The Impact of Collateral Registries in Developing Countries

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An essay submitted to the Department of Economics in partial fulfillment of the requirements for the degree of Master of Arts

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Kingston, Ontario, Canada

December 2011

Acknowledgments

I would like to thank my research supervisor, Sumon Majumdar, for his guidance and support.

I am also grateful to my family and friends for their encouragements throughout my studies at Queen's.

A special thank you to my husband Joseph and my daughter Joyce. Without your love and support, this project would not have been possible. Merci milles fois.

Contents

1	Intr	roduction	1
2	Lite	rature Review	5
3	Collateral Registry: Policy Overview		8
	3.1	Secured lending, collateral and	
		collateral registry	9
	3.2	Collateral registries in the world:	
		recent examples	12
4	Mod	del and results	16
	4.1	The model	16
	4.2	Contracting	18
		4.2.1 First-best contracts	19
		4.2.2 Second-best contracts	19
	4.3	No registry	22
	4.4	Introduction of a collateral registry	25
	4.5	Numerical results	27
		4.5.1 Impact of introducing a collateral registry	28
	4.6	Optimal policy	31
5	Conclusion		34
\mathbf{A}	Res	${ m ults}$	39

1 Introduction

For a majority of the population and for most of the firms in developed countries, opening a bank account and getting a loan from a financial institution are not unusual events. While the financial sector and financial services are well developed in these countries, this is not the case for developing countries.

The difficulty to access credit markets in developing countries is well acknowledged by researchers and policy makers, who also understand its negative implications on economic growth and development. In fact, credit services are crucial for the poor population to send the children to school, to start businesses and create employment, or even for a farmer to stabilize revenues between seasons.

However, identifying and implementing effective policies to encourage financial development and to enhance access to credit remain a challenging task. Among different policy responses to this issue, reforming collateral laws is a solution that multilateral institutions are looking into as a successful way to make progress in that direction.

Before trying to understand what could be the benefits of such policy, we should know what is a *collateral* and why it is important for credit markets in developing countries. Collateral refers to the asset the financial institutions require from the borrowers to secure their loans in case of default. For example, when an individual is contracting a mortgage loan, the house is considered as the asset securing the loan and which will be seized if the borrower is not able to fulfill his payment obligations. Credit transactions involving a collateral, also called secured transactions, are the most frequent type of loans in developed countries. They are considered to be less risky for the lenders since the pledged collateral helps enforcing the repayment.

In many low and middle-income countries, a number of problems within the financial structure and legislative framework prevent the population and the small and medium enterprises (SMEs) to be eligible for getting a secured loan. One problem is the inflexibility of collateral laws, which restricts the type of assets to be used for securing loans. Secondly, the lack of information prevents lenders from verifying the borrowers' credit history and whether the same asset has already been pledged as collateral elsewhere. This results in a number of loan applications being rejected by the lenders, or higher interest rates getting charged.

The indebtedness of the borrowers and their ability to repay the loans also raise concerns due to the popularity of microcredit in developing countries. Having multiple loans in an environment where assets accepted as collateral are limited, increases the risk for lenders not to be repaid in case of default. For example, suppose that a farmer owns only one cow and he uses it to secure two different loans, with different banks. In case of default, the value of his cow would not necessarily offset the cost faced by both the lenders. The perceived risk involved in a simple loan for a lender is therefore much higher than if borrowers' information was readily available. The borrowers are also affected through high interest rates and low volume of credit.

In this context, the implementation of a collateral registry appears as a promising solution to enhance financial access. A collateral registry is a public and centered facility, usually managed by a central bank or a governmental entity. It allows financial institutions to register loans and the related collaterals. The variety of assets or securities, accepted as collateral is thus widened. Third parties are also able to verify if there is an existent interest in a particular security. Without a registry, a borrower can "over-use" the same asset to obtain many loans, but he would not necessarily be able to repay all the lenders in case of default. This would be the case for our farmer with his two loans and his only cow. This issue arises when multiple lenders have an interest in the same collateral, which creates uncertainty about the repayment in

case of default. We refer to this issue as the multiple security interests problem.

This paper studies the theoretical implications of the introduction of a collateral registry on credit markets, focusing on the multiple security interest problem. The paper uses a standard model for studying static information problems in credit markets, but with multiple lending. In order to study the overborrowing issue, we introduce two types of borrowers, where one of them has limited liability, i.e. the total value of his wealth if not sufficient to compensate for the value of the loans contracted. The parameter representing the fraction of the population with this characteristic reflects the importance of the overborrowing problem in the population. When the collateral registry is introduced, registering a collateral informs the second lender whether there is a potential claim on the asset already. Secondly, a priority rule is assigned to establish the order in which lenders are paid back in case of default. We study how the collateral registries play a role in improving access to credit, in terms of the interest rates charged and the efficiency of the effort levels implemented.

Using this model to analyze the impact of a collateral registry, we obtain the following results: i) the registry reduces the uncertainty about the riskiness of a transaction and allows lenders to price the loans accordingly; ii) the cost of credit decreases when the loan is less risky; iii) all borrowers benefit from this policy; iv) access to credit can be facilitated, by allowing more agents to obtain a loan. These findings support the hypothesis that collateral registries can have a positive impact on credit markets in developing countries, and can improve access to credit.

The remainder of the paper is organized as follows. The next section reviews the papers which have studied information problems of credit markets in developing countries. Section 3 focusses on the public policy of secured transactions reforms and collateral registries. It explains the economic rationale provided by policy makers to implement theses measures, and provides a portray of the current effort made by international organizations in that direction. Some examples of the implementation of a collateral registry in different regions of the world are also provided. Section 4 presents the model used in this paper. It studies the first- and second-best contracts and analyzes the results of the model. Section 5 concludes with comments on the policy implications of the results and discusses the potential avenues for future research.

2 Literature Review

It is well acknowledged that credit markets in developing countries suffer from several imperfections. The quest of a better understanding of the underlying causes and possible remedies has led to a quite important literature. Banerjee (2004) reviews this literature and builds a simple model to explain the main stylized facts extracted from the micro-evidences. Among these facts, the primary one is the lack of access to finance by a portion of the population: credit rationing.

The existence of credit rationing was first shown using a theoretical framework by Stigliz and Weiss (1981). They were one of the first to study information problems in credit markets and their implications. In particular, they show that in equilibrium, there is an excess demand for loanable funds. Lenders have no incentive to decrease the interest rate charged on loans, for the fear of attracting riskier borrowers.

Two important approaches are used to explain market imperfections, and they are reviewed in Ghosh, Mookherjee and Debraj (1999): adverse selection and moral hazard. The first one shows up before the loan contract is signed and is related to the fact that lenders are not able to verify the credit history of the borrowers and to determine whether one is a good borrower or not. The second issue appears after the loan contract is signed and refers to the problem that lenders have to verify what the borrowers are actually going to do with the money that was lent. The authors show that different models for adverse selection and moral hazard in credit markets have similar features. More specifically, both type of models result in borrowing constraints, leading to welfare levels lower than the optimal one. These models are also able to explain many of the particular features of the informal banking sector that are not found in the formal sector. Ghosh et al. (1999) also look at the effect of collateral on the credit markets' information problems and find that there will be credit rationing if the amount of collateral is not large enough.

The role of the collateral in credit markets with imperfect information was also studied by Bester (1987). The author shows that collateralization works as a screening device for lenders, who can offer incentive-compatible contracts. A riskier borrower will tend to choose a contract with a higher interest rate and lower collateral requirements since they are more likely to default. Moreover, collateral also works to modify the borrowers' incentive, by bringing them to choose less risky projects. If these mechanisms are working properly, there will be no credit rationing in equilibrium. This will not be the case if, for example, a borrower is constrained in his amount of collateral, and therefore cannot reveal his own type.

Boot, Thakor and Udell (1991) were also interested in better understanding of the different aspects of secured lending by building a model with moral hazard and private information. They find that despite the costs related to its liquidation, most lenders will require collateral since it helps reducing the information problems. If that cost decreases, the collateral requirements in the loan contract are reduced. It is also found that, as the loan size increases, the lenders require a smaller amount of collateral. This is explained by a stronger incentive to make the project succeed, implying less moral hazard.

However, as raised by de Soto (2001), a major barrier to access to finance in developing countries is the lack of collateral and the inability to use the owned assets to get credit. Besley and Ghatak (2009) study different aspects of this de Soto effect. Using a theoretical framework where the borrowers' effort is subject to moral hazard, the authors study whether welfare gains can be made by improving the property rights on assets which can be used as collateral. They assume that only a fraction of the borrowers' wealth can be used as collateral, which they call effective wealth, and that policies strengthening property rights

increase this fraction of effective wealth. Their results show that these policies can indeed lead to welfare gains. However, it is also found that if the credit market competition is weak, then the benefits for the borrowers are marginal.

The present paper builds on Besley and Ghatak (2009) to study the effect of introducing a collateral registry in developing countries. The model used is also characterized by moral hazard and adverse selection, as the borrowers' effort is not contractible, and the lenders are unable to verify their credit history. However, this paper introduces multiple lending, as well as the presence of two types of borrowers, which allows us to focus on how collateral registries can fix the issue of multiple interests in the same collateral. We also analyze the conditions under which this policy is optimal, i.e. when it has the most effects of the borrowers' welfare.

This paper is also linked to the literature that looks at information sharing systems in credit markets. Bernardo, Pagano and Piccolo (2010) study the effects of the presence of multiple banks on credit markets. The authors argue that in this case, borrowers tend to overborrow and lenders tend to give more credit in order to compete with the other banks. The unaligned incentives may increase the probability of default. This problem could be mitigated through a system of credit information sharing, but the final impact depends on how well the creditors rights are protected and on the volatility of the collateral. When the creditor rights are well protected, information sharing among banks reduces the probability of default and interest rates. However, when creditor rights are poorly protected and the collateral value is sufficiently volatile, there is a market freeze, with lower credit availability.

Jappelli and Pagano (2000) look at information sharing among lenders in order to cope with information problems in the credit markets. The paper analyzes the different channels through which information sharing systems help mitigating those issues. In particular, four impacts are identified: information sharing (i) reduces the adverse selection problem by allowing lenders to better know the borrowers' characteristics; (ii) diminishes the informational rent, leading to lower interest rates on the loan; (iii) reduces the moral hazard problem, increasing the incentives for the borrowers to repay their loans; and (iv) diminishes the incentives to overborrow.

While some empirical studies such as Djankov et al. (2005) and Safavian et al. (2006) have found statistical evidence that the faction of private credit to gross domestic product (GDP) is higher in countries with improved creditor rights, the economic literature has not devoted much effort to understand the effects of the implementation of a collateral registry through a theoretical perspective. Nevertheless, this policy has interested the international institutions and the following section reviews the work done by these institutions, and provides concrete examples of collateral registries.

3 Collateral Registry: Policy Overview

The constrained access to finance, or credit rationing, represents one of the most important obstacles to private sector growth in developing countries. This difficulty of accessing financial services has caught the attention of a number of international financial institutions (IFIs). Among the various policies proposed, the improvement of secured transactions systems and collateral registries are seen as part of the solution to the lack of collateral that constrains the access to finance.

In 2006, the World Bank Group (WBG) published a book explaining the importance of collateral ¹. The European Bank for Reconstruction

¹Fleisig et al. (2006)

and Development (EBRD) also published a book on the topic of secured transactions in 2008, after more than 15 years of experience in assisting countries in transition to modern their collateral laws. The United Nations Commission on International Trade Law (UNCITRAL) also created a working group in 2002 to look at the issue of security interests. It published in 2007 a legislative guide on secured transactions "to assist States in developing modern secured transactions laws [...] with a view to promoting the availability of secured credit "². More recently, in March 2011, was held the Financial Infrastructure Week in Rio de Janeiro, a conference organized by the WBG regrouping researchers and policy makers from around the world. The objective was to discuss and share best practices around key dimensions of financial infrastructure, including secured transactions.

As we can see, a growing number of institutions and countries are interested in this idea of reforming collateral laws, notably through the implementation of modern collateral registries. Before providing an overview of different examples of registries recently introduced, this section first explains the concepts behind this policy: what it is exactly and how it affects the country that implements it.

3.1 Secured lending, collateral and collateral registry

The concept of secured lending refers to loan transactions where the borrower promises the lender to give him an asset in the case of default. The asset, or security, pledged by the borrower is called the collateral. Since the lender is able to seize the asset pledged as collateral in case of default of payment, secured lending strengthens the lenders' ability to enforce the loan repayments, which therefore reduces the risk of a default. Using data from a microcredit lender in Bolivia, we can see that this lower risk implied with secured loans generally translates into

²UNCITRAL, (2007), p.1

better lending terms. Figure 3.1 clearly shows that the interest rate charged on a secured loan is lower, the loan is larger, and the lender gives a longer period to repay.

MOVABLE **IMMOVABLE** (Unsecured loan = 100) **COLLATERAL COLLATERAL** 250 Terms on loans from a Bolivian 200 **⋖** MATURITY microcredit lender smaller borrower) 150 **▼ LOAN SIZE** March 2005 LOAN SIZE 100 MATURITY 90 INTEREST RATE 80 **▼INTEREST RATE** 70

Figure 3.1: Loan terms on secured and unsecured loans in Bolivia

Source: Fleigsig et al. (2006), p.6

Figure 3.1 above also differentiates between two types of collateral: movable and immovable. Immovable collateral is the type of assets we would naturally think of as good securities to back a loan: buildings and land. In contrast, movable collateral would include a broader variety of assets: vehicles, equipment, inventories and account receivables.

The legal framework in low and middle-income countries usually does not allow borrowers to use movable assets as collateral, which restrains access to credit. This issue is addressed by reforming the laws that control the use of collateral, so a wider range of assets can be used.

A collateral registry is part of the legal structure that facilitates the use of assets as collateral, and the introduction of a registry that accepts immovable and movable assets further encourage the realization of secured loans. A country interested in introducing a collateral registry or modernizing an existing one has to go through a long process. The private sector arm of the WBG, the International Finance Corporation (IFC) published a guide to help governments in this regard³. A comprehensive diagnosis of existing collateral laws and credit markets is necessary to identify the needs and the appropriate design

 $^{^{3}}$ IFC (2010).

for the registry. It often implies the creation of a new legislation, or to reform existing ones, and issues may arise with existing laws: sales, land, enforcement laws, etc.

When a collateral registry is introduced, it provides the lender with the ability to register the asset used to secure the loans. This is called the creation of security interest, and gives the lender the right to seize the pledged asset in case of default. Moreover, most collateral registries allow the introduction of a priority rule, determining the order in which lenders having interest in the same collateral will be repaid if there is a default. These components of the collateral registry are important to design properly for the registry to be effective. Indeed, there is a risk related to providing multiple secured loans to the same borrower if his wealth is limited. Borrowers could be tempted to use the same asset to apply for several loans. This is referred as the multiple security interests issue, or the overborrowing problem. Introduction of a collateral registry addresses this problem by allowing lenders to check whether there is already an interest in a particular asset, and also by creating a priority rule for the repayment in case of default when there is multiple security interests.

Once the collateral registry starts its operations, lenders register the loans and the associated securities. This information is made available, usually upon the payment of a fee, which is useful for lenders to check before lending to a borrower if he has already promised the same asset to another lender from a previous loan.

Reforming secured transactions laws and registries are expected to improve the access to credit by allowing borrowers to use a broader range of assets as collateral and by tackling the multiple security interests problem. This would result in an increased level of credit and a lower cost of credit⁴, as it reduces the uncertainty for the lenders to

⁴IFC (2010), p. 8.

be repaid in case of default. This policy would also have an impact on financial institutions and on financial stability⁵. By introducing a collateral registry, the financial institutions have more information on the existing security interest and diversify their portfolios by accepting a broader range of assets as collateral. This would overall benefit the financial system and reduce the risks taken while providing loans.

3.2 Collateral registries in the world: recent examples

In 1992, the European Bank for Reconstruction and Development (EBRD) created a secured transaction program to assist the countries in transition to modernizing their collateral laws and therefore encouraging the access to finance. In 2000, the bank published a report to assess the progress made in this area. In another study, the EBRD has shown by using bank lending data in twelve countries from 1994 to 2002, that allowing the use of movable collateral can increase the credit offered by banks⁶.

The map presented below shows the countries where the EBRD supports collateral laws reform. The data used is from 2000, but surveys and assessments of the secured transition law and practice in each of these countries are done on a regular basis⁷.

Apart from the EBRD, the WBG is also active in advancing the collateral laws reform, notably through the IFC. It has an important advisory program to support countries interested in implementing such reform and setting up collateral registries. It currently has projects in:

- Sub-Saharan Africa (Ghana, Malawi, Rwanda, South Sudan, Liberia)
- Middle East and North Africa (Afghanistan, Jordan, West Bank and Gaza, Yemen)

⁵Ibid, p.13.

⁶EBRD (2005)

⁷EBRD, Secured transactions, http://www.ebrd.com/pages/sector/legal/secured.shtml

Figure 3.2: Progress of collateral laws reform made by EBRD member

countries in 2000



Source: EBRD (2000), p.32.

- East Asia and Pacific (Cambodia, China, Laos, Philippines, Vietnam)
- South Asia (India, Nepal, Sri Lanka)
- Europe and Central Asia (Azerbaijan, Belarus, Moldavia, Tajikistan, Uzbekistan)
- Latin America and the Caribbean (Colombia, Haiti).

Some of the regional development banks, such as the Inter-American Development Bank and the Asian Development Bank are also developing programs in that direction.

To illustrate how the collateral laws have an impact on credit markets, we present below the cases of three countries, China, Ghana and Mexico, where collateral laws were reformed and where a collateral registry was recently introduced.

China

China approved its Property Law in 2007, which allows financial institutions to accept accounts receivable as collateral to secure a loan. A new registry for accounts receivable financing was introduced in 2008, supported by the Bank of China. Since then, the number of registra-

tions increased steadily, along with the loans secured by the registered accounts receivable as illustrated in Figure 3.3 below.

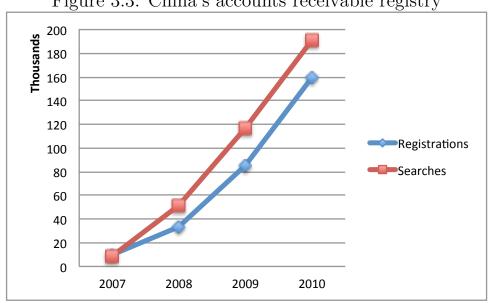


Figure 3.3: China's accounts receivable registry

Source: Credit Reference Center and People's Bank of China (2011)

The Small and Medium Enterprises (SMEs) are the major borrowers of these loans, receiving more the 50% of them, totaling more than US\$1.5 trillion.

Ghana

According to the Association of Ghana Industries (AGI) Business Barometer report for the second quarter of 2011, access to credit and high cost of credit emerged as the top two critical factors hindering growth of businesses in Ghana⁸. This is even truer for SMEs, compared to large firms, which don't have much problems to access credit.

In response to these issues, Ghana has implemented since a number of years different legislative reforms to stimulate the private sector investment and to improve access to credit. In 2008, the Borrowers and Lenders Law Act was adopted by the Ghanian Parliament, to create a secured transaction regime and to establish a collateral registry. The registry started its operation in February 2010. This new registry allows lenders to register the collateral used by the borrowers to secure loans. Reversely, other lenders are able to verify the existence of a past

 $^{^{8}}$ AGI (2011)

interest in the same asset. The registration is not compulsory and involves a fee. However, the incentive to register collaterals comes from the priority rule, which states that the first lender to register a collateral will have the priority in the case of default by the borrower. Both movable and immovable assets are accepted under the registry law, and the process to enforce the payment of the asset in case of default has been simplified and streamlined.

After one year of operation, more than 12,000 collaterals were registered, from which about 81% are movable assets that were not traditionally accepted to secure a loan. Efforts are now being made to expand the use of the registry, to computerize the system and to make it more accessible.

Mexico

After going through a number of secured transactions law reforms that started in 2000, Mexico amended in August 2009 its federal Commercial Code which allowed for the establishment of a unified Registry of Movable Property Collateral. The new registry started its operation in October 2010 and was designed by the Mexican government to facilitate the access to credit for SMEs. The registry is available online and is cost free, therefore improving the time of processing and the database is unified across the country. It allows lenders to search for any previous interest in a security and to register a new one. Moreover, as in the Ghanaian case, multiple interests in the same security are assigned different priorities.

After first six months of operations, the number of loans secured by a movable collateral is over 20,000, representing about \$US 50 billion. About 50% of these loans went to the agricultural sector.

4 Model and results

4.1 The model

The model builds on the one used by Besley and Ghatak (2009) to study contracting between lenders and borrowers. However, using the important characteristics of collateral registries explained in the previous section, it is adapted to focus on the multiple security interests issue and how this is addressed when a collateral registry is introduced. In particular, the model is built in an environment with no initial collateral registry, and where a share of the population has limited liability constraint. Then, it is extended to study the effect of the introduction of a registry.

There are n borrowers, who can all potentially benefit from getting credit from money-lenders, which may be also banks. They are endowed with the identical amount of wealth w (inclusive of assets), which can be used to secure one or multiple loans. It should be noted this paper does not differentiate between movable and immovable collateral. While we have seen that this is an important element when reforming collateral laws, this aspect is studied in Besley and Ghatak (2009). In order to focus on the overborrowing problem and how a registry can address this, we assume that the borrowers are not restrained by the type of assets they can use as collateral. This is also why there are two types of borrowers in the model, who differ by the number of projects they wish to undertake. A share of the population θ , the type-A borrowers, has only one project to invest in. The other borrowers, the type-B clients, have two projects.

All the projects are of the same size L, which produce a return $Q \geq L$ with probability p(e), and nothing otherwise. The probability of success p(e) of the project depends on the level of efforts e chosen by the borrowers.

It is assumed that type-B borrowers have to get separate financing from two sources for their two projects. They can get two loans of size L, but 2L > w, i.e.they do not have enough assets to use as collateral for both the loans. This allows them to overborrow, but this creates a risk for the lenders of not being able to recover the full value of their collateral if the type-B borrowers default on both loans. The severity of this problem is therefore captured by the parameter θ , and is more important as the share of type-B borrowers increases (i.e. θ is lower).

There are two identical lenders, with the same marginal cost of funds γ . They both maximize their expected profits, which depend on the success rates of the investments made with the credit they offer to the borrowers. If the projects are successful, the borrower repays a gross interest r, which equal to the value of the loan L, plus the interest cost. If the project fails and the borrower is unable to pay back its loan, the lender seizes its collateral c. It is assumed that the banks will be asking for the largest collateral requirement possible to fully protect them in case of default. Therefore, the value of the asset pledged has to be equal to the loan size: c = L. This assumption not only simplifies the calculations, but is in line with the high collateral requirements observed in developing countries.

The loan contract between the lender and the borrower is defined by the pair $\{r,c\}$. Since the collateral requirement is fixed in this model, the lender maximizes its expected profits by choosing the interest rate r. Given the loan contract offered by the lender, the borrower decides on the level of effort to invest on his projects, $e \in [0, e]$, to maximize his expected surplus.

The following assumptions holds for the function p(e), to ensure a well-behaved maximization problem with an interior solution:

- (i) p(e) is twice continuously differentiable, strictly increasing and strictly concave for $e \in [0, e]$;
- (ii) $p(0) \ge 0, p(\bar{e}) < 1;$
- (iii) The Inada condition holds for p(e) as $e \to 0$, i.e. $\lim p'(e) = +\infty$, when $e \to 0$.

The function $p(e) = \sqrt{e}$ respects these conditions and is used in this paper to illustrate the results.

4.2 Contracting

All borrowers pick randomly the first lender (Lender 1) to get a loan from. The type-B borrowers then visit the other lender (Lender 2) to get their second loan. The lenders offer the loan contracts that maximizes their expected profits. Given the interest rate charged in the loan contract, the borrowers choose their effort levels to invest in their projects in order to maximize their expected surplus as defined below:

• Borrower A: With probability $p(e_A)$ which depends on his effort level e_A , the borrower completes its project with success. He obtains the output Q and repays the interest rate r_A to his lender. With probability $(1 - p(e_A))$, the project fails and the borrower defaults on his loan. He pays the collateral $c_A = L$ to the bank in this case. The expected surplus for the type-A borrowers can therefore be defined as:

$$S_A = p(e_A)(Q - r_A) - (1 - p(e_A))L - e_A$$
 (4.1)

• Borrower B: With probability $p(e_{Bi})$ depending on the borrowers B's level of effort e_{Bi} invested in each of his projects $i = \{1, 2\}$, the project is a success. He gets Q in this case, and repays r_{Bi} to lender i. With probability $(1 - p(e_{Bi}))$, the projects fails and the borrower defaults on his loan. He pays his collateral $c_{Bi} = L$ to

bank i in this case. In case both his projects fails, his entire assets w are seized, to be distributed across the two lenders. We assume that the realization of the two projects' output are independent events, i.e. the default on one loan does not necessarily involve a default on the second loan. The type-B borrowers' expected surplus is therefore:

$$S_{B} = p(e_{B1})p(e_{B2}) [(Q - r_{B1}) + (Q - r_{B2})]$$

$$+p(e_{B1})(1 - p(e_{B2})) [(Q - r_{B2}) - L]$$

$$+p(e_{B2})(1 - p(e_{B1})) [(Q - r_{B1}) - L]$$

$$-(1 - p(e_{B1}))(1 - p(e_{B2})) w - e_{B1} - e_{B2}$$

$$(4.2)$$

The borrowers' outside option is assumed to be small enough not to be binding at the optimal levels of effort chosen. The payoff in autarky is 0, for both the borrowers and the lenders.

4.2.1 First-best contracts

In the first-best contract, it is assumed that the effort level is contractible, and that the borrowers have enough collateral to back their loans, i.e. $2L \leq w$. The first-best result is defined as the level of effort e^* that maximizes the joint surplus of the economy and is given by:

$$p'(e^*) Q = 1 (4.3)$$

4.2.2 Second-best contracts

The second-best contracts are characterized by non-contractible effort, and the presence of multiple security interests. These issues make the first-best no longer possible to achieve.

First, there is the moral hazard problem on the past of the borrower since the effort is no longer contractible. The lenders are not able to verify, and enforce the amount of effort the borrowers will invest on the projects after they have been financed. This explains that effort levels will most likely be lower than in the first-best case.

Second, for a share of the population, there is a multiple-lending issue arising due to a situation of limited liability. Unfortunately, lenders cannot identify the type of the borrower characterized by this problem. This creates the risk for the lenders not to be able to recover the full value of the collateral that was pledged initially. This would happen if the type-B borrowers default on both of their loans. The lenders are aware of this problem, but cannot distinguish between the different types of borrowers. Thus, it is reasonable to think that all borrowers will pay higher terms on their loans due to this overborrowing problem.

We therefore analyze the borrowers' and the lenders' problem in this case, and verify if the implementation of a collateral registry that sets a priority rule can help improve the borrowers' welfare.

The borrowers' problem

The type-A borrower randomly chooses among the two lenders and gets a loan to finance his project. He chooses his effort level e_A to maximize his expected surplus as defined in the equation 4.1:

$$Max_{e_A} S_A$$
 (4.4)

The first-order conditions give us this incentive compatibility constraint (ICC):

$$p'(e_A) \{Q - r_A + L\} = 1 \tag{4.5}$$

This defines borrower A's effort level in terms of the interest rate charged by the lender: $e_A(r_A)$.

The type-B borrower faces a similar problem, with the difference that he has to choose the two effort levels e_{B1} and e_{B2} . The default on both loans are assumed to be independent events. However, if they

were to realize at the same time, the total value of the wealth of the borrower cannot fully compensate the two lenders. In this case, the borrower gives up its full wealth w, which will be shared between the two lenders. The type-B's problem is then:

$$Max_{e_{B1}, e_{B2}} S_B \tag{4.6}$$

The first-order conditions give the ICC equations for the type-B borrowers:

$$p'(e_{B1}) \{Q - r_{B1} + p(e_{B2}) L + (1 - p(e_{B1})) (w - L)\} = 1$$
 (4.7)

and

$$p'(e_{B2}) \{Q - r_{B2} + p(e_{B1}) L + (1 - p(e_{B2})) (w - L)\} = 1$$
 (4.8)

Equations (4.5), (4.7) and (4.8) define implicitly the interest rates charged on the different loans, as functions of the effort levels the borrowers put on their projects:

$$r_A(e_A) = Q + L - \frac{1}{p'(e_A)}$$
 (4.9)

$$r_{B1}(e_{B1}, e_{B2}) = Q + p(e_{B2}) L + (1 - p(e_{B1})) (w - L) - \frac{1}{p'(e_{B1})}$$
 (4.10)

$$r_{B2}(e_{B1}, e_{B2}) = Q + p(e_{B1})L + (1 - p(e_{B2}))(w - L) - \frac{1}{p'(e_{B2})}$$
(4.11)

The lenders' problem

The efficient contracts solve the lenders' maximization problem, subject to the ICCs defined above. We will first look at the case before the collateral registry is introduced.

4.3 No registry

In the case with no registry, the lenders are not able to distinguish between the two types of borrowers. Therefore, they offer the same contract, i.e. the interest rate is the same on each loan and for each type of borrowers. Setting all the interest rates to r, the maximization problem of any lender is:

$$Max_{r} \pi = \theta \{p(e_{A}) r + (1 - p(e_{A})) L\}$$

$$+ (1 - \theta) \{p(e_{B1}) r + (1 - p(e_{B1})) \left[p(e_{B2}) L + (1 - p(e_{B2})) \frac{w}{2}\right]\}$$

$$-\gamma L$$

$$(4.12)$$

subject to the ICCs defined above.

The first part of the expected profit equation (4.12) is the profit the lenders would get from borrowers of type A: the interest rate r is paid if the project of the type-A borrower succeeds, and the lenders obtain the collateral L in case of default. The second portion $(1-\theta)$ is the expected profits from borrowers of type B. The lenders are paid r if the type-B borrowers' project 1 succeeds. However, the amount the lenders are able to recover in case of default on this first project depends on the outcome of the second project undertaken by borrowers B due to their limited liability constraint. If borrowers B complete with success the second project, the lender is able to get the full collateral L. If borrowers B default on both projects, then the two lenders have

to split among themselves the borrowers' wealth w, and obtain $\frac{w}{2}$ each.

The fact that there is only one contract and all the interest rates are equal to r, i.e. $r_A(e_A) = r_{B1}(e_{B1}, e_{B2}) = r_{B2}(e_{B1}, e_{B2}) = r$, we observe from equations (4.7) and (4.8) that the borrowers' B effort levels on both loans are equal:

$$e_{B1} = e_{B2} = e_B$$

We have therefore the following simplified incentive compatibility constraints:

$$A: p'(e_A) \{Q - r + L\} = 1 \tag{4.13}$$

B:
$$p'(e_B) \{Q - r + p(e_B) L + (1 - p(e_B)) (w - L)\} = 1$$
 (4.14)

From (4.13) and (4.14), we get a relation between the effort levels e_A and e_B is denoted $e_B(e_A)$ and is determined as follows:

$$\frac{1}{p'(e_A)} = \frac{1}{p'(e_B)} + (1 - p(e_B))(2L - w) \tag{4.15}$$

Since we have 2L > w by assumption, we find that the borrower A's effort level is greater than borrower B's :

$$p'(e_A) < p'(e_B) \quad \Rightarrow \quad e_A > e_B$$

Thus, the type-B borrower puts in less effort the project as compared to a type-A. This creates a risk for the lender.

Also, since Q - r + L < Q and comparing equation (4.13) with the first-best result in (4.3), we find that effort levels in the case with no

registry are lower than the first-best level:

$$p'(e_A) > p'(e^*) \Rightarrow e_A < e^*$$

 $\Rightarrow e_B < e^*$

Using the interest rate $r = r(e_A)$ as defined in (4.9) and $e_B(e_A)$ defined in (4.15), we can express the maximization problem as a function of the effort level of the type-A borrower:

$$Max_{e_A} \pi = \theta \{ p(e_A) r(e_A) + (1 - p(e_A)) L \}$$

$$+ (1 - \theta) \{ p(e_B(e_A)) r(e_A)$$

$$+ (1 - p(e_B(e_A))) [p(e_B(e_A)) L + (1 - p(e_B(e_A))) \frac{w}{2}] \} - \gamma L$$

$$(4.16)$$

Using the particular function $p(e) = \sqrt{e}$, the first-order conditions for this problem defining the optimal effort level the borrower A can be solved to get:

$$\sqrt{e_A^*} = \frac{Q}{4} \left[\frac{1}{\theta(2 - (2L - w)) + (1 - \theta) \left(\frac{4 - (2L - w)}{2 - (2L - w)}\right)} \right] \times \{ (\theta(2 - (2L - w)) + 2(1 - \theta)) + \frac{1}{2} \left((1 - \theta)(2L - w) \left(\frac{4 - (2L - w)}{2 - (2L - w)}\right) \right) \}$$
(4.17)

This optimal value for the borrower A's effort then allows to find the effort level for the borrower B from equation (4.15) and the interest charged on all loans from (4.13):

$$e_B^* = e_B(e_A^*)$$
 and $\mathbf{r}^* = \mathbf{r}(\mathbf{e}_A^*)$

This is plotted in Figure 4.1 presented in Section 4.5, using values for the model parameters. In the case with no registry, we can see that effort levels for both types are almost the same, with the borrower A's effort being slightly higher. Also, the effort levels with no registry stay almost constant regardless of the value of θ , meaning that the importance of the multiple security interests problem does not influence much the borrowers' choice. The same pattern is observed for the interest charged by the lenders, which stays constant at a high value relative to the loan size. This could be explained by the fact that lenders are not able to verify the borrowers' type and their riskiness. Moreover, if we compare the optimal values for the effort found in this case, of about 0.30 using parameters values, it is much below the first-best contract allocations, which is equal to 0.95. This is consistent with what we just found analytically. The uncertainty about the borrowers' type creates a risk in the economy that causes the effort levels not to be optimal.

4.4 Introduction of a collateral registry

When the collateral registry is introduced, borrowers of type A still only visit one lender, say Lender 1, to negotiate a loan. Borrowers B visit a first lender, say Lender 1, and then another lender, Lender 2, to get their second loan. The borrowers' optimization problem remains the same given the loan-repayment contracts, and thus the incentive compatibility constraints defined in (4.5), (4.7) and (4.8) above also apply.

There are two changes for the lenders with the introduction of the registry. One is with respect to the priority rule for lenders in case of default. If the priority rule is such that the Lender 1 has the first claim on the collateral in case of default, then Lender 1 is able to recover its full collateral c = L in case of default. Second, since lenders can use the

registry to identify any previous claims on an asset, Lender 2 can now identify a type-B borrower, and then offers a different loan contract to type-B borrowers.

Lender 1's problem

For Lender 1, while there is no more uncertainty about the collateral it will get back in case of default, it is still facing borrowers of different types that it is not able to differentiate due to the absence of any charge, or past loans. Lender 1 therefore offers the same contract, with the same interest rate for the type-As and on the first loan of the type-B borrowers. Lender 1's problem is now:

$$Max_{r_1}\pi_1 = \theta \{p(e_A) r_1 + (1 - p(e_A)) L\} + (1 - \theta) \{p(e_B)r_1 + (1 - p(e_B) L)\} - \gamma L$$

$$(4.18)$$
subject to
$$r_1 = r_A(e_A) = r_{B_1}(e_{B_1}, e_{B_2}).$$

Lender 2's problem

Lender 2 offers a contract for the second loan of the type-B borrowers, using his past record on the collateral registry. In case of default on the two loans, Lender 2 gets only w-L in collateral instead of w/2. This will be taken into account while pricing its loan to the riskier client. The expression for its maximization problem is now:

$$Max_{e_{B2}}\pi_2 = p(e_{B2}) r_2 + (1-p(e_{B2})) \{p(e_{B1}) L + (1-p(e_{B1}) (w-L)\} - \gamma L$$

$$(4.19)$$
subject to
$$r_2 = r_{B2}(e_{B1}, e_{B2}), \text{ as defined in 4.11, and using } e_{B1}(e_A).$$

Using the particular function $p(e) = \sqrt{e}$, Lender 1's and Lender 2's maximization problems gives us the following optimal levels of effort for the borrower A, and for the borrower B on its second loan:

$$\sqrt{e_A^{**}} = \frac{Q \left[\theta \left(2 - (2L - w)\right) + 2\left(1 - \theta\right)\left(2 + L - w\right)\right]}{4 \left[\theta \left(2 - (2L - w)\right) + 2\left(1 - \theta\right)\right]} \tag{4.20}$$

$$\sqrt{e_{B2}^{**}} = \frac{Q}{4} + \frac{(2L - w)^2 (1 + \theta)}{16}$$
 (4.21)

We then obtain $e_{B1}^{**} = e_{B1}(e_A^{**}), r_1^{**} = r_{B1}(e_{B1}^{**}, e_{B2}^{**})$ and $r_2^{**} = r_{B2}(e_{B1}^{**}, e_{B2}^{**}).$

4.5 Numerical results

The optimal effort levels resulting from the optimization problems exposed above are complex, which prevents us from making any conclusive comparisons between the allocations before and after the registry. We therefore use particular values for the model parameters, which are reasonable and respect the model assumptions and constraints. Using these parameters, we are able to get optimal values for the effort levels chosen by the two types of borrowers, and the interest rates charged on the different loans.

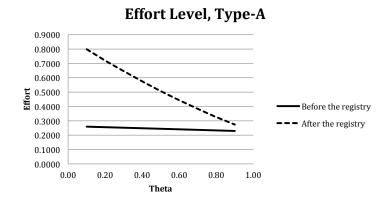
The first and the second graphs in Figure 4.1 below present the results obtained for the effort levels chosen by type-A and type-B borrowers, before and after the introduction of the registry. The third graph displays the interest rate charged on all the loans before the registry, as well as the interest rates charged after the introduction of the registry on loan 1 (which applies for type-A's loan and the first loan of borrower B) and on loan 2 (which applies on the second loan of type-B borrower). In this figure, the values for the loan size (L), the borrowers' welfare (w) and the expected output of the project (Q) are kept

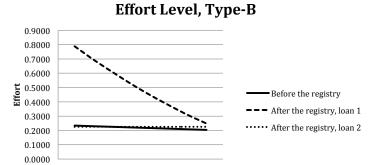
constant, while we vary the parameter θ ("Theta"). The value used are the following: L=1.90, w=0.40 and r=0.25. Since this parameter represents the share of type-A borrowers in the population, we are able to analyze the effect of the multiple security interests issue on the results and how the collateral registry helps in alleviating this problem.

4.5.1 Impact of introducing a collateral registry

When the collateral registry is introduced, the lenders are now able to verify whether the borrowers have an existing loan that they secured with their assets. We would therefore expect the lenders to best assess the risk associated with each loan compared to when this information was not available. Moreover, the priority rule ensures that the lender who has the first claim on the collateral will be able to recover the full collateral in case of default. Figure 4.1 below presents the impact this policy has on the optimal effort levels and interest rates charged. These results are presented and explained below.

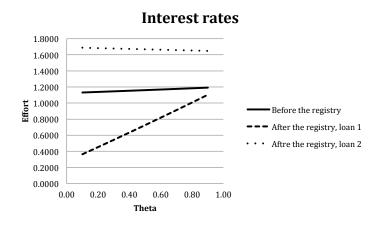
Effort levels: The results show that type-A borrowers invest more efforts on their projects after the introduction of the registry, as well as type-B borrowers on the project financed by the first loan. This leads to a more effective economy, since a higher level of effort increases the probability of success of the projects, although this imposes a cost for the borrowers. Also, we find that the increase of effort levels observed after the registry is linked to the share of type-B borrowers in the population, reflecting the importance of the multiple interests issue. In fact, when there are more borrowers B (i.e. when θ is low), the increase in effort for the type-A borrowers and for the first loan of the type B is is much higher after the introduction of the registry. This means that the impact of the policy on the effort levels is greater when the information problem is more acute.





0.80

1.00



0.20

0.40

Theta

0.60

0.00

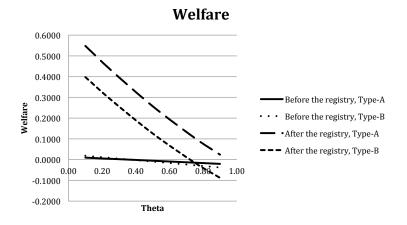


Figure 4.1: Results using L = 0.25, w = 0.40, Q = 1.90

Interest rates: After the introduction of the registry, the riskiness involved in lending to type-A and type-B borrowers with no past loan is not very high for the lenders, as they are sure to recover the full collateral value in case of default. This lower level of risk should be reflected through the interest rate charged, and this is what we find. The interest rate on "loan 1" is lower than before the registry, and the decrease is proportional to the importance of the multiple security interests problem. Again, we observe that the impact on the interest rate is more important for low values of θ .

The model also takes into account the higher riskiness related to lending a second time to the type-B borrower, since he already has a loan and his wealth cannot fully compensate the lenders in case of default on both loans. This is reflected by a higher interest rate on the second loans after the collateral registry is introduced. We can also note that the interest charged on these second loans remains higher than on type-B's first loan, and this difference is more important when there are more risks in the economy, i.e. when θ is smaller.

Before the introduction of the registry, an gross interest rate of about 1.20 was charged on all loans. After, the interest rate charged on the less risky loans, the borrowers A's loans and the first loans of the type-B borrowers, decrease to approximatively 0.40. In contrast, the interest rate on the second loans of the type-B borrowers increase to 1.70, taking up almost all the project output of 1.90.

These effects on interest rates are intuitive, and reflect the better ability for lenders to identify the risks related to each project, and to price the loans accordingly.

Introducing a collateral registry therefore helps to fix information problems observed in credit markets, which leads to efficiency gains. Moreover, the borrowers of all types benefit from this policy. The numbers obtained for the type-A and type-B borrowers' welfare show an increase after the introduction of a collateral registry. Although this policy forces the borrowers to invest more efforts in their projects, which is costly for them, these costs are compensated by a higher expected revenue on the project.

We also observe that the level of welfare is negative in some cases before the registry, mainly due to high interest rates. Given that the level of welfare in autarky is zero, this would mean that borrowers will not enter into a credit contract in this case, as they are worse off if they do. However, all the negative welfare levels for type-A borrowers become positive after the collateral registry is in place, thus increasing the number of borrowers getting a loan after the registry. This observation therefore shows that collateral registries can enhance the access to credit.

4.6 Optimal policy

According to the results analyzed above, we can say that introducing a collateral registry in developing countries can increase the access to credit, lower its cost for non-risky loans and improve the borrowers' economic conditions. However, we have seen in Section 3 that such policy involves a long process to reform the collateral laws in a first step, and then to introduce the registry. This implies significant costs for the local government, and it is therefore important to identify the conditions under which the impact of this policy is greater. This is done through the comparative statics below, analyzing which parameters have a greater effect on the optimal effort levels, the cost of credit, and the borrowers' welfare. Appendix A provides the graphs of the different results presented and explained below.

Change in the borrowers' wealth

By changing the value of the borrowers' wealth, which is assumed to be the same for all borrowers in this model, we want to analyze whether the collateral registries have more impact in a poor or rich population. As we can observe in Figure A.1, the impact of introducing a collateral registry is not influenced much by the level of wealth. The change in efforts, and in the interest rates are relatively the same, regardless of the value of w. The assumption that all the borrowers possess the same level of wealth could explain why we are not able to draw any conclusion here.

However, the results show that the borrowers' welfare is higher when the level of wealth is lower, both before and after the registry. This is an insightful result, which says that the development of the financial services and better access to these services serve the poor population and can certainly stimulate the economic growth. After the introduction of the registry, type-A and type-B borrowers' welfare increases more than four times, and we can observe that this is falling with wealth.

Change in the project size

The graphs in Figure A.2 show that the increase in effort levels for borrower A is constant for different loan sizes, and the change in interest levels for all loans does not vary much with the loan size. However, the change in the size of the project has a particular influence on the choice of effort level on the second loan of the borrower B. When the size of a project increases, the increase of his effort on the second project is more important. This would mean that the registry has more impact when the loan size is higher, i.e. when the limited liability issue is more acute. This is reflected in the borrower B's welfare, which increases more significantly after the collateral registry and when the loan is bigger. The type-A borrower's welfare does not vary much with the loan size.

Change in the expected output of the projects

While the increase in the interest rate on all loans stays almost constant regardless of the expected output of the projects, we can observe that the effort levels increases more rapidly as Q increases. This is an intuitive result, as borrowers have more incentives to work hard on their project if the related output is higher. The impact of collateral registry would therefore be more important when the project output is higher.

The results found in this section indicates that a country would benefit more from the implementation of a collateral registry when there are more borrowers using the same asset for multiple loans, and when the limited liability problem is more important for these borrowers.

5 Conclusion

To conclude, the model used in this paper lays out a framework to analyze the theoretical implications of introducing a collateral registry, using common characteristics of collateral registries introduced in various countries since a number of years. Although no general conclusions can be drawn from the model given the complexity of the variable interactions at play, the use of a functional form and reasonable numbers for the model's parameters leads to some interesting results. In particular, we are able to show that the introduction of a collateral registry can:

- reduce the uncertainty about the riskiness of secured loans;
- allow the lenders to identify the more risky borrowers and to price their loans accordingly;
- improve the borrowers' welfare;
- increase the access to finance.

These results support the generally well-accepted rationale for implementing such policy. Improvements to the financial structure and to the legal framework are crucial for enhancing the financial sector development, which in turn can support the private sector development and the overall economic growth. Further work to generalize these results would provide a stronger argument for the positive impact of the introduction of collateral registries in developing countries.

Some assumptions of the model could also provide more insights on the implications of this policy, for example by allowing the borrowers' wealth to vary. In fact, as more countries are reforming their collateral laws and are looking into introducing a collateral registry, it would be fair to ask who between the rich and the poor is benefiting the most from this policy. This would also help in advancing the growing research studying the links between financial development and inequality. The present paper suggests that introducing collateral registries in developing countries can improve access to finance. This could be further assessed by an empirical analysis, as more registries are implemented and more data become available.

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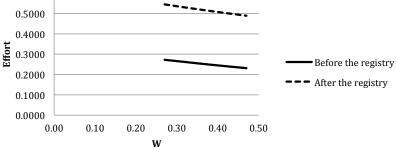
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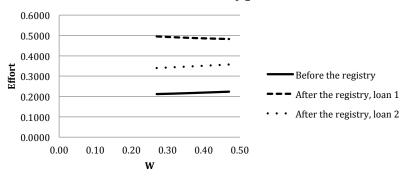
A Results

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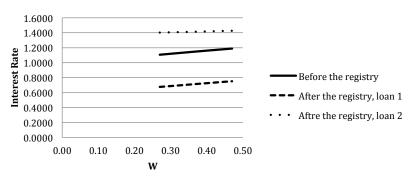
Effort Level, Type-A Borrower



Effort Level, Type-B



Interest rates



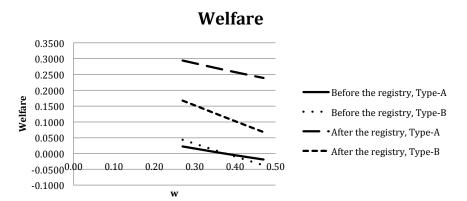


Figure A.1: Results using different values for w

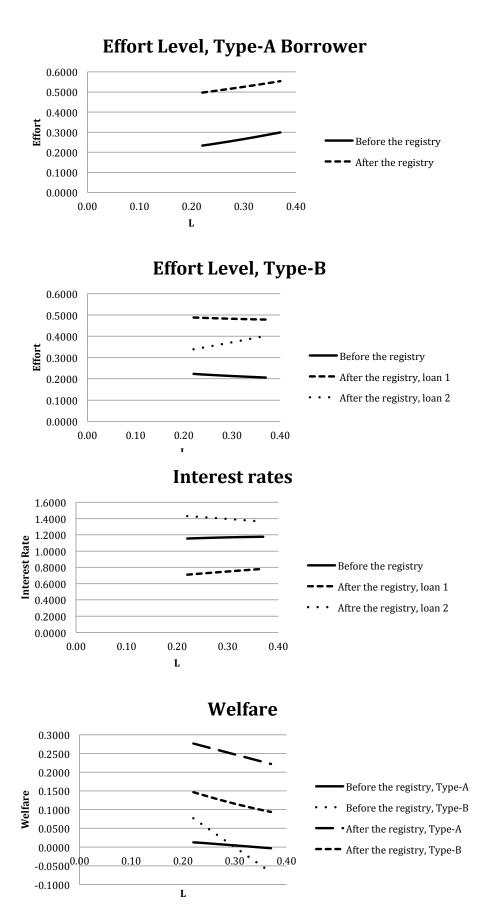
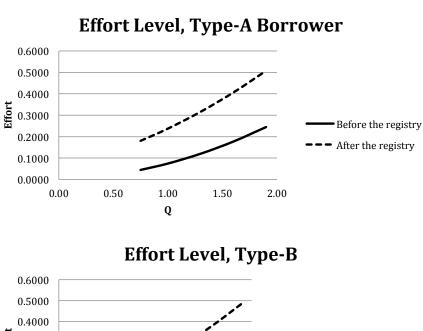
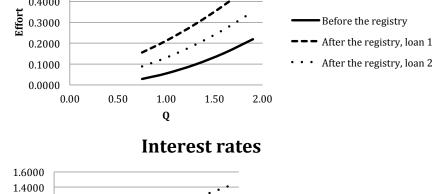
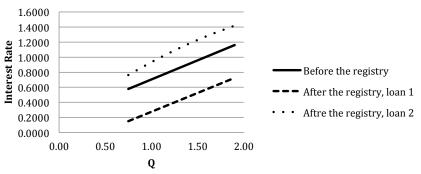


Figure A.2: Results with different values for L







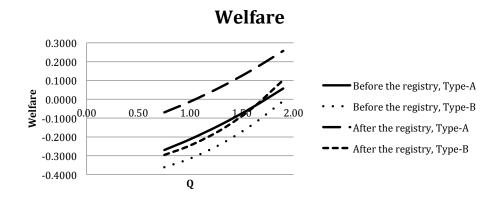


Figure A.3: Results using different values for Q