

CAPITAL REQUIREMENTS UNDER UNCERTAINTY

by

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1 Introduction

Economic decision models in finance typically assume individuals act as expected utility maximizers who have enough information to form a prior.¹ However, there are situations when this basic assumption does not hold true. Situations in which agents cannot form a single prior and instead have a set of priors such as during flight-to-quality² episodes. The recent financial crisis of 2007/2008 illustrates how behaviour changes when agents are faced with uncertainty. An approach to analyzing policy under these situations is to assume that agents are uncertainty averse, or will attempt to maximize their expected utility based on the worst-case scenario of outcomes an individual think are possible. The old saying “expect the worst and hope for the best” is a reflection of this decision rule.

The purpose of this analysis is to explore the relationship between uncertainty averse agents and the regulation of financial intermediaries. In particular I will look at government policies to regulate capital and its impact on agent’s financial decision making.

Banks are regulated at the national and sub-national levels coordinated through the Basel Committee on Banking Supervision. The most recent version of the Committee’s guidelines, Basel II, includes a recommendation for ‘Downturn LGD’ (Downturn Loss Given Default) when calculating capital holdings by banks (Basel, 2004). This regulation advises banks to increase the amount of their capital holdings as the risk of default by their borrowers increases (Basel, 2004). The implications of this policy extends beyond the continued operation of banks, to capital access by anyone using the services of regulated financial intermediaries.

Capital requirements which fluctuate with frequent changes in market values can have dramatic impacts on the aggregate supply of capital and hence the overall ability of financial intermediaries to lend funds to positive net present value projects. Lack of access to funds inhibits overall economic growth. This analysis applies to business owners and managers who are interested in the impact of aggregate liquidity shortages on their access to capital

¹A prior probability distribution (or prior) of an uncertain quantity is the probability distribution that would express one’s risk about an event before the data is taken into account. It is meant to attribute risk rather than randomness to the uncertain quantity.

²flight-to-quality episodes share common investor behaviour which includes conservatism and disengagement from risky activities episodes. (Caballero and Krishnamurthy, 2008)

in times of uncertainty. This analysis is also applicable to regulators who are faced with the challenge of acting on behalf of the public in response to distortions due to monopoly power and externalities.

At the implementation level of financial system supervision, governments and publicly appointed bodies are responsible for protecting the public interest. For example, The Ontario Securities Regulator is required to inform the Minister of Finance of events perceived to be “extraordinary and which require immediate action in the public interest” (Ontario, 2009). What constitutes an extraordinary event and how to respond to such events is not clear. What is clear is that the Ontario government believes, as do many other governments, that market failures occur and require government intervention.

For our analysis, the assumption is made that, governments have control over the supply of capital available to agents. The control is effected by varying capital requirements of intermediaries. For example, if capital requirements are tightened the overall supply of capital available for investment in the economy decreases. With capital requirements based on mark-to-market³ valuations, capital requirements change regularly with economic conditions. Based on the assumption that intermediaries asset values are correlated with the overall state of the economy, the mark-to-market value of assets held by intermediaries correspond to adjustments in the value of GDP. A central assumption in this analysis is that GDP cannot be perfectly predicted and neither can mark-to-market valuations. If GDP and mark-to-market value were predictable and known by banks and regulators capital holdings would be set according to the known probabilities of default given the macroeconomic conditions. Unfortunately, predicting GDP and mark-to-market valuations perfectly is not possible. Therefore capital requirement policy must attempt to strike a balance between ensuring minimal liquidity shortages, which could lead to a decline in asset values and drop in GDP, and allowing intermediaries to invest capital in positive net-present-value projects which encourage long-term economic growth.

Our analysis illustrates that capital requirements should be based on long-term measures and should not fluctuate with economic conditions. A broader implication of our model is

³Mark-to-market or fair value accounting refers to accounting for the value of an asset or liability based on the current market price of the asset or liability.

that government policy should focus on reducing the impact of worst-case scenarios if it is going to have any impact on the behaviour of economic agents. The basis for this argument is the assumption of economic agents' behaviour when faced with uncertainty. Typically economic agents are assumed to maximize their expected utility based on a probabilistic outcome which satisfies Savage's axioms.⁴ This is not always the case. In particular the Sure-thing Principle⁵ is not always satisfied. There is a growing body of empirical evidence that suggests asset pricing behaviour is not well described by Savage rationality. For our analysis the assumption of Savage rationality is replaced with uncertainty aversion through the use of a maximin utility function. Rationality is retained and individuals maximize their utility with respect to consumption based on a minimization (or worst-case) of outcomes for which the agent is uncertain.

The following analysis is divided into three main sections, 1) Context, 2) the Model and 3) Policy Implications. The Context provides an overview of the role of capital requirements and the debate surrounding cyclical requirements. The Model is divided into three subsections: an introduction to the use of uncertainty averse preferences, set-up of a model for uncertainty under liquidity shortages based on a paper by Caballero and Krishnamurthy (2008) and extension of the Caballero and Krishnamurthy model to demonstrate the impact of cyclical capital requirements when agents are uncertainty averse. The policy implications section provides a review of several proposed policies to deal with capital requirements based on maximin decision rules as opposed to Savage rationality. The result is that government policy should focus on reducing the severity of worst-case outcomes as opposed to focusing on the mean outcome. In situations where agents do not have complete information policies which do not reduce worst-case outcomes will be ineffective.

⁴Savage's axioms (1954) consist of seven axioms which form the basis for the typical assumption that for a rational individual all uncertainties can be reduced to risks.

⁵Savage's (1954) Postulate II from Ellsberg (1961) known as the Sure-thing principle: The choice between two actions must be unaffected by the value of pay-offs corresponding to events for when both actions have the same pay-off. If there are two events α and β where for convenience we assume they are mutually exclusive and where $\bar{\alpha}$ is not- α and $\bar{\beta}$ is not- β . The Ramsey-Savage proposal is to interpret the preferences between gamble I and II as the relative likelihood they assign to α and β , with a , b and c being payoffs. If I is preferred to II then this should hold regardless of the value of c since c is the same value for gamble I

		α	β	$\bar{\alpha} \cap \bar{\beta}$	
and II.	I	a	b	c	
	II	b	a	c	

2 Context

2.1 Financial intermediaries and regulation

The challenge of regulating financial intermediaries is to balance two competing objectives: 1) protecting the system against the cost of bank defaults, 2) encouraging the creation of positive net-present-value (NPV) loans (Kashyap and Stein, 2004). Kashyap and Stein (2004) state that it is generally supported that the “shadow value of banking”⁶ is higher during recessions. Therefore when banks lending activities are more severely capital constrained, such as during a recession, it is socially desirable to accept higher probability of bank default, all else equal. When lending activities are less capital constrained, a lower probability of bank default is acceptable.

In a model developed by Holmstrom and Tirole (1998) firms can meet liquidity needs in three ways: issuing new claims, obtaining a credit line from a financial intermediary and by holding claims on other firms. In their model, under situations of aggregate certainty these instruments are sufficient for implementing the social optimal (second-best) contract between investors and firms. However the allocation may require an intermediary to coordinate the use of scarce liquidity. When there is aggregate uncertainty, the private sector cannot satisfy its own liquidity needs. The government can improve welfare by issuing bonds that commit future consumer income. They consider two polar cases: one in which firms liquidity shocks are independent so there is no aggregate uncertainty and the opposite in which the firms liquidity shocks are identical. Aggregate uncertainty does not play a role in the model developed by Caballero and Krishnamurthy (2008) which is used as a basis for our analysis however the findings of Holdstrom and Tirole (1998) related to aggregate uncertainty are relevant. They find that individual firms will in general be unable to satisfy their liquidity needs by holding only private market instruments because lucky firms with low liquidity shocks end up holding excess liquidity. They show that optimal allocations can be achieved with intermediaries which act as liquidity pools or insurers preventing wasteful

⁶In terms of economic efficiency the key item of interest for bank lending is the shadow value which measures the scarcity of bank capital relative to positive-NPV lending opportunities. There are two competing effects for the shadow value of banking: on the one hand loan losses reduce operating income which lowers the stock of bank capital pushing up the shadow value of banking on the other hand a slowdown in aggregate economic activity means fewer positive NPV-projects

accumulation. In the model by Caballero and Krishnamurthy (2008) there is not aggregate uncertainty but intermediaries play a similar role.

I assume that financial intermediaries, such as banks, play an explicit role, similar to Diamond and Dybvig (1983): Banks transform illiquid assets into liquid liabilities by offering liabilities with a different pattern of return over time than illiquid assets offer. In the normal state of the world optimal risk sharing occurs among depositors, or agents, who need to consume at different random times. Agents face their own private risks which are not directly insurable because their knowledge is not publicly verifiable. When confidence is maintained there can be efficient risk sharing. If agents panic incentives are distorted and there can be liquidity shortages. These liquidity shortages are often referred to in the literature as coordination failure, where agents' inability to coordinate their behaviour (choices) leads to an outcome (equilibrium) that leaves all agents worse off than in an alternative situation.

2.2 Financial intermediaries and financial crises

According to Hellman, Murdock and Stiglitz (2000) in the last two decades of the 20th century the frequency of severe banking crises increased significantly. Banking crisis are important because the shock waves can effect the entire economy. A compilation of cases over the past two decades of the 20th century by the World Bank shows the cost of financial crises were up to 40 percent of GDP (Hellman, Murdock and Stiglitz 2000).

Financial system failure or near failure is often characterized by an increase of financial institution insolvencies due to depressed asset values and aggregate liquidity shortages. This may lead to contagion in the broader economy through bankruptcies, job losses and a decline in economic activity. The real economic effects can be severe. Barro (2006) measures the frequency and sizes of international economic disasters which occurred during the 20th century. The three principal events are World War I, the Great Depression and World War II. In the Canadian context, during the Great Depression (1929-1933) Canada faced a 33% decline in real GDP (Barro, 2006).

The frequency of disasters (60 occurrences for 35 countries over 100 years) implies a baseline value for probability of disaster of 1.7% per year and a contraction size of 15% to

64%. Barro's model predicts government treasury bill rates will increase in the probability of a disaster but decrease in the probability of default which can lead to an ambiguous result. This is evidence of flight-to-quality and if governments themselves are facing uncertainty, Treasury bills can also be prone to flight.

Recent flight-to-quality events include the 1987 equity crash, the Exchange Rate Mechanism (ERM) crisis of 1992 and 1993 and the fall in bond markets in the first quarter of 1994, the 1998 Russian financial crisis, the 2000 bursting of the technology stock bubble and the 2007/08 sub-prime crisis. These examples, which are provided by the Basel Committee (2009), are characterized by large price movements and a sharp reduction in liquidity. According to Caballero and Krishnamurthy (2008) flight-to-quality episodes share common investor behaviour which includes conservatism and disengagement from risky activities which indicate Knightian uncertainty.⁷ An important observation about flight-to-quality under uncertainty is that history seldom repeats itself (Caballero and Krishnamurthy, 2008). The market learns from the past with realized uncertainty becoming an expected outcome dealt with through pricing. History does repeat itself in the sense that tail risk events will occur again, but how these events unfold, and the causes of such events are unique.

2.3 Financial intermediary regulation past and present

2.3.1 Foundations of financial intermediary regulation

Regulatory intervention is justified if it: a) constrains distortionary effects due to monopoly power, b) protects the essential needs of ordinary people when information is costly to acquire or c) internalizes significant externalities (Brunnermeier in Dewatripont, 2009). Regulation is most effective when it can harness economic forces at work within the system. Neave (2009) lists broad questions to consider when evaluating whether regulation will enhance financial system performance in the long-run and over economic cycles. Two of these questions which are relevant to the analysis of cyclical capital requirements include: How can regulation encourage the financial system to finance an economy's viable economic

⁷The practical difference between risk and uncertainty is that, in the former, the distribution of the outcome in a group of instances is known (either through calculation a priori or from statistics of past experience). In the case of uncertainty this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique (Knight, 1921).

prospects? And, can public confidence in the financial system be strengthened and if so, how?

Capital requirements regulation depends on the type of institution, their scope and function. Typically regulations are at the national or sub-national level. International cooperation is crucial; without it the cost regulation can differ between countries with the least costly regulatory environment attracting risk-taking businesses. The focus in our analysis is on international standard setting through the Basel Committee related to capital requirements. Basel standards have a direct impact on the regulations of national and sub-national regulators of signatories and the requirements of institutions which fall under their regulations.

Regulators of the financial system are charged with protecting the public interest. In an extremely interconnected financial system, regulations of one market impact the objectives and regulations of others. The impact of cyclical capital requirements extends beyond the likelihood of bank runs to the broader access of capital by agents which include individuals and businesses. Businesses such as those listed on exchanges rely on capital for long-term profit generation. Limited access to capital can reduce profitability and revenue growth impacting the overall health of the economy. Liquidity shortages can be magnified by tight capital requirements, especially if these requirements are procyclical. The regulations established by the Basel Committee have implications for regulators beyond those directly impacted by the policy to the whole financial system. For example, the Ontario Securities Commission is required by the Ontario Tax Plan for More Jobs and Growth Act (2009) to inform the Minister of Finance of events constituting extraordinary circumstances, which include:

1. A major market disturbance characterized by or constituting sudden fluctuations of securities prices that threaten fair and orderly capital markets.
2. A major market disturbance characterized by or constituting a substantial disruption in the system for clearance and settlement of transactions.
3. A major disruption in the functioning of capital markets or of a significant segment of the markets, including a major disruption in the availability of capital to market

participants.

4. A major disruption in the transmission, execution or processing of securities transactions.
5. A substantial threat of such a major market disturbance or major disruption.

Ontario (2009).

Extraordinary circumstances in 1., 2., 3., and 5. above can occur in a flight-to-quality episode which can be impacted by capital requirements placed on financial intermediaries. According to Drumond (2009) capital requirements can amplify the effects of monetary and other exogenous shocks.

2.3.2 Macro- and Micro-prudential regulation

The rationale for bank capital holdings builds on the premise that banks hold capital for market and regulatory reasons (Drumond, 2009). Market capital requirements are associated with the capital ratios that maximize the value of the bank in the absence of regulation and protect the safety and soundness of the bank's operations. Market requirements can be justified by the cost of banks financial distress. These costs can be in terms of selling assets below their long-term value to meet liquidity needs, transaction costs of issuing equity coupled with substantial financial distress from low capital and the existence of agency problems between shareholders and creditors (Drumond, 2009). Regulatory bank capital requirements are part of a set of instruments used in prudential banking regulation. Regulatory mechanisms other than capital requirements include i) entry restrictions, ii) portfolio restrictions, iii) deposit insurance, iv) regulatory monitoring and v) deposit interest rate ceilings (Drumond, 2009).

“Shades of grey are best appreciated set against their two primitive components, black and white.” This analogy is used by Borio in *Current Directions in Financial Regulation*, edited Milne and Neave (2004) to define micro- and macro-prudential aspects of regulation. The objective of macro-prudential regulation is to limit the risk of episodes of financial distress with significant losses in terms of the real output for the economy as a whole. The

objective of the micro-prudential approach is to limit the risk of episodes of financial distress at individual institutions, regardless of their impact on the overall economy (Milne and Neave, 2004). The macro-prudential approach is top-down with the first step setting acceptable tail losses for a portfolio as a whole, then calibrating prudential controls on the basis of marginal contributions to the portfolio risk (Milne and Neave, 2004). A microprudentialist would argue that for a financial system to be sound it is necessary and sufficient for each individual institution to be sound.

Financial intermediary regulation traditionally consists of a mixture of monitoring individual transactions, capital requirements and entry restrictions. Many countries also impose interest-rate restrictions. Concerns about bank-runs have led many countries to provide deposit insurance and to establish central banks to serve as lenders of last resort. Over the past decade, several changes in the systems of regulation have occurred. Due to the increased number and complexity of transactions there has been less emphasis on monitoring individual transactions and greater emphasis on monitoring banks' risk-management systems such as the Internal Ratings Based (IRB)⁸ approach in Basel II. As these changes have occurred, financial crises have become more frequent, most agree that moral hazard plays an important role. Some argue that deposit insurance is part of the problem (Hellman, Murdock and Stiglitz, 2000). The moral hazard argument is based on the idea of "gambling on resurrection" in which banks choose a risky asset portfolio that pays out high profits or bonuses if the gamble succeeds but leaves depositors or their insurers with the losses if the gamble fails.

Recent empirical studies suggest it may be financial-market liberalization which has led to an increase in the frequency of banking crises. Liberalization increases competition, erodes profits, lowers franchise values and leads to a desire to gamble (Hellman, Murdock, and Stiglitz, 2000). Hellman, Murdock and Stiglitz (2000) argue that it is possible to combat moral hazard with capital requirements but they are insufficient alone because with freely determined deposit rates banks have excessive incentive to compete for deposits by offering higher rates. They show that any Pareto-efficient outcome can be implemented by

⁸Institutions subject to the Internal Ratings Based (IRB) approach in Basel II have received supervisory approval to use their own internal estimates of risk components in determining the capital requirements for a given exposure.

a combination of deposit-rate controls and capital requirements.

2.3.3 Capital, liquidity and contagion

Margin/haircut⁹ liquidity spirals, are the underlying cause of procyclicality. As asset prices drop, losses mount and margins/haircuts increase. Dewatripont (2009) highlights three reasons why asset prices lead to liquidity spirals:

- 1) Backward looking risk measures: Value-at-Risk (VaR), which is a commonly used measure of risk uses historical data. Hence, a sharp temporary price drop leads to a sharp increase in the estimates of the risk measures. When the first adverse shocks hit, the volatility estimates shoot up, leading to a deleveraging process described by the margin spiral. If the objective of individual institutions is to maintain return on equity or value at risk, leverage will be procyclical.
- 2) Time varying volatility: the price process can vary over time. A sharp price decline may signal that we are about to enter more volatile times. And,
- 3) Adverse selection: the emergence of frictions due to asymmetric-information. As losses mount, debt becomes more risky and hence more information sensitive.

Amplification due to precautionary hoarding is an important factor when considering cyclical capital requirements. Precautionary hoarding occurs if lenders are afraid that they might suffer from a shock. According to Brunnermeier in Dewatripont (2009) precautionary hoarding increases when the likelihood of a shock increases and when it is expected that it will be more difficult to obtain outside financing. The troubles in the interbank lending market in 2007/08 is an example of precautionary hoarding by individual banks (Brunnermeier in Dewatripont, 2009). Each bank was uncertain about its own funding and at the same time banks were more uncertain whether they could access funds in the interbank market because banks did not know the financial situation of other banks. These effects led to sharp spikes in the (3 months) interbank market interest rate, LIBOR, relative to the Treasury bill interest rate (Brunnermeier in Dewatripont, 2009).

⁹haircut is a percent subtracted from the book value of an asset which is being used as collateral and reflects the perceived risk for holding the asset.

According to Allen (2004) underlying risk builds up as the economy expands and leverage continues while apparent risk declines with the rise in collateral values. The risks and imbalances accumulate during the later stages of an upswing, only to materialise in the ensuing recession. Allen (2004) mentions that concern about the macroeconomic implications of the procyclical nature of risk sensitive bank capital regulations has contributed to modifications of the Basel Capital Accord. The main vector of contagion has been the liquidity shortage. This contagion mechanism combines marking-to-market and forced sales to obtain liquidity. When the price of an asset falls, marking-to-market requires the bank to take the loss immediately, decreasing its equity and thus increasing its leverage beyond the target level. To improve its capital ratio the bank may be forced to sell some assets, but this will lead to a further decrease in the price of the asset (Dewatripont et. al. 2009).

2.4 Challenges of financial intermediary regulation

Regulatory requirements that are based on estimated risk measures would be stringent during a crisis and lax during a boom. This introduces procyclicality, precisely what effective regulation is attempting to avoid (Tobias and Brunnermeier (2009), Allen (2004), Dewatripont (2009), and Drumond (2009)). Difficulties solving this problem are multi-fold; there is a lack of consensus on the relationship between the macro-economy and credit risk, there is an inability to predict contagion, and there is no consensus on how to balance the value of public insurance against the moral hazard problem.

Predicting credit risk

Lowe (2002) illustrates the lack of consensus on the issue between the macro-economy and credit risk. He uses an example where there are two economies which behave in very different ways: In economy (I) the economic activity is roughly described as a sine wave with a boom following a recession and a recession by a recovery. In this economy, a forward looking rating system would be likely to show an increase in average credit risk around the peak of the business cycle. In economy (II), while business cycles may be discernable ex post, cycles are so irregular that the economy's current performance is the best indicator of the future. In this economy a forward looking ratings system would be likely to show a decline in credit risk when macroeconomic conditions are strong (Lowe, 2002). The real

economic environment does not fit either of these extreme cases. A common view is that economic forecasters have such a poor record that the current performance can be taken as the best guess for the future which leads to a rating system that heavily relies on the current state of the economy and firms financial position.

The existing empirical literature provides mixed guidance. There is considerable statistical evidence that most movements in GDP can be viewed as permanent rather than temporary fluctuations around a trend (Lowe, 2002). There should be no presumption that simply because an expansion has gone on for a number of years, credit risk has increased. In contrast to these findings a number of authors suggest that useful indicators of banking system stress can be developed using only ex ante information. This research does not suggest that the business cycle can be forecast but makes the more modest claim that the combination of fast credit growth and rapidly increasing asset prices makes an episode of financial stress more likely (Lowe, 2002). An implication of this is that when such developments occur the level of credit risk should be judged to have increased. Lowe's interpretation is that during a period of strong growth there need be no presumption that a period of weak growth will follow. At the same time, during strong growth uncertainty about the future can be said to be higher than average due to the emergence of imbalance in the financial system or real economy.

Predicting contagion

Nobody knows what effects the default of a particular institution would have as it ripples through the financial system. This lack of information significantly increases uncertainty and counterparty credit risk (Dewatripont, 2009). The problem is exacerbated as most credit derivatives are traded over-the-counter. If all credit derivatives were traded via a clearing house, exposures could be netted out and the clearing house would know the exposure of each financial player (Dewatripont, 2009).

Moral hazard and adverse selection

Moral hazard and adverse selection are a result of information asymmetries in the lending/borrowing relationship. In a literature survey by Drumond (2009) evidence shows two sources of moral hazard in bank regulation. The first is between banks and borrowers. Borrowers can choose between different projects and have an incentive to undertake riskier

projects in order to enjoy private benefits. To deter entrepreneurs from going after those private benefits banks require entrepreneurs to invest their own funds in the project. This case of moral hazard is also related to adverse selection such as the model developed by Stiglitz and Weiss (1981) where increases in the interest rate can lead to a riskier portfolio of borrowers. The second source of moral hazard is the relationship between banks and depositors. Banks may not monitor entrepreneurs and therefore banks must be well-capitalized to convince depositors that they have enough stake in entrepreneurs' projects.

Hellman, Murdock and Stiglitz (2000) examine the moral-hazard problem of banks in a dynamic setting. Banks can either invest in a prudent asset yielding high expected returns or in an inefficient gambling asset that can yield high private returns if the gamble pays off but imposes costs on depositors if the gamble fails. If there is sufficient competition the bank earns relatively little from prudential investment but can always capture a one-period rent from gambling. The benefit of capital requirements is that they force banks to have more of their own capital at risk so that they internalize the inefficiency of gambling. According to Hellman, Murdock and Stiglitz (2000) there always exists a policy regime consisting of both a capital requirement and a deposit-rate control that Pareto-dominates any policy regime that only uses capital requirements.

2.5 Current directions of regulation

2.5.1 Basel and evolution of Basel

The Basel Accord of 1988 (Basel I) was adopted by more than 100 countries around the world. The regulations include the requirement that each bank must maintain a total risk-weighted capital ratio of at least 8% with weights depending on the institutional nature of the borrower (Drumond, 2009). The international nature of the financial system and therefore the broader impact of cyclical capital requirements on liquidity lead to the important role of principles and guidelines set by the regulating bodies. These bodies include the Basel Committee on Banking Supervision and organizations within nations such as the Office of the Superintendent of Financial Institutions in Canada and the provincial securities regulators. Basel II seeks to align regulatory capital requirements more closely to the un-

derlying risks that banks encounter. The fundamental objective of the Basel II committee was to revise the 1988 Accord, by developing a framework that would further strengthen the soundness and stability of the international banking system. Simultaneously, a goal was to maintain sufficient consistency so capital adequacy regulation would not be a source of competitive inequality among internationally active banks. The Committee retained key elements of the 1988 capital adequacy framework, including the general requirement for banks to hold total capital equivalent to at least 8% of their risk-weighted assets and including the basic structure of the 1996 Market Risk Amendment regarding the treatment of market risk and the definition of eligible capital (Basel, 2004).

Basel II is based on three pillars. Pillar I attempts to link capital requirements for large, internationally active banks more closely to the risks they assume. Therefore, pillar I guides calculations for the amount of capital banks must set aside for credit risk, market risk and operational risk. According to Jones (2000) banks attempt to boost their risk-based capital by increasing the total capital holdings or decreasing risky capital. Apart from these traditional (on-balance sheet) adjustments there is also evidence that in some circumstances banks may attempt to boost reported capital ratios through cosmetic adjustments which include inflating the measures of capital appearing in the numerators of regulatory capital ratios or artificially deflating the measures of total risk appearing in the denominators such as through securitization and other financial innovations (Jones, 2000). This process has been termed regulatory capital arbitrage (RCA). This raises the possibility that minor adjustments of capital requirements are not an effective policy tool and only a range of requirements is under the control of regulators. Pillar II focuses on supervisory activity. Pillar III attempts to strengthen market discipline of banks by requiring more public disclosure of bank lending activity and risk management activities. The focus of our analysis is encompassed within the regulations under Pillar I but it is also important to recognize the importance of supervision and public disclosure (Pillar II and III) to the success of Pillar I.

As an example of how the Basel II requirements are implemented at the national level, US banks are required to maintain balances at a certain level during two-week periods known as reserve maintenance periods (McAndrews, 2002). This policy is micro-prudential as it focuses on risk at the individual institution level. In addition, the Federal Reserve can

supply funds to the banking market through open market operations. Overnight balances at the Federal Reserve are costly to maintain because they do not earn interest. If bank balances fall below the target on average for the two-week period, the banks face a penalty rate and must hold a higher level of balances during the next two-week period (McAndrews, 2002). In addition, if banks fall into overdraft positions on any given night, they must pay a substantial penalty of four percentage points in excess of the effective federal funds rate for that day. As a result of the disincentives to falling short of required balances and to holding excessive balances banks try to target their overnight balance within a narrow band.

2.5.2 Basel II - internal ratings based (IRB) approach

Subject to certain minimum conditions and disclosure requirements, banks that have received supervisory approval to use the IRB approach may rely on their own internal estimates of risk components in determining the capital requirement for a given exposure. A qualifying rating system must have two separate and distinct dimensions: i) the risk of borrower default, and ii) transaction-specific factors.

There are different model structures used in the IRB approach, with all of them trying to measure potential losses using a specified time horizon and confidence level. The common building blocks include: a system for rating loans (generally a probability the borrower will default), assumptions about the correlation of default probabilities (PD) across borrowers, assumptions about the loss incurred in the case of default (LGD) and assumptions regarding the correlation between PD and LGD (Lowe, 2002). Each of these elements can be found in the IRB approach to calculating regulatory capital.

For each of the asset classes covered under the IRB framework there are three key elements:

- i) Risk components: estimates of risk parameters provided by banks some of which are supervisory estimates.
- ii) Risk-weighted functions: the means by which risk components are transformed into risk-weighted assets and therefore capital requirements.

iii) Minimum requirements: the standards that must be met in order for a bank to use the IRB approach.

The most common risk measurement is using value-at-risk (VaR) based models (Lowe, 2002). Basel (2004) requires that banks must hold capital equal to the potential loss on the institution's equity holdings as derived using internal VaR models subject to the 99th percentile, one-tailed confidence interval of difference between quarterly returns and an appropriate risk-free rate computed over a long-term sample period. Using VaR is criticized by Brunnermeier (2009) because it only captures an individual bank's risk in isolation. Brunnermeier's view is that regulation based on VaR reduces likelihood of the failure of an individual bank, irrespective of whether this bank causes, or is correlated with distress in other financial institutions. VaR is a means of micro-prudential regulation and is useful for individual investor protection but it may not be effective for regulating against contagion. Another important consideration when using VaR as a risk measure is the length of time considered, which comes back to the debate whether changes in GDP and associated changes in asset values are permanent or viewed as temporary fluctuations around a trend.

2.5.3 Cyclicalities and stress-testing

Stress testing processes is part of the assessment for capital adequacy. According to Basel (2004) stress testing must involve identifying possible events or future changes in economic conditions that could have unfavourable effects on a bank's credit exposure and assessment of the bank's ability to withstand such changes. Examples used by Basel (2004) of such events include i) economic or industry specific downturns, ii) market-risk events, and iii) liquidity conditions.

A literature review investigating business cycle fluctuations and the Basel Accords by Drummond (2009) finds that the introduction of bank capital requirements for market or regulatory reasons tends to amplify the effects of monetary and other exogenous shocks. The amplification effect arises from imperfections in the markets for bank capital: either banks cannot raise capital in the open markets or they face an issuance cost that tends to increase during economic downturns.

The Basel Committee has recognized the importance of correlations. It assumes that there is only a single systematic risk factor that drives correlations (Lowe, 2002). A major advantage of the single factor model is that the capital requirements assigned to a given borrower is determined solely by the characteristics of that borrower. In a multi-factor model by contrast, the marginal capital requirement might also depend on the characteristics of other borrowers in the portfolio such as the recommendation by Brunnermeier in Dewatripont (2009) for CoVaR.¹⁰

Procyclical effects of capital requirements depend on i) the composition of banks asset portfolio, ii) the approach adopted by banks to compute their minimum capital requirements, iii) the rating system¹¹ used by banks, iv) views adopted concerning how credit risk evolves through time, v) capital buffers over regulatory minimum, vi) improvements in credit risk management and vii) the supervision and market intervention under Basel II (Drumond, 2009).

Under Basel II, it is recommended that banks and other financial institutions calculate ‘Downturn LGD’ (Downturn Loss Given Default), which reflects the losses occurring during a ‘Downturn’ in a business cycle for regulatory purposes. Downturn LGD is interpreted in many ways, and most financial institutions that are applying for internal ratings based approval under Basel II often have differing definitions of what downturn conditions are. One definition is at least two consecutive quarters of negative growth in real GDP. Often, negative growth is also accompanied by a negative output gap in an economy (where potential production exceeds actual demand) (Basel II, 2004).

Basel II expects conservative assumptions as well as consideration of cyclical variation in loss given default for their capital reserve ratios. Banks may use averages of loss severities observed during periods of high credit losses with appropriately conservative assumptions using internal and/or external data. Under section 468 “a bank must estimate an LGD for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks” (Basel, 2004). LGD estimates cannot be less than the long-run

¹⁰CoVaR is defined as the value at risk (VaR) of the financial system conditional on institutions being under distress.

¹¹Rating system comprises all of the methods, processes, controls and data collection and IT systems that support the assessment of credit risk, the loss assignment of internal risk ratings and the quantification of default and loss estimates (Basel, 2004).

default weighted average loss within the data and banks must take into account higher than default-weighted average during periods when credit losses are higher than average. The minimum data observation period is considered to be one complete economic cycle or no shorter than seven years (Basel, 2004). Exposure at default (EAD) is also expected to be calculated based on a complete economic cycle and can be no shorter than seven years. Cyclical capital requirements are based on expected business cycles under normal operating environments but do not address severe liquidity shortages under a flight-to-quality episode.

Drumond (2009) provides a thorough review of literature on capital requirements under Basel II and business cycle fluctuations. His goal was to bring together theoretical literature on the bank channel of propagating exogenous shocks and the literature on the regulatory framework of capital requirements under Basel. His general conclusion is that the existing theoretical models that look at Bank capital under Basel II generally support the hypothesis that Basel II is procyclical and may have an amplifying effect.

Lowe (2002) discusses how macroeconomic considerations are incorporated into credit risk models and asks what effect these measurement approaches are likely to have on the macro-economy. Lowe (2002) suggests that a system of risk-based capital requirements is likely to deliver large changes in minimum requirements over the business cycle, particularly if based on market prices. Lowe finds that some business cycles are characterised by low levels of credit risk while others have high levels of risk. The latter is more relevant when the expansion is associated with rapid credit growth and large increases in asset prices and investment. These developments are often symptomatic of the emergence of financial imbalances. Lowe (2002) also finds that most credit risk measures pay little attention to the business cycle. Both internal and external ratings improve during expansions and deteriorate during contractions which indicate that rating agencies view changes in firm credit or asset values as permanent as opposed to part of a cycle. Given the way risk is measured the level of capital suggested by credit risk models under Basel II is that requirements will fall in booms and increase in downturns.

The Committee clarifies that regulators retain the ability to require adjustments to current value beyond those required by financial reporting standards, in particular where there is uncertainty around the current realisable value of a position due to illiquidity (Basel,

2009). This guidance focuses on current valuation and is a separate concern from the risk that market conditions and/or variables will change before the position is liquidated causing a loss of value to positions held.

The lenders and regulators are faced with a trade-off between making positive net-present-value (NPV) loans, and the expected default costs. One can always reduce the expected default costs by raising the capital requirements (Kashyap and Stein, 2004).

To explore the regulation of capital and the trade-off between net-present-value loans and expected default costs our model will explore the actions of agents who are uncertainty averse and act according to maximin preferences. This preference function is introduced in section 3.1 followed by use of a maximin utility function under Knightian Uncertainty, section 3.2, which is the basis of our model and was originally developed by Caballero and Krishnamurthy (2008). In section 3.3 our model explores an application of the Caballero and Krishnamurthy (2008) model to capital requirements.

3 Model

3.1 Model Overview

Forming regulation to balance positive NPV projects versus the chance of economic crisis and its broad economic implications is a difficult task. There is no clear consensus and opinion evolves over time. There is general consensus that cyclical requirements under Basel II can have amplification effects on the procyclical nature of banking. How to deal with this situation is of continual debate. This analysis takes one narrow perspective: the impact of capital requirements during a flight-to-quality. Whether initiated by a financial institution's actions or beyond the financial sector, contagion occurs. Stress-testing should take up the majority of debate on financial institution regulation as demonstrated by the contagion experienced during 2007/08 credit crisis. Following the credit crisis, revisions to Basel II in July 2009 and December 2009 place greater emphasis on stress-testing. Of secondary importance to intermediary regulation is the exact level of the requirement and what is included in the requirement because as Jones (2000) points out, regulatory capital arbitrage can limit the success of capital requirements.

The rationale under Basel II for the consideration of downturn losses in capital requirements is clear; banks should hold more capital when risks are higher, or when there is a greater chance that their borrowers will default (Basel, 2004). Do cyclical capital requirements encourage the financial system to finance an economy's viable economic prospects? And, do cyclical capital requirements strengthen public confidence in the financial system? To evaluate these questions the assumption is made that agents behave according to a maximin utility function as opposed to expected utility maximization when faced with uncertainty.

Mark Carney, Governor of the Bank of Canada (2009) classifies three states of the world:

“In the normal state, financial agents balance macroeconomic and idiosyncratic risks in their investing, lending, and financing decisions. In the exuberant state, agents become complacent about macroeconomic risks and seek to exploit more idiosyncratic or obscure opportunities. In the panicked state, macroeconomic risks dominate and all idiosyncratic risks are shunned. The normal state is just that, normal. The other two extremes are the tails that we have just lived through.”

Modeling the behaviour of agents under uncertainty, similar to the states of the world indicated by Mr. Carney, can best be illustrated by an example known as the Ellsberg Paradox in a paper by Ellsberg (1961). Ellsberg (1961) provided examples which show Savage's (1954) postulate II, the Sure-thing Principle may not apply. Following the example, which is reproduced below, Ellsberg demonstrates a decision rule that can be used as a simple model of how individuals may respond to decisions under uncertainty. This model is later applied to the concept of capital requirements under uncertainty. The model followed in our analysis was developed by Caballero and Krishnamurthy (2008) but reflects the ideas of Ellsberg (1961) with much more detail and it is specific to liquidity during a flight-to-quality episode.

The example from Ellsberg (1961) goes as follows: Suppose there is an urn known to contain 30 red balls and 60 black and yellow balls, the latter is an unknown proportion. One ball has to be drawn at random from the urn with a prize of either \$0 or \$100; the following actions are considered:

	30	60	
	Red	Black	Yellow

I \$100 \$0 \$0

II \$0 \$100 \$0

Action I is "bet on red" and action II is "bet on black". Which do you prefer? Now consider the following two actions:

	30	60	
	Red	Black	Yellow

III \$100 \$0 \$100

IV \$0 \$100 \$100

Action III is "bet on red and yellow" IV is "bet on black and yellow" A typical response is I is preferred to II and IV is preferred to III. This violates the sure-thing principle. The Principle would require ordering of I to II and III to IV since the two actions only differ by the yellow balls. Ellsberg asked people, including several economists to answer the above. Savage himself violated the Principle! The actions from the example form the basis for a simple decision rule under uncertainty.

Consider an individual's decision over a gamble for which they consider two factors: the mean outcome (est_x) and a minimum expected payoff (min_x). The individual's level of confidence in a state of uncertainty is denoted ρ . The simplest decision rule associated with event x would be:

$$\rho est_x + (1 - \rho)min_x \tag{1}$$

An equivalent formulation would be the following, where y^0 is the mean probability vector, y_x^{min} the probability vector in Y^0 corresponding to min_x for action x , and X is the vector of payoffs for action x . Associated with each x is the index:

$$[\rho y^0 + (1 - \rho)y_x^{min}](X) \tag{2}$$

and choose the act with the highest index.

Continuing with the original example suppose that the estimate for (red, black, yellow) is (1/3, 1/3, 1/3) but a reasonable individual would assume the distribution is between

(1/3, 2/3, 0) and (1/3, 0, 2/3). When $\rho = 1/4$ the simple formula becomes

$$\frac{1}{4}est_x + \frac{3}{4}min_x \tag{3}$$

with arbitrary utility values of 6 when the individual receives \$100 and 0 when the individual receives \$0 from the gamble the outcome is as follows:

	Red	Black	Yellow	Min_x	Est_x	Index
I	6	0	0	2	2	2
II	0	0	6	0	2	0.5
III	6	6	0	2	4	2.5
IV	0	6	6	4	4	4

Generally, an individual selects outcome I to II and IV to III which violates the sure-thing principle and reflects the typical response above. However, this is not always the case. Depending on the amount of information available, an agents confidence in the information provided and the potential impact of their decision on their utility, individuals will have different estimates for ρ . If they are confident in the estimate they use a ρ of 1 and if they are completely uncertain they use a ρ of 0, which assumes the minimum payoff.

In 2001 Chordia examined how aggregate market liquidity behaves over time. In his analysis, explanatory factors consists of short- and long-term interest rates, default spreads, market volatility and contemporaneous market moves. Chordia (2001) studied liquidity and trading activity for a comprehensive sample of NYSE-listed stocks over an 11-year period. The result of his analysis was that although a return anomaly is subject to arbitrage forces, a “liquidity anomaly” is self-perpetuating; that is, as agents find out about such an anomaly, they will avoid trading in illiquid periods, which will further reduce liquidity in those periods (Chordia, 2001). This is an important indication of how we can expect agents to behave when faced with liquidity shortages - their behaviour fits with the assumption that they behave according to maximin preferences when faced with uncertainty.

In our model, the assumption is made that agents are able to create value which cannot be created by others. In this context they require liquid assets to generate returns to lenders over time. Lenders must be patient as entrepreneurs and businesses borrow based

on the short- to medium-term cash needs of their operations. An agent's liquidity needs are prone to aggregate shocks beyond probabilistic expectations, in other words, Knightian uncertainty.

The following two sections form the basis for the model. They are divided into two sections: The first introduces the model of Knightian uncertainty under flight-to-quality developed by Caballero and Krishnamurthy with clarifying steps and omissions which are unnecessary to this analysis. The second section is an analysis of capital requirements based on the model of Caballero and Krishnamurthy (2008).

3.2 Knightian Uncertainty under a flight-to-quality

To analyze the impact of capital requirements under uncertainty, our analysis builds off a model developed by Caballero and Krishnamurthy (2008) who show a flight-to-quality can be characterized as a common feature of the most severe financial crises. The focus of their analysis is the behaviour of agents in a flight-to-quality and the value of a lender-of-last-resort. Our analysis will extend their results to illustrate the impact of capital requirements.

3.2.1 Probabilities and uncertainty

There is a continuum of competitive agents who are indexed by $\omega \in \Omega \equiv [0, 1]$. An agent may receive a liquidity shock in which he needs some liquidity immediately. Shocks are correlated across agents. Financial institutions will have their own data and models to forecast how aggregate economic activity will impact losses at default as allowed under Basel II. Our analysis assumes that forecasts of financial institutions across the economy are correlated and the losses of all institution given default (LGD) are correlated and reflect the overall state of the economy. Therefore if financial institutions project that the economy will slow down they also project that there will be an increase in bankruptcies, leading to losses given default and this projection is correlated across the international banking system.

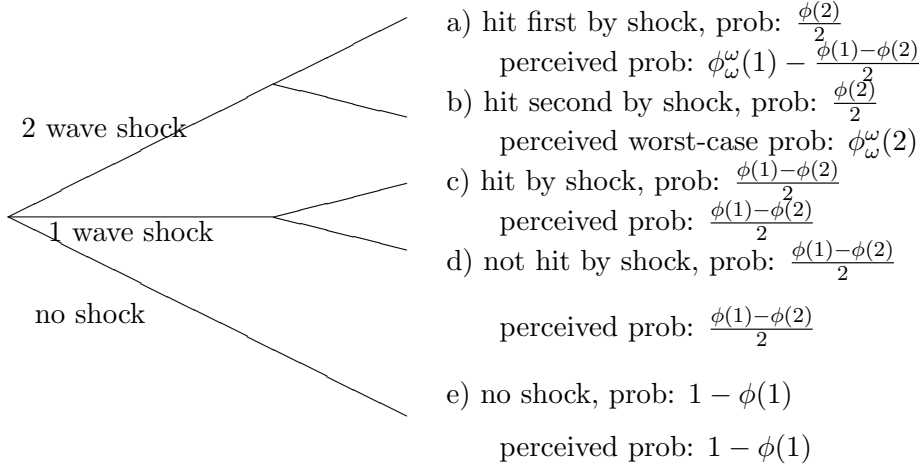
With probability $\phi(1)$, the economy is hit by a first wave of liquidity shocks. In this wave a randomly chosen group of one-half of the agents have liquidity needs. $\phi_\omega(1)$ is the probability of an agent receiving a shock in the first wave, and on average, this equals $\phi(1)/2$. With probability $\phi(2|1)$ a second wave of liquidity shocks hits the economy. In

the second wave of liquidity shocks, the agents which were not impacted by the first wave require liquidity. Let $\phi(2) = \phi(1)\phi(2|1)$. The probability with which agent ω is in this second wave is $\phi_\omega(2) = \phi(2)/2$. With probability $1 - \phi(1) > 0$ the economy experiences no liquidity shocks. See figure 1 below.

Another interpretation of the two-wave economic shock structure is that a first wave event is one which effects a specific industry and there is no contagion whereas a second wave occurs when a shock impacts industries beyond the one subject to a direct shock.

The sequential shock structure is such that $\phi(1) > \phi(2) > 0$. In aggregate a single-wave event is more likely than the two-wave event.

Figure 1



Agents know the aggregate shock probabilities $\phi(1)$ and $\phi(2)$ however agent ω does not know whether they are more likely to be in the first or second wave. Agents treat this as Knightian uncertainty. The perceived probabilities of agent ω receiving the first shock is $\phi_\omega^\omega(1)$ and second shock is $\phi_\omega^\omega(2)$. Each agent knows the aggregate which is $\phi_\omega(1) + \phi_\omega(2)$ and therefore the perceived probabilities satisfy:

$$\phi_\omega^\omega(1) + \phi_\omega^\omega(2) = \phi_\omega(1) + \phi_\omega(2) = \frac{\phi(1) + \phi(2)}{2} \quad (4)$$

Following Caballero and Krishnamurthy (2008), define:

$$\theta_\omega^\omega = \phi_\omega^\omega(2) - \frac{\phi(2)}{2}, \text{ and } -\theta_\omega^\omega = \phi_\omega^\omega(1) - \frac{\phi(1)}{2} \quad (5)$$

where θ_ω^ω reflects how much agent ω 's probability assessment of being second is higher than the average agent in the economy's true probability of being second. Agents consider a range of probability models θ_ω^ω in the set Θ where $[-K, +K](K < \phi(2)/2)$ and design insurance portfolios that are robust to their uncertainty. K captures the extent of an individuals uncertainty. $K > 0$ can be seen as an individuals perceived uncertainty when an unusual event occurs and corresponds to a flight-to-quality.

Liquidity shocks are considered as shocks to preferences. Agent ω receives utility from consumption:

$$U^\omega(c_1, c_2, c_T) = \alpha_1 u(c_1) + \alpha_2 u(c_2) + \beta c_T \quad (6)$$

Where $\alpha_1 = 1$ and $\alpha_2 = 0$ if the agent is in the early wave; $\alpha_2 = 1$ and $\alpha_1 = 0$ if the agent is hit by the second wave and $\alpha_1 = 0$ and $\alpha_2 = 0$ if the agent is not hit by a shock. The final date is date T. The utility function is assumed to be twice continuously differentiable increasing and strictly concave. Preferences are concave over c_1 and c_2 and linear over c_T . The preference over c_T captures a time in the future when the market conditions are normalized and the trader is risk neutral. The discount factor β can be thought of as an interest rate. Concave preferences over c_1 and c_2 reflect potentially higher marginal value of liquidity during market distress.

3.2.2 Budget constraints and consumption

Each agent is endowed with Z which can be interpreted as their capital or liquidity. Agents can trade financial claims that are contingent on shock realizations. These claims allow agents to insure against receiving a shock.

Consumption depends on whether there is no shock, a one-wave shock or a two-wave

shock. Consumption for an agent is $c^{i,j}$ where i is the number of waves and j is the shock faced by the agent (first or second). $c^{i,j}$ defined for i and j : $i=1$ if one wave event, $i=2$ if two wave event, $i=0$ for no shock. $j=no$ if the agent is not hit, $j=1$ if the agent is hit first and $j=2$ if the agent is hit second. Therefore,

- when agents are hit first by a two-wave shock they receive consumption $(c_1, c_T^{2,1})$;
- when agents are hit second by a two-wave shock they receive consumption $(c_2, c_T^{2,2})$;
- when agents are hit by a one-wave shock they receive consumption $(c_1, c_T^{1,1})$;
- when agents are not hit by the one-wave shock they receive consumption $(c_T^{1,no})$;
- when no shock occurs they receive consumption $(c_T^{0,no})$

For every shock realization (two-wave, one-wave, or no shock) the level of consumption cannot be more than the endowment, Z . Recall that probability of each agent being hit by a shock in the second wave is $1/2$ and being hit by the first wave is $1/2$. In the intermediation implementation, each agent deposits Z in the intermediary initially and receives the right to withdraw $c_1 > Z$ if he receives a shock in the first wave. Since shocks are fully observable, the withdrawal can be conditioned on the agents' shocks. Agents who do not receive a shock in the first wave own claims to the rest of the intermediary's assets ($Z - c_1 < c_1$). The second group of agents either redeem their claims upon incidence of the second wave of shocks, or at date T . Finally if no shocks occur, the intermediary is liquidated at date T and all agents receive Z (Caballero and Krishnamurthy, 2008).

The budget constraints are:

$$\begin{aligned}
 c^{0,no} &\leq Z \\
 \frac{1}{2}(c_1 + c_T^{1,1} + c^{1,no}) &\leq Z \\
 \frac{1}{2}(c_1 + c_T^{2,1} + c_2 + c_T^{2,2}) &\leq Z
 \end{aligned} \tag{7}$$

Where the first constraint can be dropped because if a shock does not occur each agent will consume their endowment, Z . The second budget constraint is for consumption when there is a one wave shocks and the third budget constraint is consumption when there is a two

wave shock. Agents are ex ante identical, there is no aggregate uncertainty and the true probability of being hit first or second in a two wave event is $1/2$ which is reflected in the second and third budget constraints.

3.2.3 Maximin utility function

Utility representation of Knightian uncertainty aversion:

$$\max_{(c_1, c_2, c_T)} \min_{\theta_\omega^\omega \in \Theta} E_0[U_\omega(c_1, c_2, c_T) | \theta_\omega^\omega] \quad (8)$$

The utility function under Knightian uncertainty aversion follows an axiomatic approach to preferences developed by Gilboa and Schmeidler (1989). Easley and O’Hara (2009) and Gilboa and Schmeidler (1989) refer to the Ellsberg Paradox as previously shown. The paradox illustrates the certainty-independence axiom¹² of Gilboa and Schmeidler (1989) which is a much weaker version of the Sure-thing principle developed by Savage (1954). Caballero and Krishnamurth (2008), Easley and O’Hara (2009) and Routledge and Zin (2009) base their models of uncertainty on Gilboa and Schmeidler’s axioms which are weak ordering, certainty independence, continuity, monotonicity, uncertainty aversion and nondegeneracy, see appendix I for more details. The model extends the classical expected utility model. Expected utility theory builds on previous work by Ramsey and von Neumann on risk and utility. Risk is based on probability which combines a utility function and personal probability distribution based on Bayesian probability theory¹³. The interpretation of the maximin result is a set of priors instead of a unique prior. If the set Θ is a singleton, then the decision rule is standard Savage rationality or expected utility.

A growing body of empirical evidence suggests that asset pricing behaviour is not well described by Savage rationality. Behavioural finance is one such branch of the literature where the rationality of investors is replaced by psychology-based alternatives such as over-confidence, under-reaction and loss aversion. Another branch of the literature and the focus

¹²Under the certainty-independence axiom an agent who prefers f to g can more easily visualize the mixtures of f and g with a constant h than with an arbitrary one, hence he is less likely to reverse his preferences. See appendix I for the axioms in more detail.

¹³Bayesian probability can be seen as an extension of logic that enables reasoning with uncertain statements. The Bayesian probabilist specifies some prior probability which is then updated in light of new relevant data.

of this analysis is uncertainty or learning problems. Rationality is retained but expectations are replaced with beliefs updated through learning.

Routledge and Zin (2009) investigate whether a severe reduction in liquidity can result from model uncertainty using a maximin utility function. They focus on the bid and ask prices for a proprietary derivative security, with the spread representing a measure of liquidity. They find that when uncertainty increases the bid-ask spread increases implying reduced liquidity. How Knightian uncertainty affects the optimal portfolio is very important to the understanding of the market-making problem in their model. An expected-utility market maker is willing to raise his bid price as volatility increases, whereas the market maker with aversion to Knightian uncertainty does not. It is possible in Routledge and Zin's model to completely eliminate the willingness of uncertainty averse market-makers to provide liquidity.

What matters to the uncertainty averse investor is not the variance but the largest variance. Ellsberg (1961) describes the agent as not expecting the worst, but choosing to act as though the worst were somewhat more likely than his best estimated likelihood would indicate.

Agents may have a good understanding of their own markets, but they may be unsure of the behaviour of agents in other markets. The utility function is one which represents worst-case scenario analysis. Agents are symmetric at date 0. ϕ_ω , K , Z and $u(c)$ are the same for all ω and this is common knowledge. There is no concern that an agent will pretend to have a shock and collect on an insurance claim.

Our model will only consider cases where there is uncertainty, $K > 0$. See Caballero and Krishnamurthy (2008) for the benchmark case for which there is no uncertainty. The benchmark case of no uncertainty leads to Caballero and Krishnamurthy's first proposition which is when $K = 0$ and $Z < c^*$, $c^* > c_1 > Z > c_2$ and agents are partially insured. When there is sufficient liquidity $Z > c^*$ and agents are fully insured $c_1 = c_2 = c^*$. c^* can be defined as the optimal consumption when there is no liquidity shortage.

With uncertainty there are two potential outcomes, 1) sufficient liquidity, or 2) insuffi-

cient liquidity. The agents problem is to solve:

$$\max_c \min_{\theta_\omega^\omega \in \Theta} \sum P^{i,j} U_\omega(c^{i,j})$$

subject to the resource constraints in (7). Given the probability structure this becomes:

$$V(c, \theta_\omega^\omega) = \max_C \min_{\theta_\omega^\omega \in \Theta} \left[\phi_\omega^\omega(1)u(c_1) + \beta c_T^{2,1}(\phi(2) - \phi_\omega^\omega(2)) \right. \\ \left. + \phi_\omega^\omega(2)(u(c_2) + \beta c_T^{2,2}) + \beta c_T^{1,no} \frac{\phi(1) - \phi(2)}{2} \right] \quad (9)$$

$$\text{s.t. } c_1 + c_T^{1,no} \leq 2Z$$

$$\text{s.t. } c_1 + c_2 + c_T^{2,1} + c_T^{2,2} \leq 2Z$$

Where the first two terms in the objective function represent the utility if hit by the first wave, the third term is the utility if hit by the second wave and the fourth term is utility if not hit by a shock.

The resources constraints from (7) have been simplified to the above: $c_T^{1,1}$ and $c_T^{1,no}$ enter as a sum in both the objective function and constraint, so choose $c_T^{1,1} = 0$ which results in the resource constraints seen above.

Recall definitions:

$$\theta_\omega^\omega = \phi_\omega^\omega(2) - \frac{\phi(2)}{2}, \text{ and } -\theta_\omega^\omega = \phi_\omega^\omega(1) - \frac{\phi(1)}{2}$$

rearranged

$$\phi_\omega^\omega(1) = \frac{\phi(1)}{2} - \theta_\omega^\omega, \text{ and } \phi_\omega^\omega(2) = \theta_\omega^\omega + \frac{\phi(2)}{2}$$

which are the worst case probabilities based on $K = \theta_\omega^\omega$. K is the agent's perceived probability from a range of probability models θ_ω^ω . An example can be drawn from the credit crisis of 2007/08. If agents are considered trading desks of an investment bank, retaining c_2 to cover any needs that may arise in a particular market, at date -1 the trading desk pools their capital to cover any needs that may arise. They also agree that a top-level risk manager will commit $c_1 - c_2 > 0$ to cover markets which receive a shock first. At date 0 there is a shock such as the collapse of Lehman Bros. Agents in non-related markets become concerned that the risk managers will no longer have enough capital if they are hit with a shock and therefore every trading desk has less capital for a first shock and the average market has less capital to absorb shocks which reduces liquidity in all markets.

Using equation (9) and the rearranged definitions above the equation becomes:

$$V(C, \theta_\omega^\omega) = \max_C \min_{\theta_\omega^\omega \in \Theta} \left[\left(\frac{\phi(1)}{2} - \theta_\omega^\omega \right) u(c_1) + \beta c_T^{2,1} \left(\frac{\phi(2)}{2} - \theta_\omega^\omega \right) + \left(\theta_\omega^\omega + \frac{\phi(2)}{2} \right) (u(c_2) + \beta c_T^{2,2}) + \beta c_T^{1,no} \frac{\phi(1) - \phi(2)}{2} \right] \quad (10)$$

Taking the partial derivative of equation (10) with respect to θ_ω^ω :

$$\frac{\partial V}{\partial \theta_\omega^\omega} = -u(c_1) + u(c_2) + \beta (c_T^{2,2} - c_T^{2,1}) \quad (11)$$

Note that $c_T^{2,1} = 0$ because if $c_T^{2,1} > 0$ we can reduce $c_T^{2,1}$ by ϵ and increase $c_T^{2,2}$ by ϵ and produce a utility gain of:

$$\epsilon(\phi_\omega^\omega(2) - \phi(2) + \phi_\omega^\omega(2)) > 0$$

when $\theta_\omega^\omega > 0$. With this knowledge, we can write $\partial V / \partial \theta_\omega^\omega < 0$ as

$$u(c_1) > u(c_2) + \beta c_T^{2,2} \Rightarrow c_1 > c_2$$

If $c_1 > c_2$ and $Z < c^*$ it follows from the agents problem that $c_T^{2,2} = 0$ because they do not save any resources for period T if the resources can be used earlier. Therefore we only need to consider the agents problem for $c_1, c_2, c^{1,no} > 0$ under liquidity shortages.

When there is insufficient aggregate liquidity, and with $c_T^{2,2} = 0$ and $c_T^{2,1} = 0$ the resource constraints from (9) become:

$$\begin{aligned} c_1 + c_2 &\leq 2Z \Rightarrow c_2 = 2Z - c_1, \text{ and} \\ c_1 + c_T^{1,no} &\leq 2Z \Rightarrow c_T^{1,no} = 2Z - c_1 \end{aligned}$$

which hold with equality when the resource constraints are binding.

From equation (10) and substituting in the binding constraint for $c_T^{1,no}$ we get:

$$\begin{aligned} V(C, \theta_\omega^\omega) &= \max_C \min_{\theta_\omega^\omega \in \Theta} \left[\left(\frac{\phi(1)}{2} - \theta_\omega^\omega \right) u(c_1) + \overbrace{\beta c_T^{2,1}}^0 \left(\frac{\theta(2)}{2} - \theta_\omega^\omega \right) \right. \\ &\quad \left. + \phi_\omega^\omega(2) \left(u(c_2) + \underbrace{\beta c_T^{2,2}}_0 \right) + \beta c_T^{1,no} \frac{\phi(1) - \phi(2)}{2} \right] \\ V(C, \theta_\omega^\omega) &= \max_C \min_{\theta_\omega^\omega \in \Theta} \left[\left(\frac{\phi(1)}{2} - \theta_\omega^\omega \right) u(c_1) + \left(\theta_\omega^\omega + \frac{\phi(2)}{2} \right) (u(c_2)) \right. \\ &\quad \left. + \beta(2Z - c_1) \frac{\phi(1) - \phi(2)}{2} \right] \end{aligned} \tag{12}$$

First order conditions:

$$\begin{aligned} \frac{\partial V}{\partial c_1} &: \left(\frac{\phi(1)}{2} - \theta_\omega^\omega \right) u'(c_1) - \beta \left(\frac{\phi(1) - \phi(2)}{2} \right) = 1 \\ \frac{\partial V}{\partial c_2} &: \left(\theta_\omega^\omega + \frac{\phi(2)}{2} \right) u'(c_2) = 1 \end{aligned}$$

Combining the first order conditions and where $K = \theta_\omega^\omega$, and with K as the agents

uncertainty, which is similar to assuming a worst-case probability distribution:

$$\left(\frac{\phi(1)}{2} - K\right) u'(c_1) = \left(\frac{\phi(2)}{2} + K\right) u'(c_2) + \beta \left(\frac{\phi(1) - \phi(2)}{2}\right)$$

For small values of K , a solution exists in which the agent chooses $c_1 > c_2$, and K based on the current uncertainty is $\theta_\omega^\omega = +K$. As K becomes larger c_1/c_2 decreases and at some point $c_1 = c_2 = Z$. This occurs at the point \bar{K} , Which is defined by Caballero and Krishnamurthy (2008) as the point where agents are fully insured against their uncertainty (K). \bar{K} solves:

$$\left(\frac{\phi(1)}{2} - \bar{K}\right) u'(Z) = \left(\frac{\phi(2)}{2} + \bar{K}\right) u'(Z) + \beta \left(\frac{\phi(1) - \phi(2)}{2}\right) \quad (13)$$

isolating for \bar{K} :

$$\bar{K} = \frac{\phi(1) - \phi(2)}{4} \frac{u'(Z) - \beta}{u'(Z)} \quad (14)$$

If $K > \bar{K}$ the solution $\theta_\omega^\omega = +\bar{K}$ and $c_1 = c_2$ solves the optimization problem. The agent's choice is uniquely optimal at $\theta_\omega^\omega = +\bar{K}$. The equilibrium in the robust economy under insufficient aggregate liquidity depends on both K and Z as follows:

- i) $0 \leq K \leq \bar{K}$ and in the solution $c_2 < z < c_1 < c^*$ with $c_1(K)$ decreasing and $c_2(K)$ increasing. Agents are not fully insured against uncertainty.
- ii) $K \geq \bar{K}$ and agents decisions satisfy $c_1 = c_2 = Z < c^*$. Agents are fully insured against uncertainty.

Caballero and Krishnamurthy (2008) refer to the first case as partially robust and the second case as fully robust. In the partially robust cast agents are not fully insured and are also over-insuring for the less likely second-wave shock. In the fully robust case agents are insulated against their uncertainty over whether their shocks are likely to be first or second, $c_1 = c_2 = Z < c^*$ but consumption is not optimal because agents are over-insuring for the less likely second-wave shock.

3.3 Capital requirements under uncertainty

To show the impact of capital requirements when agents are faced with uncertainty our model assumes government has direct control over the level of agents' capital or endowment (Z) by imposing capital requirements on intermediaries. The government is interested in the impact of changing capital requirements on agents who use the services of regulated intermediaries to finance their projects.

In our model capital requirements are modeled as a government control variable, δ , which increases or decreases the availability of capital in the economy. From the perspective of agents, an increase in δ reduces the supply of capital and a decrease in δ increases the supply of capital.

There are three parts to the analysis of the impact of cyclical capital requirements on agents:

3.3.1 How changes in capital impact agents' value function or utility. This will provide an overall picture of the role of capital in the value function.

3.3.2 How the cutoff for full-insurance, \bar{K} , changes with a change in capital. If \bar{K} goes up more capital is required to reach full-insurance against uncertainty of being hit second in a two-wave shock. Recall that full-insurance is in fact over-insurance because a single-wave event is more likely than a two-wave event and full-insurance reflects an individual's bias for insuring against the less likely, more severe outcome.

3.3.3) How individual bias changes with a change in capital requirement.

3.3.1 How changes in capital impact the value function

To understand the impact of capital (Z) on the value function, equation (12), apply the envelope theorem by taking the partial derivative of the value function with respect to Z . The binding constraint $c_2 = 2Z - c_1$ also needs to be substituted into equation (12) to get:

$$\frac{\partial V}{\partial Z} = 2\theta_\omega^\omega + \phi(2) + \beta(\phi(1) - \phi(2)) \quad (15)$$

which is equivalent to:

$$2\phi_{\omega}^{\omega}(2) + \beta(\phi(1) - \phi(2)) \quad (16)$$

where $\phi(1)$ is the probability the economy is hit by the first wave, $\phi(2)$ is the probability the economy is hit by the second wave and $\phi_{\omega}^{\omega}(2)$ is the perceived probability of being hit by the second wave. A change in the value function with respect to capital, Z , is positive because $\phi(1) > \phi(2)$, $1 > \beta > 0$ and $\phi_{\omega}^{\omega}(2) > 0$ which reflects uncertainty. This occurs regardless of the level of bias (θ_{ω}^{ω} or K) as long as $\phi(1) > \phi(2)$ which is assumed in the model developed by Caballero and Krishnamurthy (2008) and in our model. This is not unexpected, utility increases the greater capital available and decreases as capital falls all else equal, including uncertainty. This means that agents utility is lower the higher the capital requirement all else equal including uncertainty and in the absense of uncertainty intermediaries should not hold capital buffers. What is also evident is that Z has a larger impact on the value function the greater value of $\phi_{\omega}^{\omega}(2)$, or the perceived probability of being hit second in a two-wave shock. Therefore changes in capital have a greater impact on utility the greater the level of uncertainty.

3.3.2 How the cutoff for full-insurance changes with a change in capital

From equation (14) the effect of changing capital, Z , through adjustments in capital requirements on full-insurance against uncertainty, \bar{K} , can be shown. Rearranging equation (14):

$$\bar{K} = \frac{\phi(1) - \phi(2)}{4} \left(1 - \frac{\beta}{u'(Z)} \right)$$

and taking the partial derivative with respect to capital (Z):

$$\frac{\partial \bar{K}}{\partial Z} = \underbrace{\frac{\phi(1) - \phi(2)}{4}}_{>0} \underbrace{\frac{\beta u''(Z)}{u'(Z)^2}}_{<0} \quad (17)$$

Overall \bar{K} decreases in Z . The first term in (17) is positive because $\phi(1) > \phi(2)$. The second term is negative because utility is concave in consumption and therefore increases

but at a slower rate the greater is Z . In the robust full-insurance scenario with insufficient liquidity the cutoff for values of K such that $K > \bar{K}$ is lower the greater is the level of capital, Z . Recall that in this scenario $c_1 = c_2 = Z < c^*$. Equation (17) implies the larger the level of capital (Z) the more likely an agent will be fully-insured against uncertainty. It also implies that if individuals are already fully insured and the capital available increases they will require less insurance to be fully insured.

When capital requirements are tightened (or intermediaries are required to hold more capital) financial intermediaries decrease lending which restricts capital access by firms. In this case there is a higher value for \bar{K} , which is the cutoff value for full-insurance against uncertainty. This occurs because if an agent is hit by the second wave there is less capital available to fund consumption.

When capital requirements are relaxed (or decreased), financial intermediaries increase capital access to firms. If firms are already fully insured, then the outcome is better because less capital is required to reach full-insurance \bar{K} by agents. If individuals were close to being fully insured before a decrease in capital requirements, they will now they will be fully insured. Overall what this shows is that the tighter the capital requirement the more over-insurance by agents all else equal.

3.3.3 Impact of capital requirements on bias

Uncertainty averse individuals explicitly try to avoid the worst outcome under uncertainty. The worst outcome can be perceived as occurring less often assuming the government forecast are better than a coin toss. The downside is that the worst case outcomes will have either the same severity or there can be an increase in severity. This can be reflected in our model by a change in the previous probability structure which is illustrated in figure 1. In particular $\phi(2|1)$, or the probability of a two-wave event occurring given a first wave shock has occurred will decrease. For example the probability of being hit second in a two-wave shock could change from its current level of $\phi(2)/2$ to $\phi(2)/4$. This could result in an aggregate benefit if agents behaved according to Savage rationality because on average there are less occurrences of flight-to-quality. In contrast, what matters in an uncertainty model is the change in agents' perceptions of worst-case outcomes. If agents are unable to form

a probability structure with a reduction in the worst-case scenario, or uncertainty has not decreased, then they over-insure to the same extent or increase their over-insurance.

Recall that in the standard problem set-up from equation (7) the minimization term is: $\min_{\theta^\omega \in \Theta}, \Theta[\theta_{\omega_1}^\omega, \theta_{\omega_2}^\omega, \dots, \theta_{\omega_n}^\omega]$ which is a minimization over the set of all biases or how much agents true probability assessment of being hit second is higher than the average. Each element $\theta_{\omega_i}^\omega$ in the set Θ consists of two measures: 1) The mean, which is known $\phi(1)/2$ and $\phi(2)/2$, and 2) the agents bias, or the difference between the agents perceived probability of being hit second compared to the true average probability of being hit second, $K = \theta_\omega^\omega = \phi_\omega^\omega(2) - \phi(2)/2$.

How the bias changes with a change in capital requirements is important to the analysis. The government control variable, capital requirements (δ), can be positive or negative. These requirements have a direct effect on agents capital i.e. if the requirement increases intermediaries decrease their lending and therefore agents have less access to capital. Agents believe that capital requirements will correctly fluctuate with their businesses mark-to-market values most of the time but agents also realize that sometimes predictions will be wrong, which can effect their bias. Our model also assumes mark-to-market values are correlated with GDP. On average, capital requirements are low during downturns and high during up-turns which would be the appropriate policy for encouraging investment in positive NPV projects and expected default costs if agents behaved according to Savage rationality and/or the government was never wrong with its predictions.

To explore the impact of the government varying capital requirements, δ , on agents bias, consider Z from our model as the benchmark level of capital based on flat capital requirements of 8% similar to Basel II requirements. The level of which is determined by an IRB approach based on value-at-risk measured by mark-to-market. The capital requirements in our mode are assumed to be countercyclical and fluctuate with changes in GDP projections and therefore can be lower or higher than the benchmark. I will further assume that projections are updated quarterly and capital requirements are adjusted with these new projections. According to Allen (2004) in a survey of cyclicity in catastrophic risk of financial institutions there is substantial anecdotal evidence to suggest that macroeconomic conditions impact the probability of default (PD). Allen refers to studies by Mulder (2000)

and Reisen (2000) who find evidence that ratings agencies behave cyclically, particularly with respect to setting credit ratings for sovereign country debt. According to Allen (2004) evidence suggests that systematic factors affect loss given default (LGD) as well as probability of default. Allen also refers to Crouhy, Galai and Mark (2000, 2001) who find that the most speculative risk classifications' default probabilities are most sensitive to shifts in macroeconomic conditions. The findings of Allen et. al. suggest that banks tighten standards preceding a forecasted downturn which reflects the cyclical nature of banking and as mentioned regulation should be implemented counter to the banks reaction which supports the use of countercyclical capital requirements.

Define Z^A as the level of capital which can be controlled by cyclical requirements. Capital can be set to the benchmark level with flat capital requirements of 8%. The capital available to agents including capital requirements imposed on intermediaries would be Z^A is $1 - .08 = 0.92$. Define Z^H as the level of capital when government sets low capital requirements and therefore endowment level is high, and define Z^L as the level of capital when government sets high capital requirements and therefore endowment level is low. Since mark-to-market valued are based on GDP productions, if GDP is expected to rise by 5% in the next period predicted GDP would be 1.05 and the new level of capital after regulation would be 0.97. If countercyclical capital requirement were implemented $Z^A = 1.05 - .08(1.05) = 0.966$ moderating the business cycle. If it was a procyclical capital requirement and GDP was expected to rise by 5% GDP after the capital requirement would be $Z^A = 1.05 - 0.08(0.95) = 0.974$ amplifying the business cycle. This demonstrates why our model assumes government uses countercyclical capital requirements.

When a negative shock hits and the level of capital held by agents is Z^H they have sufficient capital to meet their business needs. With countercyclical capital requirements there is a range of new outcomes which are better than without cyclical capital requirements: all severe shocks which occur when capital is $> Z$ and $< Z^H$. If holding Z^H , or a low capital requirement, was costless during a flight-to-quality, capital would be set at this level all the time. The problem with Z^H is that it is costly with a balance between financing positive NPV projects and the possibility of a severe liquidity shortage and its resulting decline in economic output. Therefore it is not in the interest of the planner or government to regulate

capital to the level Z^H or Z^L all the time. The second problem is that it is not possible to perfectly predict the periods of time when Z^H should be implemented.

When the government assumes agents behave according to Savage rationality any action to improve the average economic output when balancing NPV projects with the cost of liquidity shortages should be taken. With uncertainty averse preferences the agent assumes the worst-case over the distributions for which they are uncertain. Unless the government is able to eliminate the most severe liquidity shortages or averse outcome, even if the mean outcome is better, the agents will still over insure and may in fact increase their over insurance for being hit second in a two wave event.

Following the full-insurance outcome under uncertainty, as defined by equation (14) the equilibrium occurred where $c^* > z = c_1 = c_2$ for all $K > \bar{K}$. With a fluctuating Z that is not completely predictable the full-insurance outcome under uncertainty will be $c^* > Z^H > Z^L = c_1 = c_2$. Agents plan for insurance purposes as if their capital is at Z^L . Agents know that the average outcome is better with cyclical requirements but in the new worst-case if they are hit second in a two-wave shock and capital is Z^L the level of capital is lower than when there is non-fluctuating capital requirements and they will be worse off if they are not insured. This outcome results in a greater individual bias even if the overall outcome achieved in aggregate appears to be better. This outcome is in stark contrast to one which assumes individuals are expected utility maximizers. If individuals were expected utility maximizers they would consider the mean outcome with some weighting on the variance of outcome.

As previously defined an agent's conservative probabilities of receiving shocks in the first and second wave are $\phi_\omega^\omega(1)$ and $\phi_\omega^\omega(2)$. Where $\phi_\omega^\omega(1) = \phi(1)/2 - K$ and $\phi_\omega^\omega(2) = \phi(2)/2 + K$. Agents also equate marginal utility of early consumption with date T consumption which is B given the linear utility over C_T , therefore $u'(c^*) = B$. Using these fact and equation (13) in the fully robust outcome where agents fully insure against uncertainty:

$$\phi_\omega^\omega(1) - \phi_\omega^\omega(2) = \beta \left(\frac{\phi(1) - \phi(2)}{2} \right) \frac{1}{u'(Z^A)}$$

$$\phi_{\omega}^{\omega}(1) - \phi_{\omega}^{\omega}(2) = \left(\frac{\phi(1) - \phi(2)}{2} \right) \frac{u'(c^*)}{u'(Z^A)} \quad (18)$$

and in the fully robust outcome $c^* > Z^A = c_1 = c_2$ which reflects full-insurance and their bias.

Agents place too high a probability on the second wave because each agent biases upward the probability of receiving a shock later than the average. For values of Z^H agents conservative probability assignment for $\phi_{\omega}^{\omega}(1)$ are relatively high and $\phi_{\omega}^{\omega}(2)$ are relatively low. The capital level Z^H is closer to the optimal outcome than Z since the true probability of an agent being hit by a one-wave shock is higher than being hit second. This can be seen because $\phi(1) - \phi(2)$ increases and $u'(c^*)/u'(Z)$ increases because both $u'(c^*)$ and $u'(Z)$ decrease but for any level of Z where there is an aggregate liquidity shortage $u'(c^*) > u'(Z)$ but by a smaller amount since $u''(Z) < 0$ and $u''(c) < 0$. Therefore $\phi_{\omega}^{\omega}(1)$ increases in Z and $\phi_{\omega}^{\omega}(2)$ decreases in Z which is a reduction in the individual bias. Eventually $Z = c^*$ and the individual bias is completely eliminated. For Z^L the opposite situation holds.

The government will forecast either a recession or a boom for the next quarter. Government capital requirements are countercyclical in our model based on the recommendations of Tobias and Brunnermeier (2009), Allen (2004), Dewatripont (2009) and Drumond (2009), i.e. government loosens restrictions in a recession and tightens them in a boom. The government forecast is based on an increase or decrease in predicted GDP. As previously mentioned our model assumes mark-to-market values for regulated intermediaries which correspond with changes in GDP and so do capital requirements.

Forecast of a recession: The government will follow a recession forecast with lower capital requirements (δ) which increases the supply of capital to firms (Z^H). Less uncertainty (K) is required to reach full insurance as shown by equation (17). Individuals prepare for the worst-case and therefore less over-insurance is needed because the cutoff \bar{K} is lower.

Forecast of a boom: The government will follow a boom forecast with a tightening of capital restrictions (δ) which decreases the supply of capital to firms (Z^L). If the forecast was correct it will be more expensive for firms to access capital but there will be no shortage of capital. If the forecasted boom was wrong the capital shortage will either be the same or worse for firms. There is a higher cutoff value of \bar{K} and therefore individuals would require

more insurance to reach the full-insurance outcome.

Individuals with a maximin utility function will focus on the uncertainty surrounding the forecast of booms when they are incorrect. Since individuals prepare for the worst and inaccurate prediction of booms increases uncertainty agents will at best maintain the same level of insurance but more likely increase their over-insurance. The uncertainty surrounding predicting GDP and its relation to mark-to-market valuation of risk leads agents to perceive either the same variance in outcomes compared to capital requirements which do not fluctuate regularly or a greater variance. Agents act as if Z is at the level Z^L , which is the same as assuming that government will forecast a boom incorrectly. From equation (18):

$$[\phi_\omega^\omega(1) - \phi_\omega^\omega(2)]|_{Z^L} = (\phi(1) - \phi(2)) \frac{u'(c^*)}{u'(Z^L)} < [\phi_\omega^\omega(1) - \phi_\omega^\omega(2)]|_Z = (\phi(1) - \phi(2)) \frac{u'(c^*)}{u'(Z)}$$

where $Z^L < Z$ and $u'(Z^L) > u'(Z)$ because of declining marginal utility of capital.

$\phi_\omega^\omega(1) - \phi_\omega^\omega(2)$ evaluated at Z^L is less than $\phi_\omega^\omega(1) - \phi_\omega^\omega(2)$ evaluated at Z which is less than $\phi(1) - \phi(2)$. When there is uncertainty agents overweigh the chance of being hit second higher with cyclical capital requirements as long as there is the potential for a worse outcome.

4 Policy Implications

Any policy which does not decrease uncertainty or the perceived worst-case scenario will not have a positive effect based on maximin preferences. This result has broad policy implications as it is typically assumed that if policies lead to higher expected utility they should be implemented. There are many ways to extend the use of maximin utility functions. The first extension below is a paper by Easley and O'Hara (2009) who model uncertainty aversion and expected utility agents behaviour in investment decisions. The extensions which follow Easley and O'Hara (2009) reviews capital requirement policies proposed by Repullo in Dewatripont (2009, section 2.7), Tobias and Brunnermeier (2009) and Julie Dickson (2010).

This analysis considers their recommendations based on agents being uncertainty averse.

4.1 Uncertainty averse and expected utility agents

The Gilboa and Schmeidler (1989) model has itself been generalized to allow for the possibility that the decision maker is not so pessimistic as to select the act that maximizes the minimum expected utility. Easley and OHara (2009) refer to models developed by Ghirardata, Maccheroni and Marinacci (2004) and Klibanoff, Marinacci and Mukerji (2004) who provide alternative approaches to separating uncertainty and the decision maker's attitude towards uncertainty. In the real world agents differ in the amount of information they have when making decisions reflecting varying levels of uncertainty. Easley and O'Hara (2009) follow Gilboa and Schmeidler (1989) to illustrate a model which consists of standard expected utility maximizers and uncertainty adverse agents in an investment decision. The parameters determining the distribution of returns to risky assets are unknown. Both the uncertainty averse and expected utility maximizer are rational but uncertainty averse traders do not have enough information about asset markets to form a prior model of returns and their investment behaviour is greatly affected by the most pessimistic model. The idea of using two type of agents can be extended to the analysis of flight-to-quality and capital requirements. Caballero and Krishnamurthy's (2008) model and our extension would include both savage agents and uncertainty averse agents if Easley and O'Hara's (2009) approach was used.

Easley and OHara (2009) use both types of agents when modeling investor behaviour to show that uncertainty aversion can effect equilibrium risk premium through its influence on the participation of investors in financial markets. If the uncertainty adverse investor considers an unfavourable return distribution to be possible he may choose not to invest in the risky asset. An expected utility maximizer places a prior on the set of distributions and invests according to his predicted distribution of returns. He will choose to invest in the risk asset if its risk-return tradeoff is favourable where risk and return are evaluated using prior over distributions. An economy with some uncertainty averse investors may price assets as if the uncertainty averse investors did not exist, but the per capital risk premium in equilibrium is higher. They argue that changing the perceived minimum mean returns and

maximum variances can be accomplished by regulation. The overall idea that the minimum mean return and maximum variance plays a greater role than if agents are assumed to be expected utility maximizers follows the same logic as the model used in this analysis which follows Caballero and Krishnamurthy (2008).

4.2 GDP-growth multiplier

According to Rafael Repullo et. al in Dewatripont (2009, section 2.7) there is broad agreement that one has to index current capital requirements to take into account macroeconomic variables to avoid the destabilizing effects of prudential regulation. Repullo et. Al. in Dewatripont (2009) use data from Credit Register of the Bank of Spain to analyze the effect of alternative lending procedures that have been proposed to mitigate the procyclical effects of the Basel II requirements. Their empirical model provides an estimate of the point-in-time (PIT) probabilities of default of a commercial and industrial loans portfolio for Spanish banks. They can then compute the corresponding Basel II capital requirements per unit of loans and estimate the credit risk profile of the Spanish bank over the sample period using the metric of Basel II. They consider the effect of two procedures to mitigate the cyclical behaviour of these requirements, namely: i) smoothing the inputs of the Basel II formula, by using through-the-cycle (TTC) adjustments in the PDs, or (ii) smoothing the output by using an adjustment of the Basel II final capital requirements computed from the PIT PDs.

Their results show that the best procedure is to use a simple multiplier of the Basel II requirements that depends on the deviation of the rate of growth of the GDP with respect to its long-run average. The requirements would be increased in expansions by 7.2% for a one standard deviation change in GDP growth.

For the period 1986 to 2006 the model looks at how PIT capital requirements would have evolved in Spain had Basel II been in place together with the Spanish GDP growth rate. Both series are highly negatively correlated which suggests that GDP growth rates may be useful to correct the cyclicity of bank capital requirements. They conclude that the best approach to smooth capital requirements is to use a GDP growth multiplier adjustment of capital requirements with respect to its long-run average. To mitigate procyclicality of Basel II they look at through the cycle probabilities of default. This is equivalent to

using through-the-cycle ratings and associated probabilities of default in computing capital requirements which is used in our model. Based on our model with uncertainty averse agents through-the-cycle ratings should be based on the long-term to reduce cyclical variance. This was reflected in our model by the impact on bias. If cyclical requirements vary and there is the chance of a wrong prediction the worst-case outcome can lead to further over-insurance against unlikely events.

Capital requirements, cyclical or not, may not be the appropriate policy to deal with financial crises for extreme events. As shown in our model cyclical requirements are worse than stable requirements, which grow based on long-term ratings. In times of severe stress, such as flight-to-quality, long-term ratings will not lead to capital requirements which are sufficiently high and therefore a buffer or other form of liquidity injection is needed which is not based on capital requirements. Through-the-cycle rating periods need to be sufficiently long to fit with long-run growth of asset values as opposed to short-run market values in a model with uncertainty aversion.

4.3 CoVaR

Tobias and Brunnermeier (2009) recommend the use of CoVaR as a means to mitigate severe flight-to-quality instead of using capital requirements. CoVaR, is defined as the value at risk (VaR) of the financial system conditional on institutions being under distress. Tobias and Brunnermeier (2009) define an institution's marginal contribution to systemic risk as the difference between CoVaR and the financial system VaR. They quantify the extent to which characteristics such as leverage, size, and maturity mismatch predict systemic risk contribution. They argue for macroprudential regulation based on the degree to which such characteristics forecast systemic risk contribution and that VaR and CoVaR are only loosely related. Therefore institution level risk based capital regulations is not the correct way to deal with contagion.

Tobias and Brunnermeier argue that the least distortionary way to recover the fiscal cost of direct support would be by a backward-looking charge such as one based on historical balance sheet variables. This would define a fixed monetary amount that each institution would owe, to be paid over some specified period of time and subject to rules limiting the

impact on net earnings. The charge would be in the form of a Pigouvian tax.

Tobis and Brunnermeier (2009) say that “CoVar focuses on the tail distribution which typically shifts the mean downwards and increases the variance in an environment with heteroskedasticity”. Looking at the proposition from the perspective that agents are uncertainty averse, individuals will be subject to potentially worse outcomes since the variance in outcomes would increase. The backward-looking charge would be an appropriate way to deal with tail-risk event when individuals are risk averse. By having a charge to deal with contributions to contagion uncertainty averse agents may perceive the worst-case outcome as being not as severe and therefore may insure closer to optimal (increase insurance for the first, more likely, wave and decrease insurance for the second-wave) which was closer to the optimal outcome in our model.

Tobias and Brunnermeier (2009) go on to construct a countercyclical risk measure from unconditional and conditional measures of covariance. Unconditional marginal CoVar is constant over time whereas conditional CoVar evolves with state variables that model the evolution of tail risk over time. The state variables include the slope of the yield curve, aggregate credit spread, implied volatility from VIX (Chicago Board Options Exchange Volatility Index). Assuming agents are uncertainty averse and the planner can err, CoVar may not correctly evolve with the state variables all of the time and there would be the potential for worse tail events, although less frequently. This is the same problem illustrated in our model when highlighting the difference between stable and counter-cyclical capital requirements. Therefore unconditional marginal CoVar would be a better measure to use than conditional marginal CoVar.

4.4 Debt-Equity SWAP provision

Julie Dickson, the Superintendent of Financial Institutions (2010), recommends embedded contingent capital as a potential way to counteract moral hazard. The principle is that if a bank took on excessive risk the subordinated debt would be converted to common equity diluting the existing shareholders. This would lead to a real incentive for investors in bank bonds to monitor risky behaviour. A key aspect of the proposal is that governments would not guarantee any bank or provide emergency capital unless contingent capital had been

used. Dickson (2010) argues that the proposal would avoid the need for a systemic risk fund. The disadvantage of the proposition is that investors may steer away from this form of debt (Dewatripont, 2009). Based on maximin preferences used in our model investors would behave as Dewatripont suggests. If embedded contingent capital reduces the likelihood of liquidity shortage without increasing the severity of a rare severe crisis it may reduce the over-insurance by economic agents. If embedded contingent capital reduced the likelihood of liquidity shortages but increases the severity of a rare crisis it could lead to over-insurance of a second-wave shock based on our model.

4.5 Deposit insurance

Diamond and Dybvig (1983) illustrate how uncertainty is reduced by government deposit insurance. Deposit insurance provided by the government allows bank to offer contracts that dominate the best they can offer without insurance. Private insurance companies are constrained by its reserves in the scale of unconditional guarantees which it can offer whereas governments are backed by their taxing abilities and therefore do not need to maintain a large capital reserve to provide credible insurance. The key social benefit of government deposit insurance is that it allows the bank to follow a desirable asset liquidation policy which can be separated from the cash-flow constraint imposed directly by withdrawal. (Diamond and Dybvig, 1983)

Deposit insurance deals with the problem of behaviour where it originates (depositors). This policy is consistent with strengthening public confidence in the financial system and encouraging the financial system to finance viable economic prospects. Deposit insurance is an example of action which impacts the individual level decisions. Easley and O'Hara (2009) mention that deposit insurance plays an augmented role by inducing uncertainty averse investors to participate in the banking system. If the investor considers several models of how banks work and one of these models has bank runs occurring with high probability, uncertainty averse investors refuse to put money in the bank. If the government introduces a guarantee on deposits worst-case losses disappear and the uncertainty averse investor deposits money in the bank. This regulatory approach expands the resources available to the banking system and facilitates greater risk sharing in the economy.

5 Conclusion

The recent credit crisis of 2007/2008 clearly illustrated that financial crises happen. In fact, times of extreme uncertainty occur as often as once every thirty years (Barro, 2006). Maximin functions are a useful alternative approach from expected utility maximization to analyzing agent's behaviour to policy under uncertainty, such as during the 2007/2008 financial crisis. Knightian uncertainty as explained by Knight (1921) followed by the Ellsberg's Sure-thing principle (1961) and the axiomatic approach to uncertainty averse preferences of Gilboa and Schmeidler (1989) has been applied to recent research related to investor behaviour as in Easley and O'Hara (2009) and Routledge and Zin (2009). The application of uncertainty averse preferences by Caballero and Krishnamurthy (2008) is a new and relevant approach to collective risk management under Knightian uncertainty applicable to events such as the financial crisis of 2007/2008. The extension of Caballero and Krishnamurthy's model in this analysis shows that policies which do not reduce the worst-case outcome can have little or no effect. Specifically, our model shows that utility increases in the amount of capital available and changes in capital have a greater impact on utility the greater the level of uncertainty K . The cutoff level for full-insurance \bar{K} is lower the more capital available and when individuals fully-insure against uncertainty there is less capital wasted on insurance the more capital available. Inaccurate predictions of booms increases uncertainty and at best individuals will maintain the same level of insurance. More likely, the result will be an increase in over-insurance under cyclical capital requirements as opposed to a more stable level of capital requirements.

While policy setters are concerned with aggregates, individual agents are uncertain about the impact of shocks on their individual outcomes. Even when the aggregate outcome is known, each agent's conservative probabilities may be biased and are individually plausible. Although regulators are concerned with the aggregate, as in macroeconomics, the micro- drives the aggregate. Therefore the focus of policy should be to consider how it will address individual decisions and the confidence of agents.

When considering the purpose of regulation to encourage the financial system to finance viable economic prospects and strengthen public confidence in the financial system,

cyclical capital requirements can do more harm than good. The harm of cyclical capital requirements under uncertainty can be improved with some clarification on policy. It should be clear that under normal conditions institutions should consider normal fluctuations in capital needs and their corresponding market capital requirements. Regulators should also consider externalities when setting requirements beyond market levels. Under flight-to-quality an alternative approach to maintaining aggregate liquidity is required.

Do cyclical capital requirements encourage the financial system to finance an economy's viable economic prospects? And, do cyclical capital requirements strengthen public confidence in the financial system? No and No. Not under uncertainty. Agents choose to minimize potential losses during times when their preferences perceive uncertainty as high. Inaccurate predictions of booms increase uncertainty and at best individuals will maintain the same level of insurance as without cyclical capital requirements but more likely they will over insure.

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Appendix I: Main result from Gilboa and Schmeidler (1989)

Let X be a set and let Y be the set of distributions over X . Let S be a set and let Σ be an algebra of subsets of S . Both sets, X and S are assumed to be non-empty. Denote L_0 the set of all Σ -measurable finite step functions from S to Y and L_c the constant functions in L_0 . Let L be a convex subset of Y^S which includes L_c . Y can be considered a subset of some linear space and Y^S can be considered a subspace of the linear space of all functions from S to the first linear space. It should be stressed that convex combinations in Y^S are performed pointwise. I.e., for f and g in Y^S and α in $[0, 1]$, $\alpha f + (1 - \alpha)g = h$ where $h(s) = \alpha f(s) + (1 - \alpha)g(s)$ for $s \in S$. In bayesian theory elements of X are deterministic outcomes, elements of Y are random outcomes, elements of L are acts, elements of S are states and elements of Σ are events.

The usual assumptions on preference relations are transitivity, completeness, continuity and monotonicity. Gilboa and Schmeidler (1989) add uncertainty aversion and certainty-independence. Certainty-independence is a new one and a weakening of the independence axiom. Gilboa and Schmeidler's (1989) axioms are:

A.1 Weak order. (a) For all f and g in L : $f \geq g$ or $g \geq f$, (b) For all f, g and h in L : If $f \geq g$ and $g \geq h$ then $f \geq h$.

A.2 Certainty-Independence. For all f, g in L and h in L_c and for all α in $[0, 1]$: $f > g$ iff $\alpha f + (1 - \alpha)h > \alpha g + (1 - \alpha)h$

A.3 Continuity. For all f, g and h in L : if $f > g$ and $g > h$ then there are α and β in $[0, 1]$ such that $\alpha f + (1 - \alpha)h > g$ and $g > \beta f + (1 - \beta)h$.

A.4 Monotonicity. For all f and g in L : if $f(s) \geq g(s)$ on s then $f \geq g$.

A.5 Uncertainty Aversion. For all f and g in L and $\alpha \in [0, 1]$: $f \succ g$ implies $\alpha f + (1 - \alpha)g \geq f$

A.6 Non-degeneracy. Not for all f and g in L , $f \geq g$.

In contrast to the certainty-independence axiom in Gilboa and Schmeidler, the standard independence axiom would be: if $f > g$ $\alpha f + (1 - \alpha)h > \alpha g + (1 - \alpha)h$ for all h and $\alpha \in [0, 1]$.