Does Appraisal Make a Difference? The Influence of Price Setting on Bargaining with Incomplete Information

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September, 2012

An essay submitted to the Department of Economics in partial fulfillment of the requirements for the degree of Master of Arts

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Abstract

This paper considers a theoretical model of bargaining under incomplete information, in which a dominant player has the opportunity to influence bargaining by price setting. The model consists of a buyer who chooses between immediately taking a guaranteed offer from a price setter, or searching (at a cost) for an additional offer from a private seller, at which point the buyer chooses a desirable offer or declines both. The paper considers different equilibrium outcomes, and the effect of the dominant price setter on welfare. Under this construction, the price setter causes total welfare in the market to increase by eliminating certain inefficiencies caused in a standard model of bargaining with incomplete information.

The author wishes to thank his supervisor, Sumon Majumdar, for his guidance and patience, all his professors who have contributed to his pursuit of knowledge, his peers in the Queen's Economics MA 2011-2012 class, specifically Michael Anthony, Christopher Penney and Nick Andexer, for their insightful discussions, and his family for their continuous support.

Contents

1 Introduction

In many markets, bilateral trading between two relatively insignificant parties occurs under the influence of a larger dominant party who affects the market through its choice of price. This kind of bargaining often occurs in durable goods markets, where two parties exchange a good under the influence of either the original seller's price, or a price setting entity in the secondary market. Many of these bargaining arrangements occur with incomplete information concerning the other party's preferences. In this paper, we consider a simple two-person bargaining model with incomplete information, under the influence of a price setter. In this model, the price setter acts as an alternative option affecting negotiations.

One motivating example for this model is the trading of certain collectibles, such as collectible sports cards, under the influence of a large price setting firm or appraiser. This large firm is different from the monopoly producer, in that they sell individual cards in the secondary market, and not packs containing a random distribution of cards. Through its choice of price, this appraiser specifies a value at which all members of the market refer to throughout negotiations. A firm could have this type of secondary market power for a variety of reasons, including controlling a large percentage of the individual card supply, or efficient marketing practices causing individuals to follow their price. Daily purchases of single cards occur privately between buyers and sellers under the influence of this large firm. These interactions traditionally occur in person (at conventions and local stores), however recently the majority of them take place over the internet (using eBay or other online shopping services). Rysman and Jin (2012) discuss this shift towards online transactions, as well as several other features of the sports card market.

The main justification for examining this type of model is to determine if the existence of this dominant party is a benefit to the smaller parties, or creates additional inefficiencies compared to a standard model of bargaining with incomplete information. The introduction of a dominant player results in additional options for individual bargainers, which may reduce the inefficiencies created by their incomplete information. However, this dominant player has the opportunity to use its position to increase its own profits, which may introduce new inefficiencies in the market. It is also a useful examination of how private sales could occur in markets where there is a large firm controlling the price. We derive several general results regarding the equilibrium behaviour of all players, as well as some explicit numerical solutions to provide some intuition regarding how changes on the model parameters effect its outcome.

Much of the literature regarding bargaining focuses on bargaining with complete information. Contributions from Nash (1951 and 1953), and Rubinstein (1982) provide a foundation for bargaining models with complete and symmetrical information. In the context of this model, complete information would imply that all players know each other player's reservation price. Each player would have full knowledge of the possible actions by each other player, and their resultant payoffs. This implies that it is known to all players before the bargaining process whether gains from trade exist, and more specifically, the precise value of the gains from trade are common knowledge. If these gains from trade exist, then the problem for the players is to negotiate a price between their reservation prices. The solution to such a model can be axiomatic in nature, where certain desired properties of a solution to the bargaining process are outlined, as in Nash (1953). Alternatively, one could take a strategic approach and model the interaction as a sequential game, as in Rubinstein and Wolinsky (1985). Binmore et al. (1986) provide a good overview of various models of both of these types. We will focus on the strategic approach.

This paper focuses on the case of incomplete information between players, as in Chatterjee and Samuelson (1983) and Satterthwaite and Williams (1989). In the context of this paper, incomplete information implies that players only know the distribution of their adversary's reservation price over some domain, and not its precise value. That suggests that players are not aware if gains from trade exist when they enter the bargaining process. However, we do allow for players to have information about each other player's payoff (as a function of their reservation price) in the negotiating process. They are aware of the discount factor incurred by each other player. We choose this approach for practical reasons. In reality, the complete information model does not accurately portray real bargaining in many situations, as negotiations between buyers and sellers may not have a possible desirable outcome for the players, or they may break down even when gains from trade exist. We also assume that the fact gains from trade are not assured is not a deterrent from players participating in this game. In other words, the buyer and seller risk nothing by participating, regardless of their reservation prices.

In a standard treatment of bargaining under incomplete information, not all mutually beneficial agreements can be attained via bargaining, even when gains from trade exist (Chatterjee and Samuelson, 1983). The existence of a price setter reduces the instances where gains from trade cannot be realized. In this model, the probability of this outcome depends solely on the price setter's choice (higher set prices make it difficult for the players to attain positive payoffs).

The model outlined in this paper considers a mechanism which allows the buyer to privately search for offers other than the available set price. There is extensive literature regarding bargaining models where players can search for alternatives. In most of these models, alternative offers cannot be recalled if they are not accepted immediately, resulting in equilibria where no search occurs. Wolinsky (1987) explores a model of this variety. Chatterjee and Lee (1998) consider a more realistic model in which players bargain with complete information about each other, and have the capability to search for outside options, which are recallable throughout the bargaining process. Another example of search is the model by Lee (1994). The model presented in this paper has many similarities with the model presented by Chaterjee and Lee, the most obvious being that both models contain search with a recallable offer. However, the treatment of several aspects of the model differ. In Chatterjee and Lee (1998), the search costs are modelled as a fixed cost c, while we model the cost of search as a discount factor δ. They also assume search offers are drawn randomly from a distribution, while the model presented here assumes reservation prices are drawn from a distribution and search results in a strategic offer from a seller to the buyer, considering the influence of the recallable offer by the price setter.

The main result from this paper is that the introduction of a price setter increases total welfare in the market, regardless of the discount factor. The reduction of inefficiencies due to more available options for the buyers outweigh the increase in inefficiencies caused by the self-serving, profit maximizing nature of the price setter. By introducing an additional option on the seller side of the market, buyers benefit from lower prices while sellers experience a decrease in profits. However, when we examine individual welfare, the existence of the price setter benefits the buyers to a greater degree than it harms the sellers.

2 The Model

We consider the following model of bargaining. There are three players, a buyer, a seller, and a price setter. We will occasionally refer to these players as B, S and PS respectively. Both B and S have reservation prices given by v_b and v_s, which are modelled as random variables with a continuous distribution function given by $F(.)$, a density function given by $f(.)$, and support $[0, 1]$. These distributions are commonly known to all players, and are not necessarily the same for B and S . We will focus on the case where $F(.)$ is uniform for both players. B and S realize their own reservation price before the game begins, but are unaware of the precise value of their rival's reservation price. PS has a reservation price $v_{ps} = 0$, which is known to all three players. PS is also only aware of the distribution of his rivals' reservation prices, and not of their precise value.

There are four periods. In the first period, PS chooses a price s at which they will sell to any buyer, to maximize their expected payoff. This choice of s is immediately known to all players, and is recallable by the buyer in the future. Then, we consider the following model of bargaining between the buyer and seller, under the influence of this price setter. In the second period, B may choose to buy the good immediately from PS, ending the game with payoffs $p_b = v_b - s$, $p_s = 0$, and $p_{ps} = s$, or incur a penalty (in the form of a discount factor δ) and solicit an offer from a seller (retaining the original offer from PS). This penalty can be considered as the cost of delay, or as the cost of finding a seller in the private market.

If B chooses to search for an offer, S makes an offer x in period 3. We can imagine the mechanism which describes the meeting of B and S as a random meeting between a buyer and seller with reservation prices described as above. In this sense, the model described here does not consider any sort of market matching as described by Wolinsky (1987). This also differs from the treatment of Chaterjee and Lee, where by searching the buyer receives an offer drawn from some distribution. The model described in this paper requires us to examine the optimal strategy for the seller. We note it is possible for the chosen seller to have a reservation greater than the price setter's offer, $v_s > s$, causing the seller to make an undesirable offer. In that sense, our model is constructed in a way such that the buyer randomly receives an offer from someone who owns the desired good, and does not solicit only those who are willing to sell.

In the final period, B ends the game by choosing to either accept the offer from S, accept the offer from PS, or reject both offers. If B accepts the offer of x, the payoffs are given by $p_b = \delta(v_b - x)$, $p_s = x - v_s$, and $p_{ps} = 0$. If B accepts the offer from PS in the final period, the payoffs are given by $p_b = \delta(v_b - s)$, $p_s = 0$, and $p_{ps} = s$. If B chooses to reject all offers, then all players receive a payoff of zero. The model is constructed in a way such that only the buyer is affected by the discount factor. The discount factor does not apply to the seller's payoff, as they only participate in the negotiating process for one period (when a buyer approaches them soliciting an offer). This assumption is without loss of generality, as applying a discount factor to the seller's payoff would simply reduce their payoff proportionally at each possible reservation price. With regard to the price setter, it is assumed that a large number of these bargaining games occur at each moment between different buyers and sellers, and the same price setter is involved in each interaction. As a result, PS will not incur a penalty when the game ends in the final period (as B does); they simply earn $p_{ps} = s$ for each interaction which results in a buyer purchasing the good from them. This choice likely does have an effect on the equilibrium outcome. If a discount factor was applied to the price setter's payoff in the final period, the optimal choice of s would likely be lower to encourage agreements which avoid the discount factor completely.

This model represents many examples of durable good bargaining under the influence of a set price. Consider how it describes the case of a collector wishing to purchase a new card for a collectible card game (a strategic game using collectible cards). The buyer will have some reservation price for the card, depending on its rarity and perceived usefulness in gameplay. They face the following choice: either they can buy the card from a large company who determines the value of new cards (the price setter) for a set price, s, or they can search the private market and attempt to find an alternative offer, x, from another collector (the seller). If they search, they decide to take either the best offer from the large firm or the private seller, or they decide that they don't value the card enough to purchase it at all (perhaps the card is easily replaceable by a similar card). Our model does not consider that in a practical setting, the collector may continue searching (at an additional cost) for more collectors to make additional offers. Such an extension would be a useful avenue for additional research.

In the following section, we first consider the decisions facing the buyer and seller when s is given exogenously, and attempt to find an equilibrium strategy for B and S consisting of a search strategy for B and an offer strategy for S. Finally, we then consider the game as whole, incorporating the decision facing the price setter. This model results in different types of equilibria, depending on the choices made for the search cost, δ, and the set price, s (for the case where it is determined exogenously). The equilibrium found will consist of a search strategy for the buyer, and an optimal offer for the seller. Utilizing these equilibrium results, we focus on welfare considerations by determining whether the presence of a price setter causes the players to be better or worse off in this market. We show results for each player individually, in addition to the market as a whole.

3 Analysis

3.1 Buyer and Seller's Behaviour

We will solve for the equilibrium of this sequential game using backwards induction.

3.1.1 Period 4: Buyer's Acceptance Decision

First, we consider the decision faced by the buyer in the final period. At this point, the buyer simply wishes to maximize their payoff given the three available options. Either they can accept an offer, and win either $p_b = \delta(v_b - x)$ or $p_b = \delta(v_b - s)$, or if both payoffs are negative, reject the offers and receive $p_b = 0$. In this period, their payoff is given by $p_b = max[\delta(v_b - x), \delta(v_b - s), 0]$. We assume that in the case of a tie between two desirable offers, $\delta(v_b - x)$ and $\delta(v_b - s)$, or $x = s$, the buyer takes the seller's offer, x. We say an offer is desirable if it gives a non-negative payoff. If the best offer gives a payoff of 0, the buyer will take it.

3.1.2 Period 3: Seller's Offer

We now consider the decision faced by the seller, if the buyer solicits an offer. First, note that depending on the values of s and δ, the seller knows that only buyers with certain reservation prices will search in this market. We denote the point at which the buyer is indifferent between searching and taking the price setter's offer by $\hat{v}$. The buyer will search with all reservation prices $v_b \leq \hat{v}$, and will accept the guaranteed offer immediately with all reservation prices $v_b > \hat{v}$. The reason for this makes intuitive sense: with low reservation prices, the buyer will solicit an offer, as either the price setter's offer is undesirable ($v_b < s$), or they risk little by searching ($(1 - \delta) \cdot (v_b - s)$ to be exact). With higher reservation prices, the risk of soliciting an offer is greater, so they may choose to take the price setter's offer immediately.

Now, the decision faced by the seller is to make an offer x, which maximizes their payoff assuming buyers search with reservation prices $v_b \leq \hat{v}$. There are two cases to consider. The seller could choose not to sell, due to their reservation price being higher than the price setter's offer, $v_s > s$. In this case, we will say that the seller offers a price $x = s + \epsilon$, $\epsilon > 0$ to guarantee rejection. The reason for this artificial construction is to simplify how we describe the seller's optimal offer, $x^{\star}$. The critical point is that in this case the offer is chosen such that it cannot be accepted.

If $v_s \leq s$, the seller chooses an $x^{\star}$ (as a function of v_s) to maximize the following. Although we require $x^{\star} \leq s$, we will ignore that aspect of the problem momentarily:

$$max_{x^{\star} \leq s} P(x^{\star} \leq v_b)(x^{\star} - v_s) \tag{1}$$

$$max_{x^{\star} \leq s} (1 - F(x^{\star}))(x^{\star} - v_s) \tag{2}$$

$$max_{x^{\star} \leq s} (1 - \frac{x^{\star}}{\hat{v}})(x^{\star} - v_s) \tag{3}$$

Examining the first order condition yields:

$$1 - \frac{x^{\star}}{\hat{v}} - \frac{x^{\star} - v_s}{\hat{v}} = 0 \tag{4}$$

$$\mathbf{x}^{\star}(\mathbf{v_s}) = \frac{\mathbf{v_s} + \hat{\mathbf{v}}}{\mathbf{2}} \tag{5}$$

Note that for some reservation prices it is possible for this solution to give $x^{\star}(v_s) > s$, which would result in the offer being rejected. However, if $v_s \leq s$, the seller wants his offer to be accepted. The payoff function used, equation (3), is in quadratic form, and has positive first derivative when $x < x^{\star}$. Therefore, in this case, the maximizing $x^{\star}$ (which can be accepted) is at $x^{\star} = s$. The full solution to the seller's problem is given by:

$$x^{\star}(v_s) = \begin{cases} \frac{v_s + \hat{v}}{2} & \text{if } \frac{v_s + \hat{v}}{2} \leq s \\ s & \text{if } \frac{v_s + \hat{v}}{2} > s, v_s \leq s \\ s + \epsilon & \text{otherwise, } (\epsilon > 0) \end{cases} \tag{6}$$

3.1.3 Period 2: Buyer's Search Decision

Now we consider the buyer's search decision in the second period, i.e., whether to purchase the good immediately from the dominant price setter at s, or to search in the private market. Trivially, when $v_b \leq s$, the buyer will always solicit an offer by searching. When $v_b > s$, the buyer may still want to search to try and find a better deal than the offer from the price setter. In this case, the buyer will search when:

$$E[search] \geq v_b - s \tag{7}$$

To find an equilibrium strategy, we must find the indifference point, $\hat{v}$, at which the buyer with reservation prices less than or equal to $\hat{v}$ will search, and take the price setter's offer s at reservation prices greater than $\hat{v}$. The seller makes an offer assuming these search tendencies of the buyer. An equilibrium in the context of this model means the seller does not find it profitable to deviate from his offer $x^{\star}(v_s)$, and the buyer does not deviate from a strategy to search when his reservation price is no greater than $\hat{v}$.

We want to solve:

$$E[\text{search}] = \hat{v} - s \tag{8}$$

The offer is accepted when $\frac{v_s+\hat{v}}{2} \leq s$, and rejected otherwise. This is the same as requiring $v_s \leq 2s - \hat{v}$. Thus, we have:

$$\int_0^{2s-\hat{v}} \delta(\hat{v} - x^{\star}(v_s))dv_s + P(v_s > 2s - \hat{v})\delta(\hat{v} - s) = \hat{v} - s \tag{9}$$

$$\int_0^{2s-\hat{v}} \delta(\hat{v} - \frac{v_s + \hat{v}}{2})dv_s + P(v_s > 2s - \hat{v})\delta(\hat{v} - s) = \hat{v} - s \tag{10}$$

$$\int_0^{2s-\hat{v}} \delta(\frac{\hat{v}-v_s}{2})dv_s + (1-F(2s-\hat{v}))\delta(\hat{v}-s) = \hat{v}-s \tag{11}$$

$$[\frac{\hat{v}v_s}{2} - \frac{v_s^2}{4}]_0^{2s-\hat{v}} + (1-2s+\hat{v})(\hat{v}-s) = \frac{\hat{v}-s}{\delta} \tag{12}$$

$$\frac{\hat{v}(2s-\hat{v})}{2} - \frac{(2s-\hat{v})^2}{4} + (1-2s+\hat{v})(\hat{v}-s) = \frac{\hat{v}-s}{\delta} \tag{13}$$

$$s^2 - s + \frac{\hat{v}^2}{4} + \hat{v} - \hat{v}s = \frac{\hat{v}-s}{\delta} \tag{14}$$

Solving this for $\hat{v}$ yields:

$$\hat{v} = \frac{2}{\delta}[\delta s - \delta + 1 \pm \sqrt{(1-\delta)(\delta s - \delta + 1)}] \tag{15}$$

Although there are two solutions, $\hat{v}_1$ and $\hat{v}_2$, we note that the greater of them, $\hat{v}_2$, is inconsistent with the equilibrium of the model. In this case, $2s-\hat{v} < 0$, meaning no individual sellers are willing to sell. All buyers will immediately buy from the dominant price setter. Thus, one equilibrium of the model is where individual sellers choose an unreasonable price, $x(v_s) = 1$, and none of the buyers search for an offer. We will focus on the more interesting equilibrium resulting from $\hat{v}_1$. The buyer's indifferent search point is given by:

$$\hat{\mathbf{v}} = \frac{\mathbf{2}}{\delta}[\delta\mathbf{s} - \delta + \mathbf{1} - \sqrt{(\mathbf{1}-\delta)(\delta\mathbf{s} - \delta + \mathbf{1})}] \tag{16}$$

The solution, $\hat{v}$, has several properties we would expect. Firstly, it is greater than s. This is expected, as there are cases where the buyer wishes to risk paying the search cost δ to find a better offer. It is increasing in δ, that is $\frac{d\hat{v}}{d\delta} > 0$. This seems reasonable, as we would expect the buyer to take more risks and search more often when the penalty associated with delay is less. It is also increasing in s, or in other words $\frac{d\hat{v}}{ds} > 0$, which is intuitively obvious as the higher the offer from the price setter, the more often the buyer must search.

3.2 Price Setter's Behaviour

We now consider the model with the price setter's choice as endogenous, in which they have an opportunity to choose s in the first period. The problem facing the price setter is:

$$max_{s^\star} s^\star[P(\text{choose s in final period}) + P(\text{no search})] \tag{17}$$

$$max_{s^\star} s^\star[P(s \leq v_b \leq \hat{v}) \cdot P(v_s > s) + P(v_b > \hat{v})] \tag{18}$$

$$max_{s^\star} s^\star[(\hat{v} - s) \cdot (1 - 2s + \hat{v}) + (1 - \hat{v})] \tag{19}$$

Solving this analytically is difficult. The price setter has to consider that the choice of s influences the equilibrium that is reached between the buyer and seller: both the search decision for the buyer and the seller's offer are affected. Attempting to solve this by substituting in the result for $\hat{v}$ results in an extremely complicated and intractable expression, the result of which is not interpretatively useful. However, we can examine the solutions from a numerical standpoint.

Figure 1 shows the optimal choice of s for each possible δ. First, we note the solution for s is decreasing in δ. In other words, if we could derive an expression for $s^\star$, $\frac{ds^\star}{d\delta} < 0$. This makes sense, as when the penalty for searching decreases, the buyer has an incentive to search with higher reservation prices. A higher search frequency results in more agreements in the private market and fewer agreements with the price setter, causing a lower offer of s that will be accepted on average. The price setter's choice is made to maximize payoff, so it stands to reason that the optimal choice of s will decrease to combat this lower acceptance rate. At $\delta = 0$, where the buyer can never search for another offer, the optimal offer for the price setter is given by $s^\star = 0.5$, the monopoly price in this setting. From this point, as δ increases, the optimal offer for the price setter decreases. Some examples: When $\delta = 0.7$, the price setter chooses $s = .438$, causing buyers with reservation prices $v_b \leq 0.514$ to search. When $\delta = 0.8$, the price setter chooses $s = 0.422$, causing buyers with reservation prices $v_b \leq 0.524$ to search. When $\delta = 0.9$, the price setter chooses $s = 0.399$, causing buyers with

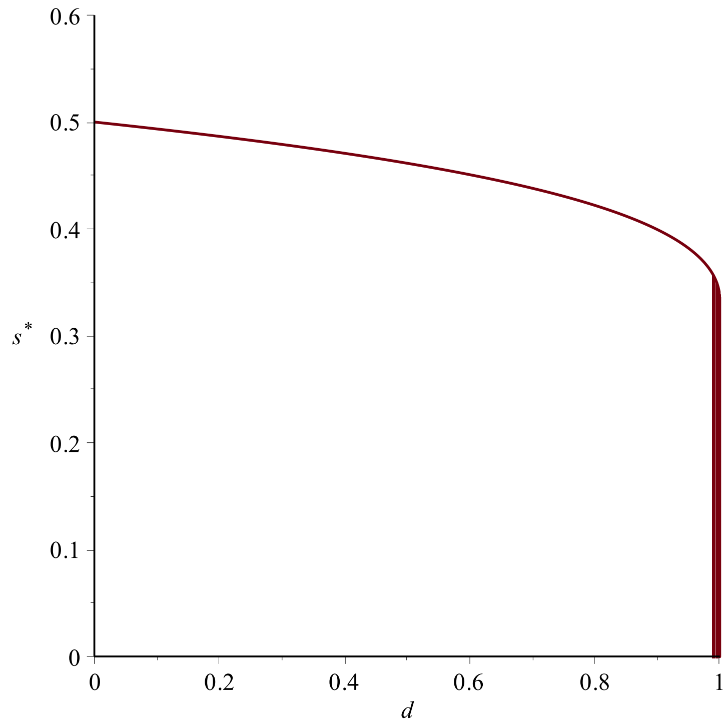

Figure 1: Price Setter's Problem: Optimal choice of $s^\star$ vs. discount factor

reservation prices $v_b \leq 0.544$ to search. When $\delta = 0.99$, the price setter chooses $s = 0.356$, causing buyers with reservation prices $v_b \leq 0.611$ to search.

3.3 Comparison with No Price Setting

One of the motivations for considering this model of price setting is to compare it with a model where a price setter is not present, to examine the dominant player's effect on welfare. The price setter may increase welfare by providing more options to the buyers, but may reduce welfare by using their dominance to choose a profit maximizing price. Conveniently, the construction of our model allows us to easily consider this case. By setting s to the highest possible price, $s = 1$, and assuming the buyer always searches, $\hat{v} = 1$, the model outlined here accurately portrays a one period bargaining model with incomplete information, where a seller makes a single take it or leave it offer to the buyer. In this case, the seller's optimal offer is given by $x^\star(v_s) = \frac{1+v_s}{2}$. This is similar to the treatment of Chatterjee and Samuelson

(1983), with the bargaining parameter k set to 0.

To compare the welfare of the players, we examine the expected welfare for the buyer and seller at each of their possible reservation prices, $W_b(\hat{v}, s, \delta, v_b)$, $W_s(\hat{v}, s, \delta, v_s)$. We then solve for the expected welfare for all the players over their possible reservation prices.

First, we consider the buyer's welfare with a price setter. The payoff to the buyer can be determined for each of the following three cases.

When $0 \leq v_b < s$, the buyer always searches, and receives a positive payoff if the seller makes a desirable offer. A desirable offer comes if $x^\star(v_s) = \frac{\hat{v}+v_s}{2} \leq v_b$, or $v_s \leq 2v_b - \hat{v}$. Also, note that it is impossible for the buyer to receive a desirable offer if $v_b < \frac{\hat{v}+0}{2}$; in this case the buyer always receives a payoff of 0. Thus, the following gives the welfare for the buyer when $\frac{\hat{v}}{2} \leq v_b < s$ (it is easy to show that $\frac{\hat{v}}{2} < s$ is always satisfied):

$$W_b(\hat{v}, s, \delta, v_b) = \int_0^{2v_b - \hat{v}} \delta(v_b - x^\star(v_s))dv_s \tag{20}$$

$$W_b(\hat{v}, s, \delta, v_b) = \delta[v_b(2v_b - \hat{v}) - \frac{\hat{v}}{2}(2v_b - \hat{v}) - \frac{(2v_b - \hat{v})^2}{4}] \tag{21}$$

When $s \leq v_b < \hat{v}$, the buyer again searches according to his optimal strategy, but in this case they receive the better of the seller's offer and the price setter's offer. The seller's offer is more desirable if $x^\star(v_s) = \frac{\hat{v}+v_s}{2} \leq s$, or $v_s \leq 2s - \hat{v}$. In this case, the buyer earns $\delta(v_b - \frac{\hat{v}+v_s}{2})$. In all other cases, the buyer pays s and earns $\delta(v_b - s)$. Considering this, the welfare for the buyer over this domain is given by:

$$W_b(\hat{v}, s, \delta, v_b) = \int_0^{2s - \hat{v}} \delta(v_b - x^\star(v_s))dv_s + \delta(1 - 2s + \hat{v})(v_b - s) \tag{22}$$

$$W_b(\hat{v}, s, \delta, v_b) = \delta[v_b(2s - \hat{v}) - \frac{\hat{v}}{2}(2s - \hat{v}) - \frac{(2s - \hat{v})^2}{4} + (1 - 2s + \hat{v})(v_b - s)] \tag{23}$$

Finally, when $\hat{v} \leq v_b \leq 1$, the buyer doesn't search and immediately accepts the offer from the price setter, earning $W_b(\hat{v}, s, \delta, v_b) = v_b - s$.

Thus, the complete welfare function is given by:

$$\mathbf{W_b}(\hat{\mathbf{v}}, \mathbf{s}, \delta, \mathbf{v_b}) = \begin{cases} \delta[v_b(2v_b - \hat{v}) - \frac{\hat{v}}{2}(2v_b - \hat{v}) - \frac{(2v_b - \hat{v})^2}{4}] & \text{if } \frac{\hat{v}}{2} \leq v_b < s \\ \delta[v_b(2s - \hat{v}) - \frac{\hat{v}}{2}(2s - \hat{v}) - \frac{(2s - \hat{v})^2}{4} + (1 - 2s + \hat{v})(v_b - s)] & \text{if } s \leq v_b < \hat{v} \\ v_b - s & \text{if } \hat{v} \leq v_b \leq 1 \end{cases} \tag{24}$$

When there is no price setter, which we represent by $s = 1$ and $\hat{v} = 1$, we have only the first case, where the buyer searches and accepts the offer $x^\star(v_s) = \frac{1+v_s}{2}$, if it is desirable. As before, they can only receive a desirable offer if $v_b \geq \frac{\hat{v}}{2}$. Their welfare is then given by:

$$W_b(\hat{v}, s, \delta, v_b) = \delta[v_b(2v_b - \hat{v}) - \frac{\hat{v}}{2}(2v_b - \hat{v}) - \frac{(2v_b - \hat{v})^2}{4}] \tag{25}$$

Solving for when $s = 1$ and $\hat{v} = 1$ yields the following when $\frac{1}{2} \leq v_b \leq 1$:

$$\mathbf{W_b}(\hat{\mathbf{v}}, \mathbf{s}, \delta, \mathbf{v_b}) = \delta(\mathbf{2v_b - 1})(\mathbf{v_b} - \frac{\mathbf{1}}{\mathbf{2}} - \frac{\mathbf{2v_b - 1}}{\mathbf{4}}) \tag{26}$$

and a payoff of $W_b(\hat{v}, s, \delta, v_b) = 0$ otherwise.

Figure 2 compares these welfare functions when $\delta = 0.8$. In this case, the price setter chooses $s = 0.422$ and buyers with reservation prices $v_b \leq 0.524$ search. Examining Figure 2, we see that for the buyer, the payoff is always better in the case where the price setter exists. When the buyer has the additional option of taking the price setter's offer, and not just the opportunity to solicit an offer from a seller, they retain the same opportunities as they did before, plus the additional options made available by the existence of an outside option. More options weakly improves the buyer's outcome, regardless of the buyer's reservation price. One point of interest, is that as a result of the price setter, the buyer has a positive expected payoff for lower reservation prices than they would if there was no price setting. Now, they experience a positive expected payoff for all reservation prices $v_b > \frac{\hat{v}}{2}$, whereas

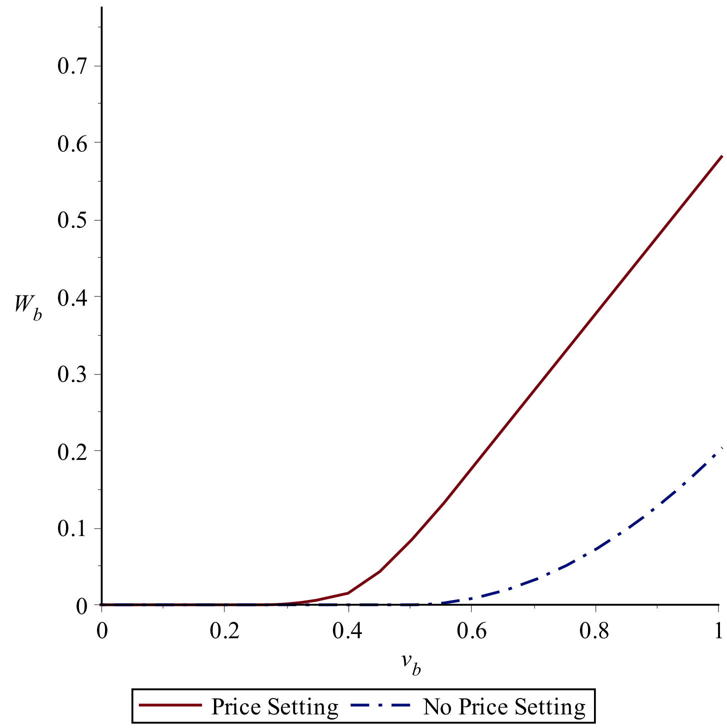

Figure 2: Payoff to Buyer vs. Buyer's Reservation Price, $\delta = 0.8$

previously they received a positive expected payoff for all reservation prices $v_b > \frac{1}{2}$. In the following section we will see that this welfare gain occurs regardless of the choice of δ.

For the seller, there are again three cases to consider. First, when the seller chooses to offer a price less than s, $x^\star(v_s) = \frac{\hat{v}+v_s}{2} \leq s$, he will earn a positive payoff when it is a desirable offer for the buyer, $x^\star(v_s) \leq v_b$. His offer is no greater than s when $0 \leq v_s \leq 2s - \hat{v}$. In this case, his welfare is given by:

$$W_s(\hat{v}, s, \delta, v_s) = (\frac{v_s + \hat{v}}{2} - v_s) \cdot P(v_b \geq \frac{v_s + \hat{v}}{2}) \tag{27}$$

$$W_s(\hat{v}, s, \delta, v_s) = (\frac{\hat{v} - v_s}{2}) \cdot (1 - \frac{v_s + \hat{v}}{2}) \tag{28}$$

As discussed before, occasionally the expression $x^\star(v_s) = \frac{\hat{v}+v_s}{2}$ gives an optimal offer which is greater than s, resulting in the offer being rejected. However, as long as the seller has a reservation price no greater than the price setter's offer, the seller will actually choose the highest acceptable offer to the buyer, $x^\star = s$, to guarantee acceptance. This situation occurs when $2s - \hat{v} < v_s \leq s$, and the seller's welfare is given by:

$$W_s(\hat{v}, s, \delta, v_s) = (s - v_s) \cdot P(v_b \geq s) \tag{29}$$

$$W_s(\hat{v}, s, \delta, v_s) = (s - v_s) \cdot (1 - s) \tag{30}$$

Finally, when the seller has a reservation price no less than the sure thing, $s < v_s \leq 1$, the seller cannot make a desirable offer, and he earns nothing.

Thus, the welfare function for the seller is given by:

$$\mathbf{W_s(\hat{v}, s, \delta, v_b)} = \begin{cases} (\frac{\hat{v}-v_s}{2}) \cdot (1 - \frac{v_s+\hat{v}}{2}) & \text{if } 0 \leq v_s \leq 2s - \hat{v} \\ (s - v_s) \cdot (1 - s) & \text{if } 2s - \hat{v} < v_s \leq s \\ 0 & \text{if } s < v_s \leq 1 \end{cases} \tag{31}$$

Solving for when $s = 1$ and $\hat{v} = 1$ yields the following:

$$\mathbf{W_s(\hat{v}, s, \delta, v_s) = (\frac{1 - v_s}{2})^2} \tag{32}$$

Figure 3 compares these welfare functions when $\delta = 0.8$. Examining Figure 3, it is clear that the seller does better when there is no dominant price setter in the market. When there is no price setter in the market, all the buyers have no choice but to solicit an offer. The introduction of a price setter gives the buyers an alternative option. This results in less agreements between the buyer and private seller, and these agreements occur at a lower agreed price. Essentially, the presence of a competitor shifts control in negotiations towards the buyer, as they now retain the outside offer from a price setter, and use this fact to force a lower offer. Figure 3 also shows that unlike the no price setting case, where the seller can always expect a positive payoff (except at the boundary case $v_s = 1$), when price setting occurs the seller can only earn a positive payoff if his reservation price is below s, $v_s < s$.

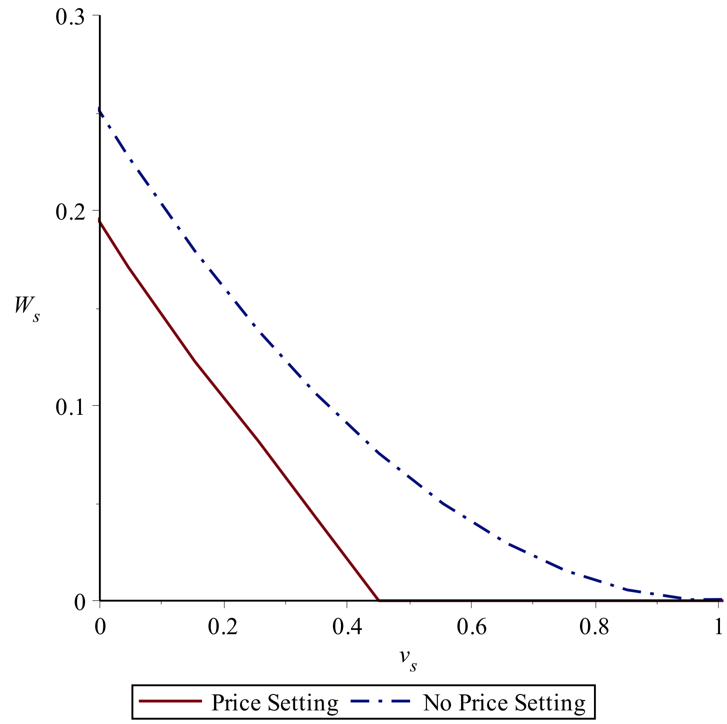

Figure 3: Payoff to Seller vs. Seller's Reservation Price, $\delta = 0.8$

This is a direct result of the buyer having an available alternative to private negotiation with a seller.

3.3.1 Examining Total Welfare Changes

In the previous section, we showed that due to a price setter, the welfare for the buyer increases (more available options), and the welfare for the seller decreases (more competition). We now consider how total welfare changes in the entire market due to this welfare shift from the seller to the buyer. To accomplish this, we consider the expected payoff to each player by taking the expectation of the payoff functions over all possible reservation prices. To find the expected welfare for the buyer, we must integrate the piecewise function which defines the buyer's payoff at each reservation price, equation (24). To find the expected welfare for the seller, we again integrate the piecewise function which defines the seller's payoff at each reservation price, equation (31).

Unfortunately, these functions don't behave in a way that is simple to analyze. They are

δ	ΔEW_b	ΔEW_s	$\Delta E\Pi_{ps}$	Change in Total Welfare
0.99	0.1664	-0.0434	0.1970	0.3200
0.9	0.1448	-0.0414	0.2167	0.3201
0.8	0.1358	-0.0413	0.2257	0.3202
0.7	0.1310	-0.0414	0.2315	0.3211
0.5	0.1264	-0.0415	0.2392	0.3241

Table 1: Differences in welfare due to price setter, for different discount factors

both high degree functions of the three parameters, $\hat{v}$, s, and δ. Thus, we use a numerical computation in MATLAB to determine these welfare comparisons.

The payoff to the price setter is simply given by:

$$\mathbf{\Pi_{ps}} = \mathbf{s}^{\star}[(\hat{\mathbf{v}} - \mathbf{s}) \cdot (\mathbf{1} - \mathbf{s}) + (\mathbf{1} - \hat{\mathbf{v}})] \tag{33}$$

We now consider some examples using optimal parameters for different choices of the discount factor. Table 1 shows the welfare change due to introducing a dominant price setter, for different choices of δ.

The main result from Table 1, is that while individual buyers gain, and individual sellers lose, the presence of the price setter increases total welfare in the market. It is also noted, that if we only consider the welfare changes between the buyer and the seller (i.e., without considering the profits for the dominant price setter), total welfare still increases. Consider the case where no price setting occurs. In this market, there are several instances where there are potential gains from trade, $v_b > v_s$, but these gains from trade are not realized. This source of inefficiency occurs when $v_b < \frac{v_s+1}{2}$. For example, if the buyer has a reservation price of $v_b = 0.6$, and the seller has a reservation price of $v_s = 0.5$, there is a possible gains of trade of $v_b - v_s = 0.1$ for the players. Unfortunately, the seller's optimal offer in this case is given by $x^{\star} = \frac{0.5+1}{2} = 0.75$, which the buyer will immediately reject. Thus, the possible gains from trade are not realized. Now, as a result of price setting, the buyer can more often realize potential gains from trade due to the available option from the price setter. Whenever $v_b > s$, the buyer is guaranteed to purchase the good from the better option of either the private seller or the price setter. In fact, the described model always guarantees the buyer trades with the player who generates the greatest possible gains from trade when $v_b > s$, as they will accept an offer from the seller when $v_s < s$, and will accept the sure thing otherwise. Another source of welfare gain is the ability to avoid paying the search cost, δ. Due to the construction of this model, the buyer can choose to immediately accept the price setter's offer and avoid the discount factor all together. Thus, the existence of a price setter will directly increase total welfare in the market by eliminating some inefficiencies due to paying this search cost.

The seller is not as fortunate. The influence of the price setter results in more cases where the seller cannot realize possible gains from trade between him and the buyer. This happens for several reasons. First, the set price, s, means the buyer sometimes does not even search for a private offer. In this case, any possible payoff for the seller is not realized, and all the welfare is distributed between the buyer and the price setter. Also, whenever $s < v_s < v_b$, the seller cannot take advantage of the possibility for welfare increase between him and the buyer. Again, in this case all welfare gains are realized between the buyer and the price setter. Finally, it's possible the seller's optimal offer is undesirable to the buyer when $v_s < v_b < s$, and nobody experiences a positive payoff. This final case indicates there is still inefficiency in this market, even though this inefficiency is more uncommon under the price setter's influence.

Table 1 also provides some insight into how the discount factor δ affects the equilibrium payoff for the players. The buyer's welfare change increases as the discount factor increases. The price setter's profit decreases as the discount factor increases. The seller's welfare loss increases in δ for large values, but decreases in δ for smaller values. Most importantly, it seems that the total welfare increase is greater for lower values of δ, indicating that the influence of the price setter on welfare is greatest when the buyer must pay a higher search cost. Note, these results are based on only a few numerical calculations, and cannot be easily verified without analytically examining the first derivative of these payoff functions.

4 Conclusions

In this paper, we considered a model of bargaining with incomplete information, under the influence of a price setter. Considering the model where price setting occurs exogenously, the equilibrium consists of an optimal search strategy for the buyer, and an optimal price choice for the seller. When factoring in the decision facing a price setter trying to maximize their own payoff, the equilibrium results in a welfare increase for the market. Individually, the seller is now worse off while the buyer is better off, as a direct result of new competition in the market.

Consider the original example of the purchase and sale of a collectible card under the influence of a large appraiser who specifies its value. The appraiser acts as a price setter, by setting a value at which the buyer can easily purchase the good from a larger firm. Our result indicates that this appraiser is a benefit to the market. Some inefficiencies that would normally occur due to incomplete information now do not exist, as the buyer has more options in bargaining and the seller has to make more competitive offers as a result. Perhaps two people negotiating over the price of a collectible card, with reservation prices $v_s < s < v_b$, will now reach an agreement, when without the appraiser such a desirable agreement may not be found. One could summarize this result by claiming the appraiser is providing more information to the other players in the game, which they can use to find an efficient outcome. Additionally, the price setter directly reduces welfare loss in the market by allowing the buyer to take a deal without paying a search cost.

The most interesting possible extension of this model is to consider the problem over an infinite amount of periods. It would be useful to allow the buyer to search in the private market repeatedly, incurring a cost of δ each time they search, with the option of taking the recallable price setter's offer in any period before they search. This would obviously be a more accurate representation of real world bargaining, as in reality a buyer could search for offers as often as they desire. Examining the resulting equilibria and performing a welfare comparison would provide insight into whether the price setter does in fact have a positive effect on this kind of market. One would expect this extension would cause lower choices of s by the price setter (since the buyer has more private options), more frequent searching by buyers (since they can search indefinitely), and lower offers by sellers (again, due to the buyer having more alternative options).

In this paper, the search cost was modelled as a discount factor δ applied to certain payoffs. Alternatively, one could examine this model with a fixed cost c. This could significantly change the results: buyers with reservation prices only marginally greater than s may no longer be able to search with positive expected payoffs, and buyers with high reservation prices may now search more easily. In this case, it is possible that the buyer's optimal search strategy will be to search only when s is not desirable (the buyer must search anyway), or when v_b is significantly greater than s (the buyer can afford to search). This would drastically change the equilibrium analysis, and could cause a different welfare result.

The model provided is basic in the sense that it assumes the distribution of reservation prices is uniform for both players. This allows us to explicitly derive an equilibrium based on any choice of parameters. However, these distributions are somewhat limited. Expanding the model to allow for any distribution would be a useful exercise, although it would likely be limited to a numerical study, as most models of real reservation price distributions would not behave as nicely.

Another potential source of analysis would be to examine under what circumstances this sort of price setting could occur, and how a large firm could maneuver themselves into a position to act as a large price setting entity which drives the negotiations in the private market. Additionally, one could analyze if members of the private market could collude and take advantage of a price setter, or force the price setter to set a different price. Ultimately, the model described here provides a starting point for further examination of the effect of price setting on bargaining models with incomplete information.

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