Public-Private Complements in a Market Setting: Mergers and Efficiency

by

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1 Introduction

Merger analysis has largely investigated the idea that mergers increase market power and that this generally leads to a decrease in social welfare. With few exceptions, the verdict is that mergers do indeed lead to a decrease in social welfare, especially in multiproduct markets where the constituent goods are substitutes. However, there are situations where mergers may actually increase social welfare. This can happen if mergers realize cost synergies that would not be realized in the absence of a merger, though they do not necessarily increase consumer surplus.[1] As a general rule, though, mergers are usually viewed with caution, and sometimes with outright disdain.

Relatively little research has been done on mergers in markets with complementary goods. The results are different with complements; mergers do not always reduce welfare, and some goods may actually be produced below cost to maximize profit. Further research should be done to understand the welfare effects of these mergers, both for academic and for antitrust policy reasons.

It is well-known that voluntary contributions equilibria for public goods produce too little of the public good to be efficient; no player takes any other player's marginal utility into account when contributing to the additional production of a public good. Taxation and public provision of these goods is the solution most commonly used. This paper suggests that in some cases, market-based solutions may lessen this problem in situations where public provision is either undesireable or impossible.

The argument is as follows: in a market with one private good and one public good, which are complementary and are each produced by a separate firm, a merger between the two firms may lead to a more efficient level of production of the public good and possibly increase overall welfare. The merged firm knows that an increased availability of the public good will increase demand for the private complement; since it now maximizes joint profit, it may lower the price of the public good to increase

[1] Farrell and Shapiro, 1990.

demand for the private good and earn additional profits in that sector.

Previous results in public economic theory and in the industrial organization literature support this argument. We know that voluntary contributions equilibria have a market-based equivalent and that the characteristics of these equilibria are unchanged in moving to a market setting. We also know that mergers between firms producing Bertrand complements tend to lower at least one price and increase production of at least one good, and that consumer welfare can increase under these mergers. Therefore, it follows that a merger between a public and a private complement can increase the production of a public good, bringing it closer to first-best levels, and this can increase overall welfare. This syllogism will be fleshed out in the following pages.

This model can easily be applied to other bodies of literature. Hardware-software models are apt examples, especially if the software is an information good or is computer software. Corporate licensing of software is another example. Since piracy is much harder to prosecute for individual users than for large-scale enterprises, allowing private users to download software illegally (making it nonrival) may make the software a more desireable product for employers, since there is already an installed userbase. Finally, pirated digital music could be seen as a nonrival good whose private complement is concert revenues. Pirated music is effectively free publicity for artists, who could reap profits off of heightened ticket sales. These markets may exist in pre-merger or in a monopolized form; the policy implications of this paper will shed light on whether these monoplies should be tolerated by the relevant competition agencies.

2 Previous Work

The body of work most closely related to this paper concerns tying or bundling of goods. Telser (1979) examines this idea in detail. Given a system of complementary goods, Telser asks whether a monopolist could be better off by selling the system as a single package or by selling each member of the system separately, and finds that a monopolist selling the system as a single package (i.e., tying) can do no worse than if they were to sell each good separately.[2] However, this set-up assumes that each consumer wants to consume the system of complements in fixed proportion; i.e., the consumers have (possibly distinct) Leontief preferences. Telser does not consider situations where a consumer, faced with a separate price of each good in the system, would change his proportional consumption given a change in prices. I want to consider a more general set of preferences that are less likely to be consumed in fixed proportions, and are thus much less likely to be tied in the marketplace. Each good is distinct, and consumers derive utility from consuming these goods even when the complement is not present.

Tirole (1988) casually examines a monopolist that produces such complements and compares the outcome to that of production by distinct firms. This is closer to the question that this paper asks. His conclusions are somewhat confusing: Tirole states that the Lerner index, which involves the price-cost margin, is less than its value before the merger "for each good", implying that the price of each good has fallen. He then mentions the possibility that some goods may be sold below marginal cost, but leaves it as an "interesting phenomenon" until he returns to the issue in his chapter on tying. The problem is that in showing that the Lerner index (and therefore price) for a given good has fallen, his math involves the price-cost margin for the other goods. If one of these is negative, as he states is possible, then the Lerner index may actually have risen for this good. Thus Tirole's conclusions are somewhat misleading. However, this is not much of a problem for him, since his main concern with systems of complements concerns tying, as above.

Other authors have noticed this discrepancy and have sought to clarify what can happen to prices as the result of a merger in this situation. Shy (1996) and Andriy-

[2]Telser 1979, p. 216.

chenko et al (2006) are the most prominent examples. Their methodology and conclusions are similar; Shy's work is concerned again with perfect complements (Leontief preferences). Andriychenko et al, however, allow the proportions of consumption to be a function of the component prices, which is the more general case that this paper will investigate. Their conclusions again allow for the possibility that some of the goods may be produced below cost, but finds that the system price paid for both goods is "unambiguously lower" under monopoly than under duopoly. Further, consumer surplus and firm profits are higher post-merger. These findings are very similar to what this paper seeks to show: prices (at least in some sense) have fallen, and this could mean that if one of the goods is nonrival, its quantity may be closer to the socially optimal level.

Hardware-software models share similarities with the markets studied in this paper as well. These goods are complements; a decrease in the price of a firm's hardware entices more consumers to adopt that platform and demand more software. As will be argued below, these goods are particularly at risk of being nonrival, and this can have dynamic effects on the market over time. However, the focus of much of the hardware-software literature concerns network effects and how competing platforms influence effciency, with particular attention to adoption externalities.[3] This paper (mainly) focuses on a single set of complements in isolation, ignoring the effects of other platforms that may exist. Further work should be done to determine how public-private complements behave in the context of platform competition, and suggestions are made below.

The appeals to public economic theory and intuition in this paper are fairly elementary; I only go so far as to show that market equilibria can exist for these goods, and that comparison of demand functions for public goods and private goods give firms certain profit incentives. It is more likely that firms will use the public good

[3]See Katz & Shapiro, 1994; Church & Gandal, 2004; and Church, Gandal, & Krause, 2008.

as a “loss leader” and make their profit from the private complement; this will be motivated below. In my discussions of voluntary-contributions models, I refer to and expand upon Laffont (1988).

I will proceed as follows. I will begin by motivating the idea of a voluntary-contributions equilibrium in a market context, specifically that of public-private complements. This will lead to demand schedules that can be used for profit maximization by firms. Second, I will show that given a demand schedule for complementary goods, firms have an incentive to lower prices. In some cases, they may lower both prices, but in many cases, only one price will be lowered. In either case, I appeal to previous work that shows that regardless, welfare will increase. This leads to the conclusion that if a merger can lower price (and then raise quantity), it may produce a more efficient level of the public good. Third, I provide a simple, illustrative numerical example that provides evidence for which of these outcomes is most likely to occur. I then discuss possible applications, the relationship of this concept to other bodies of research, and conclude.

3 Demand-Side

It is relatively simple to obtain decentralized market demands from consumers maximizing private utility, even when considering public goods. I follow the general framework of Laffont 1988;[4] my model considers a utility function with one public good, one numeraire private good, and multiple other private goods.

3.1 Voluntary Contributions Equilibrium

Consider an economy with n consumers, one public good Z, m private goods $X_1 \dots X_m$, and one private good Y. Consumer i is endowed with w_i units of good y. She may

[4] pp. 37-41.

freely transform z_i^* units of y into z_i units of the public good using the technology $z_i = h(z_i^*)$, and she may freely transform x_{ji}^* units of y into x_{ji} units of private good X_j using the technology $x_{ji} = g_j(x_{ji}^*)$, $j = 1 \ldots m$. Each consumer derives utility from the function $U_i(x_{1i}, \ldots, x_{mi}, y_i, Z)$, where $Z = \sum_{i=1}^{n} z_i = \sum_{i=1}^{n} h(z_i^*)$, since Z is a public good and therefore nonrival. In order to maximize utility, each consumer chooses how much of her endowment to transform into each good (i.e., z_i^* and x_{ji}^* $\forall j$) and how much of each of the resulting goods to consume (i.e., y_i, Z, and x_{ji} $\forall j$), given the above resource and transformation constraints.

In other words, the consumer faces the following maximization problem:

$$\begin{aligned} Max\ U_i(x_{1i}, \ldots, x_{mi}, y_i, Z) \quad s.t. \quad & g_j(x_{ji}^*) - x_{ji} \geq 0 \quad \forall\ j = 1 \ldots m, \\ & \sum_{i=1}^{n} h(z_i^*) - Z \geq 0, \\ & w_i - \sum_{j=1}^{m} x_{ji}^* - z_i^* - y_i \geq 0. \end{aligned}$$

Let $\lambda_i \ldots \lambda_m$, μ, and θ be the multipliers associated with the constraints. Then the maximization problem becomes

$$\begin{aligned} Max\ U_i(x_{1i}, \ldots, x_{mi}, y_i, Z) + & \sum_{j=1}^{m} \left(\lambda_j \cdot (g_j(x_{ji}^*) - x_{ji})\right) \\ & + \mu \cdot \left(\sum_{i=1}^{n} h(z_i^*) - Z \right) \\ & + \theta \cdot \left(w_i - \sum_{j=1}^{m} x_{ji}^* - z_i^* - y_i \right) \end{aligned} \tag{1}$$

Solving (1) leads to the following conditions:

$$\frac{\partial U_i/\partial x_{ji}}{\partial U_i/\partial y_i} = \frac{1}{g'_j} \quad \forall j \tag{2}$$

$$\frac{\partial U_i/\partial Z}{\partial U_i/\partial y_i} = \frac{1}{h'} \tag{3}$$

These equations define a voluntary-contributions equilibrium. The Samuelson conditions for this problem, which define the socially optimal outcome, are

$$\frac{\partial U_i/\partial x_{ji}}{\partial U_i/\partial y_i} = \frac{1}{g'_j} \quad \forall j \tag{4}$$

$$\sum_{i=1}^{n} \left(\frac{\partial U_i/\partial Z}{\partial U_i/\partial y_i} \right) = \frac{1}{h'} \tag{5}$$

Notice that equations (2) and (4) are identical, while equations (3) and (5) are not. This is an elementary result in public economic theory; the social planner will produce a higher amount of the public good.

3.2 Voluntary Contributions in a Market Setting

To bring this model into a market setting, we need to give some meaning to the technologies $g_j(x^*_{ji})$ and $h(z^*_i)$. In a traditional general equilibrium framework, Y is the consumers' numeraire good, since their wealth is measured in Y. To buy some good A in a market, one would take p_A units of their endowment of Y and give it to a firm, who would in turn give the consumer one unit of good A. In other words, the "transformation technology" faced by the consumer in this market setting is $a_i = a^*_i/p_A$, where a^*_i is portion of the consumer's endowment she transforms into A and a_i is the number of units of good A she receives in return.

Thus to bring this model into a market setting, we define $g_j(x^*_{ji}) = x^*_{ji}/p_{xj}$ and

$h(z_i^*) = x_i^*/p_z$. Then equations (2) and (3) become

$$\frac{\partial U_i/\partial x_{ji}}{\partial U_i/\partial y_i} = p_{xj} \quad \forall j \tag{6}$$

$$\frac{\partial U_i/\partial Z}{\partial U_i/\partial y_i} = p_z \tag{7}$$

Equations (6) and (7) can then be solved to find market demand curves for the m public goods and the private good.

It is prudent to discuss the validity of this framework in game-theoretic terms. In the usual voluntary-contributions model, the vector of contributions $(z_1^*, \ldots z_n^*)$ forms a Nash equilibrium, regardless of the disparity between the size of any two players' contributions. Indeed, a Nash equilibrium can exist where one player purchases all of the public good and every other player purchases nothing. As an example, say that player i has purchased Z' units of the public good such that $\partial U_i/\partial Z' = p_z$. Further, assume that all players' utility functions are the same. Then, taking as given the purchases of every other player, the sole contributor will not want to purchase less of the good, and the free-riders will not want to purchase any more. This defines a Nash equilibrium with a single contributor.

It may be tempting to say that since each consumer can free-ride off of his neighbor, no consumer will purchase the public good and none will be provided, especially if the number of consumers is high. This is not a Nash equilibrium; one of the consumers will purchase at least something, even if the total amount purchased is below the socially optimal amount.[5]

To summarize, the traditional voluntary-contributions framework can be easily adapted to a market context. While the free-rider problem will still exist, the market will provide a non-zero amount of the public good, regardless of the number of

[5]Obviously, for this to be true, at least one consumer's marginal benefit of the public good at zero must be greater than p_z.

consumers. We can then solve for demand curves and see how firms act when faced with said demand curves.

4 Supply-Side

A literature exists that supports the idea that a merger between two Bertrand competitors in a market with complements will lower one or both prices. One can construct examples where the new monopolist will lower one price to raise demand for the other good while raising its price as well; sometimes the firm will even price one good below cost. However, this certainly does not occur in every case. Below are three arguments that give evidence of falling prices. Note that in all of the following, we focus on a market with two complementary goods X and Z and two firms that compete in prices, both facing constant marginal costs. The two firms merge into a multiproduct monopoly and outcomes are compared in each case.

The first is due to Tirole.[6] He derives the first-order conditions for the profit-maximization problem faced by a solitary monopolist and for a multiproduct monopolist. The first case corresponds to the first-order condition for one Bertrand competitor. After some algebraic manipulation, he arrives at the following:

4.1 Tirole: Change in Lerner Index

$$\frac{p_x - c_x}{p_x} = \frac{1}{\varepsilon_{xx}} \tag{8}$$

$$\frac{p_x - c_x}{p_x} = \frac{1}{\varepsilon_{xx}} + \frac{p_z - c_z}{X \cdot \varepsilon_{xx}} \frac{\partial Z}{\partial p_x} \tag{9}$$

where (8) is the manipulated first-order condition for an isolated monopolist and (9) is the manipulated first-order condition for a multiproduct monopolist. The left-hand

[6] Tirole 1988, Ch.1.

side of both equations is the price-cost markup, or Lerner index of monopoly. The argument is thus: Compare the price-cost markup in the first and second cases. In the first, it equals the inverse elasticity of demand. In the second, it equals something that is less than the inverse elasticity of demand (since $\partial Z/\partial p_x < 0$; i.e., the goods are complements). Therefore, the price-cost markup has fallen, meaning that price has fallen.

This is evidence for falling prices post-merger. However, this is not a complete proof: it only works for demands with constant own-price elasticity. If elasticity rises as prices change from pre- to post-merger, the right-hand side of equation (9) might end up higher than that of equation (8), actually raising price. A further concern is that this does not work for situations where one good is priced below cost. Then the right-hand side of equation (9) may be positive, causing the other price to rise. Tirole mentions this as an "interesting phenomenon" and discusses it further in the context of tying, but does not mention how it affects the above result. As it stands, this argument only serves as evidence for falling prices in certain situations.

4.2 Changes in Price-Cost Margin

We can see the necessity of constant own-price elasticity in Tirole's argument more directly. Taking again the first-order conditions for profit-maximization in pre- and post-merger cases, we can solve for the price-cost margin in each case:

$$(p_x - c_x)_{pre} = \frac{-X}{\partial X/\partial p_x} = \left.\frac{p_x}{\varepsilon_{xx}}\right|_{pre} \tag{10}$$

$$\begin{aligned}(p_x - c_x)_{post} &= \frac{-X}{\partial X/\partial p_x} - \frac{\partial Z/\partial p_x}{\partial X/\partial p_x}\cdot(p_z - c_z)_{post} \\ &= \left.\frac{p_x}{\varepsilon_{xx}}\right|_{post} - \frac{\partial Z/\partial p_x}{\partial X/\partial p_x}\cdot(p_z - c_z)_{post} \qquad (11)\end{aligned}$$

where (10) is the markup pre-merger and (11) is the markup post-merger. We can find similar expressions for the price-cost margin in good Z. Then we can subtract (10) from (11) to solve for the change in price due to the merger. Note that if own-price elasticity is not constant, we cannot arrive at the following expressions and the argument breaks down. Nevertheless, if own-price elasticity is constant,

$$\Delta p_x = \frac{\partial Z/\partial p_x}{\partial X/\partial p_x} \cdot \left(1 - \frac{1}{\varepsilon_{xx}}\right)^{-1} \cdot (c_z - p_z)_{post} \tag{12}$$

$$\Delta p_z = \frac{\partial X/\partial p_z}{\partial Z/\partial p_Z} \cdot \left(1 - \frac{1}{\varepsilon_{zz}}\right)^{-1} \cdot (c_x - p_x)_{post} \tag{13}$$

Now, if the demands are elastic, then the first two terms of the right-hand side are positive, since the goods are complements.[7] We have three cases to consider: either only X is produced below cost, only Z is produced below cost, or both goods are produced above cost. In the first case, $\Delta p_x < 0$ and $\Delta p_z > 0$. In the second, $\Delta p_x > 0$ and $\Delta p_z < 0$. In the third, equations (12) and (13) are negative, so both prices fall.

In short, at most one good is produced below cost and at most one price rises. If neither good is produced below cost, then both prices fall.

Again, neither of the above arguments are robust to changing own-price elasticity. If own-price elasticity is not constant, then we may have other outcomes. For example, with linear demand both prices may be above cost, but one price may still rise. We will see this below.

4.3 Profit Incentives at Pre-Merger Prices

Another argument that supports falling prices involves signing the change in profit due to a change in price for the multiproduct monopolist at pre-merger prices.[8] The first

[7] If the demands are inelastic, the conclusions are reversed.

[8] I am grateful to Roger Ware for this argument.

derivatives of the profit functions pre- and post-merger, respectively, are as follows:

$$\frac{\partial \pi_x}{\partial p_x} = X + (p_x - c_x) \cdot \frac{\partial X}{\partial p_x} \tag{14}$$

$$\frac{\partial \pi_M}{\partial p_x} = X + (p_x - c_x) \cdot \frac{\partial X}{\partial p_x} + (p_z - c_z) \cdot \frac{\partial Z}{\partial p_x} \tag{15}$$

where π_x and π_M are the pre-merger and post-merger profit functions, respectively. If we evaluate equation (15) at pre-merger prices $(p_x, p_z) = (\hat{p_x}, \hat{p_z})$, noting that equation (14) equals zero at these prices, we get

$$\begin{aligned} \frac{\partial \pi_M}{\partial p_x} &= \frac{\partial \pi_x}{\partial p_x} + (p_z - c_z) \cdot \frac{\partial Z}{\partial p_x} \\ \left.\frac{\partial \pi_M}{\partial p_x}\right|_{p=\hat{p}} &= 0 + (p_z - c_z) \cdot \frac{\partial Z}{\partial p_x} \end{aligned} \tag{16}$$

Equation (16) tells us that, at pre-merger prices, increasing p_x a little will decrease profits, since p_z will not be below cost pre-merger and the goods are complements. Therefore, there is an incentive post-merger to decrease prices. However, this only works if the profit function is concave in prices. If this is not true, the sign of the derivative could change when prices are significantly different from pre-merger levels, and the prices that maximize profit may well be above the old prices. Still, this argument is further evidence for falling prices.

4.4 Pricing Below Cost and Ambiguous Price Changes

Indeed, work has been done to clarify these exceptions to the "all prices fall" rule, when it occurs, and what its effects on welfare are. Davis & Murphy (2000) show using a simple linear demand system that, instead of acting anticompetitively, Microsoft may have actually fostered competition and increased efficiency by packaging Internet

Explorer, its web browser, with versions of Windows.[9] Shy (1996) and Andriychenko et al (2006) examine systems of demands for complements and compare pre- and post-merger prices, as well as consumer surplus changes. Andriychenko et al extend Shy's analysis from perfect complements and linear demand to a more general setting of imperfect complements and general demand specifications.

Andriychenko et al find that when goods form a system of complements (i.e., consumers purchase α units of good two for every unit of good one they purchase, and α is a function of prices), a merger from two firms to a multiproduct monopoly decreases the system price (the total price paid by consumers for one unit of good one and α units of good two), increases total firm profits, and increases consumer surplus. However, the change in component prices is ambiguous; while at least one price will fall (possibly below cost, but possibly not), the other may rise.

These caveats are not completely disheartening; Andriychenko et al show that consumer welfare will increase post-merger. Since this result is obtained using arbitrary demand curves, and we can obtain market demand curves for public goods, Andriychenko et al's result should hold with public and private complements.

Finally, just because there exists the possibility that the post-merger price of the public good is higher, it is not necessarily true that this happens often. I will argue below that this is actually fairly unlikely; it probably benefits firms more to sell the public good at a lower price than to sell the private good at a lower price. The logic behind this is made apparent through the following numerical exercise.

[9]Davis & Murphy use a linear demand system with specific parameter values to show that Microsoft could have chosen a price of zero for Internet Explorer simply to maximize profits, not to engage in predatory pricing. They argue the plausibility of this setup: software often involves very low marginal costs of production, so a zero price is not too much of a loss, and Windows typically has a larger market than a single component program. Since Microsoft typically engages in this activity with a myriad of other software they develop, it is more likely that they were simply maximizing profit as a static problem.

5 A Numerical Example

Because the outcome of this merger is somewhat ambiguous, it is useful to have an illustrative numerical example to provide context for the various possible results. I use a linear demand specification for two goods, one public and one private; I assume that these demands are derived from utility maximization by a representative agent and lead to equations (2) and (3), letting the number of private goods $m = 1$. We can solve these equations for x_i and for Z. Since x_i is not an aggregate quantity, we must sum over all n consumers to find the aggregate demand. Since we are dealing with a representative agent, $X = n \cdot x_i$. Then the specification I will work with is

$$Z = A - b \cdot p_z - d \cdot p_x \tag{17}$$

$$X = n \cdot (E - f \cdot p_x - g \cdot p_z) \tag{18}$$

where A, b, d, E, f, and g are parameters and are nonnegative. Here, A and E are measures of market size, b and f are the own-price derivative coefficients, and d and g are complementarity parameters. Since these parameters are nonnegative, any increase in either price will decrease the quantity of the good demanded. Note that Z does not depend on n while X does. This is due to the nonrival nature of Z: as more consumers enter the market, they will free-ride on Z unless they have a higher marginal benefit, but these consumers have to purchase their own X. This is a key observation in my analysis.

There are five qualitatively different post-merger outcomes: p_x could rise while p_z falls (but not below cost), p_x could rise while p_z falls below cost, p_z could rise while p_x falls (but not below cost), p_z could rise while p_x falls below cost, or both prices could fall but stay above their respective costs.[10] Given specific parameter values, all

[10] If the price of a good falls below cost, as in the second and fourth cases, we refer to that good as a "loss leader" for the other good.

of these outcomes are possible, depending on the number of consumers n. However, some of these outcomes may result in negative quantities or other nonsensical results if we make no assumptions about the parameter values; this is an artifact of linear demand.

For the following illustrative examples, we use the following parameter values: $A = 12$, $b = 8.4$, $d = 1.2$, $E = 1$, $f = 0.3$, and $g = .15$.[11] The marginal costs are constant and set at $c_x = 0.5$ and $c_z = 0.05$. Note that the demand for Z is in some sense much larger than the demand for X, at least when n is small.

The results of a merger of Bertrand competitors for this demand specification is as follows:

n	Δp_x	Δp_z	Loss leader	Qty. X	Qty. Z
1	Down	Up	X	Up	Up
2, 3	Down	Up	None	Up	Up
4…16	Down	Down	None	Up	Up
17…44	Up	Down	None	Up	Up
$n > 44$	Up	Down	Z	Up	Up

Table 1: Changes in Prices and loss leaders for different n

Table 1 shows that if n is somewhat large, we should expect to see firms using the public good as a loss leader and reaping profits from the private good. Interestingly, for any value of n, both X and Z rise; this happens regardless of the directions of price changes. We cannot say with certainty what the welfare effects are since prices move in different directions when n is large.

Figure 1 plots the change in each price as a function of n, for $n \in \{1, 30\}$. This clearly shows the range for which both prices have fallen, and that p_x is increasing and p_z is decreasing as n becomes large.[12]

[11] I chose these values to illustrate the various possibilities and because they seem reasonable: own-price effects are larger than cross-price effects and Z is larger than X for small n. There is no econometric evidence behind the choice.

[12] These increases and decreases are monotone for $n \in [1, 431]$. Beyond that, there is a vertical asymptote as Δp_z and Δp_x approach $-\infty$ and ∞, respectively. I dismiss this as an artifact of the linear demand specification.

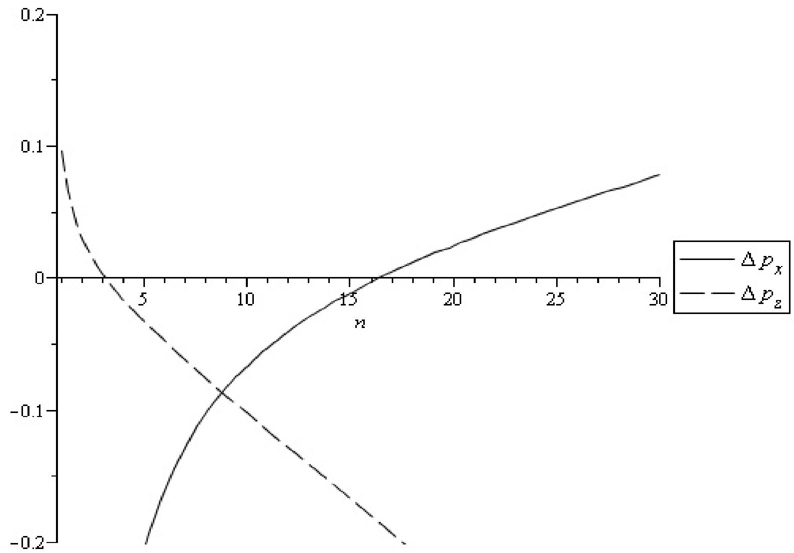

Figure 1: Changes in Prices as a function of n

This result should not be surprising. As noted above, the demand for Z is larger than the demand for X for small values of n. However, as n increases, this is no longer true. Since X is rival and Z is nonrival, a new consumer entering the market will purchase additional X, but will likely free-ride off of the existing Z. From the firm's perspective, then, lowering p_z enough to induce the sale of one extra unit of Z (i.e., lowering price by $1/d$) will allow all n consumers to consume one extra unit of Z, since it is nonrival. Then, since the goods are complements, each consumer will buy g/d more units of X, raising sales by $\frac{ng}{d}$. Intuitively, each additional sale of Z will make all n consumers consider buying an additional X for themselves, even if this means selling Z at a lower price or even below cost. As n becomes large, this tradeoff cannot be ignored, and Z becomes the obvious choice for a loss leader.

This example shows that while many combinations of price changes are possible, it seems that after a merger, quantities of both goods will rise, and that p_z will fall for any reasonable value of n. This may not hold exactly true for other kinds of demand systems, but it is a good guideline for future work.

6 Applications

The above arguments provide a fairly general framework that can be applied to numerous situations, either in terms of policy recommendations or profit motives. Many involve information goods that share characteristics with public goods, while others involve a conscious decision to treat digital goods as public rather than private. This is certainly not an exhaustive list, and each of these topics deserve deeper treatment than the few pages I am able to give them at present.

6.1 Hardware-Software Models

Consider a set of firms, each of which produces a system of complements (hardware and software) such that each firm's software is incompatible with any other firm's hardware, and assume that consumers have unit demand for hardware. Usually, these markets reach an equilibrium where a given consumer adopts one firm's hardware platform and then only buys software from this firm. Then the firms compete by incentivizing consumers to buy their hardware, often by selling the hardware at or below cost. The firms reap profits on software sales once consumers are locked into their platform.

However, it may be possible that if the software is (or could be made to be) nonrival, then this may be an extra incentive for consumers to choose one particular platform over another. Consider the two-firm case, where one firm's software is a traditional private good, and the other firm's software is nonrival. This creates additional network effects: upon the purchase of the second firm's hardware, the consumer immediately has access to all software purchased by the firm's other customers. This is a large incentive to adopt the second platform over the first. The new consumer may or may not purchase additional software, but they will purchase the firm's hardware. It could be that by making their software nonrival, one firm attracts more subscribers than the other and profits accordingly.

Note that this is, to some extent, the opposite of what usually happens in hardware-software models; i.e., with private software, firms sell hardware cheaply and make up the difference in software sales. Here, firms could forego software sales to free-riders and make up the difference in hardware sales.

While classic examples of hardware-software systems simply do not make sense in this context, other examples may work, given recent technological advances. A razor company certainly cannot make its razorblades nonrival, but Amazon (the manufacturer of the Kindle e-reader) could choose to ignore piracy of its e-books to increase demand for Kindle.[13] If piracy of e-books is rampant, the production of one additional title makes the consumption of this title nonrival. If Amazon does this with its software before Sony does with its Sony Reader, it could gain a market advantage by snatching up new customers.

Consider the following static, two-firm game. Each firm produces hardware and software, and each firm chooses whether or not to make its software nonrival. If both firms choose rival, they split the market and each receives a payoff of 20. If both firms choose nonrival, they split the market again, since the network effect discussed above is present with either firm. However, sales of software are foregone, and each firm receives a payoff of 15. However, if one firm chooses rival and the other chooses nonrival, the network effect exists for only one firm and more consumers choose that platform. Then the firm that chose rival has a smaller market share and receives a payoff of 10, while the firm that chooses nonrival gains market share and receives a payoff of 25. This is a classic Prisoners' Dilemma, where "rival" is mum and "nonrival" is fink. If this is true, then many of the digital hardware-software markets would be in a non-equilibrium outcome from this static point of view, but dynamic considerations could change this. Obviously, it is possible that the payoffs from (nonrival, nonrival)

[13] Additionally, Amazon would need to encrypt its e-books or otherwise prevent them from being readable by competitors' hardware.

are smaller than those from (rival, rival), since the sales to free-riders are foregone. A fuller treatment of this game would have to examine cost structures to discern the profitability of selling hardware significantly above cost.

Now consider the following dynamic, two-firm game. Each firm produces its hardware and software as before, but play is repeated for an infinite number of periods. At the outset, each firm's software is considered rival. Each period, the firm (if it has not already done so) may make its software nonrival, chooses a market price for its software and one for its hardware, and sells its goods to willing consumers. If it has already made its software nonrival in a previous period, it cannot revert to rival in a subsequent period.[14] Note that in a given period, consumers who previously purchased one firm's hardware are more likely to stay with that firm than switch. Now, assume that in each period t, the number of potential consumers in the market is $S(t)$, and that firms do not know the value of $S(t)$ until period t. These are consumers whose demand for hardware is greater than zero for some price.[15] Also assume that $S(t)$ is nondecreasing in t, that is, as time passes, more consumers become interested in a type of product. Finally, assume that all possible payoffs are increasing in $S(t)$ and are structured similarly to a repeated Prisoners' Dilemma, where "rival" is mum and "nonrival" is fink.

One possible outcome from this game is that while t is small, both firms will choose "rival", since they know that defection would gain them a small amount of profit (since $S(t)$ is still small). However, as t increases, so does $S(t)$, and eventually the gain from defection (which, again, is an increasing function of $S(t)$) is too large to ignore. When this happens, firms defect, make their software nonrival, and the game collapses to the (fink, fink) equilibrium.

In the real world, one firm will announce its plans to make its software nonrival

[14] This could plausibly be due to public relations reasons: once given the ability to download software freely, consumers would be very put off if this were taken away from them.

[15] Note that these consumers may not purchase if hardware prices are too high. This is the set-complement of the set of consumers that will not purchase hardware at any price.

before the other, and this firm will gain more market share than the lagging firm, giving it more payoffs in all subsequent periods. The game above could (and should) be refined to reflect this.[16]

6.2 Corporate and Personal Software Licensing

Consider a firm such as Microsoft, which produces software suites like Microsoft Office. This software has a somewhat significant learning curve to it, so once users feel comfortable with it, they are disinclined to switch to another platform. An example that is perhaps a bit closer to home is the problem of choosing MATLAB, Stata/Mata, Ox, etc., to do modeling. Professor X prefers MATLAB despite the fact that the task he wants to perform is best done in Ox, and he will not switch because it is costly to learn Ox.

Consider a university or another large-scale enterprise with dozens, possibly hundreds of computers that each needs software capable of wordprocessing (or nonlinear regression). Training employees to use software is a costly endeavor, and these firms want to keep training costs as low as possible. If the majority of Statistics Canada employees are more comfortable with EViews than with Stata, StatsCan has an incentive to buy licenses for EViews rather than Stata.

Now consider the 21^{st}-century problem of a firm that produces software like this. Software piracy is rampant, and firms spend sizeable portions of their profits in making their software harder to steal and in suing those that do pirate. Further, it is much more difficult to successfully prosecute individual consumers for piracy of a single copy of software than it is to police large corporations and universities.

These firms want as many people using their software as possible, since the network effect will net them additional sales. However, they attempt to stop people from using

[16] Finally, consider the dynamic game where each firm produces new "generations" of its hardware and software, each generation's production involves the repeated play of the above dynamic game, and consumers have some intergenerational loyalty to platforms.

their software without paying for it. An alternative option is to price-discriminate: charge corporations and universities, who are easy to keep legal, for software, and give away the software to individual users. Then users will become familiar with the software at home, while corporations save on training costs since their employees are playing with their free copy of Stata in their free time.

Another possibility, however, is to keep selling software to corporations and to individuals at a non-zero price, but to only prosecute corporations and other large groups for piracy. Individuals could freely download a copy of Microsoft Excel from The Pirate Bay without fear of prosecution (or buy a copy from Microsoft), learn how to use it effectively, and then use it more productively at work. Their employer saves on training costs and pays for corporate licenses.

It is difficult to say whether any individuals will still purchase software, since they could possibly pirate the corporate version. This may result in a larger free-rider externality.

6.3 Digital Music and Concert Revenues

Piracy looms large on the radar of musicians and record labels, not just software manufacturers. Companies like Universal Music Group and lobbying associations like the RIAA and IFPI spend millions to combat music piracy, blaming it for the decline of record sales over the past two decades. The problem is that as soon as a new album is released (and often before), one consumer buys the CD, rips the music to his computer's hard drive, and uploads the songs to a filesharing site online. From that point on, everyone with access to that website has unfettered, nonrival access to these songs. Many choose not to purchase the new album and instead free-ride by downloading the album online.

Consumers that listen to CDs and to MP3s of their favorite artists also buy concert tickets to see their favorite artists live. While concerts are technically nonrival, they are definitely excludeable, and they are complementary to album consumption. Further, free-riders that download their music online will pay for concert tickets, buy the band's t-shirts, and (importantly) tell others about this band so that they too will buy concert tickets and t-shirts.

Record labels could harness this community of filesharing to increase their profits from concert and merchandise revenues. By choosing not to prosecute music piracy, more people consume music. These people tell their friends, who discover new artists to enjoy. Since piracy lowers the cost of sampling new music, these new listeners become fans of artists they would have otherwise never found. This greatly increases the number of people consuming music, which increases demand for concert tickets and artist merchandise, which finally increases revenue to the record label. In short, allowing music consumption to be nonrival could increase record label profits through increased concert and merchandise revenue.

This line of reasoning can work in some markets in which piracy is a problem. Digitally pirated movies, if they are leaked before the in-theater release date, can serve as free publicity for the movie, resulting in more box-office traffic. However, it may not work in others. It is difficult to think of a private complement to goods such as computer games. Some publishers may release the game for free and charge for premium in-game features, but this is selling below cost instead of making the good nonrival.

7 Concluding Remarks

We have seen that a merger between two Betrand competitors in a market with two complementary goods, one public and one private, will lower at least one price, and this will most likely be the price of the public good. The merged firm will often sell the good at a loss, and the provision of the private good will very likely be higher.

What can this tell us about efficiency? If we consider this problem in the context of policy analysis, the question becomes, "Should we allow this merger to take place?" As Andriychenko et al showed, the answer should probably be yes, at least for two private complements. If one of the goods is public, does this change our answer?

Compare the voluntary-contributions equilibrium outcome to that faced by a quasi-social planner, an authority who has power over how much consumers contribute, but not over how much firms charge. Just as in elementary public economic theory, the quasi-social planner will "demand" more of the public good than would be demanded in the voluntary-contributions equilibrium. However, if a policymaker only has authority to allow or forbid mergers, and not to dictate the contributions of consumers, the quasi-social planner's outcome cannot be easily obtained. Since this merger will increase the quantity of the public good in a market, this will bring it closer (or possibly surpass) the amount prescribed by the quasi-social planner. Not only is this merger likely to increase welfare, it is likely to provide a more optimal level of provision.

More work should be done to see when the answer to the above question is a conclusive yes, at least in the case of public-private complements. It should be determined when such a merger would increase social efficiency; i.e., taking into account that prices are not equal to marginal cost. If the anticompetitive inefficiency due to the merger is non-negligible and outweighs the efficiency gain due to the increase in public good provision, the merger may not be such a good idea.

Further, since the price of the public good we end with post-merger is generally not the efficient one, and since demand for the private good depends on the public good's price, the private good is probably not provided in efficient amounts either. More work is needed to determine the efficient production level of the private complement, and whether post-merger production is closer to this level.

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