The Impacts of Population Ageing on Personal Savings in Canada:
Examining the impacts of declining fertility, rising life expectancy and the post-WWII baby boom

by

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An essay submitted to the Department of Economics
in partial fulfillment of the requirements for
the degree of Master of Arts

Queen's University
Kingston, Ontario, Canada
August 2010

Acknowledgements

First and foremost, I would like to thank Professor Huw Lloyd-Ellis for his guidance and support over these past few months. I am greatly appreciative for the countless hours he spent helping me to work through the difficulties I had when simulating my model. He encouraged me to stretch my limits and attempt analysis I originally thought was out of my reach.

I am grateful to Professor David K. Foot, Professor Emeritus of Economics at the University of Toronto for fostering my interest in demographics and its important relationship to economics.

I also want to thank my colleagues and friends for always being the silver lining during the challenging times as I wrote this essay and throughout the year.

I am indebted to my parents, Dharam and Vida Boodhoo for their love and guidance. They have always supported me in my academic pursuits and encouraged me when things were stressful.

All errors and omissions are my own.

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Abstract

Canada's population is ageing. Over the next few decades, the significant changes that will be occurring to the age structure of the population will be a significant component of our economic landscape. Over the next fifty years, the share of the population aged 65 and over is expected to more than double. In this paper, a 3-period Overlapping Generations model (OLG) is used to analyze the impact of population ageing on the aggregate private savings rate in Canada. The OLG model incorporates the life-cycle savings hypothesis as well as the general equilibrium effects due to exogenous changes. Simulations of the model consider 3 demographic shocks that are the cause of an ageing population: the post-WWII baby boom, the falling birth rate and finally an increase in life expectancy. Each shock is considered in turn and then combined to understand the full demographic transition occurring in Canada. The model predicts that the aggregate private savings rate will decline due to population ageing, reaching its lowest point around 2040. The stock of savings of the young and middle-aged differ in their responses, declining and rising respectively.

1 Introduction

Canada is in the midst of a historically unprecedented demographic transition. Not only is the rate of population growth declining, but the age structure of our population is changing. The median age of the population, as well as the fraction of the elderly are climbing to levels never previously witnessed. Over the past 50 years, the fraction of the population aged 65 and over grew by 225%, while the proportion aged 19 and under grew by only 6%. Population ageing in Canada can be explained by a combination of demographic factors: the post-WWII baby boom, declining fertility rates and rising life expectancy. Demographic changes are unavoidable and thus there is mounting concern that these changes will have significant macroeconomic impacts and pose serious fiscal policy challenges.

It has become increasingly important to examine and discuss the impact of demographic change on macroeconomic variables over the past decade. Personal savings in Canada has been declining over the past two decades, in part, due to demographic changes. Considering that population ageing has not hit our economy “full force”, how will savings continue to be affected in the future? There have been many studies that look at the impacts of the change in certain demographic factors on the economy. From general equilibrium models to panel-data regressions, economists have attempted to predict what will happen to savings in the next few decades due to the ageing phenomenon. However, there have been very few studies that consider *all* the factors leading to population ageing. This study attempts to examine the full impact of population ageing by incorporating the 3 main variables that are its source. Specifically, I closely consider the impact of population ageing on the aggregate private savings rate in the economy. Simulations were performed using a general equilibrium model with an overlapping generations structure to analyze the impact of demographic changes occurring in the Canadian economy. In turn, the change to each demographic variable is considered and then finally all aspects are combined to assess the effects of all the contributors to the ageing population.

Section 2 of this paper discusses the current and projected demographic changes that are occurring in the Canadian population. Section 3 discusses the savings rate in Canada and the life cycle hypothesis. Section 4 summarizes the results of previous studies analyzing the impacts of population ageing. Section 5 describes the OLG model used in this study, whilst Section 6 explains the calibration and simulation of the model. The results of the simulations are then presented and discussed in Section 7, and section 8 concludes.

2 Ageing and Demographics in Canada

Canada's ageing population presents new challenges to the Canadian economy. It is important to understand the magnitude of this demographic shift and the triggers behind it in order to model these variables. The characteristics of the Canadian population have largely been shaped by its industrialized economy and experience with the post-WWII baby-boom.

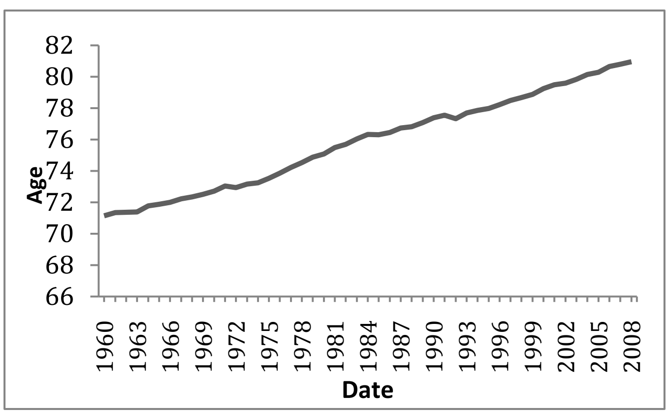
The Demographic transition describes the change in a population from one in which life expectancy is short and family size is large to one in which people live longer and family size is small. There are 4 generally accepted stages of the demographic transition. In stage one, the population is characterized by a high birth and death rate which results in a young population. During the next two stages the death rate falls, leading to large population growth. After this, the birth rate declines which causes the youth dependency ratio to fall and the average age of the population to rise. In stage four of the transition, there are both low birth and death rates which causes population growth to stagnate. In some cases the birth rate falls below the death rate and population growth begins to decline. The decline in the birth and death rates that occurs during these stages leads to dramatic changes in the age structure of the population. With these characteristics, the proportion of people ages 0-17 will continue to decline while the proportion of those aged 65 and over will swell. Canada is in stage four of this transition which means it is, for the first time, beginning to experience the effects of population aging.

2.1 Characteristics of Population Ageing[1]

Like many other industrialized counties, Canada is facing an ageing demographic due to certain drivers of ageing. These include an increase in life expectancy, a drop in the fertility rate and the post-WWII baby boom.

Over the last fifty years in Canada, life expectancy at birth has been steadily increasing (see Fig. 1.0). In 2008 the average life expectancy at birth for men and women was about 81 years of age, rising from 71 years in 1960.

Figure 1.0 Life Expectancy at Birth in Canada 1960-2008



Source: The World Bank Group, International Database

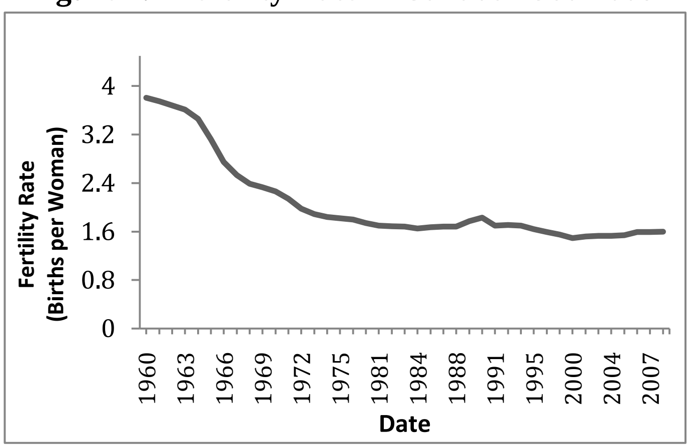
The average life expectancy measures the expected number of years a person will live at the time of their birth. An increase in longevity raises the average age of the population by changing the relative number of years that an individual is old to the number of years that they are young.

[1] Additional relevant graphs for this section can be found in Appendix 1

Assuming that the average age of retirement is approximately 65 years of age, as life expectancy increases, individuals will be spending a longer period of their lives in retirement. Another important measure is life expectancy at age 65, which measures how much longer an individual is expected to live at the age of 65. According to Statistics Canada, in 2005, life expectancy at age 65 in Canada was 19.8 years; predicting an individual will on average live until about the age of 85.. The increase in life expectancy over the past few decades has been a result of advancements in health care and increasing economic prosperity.

Unlike the trend with life expectancy, the fertility rate in Canada has been steadily declining over the past 50 years (see Fig. 1.1). The fertility rate measures the total births per woman if she was to live to the end of her child-bearing years. In 1960, the fertility rate was 3.81, it has since fallen to 1.6 in 2008.

Figure 1.1 Fertility Rate in Canada 1960-2008

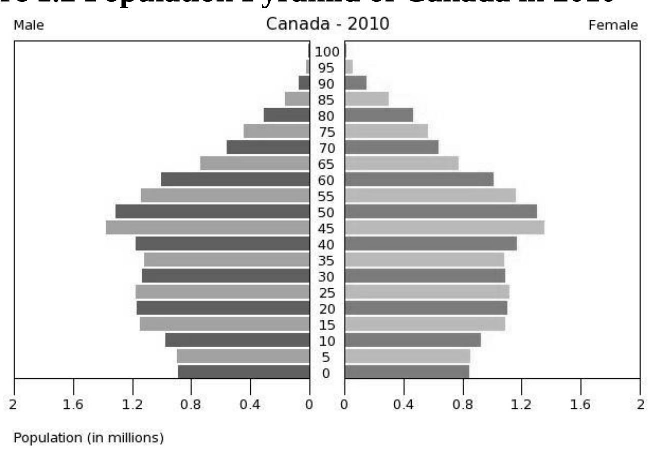


Source: The World Bank Group, International Database

The falling fertility rate causes population ageing since it changes the relative number of children born recently to the number born in the past. In order to maintain a constant population, a country must have a fertility rate of 2.1 births per woman. Like many developed nations, Canada's fertility rate is lower than this replacement rate, which means that without sufficient immigration Canada's total population will fall. The trend in the fertility rate is also mimicked by the crude birth rate, which measures the annual number of births per 1000 people. The crude birth rate has fallen from 26.7 to 11.25 in 2008. The fall in the fertility and birth rate is primarily due to improvements in contraceptives and an increased number of women in the workplace. As more people survive into their 60s and beyond, the absolute number of elderly will rise, combined with the fertility declines, this will cause the share of the elderly in the overall population to rise dramatically.

Canada's demographic history has also been characterized by dramatic changes in the birth rate. The population of Canada in 2009 was 33,739,859, with those aged 45-to-49 and 50-to-54 making up the largest proportion of the population. These age groups are unprecedentedly large due to the post-WWII baby-boom that resulted in a large number of births from1947-to-1966. In 2009, the boomers ranged from age 43 to 62. The largest single age group in Canada is those born in 1961 (Foot & Stoffman, 2004), over the next 2 decades all these boomers will be entering their retirement years.

Figure 1.2 Population Pyramid of Canada in 2010



Source: U.S. Census Bureau, International Database

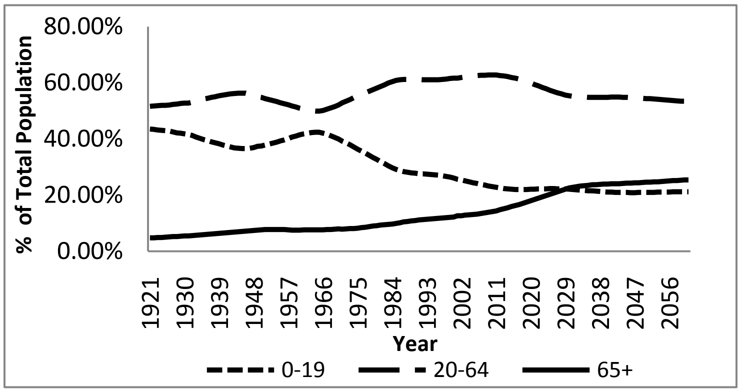
The relative size of this birth cohort can be clearly seen by examining population pyramids, which show a large bulge around ages 45-55 representing the boomers (see Fig. 1.2). The baby boomers will contribute significantly to the dramatic population aging that Canada will see in the near future. The baby boomers currently make up around 30% of the Canadian population. The baby-boom generation was followed by a baby-bust from 1967-to-1979 when the number of births declined due to falling fertility rates. Following the bust generation came the baby-boom echo from 1980-1995. The echo is now aged 14-29, and make up around 22% of the population in 2009.Population growth has steadied at about a 1% a year, with rising net migration rates helping to counter act the declining birth rate over the past ten years.

A consequence of population ageing is a change in the old-age dependency ratio. Over the past 50 years, the ratio of the old to the working population has been steadily rising. In 1960, the old-age dependency ratio was 12.72 % while in 2008 it rose to 19.56 %; a 53 percent increase. Interestingly, over the past 50 years, the total dependency ratio (young and old) has been consistently falling. This could be attributed to the fall in the fertility rate and the movement of the baby-boomers into the working stage of their lives.

2.2 Population Estimates and Projections

In order to understand the magnitude of the effects of a falling fertility rate, rising life expectancy and the baby boomers, it is important to look at population estimates and projections by age. Looking back to 1971, when the population of Canada was 21,924,082, the 60+ age group made up around 11% of the total population (see Fig. 1.3). Thirty years later, in 2001, the population has begun to shift; about 18% of the population was aged 60 and older.

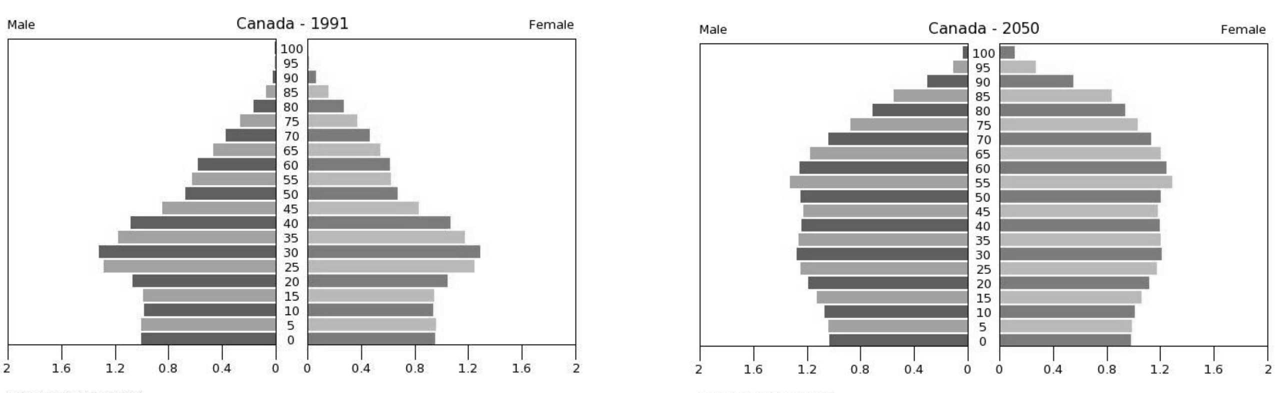
Figure 1.3 Proportion of Total Population by Age Group 1921-2060



Source: Statistics Canada, CANSIM database

Statistics Canada produces population projections based on three different growth scenarios: low, medium and high. The high growth scenario has the highest fertility and immigration rate and longest life expectancy for both males and females. The medium growth scenario chooses it underlying growth assumptions by looking at historical trends from 1981 to 2008. In 2031, the total population of the medium growth scenario is 42,100,300 with those aged 60 years and older making up around 29% of the total population. This 11% jump in this age group is due largely to the demographic effects above. In 2061, the 60+ age group is projected to make up just over 30% of the total population.

Figure 1.4 Canadian Population Pyramids 1991 & 2050



Source: U.S. Census Bureau, International Database

The ageing of the Canadian population can be seen by comparing the population pyramids of the past, present and future (see Fig. 1.4). The narrowing base and large bulge at the older age groups demonstrate the demographic transition that we are and will continue to experience. From 1960 to 2009 the 65+ age group grew 255.21% compared to the 19 and under age group which grew only 6.16% in the 50 year period. In the next 50 years, although the change is not as dramatic, it still demonstrated the population will continue to age. The proportion of the population aged 19 and under will grow by 40.76% and those aged 65 and over will grow by 175.01%.

Table 1 Growth Rates of Select Age Groups over 50 year periods

	Age Groups		
	0-19	**20-64**	**65+**
1921-1960	93.44%	100.63%	223.29%
1960-2010	6.16%	135.57%	255.21%
2010-2060	40.76%	30.16%	175.01%

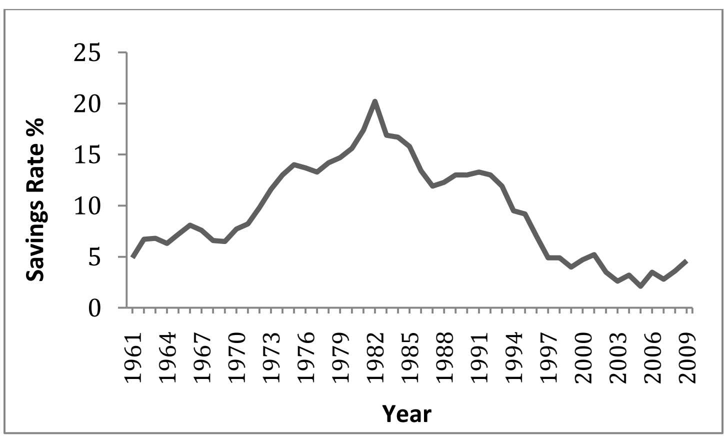
Source: Statistics Canada, CANSIM database

Demographic echo effects are an important consideration as the population continues to age. Demographic changes that have occurred in the past and that are occurring now will have a long-lasting impact on the Canadian population. Even if the fertility does rise in the future, it will only serve to slow the decline in the population. As the number of children born falls, the subsequent generation of children will also fall. Despite the fact that the children of the baby-boomers, the baby-boom echo, will help to lessen the impact of the population ageing, the trend in the birth and death rates are essentially irreversible. Population ageing will continue to perpetuate itself for some time.

3 Savings and the Life Cycle-Hypothesis

Significant changes to the savings rate in the future is an important consideration on a national and global level. Personal savings in Canada has been on the decline over the last three decades. Peaking in 1982 at 20.2%, it has since fallen to 4.6% in 2009(see Fig. 1.5). The decline in savings has been attributed to higher personal consumption and more mandatory transfers (Chawla, 2007). The close link between savings and investment and their relationship to economic growth is of special concern. Deaton (1997) summarizes that there is considerable econometric support to show a significant positive and robust relationship between savings and economic growth.

Figure 1.5 Personal Savings Rate in Canada 1961-2010



Source: Statistics Canada, CANSIM database

Modigliani (1986) argues that the positive relationship between growth and savings is a central prediction of his life-cycle model. Lower savings generally means a slower rate of capital formation which may impose a drag on the economy. A falling savings rate may have negative long-term impacts on government budgets, corporate investment and overall quality of life if the appropriate policy measures are not undertaken. The potential impact of an ageing demographic on savings is thus an important consideration to all stakeholders in the economy.

The relationship between the amount that a person saves and that person's age is one of the key questions in macroeconomics (Weil, 1994). The economic impact of the imminent changes to the age structure in Canada will be dependent on how savings varies with age. The Overlapping-Generations (OLG) model used in this study embodies the life cycles hypothesis of saving which was first developed by Ando and Modigliani (1963). In the OLG model, the savings rate is determined endogenously by the decisions made by the household. The model predicts that individuals should smooth consumption over their life time by holding their marginal utility constant. One of the main implications of the life-cycle framework is that households can separate their consumption profile from their income profile (Bérubé & Côté, 2000). Consumption smoothing occurs by varying savings over one's lifetime.

One version of the Life-Cycle model segments an individual's life into 3 stages; education, work and retirement. The model hypothesizes that borrowing occurs in the first stage, wealth accumulation during working life and dis-saving during retirement. (Browning & Crossley, 2001). The traditional 2-period OLG model however, only considers two periods; the working and retirement phase. It is normally assumed that children and thus, the education phase, is factored into the working life of the parent. As the OLG model is extended into more periods, there are simply additional periods in each stage of life which may or may not be identical. Thus, in order to achieve smooth consumption households must save during their work phases and then use these savings during their retirement phases. The basic motivation for savings in the life-cycle framework is to provide for consumption during retirement. The age distribution of the households affects personal savings because the savings rates of individuals are assumed to vary with their age. Based on the life cycle hypothesis, an ageing population will mean that more individuals will be in their retirement phase. On this conjecture alone we should see a larger proportion of the population dis-saving causing the amount of savings in the economy to fall. The OLG model factors in the general equilibrium effects of the demographic shocks which will lead to changes in the consumption and savings behaviour of individuals. Savings behaviour will change in accordance with expectations of future population growth and on the expected path of life-time earnings.

3.1 Literature Review: Age and Savings

Population ageing has renewed the controversy surround the life cycle-hypothesis. Considering the magnitude of the demographic change, economists have become more interested in the age-savings profile. To better understand the effect that ageing will have on the savings rate, many studies have looked at the validity of the life-cycle hypothesis of saving.

Using the 1990 Canadian Family Expenditure Survey (FAMEX), Burbridge and Davies (1994) show that the age pattern of the savings rate is quite different from what would be predicted in the life-cycle model. The median savings rate is around 0 at age 29 and younger and it then increases until it peaks at ages 55-59 at 0.11. Thereafter, it declines to 0.06 during 70-74 and then rises again to 0.08 at age 75 and above. The prediction of dis-saving in retirement does not apply to Canadian retirees, based on this survey. This savings behaviour could indicate that population ageing will actually cause a rise in national savings.

Given this discrepancy between household data and life-cycle models, David Weil (1994) analyzes both micro and macro data. Using American micro data he finds that the pattern of savings is partially in accordance with the prediction of the life cycle model. He finds that savings is indeed the lowest for the young and the elderly; however he finds no evidence of dis-saving by the old. However with macro data, he does finds that moving one percent of the working population to the retired population does indeed reduce savings. Unlike micro results, the conclusions of the macro data are in accordance with the life-cycle hypothesis. To reconcile the discrepancy between the two types of data, Weil adds intergenerational dependencies and finds that the expectation of bequests may cause the young to save less and compassion for the young may cause the elderly to save more.

Econometric analysis has also been used to test the main conclusions of the life-cycle model. Using panel cointegration and integration tests, Sarantis and Stewart (2001), characterize a long run equilibrium savings function for various OECD countries. The savings function that they estimated included dependency and retirement ratios, for which Canada had a significant positive and negative coefficient respectively. Their results provide strong evidence for the existence of a long-run equilibrium savings function that is based on an extended version of the life-cycle hypothesis.

Thomas Lindh argues that economic behaviour "changes profoundly over the life-cycle". His panel regressions of the OECD countries from 1950-1990 show the patterns expected by the life-cycle model. It is found that although direct age structure effects only explain a small proportion of the variation in savings, there is a delayed feedback from age effects on growth (Lindh, 1991).

Both Leff (1969) and Masson et al.(1998) found that a statistically significant negative relationship between dependency ratios and savings rate. This conclusion held over all types of regression based estimation techniques used in the studies. The negative effects of the old-age and youth dependency ratios on the savings rate indicates that any increase in the young and old age groups would causes savings to fall. This relationship demonstrates the validity of the life-cycle hypothesis.

Despite the support for the life-cycle hypothesis of savings, there have been some studies that dispute its claims. In a study of international savings rates, one of the principle findings was that there was very little evidence supporting the life-cycle model among various countries. Using household level data of several OECD countries, a National Bureau of Economic Research study examines the correlation between age-specific personal savings rates and the life-cycle hypothesis (Poterba, 1994). It was found that international demographic differences did little to explain the difference in the savings rates. Of the 6 OECD countries considered, savings after retirement is positive and in most cases higher than the savings rate during ages 30-40. These savings patterns are quite contrary to those predicted by the life-cycle hypothesis and even suggest savings may rise dramatically in the future.

In an attempt to explain the decline in the savings rate in the United States in the 1980s, Bosworth et al. (1991), consider the correlation of demographic factors with this phenomenon. They concluded that the predictions of the life-cycle model could not help to explain the decline in savings. The decline was largely due to a fall in savings among all age groups, rather than the age distribution of households. These results also stood when they considered Canadian FAMEX data; the change in the age structure played a very minor role in accounting for the change in aggregate savings over time.

4 Literature Review: Population Ageing and the Savings Rate

With the debate about the life-cycle hypothesis of savings still undecided, many economists have taken to looking at the predicative power of demographic change on future macroeconomic variables. There is a general consensus on how the population ageing phenomenon will affect the savings rate in the future. Population ageing will cause the savings rate to fall in the next forty years. Economists have evaluated the impact of population ageing on savings through general equilibrium simulations and partial equilibrium analysis. This section proceeds by examining the conclusions from each type of analysis.

General equilibrium analysis allows one to examine the impact of population ageing on the savings rate taking into account all aspects of the economy that are affected by a change in demographic variables. Models used in these studies take into account behavioural changes that result in response to an exogenous shock. Auerbach and Kotlikoff (1987) pioneered the use of overlapping generations models (OLG), a type of general equilibrium life cycle model, in determining how differing policy measures affect macroeconomic variables. The foundations of their model have been emulated by many economists who aim to simulate the impact of demographic change on the economy.

In their original model, Auerbach and Kotlikoff simulate the effects of demographic change and find that demographic conditions are potentially significant determinants of economic performance and welfare. It is possible to analyze long-term general equilibrium effects in their model in which individuals live for 75 periods, 20 of them as dependent children whose consumption is determined by their parents. A key feature of the model is that the endogenous labour supply is determined by the wage, interest rate and by a lifetime productivity profile that decline with age. This characteristic leads to individuals to retirement and life cycle savings. Fertility change is introduced into their model by exogenously changing the number of births per adult. Auerbach and Kotlikoff consider the effects of a transition from the baby-boom to a baby bust, in which the birth rate falls from 3% annual population growth to 0. Over the transition, the savings rate falls in the initial stage due to an expected increase in after tax wages. Twenty years later the savings rate rises in response to the fall in child dependency and then by year 70 the savings rate falls as the proportion of retired to working-age adults increases.

Using an OLG model similar to Auerbach and Kotlikoff (1987) but that allows for technological progress and a Pay-as-you-go pension scheme; David Miles (1999) simulates the effects of population ageing on the European economy. His results are consistent with the above conclusions; ageing drives the savings rate down sharply. In his base scenario, the savings rate peaks at around 15% in 2008 and then falls to a low of 6% in 2040 before rising slightly to 7% in 2060. Fougère and Mérette (1999) also use an OLG model with a similar framework that incorporates endogenous growth generated by the accumulation of physical and human capital. In the baseline simulation of seven OECD countries including Canada, a fall in the birth rate causes the labour force to become relatively smaller and require less capital. This in turn reduces the need for private savings which causes the national savings rate to fall. The national savings rate in the Canadian simulation declines from 1954 and reaches its lowest point around 2040. With the addition of human capital, the reduction in the national savings rate deepens in the short run but is less pronounced in the long run compared with the baseline simulation.

Other types of general equilibrium models have also been used to examine the effects of population ageing on the savings rate. Auerbach and Kotlikoff (1990) also use three different savings models to examine the impact of the changes in U.S. demographics on savings. The three models include the infinite horizon altruism model, the life-cycle model and a reduced form model intended to capture the effects not present in the other two models. The effect of population ageing on the savings rate is accomplished by predicting the change in the age-consumption profiles and then incorporating this change into the models. All three of the models predict that savings will decline over the period from 1990 to 2040. The life-cycle and altruism model both predict that savings will peak in the year 2000 and then steadily decline thereafter. The reduced-from model however predicts that savings will fall consistently over the simulated time period. A negative savings rate is predicted by all three of the models by around 2030.

Federal Reserve Bank economist Peter Yoo (1994) examines the economic effects of the baby boom, one of the causes of population ageing, by analyzing the results of three different general equilibrium models. In addition to the OLG model, he examines and simulates the neoclassical growth model of Ramsey and the dependency ratio growth model. In all 3 simulations, it is found that the baby-boom caused a temporary increase in the savings rate from its steady state value. What differed was the timing of the economic effects of the baby boom. The Ramsey and Dependency-ratio growth models, both infinite horizon models, exhibit declining savings once the baby boom ends. The OLG model shows rising savings until the time that the first baby-boomers enter retirement.

Masson and Tyron (1990) use simulations of the International Monetary Fund's multiregion econometric model (MULTIMOD) to gauge the effects of demographic change on macroeconomic variables. Unlike other models, MULTIMOD allows one to capture the general equilibrium, multicounty effects of demographic change. Ageing is simulated using three channels of the model: a consumption equation that has an estimated positive coefficient on the dependency ratio, potential output which depends on the size of the labour force and government spending which rises as the population ages. The simulations are from 2000-2025. The results seem to be consistent with other general equilibrium studies; the private savings rate for industrial countries peaks around 2010 and then falls steadily until 2025.

All of the general equilibrium models agree, savings will be negatively impacted by an ageing population in the next forty years. The magnitude of the fall in savings and the expected timeline of changes, however, are still debatable.

Partial equilibrium analysis attempting to forecast the effect of demographic change on the savings rate begins with micro household-level data on savings rates and income levels by age. This type of analysis assumes that the age-savings profile will remain constant as the population age structure changes. An important caveat is that this type of analyses does not consider how the behaviour changes due to population ageing will affect the economy. There has been some disagreement about the effects of population ageing on savings in partial equilibrium studies.

Using the results of panel-data regressions from four different studies, Peter Heller (1989) simulates the effects of population ageing on the savings rate for the G-7 countries. Peter Heller used the coefficients from four cross-country econometric studies by Modigliani (1970), Modigliani and Sterling (1980), Feldstein (1980) and Horioka (1986) which estimate the impact of changes in youth and elderly dependency ratios on aggregate private savings to predict the savings rate from 1980-2025. Each of the specifications uses the ratio of aggregate private savings to private income as the dependent variable and use demographic and non-demographic explanatory variables. Modigliani's (1970) specification includes only 3 explanatory variables while Horoika (1986) involves the most. Using the forecasted change in the youth and elderly dependency rates, Heller finds that the savings rate is expected to fall drastically based on the conclusions of all the models from 2010-2025. Horioka's specification predicts the largest fall in the savings rate while the Modigliani and Sterling models predicts the change will be 3 times less.

In a more recent study by Matthew Higgins (1998), he finds that the results from his econometric study of the savings rate implies that expected changes to the dependency ratio will cause modest to substantial declines in the savings rates of the developed world. His study projects the savings rate in the U.S and U.K to fall around 1% and the savings rate in Germany and Japan to fall by more than 5% in the next fifteen years.

In addition to their general equilibrium studies, Auerbach and Kotlikoff (1990b) use partial equilibrium analysis to forecast the U.S. savings rate from 1950 to 2050. The U.S. is experiencing an ageing demographic due to the same factors as Canada. Under various fiscal policy assumptions, the savings rate is projected to rise and peak during 2000-2020.The savings rate declines thereafter but remains at a value higher than before the peak. The predicted pattern of saving rates reflects the ageing of the population; the savings rate peaks when the proportion of the population in its prime working age is the highest and then falls as proportion of those aged 65 and over swells. Unlike many of the other models, this study actually suggests that savings will increase significantly over the next 40 years.

The model used in this study is calibrated specifically to model the Canadian economy. Unlike other studies examining the impact of population ageing on Canada, two important demographic features are incorporated into this model, the birth rate and life expectancy. With these demographic variables and the incorporation of the baby boom, *all* factors contributing to population ageing in Canada are modelled. By incorporating all aspects of population ageing, it will be possible to better understand the magnitude of population ageing on the aggregate savings rate in Canada.

5 The Model

The advantages of using general equilibrium models in understanding the effects of demographic change have motivated the use of an overlapping generations model (OLG) to simulate the effects of population ageing.. There are two main reasons why OLG models are helpful in modelling the impact of demographic change (Graf & Schattenberg, 2006). The first is that OLG models enable demographic developments to be taken into account explicitly. At any one time in the model there are multiple generations living side by side and with each new period the generations advance in age. Secondly, OLG models focus on behavioural change and the interactions within generations. As demographic variables are altered, the model will show explicitly how all factors will adjust to that change. That is, it goes beyond pure life-cycle effects.

In order to ensure that the simulated model is tractable, several simplifications are made regarding the structure and assumptions about economic behaviour. Despite its limitations, the OLG model provides a rich description of the generational structure of the population by characterizing different age groups. In this simple finite horizon OLG model, there are 3 generations living side by side at each period of time. In each period the oldest generation dies and a new one enters the labour force. Each new generation has 3 periods to live, with each period corresponding to 24 years of life. The 3 generations in the model correspond to the young, middle-aged and old. In this model, it is assumed that children are supported by their parents and thus are not taken in to consideration explicitly. Each new (young) generation begins at age 18, the middle generation at age 42 and the old at age 66. The model is based on a closed economy with 3 sectors – households, production and the government which acts solely as the administrator of a pension scheme. In the following equations, subscript t represents the time period.

5.1 Household behaviour

There is a representative individual for each generation in the household sector. Each generation retires after 2 periods (48 years) and is assumed to be able to live up to the age of 90. Individuals maximize their intertemporal utility function subject to their lifetime budget constraint. Households are assumed to be rational, have perfect foresight and have no liquidity constraints. In accordance with the life-cycle hypothesis, they spread their consumption over their life-time by varying their savings pattern. Individual labour supply is fixed in this model, and thus agents do not have a labour-leisure decision. Utility is logarithmic and time separable:

$$U_t = \ln c_t^y + \left(\frac{1}{1+\rho}\right) \ln c_{t+1}^m + \left(\frac{1}{1+\rho}\right)^2 \varphi \ln c_{t+2}^o$$

where c^y, c^m and c^o are the consumption of the young, middle and old respectively. The rate of time preference,ρ, signifies the individuals preference for current over future consumption. A relatively new feature that is used in the model is,φ, the survival rate. This exogenous variable represents the probability that the individual survives into period 3[2]. Individuals maximize their utility subject to the following budget constraint:

$$c_t^y + \frac{c_{t+1}^m}{(1+r_{t+1})} + \frac{c_{t+2}^o}{(1+r_{t+1})(1+r_{t+2})} = (1-\tau_w)\left(w_t + \frac{w_{t+1}}{(1+r_{t+1})}\right) + bq_t + \frac{p_{t+2}}{(1+r_{t+1})(1+r_{t+2})}$$

[2] In the quantitative exercise below, I account for heterogeneity in survival by measuring this as the fraction of period 3 that the individual is alive.

where bq_t is the bequest given to the young generation by the old generation of the same time period. The tax rate, τ_w, is the tax on labour income that funds the pension, p_t received in retirement.

In this model, the assets of the representative individual in the young and the middle generations at time t are defined as:

$$a_t^y = ((1 - \tau_w)w_t + \ bq_t - c_t^y)$$

$$a_t^m = (1 + r_t)a_{t-1}^y + ((1 - \tau_w)w_t - c_t^m)$$

The total assets per capita at any given point in the economy are:

$$A_{t+1} = \frac{a_t^y}{(1+n)} + \frac{a_t^m}{(1+n)^2}$$

where the first term are the assets of the young and the second term are the assets of the middle-aged. The old generation is assumed to consume or bequeath all of their assets before death, $a^o = 0$, and thus are not considered in the above equation.

The bequest of a representative individual in this model is explicitly related to the survival rate by the following equation:

$$bq_t = \frac{(1 - \varphi)c_t^o}{(1+n)^2}$$

I assume that bequests are allocated equally to each member of the young generation. The bequest per young individual is equal to the amount of unconsumed savings of the old. Bequests are assumed not to enter the utility function explicitly, but similar to the old-age pension, appear in the intertemporal budget constraint of the representative individual. That is, all bequests are assumed to be accidental.

The old-age pension scheme is modeled similarly to the social security system in Auerbach and Kotlikoff (1987). The government in this model is limited to its function as the administrator of the pension. The pension system is fully funded by the tax on income. In this model the tax rate on labour income is held fixed, while the average pension payment,p_t, is allowed to vary according to the following equation:

$$p_t = \frac{\tau_w \times w_t\left((n+1)(n+2)\right)}{\varphi}$$

The average pension payment in the model is dependent on both the size of the working-age population whose income is taxed and the size of the retired age group.

Maximizing of household utility with respect to the consumption of the young, middle and old, subject to the intertemporal budget constraint yields the following first order conditions which describe the consumption relationship between each generation:

$$\frac{1}{c_t^y} = \frac{(1+r_{t+1})}{(1+\rho)c_{t+1}^m} \quad ; \quad \frac{1}{c_{t+1}^m} = \frac{(1+r_{t+1})\varphi}{(1+\rho)c_{t+2}^o}$$

The flow of savings of each generation at a particular point in time are characterized by the following equations

$$s_t^y = (1-\tau_w)w_t + bq_t - c_t^y$$

$$s_t^m = (1-\tau_w)w_t + ra_t^y - c_t^m$$

$$s_t^o = ra_{t-1}^m + p_t - \varphi c_t^o$$

In keeping with the convention of the savings ratio as defined in Graf & Schattenberg (2006), the key variable in this model, the aggregate private savings rate, is defined as

$$S_t = \frac{s_t^y(1+n)^2 + s_t^m(1+n) + s_t^o}{\left((1-\tau_w)w_t + bq_t\right)(1+n)^2 + \left((1-\tau_w)w_t + ra_{t-1}^y\right)(1+n) + ra_{t-1}^m + p_t}$$

This equation characterizes the flow of aggregate savings rate in the economy at a given period of time, rather than the stock of savings. The aggregate savings rate in the model is dependent on the chosen life-time consumption profile and the disposable income of each generation, paralleling the developments in factor returns.

5.2 The Production Sector

The production technology in this economy is given by a 2-factor Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}$$

and in per capita terms: $$y_t = k_t^{\alpha}$$

where K_t is the capital stock and H_t is the supply of labour at time t. The capital income share is denoted by α. The market is assumed to be perfectly competitive with output being a homogenous good. The aggregate labour supply is dependent on the size of the working-age population.

Factor demands are determined by profit maximization of the firms. Firms rent capital and hire labour until their marginal products are equal to their marginal costs:

$$r_t = \alpha k_{t-1}^{\alpha-1} - 1$$

$$w_t = (1-\alpha) k_{t-1}^{\alpha}$$

where r is the interest rate (net of depreciation) of physical capital and w is the wage. 100% depreciation is assumed in this economy due to the fact that the length of each period is assumed to be 24 years long.[3]

[3] With an annual depreciation rate of 10%, the depreciation rate over 24 years would be 93%.

5.3 Equilibrium Conditions

In order to ensure that resources are not wasted in the economy, two equilibrium conditions are necessary. First the available capital stock per person is equal to the total assets per capita such that:

$$A_t = k_t$$

Secondly, the final goods output equals total household consumption plus net investment:

$$Y_t = C_t^y + C_t^m + C_t^o + I_t$$

where $C_t^i = c_t^i L_t^i$ for $i = y, m, o$ and $I_t = K_{t+1}$.

6 Calibration and Simulation

In order to obtain the values of the endogenous variables as population ageing occurs, the OLG model is linearized around its steady state and simulated.[4] The calculation of the equilibrium path of the economy proceeds in 3 stages: solving for the initial long-run steady state of the model before exogenous shock to demographic variables, solving for the final steady-state to which the economy converges and then finally solving for the transition path between the two steady states. Various scenarios based on different demographic shocks are simulated in order to understand how each determinant of ageing affects the economy. The effects of both temporary and permanent shocks are analyzed. It is important to note that a temporary shock causes the economy to deviate from its steady state during the period in which it occurs and then returns to the original steady state. A permanent shock, however, changes the path of the economy once it occurs and also affects the final steady state. In the following simulations, both unanticipated and anticipated shocks are considered given the nature of the demographic shock considered.

[4] Simulations of the model were performed using DYNARE

Calibration of the model thus begins with choosing parameter values so that a particular set of initial steady state values are obtained. Some of the parameters in the model are chosen based on those used by Hviding and Mérette (1998), where the steady state values are similar to a set of Canadian macroeconomic variables for the 1950s/1960s. The three parameters in the model are the capital income share, the consumer time preference and the tax rate on labour income. The capital income share is equal to 0.345, which is similar to most industrial countries. The consumer time preference in Hviding and Mérette is 0.0047 reflecting a 4-year period; in order to accommodate the 24-year time periods used in this model, the compounded rate of 0.0285 is used. The tax rate on wage income was chosen so that the steady state average pension payment was around 50% of the wage, in this case 10%. The steady state values of the endogenous variables are detailed in Table 2. All of the simulations begin at these values. It is interesting to note the difference in the assets of the young and the middle-aged generation. The assets of the young are higher than the middle aged. This can be explained by the fact the middle-aged are expecting to receive a pension to finance consumption when they are old.

Table 2 Results of Calibration: Steady State Values

	apsr	**ay**	**am**	**cy**	**cm**	**co**
Steady State Values	0.1507	0.0979	0.0602	0.2531	0.3585	0.2640
	w	**r**	**k**	**y**	**bq**	**p**
	0.3067	0.4566	0.1109	0.4683	0.0750	0.1764

Population ageing is simulated in this model by way of two demographic features: the rate of population growth n, and the survival rate, φ. The rate of population growth is used to model the change in the birth rate, while the survival rate is a proxy for the change in life expectancy. By changing the values of these exogenous variables to mimic population ageing including the baby-boom, it is possible to understand the "pure" demographic effect on the savings rate. Changes in the population age structure which will change the steady-state level of capital per worker which will lead to dynamic adjustments in the level of capital, wages and interest rates along the path to a final steady state. The movement in these variables will subsequently impact generational and aggregate savings. The initial and subsequent values for the rate of population growth are based on the population estimates and projections from Statistics Canada's medium growth scenario. The values of the survival rate over the simulation periods are based on life-expectancy at age 65 according to Statistics Canada.

Four different scenarios are simulated to understand the different generational effects of population ageing on aggregate savings in the economy. The first scenario looks at a temporary increase in the rate of population growth, n, in order to understand the pure effect of the baby boom. Scenario 2 considers a permanent fall in the population growth rate to proxy the decline in the fertility rate. Scenario 3 involves a permanent increase in the survival rate, φ, in order to understand the impact of rising life expectancy. The final scenario involves a combination of all three demographic shocks to model the entire impact of population ageing on the Canadian economy.

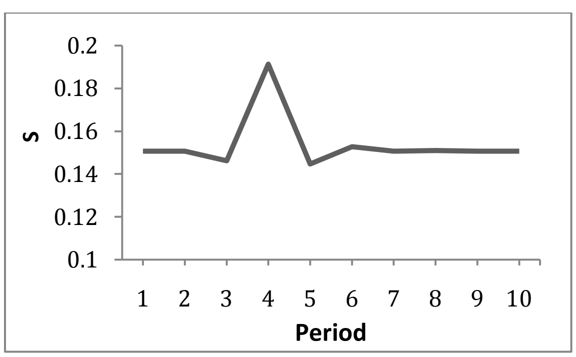
7 Results of Baseline Simulations

In order to understand the movements of the aggregate private savings rate in the economy, it is essential to analyze the impact of the mechanisms that cause these changes. When a demographic shock occurs in the economy, it translates into changes in present and future generations as the birth cohorts age. Thus, it will take some time for the economy to return to its steady state value. If households can anticipate demographic changes they can also anticipate future changes to their income and hence make changes accordingly. Movements in certain variables today may be dependent on the past and future adjustments of other variables. I adopt the convention that the values of the variables used in this model represent the beginning of the period. For instance, the wage and interest at time t are dependent on the capital stock chosen at time $t\text{-}1$. We proceed by examining the scenarios with only one shock with the aim of understanding the combination of the effects in the next section.

7.1 Scenario 1: A Temporary increase in *n*[5]

In this scenario, there is an increase in the rate of population growth that occurs for only one (24 year) period. This increase can be likened to the rise in the birth rate due to the post-WWII baby boom between 1944 and 1967. In the simulation the shock occurs in period 2 and the model was simulated for 10 periods in order for the economy to return to its steady state. The aggregate private savings rate declines slightly at first between period 2 and 3 and then rises by around 30% between periods 3-4. It then declines during period 4 before rising slightly to its original steady state value (see Fig. 2.0).

Figure 2.0 Scenario 1: Aggregate Private Savings Rate



[5] Additional relevant graphs for this scenario can be found in Appendix 2

The movement in the aggregate savings rate occurs due to changes in the variables as described in the equation detailed earlier. The initial decline in the savings rate occurs due to changes in the stock of savings of the young and middle-aged generations. The savings of the young generation (baby-boomers) rise in anticipation of the fall of the pension they will receive when in retirement two periods later. The pension of the baby-boomers will be smaller relative to previous generations for 2 reasons. In addition to the growth of retirees, the tax base will have shrunk proportionately as the rate of population growth reverts to its initial value. The stock of savings of the middle generation falls during period 2 and 3 in anticipation that the pension they will receive in the next period will be larger than previously expected. The baby-boomers, who will be middle-aged in the next period results in a larger tax base. The savings of the young rise by 6% while the stock of savings of the middle declines by 21%. Despite the rise in population growth, the decline in middle-aged assets outweighs the rise in the saving causing the aggregate savings rate to fall. The capital-labour ratio falls slightly in response to the changes in the assets of each generation.

Between period 3 and 4, the aggregate private savings rate rises drastically due to delayed effects of the baby-boom. The rate of population growth reverts back to its initial value and the baby-boomers are now middle-aged. The capital stock declines further in response to the rise in population growth in the previous period. The interest rate rises and the wage rate falls as a result of the decline in the capital stock in the previous period. The savings of the young falls in response to the rising interest rate in the next period; the agent does not need to save as much to ensure a certain level of future consumption. The bequest in this period also falls due to the decline in n, which reduces the available income to be saved. The stock of savings of the middle-aged rises in expectation of the declining pension payment in the next period, this thus decreases their consumption. The rise in the interest rate and the increase in savings when they were young also contribute to the rise in the stock of savings of the middle-aged. The decline of savings by the young is outweighed by the size of the rise in assets of the middle-aged resulting in the aggregate savings rate rising.

The aggregate private savings rate sharply declines between period 4 and 5 primarily due to the fact that the baby-boomers are now in retirement. The capital-labour ratio rises as a result of the overall rise in savings in the previous period. The interest rate and the wage rate continue their trend in the previous period, rising and falling respectively. The savings of the young continues to fall in expectation of the rise in the wage that will occur in the next period. The stock savings of the middle-aged declines as a result of the falling wage rate. The rise in the interest rate causes the consumption of the retired generation to increase. The movements in these variables complement one another and result in the drop of the aggregate private savings rate.

Subsequently, the economy begins to transition back to its steady state during period 5-6. The interest falls and the wage rate rises, increasing the saving of the young cohort. The assets of the middle-aged declines in response to the falling interest rate and the expectation their pension in the next period will be higher. After period 6, there are no major movements in the variables, as the demographic shock has made its way through each generation.

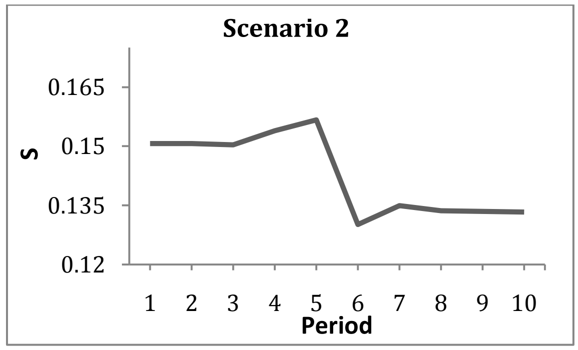
7.2 Scenario 2: A permanent decrease in *n*[6]

Similar to the decline in the birth rate that has occurred in Canada over the past few decades, this scenario considers a permanent decline in the rate of population growth. The decline in n occurs in period 4 and remains at the lower level as the economy transitions to a new

[6] Additional relevant graphs for this scenario can be found in Appendix 3

steady state. The shock is assumed to occur at period 4 in order to analyze the anticipatory behaviour starting from period 3 of the agents to the decline in n.[7]

Figure 3.0 Aggregate Private Savings Rate (Scenario 2)



The permanent change in this exogenous variable causes agents to adjust their behaviour accordingly. The aggregate private savings rate changes in accordance with the change in behaviour and then arrives at a lower steady state value (see Fig. 3.0). En route to the new steady state, there are some large changes during the transition period.

In expectation of the decline in n, the aggregate private savings rate climbs between period 3 and 4. The savings of the young increases in anticipation of the lower pension this cohort will receive when old. The effects of a falling tax-base that will occur due to a fall in n will not affect the middle-aged in to the same degree as it will the young during period 3. There is a slight decline in the savings of the middle-aged in response to the rise in the pension they will receive as the wage rate is expected to rise. The rise in the assets of the young outweighs the fall in the stock of savings of the middle-aged causing the aggregate private savings rate to rise.

[7] Since demographic movements over a couple of decades are fairly easy to forecast, it seems reasonable to assume that agents could anticipate their effects. In fact, these anticipatory effects are not that large in this scenario.

There are very slight changes to the other variables in anticipation of the decline in n, notably; the capital-labour ratio begins to rise.

Once the decline in n occurs in period 4, there are significant changes that occur in the variables. Between periods 4 and 5, the aggregate private savings rate continues upward, as the savings of the young declines and the savings of the middle generation rises. Due to the fall in n, the size of the labour force declines, causing the marginal product of labour to rise. This results in a fall of the interest rate and a rise in the wage rate. With an expectation of a high wage next period and a decline in the bequest they receive, the savings of the young declines. Middle-aged savings rise in anticipation of lower income next period when their pension falls. The pension paid in this period actually rises since the rise in the wage rate outweighs the proportionately smaller labour force.

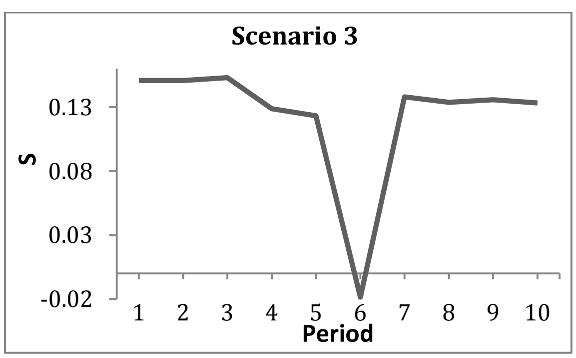
During periods 5 and 6 there is a large decline in the aggregate private savings rate as the savings of both the young and middle-aged fall. The savings rate reaches its lowest point in period 6. The capital-labour ratio continues to rise causing a further decline in the interest rate and a rise in the wage rate. This translates in to a lower stock of savings for both generations. The average pension payment declines as the size of the labour force decreases relative to the retired. At this stage, the average bequest rises despite the fall in the consumption of the old since the rate of population growth has caused the size of the young generation to decline.

Within the next period the variables transition to their new steady state values as a result of the permanent shock. The new steady value of the aggregate private savings rate is lower than its initial value. The assets of the young falls to a slightly lower steady state, while the assets of the middle-aged rises. The savings of the young is lower in response to a lower interest rate and higher wage than in the previous steady state. In response to the relatively lower pension that they will receive in the next period, the stock of savings of the middle-aged is higher compared with its initial steady state value. The permanent decline in n also causes the consumption in the old generation to reach a lower steady-state value. The capital-labour ratio also rises to a higher steady state and thus per capita output follows suit.

7.3 Scenario 3: A permanent increase in φ[8]

In order to analyze the increase in life expectancy in Canada that has occurred over the past few decades, I consider a permanent increase to the survival rate of the model. This anticipated shock occurs in period 4 of the simulation. The increase in the survival rate means that people live longer on average in the retirement stage. As the economy transitions to its new steady state, each generation will make subsequent changes to their behaviour to incorporate their longer expected life. As this occurs, the aggregate private savings rate dips below zero and then rises to a new positive steady state value (see Fig. 4.0). The final steady state value of the aggregate savings rate in this scenario has moved in the same direction as the previous.

Figure 4.0 Aggregate Private Savings Rate (Scenario 3)



[8] Additional relevant graphs for this scenario can be found in Appendix 4

In anticipation of the increase in life expectancy, the aggregate private savings rate falls between periods 3 and 4. The saving of the young falls during this time in anticipation of a higher interest rate in the next period. The stock of savings of the middle-aged however rises to compensate the rise in life expectancy that will occur as they retire in the next period. They also take into consideration the lower pension they will receive next period. The fall in the assets of the young outweighs the increased assets of the middle-aged and thus the aggregate private savings rate falls. The capital stock rises slightly between periods 2 and 3 and then declines during periods 3 and 4, as a result of the change in the assets of both generations.

Once the shock occurs in period 4, there are some dramatic changes in the assets of the young and the middle-aged. The savings of the young rises by 62%, in anticipation of a lower pension when they retire. They also respond to a larger bequest by increasing their savings. On the contrary, middle-aged assets are lower in response to a falling wage rate and low savings in the previous. The aggregate private savings rate continues to decrease as the consumption of the retired rises due to their longer life.

Between period 5 and 6, the aggregate private savings rate falls to a negative value, despite the rise in the assets of the middle-aged. The stock of savings of the middle-aged rises dramatically as a result of the higher interest rate. The savings of the young falls due to an expected increase in the wage during the next period. The bequest rate also falls in direct response to the higher life expectancy. The capital-labour ratio rises in this period, which results in a higher wage and lower interest rate in the next period.

After reaching its lowest point in period 6, the aggregate private savings rate climbs dramatically to a positive value. The assets of the young rises as a result of the rising wage, while the assets of the middle-aged falls in expectation of a higher pension in the next period. There is also a large fall in the consumption of the old which contributes to the large rise in the savings rate.

The variables finally transition to their new steady state values after some adjustment periods. As life expectancy rises, the agents make changes to their savings and consumption in order to accommodate more years of life. The final steady state of the aggregate private savings rate is just below the original. As each individual lives longer into their retirement period, the new steady state value of both the pension and bequest fall. The steady value of each generation's stock of savings changes in opposite directions. The saving of the young transitions to a slightly lower steady state as the interest rate falls and the wage rate rises. The assets of the middle-aged rises compared to its previous steady state value in order to accommodate a lower steady state average pension. The larger increase in the assets of the middle-aged dominate the fall in the savings of the young generation and thus the capital-labour ratio transitions to a higher steady state value.

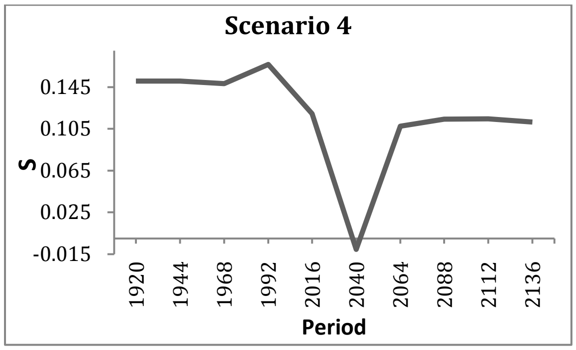
8 Simulating the total effects of population ageing (Scenario 4)[9]

This final scenario incorporates all of the causes of population ageing in Canada. By including all these changes it will be possible to see how each of the demographic shocks interact and ultimately affect the final steady state value of the aggregate private savings rate in this economy. In this simulation, each period corresponds to a particular 24 year period, in order to compare and predict how the economy will move in the "real world". The baby-boom occurs in period 2 which corresponds to the time period from 1944 to 1967. The permanent increase in life expectancy occurs in 1992 (period 4) whereas the fall in the population growth rate occurs in 2016 (period 5). There are combinations of behavioural changes from the exogenous shocks that

[9] Additional relevant graphs for this scenario can be found in Appendix 5

will impact the economy's transition to its new steady state. The movements in the aggregate private savings rate mimic that found in scenario 2 and 3; rising slightly and then falling to a negative value before returning to a lower positive steady state value(see Fig. 5.0).

Figure 5.0 Aggregate Private Savings Rate



From 1944 to 1967 there are behavioural changes due to the baby-boom as well as changes due to the anticipated increase in life expectancy that will be occur in the next 2 periods. There is a very slight decline in the capital-labour ratio as a result of to the changes in the assets of the young and middle. In expectation of a lower pension, once n reverts to its initial value, the savings of the young rises. Middle-aged agents foresee a rise in the pension in the coming periods and thus their stock of savings falls. The rise in n causes the labour force to temporarily increase in size relative to previous periods and thus the tax base will increase. The aggregate private savings rate falls slightly as the drop in the stock of savings of the middle-aged outweighs the rise in the savings of the young.

From 1968 to 1991 there are 3 main scenarios that are affecting the behaviour of each generation. In addition to the delayed effects of the baby boom, there is the anticipation of the rise in life expectancy and the fall in the rate of population growth. There is a slight rise in the aggregate private savings as the assets of each generation changes. The savings of the young falls due to a higher expected interest rate in the next period. The fall in the bequest also negatively affects the assets of the young. In order to compensate for a low pension in the next period, the savings of the middle-aged (baby-boomers) rise. The capital-labour ratio declines due to the rise in population growth that occurred last period.

In 1992, the permanent increase in life expectancy occurs, meaning that agents will live a longer period of their lives as retirees. The baby-boom generation is now retired and there is further anticipation of a fall in the rate of population growth. The aggregate private savings rate declines dramatically from 1992 to 2015. During this time, there is a rise in the savings of the young as they prepare for the low pension they will receive. Their savings also rises as the bequest during this period rises. There is a decline in the stock of savings of the middle-aged due to the fall in the wage rate as well as lower savings from when the individual was young. The interest and wage rate react to the fall in the capital-labour ratio that occurred in the previous period.

The final demographic shock, lower population growth, occurs in 2016. Combined with the previous shocks, there is a further decline in the aggregate private savings rate to a negative value. At this stage the behavioural effects of both the permanent shocks come together. As in both scenarios 2 and 3, the stock of savings of the young declines from 2016 to 2039. The higher wage expected in the next period means that the young do not need to save as much in the current period. The stock of savings of the middle-aged follows the previous 2 scenarios as well, rising to compensate for a lower pension next period. There is also a large rise in the consumption of the old generation during this time period due to the rise in life-expectancy. A large rise in the capital-labour ratio occurs in response to lower population growth.

From 2040-2064, the economy makes its final significant changes and then transitions to its new steady state. There is a rise in the aggregate private savings rate as the savings of the young rises and the savings of the middle-aged falls. This occurs due to a higher wage rate and lower pension in the next period respectively. The capital-labour ratio rises in accordance with a lower population growth rate and a higher life expectancy.

Ultimately, the economy transitions to a new steady state where the aggregate private savings rate is lower than its original value. This occurs primarily as a result of a fall in the interest rate and the rise in the wage rate that occurs relative to their original steady states. The rise in life expectancy and the fall in the population growth rate underpin each other and the aggregate private savings rate falls to a lower steady state value. The final steady state values of the assets of the young and middle-aged mimic that in the previous two scenarios, falling and rising respectively. The average pension, bequest and the interest rate are all lower than their original steady state values. While the wage and the capital-labour ratio rises. The baby-boom simply causes a temporary rise in the aggregate savings rate between 1968 and 1991.

Despite the simplicity of the model, there are similarities between the data on the personal savings rate in Canada and the results of this simulation. Although this model predicts future changes to the aggregate private savings rate, we can still compare the results with the data over the last 50 years. The personal savings rate in Canada rose consistently from 1961 to 1982 when it peaked at 20.2%. In this model, the aggregate private savings rate rises from 1968 to its peak of 16.6% in 1992. After 1982, the personal savings rate steadily declines reaching a value of 4.6% in 2009. The simulated savings rate also follows this trend, declining steadily after its peak, however it does not reach a similar value as the actual data in 2009. Notwithstanding the difference in the values of the savings rate, the general trend in personal savings over the past 50 years in Canada is consistent with the results of this simulation.

Limitations of the Model

Using a three period OLG model made it possible to determine the generational effects of population ageing on the economy. The simulations provided a more comprehensive insight into how all the effects of population ageing will affect the economy. However, a number of caveats should be stressed with regard to the simplicity of the model. The model does not include the full range of taxes and government expenditures in the Canadian economy. While this will affect the quantitative results of the simulations, it will not likely affect the qualitative results compared to previous studies. The pension system used in this model is not fully representative of the Canadian pension system, which is partially pay-as-you-go and partly funded by the government. In this model, taxes fully fund the pension payment received in retirement and there is no guaranteed amount that will be received. Incorporating Canada's pension system will likely mean that the ageing shock would exert less pressure on national savings (Fougère & Mérette, 1999). Third, there are only 3 generations in this model each representing 24 year periods. Expanding the number of generations in each stage of life would allow for less abrupt changes in the variables as the demographic shocks reverberate, however it would not affect the qualitative results of the simulations. Finally, the closed economy assumption may not be realistic for Canada whose economy is highly dependent on trade.

9 Conclusion

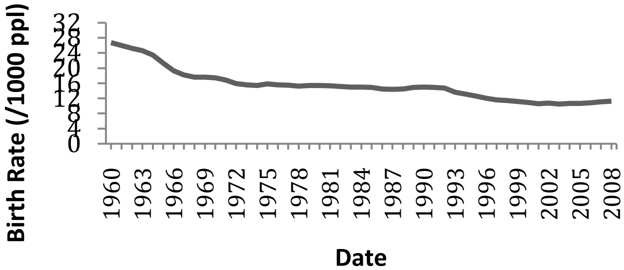
By examining all of the causes of population ageing on the economy it is possible to understand the full magnitude of this phenomenon. This essay looks at the impact of each driver of ageing as well as their combined effects to determine the effects of population ageing on the aggregate savings rate in the Canadian economy. Simulations of a general equilibrium three-period overlapping generations model are used to examine all the behavioural changes that occur in the economy in response to the demographic shocks. Scenario 1 considers a temporary rise in rate of population growth to mimic the post-WWII baby boom. The results of the simulation show a temporary rise in the aggregate private savings rate when the baby-boomers are middle-aged and then a large decline once they have entered their retirement period. There are no long-run effects of this shock considering the nature of the model as it returns to its steady state. Scenario 2 and 3 consider lasting changes to the demographic variables in the economy. A permanent decrease in the rate of population growth and an increase in the survival rate have similar effects on the final steady state values of the aggregate savings rate. The capital-labour ratio to rises in response to both anticipated shocks. This in turn leads to a lower interest rate and higher wage which deters savings. Rising life expectancy results in a need for more savings to accommodate for more years of life thus the aggregate savings rate rises compared to its initial steady state. It is important to note however that the final steady state values of key endogenous variables move in the same direction under the two scenarios. For instance the capital-labour ratio, the wage rate and the stock of savings of the middle-ages are all higher than their original steady state values. While the savings of the young, the interest rate and the pension are all lower. The only other endogenous variable that differs in the direction of movement of their final steady state is the bequest per young person. The average bequest rises in scenario 2 and falls in scenario 3. The ultimate difference between the two scenarios is the difference in the proportion of the working-aged to the old, or the dependency ratio. The size of each generation indicates that certain behaviours will dominate the changes that occur to the aggregate private savings rate.

The similarities in the movements of the steady states reinforce each other and thus most of the endogenous variables in scenario 4 move in their expected direction. The final scenario considers all of the demographic shocks over the simulation period, as they are occurring in the Canadian population. The saving rate peaks in 1992 and reaches its lowest value in 2040. The aggregate private savings rate falls, and remains at a lower value than its original steady state. The model predicts a falling aggregate personal savings rate over the past 2 decades, mimicking the trend in the actual data on the personal savings rate in Canada. The final steady state value of this scenario is lower than each of the values reached when only one demographic shock is considered. The increase in life expectancy reinforces the effects of a fall in population growth causing a deeper decline. This would seem logical considering the echo effects of demographic shocks, a fall in the birth rate perpetuates itself into the future making every generation proportionately smaller.

Considering the predictions of this model and those in previous years, the Canadian economy must prepare itself for the macroeconomic impacts of population ageing. It will now be important to study the impact of different fiscal policy measures in response to the all the aspects of population ageing.

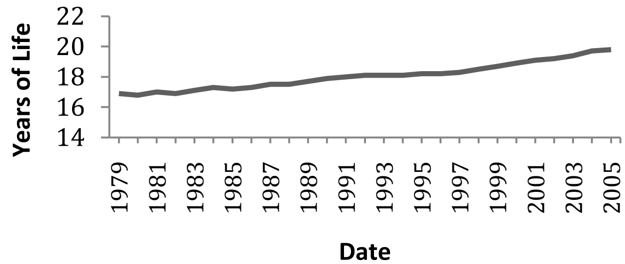
Appendix 1: Population Characteristics

Figure 1.6
Crude Birth Rate in Canada
1960-2008



Source: World Bank

Figure 1.7
Life Expectancy at age 65 in Canada
1979-2005



Source: Statistics Canada CANSIM Database

Figure 1.8
Old-Age Dependency Ratio in Canada
1960-2008

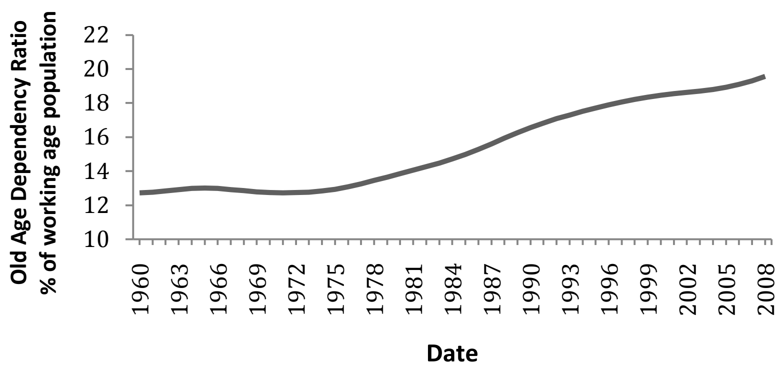
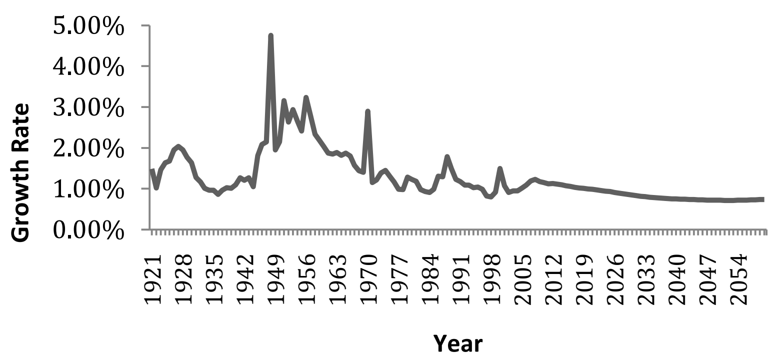


Figure 1.9
Yearly Population Growth Rate in Canada
1921-2060



Source: Statistics Canada CANSIM Database

Appendix 2: Scenario 1 Simulation Graphs

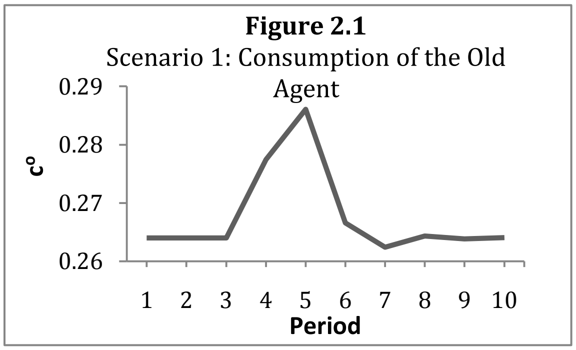


Figure 2.1
Scenario 1: Consumption of the Old Agent

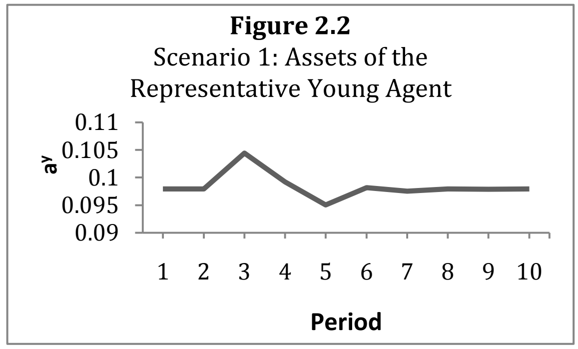


Figure 2.2
Scenario 1: Assets of the Representative Young Agent

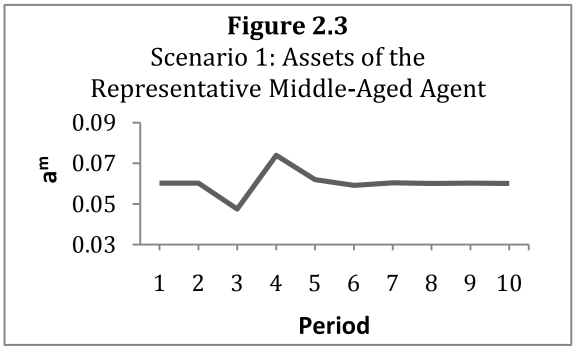


Figure 2.3
Scenario 1: Assets of the Representative Middle-Aged Agent

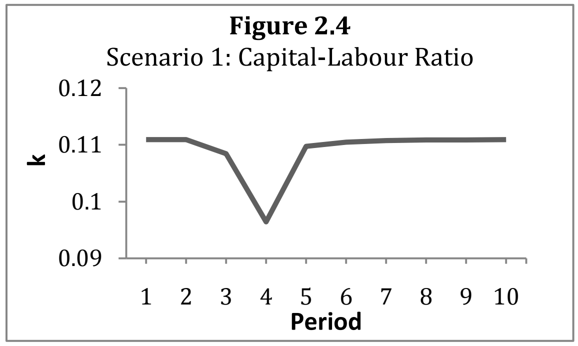


Figure 2.4
Scenario 1: Capital-Labour Ratio

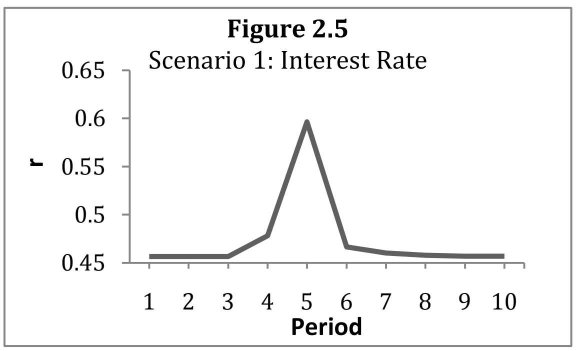


Figure 2.5
Scenario 1: Interest Rate

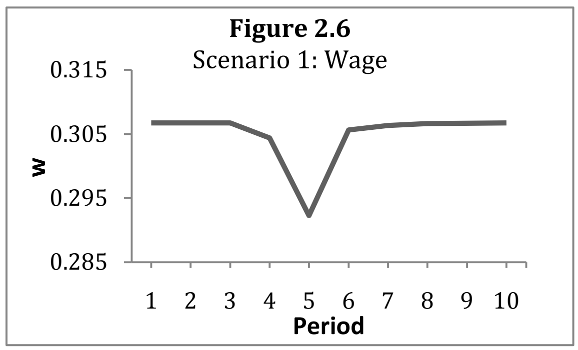


Figure 2.6
Scenario 1: Wage

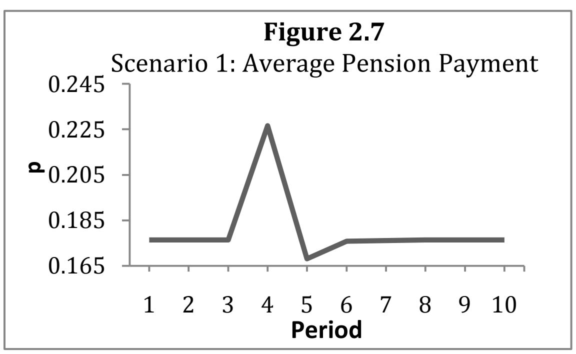


Figure 2.7
Scenario 1: Average Pension Payment

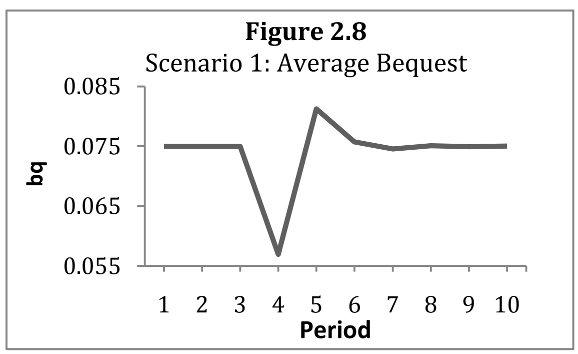
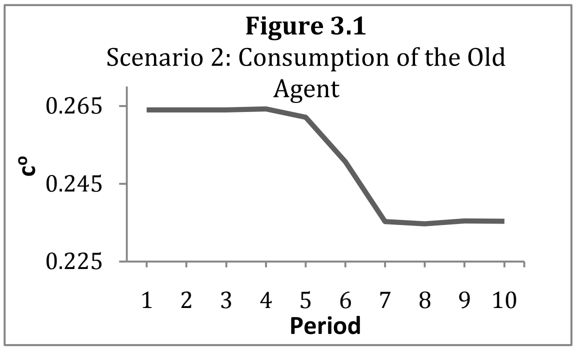
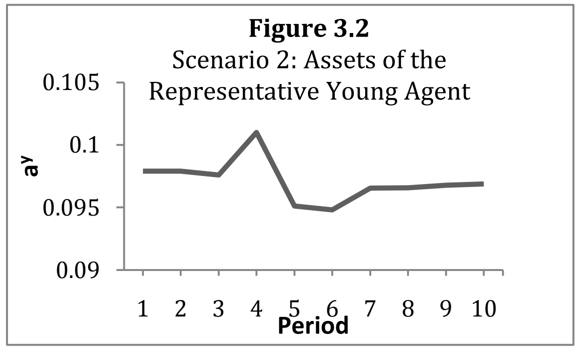
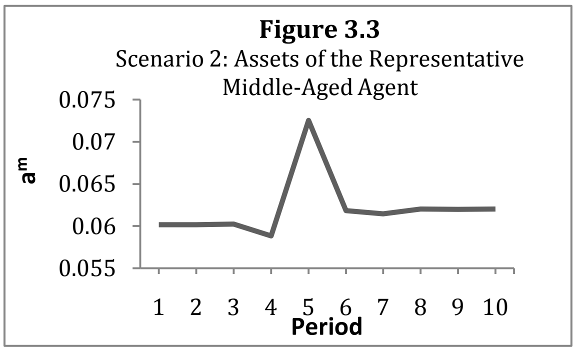


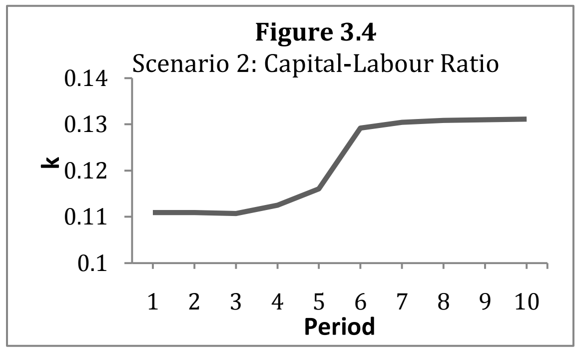
Figure 2.8
Scenario 1: Average Bequest

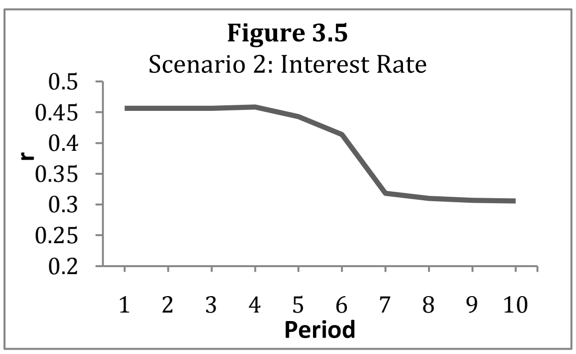
Appendix 3: Scenario 2 Simulation Graphs

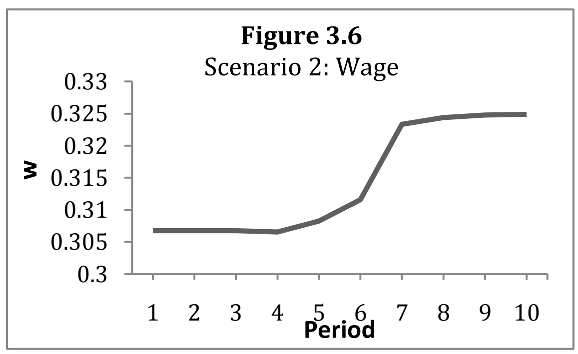


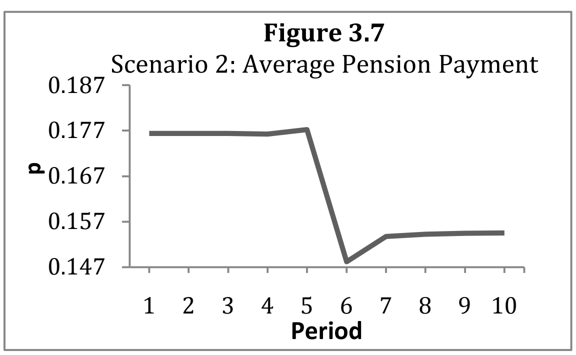


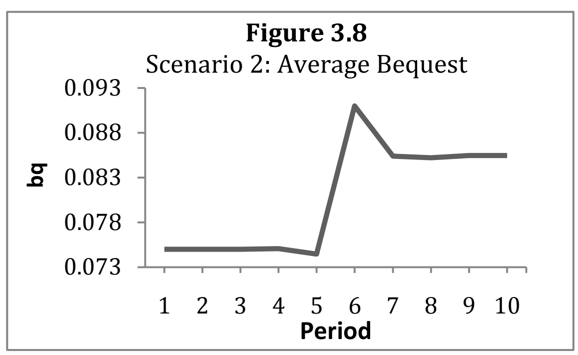












Appendix 4: Scenario 3 Simulation Graphs

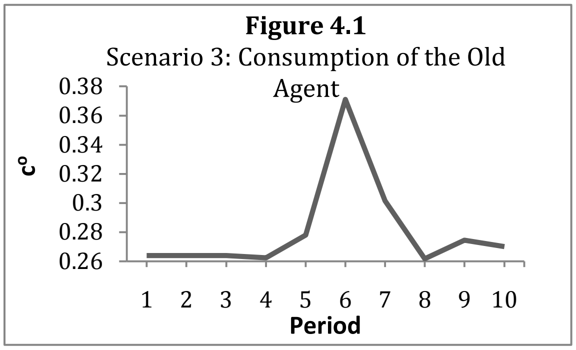


Figure 4.1
Scenario 3: Consumption of the Old Agent

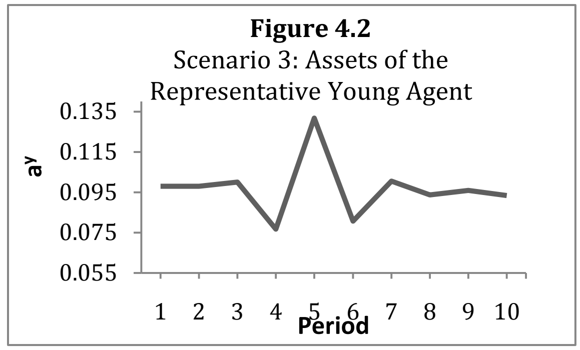


Figure 4.2
Scenario 3: Assets of the Representative Young Agent

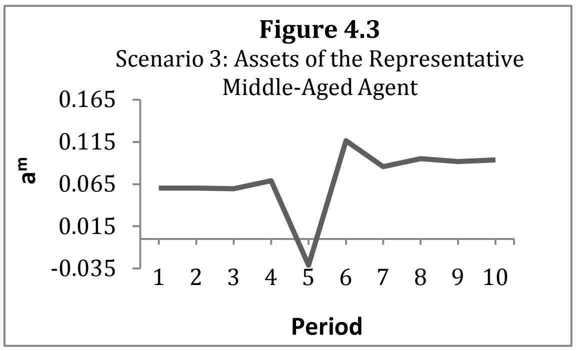


Figure 4.3
Scenario 3: Assets of the Representative Middle-Aged Agent

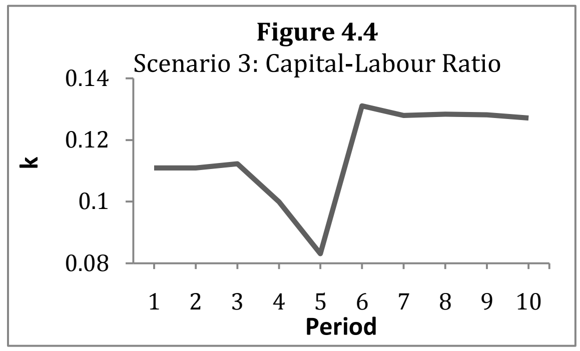


Figure 4.4
Scenario 3: Capital-Labour Ratio

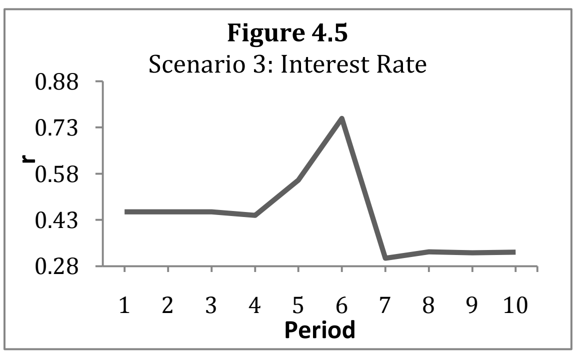


Figure 4.5
Scenario 3: Interest Rate

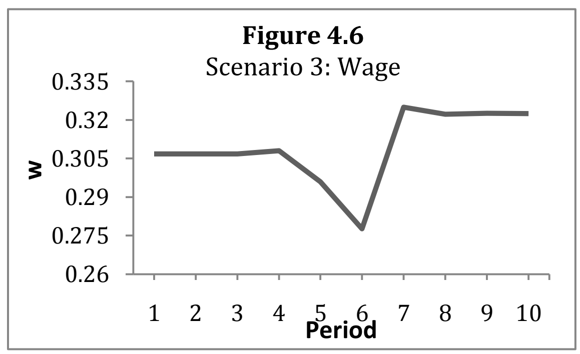


Figure 4.6
Scenario 3: Wage

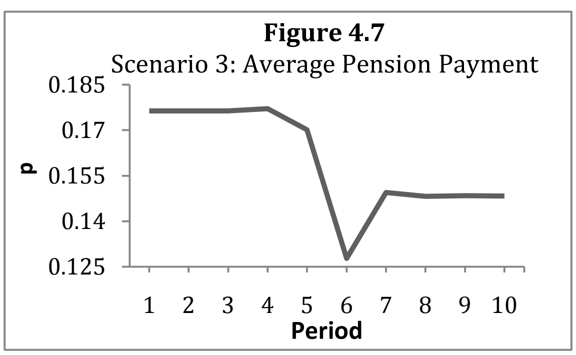


Figure 4.7
Scenario 3: Average Pension Payment

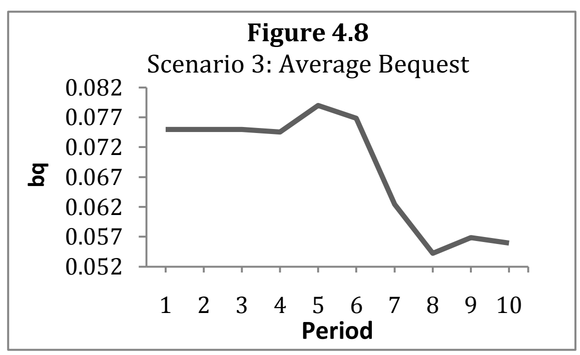


Figure 4.8
Scenario 3: Average Bequest

Appendix 5: Total Effects of Population Ageing Simulation Graphs

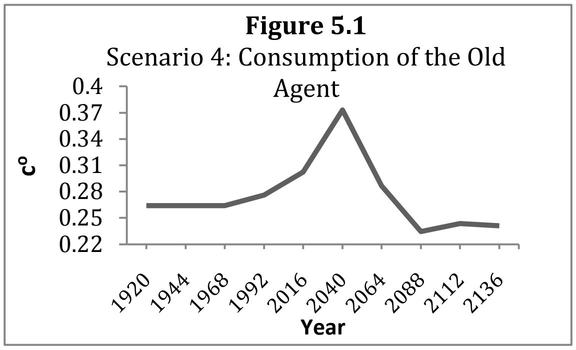


Figure 5.1
Scenario 4: Consumption of the Old Agent

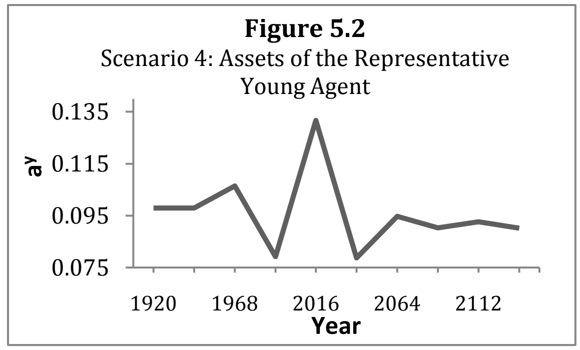


Figure 5.2
Scenario 4: Assets of the Representative Young Agent

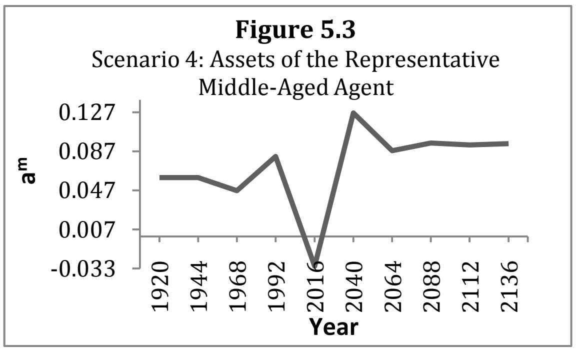


Figure 5.3
Scenario 4: Assets of the Representative Middle-Aged Agent

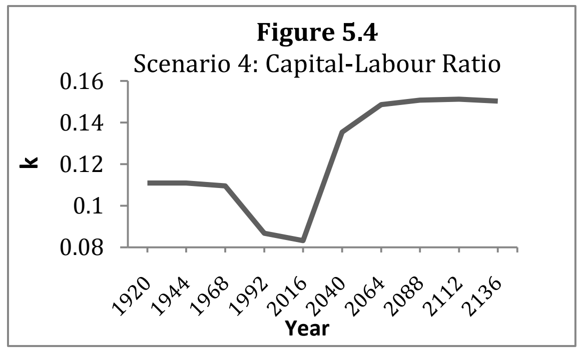


Figure 5.4
Scenario 4: Capital-Labour Ratio

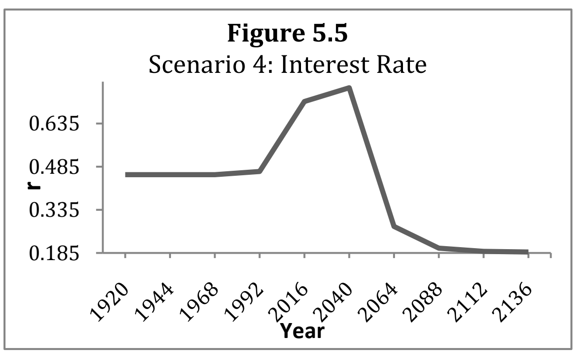


Figure 5.5
Scenario 4: Interest Rate

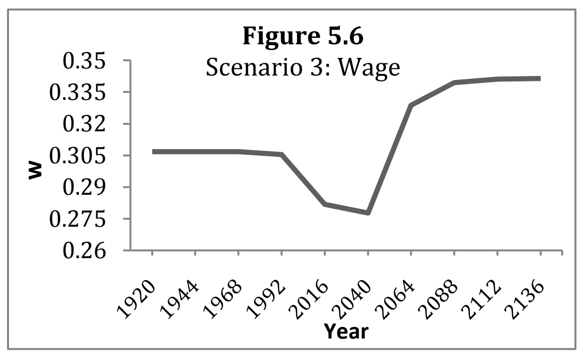


Figure 5.6
Scenario 3: Wage

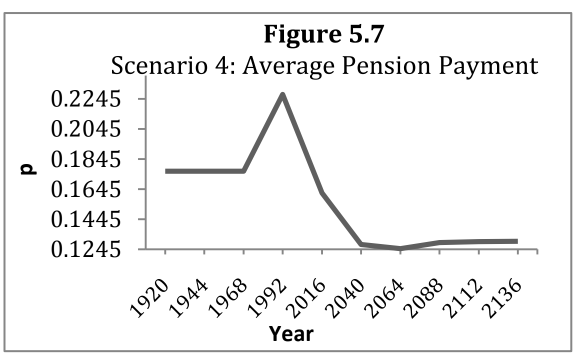


Figure 5.7
Scenario 4: Average Pension Payment

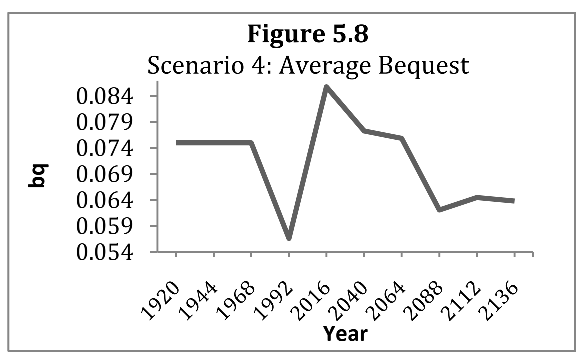


Figure 5.8
Scenario 4: Average Bequest

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