DATING CANADIAN RECESSIONS: AN APPLICATION OF THE STOCK & WATSON SINGLE INDEX MODEL

by

Thomas C. Feltmate

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Queen's University
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ABSTRACT

The current method for identifying recessions in Canada is two consecutive quarters of negative growth in GDP. This paper presents an explicit time series model, known as a dynamic factor or ‘single index’ model that implicitly estimates an unobserved common variable, which can be thought of as the overall state of the Canadian economy. Upon estimating this model, using data from 1976:1-2011:2, it is found that if the unobserved factor growth rate declines below the identified -2% threshold, then the economy is in recession. The ‘official’ recession dates used for the analysis were those posted by Statistic Canada, under the classification of two successive quarters of decline in GDP. The results show that for two of the four recessions that occurred over the period of interest, the proposed framework dates the recessions before the current Statistics Canada method would have.

1. A Brief History of Coincident Indicators & Introduction

Cyclical coincident macroeconomic indicators are widely covered throughout the media as a means of portraying the overall 'state' of the economy to the public. The evolution of how and what indicators should be used when trying to forecast business cycles has been an area of research since the era of the Great Depression. The National Bureau of Economic Research (NBER), led by Wesley C. Mitchell and Arthur F. Burns were the first to publish a list of business cycle indicators in 1938 (Mitchel and Burns 1938). The compiled list was based upon a study of nearly 500 monthly series, covering varying historical periods and ending in 1933 with the business cycle reaching a very distinct trough. Mitchell and Burns concluded by purposing a set of 21 'most trusted' indicators and another more enriched set of 71 series that proved to, "have been tolerably consistent in their timing in relation to business cycle revivals and at the same time of sufficiently general interest to warrant some attention by students of current economic conditions".

In 1950, the NBER released a second, more detailed list of business cycle indicators. In this study, approximately 800 series were included, utilizing measures of cyclical behavior from the time period used in the previous study, through 1938. In the detailed report, Geoffrey Moore explained that this study went beyond the first in several ways. First, indicators of the recession as well as revival were covered. Second, probability standards against which the historical records of timing and conformity could be judged were introduced. Third, comprehensive economic classification of the 800 series was used in making the final selection of indicators. Finally, selected series were classified into three categories reflecting their timing at business cycle peaks and troughs:

leading, roughly coincident and lagging. The study concluded by again suggesting 21 of the ‘best’ macroeconomic series, including 8 leading series, 8 roughly coincident series and 5 lagging series. (Moore 1950)

In the fall of 1957, the chairman of the Council of Economic Advisers, Raymon J. Saulnier, requested that the Bureau of the Census start a research program to develop a monthly report on economic indicators that would take advantage of new findings about the relations of economic processes over time, the availability of a great many economic time series in seasonally adjusted form and large-scale electronic computing facilities. NBER economists, Julis Shiskin and Geoffery H. Moore led the project and in November 1968, the composite indexes were released to the public for the first time in the Business Conditions Digest published by the US Bureau of the Census (Shiskin 1961). The development of the US composite index by the NBER spurred Canadian organizations such as Finance Canada and Statistics Canada to create similar indexes pertaining to the Canadian economy.

Cyclical indexes released by public organizations again underwent changes in the 1970’s and 1980’s. The modifications were made to more reflect changes in the economy, to take into account new data availability, to update the weighting scheme and to incorporate research results by economists, notably on the leading relationship of the yield curve. Around the same time, an influential paper published by Stock and Watson (1991) introduced a completely new methodology for estimating coincident indices. However, it should be noted that Stock and Waton’s (1991) theoretical model was never widely used by the NBER when trying to forecast coincident indices, even though it yields very similar coincident indices to that of the weighted methodology approach.

In the paper entitled "New Indexes of Coincident and Leading Economic Indicators" (Stock and Watson 1991) Stock and Watson present a framework whereby co-movements in many macroeconomic variables have a common element, which can be linked by a single common underlying unobserved variable. That is, assuming a set of coincident macroeconomic series, one can estimate a latent factor, which represents the overall 'state' of the economy. Stock and Watson criticize prior formulations of coincident indicators of being too limited in scope. That is, coincident variables such as industrial production, unemployment and GDP on their own, only measure specific facets of the economy, but says nothing about the overall 'state' of the economy. However, the single-index model proposed by Stock and Watson allows for multiple coincident variables to be included and assigns a weight to each of the variables against a common factor or indicator. Under this framework, series that provide more explanatory power with respect to the latent variable are given a greater weight, relative to those, which have less explanatory power. Since this paper was published, many countries including Canada have used a similar framework to that of Stock and Watson to compose their own 'composite index' (Gaudreault et al. 2003). However, to our knowledge, no one has used this index to try and date previous recessions in Canada.

In a 1975 New York Times article titled, "The Changing Business Cycle", economic statistician Julius Shiskin suggested several 'rules of thumb' to consider when trying to classify whether a country was in a recession. In particular, Shiskin suggested that if an economy experiences two or more successive quarters of negative growth in GDP, then that country is most likely a recession (Shiskin 1974). Shortly after the article was published, many countries worldwide, including Canada, adopted Shiskin's recession identifying 'rule of thumb'. However, an immediate problem with this definition is it only accounts for one measure of economic activity, namely GDP. This is a problem because GDP alone is dependent on many random and uncontrollable shocks. For example, weather droughts can severely impact the amount of agricultural output Canada has in a given year. As a result, the above definition alone, may mislead people into thinking a recession is going to happen when in fact it is not. A second problem with the current method for assessing recessions is there can be a large lag between when the recession is made 'official' and when it actually started. Identifying a recession in the early stages is particularly beneficial as it becomes much easier to contain and minimize its effects. Global wide recessions become much more difficult to deal with, as multiple countries can be affected differently by the same recession. Therefore, it is essential to ensure that a timely and flexible means to identify recessions is pursued. Even though the two successive quarters of negative GDP growth has many drawbacks it is still widely used in many countries, with the exception of the US.

In the past, different time series techniques have been studied to try and better explain movements in the business cycle and the timing of recessions. Here, we will only briefly discuss two similar techniques, which have become accepted over the years.
In the early 1980's Stephen Beveridge and Charles Nelson introduced a general procedure for the decomposition of non-stationary time series into two components, permanent and cyclical (Beveridge and Nelson 1981). For the purpose of their study, Beveridge and Nelson allowed for the permanent component to follow a random walk with drift and the cyclical component to be a stationary process with mean zero. Using a composite index formed from thirty-eight coincident indicators, the authors tried

measuring US business cycle and dating recessions that occurred in the post-war era. Their results showed that the measured expansions and contractions of the economy were approximately of equivalent duration to those predicted by the NBER. Under Beveridge and Nelson's framework, a recession was defined as any decline in the measured coincident index below the 4% threshold. Furthermore, Beverirdge and Nelson were able to identify four other 'mini-recessions' that occurred over the time period of 1947 through 1977. The authors described a mini-recession as one where there was a noticeable drop in the growth of the measured index, but never substantial enough to be warranted as a recession (i.e. never fell below the 4% threshold).

In 1997, Robert Hodrick and Edward Prescott published a paper, where they too examined aggregate fluctuations in the post-war US economy, using a similar common 'trend-cycle' decomposition (Hodrick and Prescott 1997). For the purpose of their paper, GNP was decomposed into two parts: a sum of slowly evolving secular trend and a transitory deviation also known as a cycle. The problem with any trend-cycle decomposition is the trend and cycle components are not readily observable. Therefore, any de-trending method must start out by arbitrarily defining the trend and the cycle components, before these elements can actually be observed from the data. Hodrick and Prescott suggested a technique (Hodrick-Prescott filter) to extract the trend component from the series. The benefit to using the technique purposed by Hodrick and Prescott when analyzing business cycles is a smoothed non-linear representation of the variable of interest is obtained, which is more sensitive to long-term rather than short-term fluctuations. To date, the Hodrick-Prescott filter is still a favorite empirical technique among researchers who want to separate cyclical behavior from the long run path of economic series. Applied to both true and artificial data, filtered series have been studied mainly to discover 'stylized facts' in business cycles by observing and comparing univariate and cross moments variability, correlation and autocorrelation.

The framework proposed in this paper uses the Stock and Watson (1991) technique to create a Canadian coincident leading indicator and then uses the index to identify the four official recessions that have occurred over the past 35 years. Our hope is that by accounting for more overall macroeconomic variability, a more reliable and realistic measure of recessions and their timing can be tailored to the Canadian economy. The coincident variables which proved to be the most effective in our framework were industrial production, total manufacturing and retail sales, total residential and non-residential housing permits, unemployment rate, total amount of available private credit and US industrial production. The results proved to be particularly satisfactory, dating each of the four recessions 2-6 months after their official start dates of 1980:2, 1981:10, 1990:4 and 2008:8, respectively. Our findings suggest that the Canadian economy will be considered to be in recession if the estimated unobserved coincident variable ever falls below the 2% level. In 2001:2, the estimated common factor almost falls below the 2% threshold, even though, under the Statistics Canada definition, an official recession never occurred. However, we attribute these effects to the DOT Com bubble, which had occurred in the US approximately a year before. Since this was mainly an equity driven recession, the spillover effects into Canada were not nearly as strong as previous recessions.

The remainder of this paper is structured as follows. Section 2 describes the dynamic factor model used to estimate the unobserved coincident factor. Section 3 discusses data used in this paper. Section 4 presents the empirical results. Section 5 discuses the findings, suggests several directions for refinements and further measurement and presents the concluding remarks.

2. Single Index Model

2.1 Introduction to Single Index Model

The single index model proposed by Stock and Watson (1991) assumes that there is a single unobserved variable, common to many macroeconomic time series. Let $\Delta y_{i,t}$ be an *nx1* vector of stationary variables (*i* denotes a particular macroeconomic series, *t* denotes time and variables are cast as natural change in logarithms) of which is assumed to fluctuate contemporaneously with the overall economic conditions. The single index model is therefore defined as follows:

$$\Delta y_{i,t} = \beta_i + \gamma_i C_t + u_{i,t} \qquad (i=1,2,\ldots,k) \qquad (1)$$

$$C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + \ldots + \phi_p C_{t-p} + \eta_t \qquad (2)$$

$$u_{i,t} = d_i u_{i,t-1} + d_{i,2} u_{i,t-2} + \ldots + d_{i-q} u_{i,t-q} + \varepsilon_{i,t}, \qquad (i=1,2,\ldots,k) \qquad (3)$$

where $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$ and C_t is an *nx1* matrix that denotes the stationary growth rate of the economic activity variable. This single-index model states that the growth rate of the i^{th} macroeconomic variable, $\Delta y_{i,t}$, consists of two stochastic components: the common unobserved index and an idiosyncratic shock, $u_{i,t}$. In the above representation and for the purposes of this paper, both the unobserved factor and idiosyncratic shocks are modeled by having autoregressive stochastic processes. More specifically, the unobserved factor and idiosyncratic components are said to follow separate and uncorrelated AR(p) and AR(q) processes, respectively.

In the model given above, the main identifying assumption expresses the core notion of the dynamic factor model that the co-movements of the multiple time series arise from the single source C_t. This is made precise by assuming that $(u_{1,t},\ldots,u_{n,t}, C_t)$ are mutually uncorrelated at all leads and lags, which is achieved by making the matrix that contains $(u_{1,t-p},\ldots, u_{n,t-p})$ diagonal and the *n+1* disturbances $(\varepsilon_{1,t},\ldots, \varepsilon_{n,t}, \eta_t)$ mutually and serially uncorrelated. Equation (2) implies that the mean of C_t is restricted to zero in the model. Since the first difference of C_t is imposed, the estimations of the unobserved factor will be presented in deviation form. Finally, the scale of C_t is identified by the normalization $var(\eta_t)=1$.

2.2 Estimation Using Kalman Filter

In order to estimate equations (1)-(3) the model is transformed into a state-space form so that the Kalman filter can be used to evaluate the maximum likelihood function. State-space representation is comprised of two sets of equations: state equations and measurement equations. The measurement equations relates the observed macroeconomic variables, $\Delta y_{i,t}$ (i=1,2,…,M), to the unobserved state vectors which, in this case are, C_t and $u_{i,t}$. The state or transition equations, describes the evolution of the state vector.

The discussion below follows the derivation in Stock and Watson (1989), which shows how the Kalman filter can be used to evaluate the maximum likelihood function. Following the derivation is a numerical example to help aid the somewhat abstract derivation. Kalman filtering involves prediction and updating using both the measurement and transition equations. Before proceeding, it is useful to rewrite equations (2) and (3) in a more compact form:

$$\phi(L)C_t = \eta_t \tag{4}$$

$$D(L)u_t = \varepsilon_t , \tag{5}$$

where L denotes the lag operator, $\phi(L)$ is a scalar lag polynomial and D(L) is a lag polynomial matrix. The transition equation can be obtained by combing equations (4) and (5). The goal is to estimate the unobserved factor, C_t, using information up to time *t.* In matrix form, the transition equation can be compactly written as follows:

$$\begin{bmatrix} C_t^* \\ u_t^* \\ C_{t-1} \end{bmatrix} = \begin{bmatrix} \Phi^* & 0 & 0 \\ 0 & D^* & 0 \\ Z_c & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{t-1}^* \\ u_{t-1}^* \\ C_{t-2} \end{bmatrix} + \begin{bmatrix} Z_c & 0 \\ 0 & Z_u \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \tag{6}$$

where,

$$C_t^* = [C_t \;\; C_{t-1} \ldots C_{t-p+1}]'$$

$$\Phi^* = \begin{bmatrix} \phi_1 & \cdots & \phi_{p-1} & \phi_p \\ & I_{p-1} & & 0 \end{bmatrix}$$

$$D^* = \begin{bmatrix} D_1 & \cdots & D_{k-1} & D_k \\ & I_{n(k-1)} & & 0 \end{bmatrix}$$

$$Z_c = [1 \quad 0_{1x(p-1)}]$$

$$Z_u = [I_n \quad 0_{nxn(k-1)}].$$

In the notation given above, I_n denotes the *nxn* identity matrix, 0_{nxk} denotes a *nxk* matrix of zeros, and $D_i = (d_{1i}, \ldots, d_{ni})$, where $d_j(L) = 1 - \sum_{i=1}^{k} d_{ji} L^i$.

The measurement equation can be obtained by writing (1) as a linear combination of the state vector:

$$\Delta y_t = \beta + \begin{bmatrix} \gamma Z_c & Z_u & 0 \end{bmatrix} \begin{bmatrix} C_t^* \\ u_t^* \\ C_{t-1} \end{bmatrix} \tag{7}$$

Finally, equations (6) and (7) can be expressed in a slightly more compact form where the transition and measurement equation are rewritten as follows:

$$\alpha_t = T_t \alpha_{t-1} + R\varsigma_t \tag{8}$$

$$\Delta y_t = \beta + Z\alpha_t + \xi_t \ , \tag{9}$$

where

$$\alpha_t = \begin{bmatrix} C_t^* & u_t^* & C_{t-1} \end{bmatrix}'$$

$$\xi_t = \begin{bmatrix} \eta_t & \varepsilon_t \end{bmatrix}',$$

and the matrices T_t, R and Z denote the transition matrix in (6), the selection matrix in (6) and the selection matrix in (7), respectively.

We use the Kalman filter to evaluate the Gaussian Likelihood function for a given set of parameters (Harvey 1991). The filter recursively constructs the minimum mean square error (MMSE) estimates for the unobserved state vector, given a set of observations, y_t. This is an iterative process consisting of two sets of equations known as the prediction and updating equations. The prediction equations are given by:

$$\alpha_{t|t-1} = T_t \alpha_{t-1|t-1} \tag{10}$$

$$P_{t|t-1} = T_t P_{t-1|t-1} T_t' + R\Sigma R'. \tag{11}$$

Given the above notation, $\alpha_{t|r}$ denotes the estimate of α_t based on the information set $(y_1, \ldots, y_n)$ and $\Sigma = E[\zeta_t \zeta'_t]$ denotes the estimated covariance matrix of ζ_t. Finally, $P_{t|r} = E[(\alpha_{t|r} - \alpha_t)(\alpha_{t|r} - \alpha_t)']$, which denotes the covariance matrix of the differences between the estimates of α_t and its' true value. The updating equations for the filter are given by:

$$\alpha_{t|t} = \alpha_{t|t-1} + P_{t|t-1} Z' F_t^{-1} \upsilon_t \tag{12}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} Z' F_t^{-1} Z P_{t|t-1} \ , \tag{13}$$

where $F_t = E[\nu_t \nu'_t] = ZP_{t|t-1}Z' + H$, the forecast of y_t at time *t-1* is given by $y_{t|t-1} = \beta + Z\alpha_{t|t-1}$ and the forecast error is given by $\nu_t = y_t - \beta - Z\alpha_{t|t-1}$.

The Kalman filter representation, equations (10)-(13), permit recursive calculations of the predicted state vector, $\alpha_{t|t-1,}$ and the covariance matrix, $P_{t|t-1}$, given T_t, R, Σ, H and Z as well as initial values for both $\alpha_{t|t}$ and $P_{t|t}$. When performing maximum likelihood estimation, the initial values for $\alpha_{t|t}$ and $P_{t|t}$ are typically taken to be the unconditional expectation of α_t and its' covariance matrix. More specifically, $\alpha_{0|0}=0$ and $P_{0|0}=\Sigma T^j_{t-j}\Sigma T^j_{t-j}$.

With the initial estimates, the Gaussian log likelihood is computed via:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\upsilon_t ' F_t^{-1}\upsilon_t - \frac{1}{2}\sum_{t=1}^{T}\ln(\det(F_t)), \tag{14}$$

where the Gaussian maximum likelihood estimates of the parameters are found by maximizing *L* over the parameter space.

2.3 Numerical Example

A numerical example of the single index model will now be discussed to help aid the discussion above. For the purposes of this example assume there are five macroeconomic coincident variables included in the estimation of the unobserved factor. Let these variables be Canadian industrial production and retail sales (INDP), housing permits (PERM), manufacturing of durable goods (MANF), unemployment rate (UNRT) and the amount of privately available credit (CRED). Hence the notation, $\Delta y_{i,t}$ (i=INDP,PERM,MANF,UNRT,CRED), denotes the stationary log differences of each of the variables. Further assume that both the unobserved factor and idiosyncratic error term follow autoregressive processes of order 1, that is p=1 and q=1 respectively. The measurement and state equations are therefore given as follows:

$$\begin{bmatrix} \Delta y_{INDP,t} \\ \Delta y_{PERM,t} \\ \Delta y_{MANF,t} \\ \Delta y_{UNRT,t} \\ \Delta y_{CRED,t} \end{bmatrix} = \begin{bmatrix} \gamma_{INDP} & 1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_{PERM} & 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma_{MANF} & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_{UNRT} & 0 & 0 & 0 & 1 & 0 & 0 \\ \gamma_{CRED} & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_t \\ u_{INDP,t} \\ u_{PERM,t} \\ u_{MANF,t} \\ u_{UNRT,t} \\ u_{CRED,t} \\ C_{t-1} \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} C_t \\ u_{INDP,t} \\ u_{PERM,t} \\ u_{MANF,t} \\ u_{UNRT,t} \\ u_{CRED,t} \\ C_{t-1} \end{bmatrix} = \begin{bmatrix} \phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_{INPD,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_{PERM,1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{MANF,1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{UNRT,1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{CRED,1} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{t-1} \\ u_{INDP,t-1} \\ u_{PERM,t-1} \\ u_{MANF,t-1} \\ u_{UNRT,t-1} \\ u_{CRED,t-1} \\ C_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \upsilon_{INDP,t} \\ \upsilon_{PERM,t} \\ \upsilon_{MANF,t} \\ \upsilon_{UNRT,t} \\ \upsilon_{CRED,t} \\ 0 \end{bmatrix} \tag{16}$$

With the above sets of equations in hand, the following analysis uses the Kalman filter to construct the likelihood function of the state space form and to estimate the new unobserved index of coincident indicators, C_t.

3. Data

3.1 Description of Variables

The purpose of this section is to explain why each of the key macroeconomic time series variables was used to construct a new composite index for Canada. The data used in this paper was pooled from two different sources namely The Organization For Economic Co-operation and Development (OECD) and Computing in the Humanities and Social Sciences (CHASS), University of Toronto website. Unless otherwise specified, all variables were previously seasonally adjusted and accumulated on a monthly basis. Due to the limited availability of several variables, the time period used for this paper was 1976:1 through 2011:2.

The six core variables used in this project included total industrial production and retail sales (INDP), total manufacturing output (MANF), number of residential and non-residential housing permits issued (PERM), unemployment rate (UNRT), the total amount of available household credit (CRED) and US industrial production (USIP). These variables are considered to be the core variables, as they seemed to be the most plausible to associate with Canadian recessions. Other variables considered, were total disposable income (DINC), an average hourly wage rate (HRWG) and the total number of people employed in Canada (EMPY), but proved less important.

The strategy used when selecting variables for the single index model was to try and use a wide spectrum of coincident variables, which capture the important elements of Canadian economic activity. The industrial production and retail sale index measures the total amount of output in the industrial and retail sector of the economy. Even though industrial production only contributes to a small portion of the annual Canadian GDP, it is of particular interest as it is highly sensitive to interest rates and therefore consumer demand.

The manufacturing variable included consists of the total value for new orders of durable goods, measured in 1992 dollars. Since many of the manufacturing companies in Canada use raw materials to produce semi-finished or finished goods, this variable is useful as it also provides an implicit measure of the demand for raw materials. Housing permits are the number of residential and non-residential permits issued in Canada in a given month. This variable is useful as it is highly sensitive to changes in many economic factors including: interest rates, levels of available disposable income, price of raw materials, etc. Since residential home construction accounts for approximately 80% of the Canadian housing market, it was assumed that this variable would help provide an aggregate measure of consumer confidence. The amount of available household credit was used as another measure of economic activity at the aggregate household level. Typically, the amount of credit depends on the state of the economy. More prosperous times usually bring higher levels of credit, as the probability of default is lower relative to times of economic hardship. The Canadian unemployment rate is defined as the number of people who are currently unemployed, but looking for a job. During times of economic prosperity, low unemployment rates typically prevail; however the converse statement is true during poor economic times. Finally, we know that changes in the US economy are usually indicative of similar forthcoming changes to the Canadian economy. By including US industrial production in our analysis, we attempt to capture the impact of US economic activity on the Canadian economy.

The table listed below, summarizes all of the coincident variables considered throughout the paper, and the transformation applied to each:

Coincident Variables	Transformation	Description
INDP	growth rate	Total industrial production and retail sales
MANF	growth rate	Total value of new orders of durable goods, measured in 1992 dollars
PERM	growth rate	Total residential and non-residential housing permits issued
UNRT	growth rate	Unemployment rate, both sexes, 15 years and older
CRED	growth rate	Amount of available household credit
USIP	growth rate	US industrial production
DINC	growth rate	Household disposable income
HRWG	growth rate	Hourly wage rate in the manufacturing sector
EMPY	growth rate	Total number of people employed, both sexes, 15 years and older

Table 1: The table above lists the coincident variables used throughout this paper. The first six listed are considered to be the core variables, which are used, in the base model.

3.2 Stationarity/Unit Root Testing

Before performing any sort of analysis on the transformed data, the first thing that needs to be checked, is the stationarity of each of the variables. Stationary data is particularly important because it ensures that constant means, variances and autocorrelations (AC) occur through time. For the purposes of this paper, the Augmented Dickey Fuller test was used to check for mean reversion (Dickey and Fuller 1979). The Akaike Information Criteria (AIC) (Akaike 1981) was used to determine the appropriate number of lags on each of the variables for the Augmented Dickey Fuller test. The table below summarizes the results for the suggested number of lags and the test statistic/p-values from each of the respective Augmented Dickey Fuller tests:

Variable	Suggested number of lags (AIC)	Dickey Fuller Test Statistic/P-Value	P-Value
INDP	4	-1.803	0.3791
PERM	3	-1.000	0.7152
UNRT	4	-2.238	0.1927
CRED	4	-0.993	0.7558
DINC	4	-1.431	0.5673
USIP	2	-2.249	0.1338
HRWG	3	-5.626	0.0000
EMPY	4	-3.123	0.0249
MANF	4	-1.440	0.5631

Table 2: Summarizes the results from running the Augmented Dickey Fuller test on each of the variables. From the results above, it can be seen all variables with the exception of HRWG prove to be non-stationary at the 1% level.

The results above show that at the 1% level, all of the variables with the exception of HRWG prove to be non-stationary and hence need to be first differenced. In contrast to the results obtained for HRWG, it is believed that the variable may not be stationary. The graph below depicts the natural logarithm of HRWG over the given time interval:

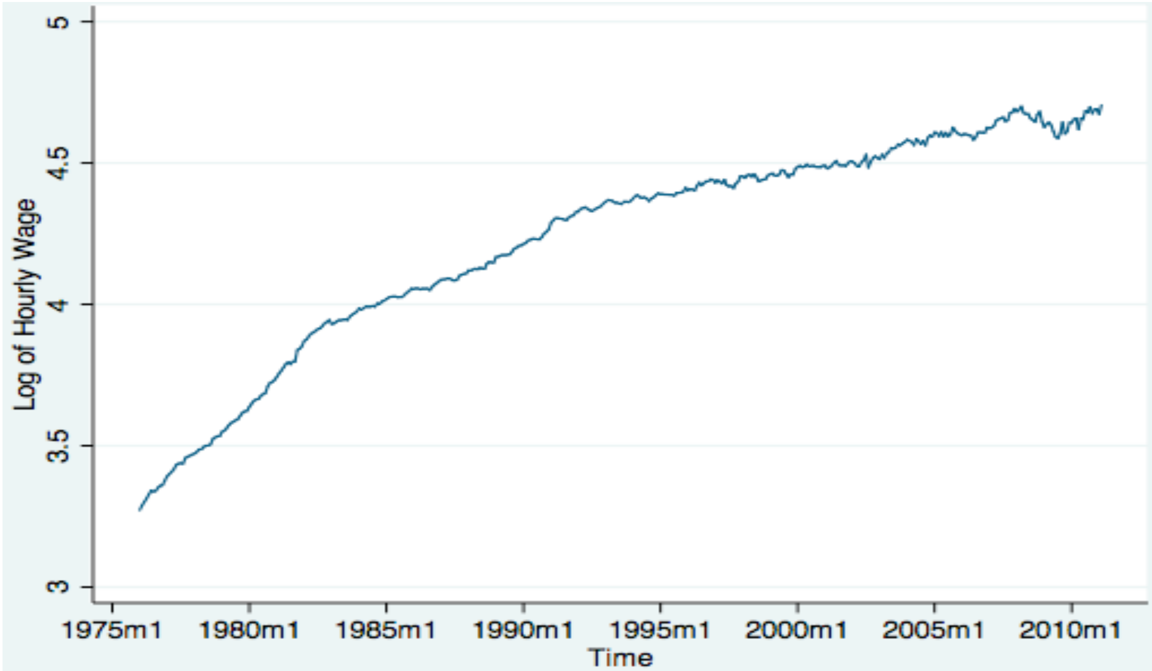

Figure 1: The figure above shows the series for the natural logarithm of hourly wage. From the above figure, one can see that the series is clearly non-stationary.

The figure above shows how the variable HRWG is increasing over time and therefore cannot have a stationary mean or variance. As a result, the variable was treated as if it were non stationary. After first differencing each of the series listed in Table 2 and performing the Augmented Dickey Fuller test again, it was found that each of the variables listed above were stationary.

4. Results

4.1 Estimation of Model

The results reported are the maximum likelihood estimations obtained from equations (1)-(3), using the base model variables. The results show that the unobserved variable was assumed to follow an AR(1) process, while the idiosyncratic errors, $u_{i,t}$ were assumed to follow an AR(2) process (p=1,q=2). The time horizon over which the above estimations were made was 1976:1 through 2011:2. Prior to estimation, each of the six coincident variables was standardized to have zero mean and unit variance. This standardization is important to ensure that the index reflects the components symmetrically without an undue influence by some high variance variable or variables. Also, for purposes of estimation, the variance of η_t was set to unity. The sign of the estimates of C_t are unidentified (i.e. we could put a negative sign in front of C_{t-1} and the C_t would change signs to accommodate).

Measurement Equations

$$\Delta \hat{\text{INDP}}_t = \underset{(0.0324)}{0.325}\Delta\hat{C}_t + \hat{u}_{INDP,t}$$

$$\Delta \hat{\text{MANF}}_t = \underset{(0.0401)}{0.353}\Delta\hat{C}_t + \hat{u}_{MANF,t}$$

$$\Delta \hat{\text{PERM}}_t = \underset{(0.0206)}{0.062}\Delta\hat{C}_t + \hat{u}_{PERM,t}$$

$$\Delta \hat{\text{UNRT}}_t = \underset{(0.0338)}{-0.261}\Delta\hat{C}_t + \hat{u}_{UNRT,t}$$

$$\Delta \hat{\text{CRED}}_t = \underset{(0.0195)}{0.039}\Delta\hat{C}_t + \hat{u}_{CRED,t}$$

$$\Delta \hat{\text{USIP}}_t = \underset{(0.0389)}{0.387}\Delta\hat{C}_t + \hat{u}_{USIP,t}$$

Transition Equations

$$\Delta\hat{C}_t = \underset{(0.0406)}{0.801}\Delta\hat{C}_{t-1} + \hat{\eta}_t \;;\; \hat{\sigma}_\eta = 1.0 \text{ (normalized)}$$

$$\hat{u}_{INDP,t} = \underset{(0.0642)}{-0.503}\,\hat{u}_{INDP,t-1} - \underset{(0.0592)}{0.286}\,\hat{u}_{INDP,t-2} + \hat{\varepsilon}_{INDP,t} \;;\; \hat{\sigma}_\varepsilon^{INDP} = \underset{(0.0522)}{0.544}$$

$$\hat{u}_{MANF,t} = \underset{(0.0542)}{0.132}\,\hat{u}_{MANF,t-1} + \underset{(0.0534)}{0.042}\,\hat{u}_{MANF,t-2} + \hat{\varepsilon}_{MANF,t} \;;\; \hat{\sigma}_\varepsilon^{MANF} = \underset{(0.0496)}{0.633}$$

$$\hat{u}_{PERM,t} = \underset{(0.0484)}{-0.374}\,\hat{u}_{PERM,t-1} - \underset{(0.0483)}{0.164}\,\hat{u}_{PERM,t-2} + \hat{\varepsilon}_{PERM,t} \;;\; \hat{\sigma}_\varepsilon^{PERM} = \underset{(0.0599)}{0.866}$$

$$\hat{u}_{UNRT,t} = \text{-}0.063\,\hat{u}_{UNRT,t-1} - 0.053\,\hat{u}_{UNRT,t-2} + \hat{\varepsilon}_{UNRT,t} \;;\; \hat{\sigma}_{\varepsilon}^{UNRT} = 0.804$$
$$\quad (0.0517) \qquad (0.0516) \qquad\qquad\qquad\qquad (0.0587)$$

$$\hat{u}_{CRED,t} = \text{-}0.447\,\hat{u}_{CRED,t-1} - 0.118\,\hat{u}_{CRED,t-2} + \hat{\varepsilon}_{CRED,t} \;;\; \hat{\sigma}_{\varepsilon}^{CRED} = 0.822$$
$$\quad (0.0485) \qquad (0.0487) \qquad\qquad\qquad\qquad (0.0568)$$

$$\hat{u}_{USIP,t} = \text{-}0.078\,\hat{u}_{USIP,t-1} - 0.0001\,\hat{u}_{USIP,t-2} + \hat{\varepsilon}_{USIP,t} \;;\; \hat{\sigma}_{\varepsilon}^{USIP} = 0.575$$
$$\quad (0.0605) \qquad (0.0586) \qquad\qquad\qquad\qquad (0.0488)$$

The negative estimates of u_{INDP}, u_{PERM}, u_{UNRT}, u_{CRED} and u_{USIP} indicate that the idiosyncratic component of these series exhibits negative serial correlation. Similarly, the positive estimates for u_{MANF} shows that the idiosyncratic term exhibits positive serial correlation.

4.2 Examining Model Fit

Statistics that examine the fit of the model are shown below in Table 3. The tests shown describe whether the disturbances in the observed variables are predictable. This is done by regressing e_Y on lagged estimations of e_Y, where e_Y denotes the one step ahead forecast errors from the single index model *(y = INDP, MANF, PERM, UNRT, CRED, USIP)*. More specifically, $e_Y = Y_t - Y_{t|t-1}$, where $Y_{t|t-1}$ is computed by applying the Kalman filter to the estimated model above. The overall model is correctly specified if, all results prove to be serially uncorrelated.

	- - - - - - - - - - - - - - - - - - -Dependent Variables - - - - - - - - - - - - - - - - - -					
Regressor	e_{INDP}	e_{MANF}	e_{PERM}	e_{UNRT}	e_{CRED}	e_{USIP}
e_{INDP}	0.6082	0.0219	0.2597	0.8152	0.0027	0.9303
e_{MANF}	0.4188	0.1814	0.0059	0.5774	0.1431	0.0850
e_{PERM}	0.2877	0.1423	0.7772	0.3768	0.7085	0.6489
e_{UNRT}	0.5645	0.0364	0.006	0.9448	0.8979	0.1020
e_{CRED}	0.5955	0.1742	0.0121	0.0055	0.4021	0.0908
e_{USIP}	0.9300	0.6918	0.1710	0.9797	0.0347	0.8419

Table 3: The table above summarizes the p-values from the regressions of e_Y against a constant and two lags of the indicated regressor. The p-values given correspond to the F-test of the hypothesis that the lags of each respective regression are jointly statistically insignificant.

From the table above, it can be seen that satisfactory specifications of the variables e_{INDP}, e_{MANF} and e_{USIP} as the null hypothesis of joint statistical insignificance of the lags cannot be rejected at the 1% level. For the other three variables, it can be seen that the null is rejected once when e_{CRED} and e_{UNRT} are used as dependent variables and twice when e_{PERM} is used as the dependent variable. However, because the null is not rejected more often than not at the 1% level, it is assumed that there is no serial correlation present. At this stage it should be noted that alternative orders for the idiosyncratic term were also subjected to the same serial correlation test. However, the AR(2) idiosyncratic error seemed to yield the best results with respect to the test above. The fact that there are no immediate signs of serial correlation provides evidence that each of the variables used in the model are likely to be coincident variables and not leading or lagging variables.

As previously stated, the structure of the error term for each of the coincident variables is given to be $u_{i,t} = u_{i,t-1} + u_{i,t-2} + \varepsilon_{i,t}$ (i=INDP, MANF, PERM, UNRT, CRED,USIP), where it is assumed that the innovations, $\varepsilon_{i,t,}$ are idiosyncratic. We now need to empirically test whether the above assumption of white noise errors. There are many tests that could be used to check for white noise errors, however, only the Portmanteau test (Box Pearce Q-Statistic) was used in this paper. The null hypothesis of the Portmanteau test is that the innovations are white noise, while the alternative hypothesis states that the errors are not white noise. Table 4 below shows the results of the test:

Innovation	Portmanteau (Q) Statistic	P-value
ε_{INDP}	1.1448	0.2846
ε_{MANF}	3.1163	0.0775
ε_{PERM}	0.0234	0.8784
ε_{UNRT}	0.0011	0.9737
ε_{CRED}	0.0750	0.7841
ε_{USIP}	0.1402	0.7081

Table: 4 Shows the results of the Portmanteau test from each of the respected estimated innovations. The results show that the estimated innovations are in fact white noise.

The results of the Portmanteau test show that the null hypothesis of white noise errors cannot be rejected in each case at the 1% significance level. As a result, the original assumption made in the modeling section regarding white noise innovations is in fact verified. That is, each of the innovations described above are random and also have a mean of 0, and variance of 1. The graph below depicts the estimated innovations for the variable INDP.

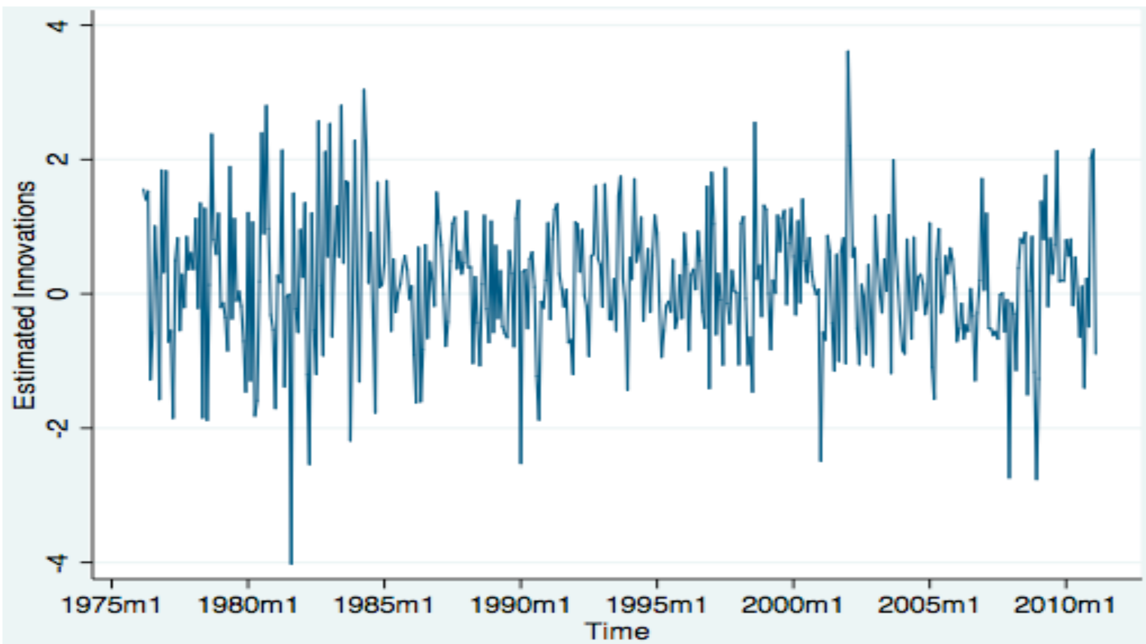

Figure 2: The figure above shows the estimated innovations of the variable INDP obtained by estimating equations (1)-(3). The figure verifies the results of the Portmanteau test that the estimated innovations are in fact white noise.

4.3 Variance Decomposition/Rolling Correlation

Given the estimates above for the measurement and transition equations, we are now able to measure the quantitative influence of variations in the common factor with changes in the individual coincident variables. Recalling equations (1)-(3) and the fact that the unobserved variable is orthogonal to the series-specific factors, the variance of each series can be decomposed into two terms:

$$\sigma_i^2 = \gamma_i^2 \sigma_C^2 + \sigma_{u,i}^2 , \quad i \in \{INDP, MANF, PERM, UNRT, CRED, USIP\}. \qquad (17)$$

In the above equation, γ_i denotes the estimated coefficients from equation (1), σ_C^2 is the variance of the unobserved factor and $\sigma_{u,i}^2$ denotes the estimated variance of the residuals given by equation (3). Keeping in mind, the variance of each series, σ_i^2, is normalized to one.

As explained in the Gregory, Head and Raynauld (1997) paper, we can compute the estimates of R_c^i which measures the variance in the individual macroeconomic coincident index, accounted for by the variation in the common factor. More specifically, R_c^i is defined as the ratio of the variance in the common factor weighted by the appropriate estimated coefficient, plus the sum of the variances of the weighted common factor and the variance of the idiosyncratic component, $\sigma_{u.i}^2$, where *(i = INDP, MANF, PERM, UNRT, CRED, USIP)*. Using the variances discussed above, we can compute the estimates of R_c^i as follows:

$$\hat{R}_C^i = \frac{\dfrac{\hat{\gamma}_i^2}{1-\hat{\phi}_C^2}}{\dfrac{\hat{\gamma}_i^2}{1-\hat{\phi}_C^2} + \dfrac{\hat{\sigma}_{i,\varepsilon}^2}{1-\hat{d}_{i,1}^2}}, \tag{18}$$

where $\hat{\sigma}_{i,\varepsilon}^2$ is the estimated variance of the innovations for the idiosyncratic component of each coincident index used. Table 5 below, shows the results of the variance decomposition, for the given model:

Coincident Variable	Share of Variance (R_c^i)
PROD	0.2167
MANF	0.3138
PERM	0.0102
UNRT	0.1664
CRED	0.0039
USIP	0.3369

Table 5: Shows the share of variance of each coincident variable, (i=INDP, MANF, PERM, UNRT, CRED, USIP) accounted for by variation in the common factor, C_t.

The estimated variance shares given in Table 5 above, give quantitative meaning to the estimates of the impact coefficients given at the beginning of this section. The results suggest that fluctuations in the common unobserved factor accounts for approximately 16.64%, 21.67%, 31.38% and 33.69% of the variation in the variables UNRT, PROD, MANF and USIP, respectively. However, for the variables PERM and CRED, fluctuations in the unobserved factor only account for 1.02% and 0.39% of their variation, respectively. This result is not surprising, as the serial correlation test earlier, suggested that both of these variables might be incorrectly specified with an AR(2) idiosyncratic error term. Alternatively, it is also possible that both PERM and CRED are not perfectly coincident variables, but rather leading or lagging variables. As a result, the unobserved coincident factor does a particularly poor job of capturing any variation of these variables.

In addition to computing the amount of variation the unobserved common factor contributes to each of the coincident variables, the correlation between the common factor and individual coincident variables was also estimated. For the readers' interest, the correlation between each of the coincident variables was also included.

	INDP	MANF	PERM	UNRT	CRED	USIP	FACTOR (C)
INDP	1.000	-----	-----	-----	-----	-----	-----
MANF	0.339	1.000	-----	-----	-----	-----	-----
PERM	0.097	0.026	1.000	-----	-----	-----	-----
UNRT	-0.243	-0.267	-0.082	1.000	-----	-----	-----
CRED	0.049	0.080	0.072	0.001	1.000	-----	-----
USIP	0.374	0.401	-0.017	-0.295	-.048	1.000	-----
FACTOR (C)	0.245	0.536	0.017	-0.262	-0.155	0.449	1.000

Table 6: Correlation matrix consisting of the six coincident variables used, as well as estimated unobserved factor.

The table above shows the correlation between the estimated unobserved factor and the variables INDP, MANF and USIP to be 0.245, 0.536 and 0.449, respectively, whereas UNRT has a correlation of -0.262 with the unobserved coincident index. For the variables PERM and CRED, the degree of correlation with the estimated unobserved coincident index is quite small in both cases. This result is not surprising as the variance decomposition suggested little variation in these variables was coming from the unobserved coincident index. As a result, the correlation between these two variables and the unobserved coincident index is also small. Figure 3 below, shows the average two-year rolling correlation between the common factor and the coincident variables used. As shown in the figure, on average, the correlation between the common factor and the coincident variables slightly increased over the sample period used. The shaded regions in the figure below show the duration of the four recessions that occurred over the period of interest (discussed in more detail in the next section). The figure shows that the average correlation between the six coincident variables and the common factor reaches a local maximum during each of the four recessions. However, we cannot draw any conclusions from this, as there are other times when the average correlation reaches a local maximum even though the economy is not in recession. Further investigation into this property is beyond the scope of this paper. However, for the time horizon considered, we can conclude that when a recession occurs, the degree of correlation between the coincident variables and the common factor increases.

Figure 3: Shows the plot of the average two-year rolling correlation between the common factor and the coincident variables used. The figure appears to have a small upward trend. The shaded regions on the figure show the four recessions, which occurred over the time horizon.

4.4 Canadian Quarterly GDP and Recessions

Figure 4 shows quarterly Canadian GDP, over the period 1970:1 through 2011:2. The shaded regions shown on the graph represent the four official Canadian recessions, which were dated using the 2 successive quarters of negative growth in GDP. The four 'official recession' dates (as given on Statistics Canada website) that occurred over the studied time horizon are 1980:2-1980:6, 1981:7-1982:10, 1990:4-1992:4 and 2008:8-2009:7. Notice, the recession dates are defined in monthly terms instead of quarterly terms. This will make for an easier comparison later in the paper, as monthly data was used in our analysis. From the figure, we can see that the recessions of 1981 and 1990 are the longest in duration. However, the recession of 2008 appears to be the most severe as GDP declines sharply by over 2%, whereas the recessions of 1980, 1981 and 1990 experience a decline in GDP of approximately 0.5%, 1% and 1.5%, respectively.

Figure 4: Shows the plot of quarterly Canadian GDP, and the corresponding recessions over the time horizon being studied. Here, the recessions were dated using the 2 successive quarters of decline in Canadian GDP.

4.5 Common Factor and Recession Dating

Figure 5 below shows the estimated unobserved component from 1976:1 to 2011:2. Again, the shaded vertical lines on the graph represent the various recessions that occurred in Canada over the time horizon considered, under the definition of two successive quarters of negative growth in GDP.

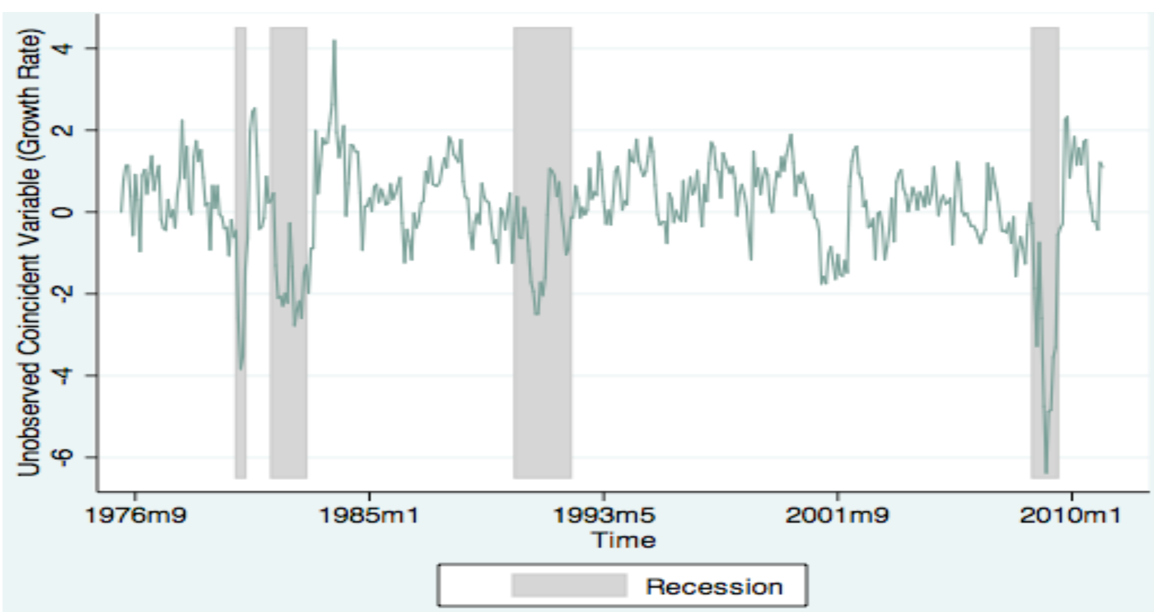

Figure 5: Depicts the growth rate of the unobserved coincident variable from 1976:1-2011:2. The figure also shows the four recessions that occurred in Canada in the timeline considered.

At first glance, it is apparent that the unobserved series takes a noticeable drop during each of the four recessions. However, what is even more interesting is the fact that the unobserved coincident series only ever exhibits a 2% (or greater) decline when Canada is in a recession. From the figure we can see that the recession of 1980:2 and the recession of 2008:8 are fairly similar in behavior. That is, even though they are shorter than the other two recessions, they both exhibit sharp declines in a matter of months. Following each of the above respected recessions, there is a sharp rebound to the common factor where growths of 2.5% and 2.25% are estimated. With regards to the recessions beginning in 1981:7 and 1990:4 it can be seen that they too exhibit similar patterns. In both of these cases, slower and less severe declines in the estimated unobserved coincident variable are exhibited. Both recessions show signs of recovery as the common factor exhibits growth rates above the -2% threshold. The recession of 1990:4 is an especially interesting case as the well-documented 'double dip' recession occurred. During the recession, the economy showed signs of recovery, before a second downturn in GDP occurred. The results of the unobserved coincident index capture the same effect, albeit to a lesser extent. Following the drop below the 2% threshold, the latent factor exhibits positive growth, before again exhibiting negative growth for the remainder of the recession. Economists believe the main reason for the second dip was a result of a very slow recovery to unemployment rates. It is possible that the high unemployment effects in our model were dominated by other series, which performed better in that time and were also assigned higher weights, relative to the unemployment variable. As a result, the 'second dip' did not force the unobserved factor back below the 2% threshold. Finally, the figure shows that the estimated coincident index almost predicted a recession in the early 2000's. This was around the time of the 'DOT Com' bubble that occurred in the US, sending their economy into a recession. Since most of the Canadian macroeconomic coincident variables are so closely tied to changes in the US economy, it is not surprising that the model, at least partially, captured this effect. However, since the Dot Com recession was mainly equity driven, the spillover effects were not nearly as severe as they have been for past recessions.

Comparing the figure containing Canadian GDP against the estimated common coincident factor shows some striking resemblances. As previously discussed, during the recessions of 1980 and 2008, GDP takes a sharp decline, but rebounds shortly after. As seen in the above figure containing our estimated unobserved coincident index, it too declines during these recessions. For the recessions of 1981 and 1990, the fall in GDP is slightly more gradual compared to the other recessions, and does not immediately rebound. Again, we see similar behavior in our estimated unobserved factor.

4.6 An Alternative Model

The other variables described in the data section were also used when trying to determine which set of coincident variables yield the best estimates. It was found that, models, which used UNRT instead of EMPY, always yielded more reasonable results. Furthermore, models where HRWG was included instead of either UNRT or EMPY or in combination with of one these variables also yielded poorer estimates. Figure 6 below, shows the estimated unobserved factor for a model that includes the variables INDP, EMPY, DINC, MANF and USIP. Both the unobserved factor and error terms were said to follow AR(1) processes.

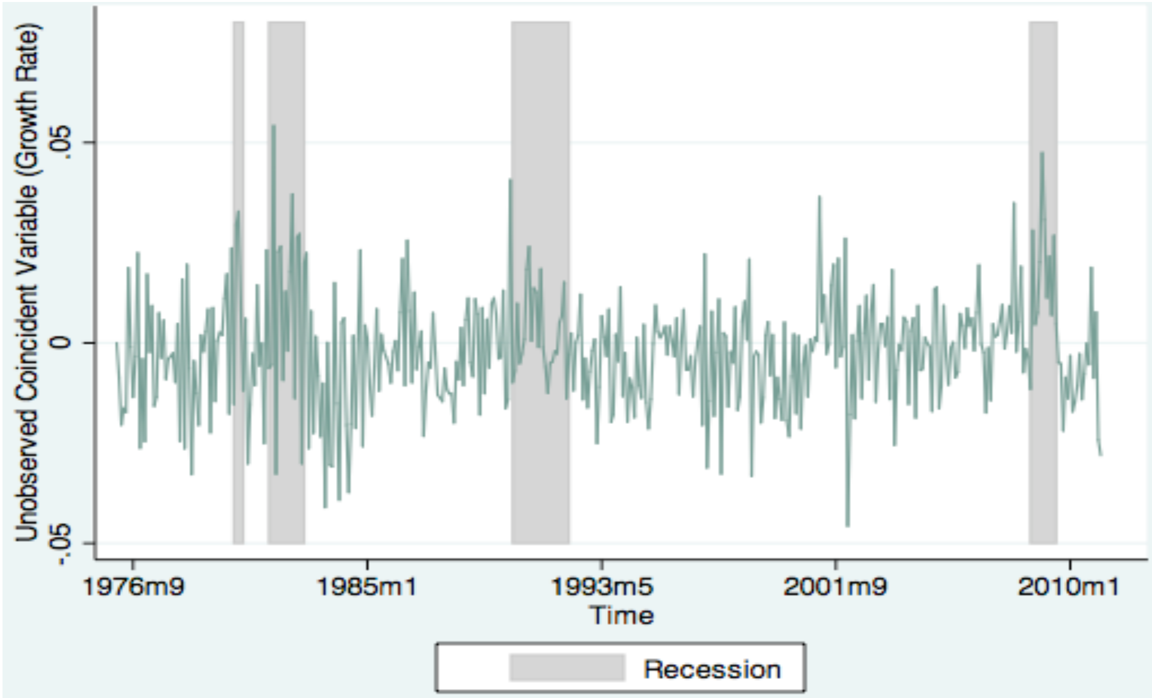

Figure 6: Depicts the results of the unobserved factor estimated from the single index model. The variables included were INDP, EMPY, DINC, MANF and USIP and both the unobserved factor and error terms were said to have followed an AR(1) process.

Figure 6 above, shows that the variables used for the given model, did a particularly poor job of dating the recessions. In all four of the recessions, the model estimated times of relatively high positive growth with respect to the unobserved factor. Subsequent serial correlation tests show that at least three the variables have serial correlation present. This result is not surprising as the variable DINC is most likely a lagged variable as apposed to a coincident variable. The main reason for including these results was to show the reader how sensitive the model can be to the choice of coincident variables.

5. Discussion/Conclusion

The findings in this paper suggest that the single index model provides an alternative method to be used when dating Canadian recessions. Up to now, Statistics Canada and Finance Canada classify a recession as two consecutive quarters of negative growth in GDP. However, this definition may not be the optimal measure of a recession for two reasons. First and foremost, GDP only measures one facet of the economy. Declines in output alone are not always indicative of a recession. That is, a decline in one coincident macroeconomic variable does not necessarily mean a decline in all macroeconomic variables. Second, using the current recession identifying method, a recession in Canada can not be 'officially' declared until at least 2 quarters (6 months) have passed. The framework purposed in this paper address the first issue by creating an unobserved coincident index that depends on several coincident macroeconomic factors. This estimated unobserved index, represents the overall 'state' of the economy. Since our definition of a recession only depends on a certain threshold being crossed (i.e. decline of 2% in the estimated common factor), recessions can theoretically be identified sooner compared to the current Statistics Canada method.

The previous section demonstrates that the proposed model accurately dates all four of the recessions that occurred in Canada over the time period 1976:1 – 2011:2. Going by Statistic Canada's official recession dates, we were able to see exactly how many months (into the 'official' recession) it took our model to date the recession. The results show that for the recessions of 1981:7 and 1990:4 it took approximately 6 months, while for the recessions of 1980:2 and 2008:8 it took four and two months respectively. Hence, for the recessions of 1981:4 and 1990:4, our model does not date the recessions any faster than the current 2 quarters of consecutive decline in GDP. Intuitively, these results are not totally surprising as no two recessions are ever alike and therefore its' affects on the set of coincident variables is never the same. Because our model places a higher emphasis on some coincident variables relative to others, the recessions that affect the more emphasized coincident variables, will be identified sooner. This result emphasizes the importance of building models, which incorporate a broad spectrum of macroeconomic coincident variables. By doing so, the researcher can be confident that all aspects of an economy are being captured within the model.

Future research may want to consider using a similar model to that devised in this paper and try to examine the severity of the different recessions that have happened in Canada. Our analysis only goes as far showing that the model can be used to accurately date recessions in Canada, but said little about the duration of the recessions. The recessions beginning in 1981:7 and 1990:4 show particularly alarming cases where the economy shows signs of recovery but then suffer a second decline. With the exception of the recession beginning in 1990:4, all recessions show signs of rapid expansion immediately occurring post recession. It would seem logical that examining this period of high growth could be the key to identifying whether or not a ‘second dip’ will occur.

Throughout the course of this paper, our analysis has remained focused on purely creating a new index, which could be used to date recessions. However, it is entirely possible that this same analysis can be used to identify different ‘boom’ periods in the economy as well. This might be a particularly useful tool for economists when trying to analysis how fast an economy is growing.

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