A Stochastic Population Projection for Canada's First Nations

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August 19, 2011

An essay submitted to the Department of Economics

in partial fulfillment of the requirements for

the degree of Master of Arts

Queen's University

Kingston, Ontario, Canada

Abstract

A stochastic population projection model for First Nations in Canada is produced and demonstrated for the years 2002 through 2032. The projection model uses historical time series and expert knowledge from traditional cohort-component based projections to specify parametric distributions for the stochastic processes governing population growth or decline. Probabilistic intervals for the future course of Canada's First Nations' population trajectory are calculated using computer simulations. A method for small-area population projections is demonstrated and a National forecast is produced as an example of the model.

Acknowledgements: I would like to thank Eddie Hunsinger at the Department of Demography, University of California Berkeley, and Rebecca Graziani at Universita Bocconi for their correspondence and insight into the structure of a stochastic population forecast program. Eddie Hunsinger maintains "An Applied Demography Toolbox" at http://www.demog.berkeley.edu/~eddieh/toolbox.html.

Contents

1 Introduction

Using classical methods, population projection is fundamentally an arithmetic accounting exercise. Sets of non-parametric, deterministic assumptions are made about current population counts, survival and fertility over time, adjustments are made for net migration, then a vector of population counts by age is advanced through time using simple matrix multiplication. Modern methods like stochastic population projections retain the classical framework while taking advantage of advancements in time series methods and the decrease in the cost of computing. All population projection methods are fundamentally about producing plausible scenarios for the trajectory of the counts and age-sex distribution of a population.

Population projections are an important tool in the formation of public policy, business planning and the social sciences. Their uses include forecasting environmental pressures, housing, educational and infrastructure needs, even ethnic mobility.[1] Debates over the growth or decline of regional, national and indeed global populations and the nature of these changes rest on population projections of the type examined in this paper.

For First Nations[2] in Canada engaging in the self-government process, population projection is a vital yet lacking tool. Many are forming new governments and negotiating funding agreements that stretch into the future without reliable estimates of their future needs. Governance involves long-run planning processes and nations with statistical offices readily employ population projections to inform their policies. First Nation governments, many without statistical capacity, are unable to produce reliable estimates of their future populations, which poses a challenge for the effectiveness of their long term planning. An objective of this

[1] Ethnic mobility refers to mobility within defined ethnic groups. For the aboriginal population in Canada, it generally refers to movements of the same people among Metis, Inuit, First Nations with status and First Nations without status.

[2] In this paper First Nations are defined as registered (status) Indians.

paper is to provide a flexible population projection model for First Nations in Canada to estimate their future populations, as any nation would, in order to inform their policy and planning.

As an example of the utility of population projections for First Nations, consider future educational and housing requirements, including infrastructure investment and program development. These investments require an informed estimate of a First Nation's future population and age structure. An accurate population projection will provide just that. Without a population projection, estimates of future need are uninformed and the misallocation of resources today will result in unmet need in some areas and surplus provision in others. First Nations communities face some of the toughest living conditions in Canada and the efficient allocation of scarce resources is a necessary condition for improvement.

The projections carried out in the final section of this paper show that, as a fraction of the total First Nations population, First Nations youth are in decline. This is contradictory to the popular belief that First Nations, as a population, are getting younger. This belief may be based on the fact that the number of First Nations youth has historically constituted an increasing fraction of the total population. If policies were to be put in place for an ever-increasing young First Nations population it would result in the misallocation of resources. Additionally, the prediction intervals implied by the realizations of the simulations for population totals and component indices are wider than the "high, low" range assumed by the forecasts using classical methods produced by Statistics Canada. Long-range policy decisions should consider the full range of possible population trajectories suggested by stochastic forecasts because even these forecasts may understate the range of possible outcomes if they do not model all the relevant sources of uncertainty.

Using the classical tools of population projection, Statistics Canada has produced an estimate of the total aboriginal population and its components (North American Indians[3], Metis and Inuit) in Canada for 2001 to 2017 (Statistics Canada, 2005). These projections use a regional cohort component approach. That is, they classify populations according to three dimensions: age, sex and geography and assume deterministic trajectories for representative indices governing the population counts for each dimension. This method is unable to produce probabilistic statements about projections, nor does it force the producer to acknowledge the stochastic nature of the underlying demographic indices.

The Demography division at Statistics Canada has since developed a microsimulation platform "Modgen" and it is not pursuing the development of capacity in stochastic methods for population projections. Microsimulation is resource intensive and its utility over stochastic methods has not been fully established, particularly for small area forecasts (Malenfant, 2011). The limitation of microsimulation for First Nations in Canada in the present context is that Statistics Canada has estimated all the life-event hazard models used in microsimulation for all Canadians. Even if they were to re-estimate the hazard models for First Nations at a national level, it is unknown if this would be an improvement over a stochastic forecast within which expert users can embody local knowledge.

This paper also seeks to address the gap between the projections from classical models and the newly developed microsimulation models by contributing a stochastic population projection model for Canada's First Nations. The model seeks to be adaptable to a regional scale where the benefits of microsimulation are uncertain and stochastic projections may be a reliable alternative.

The rest of this paper is organized in the following way: section 2 presents

[3]Referred to as First Nations in this paper, though our projections are inclusive of only First Nations with Registered Indian status.

background on population projection methods and the research on First Nations populations in Canada. Section 3 discusses the sources of data used in the projection and section 4 presents the method used to forecast each of the components underlying the projection. Section 5 presents a method for small area forecasts and in section 6 we demonstrate the model using computer simulations for a national forecast. Finally, section 7 offers some concluding remarks.

2 Background

2.1 Classical Method: Cohort-Component Approach

Until recently, demographic forecasting was conducted almost exclusively using the cohort-component method (CCM) (Keyfitz, 1977). The CCM is conducted by estimating demographic components from vital statistics and subsequently making assumptions about their expected trend. Specifically, a population is divided by age and sex, though additional dimensions may be included such as geography.[4] Then components relating to males' and females' age specific mortality, migration, and females' reproductive likelihood are estimated. These components are projected through time deterministically by extrapolating trends or assuming no change from their currently observed levels. Future populations are derived from advancing the current population through time based on assumptions about future components. Each component is used in the creation of Leslie Matrices (Leslie, 1945). The literature has focused on correcting biases is estimation methods and devising better ways to estimate the respective components given data limitations.

As an example of the use of Leslie Matrices and the CCM, consider an initial population by age vector at time t, N_t. N_t is advanced through time using

[4] For example: Statistics Canada, 2005.

matrix products in the following way: suppose A is a Leslie Matrix containing survivorship probabilities by age category on its left off diagonal and fertility in its first row. Then the population by age vector at time $t+1$, N_{t+1}, is given by the product:

$$N_{t+1} = A * N_t \tag{1}$$

A numerical example demonstrates the simplicity of this accounting approach, suppose: $N_t = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 0.8 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ where each row of N and column of A corresponds to an age category. The off-diagonal elements of A in the second and third rows are survival probabilities and the first row contains fertility rates. Then $N_{t+1} = A * N_t = \begin{bmatrix} 8 \\ 5 \\ 0 \end{bmatrix}$. The total population is the sum over the entries in N so with an initial population of 20, the Leslie Matrix has left us with a total of 13 at time $t+1$.

The mechanics are worth discussing in detail. Suppose the rows of N are categories corresponding to young, middle-aged and old people. The matrix product $A * N_t$ produces a transition for the number of people in each age category in N_t to the subsequent age category.[5] All people in each category are exposed to two possible events in a transition: reproduction and death.[6] Given the respective probabilities of reproducing and dying during the transition, corresponding to the elements of A, the vector N_{t+1} contains the expected number of people in each age category after the transition. For our example, because $A_{3,2} = 0$, no one survives the final transition (all ten from the middle-age category die). Because $A_{2,1} = 0.5$, half the young people survive to the middle-age

[5] For example, all young people transition to middle-age.

[6] These events are not mutually exclusive, births occur before deaths.

category. Finally, because $A_{1,2} = 0.8$, eight are born to the middle category as they transition into old age and die. With adjustments made for migration and sex, this is how population projections were done for much of the 20th century.

2.1.1 Mortality

Mortality has been a central focus. A recent review of the literature can be found in Kielman (2005). The most common approach relies on a mortality index: a level parameter that can summarize a survival-by-age schedule like the off-diagonal entries in the second and third rows of the matrix A from the previous example. A mortality index is generally a derivative of the life table, for example life expectancy at birth. A life table is usually sex specific and it describes mortality by different schedules. Examples of these schedules include conditional life expectancy: the expected number of years remaining conditional on having survived to a certain age and cohort survivorship: the expected number of survivors remaining in each age category if a cohort of 100 000[7] were to advance through their lifetime together. A full discussion of life tables and their construction can be found in Keyfitz (1977). The mortality index is projected forward in time using expert opinions about future realizations or historical patterns. Life tables are then estimated at each projection step, given the level of the index at that step, using a standard life table estimated empirically or using standard life tables estimated by Coale and Demeny (1983) for different regions around the world. Better fits are achieved by increasing the number of parameters used in the model,[8] though due to their complexity, multi-parametric approaches lose their appeal when it comes time to make projections.

The most popular method for forecasting mortality is the Lee-Carter method (1992) whereby a single level parameter (the index) describing life table mor-

[7] The radix, though sometimes a radix of 1 is used.
[8] Up to nine! eg: Rogers and Planck, 1983.

tality is derived using a least squares singular value decomposition (SVD) of a matrix whose columns correspond to centered estimates of the mortality rate by age and whose rows correspond to the year of the mortality estimates. Specifically, if $m_{a,t}$ is the log mortality rate at time t for the a^{th} age group and $\epsilon_{a,t}$ is an idiosyncratic disturbance term, Lee and Carter propose we model mortality in the following way:

$$m_{a,t} = \alpha_a + \beta_a \gamma_t + \epsilon_{a,t} \tag{2}$$

where α_a, β_a and γ_t are parameters to be estimated. The parameterization in (2) is not unique which poses a problem for estimation by maximum likelihood as there will be multiple maxima. As a solution, Lee and Carter impose two restrictions: $\sum_t \gamma_t = 0$ and $\sum_a \beta_a = 1$. From the first restriction it follows that $\alpha_a = \bar{m}_a$, the time average of log mortality. The model is rewritten in terms of the centered mortality rates: $\tilde{m}_{a,t} = m_{a,t} - \bar{m}_{a,t}$. Estimates of β_a and γ_t are derived from the SVD of the $a \times t$ matrix M whose elements correspond to $\tilde{m}_{a,t}$. Specifically, M is factorized such that $M = BLU^*$ where B is a unitary matrix, L is a diagonal matrix with the singular values of M along its diagonal and U^*, the conjugate transpose of U, is a unitary matrix. The estimate $\hat{\beta}_a$ is found in the a^{th} element of the first column of B and $\hat{\gamma}_t = \hat{\beta}' \bar{m}_t$. Alternative methods of estimation include an eigen-decomposition of the matrix M. A full discussion of estimation techniques can be found in Girosi and King (2007).

The level parameter $\hat{\gamma}_t$ is then modeled using a time series approach and at each projection interval a new life table is derived from the realization of the level parameter. Generally the best fitting time series model is a random walk with a drift (Girosi and King, 2006). Variants and adjustments to the Lee-Carter method are sufficient to comprise a literature on their own. For example, Wilmoth (1993) provides a weighted SVD approach using maximum likelihood and Lee (2000) discusses further extensions to the model. The Lee-Carter approach is relatively robust to structural changes (Carter, 1996, 2000) making it ideal for long run mortality forecasts, however Lee and Miller (2001) propose using only post 1950 data to limit the influence of structural shifts.

An alternative to the Lee-Carter method, used for shorter projection horizons, is the relational model of mortality (Brass, 1971). Brass' insight was to realize that survival functions are linearly related, specifically:

$$l_x \approx ((1 + exp(\alpha)) * ((1 - l_x^s)/l_x^s)^\beta)^{-1} \tag{3}$$

where l_x and l_x^s are two survivor schedules with x denoting the age category and s denoting the standard schedule. With a little manipulation (3) can be written as:

$$logit(l_x) = \alpha + \beta(logit(l_x^s)) \tag{4}$$

and α and β can be estimated using OLS. That is, with a logit transformation, two survivor schedules can be linearly related with α governing the level of mortality and β governing the relationship between childhood and adult mortality. These parameters can be forecast using time series methods or expert opinions and life tables at projection intervals can be derived from them, though usually only α is forecast. This is the approach taken by both Hunsinger (2011) and Hartmann and Strandell (2006).

In this study, I adopt a bi-parametric approach, forecasting α and allowing β to change because I believe the relationship between childhood and adult mortality will not remain constant. Instead I predict that a slight shift will occur whereby the First Nations survival schedule starts to converge in tempo to the national average. I allow β to change through the forecast in a deterministic

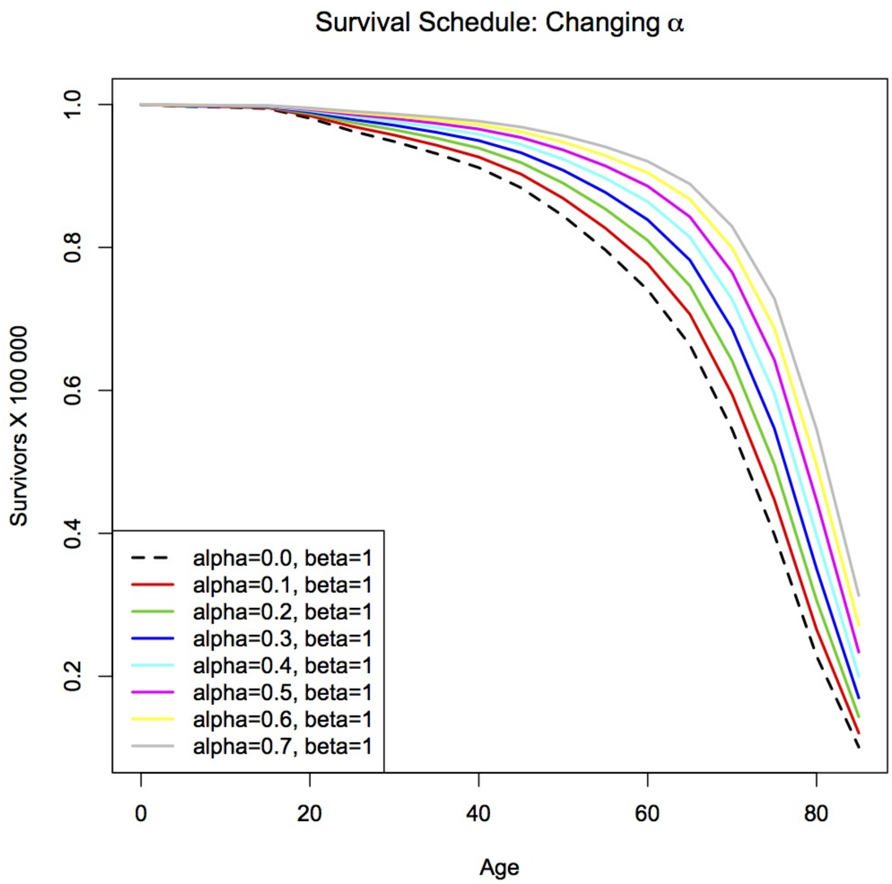


Figure 1: Survival schedule allowing α to change from 0 to 0.7

fashion. This is because historical data and expert opinions about the distribution of β, beyond a general trend, is unavailable thus an informed parametric specification is impossible. Evidence for this convergence comes from survival schedule estimates in Williamson and Roberts (2004), Miller (1982), and Health Canada (2002) for the years 1956, 1978 and 2002.

As a demonstration of the effects of varying the mortality indices, Figures 1 and 2 shows how a survival schedule changes in location and shape for given changes in α and β.

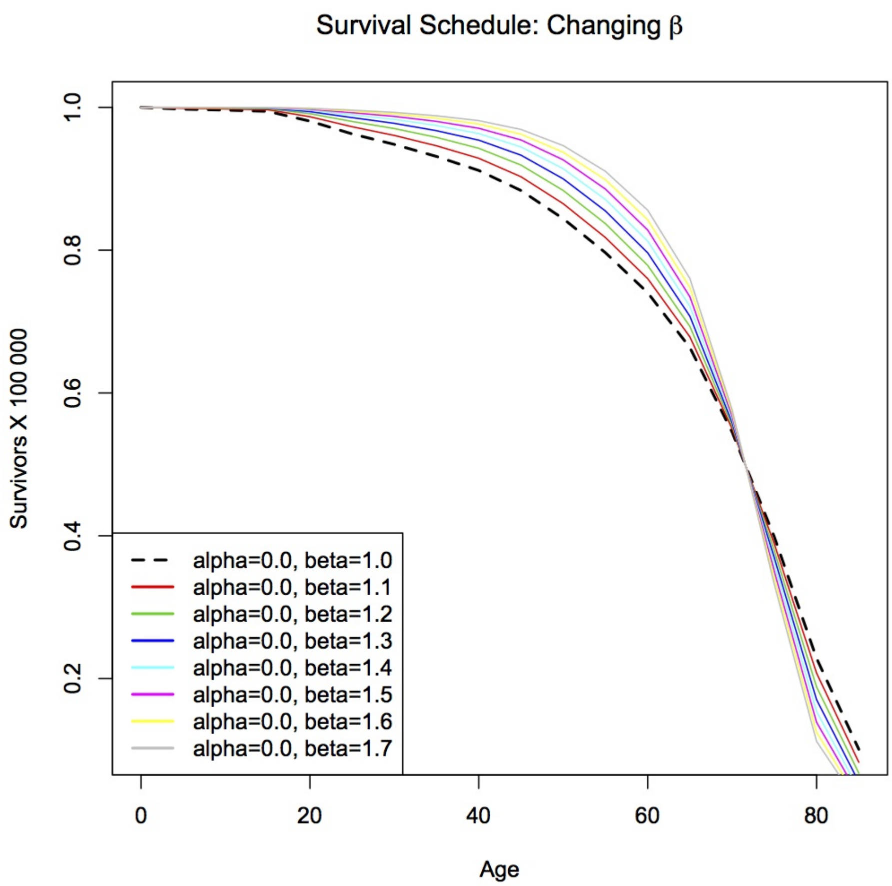


Figure 2: Survival schedule allowing β to change from 1.0 to 1.7

2.1.2 Fertility

Fertility, particularly for First Nations, has proven difficult to forecast (Loh and George, 2003). Like mortality, representative indices of fertility are generally modeled as non-stationary series. Forecasting fertility instead of births is preferred and often the total fertility rate (TFR) is estimated and forecast (Hunsinger, 2010). The total fertility rate is the sum over age specific fertility and it is equal to the expected value of the number of children born to a single woman over her lifetime.

Sometimes a bi-parametric approach is taken to modeling reproduction, for example, by forecasting the total fertility rate and mean maternal age at birth. Age specific fertility and births can then be estimated at each forecast interval (Billari, *et al.*, 2010). Otherwise we must assume that the fertility schedule (the tempo of fertility) remains unchanged and that only the TFR (quantum of fertility) is dynamic. The distribution of births over a First Nations woman's lifetime is expected to shift (Bali, 2004) and in an effort to keep my model as flexible as possible, I model both the tempo and quantum of fertility by employing Brass' Relational Gompertz Fertility Model.

According to Goldstein (2010), the Gompertz model's use in studying fertility is motivated by its initial application to modeling mortality rather than behavioral reasons. This does not mean the model is unable to support behavioral underpinnings and the objective of Goldstein (2010) is to demonstrate the applicability of the Gompertz model when fertility is modeled as a function of social diffusion. His specification allows for the tempo of fertility to be a function of a parameter $A(x)$[9] that represents social diffusion. Specifically, if $H(x)$ is the cumulative fertility up to age x as modeled with the Gompertz function, and $h(x) = H'(x)$, Goldstein models the tempo of fertility in the following way:

[9] $\frac{dA}{dx} < 0$, $\frac{d^2A}{dx^2} < 0$

$$h(x) = A(x)H(x) \tag{5}$$

Social diffusion in this context is roughly the peer effects (inclusive of cultural pressures) responsible for behavioral choices made with respect to fertility at age x. In the context of the tempo of First Nations and Canadian fertility, Goldstein's specification would show the divergence in fertility schedules as a behavioral response to different peer and cultural pressures, as modeled with the $A(x)$ parameter.

Though it could be argued that they are present in a reduced form,[10] Brass' model for fertility is ignorant of behavioral underpinnings. It proposes the following: let $H(x)$ be the cumulative fertility rate up to age x, and let TFR denote the total fertility rate: the sum of fertility over all ages. Brass' model assumes that their ratio follows the Gompertz distribution:

$$H(x)/TFR = exp(A * exp(B * x)) \tag{6}$$

If $Y(\centerdot)$ is the complementary log-log transformation then we can write (6) as:

$$Y(H(x)/TFR) = -ln(-A) - B * x \tag{7}$$

implying a linear relationship in x between the left and right hand sides of (7). Since this relationship holds for any $H(x)$ and TFR, it hold for a standard schedule (usually estimated from a rich data set) $H(x)^s$, TFR^s. With some manipulation we can relate a standard fertility schedule expressed in the form of (7) to any other fertility schedule using the Brass Relational Gompertz Fertility Model:

[10] Changes in fertility and mortality's tempo and quantum are at least in part a function of behavioral effect and though they are not explicitly modeled in Brass' specification, the changes that do exist could be interpreted as behavioral effects.

$$Y(H(x)/TFR) = \alpha + \beta * Y^s(H^s(x)/TFR^s) \quad (8)$$

where α and β can be estimated by OLS. This allows the user to relate two separate fertility schedules using only two parameters for adjustment. The parameter α governs the age location (with smaller values implying a shift to the left and larger values implying a shift right) while β governs the shape of the distribution (with smaller values implying a relatively platy-kurtic distribution and larger values implying a relatively lepto-kurtic one).[11] Although the teen fertility rate has not experienced much of a change in the most recent period (Bali, 2004 & Guimond *et al.*, 2009) for which we have data (1997-2004), I include Brass' model for fertility as an extension to keep the model as flexible as possible.

Fertility schedules have been subjected to multi-parameter estimates that consistently find better fits than the univariate or zero-factor models that are often employed in forecasts. The trade-off, again, is that multivariate approaches tend to have less regular time series, making them harder to forecast (Keyfitz, 1991).

Figure 3 shows the fertility schedule for First Nations and all Canadians. It also shows how adjustments to α and β in the Relational Gompertz Model affect the First Nations fertility schedule.

2.1.3 Migration

Relative to mortality and fertility, migration is highly variable and there is no obvious method that applies to a model for First Nations population projec-

[11] Kurtosis is a measure of how "peaked" a distribution is. For the distribution of lifetime fertility, a lepto-kurtic one would imply that women have all of their children in a narrow age window. A platy-kurtic distribution of lifetime fertility would imply that women have their children over a wide age window. The parameters α and β are therefore able to affect the skewness of the distribution: for small α, β the distribution would be skewed to the left, for large α, β the distribution would be skewed to the right.

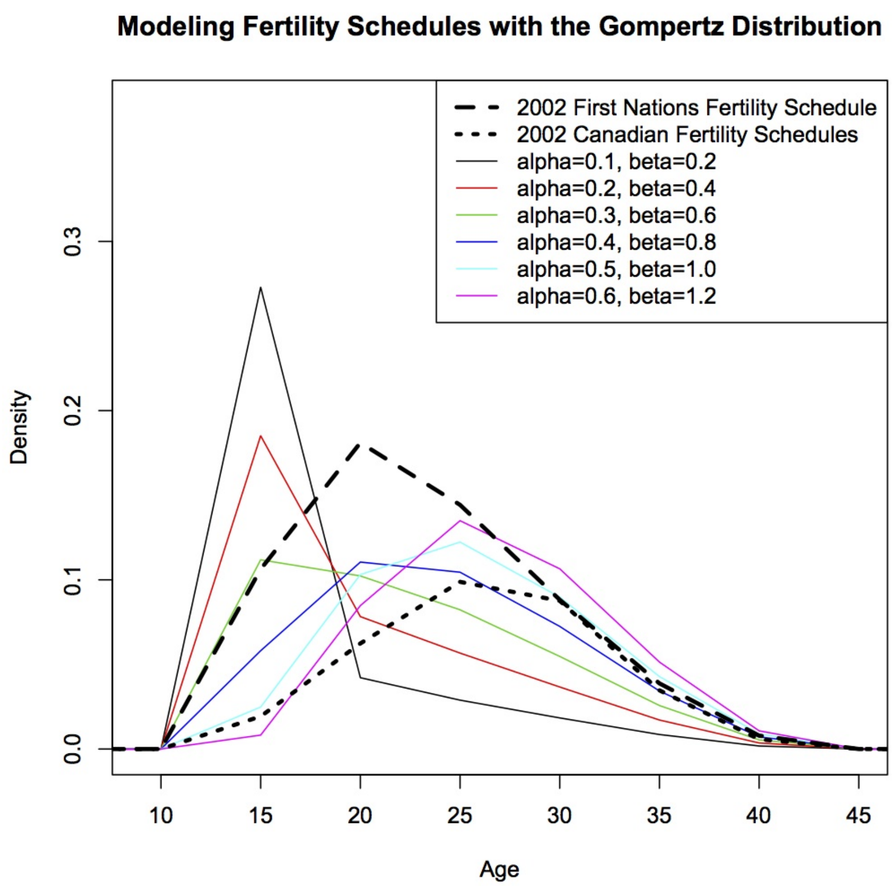


Figure 3: First Nations and Canadian fertility schedules and changes in parameters for the Gompertz Model of Fertility.

tions. When making national level forecasts I assume that international migration is negligible and that the effects of intra and inter-regional migration are manifested in the changes in mortality and fertility already accounted for in the assumptions about the movements of their respective representative indices. For example, it is expected that a family moving from a high fertility remote region to a low fertility urban environment will experience two relevant effects for the projection: their lifetime fertility will more closely approximate their new living environment and their improved access to healthcare (urban vs. rural) will manifest itself in longer life expectancies. Both of these effects are accounted for in the trends of total fertility and mortality indices implying lower total fertility and longer life expectancies at birth (CMHC, 1996). Indeed it is exactly these sorts of adjustments, present in a reduced form, that drive the component forecasts.

2.2 Uncertainty in Forecasting

A projection produces the true population counts at a future time if the sets of assumptions underlying the projection are realized. As Keyfitz (1972) points out "[population projections] can be incorrect only in the trivial sense that the author made an arithmetic error that prevents his final number from being consistent with his initial assumptions."

Users of projections often interpret them as predictions: statements about the *defacto* future state of populations independent of whether the realized fertility, survival and migration coincide with their assumed counterparts in the projection. Both demographers and users of projections seek the same outcome: an accurate description of a future population. However, it is usually only the demographer that intimately knows the assumptions underlying the projection model. Indeed projections rarely hit their mark and sometimes forecasted vital rates will lie outside their high-low ranges even *before* the projection is published. For example, births in 1990 were higher than the highest projection scenario in the 1989 forecasts from the U.S. Bureau of the Census and they also missed their mark from 1971 to 1990 by 15-20% (Lee and Tuljapurkar, 1994). This is not entirely surprising, as fertility is difficult to forecast and errors are compounded in long-range forecasts after 15 years, the point in the forecast when the uncertain fertility rates are themselves being applied to an uncertain cohort of reproductive age females. Furthermore, the assumptions embodied in projections are complex and users are inclined to disregard the assumptions and interpret final counts as unconditional statements about the future.

For much of the history of population projections, demographers were unable to make clear probabilistic statements about their projections. As an alternative many national statistical agencies would produce "high, medium, low" estimates based on three different sets of assumptions about the drivers of population growth or decline without reference to their respective probabilities.

With respect to the lack of probabilistic statements in demographic projections, progress has been made. The classical methods are now under revision with two major alternatives taking their place: stochastic population projection and microsimulation.

The theory of stochastic processes had its genesis in the 1930's (for example, A.A. Markov and A.N. Kolmogorov). The theory progressed rapidly and starting in the 1970's, with the development of time series methods (Box and Jenkins, 1976), new tools became available to users of time series data. It is only in the past twenty years that these tools have been applied to the field of demography. Presently some national statistical agencies take advantage of these new tools to produce probabilistic population projections (eg: Hartman and Strandell, 2006). These new methods retain the classical projection framework. However, they require the producer to specify parametric distributions over the possible courses of demographic indices at each step in the projection. Using Monte Carlo simulations, many realizations of the population trajectory are traced and probabilistic intervals can be estimated based on the realizations of the simulations. The transition from the classical methods has been slow for many statistical offices because the classical methods are elegantly simple, while stochastic methods are complex; they require customized computer programs and the capacity to use sophisticated statistical software.

While statistics, and forecasting in particular, has been derided as the practice of making errors, practitioners might suggest it is the practice of keeping track of the errors they make. It is exactly this statistical framework that has been adopted by demographers taking the stochastic population projection approach. Billari *et al.*, (2010) suggest that linking stochastic population projections to the CCM scenario approach, the approach still employed by most official statistical offices, would facilitate the wide scale adoption of the modern techniques.

The literature on stochastic forecasting has approached the problem of uncertainty and forecasting three different ways. The first is to use time series methods to extract the persistence and volatility of component indices over time and then to extrapolate that persistence and volatility into the future (eg: Lee and Tuljanpurkar, 1994), the approach taken in this paper. There are two major limitations to this approach: structural breaks limit the usefulness of long time series when it is available and long time series are frequently unavailable. The second approach uses observed errors from historical data and extrapolates measures of historical error as a measure of future uncertainty (eg: Stoto, 1983). Finally, in the random scenario approach, the traditional cohort component framework is employed, but based on expert opinion, parametric distributions are specified for component indices. These indices are then projected forward in time using Monte Carlo simulations to draw index values from the specified distributions at each step in the projection. The trend towards using stochastic projection models of type presented in this paper or more aptly the trend towards using microsimulation models in demographic forecasting parallels the adoption of micro-based dynamic stochastic general equilibrium models in macroeconomic forecasting. By modeling micro behavior explicitly before aggregating, microsimulation addresses the Lucas critique, while the purely stochastic approach does not. This means that if the policy conditions affecting behavior surrounding First Nations fertility and mortality were to drastically change, the method employed in this paper would no longer apply.

Goldstein (2004) presents a modified random scenario approach as a compromise between fully stochastic forecasts like Lee & Tuljapurkar (1994) and the CCM scenario approach employed in most national statistical offices. He calibrates the stochastic component forecasts of his projection to reflect the scenarios presented in a national forecast by calibrating confidence intervals for the cumulative average of the component to correspond to the CCM's "high, low" scenarios. This approach produces forecasts that are probabilistically consistent in their forecasts of population size, age groups, fertility, mortality and age structure. This is different than the CCM approach and a major advantage because it capable of producing probabilistically consistent estimates of rates of interest such as the dependency ratio or the working age population. Goldstein shows that these calibrations can produce similar estimates of uncertainty as their fully stochastic counterparts.

Another method currently employed in population projections is microsimulation, widely considered the state of the art. It uses linked Census data to estimate hazard models for life-events. Life-events can include things like births, deaths, marriage, ethnic mobility and migration. Indeed the number of events is limited only by the demographer's imagination and size and richness of the data set. Just as the classical method has two dimension: age and sex, microsimulation has a number of dimensions equal to the number of modeled life-events. After estimating hazard models for life-events, a computer simulation is populated with the individuals from the most recent census. Each individual in the simulation is assigned attributes corresponding to their responses in the census for the relevant life-events that are included in the simulation. Individuals are then advanced through time using Monte Carlo simulations to assign realizations of life-events to individuals (Malenfant, 2011).

2.3 Behavioral Considerations

Much of the success of population projection models is due to the smooth, regular movements of demographic indices at the macro level. Demographers are able to abstract away from the behavioral considerations that surely guide individual choices that affect fertility and mortality because in the aggregate, the data series they deal with are quite regular. While there is a literature, largely in economics, investigating the behavior related to fertility and mortality on a micro level, demography has only recently started to include these insights in the form of microsimulation models. This literature explains the observed variation in fertility by appealing to the co-variation in the opportunity cost of parenting. As Becker (1965) points out, the cost of producing "commodities"[12] in the household increases with the price of time. In another example, Hotz and Miller (1988) investigate the relationship between life cycle fertility and the female labor supply finding that variation in childcare costs do affect life cycle

[12]True to tradition, this part of the economics literature treats children as commodities produced in the household

spacing of births.

It can be argued, as I do in this paper, that these behavioral effects are present in a reduced form in the projections of demographic indices. That is, the trends and volatility imposed on my projections are a function of behavioral responses to individual incentives at the micro level but I am able to abstract from this level because in the aggregate these responses generate smooth, regular movements. As an example of these smooth trends and a suggestive correlation, Figure (4) presents two data series from a set of high growth countries, their TFR and their GDP per capita. GDP per capita is a measure of the price of time and, at least in this sample, it is negatively correlated with the TFR.

3 Data

The major difficulty in building a demographic projection model for Canada's First Nations is the lack of data. Up-to-date vital statistics are frequently missing or incomplete. To further complicate the process, Canada's First Nations have markedly different mortality, fertility and migration patterns than other Canadians sometimes making imputing national level statistics for First Nations unrealistic. For example, Canada's First Nations have higher teen fertility than national averages (Guidmond, et. al, 2009) implying that the distribution of fertility by age for First Nations is skewed to the left when compared to the national average. The complication that this creates arises from the use of standardized distributions of fertility by age to extract births and fertility by age from an index such as the TFR, which has been forecast. A standard distribution must be estimated for First Nations specifically.

Our primary source of data is Health Canada's publication "A Statistical Profile on the Health of First Nations in Canada: Vital Statistics for Atlantic and Western Canada, 2001/2002." This publication includes vital statistics

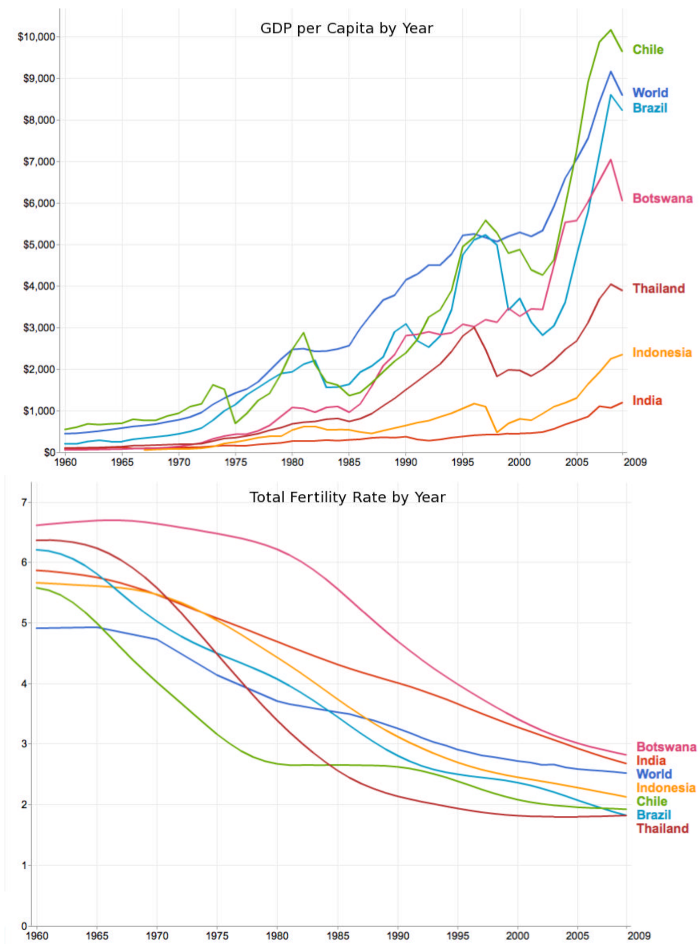


Figure 4: The total fertility rate and GDP per capita for a set of high growth countries, 1960-2009. Data from: World Bank, World Development Indicators, last updated: Jul 28, 2011

for the Registered (Status) First Nations population in Canada from regions where valid data are available. Population counts are national and calculated using the Indian Register from Indian and Northern Affairs Canada, now called Aboriginal and Northern Development Canada (AANDC). Data on age specific births and deaths, from which I calculate survival and fertility schedules are aggregated from five of Health Canada's regional offices: Atlantic, Manitoba, Saskatchewan, Alberta and British Columbia. Data from British Columbia and Alberta are for both on and off-reserve populations, while the data from the Atlantic, Manitoba and Saskatchewan offices are for on-reserve populations only. For a full description of collection methods and data limitations the reader is encouraged to visit Health Canada's website.[13] It should be noted that this data is currently under revision and being refreshed. As the program written for this paper is automated in the sense that it takes data of the form produced in this publication, new projections using the refreshed tabulations will be straightforward.

The time series of life expectancy at birth used to calibrate my forecasts of mortality are from Statistics Canada publication "Vital Statistics and Health".[14]The time series used in the forecast of the First Nations total fertility rate is from Loh and George (2003), though the raw data is from the Indian Register. The complexity of estimating and adjusting First Nations fertility rates is presented in detail in Loh and George (2003). For my purposes it is sufficient to note the TFR series has been carefully adjusted for errors present in many other First Nations TFR estimates.

[13]http://hc-sc.gc.ca/fniah-spnia/pubs/aborig-autoch/stats-profil-atlant/index-eng.php#a5

[14]http://www.statcan.gc.ca/pub/11-516-x/sectionb/4147437-eng.htm

4 Method

I follow Hunsinger (2010), Goldstein (2004), Billari *et al.* (2010) and Hartmann and Strandell (2006) in their construction of stochastic population projections for Alaska, Italy, the United States and Sweden respectively. Like Hartmann and Strandell (2006) and Billari *et al.* (2010), assuming that components are independent, I forecast the Total Fertility Rate (TFR) and sex specific mortality indices. I employ the CCM method with a modification. I advance the vector of age specific population counts in five-year intervals in the following way:

$$N_{t+1} = A_{t+1} * N_t \tag{9}$$

Different than (1), the matrix A now depends on time because its entries are derived from the stochastic forecasts of TFR and mortality indices. Ages are broken into five-year categories, corresponding with the five-year census and projection intervals. Like Hunsinger (2010), I combine the findings from several relevant sources to inform the specification of the distributions governing the stochastic processes in the projection.

4.1 Mortality

In order to project an age and sex specific survival schedule we require a standard schedule from which we can derive subsequent schedules in the projection. We estimate a standard survival schedule for First Nations in Canada using age and sex specific mortality rates published by Health Canada (2002). Since no historical time series of First Nations mortality exists, we inform our projection using several publications on First Nations mortality.

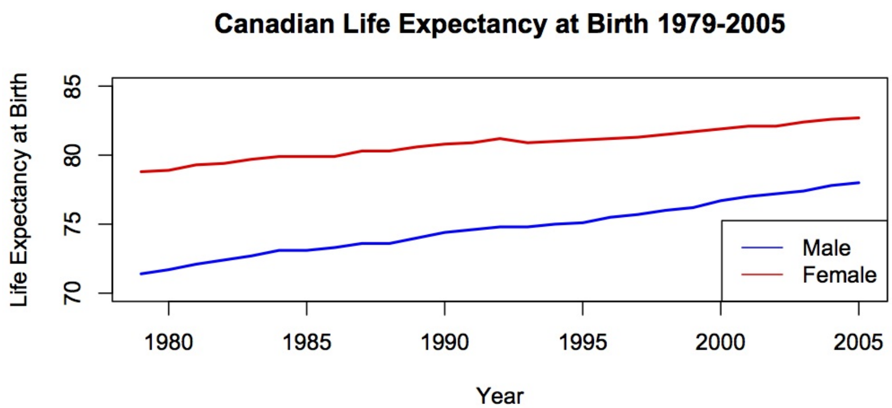
From Statistics Canada (2005), the 2017 estimates for male and female life expectancy at birth are 73.3 and 78.4 years respectively. Estimates of the 2001 male and female life expectancies at birth are 71.1 and 76.7 years implying an annual increase of 0.06 for men and 0.11 for women. They arrive at these estimates by assuming the difference between provincial averages and the First Nations average will not change over the projection period and they impute the First Nations life expectancies from provincial projections. Life expectancies estimated from my survival schedule using 2002 data are 69.62 and 74.70 for males and females respectively. I use my estimates of life expectancy for the initial condition of the projection, to be consistent with the same data source used for initial population counts and fertility.

Specifying the distribution from which consecutive draws form the stochastic process governing First Nations mortality is more difficult. Again, there are several sources to inform the specification.

First, I examine a historical (1979 – 2005) time series of Canadian life expectancy at birth and estimate a drift parameter of 0.25 for men and 0.15 for women as the mean first difference of each series.[15] The variance of the first difference of each series provides an estimate for the variance of the distribution of errors for First Nations' annual change in life expectancy. Since these are annual estimates and the projection interval is five years, by assuming that errors are independent, the variance for the distribution of errors at five year intervals is five times the annual variance. Figure 5 shows the male and female life expectancy at birth time series, along with their first difference.

Second, I consider two publications on First Nations life expectancy: Miller (1982) and Roberts and Williamson (2004). From these papers I can estimate annual gains in life expectancy simply by interpolating a straight line between historical estimates and my current estimate. In terms of annual gains, these are between 0.38 and 0.57 for males, and 0.34 and 0.71 for females. The more

[15] Approximate 95% confidence interval for males: [-0.008,0.515] and for females: [-0.139,0.439], one sided t-test statistics of $Ho : \mu < 0$ are 5.18 for males and 9.71 for females therefore we reject the null at $\alpha = 1\%$ level.



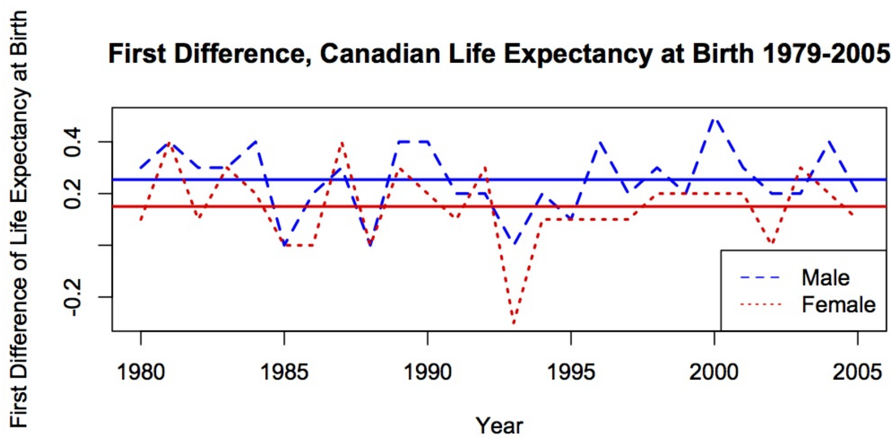


Figure 5: Canadian life expectancy at birth in levels and first differences, 1979-2005

conservative estimates are from the period 1978 to 2001 (Miller), and the larger estimates, the period 1956 to 2001 (Roberts and Williamson).

Together, these estimates provide evidence that life expectancy for First Nations is increasing, though at a decreasing rate. This observation is theoretically appealing. This is because there are diminishing returns to healthcare intervention with respect to life expectancy (Schoder and Zweifel, 2009). That is to say, for a given increase in the level of healthcare intervention, First Nations can expect higher gains in life expectancy than populations that currently have higher life expectancy. Therefore even if First Nations' exposure to health care increases linearly at historical rates we can expect declines in the rate of increase in life expectancy. The concave nature of life expectancy over time is also convenient for the way we model life expectancy. Roughly linear increases in Brass' α correspond to a concave increase in life expectancy.

I model Brass' α as a random walk with a drift because mortality has very high serial correlation (Lee and Carter, 1992):

$$BA_t = BA_{t-1} + c + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_{BA}) \tag{10}$$

where c is calibrated to reflect estimates of the annual gains in life expectancy at birth: the average of Statistics Canada's implied estimates, estimates from Miller (1982), Roberts and Williamson (2004) and my estimates of the Canadian average between 1979 and 2005. Figure 6 shows the average survival schedule for First Nations males and Canadian males. The intermediate schedules show how they converge when modeled using Brass' relational model and both α and β are allowed to change.

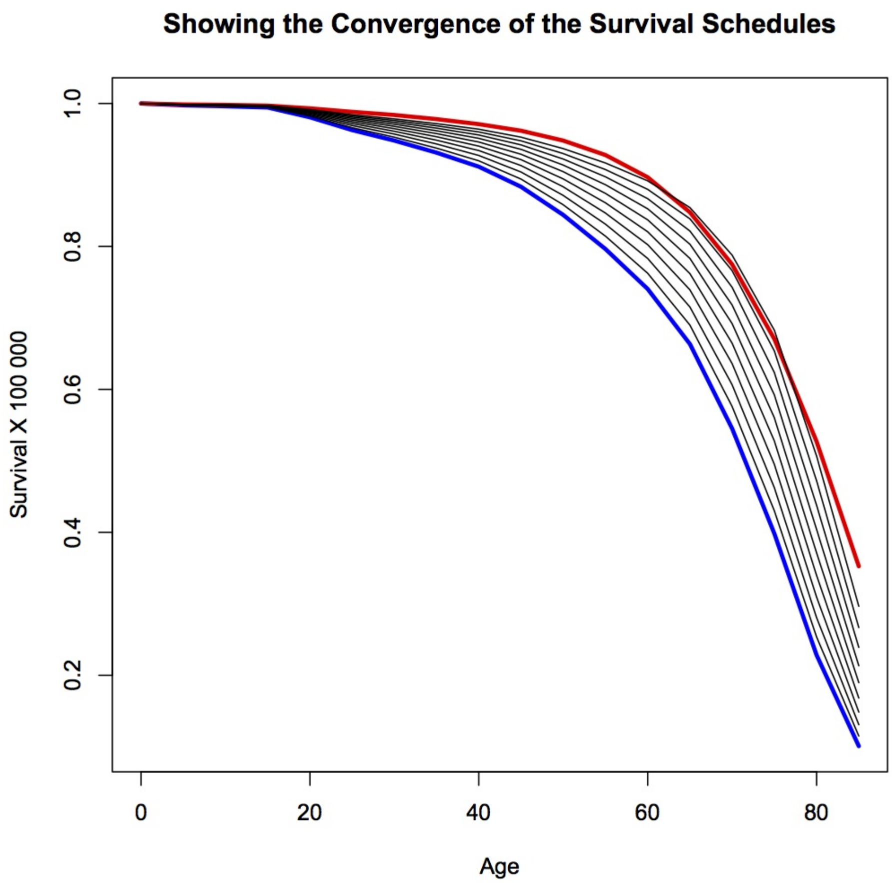


Figure 6: Convergence of survival schedules: average First Nations males and Canadian males

4.2 Fertility

Governing my forecast of the TFR is a time series from 1974-1996 constructed by Loh and George (2003). Figure 9 presents the TFR time series from 1974-1996 in addition to my TFR forecasts. Inspection of Figure 9 suggests a non-linear time trend or a structural break in the series. In 1985 Bill C-31 was passed which effectively redefined membership rules for First Nations. I believe this caused a structural break in the series. Children with maternal First Nation ancestry whose paternal ancestry was not First Nations who had previously been excluded from the population were now included. The effect of this for the TFR was a non-random redefinition of the population to include members with systematically different (lower) fertility rates. I formally test this hypothesis with Chow's (1960) $F - test$ of structural change over the interval 1980 to 1994. Figure 7 presents the results of the test indicating that there is indeed a structural break in the 1980's.

It is the post-Bill C-31 population that I am forecasting therefore I exclude information prior to the structural break in my forecast. Following Lee (1993) and Hartmann and Strandell (2010) I model fertility as a random walk with a drift. It is the general consensus that both fertility and births are non-stationary (Booth, 2006).[16] I treat the TFR as the index value and I derive age specific fertility at each projection step. The TFR is modeled quite simply as:

$$TFR_t = TFR_{t-1} + c + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_{TFR}) \tag{11}$$

where c is the drift parameter and both c and σ_{TFR} are estimated from the time series of TFR after the structural break.[17]It should be noted that the estimate of σ^2_{TFR} used in the forecasts is conditional on the assumption of no further

[16]Under the null hypothesis of a unit root, the Augmented Dickey Fuller test fails to reject the null at the $\alpha = 0.01$ level for the First Nations TFR series.

[17]$\hat{c} = \frac{\sum_{t=1990}^{1996} d(TFR)_t}{1996-1990} = -0.017$, $\sigma^2_{TFR} = \frac{\sum_{t=1990}^{1966} (d(TFR)_t - d(\bar{TFR}))^2}{(1996-1990-1)} = 0.001$

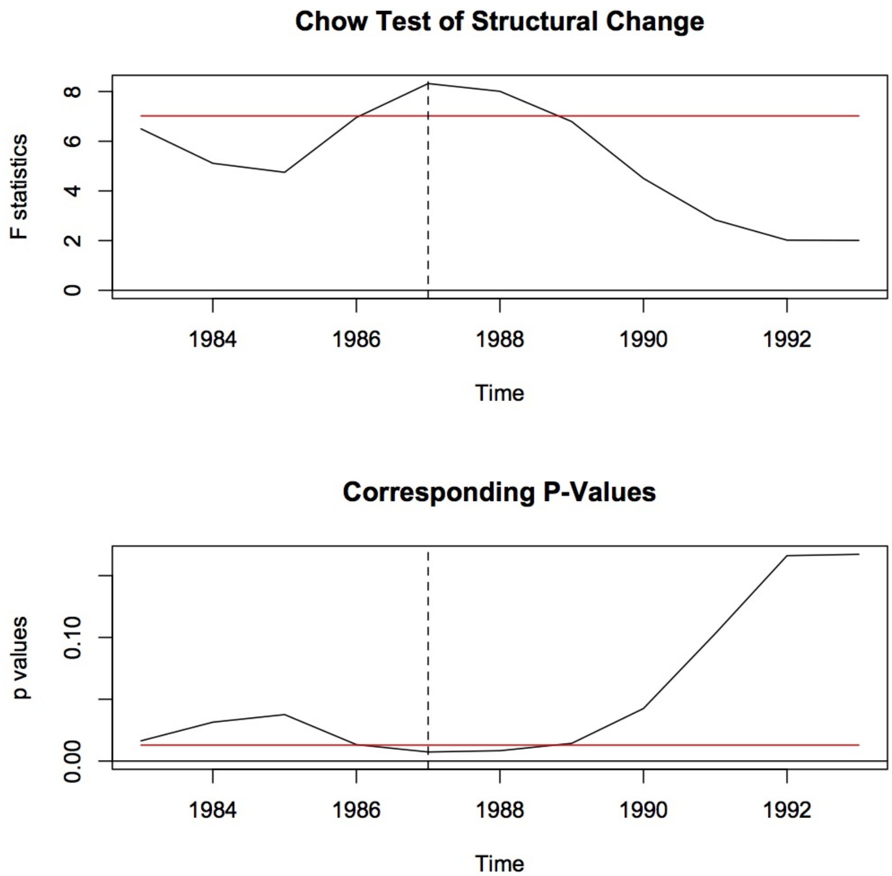


Figure 7: Testing for a structural change in the total fertility data under H_0 : no structural change. The breakpoint is identified by the dashed line, which corresponds to two years after Bill C-31 was passed. As the effects from the change in the population's definition would have taken some time, this is an intuitive breakpoint.

structural breaks.

The fertility schedule at each projection interval is constructed in the following way: let F_t be the fertility schedule at time t: a vector whose entries contain age specific rates of fertility. It follows that $\sum_a^b F_t = TFR_t$ where a and b are the minimum and maximum ages of observed reproduction. If F_t^s is the standardized schedule from (12) it follows that $\sum_a^b F_t^s = 1$.

$$F_t = F_t^s * TFR_t \tag{12}$$

4.3 Migration

As suggested in section 2, migration for First Nations at a national level is negligible because the relevant effects of migration are accounted for in the component forecasts. At a regional scale, however, migration becomes more important. Mortality and fertility patterns change at a provincial and even a regional scale. Migration affects population projections for small areas more dramatically than for national forecasts. Migration parameters should be estimated locally when making small area forecasts. Following Hunsinger (2010) I suggest the following: suppose we have a vector of counts of in-migration and out-migration by age category for the past five years, $M_m,\ m \in \{In,\ Out\}$. The i^{th} entry in the vectors corresponding to migration proportions are given by: $M_m^{i,s} = M_m^i / \sum M_m$, and these are treated as the standard, M_m^s. We can estimate net migration by projecting the percentage of the population expected to migrate at each projection interval. Let γ_m denote that percentage, then:

$$N_t^{i,*} = N_t^i + \{[(\sum N_t * \gamma_{In,t}) * M_{In}^s] - [(\sum N_t * \gamma_{Out,t}) * M_{Out}^s]\} \tag{13}$$

where $N_t^{i,*}$ is the i^{th} entry in the migration-adjusted population count vector. The sum of its elements provides the estimate of the migration-adjusted total population. This method assumes continuity in the age profile of migration and that the relevant effect of migration is the percentage of the total population that migrates.

5 Application to Small Area Forecast

While it would be hard to find a demographer willing to hang their hat on a population projection for a small community, this does not mean it is not a worthwhile endeavor. There will be two major components to our projection: a base projection and a net migration projection. Difficulty in dealing with migration is inversely proportional to the size and stability of the population under study, hence migration in small, volatile populations like First Nations communities pose a real challenge. The base projection is ignorant of migration and simply advances a vector of population counts through time, though the exact method depends on the availability and quality of data.

5.1 Counts-by-Age from Population Total

In the worst-case scenario, local death, birth and population counts by age category are unavailable. This requires one to make some strong assumptions with respect to the vital rates of the small area. The smallest amount of information required to make a projection is simply a total population count. From that, we can build a counts-by-age schedule in the following way:

Let N_x^s be the standard counts-by-age vector from national First Nations data (x refers to the age category). We can estimate N_x^{sa} , the small area schedule, by assuming a congruent distribution of the population. That is: $N_x^{sa} = (N_x^s / \sum N_x^s) * Community\, population$. With N_x^{sa} in hand we can proceed to make the base projection. Given the paucity of data in this scenario, the survival and fertility schedule must be constrained to the First Nations standard schedules.

5.2 Base Projections Assuming Constant Proportions

For this type of projection, I assume that the community's population will remain a constant proportion with respect to the First Nation to whom it belongs and that the community's survival and fertility schedules are not significantly different from that of the First Nation or alternatively from the nationally estimated First Nations standard schedule.

If N_o is the initial population vector of the First Nation, the total population of the community in ten years is given by the following:

$$\frac{Community\ Total}{First\ Nation\ Total} * (\sum N_{t+2}^{i}) = A_{t+2} * (A_{t+1} * N_o) \tag{14}$$

The A matrices are stochastic Leslie Matrices constructed as in section 3 using either the national standard schedules or schedules estimated for the First Nation.

5.3 Base Projection Assuming Congruent Age Distributions

The second method is simply an application of 5.1. I assume the distribution of people in each age category in the community is not different from the national distribution and that national level estimates apply to the community.

Using this method, the total population in ten years is given by the following:

$$\sum N_{t+2}^{i} = A_{t+2} * A_{t+1} * (N_x^s / \sum N_x^s) * Community Total \tag{15}$$

Some First Nations communities are very small and while long run forecasts will certainly be of limited utility, if informed estimates of migration can be made (as described in section 4) in consultation with community leaders, population projections of two iterations (ten years) may be informative. As discussed in section 2, after three intervals (15 years), uncertain fertility rates are being applied to an uncertain cohort of reproductive age females. This compounding of error and variability increasing in the number of intervals, particularly for migration, will limit the usefulness of long-term forecasts.

6 Results For A National Projection: 2002-2032

Projections for 2002 through 2032 for the First Nations population in Canada are produced using stochastic forecasts for both life expectancy and the TFR. Estimates of uncertainty should be interpreted as prediction intervals reflecting the uncertainty in the processes specified in section 4. There is still unaccounted for uncertainty that will not be reflected in the estimates, for example the initial population vector is itself an estimate though in the projection it is treated as the true value. Even for the modeled uncertainty, I must assume that relatively recent measures of variability are good measures of future uncertainty. All figures and tables are produced from ten thousand simulations. For exposition purposes, I present a smaller number in the figures, though prediction intervals and summary statistics are calculated using all the simulations.

6.1 Mortality

Figure 8 shows twenty forecasts of life expectancy at birth for First Nations males and females from 2002-2032. Ninety five percent of the forecasts are within the dashed lines giving prediction intervals for the realization of life expectancy at birth at each projection interval. On average, life expectancy increases across the period at a decreasing rate. Table 1 provides a summary of the simulated life expectancies at each projection interval. The ratio of female to male life expectancy in 2002 is 1.07 which falls to 1.03 in 2032.

Year	Mean (M)	97.5%	2.5%	Mean (F)	97.5%	2.5%
2002	69.55	69.55	69.55	74.63	74.63	74.63
2007	72.31	74.16	70.35	76.98	78.87	74.95
2012	74.86	77.47	72.04	79.11	81.65	76.32
2017	77.17	80.28	73.69	81.00	83.90	77.68
2022	79.29	82.70	75.31	82.67	85.63	78.90
2027	81.17	84.60	76.85	84.09	86.93	80.15
2032	82.81	86.15	78.32	85.26	87.85	81.35

Table 1: Male and female life expectancy at birth for First Nations in Canada 2002-2032 from ten thousand simulations. Statistics Canada's 2017 estimates are 73.3 and 78.4 for males and females respectively.

6.2 Fertility

Figure 9 shows one hundred realizations of the TFR forecast. Nintey five percent of the realizations fall within the red dashed lines providing prediction intervals for the forecast. Table 2 provides a summary of the TFR forecasts at each projection interval. Statistics Canada's TFR "high, low" scenarios for 2017 are almost in alignment with the 95% prediction intervals implied by the realizations of the TFR in my forecast.

Year	Mean	97.5%	2.5%
2002	2.65	2.79	2.50
2007	2.57	2.77	2.36
2012	2.48	2.73	2.23
2017	2.40	2.69	2.10
2022	2.32	2.65	2.00
2027	2.24	2.60	1.89
2032	2.15	2.54	1.77

Table 2: Summary of ten thousand simulations the TFR. Statistics Canada's predictions for 2017 are 2.56, 2.18, 2.71 for moderate, high and low decline.

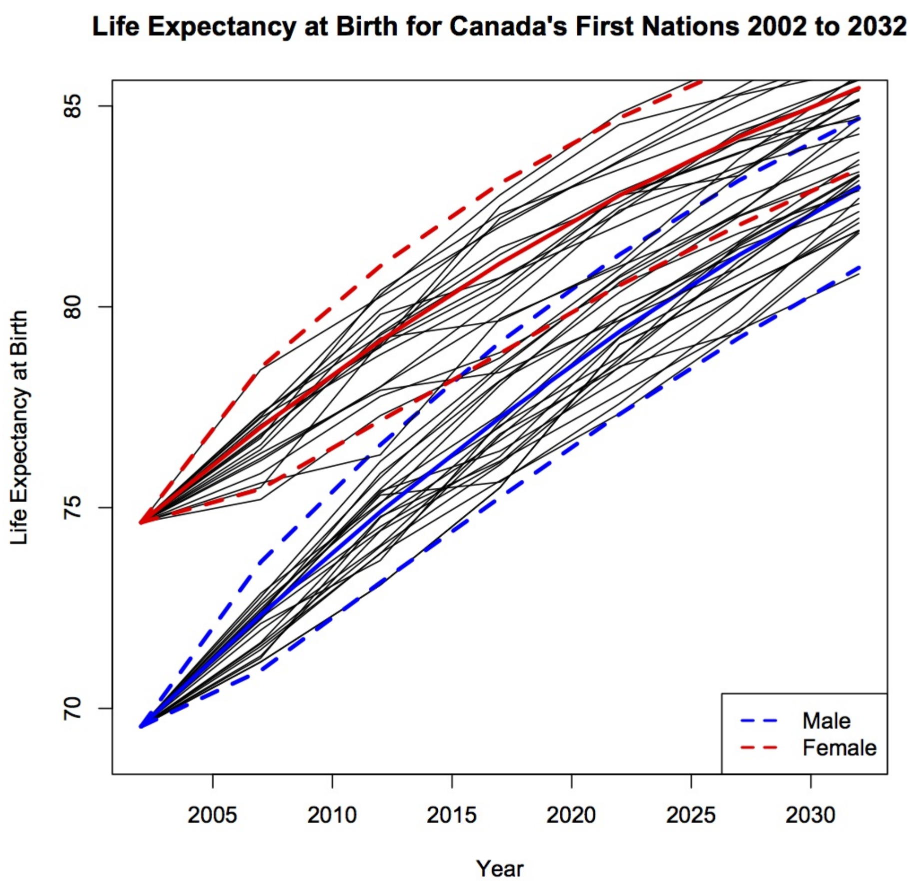


Figure 8: Twenty realizations of First Nations' life expectancy at birth for males and females, 2002-2032.

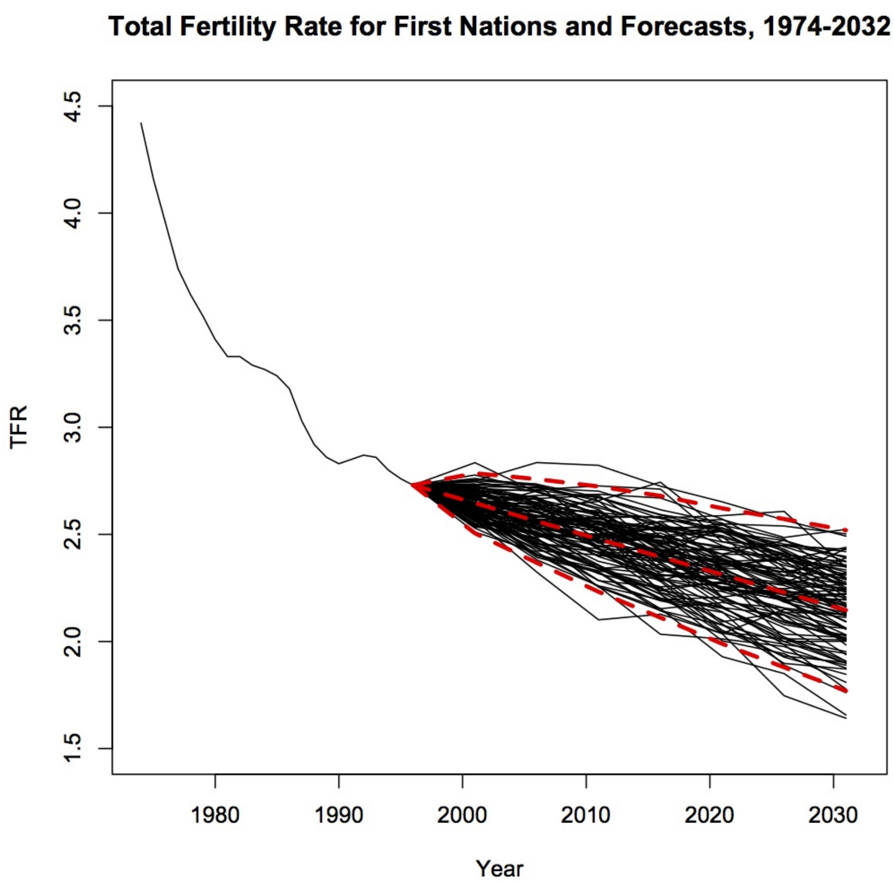


Figure 9: One hundred realizations of the total fertility rate for First Nations, 1996-2032 and the total fertility rate from 1974-2002.

6.3 Total Population

Figure 10, the total population projection, shows, on average, a roughly linear increase in Canada's total First Nations population. Uncertainty increases with the number of projection intervals, though not in the same way as the component forecasts, fanning out over the projection interval as compared with the concave curvature of the component confidence intervals. At the limit of the projection some curvature begins to appear as the large generations from the current high fertility rates are replace with younger cohorts with lower fertility rates. Table 3 presents a summary of the total population projections. Interestingly, the confidence intervals for the total population are wider than the "high, low" range in traditional CCM projections (46000 vs. 34000). This suggests that the CCM approach leads forecastsers to overstate their confidence or that the "high, low" range is not intended to correspond to 95% confidence intervals.

Year	Mean	97.5%	2.5%
2002	717276	717276	717276
2007	770010	776021	764117
2012	828286	841896	814768
2017	889094	912072	866213
2022	949949	983730	916044
2027	1008792	1056198	961770
2032	1064243	1129094	1001174

Table 3: Total population projections 2002-2032. The 95% prediction interval in 2017 is lower than the medium growth scenario produced by Statistics Canada for 2017: 971 000.

6.4 Population Distribution

Figure 11 presents four population pyramids extracted at different intervals of the projection to demonstrate the changing age distribution of First Nations in Canada. Note that the pyramids are quite base heavy, reflecting the relatively high fertility rates among First Nations, but that the distribution becomes

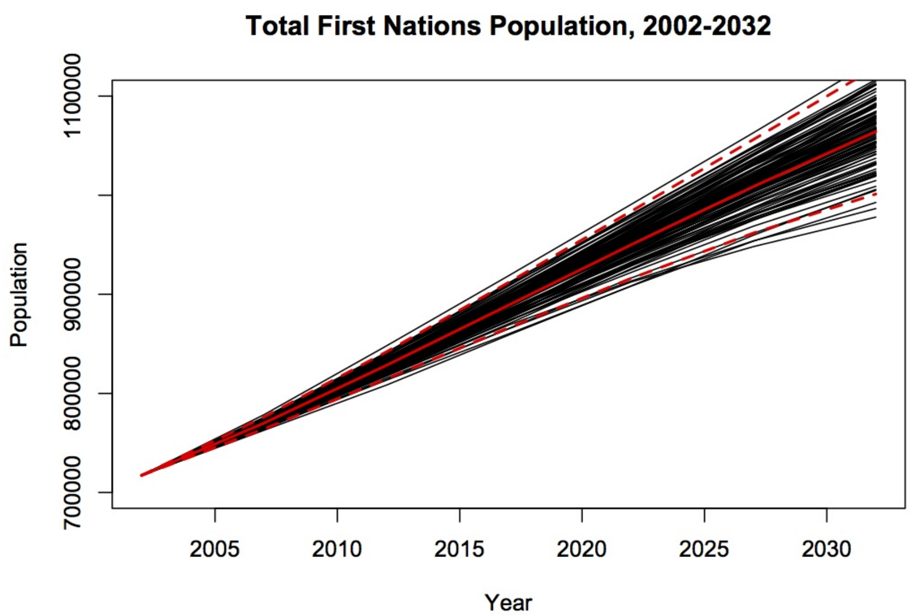


Figure 10: Total population: First Nations in Canada, 2002-2032

smoother through the projection as lower fertility rates even out the population age categories.

6.5 Percentage Growth and Youth

Figure 12 shows a histogram of the 2032 population as a percentage of the 2002 population. The histogram includes a kernel density curve showing the approximately Normal distribution of the realizations of relative size of the 2032 to 2002 population. Most realizations of the relative size over the thirty-year interval fall within the 140 to 155 percent range. This is impressive growth, especially considering the Canadian born population as a whole is expected to decline over the same thirty-year interval.

Figure 13 shows the trajectory of both the under-25 First Nations population and the percentage of the total population that this cohort comprises, over the projection range. The projection shows that as a fraction of the total population this youth cohort is presently at its highest point and, in absolute terms, it will be relatively stable over the projection period, increasing only slightly.

7 Conclusion

I outline a method for stochastic population projections using historical time series and expert informed forecasts of vital demographic rates for Canada's First Nations. I find that life expectancy will increase more rapidly for First Nations than the average Canadian and more rapidly for males than for females. Average life expectancy in 2032 is estimated to be 82.81 and 85.26 years for males and females respectively. The ratio of female to male life expectancy falls over the entire projection interval. The mean total fertility rate in 2032 is 2.15, just above the replacement rate. Contrary to a popularly held belief, the proportion of First Nations under 25 is projected to decline, and in absolute terms,

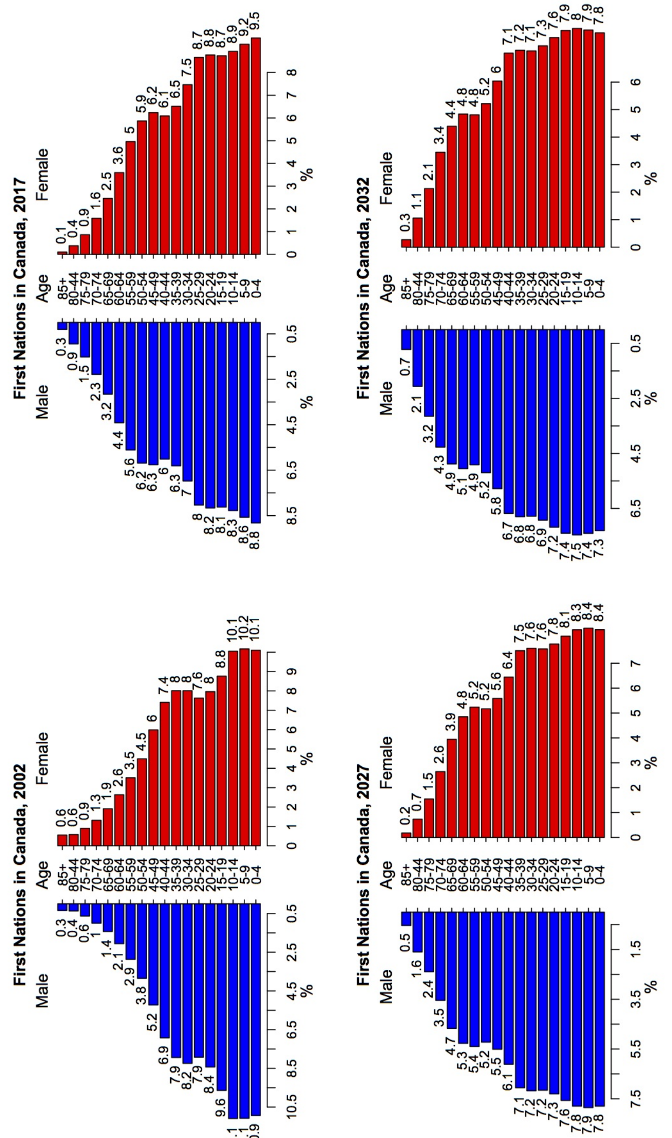


Figure 11: Population pyramids for First Nations in Canada, 2002-2032

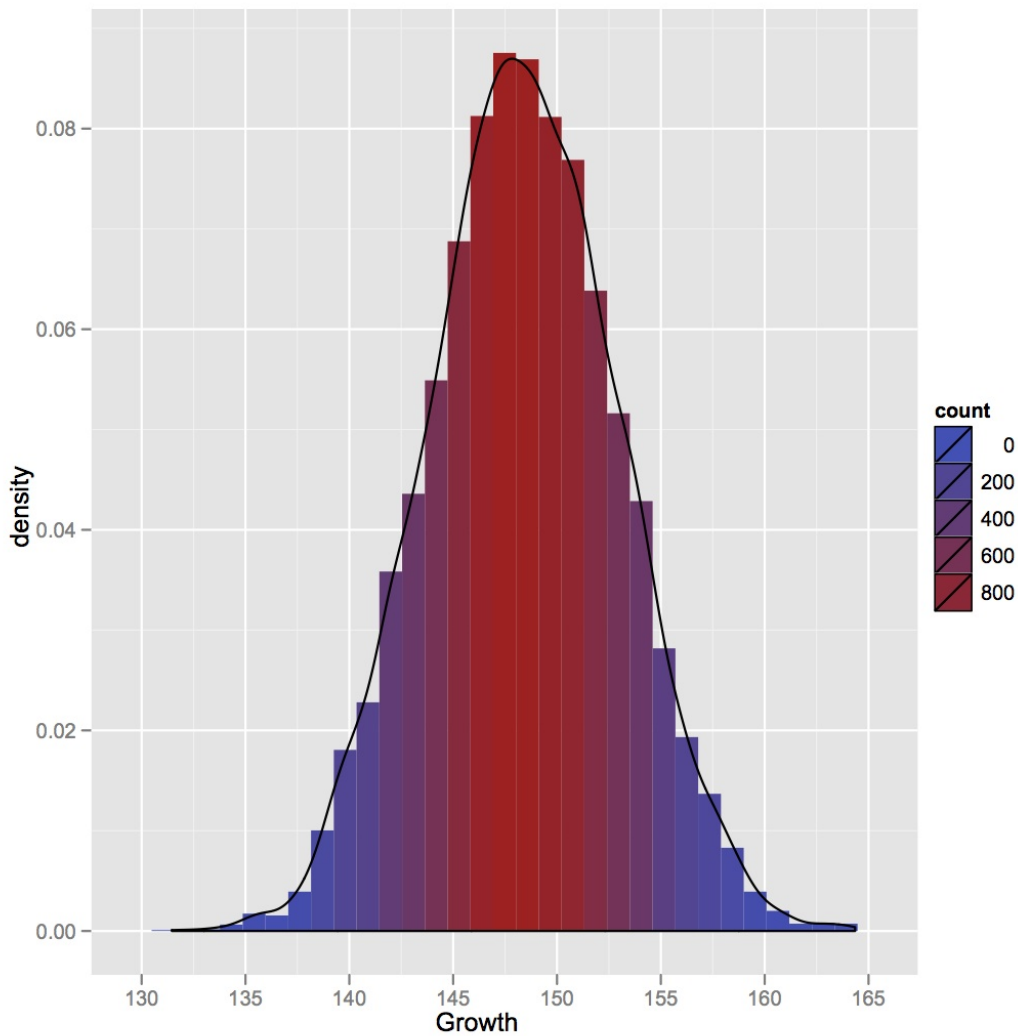


Figure 12: First Nations population in 2032 as a percentage of the population in 2002, from ten thousand simulations.

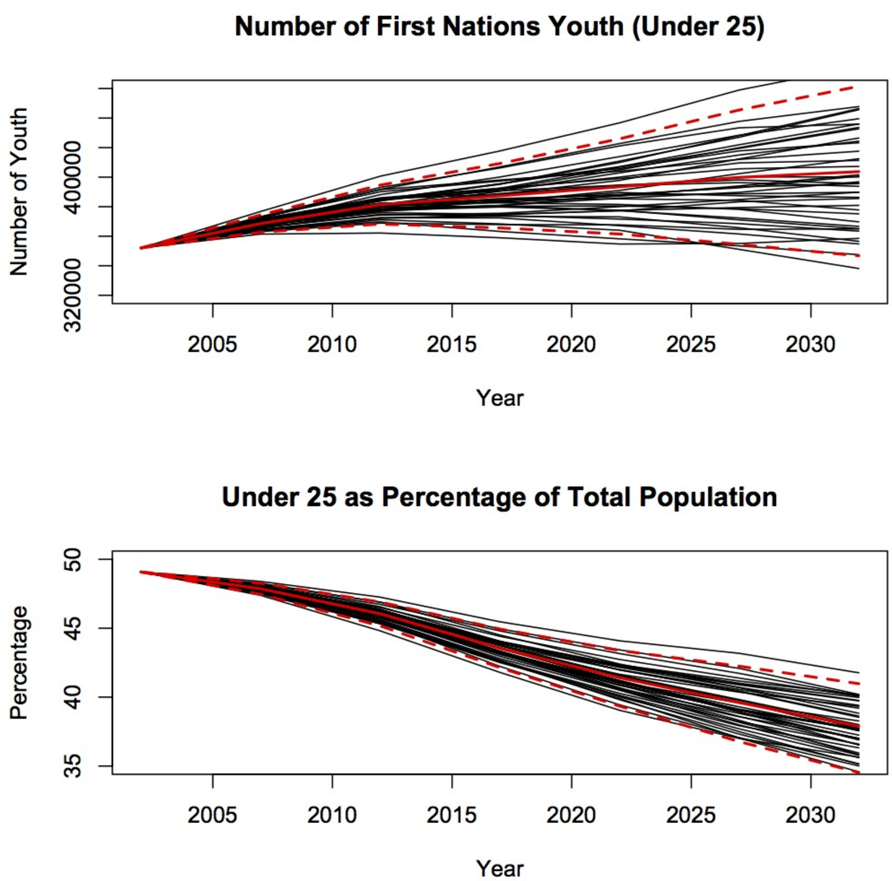


Figure 13: Under 25 First Nations population 2002-2032 in levels and as a percentage of the total First Nations population.

this youth cohort is expected grow only slightly within the 30 year projection. The First Nations population in Canada is expected to grow to 1064243 people by 2032 with 95% of the projections falling in the interval (1001174, 1129094). The most important finding in this projection, beyond the forecasts of life expectancy, fertility and the youth cohort population, is the range of realizations of the total population and its components. Traditional CCM forecasts have understated the width of the relevant range for the First Nations population suggesting a width of 34000 people by 2017 while the stochastic forecast's 95% prediction interval is over 46000 for the same year. The stochastic approach makes the uncertainty in population forecasting explicit for the end user. Policy informed by population forecasts taking into account forecast uncertainty is able to be cognisant of the limitations of forecasts and it is therefore an improvement over the traditional scenario based approach.

I provide and example of how a small First Nation can make short-run population projections to inform their policy making. Though these estimates depend heavily on the underlying assumptions, some being much stronger than those in the National forecast, hopefully this will provide a useful tool for First Nations communities in their planning processes. Many First Nations in Canada are in the process of preparing, or are already negotiating, self-government agreements. Negotiating self-government agreements includes establishing a funding framework that stretches into the future for things like education, housing and infrastructure. The population projection tool outlined in this paper can inform First Nations governments in the negotiation process with respect to their anticipated needs. Accurate measures of future needs, and importantly accurate measures of future uncertainty, will help in the negotiation of funding agreements that allow for the efficient allocation of resources today such that future needs are met and program funding is in accord with program demand.

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