between two countries could be due to falling per-unit costs, falling fixed costs, or some combination of the two, it is not clear whether gender wage gaps will rise or fall when trade increases according to this simple model.

However, a word of caution on comparing this particular model to empirical studies. While decreasing $f_x$ and $\tau$ will certainly increase the absolute volume of trade between two countries, both Black and Brainerd (2004) and Menon and Meulen Rodgers (2009) measure an increase in trade as an increase in either ratio of imports to exports, or imports to GDP.[40] According to the model developed in this paper, a decrease in $f_x$ will increase the absolute volume of trade, but will not change the value of either of these measures of trade volume when both types of firms export.[41] This result directly follows from the fact that $y^{fX}$ and $y^{dX}$ are always the same fraction of each firm's total output. This means that the fraction of GDP shipped abroad depends entirely on the iceberg trade costs, and therefore any effect that $f_x$ has on the absolute volume of trade will not be captured by an imports to GDP measure of trade volume.

However, this is largely due to the highly simplified nature of my model. If I were to

[40] As does Ben-Yahmed (2011), but she also supplements these regressions with alternative measures of "increased trade," namely, ease of international market access.

[41] This is quite tedious to show formally, but can be easily seen for the special case where $\tau = 1$. Let $X \equiv p^{dX} N^{dT} y^{dX} + p^{fX} N^{fT} y^{fX}$, and let $Y \equiv p^{dX} N^{dT} y^{dT} + p^{fH} N^{fT} y^{fT}$. The ratio of exports to GDP is $\frac{X}{Y} = \frac{p^{dX}}{p^{dH}} \frac{N^{dT} y^{dX} + \frac{p^{fX}}{p^{dX}} N^{fT} y^{fX}}{N^{dT} y^{dT} + \frac{p^{fH}}{p^{dH}} N^{fT} y^{fT}}$. When $\tau = 1$, this can be rewritten as: $\frac{1}{2} \frac{N^{dT} y^{dT} + \frac{w_m}{w_f} N^{fT} y^{fT}}{N^{dT} y^{dT} + \frac{w_m}{w_f} N^{fT} y^{fT}} = \frac{1}{2}$, i.e. half of the output produced is shipped abroad, while the other half remains in the home country.

account for, say, domestic industries that never export, such as hairdressers, a fall in the fixed costs of trade should increase the ratio of imports to GDP, since decreasing trade barriers should not affect the output of the purely domestic industry. This would perhaps be a useful addition to the model, but will not be pursued in detail here. Rather, I simply note that even though my model does not *directly* predict that a decrease in fixed export costs will increase import to GDP ratios, this model still suggests that decreasing fixed export costs will increase the gender wage gap. Therefore, if one has reason to believe that a decrease in fixed export costs should lead to an increase in import to GDP ratios in the real world, changes in fixed export costs are still worth considering as a possible explanation for why some of the empirical work on gender wage gaps has shown that increased trade is associated with an increase in the gender wage gap.

# 5 Consumer Welfare

Having now shown that trade can either increase and decrease gender wage gaps, I now consider the effect of trade on consumer welfare. Unfortunately, while trade theory involving imperfect competition and increasing returns to scale has been successful in identifying new sources of gains from trade, the welfare gains from trade tend to be ambiguous, depending on the exact form of imperfect competition and the size of trade costs.[42] The model developed in this paper is no different. It turns out that trade always makes women better off, but only sometimes makes men better off as well, depending on the size of the trade costs.

## 5.1 Consumer welfare as a function of the gender wage gap

Following Melitz (2003), I use the indirect utility function to measure the relative welfare of each consumer. By definition:

$$W_i \equiv U_i(d_i(\upsilon)|_{\mathbf{p}*}) = \left( \int_0^N \left( \frac{I_i}{(p(\upsilon))^{\frac{1}{1-\rho}} \int_0^N p(\psi)^{\frac{-\rho}{1-\rho}} \, d\psi} \right)^{\rho} d\upsilon \right)^{\frac{1}{\rho}} \Bigg|_{\mathbf{p}*} \tag{80}$$

Where $\mathbf{p}*$ is a vector of equilibrium prices.

Rearranging the above yields:

$$W_i = I_i \left( \int_0^N p(\upsilon)^{\frac{-\rho}{1-\rho}} \, d\upsilon \right)^{\frac{1-\rho}{\rho}} \Bigg|_{\mathbf{p}*} \tag{81}$$

[42]See, for example, Helpmann and Krugman (1985).

Let $W_i^A$ denote the welfare of consumer $i$ in autarky, let $W_i^{T1}$ denote the welfare of consumer $i$ in trade equilibrium where only fair firms export, and let $W_i^{T2}$ denote the welfare of consumer $i$ in trade equilibrium where both firms export. Since each firm types charges the same price, I can evaluate the above integral, and then write consumer welfare in each case as:

$$W_i^A = I_i \left( \left(p^f\right)^{\frac{-\rho}{1-\rho}} N^f + \left(p^d\right)^{\frac{-\rho}{1-\rho}} N^d \right)^{\frac{1-\rho}{\rho}} \tag{82}$$

$$W_i^{T1} = I_i \left( \left(p^{fH}\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(p^{fX}\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(p^{dH}\right)^{\frac{-\rho}{1-\rho}} N^{dT} \right)^{\frac{1-\rho}{\rho}} \tag{83}$$

$$W_i^{T2} = I_i \left( \left(p^{fH}\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(p^{fX}\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(p^{dH}\right)^{\frac{-\rho}{1-\rho}} N^{dT} + \left(p^{dX}\right)^{\frac{-\rho}{1-\rho}} N^{dT} \right)^{\frac{1-\rho}{\rho}} \tag{84}$$

Using the price ratio expressions given by (20), (19), (56) (57), (58), and (59) it follows that, $p^f = \frac{w_f}{\rho}$, $p^d = \frac{w_m}{\rho}$, $p^{dH} = \frac{w_m^T}{\rho}$, $p^{dX} = \tau \frac{w_m^T}{\rho}$ , $p^{fH} = \frac{w_f^T}{\rho}$, $p^{fX} = \tau \frac{w_f^T}{\rho}$. Substituting these expressions in the above welfare expressions yields after some minor rearranging:

$$W_i^A = I_i \rho \left( \left(w_f\right)^{\frac{-\rho}{1-\rho}} N^f + \left(w_m\right)^{\frac{-\rho}{1-\rho}} N^d \right)^{\frac{1-\rho}{\rho}} \tag{85}$$

$$W_i^{T1} = I_i \rho \left( \left(w_f^T\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(\tau w_f^T\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(w_m^T\right)^{\frac{-\rho}{1-\rho}} N^{dT} \right)^{\frac{1-\rho}{\rho}} \tag{86}$$

$$W_i^{T2} = I_i \rho \left( \left(w_f^T\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(\tau w_f^T\right)^{\frac{-\rho}{1-\rho}} N^{fT} + \left(w_m^T\right)^{\frac{-\rho}{1-\rho}} N^{dT} + \left(\tau w_m^T\right)^{\frac{-\rho}{1-\rho}} N^{dT} \right)^{\frac{1-\rho}{\rho}} \tag{87}$$

I now turn the above expressions into functions of the equilibrium wage gap, $\omega$, by specifying the income of each consumer. Since fair firms earn positive profits whenever $\omega > 1$, these profits should be included as a part of each consumer's income. However, this complicates the consumer welfare algebra without providing any important insights.

Therefore, I make the following assumptions concerning the size of the labour force to make the effect of profits on income asymptotically negligible. Suppose that each consumer shares the entirety of the aggregate profits earned by fair firms, so that each consumer receives $\frac{\pi^f N^f}{L_m + L_f}$ in profits. Note that $\pi^f$ and $N^f$ do not directly depend on $L_m$ in equilibrium. With this in mind, I assume that $L_m$ is arbitrarily large, so that the profits received by each consumer is arbitrarily close to zero. As a result, I obtain a good approximation of consumer welfare by simply considering each consumer's labour income.[43]

Since each consumer is endowed with one unit of male labour or female labour, respectively, the income of a male worker is $w_m$, while the income of a female worker is $w_f$. Let $W_m$ denote the welfare of a male consumer, and $W_f$ denote the welfare of a female consumer. Substituting each consumer's respective income into the appropriate welfare measures yields, after some minor rearranging:

$$W_m^A(\omega) = \rho \left( (\omega)^{\frac{\rho}{1-\rho}} N^f + N^d \right)^{\frac{1-\rho}{\rho}} \tag{88}$$

$$W_f^A(\omega) = \rho \left( N^f + (\omega)^{\frac{-\rho}{1-\rho}} N^d \right)^{\frac{1-\rho}{\rho}} \tag{89}$$

$$W_m^{T1}(\omega^T) = \rho \left( \left(\omega^T\right)^{\frac{\rho}{1-\rho}} N^{fT} \left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right) + N^{dT} \right)^{\frac{1-\rho}{\rho}} \tag{90}$$

$$W_f^{T1}(\omega^T) = \rho \left( N^{fT} \left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right) + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}} N^{dT} \right)^{\frac{1-\rho}{\rho}} \tag{91}$$

$$W_m^{T2}(\omega^T) = \rho \left( \left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right) \left( \left(\omega^T\right)^{\frac{\rho}{1-\rho}} N^{fT} + N^{dT} \right) \right)^{\frac{1-\rho}{\rho}} \tag{92}$$

$$W_f^{T2}(\omega^T) = \rho \left( \left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right) \left( N^{fT} + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}} N^{dT} \right) \right)^{\frac{1-\rho}{\rho}} \tag{93}$$

Since the case where $\omega = 1$ is no different from the Krugman (1980) model with a single type of labourer, I only consider the case where $\omega > 1$. I also assume that $N^f = N^{fT} = N^{f*}$ for simplicity.[44] Moreover, I assume that $\omega < \beta$ so the equilibrium expressions for $N^d$ and $N^{dT}$ given by (30) and (77) are valid. The welfare measures then can be written as:

$$W_m^A(\omega) = \rho \left( (\omega)^{\frac{\rho}{1-\rho}} N^{f*} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}} \tag{94}$$

$$W_f^A(\omega) = \rho \left( N^{f*} + (\omega)^{\frac{-\rho}{1-\rho}} \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}} \tag{95}$$

[43]Note that I could not make profits negligible if I were to consider aggregate welfare gains by making use of the indirect utility function of a single consumer who received the economy's aggregate income. Since having to include profits would needlessly complicate the algebra, I only consider welfare gains for *individual* consumers. As shall be shown, this is sufficient to show that aggregate welfare gains exist for some cases.

[44]See the Mathematical Appendix, Appendix C, for formal proofs that show when this will occur in equilibrium.

$$W_m^{T1}(\omega^T) = \rho\left(\left(1+(\tau)^{\frac{-\rho}{1-\rho}}\right)N^{f*}\left(\omega^T\right)^{\frac{\rho}{1-\rho}} + \frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}} \tag{96}$$

$$W_f^{T1}(\omega^T) = \rho\left(\left(1+(\tau)^{\frac{-\rho}{1-\rho}}\right)N^{f*} + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}} \tag{97}$$

$$W_m^{T2}(\omega^T) = \rho\left(\left(1+(\tau)^{\frac{-\rho}{1-\rho}}\right)\left(N^{f*}\left(\omega^T\right)^{\frac{\rho}{1-\rho}} + \frac{L_m(1-\rho)}{\alpha+f_x}\right)\right)^{\frac{1-\rho}{\rho}} \tag{98}$$

$$W_f^{T2}(\omega^T) = \rho\left(\left(1+(\tau)^{\frac{-\rho}{1-\rho}}\right)\left(N^{f*} + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha+f_x}\right)\right)^{\frac{1-\rho}{\rho}} \tag{99}$$

Casual inspection of the above welfare measures show that the male welfare measures increase with $\omega$, while the female welfare measure decreases with $\omega$, as one would expect.[45] With this in mind, I now consider changes in each type of consumer's welfare as one moves from autarky to trade.

## 5.2 Women's welfare

Intuitively, women stand to gain the most from moving from autarky to trade, since they get to enjoy an increase in the number of varieties as in Krugman (1980), but also enjoy further gains since the female wage appreciates relative to the male wage. As a result, one would expect women to always gain from moving to trade from autarky. I now prove that this is indeed true if we are in the "middle case" where $1 < \omega^T < \beta$ in trade, $1 < \omega < \beta$ in autarky, and $N^{fT} = N^{f*}$.

**Proposition 20.** *If $1 < \omega^T < \beta$ in trade, $1 < \omega < \beta$ in autarky, $N^{fT} = N^{f*}$, and only fair firms export, then there are welfare gains from trade for women.*

*Proof.* Suppose not, so that $1 < \omega^T < \beta$, $1 < \omega < \beta$, and women are either just as well off or worse in trade equilibrium, i.e. $W_f^A(\omega) \geq W_f^{T1}(\omega^T)$. By (95) and (97) This means that:

$$\rho\left(N^{f*} + (\omega)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}} \geq \rho\left(\left(1+(\tau)^{\frac{-\rho}{1-\rho}}\right)N^{f*} + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}}$$

Rearranging the above yields:

[45] One can see this by noting that $\omega$ enter the male welfare measure as $(\omega)^{\frac{\rho}{1-\rho}}$, which obviously increases with $\omega$, while the female welfare measure involves a $(\omega)^{\frac{-\rho}{1-\rho}}$ term, which clearly decreases with $\omega$

$$0 \geq N^{f*}\tau^{\frac{-\rho}{1-\rho}} + \frac{L_m(1-\rho)}{\alpha}\left(\left(\omega^T\right)^{\frac{-\rho}{1-\rho}} - \omega^{\frac{-\rho}{1-\rho}}\right)$$

Which is a contradiction since right hand side of the above expression is clearly positive, since $\omega^T < \omega$ by Proposition 21, which implies that $\left(\omega^T\right)^{\frac{-\rho}{1-\rho}} - \omega^{\frac{-\rho}{1-\rho}} > 0$. □

Proposition 22 is almost trivially true, since a movement from autarky to trade equilibrium when only fair firms export does not involve any loss in the number of varieties, and women are richer in this equilibrium since their wages rise. So it is no surprise that women are better off.

I now show that women are also better off in trade equilibrium when both types of firms export. To do this, I first need to prove this intermediate result.

**Proposition 21.** *If $1 < \omega^T < \beta$ and both types of firms export in trade equilibrium, a woman's welfare is strictly decreasing in $\tau$ and $f_x$.*

*Proof.* If both types of firm's export in trade equilibrium, a women's welfare in given by (99). Since $\omega^T$ does not depend on $\tau$ by (75), directly differentiating this expression with respect to $\tau$ yields:

$$\frac{\partial W_f^{T2}}{\partial \tau} = \rho\left(N^{f*} + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha + f_x}\right)^{\frac{1-\rho}{\rho}}\left(\left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right)\right)^{\frac{1-2\rho}{\rho}}(-1)\tau^{\frac{-1}{1-\rho}} < 0$$

Since $\omega^T$ does depend on $f_x$, I substitute the equilibrium gender wage gap given by (75) into (99), which yields after some rearranging:

$$W_f^{T2}(\omega^T) = \rho\left(\left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right)\left(N^{f*} + L_m\rho^\rho(1-\rho)^{1-\rho}\frac{1}{\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)^\rho(\alpha + f_x)^{1-\rho}}\right)\right)^{\frac{1-\rho}{\rho}}$$

To save space, let $A \equiv \left(\left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right)\left(N^{f*} + L_m\rho^\rho(1-\rho)^{1-\rho}\frac{1}{\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)^\rho(\alpha + f_x)^{1-\rho}}\right)\right)$ Differentiating the above with respect to $f_x$ yields:

$$\frac{\partial W_f^{T2}}{\partial f_x} = A^{\frac{1-2\rho}{\rho}}(1 + \tau^{\frac{-\rho}{1-\rho}})L_m\rho^\rho(1-\rho)^{2-\rho}\frac{(1-\rho)\frac{L_f}{N^{f*}} - \alpha - f_x}{\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)^{\rho+1}(\alpha + f_x)^{2-\rho}}(-1) < 0$$

The above inequality follows from the fact that $1 < \omega^T < \beta$, which, as is shown in the Mathematical Appendix, Appendix D, only occurs when $\frac{L_f}{N^{f*}} > \frac{\alpha + f_x}{1-\rho}$, which is equivalent to $(1-\rho)\frac{L_f}{N^{f*}} - \alpha - f_x > 0$.

□

The above proposition is of interest, since it shows that a woman's welfare is strictly decreasing in $f_x$, even though increasing $f_x$ decreases the relative female wage. The reason for this is that increasing $f_x$ causes some discriminatory firms to exit the market altogether, which decreases the number of varieties available to each consumer. Proposition 21 then shows that the welfare losses due to a decrease in the number of varieties outweigh each woman's gain in relative income due to an increase in $f_x$. This is simply because consumers primarily desire variety over quantities of a particular good, and thus a loss in the number of varieties available to a consumer hurts more than a loss in income.

More importantly, I can use Proposition 21 to show that women are always strictly better off in trade equilibrium than in autarky if both types of firms export.

**Proposition 22.** *If $1 < \omega^T < \beta$, $1 < \omega < \beta$, and both types of firms export in trade equilibrium, women are strictly better off in trade than in autarky.*

*Proof.* Both types of firms will only export in trade equilibrium if $f_x \tau^{\frac{\rho}{1-\rho}} \leq \alpha$. Call any combination of $f_x$ and $\tau$ that satisfies this expression "admissible." If one lets this inequality hold with equality and rearranges the expression, one obtains $\tau^{\frac{\rho}{1-\rho}} = \frac{\alpha}{f_x}$. Call any combination of $f_x$ and $\tau$ that satisfies this expression the maximal admissible set of trade costs. By Lemma 1, any admissible combination of $f_x$ and $\tau$ that *does not* belong to the maximal admissible set of trade costs must leave women strictly better off than at least one combination of $f_x$ and $\tau$ that *is* in the maximal admissible set of trade costs, since welfare is strictly decreasing in $f_x$ and $\tau$. As a result, the combination of $f_x$ and $\tau$ that leaves women as worse off as possible must be found in the maximal admissible set of trade costs.

It follows that if women are better off in trade equilibrium when $\tau^{\frac{\rho}{1-\rho}} = \frac{\alpha}{f_x}$, then they are better off for all admissible combinations of $f_x$ and $\tau$. Therefore, the proposition follows if I can show that $W_f^{T2}(\omega^T) > W_f^A(\omega)$ when $\tau^{\frac{\rho}{1-\rho}} = \frac{\alpha}{f_x}$. By (95) and (100), $W_f^{T2}(\omega^T) > W_f^A(\omega)$ whenever:

$$\rho\left(\left(1 + (\tau)^{\frac{-\rho}{1-\rho}}\right)\left(N^{f*} + \left(\omega^T\right)^{\frac{-\rho}{1-\rho}} \frac{L_m(1-\rho)}{\alpha + f_x}\right)\right)^{\frac{1-\rho}{\rho}} > \rho\left(N^{f*} + (\omega)^{\frac{-\rho}{1-\rho}} \frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}}$$

Substituting $\tau^{\frac{\rho}{1-\rho}} = \frac{\alpha}{f_x}$ into this expression yields:

$$\rho\left(\left(\frac{\alpha+f_x}{\alpha}\right)\left(N^{f*}+\left(\omega^T\right)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha+f_x}\right)\right)^{\frac{1-\rho}{\rho}} > \rho\left(N^{f*}+(\omega)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}}$$

Rearranging yields:

$$\rho\left(\left(\frac{\alpha+f_x}{\alpha}\right)N^{f*}+\left(\omega^T\right)^{\frac{-\rho}{1-\rho}}\left(\frac{L_m(1-\rho)}{\alpha}\right)\right)^{\frac{1-\rho}{\rho}} > \rho\left(N^{f*}+(\omega)^{\frac{-\rho}{1-\rho}}\frac{L_m(1-\rho)}{\alpha}\right)^{\frac{1-\rho}{\rho}}$$

$$\left(\frac{\alpha+f_x}{\alpha}\right)N^{f*}+\frac{L_m(1-\rho)}{\alpha}\left(\left(\omega^T\right)^{\frac{-\rho}{1-\rho}}-\omega^{\frac{-\rho}{1-\rho}}\right) > 0$$

Which is must be true since $\omega^T < \omega$, which means that $\left(\omega^T\right)^{\frac{-\rho}{1-\rho}}-\omega^{\frac{-\rho}{1-\rho}} > 0$. □

Thus, by Propositions 20 and 22, women are always better off in trade equilibrium, no matter the size of the trade costs.

## 5.3 Men's welfare

While men get to enjoy an increased number of varieties in trade equilibrium, since the gender wage gap falls when a country moves from autarky to trade, each man's share of the economy's aggregate income will fall. As a result, men will not *always* be better off when a country moves from autarky to trade. Unfortunately, the interaction between the welfare effects due to an increased number of varieties and a decreased income are quite complex, since they depend on almost all of the model's underlying parameters. As a result, it is much more difficult to make general statements concerning male welfare, than it was to make general statements concerning female welfare.

Instead, I shall simply demonstrate that it is possible for men to make both welfare gains, as well as welfaire losses, when moving from trade to autarky. To prove this, I shall have to make use of the following Lemma.

**Lemma 1.** *If* $0 < \rho < 1$, *then* $\frac{1}{1-\rho} \geq \frac{2^{\frac{1-\rho}{\rho}}-0.5}{2^{\frac{1-\rho}{\rho}}-1}$

*Proof.* See Mathematical Appendix, Appendix F. □

I can then prove that there are welfare gains from trade for men when $\tau$ approaches 1.

**Proposition 23.** *If* $1 < \omega^T < \beta$, $1 < \omega < \beta$, $\tau$ *approaches 1, and both firms export, then men will make welfare gains in trade equilibrium for all feasible values of* $f_x$.

*Proof.* Note that the male welfare expression given by (98) is strictly decreasing in $f_x$ since $\omega^T$ is strictly decreasing in $f_x$, [46] as is $\frac{L_m(1-\rho)}{\alpha+f_x}$.[47]

Further note that both firms export only if $f_x \tau^{\frac{\rho}{1-\rho}} \leq \alpha$. Since I am only considering the case where $\tau$ approaches 1, the largest value of $f_x$ consistent with both firms exporting is $\alpha$, since $\lim_{\tau \to 1} f_x \tau^{\frac{\rho}{1-\rho}} = \alpha$. Since male welfare is strictly decreasing in $f_x$, this means that if male welfare is greater in trade equilibrium than autarky when $f_x = \alpha$, then male welfare is always greater in trade equilibrium for any feasible value of $f_x$ that is consistent with both firms exporting.

Taking the limit at $\tau$ goes to 1 and substituting $f_x = \alpha$ into (98) yields:

$$\lim_{\tau \to 1} W_m^{T2} = \rho \left( 2 \left( \lim_{\tau \to 1} \left( \omega^T|_{f_x=\alpha} \right) \right)^{\frac{\rho}{1-\rho}} N^{f*} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}}$$

There will be welfare gains for men, i.e. $\lim_{\tau \to 1} W_m^{T2}(\omega^T) > W_m^A(\omega)$ whenever:

$$\rho \left( 2 \left( \lim_{\tau \to 1} \left( \omega^T|_{f_x=\alpha} \right) \right)^{\frac{\rho}{1-\rho}} N^{f*} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}} > \rho \left( (\omega)^{\frac{\rho}{1-\rho}} N^{f*} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}}$$

Which, when rearranged yields:

$$2 \left( \lim_{\tau \to 1} \left( \omega^T|_{f_x=\alpha} \right) \right)^{\frac{\rho}{1-\rho}} > (\omega)^{\frac{\rho}{1-\rho}}$$

Note that $\omega^T$ does not depend on $\tau$ when both firms export, and as a result $\lim_{\tau \to 1} \left( \omega^T|_{f_x=\alpha} \right) = \omega^T|_{f_x=\alpha}$ . By the equilibrium wage gap equations given by (37) and (75), the above can be written as:

$$2 \left( \left( \frac{L_f}{N^{f*}} - 2\alpha \right) \frac{1-\rho}{2\alpha\rho} \right)^{\rho} > \left( \left( \frac{L_f}{N^{f*}} - \alpha \right) \frac{1-\rho}{\alpha\rho} \right)^{\rho}$$

Rearranging yields:

$$\frac{L_f}{N^{f*}} > 2\alpha \left( \frac{2^{\frac{1-\rho}{\rho}} - 0.5}{2^{\frac{1-\rho}{\rho}} - 1} \right)$$

The above inequality has to hold if $1 < \omega^T < \beta$. To see this, note that $1 < \omega^T < \beta$ only if $\frac{L_f}{N^{f*}} > \frac{\alpha+f_x}{1-\rho}$. Since $f_x = \alpha$ by assumption, this means that $\frac{L_f}{N^{f*}} > 2\alpha\frac{1}{1-\rho}$. Since by Lemma 1 I know that:

[46] See Figure 5.

[47] $\frac{\partial \frac{L_m(1-\rho)}{\alpha+f_x}}{\partial f_x} = -\frac{L_m(1-\rho)}{(\alpha+f_x)^2} < 0$

$$\frac{1}{1-\rho} \geq \frac{2^{\frac{1-\rho}{\rho}} - 0.5}{2^{\frac{1-\rho}{\rho}} - 1}$$

it follows that whenever $1 < \omega^T < \beta$:

$$\frac{L_f}{N^{f*}} > 2\alpha\frac{1}{1-\rho} \geq 2\alpha\left(\frac{2^{\frac{1-\rho}{\rho}} - 0.5}{2^{\frac{1-\rho}{\rho}} - 1}\right)$$

$\square$

The following Corollary follows from Proposition 23, since the male welfare measure given by (98) is continuous and strictly increasing in $\tau$.

**Corollary 4.** *If $1 < \omega^T < \beta$, $1 < \omega < \beta$, and both firms export, there exists an interval of iceberg trade costs, $\tau \in (0, \tau*)|\ \tau* \in (0, \frac{\alpha}{f_x}]$, where there are welfare gains for men from moving to trade equilibrium from autarky.*

Since there are welfare gains for women for any feasible values of $f_x$ and $\tau$ when both firms export in trade equilibrium, Corollary 4 implies that there must be *aggregate* welfare gains if $\tau \in (0, \tau*)$, since every consumer is better off in trade equilibrium than in autarky.

This is an important result, since it provides a stronger case for trade liberalization than many trade models, such as the workhorse Hecksher-Ohlin-Samuelson model. While a detailed discussion of this model is outside the scope of this paper, one of the key theorems in this trade environment is that while there are always *aggregate* welfare gains from trade, some individual consumers may be made worse off from trade unless there is income redistribution to compensate the owners of the factors of production whose real income falls due to trade. Note, however, that in the trade environment developed in this paper, all consumers will be better off in trade equilibrium if trade costs are sufficiently small, *even if there is no redistribution of income towards the factors of production whose relative income share has fallen due to trade.* Even though the gender wage gap falls due to trade, which means that men receive a smaller share of the total aggregate output than they did in autarky, men can *still* be better off due to the increased access to new varieties provided by trade.

However, while the case for trade liberalization is strongest when fixed costs of trade are low, if fixed are relatively high so that only fair firms export in trade equilibrium, it can also be shown that there will be no welfare gains for men if $\tau$ becomes too large. Consider the following proposition:

**Proposition 24.** *If $1 < \omega^T < \beta$, $1 < \omega < \beta$, $N^{fT} = N^{f*}$, and only fair firms export, men are made worse off by trade if iceberg trade costs are sufficiently large.*

*Proof.* Suppose not, so that men are either just as well off or better from trade, i.e. $W_m^{T1}(\omega^T) \geq W_m^A$ for any possible choice of $\tau$. Fix $f_x = \frac{\alpha+\epsilon}{\tau^{\frac{\rho}{1-\rho}}}$, where $\epsilon > 0$ and is arbitrarily small, so that $f_x \tau^{\frac{\rho}{1-\rho}} = \tau^{\frac{\rho}{1-\rho}} \frac{\alpha+\epsilon}{\tau^{\frac{\rho}{1-\rho}}} = \alpha + \epsilon > \alpha$ for all $\tau > 0$

Taking the limit of $W_m^{T1}(\omega^T) \geq W_m^A$ at $\tau$ goes to infinity yields:

$$\lim_{\tau\to\infty} W_m^{T1}(\omega^T) = \rho \left( N^{f*} \left( \lim_{\tau\to\infty} \left(\omega^T\right)\right)^{\frac{\rho}{1-\rho}} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}}$$

By (96) and (94), $W_m^{T1}(\omega^T) \geq W_m^A$ only if:

$$\rho \left( N^{f*} \left( \lim_{\tau\to\infty} \left(\omega^T\right)\right)^{\frac{\rho}{1-\rho}} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}} \geq \rho \left( (\omega)^{\frac{\rho}{1-\rho}} N^{f*} + \frac{L_m(1-\rho)}{\alpha} \right)^{\frac{1-\rho}{\rho}}$$

Rearranging the above yields:

$$\lim_{\tau\to\infty} \left(\omega^T\right) \geq \omega$$

Which is a contradiction by Proposition 19, i.e. gender wage gaps fall in trade equilibrium relative to autarky when $1 < \omega^T < \beta$, $1 < \omega < \beta$ and $N^{fT} = N^{f*}$.

□

The intuition for the above result is simple; if iceberg trade costs become sufficiently large, men do not get to enjoy sufficient quantities of each variety produced in the foreign country to compensate them for their loss of relative income compared to autarky. As a result, men are made worse off by trade. This means that I cannot make the strong claim that trade always makes everyone better off, but instead I can only claim that in this model, everyone is made better off by trade if trade costs are sufficiently small. Thus, small trade costs are a sufficient, but not necessary,[48] condition for there to aggregate welfare gains in this model.

# 6 Conclusions and Extensions

The monopolistically competitive trade model developed in this paper shows how increased trade may increase or decrease the gender wage gap, depending on whether iceberg trade costs, or fixed export costs, are responsible for the change in equilibrium trade volume. Moreover, this model shows that transitions from autarky to trade will decrease the gender

[48]There may still be aggregate welfare gains when trade costs are high, under a suitably chosen redistribution of income. However, determining whether this is indeed the case is left for future research.

wage gap whenever there are enough fair firms competing with discriminatory firms. This then generates welfare gains for *both* men and women if trade costs are sufficiently small.

The ambiguous effect that a change in observed trade volume may have on gender wage gaps is one of the key insights obtained by this model. As this model has demonstrated, different types of trade costs can result in a variety of different competitive effects, which then affect the gender wage gap in different ways. For example, fixed export costs tend to hurt discriminatory firms more than fair firms, since discriminatory firms only hire men who are overvalued relative to women. As a result, decreasing the fixed costs of trade will result in greater discriminatory firm entry, which increases the gender wage gap. On the other hand, if fixed costs are sufficiently high, discriminatory firms cannot afford to enter the export market, which means that any fall in iceberg trade costs will only benefit fair firms, which increases the equilibrium wages paid to women, and decreases the gender wage gap.

Even though the above insights are quite useful, I must be careful not to overstate their importance. Since the model environment from which these insights are gleaned involves a variety of very restrictive assumptions concerning market structure, firm entry, and the pattern of trade, more models of a similar nature are needed if these results are to help inform trade policy. In particular, subsequent modifications to the model will be necessary to verify that these results are robust.

For example, one of the key mechanisms that lead to the "middle case" I focused on for the vast majority of this paper, was the fact that men and women were segregated between fair firms and discriminatory firms, respectively, due to the linearity of the production function. This made the goods pricing behaviour of different types of firms equivalent to the difference in pricing between male and female labour, which in turn made the model very simple to solve. However, since individual firms tend to hire both men and women in the real world, this segregation result was more a "useful fiction", rather than an accurate model of a firm's hiring decisions.

Therefore, it would be beneficial if future research considered the effect of non-linear technologies in this sort of trade environment. A recent working paper by Namini, Facchini, and Lopez (2012) considers a trade environment that could easily be modified to accomplish this goal. In this paper, firms choose bundles of unskilled and skilled labour to produce using a CES production function. Since each type of labour exhibits diminishing returns, and has a marginal product of infinity if zero units of that particular input are used, then each type of labour will be used by each firm in equilibrium, even if some firm "values" a particular input less than another. Aside from this modification, the model is quite similar to that developed in Melitz (2003), and therefore, could easily be modified to account for discrimination against women as in the current paper.

Moreover, alternative specifications of the set of discriminatory preferences could prove useful. Unfortunately, the model developed in this paper only considered distributions of discriminatory preferences that were quite simple; namely, there was a set of firms with no taste for discrimination, and a set of firms that had identical tastes for discrimination. It would be useful to see how allowing for a continuum of discriminatory preferences, as in Ben Yahmed (2011), would modify this model's predictions. This would have the added advantage of ruling out the corner solutions that were a tricky feature of the model considered in this paper.

It would also be useful to consider whether allowing for multiple industries, such as an industry that never exports, modifies this model's predictions concerning changes in fixed export costs and their effect on an economy's import to GDP ratio. If such a model were to show that a decrease in fixed export costs increases an economy's import to GDP ratio, as I alluded to in the main text, this would provide stronger evidence for my claim that decreasing export costs may be responsible for the empirically observed relationship between increased trade and increased gender wage gaps.

In addition, empirical work directly testing the predictions of the model developed in this paper would be a useful contribution to the literature. For example, an empirical study relating changes in absolute trade volumes to changes in fixed export costs and per unit trade costs would help determine if the insights gained from this model are of any empirical relevance. This could, however, be a tricky empirical problem, since finding examples of falling fixed export costs could prove to be much more difficult than finding examples of falling per-unit trade costs, which are often observed through tariff reduction.

At the very least, I hope that the insights provided by this model are able to help empirically minded economists reach a more definitive verdict on trade's impact on gender wage gaps. By pointing out the heterogenous effect that different types of trade costs can have on gender wage gaps, future empirical studies may be able to obtain better estimates of the effect of trade on discriminatory practices by controlling for changes in trade costs over time. It may turn out that modifying the correct trade barriers, as opposed to simply increasing trade, is what will have the greatest impact on the gender wage gap.

# 7 Mathematical Appendix

## Appendix A: Bounds for the wage gap in autarky

Recall (38) and (39) from the main text, which define the variables:

$$\phi_L \equiv \frac{\alpha}{1-\rho}$$

$$\phi_H \equiv \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$$

I now restate and prove Proposition 9 from Section 3 in terms of the model's fundamental parameters, as opposed to $\phi_H$ and $\phi_L$:

**Proposition 9.** *There will be no wage gap, i.e.* $\omega = 1$, *whenever* $\frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho}$, *while the wage gap satisfies* $1 < \omega < \beta$ *whenever* $\frac{\alpha}{1-\rho} < \frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$, *and satisfies* $\omega = \beta$ *whenever* $\frac{L_f}{N^{f*}} \geq \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$.

*Proof.* First, I prove that if $\frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho}$, then $\omega = 1$.

If $\frac{L_f}{N^{f*}} \leq \frac{\alpha}{1-\rho}$, then by (35), $y^f = \frac{\alpha\rho}{1-\rho}$. By (22), $y^d = \frac{\alpha\rho}{1-\rho}$. Then by equation (36):

$$\omega = \left(\frac{y^f}{y^d}\right)^{1-\rho}$$

Substituting in these equilibrium outputs yields:

$$\omega = \left(\frac{\frac{\alpha\rho}{1-\rho}}{\frac{\alpha\rho}{1-\rho}}\right)^{1-\rho}$$

Simplifying the above yields:

$$\omega = 1$$

Next, I prove that if $\frac{\alpha}{1-\rho} < \frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$, then $1 < \omega < \beta$.

If $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$, then by (35), $y^f = \frac{L^f}{N^{f*}} - \alpha$, while $y^d = \frac{\alpha\rho}{1-\rho}$ as before. Then by equation (36):

$$\omega = \left(\frac{y^f}{y^d}\right)^{1-\rho}$$

Substituting the equilibrium outputs into the above yields:

$$\omega = \left( \frac{\frac{L^f}{N^{f*}} - \alpha}{\frac{\alpha\rho}{1-\rho}} \right)^{1-\rho}$$

Rearranging the above yields:

$$\omega = \left[ \left( \frac{L_f}{N^{f*}} - \alpha \right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} \tag{100}$$

Note that (100) is only valid if $\omega < \beta$ since (35) only holds under that assumption. However, I now prove that $\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho} \left(1 + \rho(\beta^{\frac{1}{1-\rho}} - 1)\right)$ implies that $\omega < \beta$.

To see this, suppose that $\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho} \left(1 + \rho(\beta^{\frac{1}{1-\rho}} - 1)\right)$. Rearranging this expression yields:

$$\left[ \left( \frac{L_f}{N^{f*}} - \alpha \right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} < \beta$$

Which, by (100), is equivalent to:

$$\omega < \beta$$

Next, I show that $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$ implies that $\omega > 1$. So see this, suppose that $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$. Subtracting $\alpha$ from both sides yields:

$$\frac{L_f}{N^{f*}} - \alpha > \frac{\alpha}{1-\rho} - \alpha$$

Rearranging the above yields:

$$\left( \frac{L_f}{N^{f*}} - \alpha \right) \frac{1-\rho}{\alpha} > 1$$

Taking both sides to the power of $1-\rho$ yields:

$$\left[ \left( \frac{L_f}{N^{f*}} - \alpha \right) \frac{1-\rho}{\alpha} \right]^{1-\rho} > 1$$

Which, by (100) is equivalent to:

$$\omega > 1$$

Finally, I show that $\omega = \beta$ whenever $\frac{L_f}{N^{f*}} \geq \frac{\alpha}{1-\rho} \left(1 + \rho(\beta^{\frac{1}{1-\rho}} - 1)\right)$.

Suppose not, so that $\frac{L_f}{N^{f*}} \geq \frac{\alpha}{1-\rho} \left(1 + \rho(\beta^{\frac{1}{1-\rho}} - 1)\right)$, and either $\omega > \beta$ or $\omega < \beta$. $\omega > \beta$ is impossible by Proposition 2. Therefore, it must be that $\omega < \beta$.

Since $\frac{L_f}{N^{f*}} \geq \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$, it follows that $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$ because $\beta > 1$ implies that $\rho(\beta^{\frac{1}{1-\rho}}-1) > 0$. Therefore, I may use the expression for $\omega$ given by (100). Since $\omega < \beta$:

$$\left[\left(\frac{L_f}{N^{f*}} - \alpha\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} < \beta$$

Which after some algebraic manipulation yields:

$$\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$$

Which is a contradiction. □

## Appendix B: Proofs concerning the shape of Figure 1

**Proposition 25.** *If $\phi_L < \frac{L_f}{N^{f*}} < \phi_U$ then* increasing $\frac{L_f}{N^{f*}}$*, holding everything else constant, increases the wage gap.*

*Proof.* Partially differentiating (40) over the appropriate interval yields:

$$\frac{\partial\,\omega}{\partial\left(\frac{L_f}{N^{f*}}\right)} = \frac{(1-\rho)^{2-\rho}}{(\alpha\rho)^{1-\rho}}\left[\frac{L_f}{N^{f*}} - \alpha\right]^{-\rho} > 0$$

Since $0 < \rho < 1$, it follows that $\phi_L = \frac{\alpha}{1-\rho} < \frac{L_f}{N^{f*}}$ means that $\frac{L_f}{N^{f*}} - \alpha > 0$ □

**Proposition 26.** *The equilibrium wage gap function, $\omega\left(\frac{L_f}{N^{f*}}\right)$, is continuous over its entire domain.*

*Proof.* Casual inspection of the equilibrium wage gap function over it's entire range, given by (40), shows that the function is continuous whenever $\frac{L_f}{N^{f*}} < \phi_L$, $\phi_L < \frac{L_f}{N^{f*}} < \phi_H$, and $\frac{L_f}{N^{f*}} > \phi_H$. This means that the proposition will follow once it is shown that $\lim\limits_{\frac{L_f}{N^{f*}}\to\phi_L} \omega\left(\frac{L_f}{N^{f*}}\right) = 1$ and $\lim\limits_{\frac{L_f}{N^{f*}}\to\phi_H} \omega\left(\frac{L_f}{N^{f*}}\right) = \beta$.

Taking the limit of (100) as $\frac{L_f}{N^{f*}}$ approaches $\phi_L = \frac{\alpha}{1-\rho}$ yields:

$$\lim_{\frac{L_f}{N^{f*}}\to\phi_L} \omega\left(\frac{L_f}{N^{f*}}\right) = \left[\left(\frac{\alpha}{1-\rho} - \alpha\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} = \left[\left(\frac{\alpha\rho}{1-\rho}\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} = 1$$

Taking the limit of (100) as $\frac{L_f}{N^{f*}}$ approaches $\phi_H = \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$ yields:

$$\lim_{\frac{L_f}{N^{f*}}\to\phi_H} \omega\left(\frac{L_f}{N^{f*}}\right) = \left[\left(\frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)-\alpha\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} = \left[\left(\frac{\beta^{\frac{1}{1-\rho}}\alpha\rho}{1-\rho}\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} = \beta$$

□

# Appendix C: Proofs concerning firm entry and trade equilibrium

## Appendix C1: Fair firm entry when $\omega = \beta$

**Proposition 27.** *If $\omega = \beta$ in autarky, then fair firms firms will enter the export market if and only if $\beta^{\frac{1}{1-\rho}}\alpha \geq f_x\tau^{\frac{-\rho}{1-\rho}}$.*

*Proof.* Note that when $\omega = \beta$ in autarky, each other fair firm's scale is given by $y_o^f = \frac{\beta^{\frac{1}{1-\rho}}\alpha\rho}{1-\rho}$. Recall that $\frac{\hat{y}_u^{fX}}{y_o^{fH}} = \left(\frac{p^{fH}}{p^{fX}}\right)^{\frac{1}{1-\rho}} = \tau^{\frac{-1}{1-\rho}}$, where the last equality follows from (61). It follows that:

$$\hat{y}_u^{fX} = \tau^{\frac{-1}{1-\rho}}\frac{\beta^{\frac{1}{1-\rho}}\alpha\rho}{1-\rho} \tag{101}$$

Note by Assumption 3, each fair firm will export if and only if such a decision results in them earning non-negative profits in the export market. Therefore, the following must hold:

$$\pi_u^{fX} = p^{fX}\hat{y}_u^{fX} - w_f(f_x + \tau\hat{y}_u^{fX}) \geq 0$$

Rearranging yields:

$$\frac{p^{dX}}{w_f}\hat{y}_u^{fX} - \tau\hat{y}_u^{fX} \geq f_x$$

Substituting in the expression for $\frac{p^{dX}}{w_f}$ given by (58) yields, after some rearranging:

$$\tau\hat{y}_u^{fX}\frac{1-\rho}{\rho} \geq f_x$$

Substituting the expression for $\hat{y}_u^{fX}$ given by (102) yields:

$$\tau^{\frac{-\rho}{1-\rho}}\left(\frac{\beta^{\frac{1}{1-\rho}}\alpha\rho}{1-\rho}\right)\frac{1-\rho}{\rho} \geq f_x$$

Rearranging yields:

$$\beta^{\frac{1}{1-\rho}}\alpha \geq f_x \tau^{\frac{-\rho}{1-\rho}}$$

□

Note that when $\omega = \beta$, discriminatory firms enter the export market if and only if $\alpha \geq f_x \tau^{\frac{-\rho}{1-\rho}}$. Since $\beta > 1$, this means that $\beta^{\frac{1}{1-\rho}}\alpha > \alpha \geq f_x \tau^{\frac{-\rho}{1-\rho}}$. This means that the above proposition also establishes that fair firms *always* enter the export market if discriminatory firms enter the market, i.e. whether $\omega = \beta$, or $\omega < \beta$ as was proven in the main text.

**Appendix C2: Fair firm measures in trade equilibrium**

**Proposition 28.** *If* $\frac{L_f}{N^{fT}} > \frac{\alpha+f_x}{1-\rho}$ *and* $1 < \omega^T < \beta$, *then all fair firms earn positive profits in trade equilibrium.*

*Proof.* Suppose not, so that $\frac{L_f}{N^{fT}} > \frac{\alpha+f_x}{1-\rho}$, $1 < \omega^T < \beta$, and fair firms earn profits less than or equal to zero. It follows that:

$$\pi^{fT} = p^{fH}y^{fH} + p^{fX}\hat{y}^{fX} - w_f^T(\alpha + f_x + y^{fH} + \tau y^{fX}) \leq 0$$

Rearranging:

$$\frac{p^{fH}}{w_f^T}y^{fH} + \frac{p^{fX}}{w_f^T}\hat{y}^{fX} - y^{fH} - \tau y^{fX} \leq \alpha + f_x$$

Plug in optimal pricing given by (56) and (58)

$$\frac{y^{fH} + \tau\hat{y}^{fX}}{\rho} - y^{fH} - \tau y^{fX} \leq \alpha + f_x$$

Rearranging:

$$\left(y^{fH} + \tau\hat{y}^{fX}\right) \leq (\alpha + f_x)\frac{\rho}{1-\rho}$$

Since $y^{fT} = y^{fH} + \tau\hat{y}^{fX}$:

$$y^{fT} \leq (\alpha + f_x)\frac{\rho}{1-\rho} \tag{102}$$

If $1 < \omega^T < \beta$ in trade equilibrium, then for the female market to clear, it must be that $L_f = N^{fT}(\alpha + f_x + y^{fT})$, which when rearranged yields $y^{fT} = \frac{L_f}{N^{fT}} - \alpha - f_x$. Substituting this into the above expression yields:

$$\left(\frac{L_f}{N^{fT}} - \alpha - f_x\right) \leq (\alpha + f_x)\,\frac{\rho}{1-\rho}$$

Rearranging yields:

$$\frac{L_f}{N^{fT}} \leq \frac{\alpha + f_x}{1-\rho}$$

Which is a contradiction. Therefore, it must be that fair firms earn positive profits. □

**Corollary 5.** *If* $\frac{L_f}{N^{fT}} < \frac{\alpha+f_x}{1-\rho}$*, then fair firms earn negative profits. If* $\frac{L_f}{N^{fT}} = \frac{\alpha+f_x}{1-\rho}$*, fair firms earn zero profits.*

*Proof.* The corollary follows by doing the derivation in the above proposition backwards with a strict inequality and then a strict equality. □

**Proposition 29.** *If* $\frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}$*, then* $N^{fT} < N^{f*}$ *and fair firms earn zero profits. Likewise, if* $\frac{L_f}{N^{f*}} = \frac{\alpha+f_x}{1-\rho}$*, then* $N^{fT} = N^{f*}$*.*

*Proof.* By Assumption 2, $N^{fT} = N^{f*}$ and fair firms earn non-negative profits, or $N^{fT} < N^{f*}$ and fair firms earn zero profits. If $\frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}$, then By Corollary 5, fair firms earn negative profits when $N^{fT} = N^{f*}$. Since this is impossible by Assumption 2, it must be that $N^{fT} < N^{f*}$ and fair firms earn zero profits. Likewise, if $\frac{L_f}{N^{f*}} = \frac{\alpha+f_x}{1-\rho}$, then fair firms earn zero profits if $N^{fT} = N^{f*}$. Since fair firms would earn positive profits if $N^{fT} < N^{f*}$, which is not possible by Assumption 2, it follows that $N^{fT} = N^{f*}$ whenever $\frac{L_f}{N^{f*}} = \frac{\alpha+f_x}{1-\rho}$. □

**Proposition 30.** *If* $\frac{L_f}{N^{f*}} > \frac{\alpha+f_x}{1-\rho}$*, all possible fair firms enter the market, i.e.* $N^{fT} = N^{f*}$*, and each fair firm earns positive profits.*

*Proof.* Suppose that $\frac{L_f}{N^{f*}} > \frac{\alpha+f_x}{1-\rho}$. By Assumption 2, either fair firms earn zero profits and $N^{fT} < N^{f*}$, or fair firms earn profits greater than or equal to zero and $N^{fT} = N^{f*}$. The second case is possible if $\frac{L_f}{N^{f*}} > \frac{\alpha}{1-\rho}$, since if $N^{fT} = N^{f*}$, then by Proposition 28 each fair firm earns positive profits. I rule out the second case by noting that if $N^f < N^{f*}$ then $\frac{L_f}{N^{fT}} > \frac{L_f}{N^{f*}} > \frac{\alpha+f_x}{1-\rho}$, which means that fair firms earn positive profits. □

## Appendix D: Corner solution bounds with both firms exporting

Recall from the main text that whenever both types of firms export, $1 < \omega^T < \beta$ and $N^{fT} = N^{f*}$ one can make use of (75) to define the gender wage gap, i.e.:

$$\omega^T = \left[\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha + f_x)\rho}\right]^{1-\rho} \tag{103}$$

I use this to prove the following proposition:

**Proposition 31.** *If both firms export in trade equilibrium and* $\frac{\alpha+f_x}{1-\rho} < \frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$, *then* $1 < \omega^T < \beta$.

*Proof.* If $\frac{\alpha+f_x}{1-\rho} < \frac{L_f}{N^{f*}}$, then by Proposition 30, the number of fair firms in equilibrium satisfies $N^{fT} = N^{f*}$. This means (103) defines the gender wage gap whenever $1 < \omega^T < \beta$. I now show that $\frac{\alpha+f_x}{1-\rho} < \frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$ implies that $1 < \omega^T < \beta$.

To see this, first suppose that $\frac{\alpha+f_x}{1-\rho} < \frac{L_f}{N^{f*}}$. Subtract $\alpha + f_x$ from both sides, yielding:

$$\frac{L_f}{N^{f*}} - \alpha - f_x > \frac{\alpha+f_x}{1-\rho} - \alpha - f_x$$

Rearranging the above yields:

$$\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha+f_x)\rho} > 1$$

Taking the power of $1-p$ for both sides yields:

$$\left[\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha+f_x)\rho}\right]^{1-\rho} > 1$$

Which, by (103), is equivalent to:

$$\omega^T > 1$$

Next, suppose that $\frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$. Rearranging this inequality yields:

$$\left[\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha+f_x)\rho}\right]^{1-\rho} < \beta$$

Which by (103), is equivalent to:

$$\omega^T < \beta$$

□

**Proposition 32.** *If both firms export in trade equilibrium and* $\frac{L_f}{N^{f*}} \geq \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$, *then* $\omega^T = \beta$

*Proof.* Suppose not, so that $\frac{L_f}{N^{f*}} \geq \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$ and $\omega^T > \beta$ or $\omega^T < \beta$. Since $\omega^T > \beta$ is impossible by Proposition 2, it must be that $\omega^T < \beta$. Moreover, since $\frac{L_f}{N^{f*}} \geq \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right) > \frac{\alpha+f_x}{1-\rho}$, it follows that $N^{fT} = N^{f*}$ by Proposition 30. Since $N^{fT} = N^{f*}$ and $\omega^T < \beta$, (103) defines the equilibrium gender wage gap, which means that:

$$\left[\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{(\alpha+f_x)\rho}\right]^{1-\rho} < \beta$$

Which when rearranged, yields:

$$\frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$$

Which is a contradiction. □

**Proposition 33.** *If both firms export in trade equilibrium and* $\frac{L_f}{N^{f*}} \leq \frac{\alpha+f_x}{1-\rho}$, *then* $\omega^T = 1$.

*Proof.* Whenever $\frac{L_f}{N^{f*}} \leq \frac{\alpha+f_x}{1-\rho}$, then by (5), it follows that fair firms earn zero profits. Setting a fair firm's overall profits equal to zero yields

$$\pi^{fT} = p^{fH}y^{fH} + p^{fX}\hat{y}^{fX} - w_f^T(\alpha + f_x + y^{fH} + \tau y^{fX}) = 0$$

Rearranging this expression and then substituting the pricing equations given by (56) and (58) into the expression yields:

$$y^{fH} + \tau\hat{y}^{fX} = \frac{(\alpha+f_x)\,\rho}{1-\rho}$$

Since $y^{fT} = y^{fH} + \tau\hat{y}^{fX}$:

$$y^{fT} = \frac{(\alpha+f_x)\,\rho}{1-\rho} \tag{104}$$

Substituting the above into (74) yields:

$$y^{fH} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\,\rho}{1-\rho} \tag{105}$$

Note that (36) must still hold in equilibrium i.e. $\omega^T = \left(\frac{y^{fH}}{y^{dH}}\right)^{1-\rho}$. Since $y^{dH} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\rho}{1-\rho}$ by (69) it follows that:

$$\omega^T = \left(\frac{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\rho}{1-\rho}}{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\rho}{1-\rho}}\right)^{1-\rho} = 1$$

□

## Appendix E: Corner solution bounds with only fair firms exporting

Recall from the main text that whenever both types of firms export, $1 < \omega^T < \beta$, and $N^{fT} = N^{f*}$, one can make use of (79) to define the gender wage gap, i.e.:

$$\omega^T = \left[ \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \left( \frac{L_f}{N^{f*}} - \alpha - f_x \right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} \tag{106}$$

Note, however, that $N^{fT}$ will not necessarily equal $N^{f*}$ whenever $1 < \omega^T < \beta$ , as is the case when both firms export. In this case, I instead write:

$$\omega^T = \left[ \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \left( \frac{L_f}{N^{fT}} - \alpha - f_x \right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} \tag{107}$$

to account for the case where $N^{fT} < N^{f*}$ and $1 < \omega^T < \beta$.

With the above expressions, I can prove the following propositions:

**Proposition 34.** *If only fair firms export in trade equilibrium, then $\omega^T > 1$.*

*Proof.* If only fair firms export in trade equilibrium, then by Proposition 16, it must be that $f_x\tau^{\frac{\rho}{1-\rho}} > \alpha$. By (107), $\omega^T > 1$ is satisfied only if:

$$\left[ \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \left( \frac{L_f}{N^{fT}} - \alpha - f_x \right) \frac{1-\rho}{\alpha\rho} \right]^{1-\rho} > 1$$

Rearranging this expression yields:

$$\frac{L_f}{N^{fT}} > \frac{\alpha+f_x}{1-\rho} + \frac{\rho\left(\alpha - f_x\tau^{\frac{\rho}{1-\rho}}\right)}{(1-\rho)\tau^{\frac{\rho}{1-\rho}}}$$

Note, however, that $f_x\tau^{\frac{\rho}{1-\rho}} > \alpha$, which means that $\frac{\rho\left(\alpha - f_x\tau^{\frac{\rho}{1-\rho}}\right)}{(1-\rho)\tau^{\frac{\rho}{1-\rho}}} < 0$. This means that whenever $\frac{L_f}{N^{fT}} \geq \frac{\alpha+f_x}{1-\rho} > \frac{\alpha+f_x}{1-\rho} + \frac{\rho\left(\alpha - f_x\tau^{\frac{\rho}{1-\rho}}\right)}{(1-\rho)\tau^{\frac{\rho}{1-\rho}}}$, it must be that $\omega^T > 1$.

I now consider values of $\frac{L_f}{N^{fT}}$ that satisfy:

$$\frac{L_f}{N^{fT}} \leq \frac{\alpha+f_x}{1-\rho}$$

By Proposition 29, whenever $\frac{L_f}{N^{fT}} \leq \frac{\alpha+f_x}{1-\rho}$, fair firms earn non-positive profits. Since negative profits are impossible by Assumption 2, it must be that fair firms earn zero profits when $\frac{\alpha+f_x}{1-\rho} \leq \frac{L_f}{N^{fT}}$. By (105), this means that $y^{fH} = \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}} \frac{(\alpha+f_x)\rho}{1-\rho}$.

Note that since fair firms do not export, $y^{dT} = y^{dH} = \frac{\alpha\rho}{1-\rho}$. Since (36) must still hold in equilibirum, i.e. $\omega^T = \left(\frac{y^{fH}}{y^{dH}}\right)^{1-\rho}$, it follows that $\omega^T > 1$ holds whenever:

$$\omega^T = \left(\frac{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\rho}{1-\rho}}{\frac{\alpha\rho}{1-\rho}}\right)^{1-\rho} > 1 \tag{108}$$

Rearranging yields:

$$f_x\tau^{\frac{\rho}{1-\rho}} > \alpha$$

Which has to hold whenever only fair firms export in trade equilibrium. Thus, whenever $\frac{L_f}{N^{fT}} \leq \frac{\alpha+f_x}{1-\rho}$ it must be that $\omega^T > 1$, and the proposition follows. . □

**Proposition 35.** *If only fair firms export and* $\frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \frac{\alpha}{1-\rho} \leq \frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}$ *then* $\omega^T = \left(\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}\right)^{1-\rho}$

*Proof.* By Proposition 15, fair firms export only when $\frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \frac{\alpha+}{1-\rho} \geq \frac{L_f}{N^{f*}}$. By Proposition 29, whenever $\frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}$, it must be that $N^{fT} < N^{f*}$ and fair firms earn zero profits. This means that $\omega^T$ is given by (108), which means that $\omega^T = \left(\frac{\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{(\alpha+f_x)\rho}{1-\rho}}{\frac{\alpha\rho}{1-\rho}}\right)^{1-\rho}$. Rearranging yields:

$$\omega^T = \left(\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}\right)^{1-\rho}$$

□

Note that $\frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \frac{\alpha}{1-\rho} \leq \frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}$ is not always possible. Specifically, if $f_x > \rho(f_x\tau^{\frac{\rho}{1-\rho}} - \alpha)$, then $\frac{\rho(f_x\tau^{\frac{\rho}{1-\rho}}-\alpha)}{1-\rho} + \frac{\alpha}{1-\rho} > \frac{\alpha+f_x}{1-\rho}$, which means that the aforementioned interval on $\frac{L_f}{N^{f*}}$ does not exist.

**Proposition 36.** *If* $\frac{\alpha+f_x}{1-\rho} \leq \frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right) + \beta^{\frac{1}{1-\rho}}\frac{\alpha\rho}{\tau^{\frac{\rho}{1-\rho}}(1-\rho)} + f_x$ *and only fair firms export, then* $\left(\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}\right)^{1-\rho} \leq \omega^T < \beta$

*Proof.* If $\frac{\alpha+f_x}{1-\rho} \leq \frac{L_f}{N^{f*}}$, then by Corollary 5 and Proposition 30, it must be that all fair firms remain in the market so $N^{fT} = N^{f*}$, which means (106) defines the gender wage gap whenever $1 < \omega^T < \beta$. I now show that $\frac{\alpha+f_x}{1-\rho} < \frac{L_f}{N^{f*}} < \frac{\alpha+f_x}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)$ implies that $\left(\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}\right)^{1-\rho} \leq \omega^T < \beta$

To see this, first suppose that $\frac{\alpha+f_x}{1-\rho} \leq \frac{L_f}{N^{f*}}$. Subtracting $\alpha + f_x$ from both sides yields:

$$\frac{L_f}{N^{f*}} - \alpha - f_x \geq \frac{\alpha + f_x}{1-\rho} - \alpha - f_x$$

Rearranging the above inequality yields:

$$\frac{L_f}{N^{f*}} - \alpha - f_x \geq \frac{(\alpha + f_x)\rho}{1-\rho}$$

Multiplying both sides of the above inequality by $\frac{1}{\alpha}\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}$ yields after rearranging:

$$\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{\alpha\rho} \geq \frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}$$

Taking both sides to the power of $1-\rho$ yields:

$$\left[\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} \geq \left(\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}\right)^{1-\rho}$$

Which by (106), is equivalent to:

$$\omega^T \geq \left(\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\frac{\alpha+f_x}{\alpha}\right)^{1-\rho}$$

To complete the proof, note that $\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right) + \beta^{\frac{1}{1-\rho}}\frac{\alpha\rho}{\tau^{\frac{\rho}{1-\rho}}(1-\rho)} + f_x$, can be rearranged as follows:

$$\left[\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}} - \alpha - f_x\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} < \beta$$

Which by (106), is equivalent to:

$$\omega^T < \beta$$

□

**Proposition 37.** *If* $\frac{L_f}{N^{f*}} \geq \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right) + \beta^{\frac{1}{1-\rho}}\frac{\alpha\rho}{\tau^{\frac{\rho}{1-\rho}}(1-\rho)} + f_x$ *and only fair firms export in trade equilibrium, then* $\omega^T = \beta$.

*Proof.* Suppose not, so that $\frac{L_f}{N^{f*}} \geq \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)+\beta^{\frac{1}{1-\rho}}\frac{\alpha\rho}{\tau^{\frac{\rho}{1-\rho}}(1-\rho)}+f_x$ and $\omega^T > \beta$ or $\omega^T < \beta$. Since $\omega^T > \beta$ is impossible by Proposition 2, it must be that $\omega^T < \beta$. Therefore, (106) defines the equilibrium gender gap. It follows that $\omega^T < \beta$ only if

$$\omega^T = \left[\frac{\tau^{\frac{\rho}{1-\rho}}}{1+\tau^{\frac{\rho}{1-\rho}}}\left(\frac{L_f}{N^{f*}}-\alpha-f_x\right)\frac{1-\rho}{\alpha\rho}\right]^{1-\rho} < \beta$$

Which, when rearranged, yields:

$$\frac{L_f}{N^{f*}} < \frac{\alpha}{1-\rho}\left(1+\rho(\beta^{\frac{1}{1-\rho}}-1)\right)+\beta^{\frac{1}{1-\rho}}\frac{\alpha\rho}{\tau^{\frac{\rho}{1-\rho}}(1-\rho)}+f_x$$

Which is a contradiction. □

**Proposition 38.** *If fair firms earn zero profits and exports in trade equilibrium, while discriminatory firms do not export, then* $\frac{N^{dT}}{L_m} > \frac{N^{fT}}{L_f}$.

*Proof.* If discriminatory firms do not export, $y^{dT} = \frac{\alpha\rho}{1-\rho}$. Likewise, if each fair firm earns zero profits, then by (104), $y^{fT} = \frac{(\alpha+f_x)\rho}{1-\rho}$

By male labour market clearing:

$$N^{dT}\left(\alpha+y^{dT}\right) = L_m$$

Substitute each discriminatory firm's output into the above and then rearrange, yielding:

$$\frac{N^{dT}}{L_m} = \frac{1-\rho}{\alpha} \tag{109}$$

Likewise, by female labour market clearing:

$$N^{fT}\left(\alpha+f_x+y^{fT}\right) = L_f$$

Substitute each fair firm's output into the above and then rearrange, yielding:

$$\frac{N^{fT}}{L_f} = \frac{1-\rho}{\alpha+f_x} \tag{110}$$

Then by (109) and (110):

$$\frac{N^{dT}}{L_m} = \frac{1-\rho}{\alpha} > \frac{1-\rho}{\alpha+f_x} = \frac{N^{fT}}{L_f}$$

□

## Appendix F: Male welfare lemma

**Lemma 1.** *If* $0 < \rho < 1$, *then* $\frac{1}{1-\rho} \geq \frac{2^{\frac{1-\rho}{\rho}}-0.5}{2^{\frac{1-\rho}{\rho}}-1}$

*Proof.* Let $x \equiv \frac{1-\rho}{\rho}$. It follows that $\frac{1}{1-\rho} = \frac{1+x}{x}$, and $0 < x < \infty$ if $0 < \rho < 1$. With this in mind, I rewrite $\frac{1}{1-\rho} \geq \frac{2^{\frac{1-\rho}{\rho}}-0.5}{2^{\frac{1-\rho}{\rho}}-1}$ as:

$$\frac{1+x}{x} \geq \frac{2^x - 0.5}{2^x - 1}$$

The Lemma follows once it is shown that the above is true for all $x > 0$

Rearranging the above inequality yields:

$$2^x - 0.5x - 1 \geq 0 \tag{111}$$

The right hand side of the above inequality is strictly increasing in $x$ on the appropriate interval since $\frac{\partial(2^x-0.5x-1)}{\partial x} = 2^x \ln(2) - 0.5 > 0$, where the strict inequality follows from the fact that $x > 0$, which means that $2^x > 1$.

Note that $\lim_{x\to 0}(2^x - 0.5x - 1) = 0$. Since $2^x - 0.5x - 1$ is strictly increasing when $x > 0$, it follows that (111) must hold, and the proposition follows. □

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