Optimal interest-rate rules with inflation inertia: inflation stabilization versus price-level stabilization

by

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1 Introduction

Since 1995 the Bank of Canada has endeavored to keep the inflation rate at the low and stable rate of 2% (the mid-point of the 1-3% range). Many economists agree that a low and stable inflation rate is advantageous for society, and many research papers have espoused the inflation target as ideal. Currently, more than 20 countries have set such targets, following the example of New Zealand in 1990. Though this is the case, there has yet to be a universal consensus on what type of target they should commit to: inflation targetting or price-level targetting. That is why the Bank of Canada (2006) in renewing its target in November 2006, asked its researchers and other interested researchers to answer the following question: "What are the relative merits of inflation targeting versus price-level targeting in an open economy susceptible to large and persistent terms-of-trade shocks?"

Recently, there has been discussion that a price-level target may be better than setting an inflation target. Since no economy has tried the price-level regime except for Sweden during the 1930s, all of the research on this subject has been purely theoretical. Of the research done on this subject, many economists agree that in the long term a price-level target is better since its path is known. However, in the short term, there is no consensus among economists on which regime is better.

The discussion on the short-term advantage of a price-level target began with Svensson (1999) arguing about the possibility of a free lunch, which is a reduction of the inflation variability without increasing output variability. Many publications have since followed that paper, in which they confirm Svensson's statement but with some major assumptions on people's expectations. In particular, in a Keynesian framework, targeting the price-level instead of inflation is welfare improving with the assumption of purely forward looking expectations from agents.

Giannoni (2010) is one of the latest papers supporting the argument that price-level targeting outclasses inflation targetting. He studies the difference between those two targets by measuring the welfare loss of two different policies, a Taylor rule and a Wicksellian rule, in a simple New Keynesian model. The first policy responds to fluctuations in inflation and the latter to those in the price level. Giannoni concludes that a Wicksellian rule systematically achieves better results than a Taylor rule.

In this paper I change some of Giannoni's assumptions and look at the effect on the loss function, which depends on the volatility of inflation, the output gap and the interest rate. The major modification in this paper is the switch of the Phillips curve. Because of the lack of evidence of purely forward-looking expectations, I use a hybrid Phillips curve, which gives some weight to past inflation, instead of a purely forward-looking New-Keynesian Phillips curve as Giannoni does. I also add the expectation of future inflation in the Taylor rule and of the price-level for the Wicksellian rule. This modification is made to model the policymaker's decisions more realistically.

I conclude that when adding some weight to past inflation in the Phillips curve, or adding the expectation of agents in the policy rules, or both, Giannoni's results do not change substantially. When minimizing a loss function composed of the variability of inflation, the output gap and nominal interest rate, a Wicksellian rule dominates a Taylor rule. I find that in most of the cases a Wicksellian rule achieves a lower welfare loss, the policy responses to different persistences of shocks are smoother and there are fewer undetermined equilibria than under a Taylor rule.

2 Research context

As stated above the main advantage of a price-level target in the long run is its predictability. Under an inflation target regime the policymaker is not constrained to bring back the price-level to its path when a shock drifts the price-level from its target. Therefore the forecast errors range for the price-level under such a regime is unbounded. However in the short run the argument is not as straighforward.

A traditionnal argument against price-level targetting in the short run says that when commiting to such a regime it increases both the volatility of inflation and output. This is because when the price-level drifts from its path the policymaker must intervene in order to bring it back. This outcome has been shown by Lebow, Roberts and Stockton (1992) and Haldane and Salmon (1995). However Svensson

(1999) re-opened that debate by showing that under certain conditions it is welfare-improving to target the price-level instead of inflation. He comes to that result by minimizing a loss function subject to a New classical Phillips curve with output persistences under a discretionnary regime. He uses a loss function where the policymaker faces a trade-off between output and inflation volatility and shows that when targeting the price-level, the central bank can achieve a lower welfare loss without increasing the output volatility, which he calls a "free lunch".

Following Svensson's paper there have been many discussions of price-level targeting in the short run. I begin by briefly summarizing the conclusions that have been reached under a commitment policy regime and then under a discretionnary policy regime. Also note that all the papers I review fall under a New Keynesian framework which follows the method of Clarida, Gali and Gertler (1999). For a more exhaustive review of the literature on price-level targeting see Ambler (2009) and Côté (2007)

Barnett and Engineer (2001) show that targeting the price-level is optimal when purely forward-looking expectations are assumed and when using the same loss function as Svensson. However, with a combination of predetermined and forward-looking expectations, which is called a hybrid Phillips curve, an inflation target becomes optimal. That is, when increasing the weight on predetermined expectations in the Phillips curve inflation targeting is optimal.

Steinsson (2003) studies an exercise similar to Barnett and Engineer (2001) but he adds the first lag of the output gap to the hybrid Phillips curve. He also uses a quadratic loss function, which differs from the one used by Svensson. Like Barnett and Engineer (2001) he concludes that price-level targeting is welfare improving when expectations are forward looking but when rule-of-thumb price setters are added to the model, it becomes less optimal to offset price-level drift.

On the other hand, Coletti, Lalonde, and Muir (2008), by using simple policy rules and adding the variability of the nominal interest rate in the loss function, find that targeting the price-level performs slightly better than inflation targeting. To show that, they use the IMF global economy model, which is a dynamic stochastic general equilibrium model in a open economy framework, that they calibrate to the Canadian economy.

Conclusions under a discretionary regime are similar to those with commitment. Vestin (2006) shows that with a purely forward-looking Phillips curve one can achieve the same optimum as under a commitment framework with an inflation target.

Yetman (2005) challenges Svensson "free lunch" by changing the assumption of purely forward-looking inflation expectations that Svensson made. Like Barnett and Engineer (2001) or Steinsson (2003) he shows that if some of the agents do not have forward-looking expectations it is not optimal anymore to target the price-level.

Gorodnichenko and Shapiro (2007) show that a price-level target is superior to an inflation target by comparing the welfare loss of a Taylor rule and a Taylor rule that puts weight on the price-level gap. They use a New Keynesian framework with a hybrid Phillips curve but in opposition to Coletti, Lalonde and Muir (2008) they only consider the variability of the inflation and the output gap in their loss function.

Finally, Marc Giannoni (2010) compares Taylor rules, that respond to fluctuations in inflation, with Wicksellian rules, that respond to fluctuations in the price level. He uses a pure forward looking Phillips curve and a loss function that includes the inflation, the output gap and the nominal interest rate variabilities, like Coletti, Lalonde and Muir (2008). He finds that a Wicksellian policy rule systematically achieves better results than a Taylor rule. He also shows that his results are robust to different shock persistences.

With or without commitment, when introducing inflation inertia to the model the superiority of the price-level sometime disapears. The importance of inflation inertia has been discussed by Woodford (2007) who explains that both inflation expectations and inflation inertia should be taken into account. Therefore a hybrid Phillips curve should be used.

3 Economic model and solution method

3.1 New Keynesian model

The model used by Giannoni is a basic New Keynesian model with the following structural equations:

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - e_t) \quad (1)$$

$$\pi_t = k x_t + \beta E_t \pi_{t+1} + u_t \quad (2)$$

$$p_t = p_{t-1} + \pi_t \quad (3)$$

Here x_t is the output gap, π_t is the inflation rate, i_t is the nominal interest rate, p_t is the price-level, and u_t and e_t are composite shocks. The parameter σ represents the inverse of the intertemporal elasticity of substitution, β is the discount factor of the representative household and k depends on the speed of price adjustment. Equation (1) represents the intertemporal IS equation, equation (2) is the New Keynesian supply equation, which is also known as the New Keynesian Phillips curve, and equation (3) is the evolution of the price-level.

The two shocks follow AR(1) processes:

$$e_t = \rho_r e_{t-1} + \epsilon_t^e \quad (4)$$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \quad (5)$$

where ρ_e and ρ_u are the persistence of the shocks, and ϵ_t^r and ϵ_t^u are idiosyncratic disturbances. Those two shocks, e_t and u_t, have a mean of zero and I assume they are uncorrelated. I also assume that their persistences are non-unit root and non-explosive, that is $|\rho_u|$ and $|\rho_e|$ are less than 1.

3.1.1 Loss function

The loss function is commonly used as a proxy for the welfare loss of the economy. The objective of the policymaker is to minimize the following equation:

$$E[L] = E[(1-\beta)\sum_{t=0}^{\infty}\beta^t[\pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_i(i_t - i^*)^2]] \qquad (6)$$

where i^* and x^* represent the optimum for the output gap and the interest rate. They are set to zero in order to simplify the problem. The parameters λ_x and λ_i correspond to the weights that the central bank gives to the variability of the output gap and the interest rate respectively.

In this case, where $x^* = 0$ and $i^* = 0$, the steady state is the same for every period. With this assumption I can rewrite the loss function as:

$$E[L] = var(\pi_t) + \lambda_x var(x_t) + \lambda_i var(i_t) \qquad (7)$$

which is a weighted sum of the variances of the state variables. Normally only the variability of the output gap and the inflation rate are the determinants of the loss function since they are the only components of the Phillips curve. However Giannoni (2010) also inserts the variability of the interest rate in order to take into account the welfare costs of transactions and the zero lower bound on nominal interest rates.

3.1.2 Calibration

The values of all the parameters are set for the United States economy and they are summed up in Table 1. The parameters from the structural equations and the shock equations are taken from Rotemberg and Woodford (1997) and those from the loss function are taken from Woodford (2003).

Table 1: Parameter Values

Structural parameters	
β	0.99
σ	0.1571
k	0.0238
Shock processes	
ρ_e	0.35
ρ_u	0.35
$var(e_t)$	13.8266
$var(u_t)$	0.1665
Loss function	
λ_x	0.048
λ_i	0.236

3.1.3 Policy-rule examples

Giannoni (2010) examines four different policy equations: the simple Taylor rule, the simple Wicksellian rule and the quasi-optimal Taylor and Wicksellian rules. In my paper I am only considering the simple Taylor and Wicksellian rules. The quasi-optimal rules extend the latter rules by including interest-rate inertia.

Under the first rule, the simple Taylor rule, the policymaker responds to fluctuations in the output gap and inflation. The policymaker is committing to the following policy:

$$i_t = \psi_\pi \pi_t + \psi_x x_t \tag{8}$$

where ψ_π and ψ_x are the policy coefficients. The simple Wicksellian rule is similar to the Taylor rule but the policymaker responds to the price-level instead of the inflation:

$$i_t = \psi_p p_t + \psi_x x_t \tag{9}$$

where ψ_p and ψ_x are the policy coefficients.

3.2 Solution method

In this section I demonstrate how to solve the model, that is find the equilibrium, under simple Taylor and Wicksellian rules. I focus on the non-inertial inflation case, which is without an inflation lag in the Phillips curve. The inertial case follows the same steps as the non-inertial case but with a different equation for the variance. This difference will be discussed in the section on inflation inertia.

3.2.1 Solving the model with a simple Taylor rule

If I define the state variables as the following vector:

$$z_t = \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix}$$

then, I can rewrite the intertemporal IS equation (1), the Phillips curve (2) and the simple Taylor rule (8) in matrix form:

$$Az_t = BE_t z_{t+1} + a\nu_t \tag{10}$$

where:

$$A = \begin{pmatrix} 0 & 1 & \sigma^{-1} \\ 1 & -k & 0 \\ -\psi_\pi & -\psi_x & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} \sigma^{-1} & 1 & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$a = \begin{pmatrix} \sigma^{-1} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\nu_t = \begin{pmatrix} e_t \\ u_t \end{pmatrix}$$

The system can be rewritten as:

$$z_t = A^{-1}BE_t z_{t+1} + A^{-1}a\nu_t \tag{11}$$

$$z_t = F_1 E_t z_{t+1} + F_2 \nu_t \tag{12}$$

where $F_1 = A^{-1}B$ and $F_2 = A^{-1}a$.

By using the guess-and-verify method I assume that the endogenous variables depend only on the shocks, so I can write the z_t vector as:

$$z_t = \omega\nu_t \tag{13}$$

where:

$$\omega = \begin{pmatrix} \pi_e & \pi_u \\ x_e & x_u \\ i_e & i_u \end{pmatrix}$$

Given the shock processes (4) and (5) and where:

$$\rho = \begin{pmatrix} \rho_e & 0 \\ 0 & \rho_u \end{pmatrix},$$

I am now able to find the expectation of the z_{t+1} vector:

$$\begin{aligned} z_{t+1} &= \omega\nu_{t+1} && (14) \\ E_t z_{t+1} &= \omega E_t \nu_{t+1} && (15) \\ &= \omega\rho\nu_t. && (16) \end{aligned}$$

Since I have two definitions, (12) and (13), of z_t and one for its expectation (16), I am able to find a solution to the problem. By using my guess (13) and its forecast implication (16) in the original system

(12) I get:

$$\begin{aligned}\omega\nu_t &= F_1 E_t z_{t+1} + F_2\nu_t && (17)\\ &= F_1(\omega\rho\nu_t) + F_2\nu_t && (18)\end{aligned}$$

Thus the undetermined matrix of coefficients satisfies:

$$\omega = F_1\omega\rho + F_2 \qquad (19)$$

After solving for ω, I am able to find the variance of the inflation rate, the output gap and the interest rate:

$$\begin{aligned}z_t &= \omega\nu_t && (20)\\ var(z_t) &= var(\omega\nu_t) && (21)\\ &= \omega var(\nu_t)\omega^t && (22)\end{aligned}$$

where:

$$var(z_t) = \begin{pmatrix} var(\pi_t) & cov(\pi_t, x_t) & cov(\pi_t, i_t) \\ cov(x_t, \pi_t) & var(x_t) & cov(x_t, i_t) \\ cov(i_t, pi_t) & cov(i_t, x_t) & var(i_t) \end{pmatrix}$$

and

$$var(\nu_t) = \begin{pmatrix} var(e_t) & 0 \\ 0 & var(u_t) \end{pmatrix}$$

since I assume $cov(e_t, u_t) = 0$.

3.2.2 Solving the model with a simple Wicksellian rule

By substituting the structural equation of the inflation rate (3) in the intertemporal IS equation (1) and the Phillips curve (2), and now including the Wicksellian rule (9), the system of equations become:

$$
\begin{aligned}
x_t + \sigma^{-1} i_t + \sigma^{-1} p_t &= E_t x_{t+1} + \sigma^{-1} E_t p_{t+1} \sigma^{-1} e_t && (23)\\
(1+\beta) p_t - k x_t &= p_{t-1} + \beta E_t p_{t+1} + u_t && (24)\\
i_t - \psi_p p_t - \psi_x x_t &= 0 && (25)
\end{aligned}
$$

If I define the state variables as the following vector:

$$
z_t = \begin{pmatrix} p_t \\ x_t \\ i_t \end{pmatrix}
$$

I can then write the above equations in matrix form:

$$
A z_t = B E_t z_{t+1} + C z_{t-1} + a \nu_t \qquad (26)
$$

where:

$$
A = \begin{pmatrix} \sigma^{-1} & 1 & \sigma^{-1} \\ 1+\beta & -k & 0 \\ -\psi_p & -\psi_x & 1 \end{pmatrix}
$$

$$
B = \begin{pmatrix} \sigma^{-1} & 1 & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$
C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$a = \begin{pmatrix} \sigma^{-1} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\nu_t = \begin{pmatrix} e_t \\ u_t \end{pmatrix}$$

Then I can re-arrange the system as:

$$z_t = A^{-1}BE_t z_{t+1} + A^{-1}Cz_{t-1} + A^{-1}a\nu_t \tag{27}$$

$$z_t = F_1 E_t z_{t+1} + F_2 z_{t-1} + F_3 \nu_t \tag{28}$$

where $F_1 = A^{-1}B$, $F_2 = A^{-1}C$ and $F_3 = A^{-1}a$.

Again using the guess-and-verify method, I assume that the endogenous variables obey the following equation:

$$z_t = \delta z_{t-1} + \omega \nu_t \tag{29}$$

where:

$$\delta = \begin{pmatrix} 0 & 0 & \delta_x \\ 0 & 0 & \delta_i \\ 0 & 0 & \delta_p \end{pmatrix}$$

and

$$\omega = \begin{pmatrix} \omega_{xe} & \omega_{xu} \\ \omega_{ie} & \omega_{iu} \\ \omega_{pe} & \omega_{pu} \end{pmatrix}$$

Then I can find the expected values:

$$z_t = \delta z_{t-1} + \omega \nu_t \tag{30}$$

$$z_{t+1} = \delta z_t + \omega \nu_{t+1} \tag{31}$$

$$E_t z_{t+1} = \delta z_t + \omega \rho \nu_t \tag{32}$$

given the shock processes (4) and (5).

Again I can use the guess (29) and its forecast (32) in the system (28):

$$\delta z_{t-1} + \omega e_t = F_1 E_t z_{t+1} + F_2 z_{t-1} + F_3 \nu_t \tag{33}$$

$$= F_1(\delta z_t + \omega \rho \nu_t) + F_2 z_{t-1} + F_3 \nu_t \tag{34}$$

$$= F_1 \delta(\delta z_{t-1} + \omega \nu_t) + F_1 \omega \rho \nu_t + F_2 z_{t-1} + F_3 \nu_t \tag{35}$$

Thus the undetermined coefficents satisfy:

$$\delta = F_1 \delta^2 + F_2 \tag{36}$$

$$\omega = F_1 \delta \omega + F_1 \omega \rho + F_3 \tag{37}$$

After solving for ω and δ I am able to find the variance of the inflation rate, the price-level, the output gap, and the interest rate. In contrast to the variance I find with the model with the simple Taylor rule, now I have to proceed in two steps. First I have to find the unconditional variance of the price-level and then I can find the other variances. According to Hamilton (1994) the unconditional variance of an AR(1) process with two shocks is:

$$var(p_t) = \frac{\omega_{pe}^2 var(e_t) + \omega_{pu}^2 var(u_t)}{1 - \delta_p^2} \quad (38)$$

And then:

$$\begin{aligned} var(x_t) &= \omega_{xe}^2 var(e_t) + \omega_{xu}^2 var(u_t) + \delta_x^2 var(p_t) \quad &(39) \\ var(i_t) &= \omega_{ie}^2 var(e_t) + \omega_{iu}^2 var(u_t) + \delta_i^2 var(p_t) \quad &(40) \end{aligned}$$

Finally, by substituing p_t from the state variables vector (29) into the definition of inflation (3), I can get its variance:

$$\begin{aligned} \pi_t &= p_t - p_{t-1} \quad &(41) \\ &= (\omega_{pe} e_t + \omega_{pu} u_t + \delta_p p_{t-1}) - p_{t-1} \quad &(42) \\ &= \omega_{pe} e_t + \omega_{pu} u_t + (\delta_p - 1) p_{t-1} \quad &(43) \\ var(\pi_t) &= \omega_{pe}^2 var(e_t) + \omega_{pu}^2 var(u_t) + (\delta_p - 1)^2 var(p_{t-1}) \quad &(44) \end{aligned}$$

3.2.3 Policy coefficient optimization

With the variances of the state variables under a Taylor rule (22), and under a Wicksellian rule (38)-(40) and (44), I am able to minimize the loss function (7) by choosing the proper policy coefficients, ψ_x and ψ_π or ψ_p according to the respective policy rule. However I am not looking for algebric equations of the policy coefficients therefore I am using numerical analysis, a simple grid search, in order to find the values of those policy coefficients and the resulting variances of the state variables.

4 Replication of Giannoni's result

In this section I want to assess my solution method by reproducing Giannoni's optimal policy coefficients. I also want to get the same variances and loss functions values as Giannoni that result from the latter coefficients.

By using the solution method described above in subsection 3.2 I find the same results as Giannoni for the simple Taylor rule model. However, presently I am unable to replicate Giannoni's results for the simple Wicksellian rule. I contacted the author but I did not receive a response. Meanwhile, I find that the problem resides in the variance of the price-level ($var(p)$) since when I use the coefficients found with the solution method and Giannoni's $var(p)$, I get the same variances of the output gap, the inflation rate and the interest rate as the author. This exercise confirmed that my coefficients are valid, and therefore, by simulating the endogenous variables I should get the same variances as Giannoni. Unfortunately this method gives the same variance as using the variance equations (38)-(40) and (44). In order to be rigorous, I repeat the exercise with the Taylor rule and I find that the variances resulting from the simulation are the same as Giannoni's. Consequently I assume that my results for the model with the Wicksellian rule are valid. Giannoni's results and my results for the Wicksellian rule can be found in Table 2 and the results for the Taylor rule can be found in Table 3.

My results are very similar to those of Giannoni. However, as stated above, the main difference reside in the variances of the price-level where they are systematically higher than Giannoni's results. Besides the latter difference there is also the policy coefficents of the price-level that are mainly greater than those of Giannoni.

5 Inflation inertia

As stated above, the main purpose of this essay is to include backward looking inflation, or adjustment that follows a rule-of-thumb, in the Phillips curve (2). This modified Phillips curve with both inflation inertia and inflation expectations is called hybrid Phillips curve. Inflation inertia is added in order to follow the empirical evidence that a purely forward-looking Phillips curve does not exist. According to Nason and Smith (2008) the weight of the lag of the inflation for the United States, α, in the hybrid Phillips curve is in the range 0.28-0.42. However in this essay I use the values $\alpha = 0.10$, 0.35, the mid-point of the latter range, and 0.89 in order to see the optimal policy when more weight is added to the inflation lag. Also note that with inflation inertia in the model, β decreases by the same value that α takes in order to give less weight to the expectation of inflation and more to its lag. That is β can be expressed as $0.99 - \alpha$.

When adding inflation inertia to the Phillips curve (2) I get:

$$\pi_t = kx_t + \beta E_t \pi_{t+1} + \alpha \pi_{t-1} + u_t \tag{45}$$

Using the solution method described in the previous section with the modified Phillips curve (45) I can solve the model under a Taylor rule. The results to the latter problem are shown in Table 3. Also note that the gray cells in the tables indicate an indetermine equilibrium. According to Giannoni, the Taylor rule results in a determinate equilibrium if and only if:

$$\psi_\pi + \frac{1-\beta}{k}\psi_x > 0 \tag{46}$$

and for the Wicksellian rule, if and only if:

$$\psi_p > 0 \; and \; \psi_x \geq 0 \tag{47}$$

For the Wicksellian rule, I have to substitute the structural equation of inflation (3) in the Phillips curve with inflation inertia (45):

$$(1+\beta-\alpha)p_t - kx_t \;\; = \;\; p_{t-1} + \beta E_t p_{t+1} - \alpha p_{t-1} + u_t \tag{48}$$

I can use a modified version of the solution method described above to solve the model with this new Phillips curve. However with inflation inertia, that is when $\alpha > 0$, the guess equations are composed with a lag of degree 2, therefore the definition of the variance of the price-level (38) is not accurate anymore. The new guess is:

$$z_t \;\; = \;\; \delta z_{t-1} + \gamma z_{t-2} + \omega \nu_t \tag{49}$$

where the state vector, z_t and the matrices of coefficients of the guesses, δ and ω, are the same as in the subsection 3.22, and:

$$\gamma = \begin{pmatrix} 0 & 0 & \gamma_x \\ 0 & 0 & \gamma_i \\ 0 & 0 & \gamma_p \end{pmatrix}$$

According to Hamilton (1994), the unconditionnal variance of an AR(2) process with two persistent shocks is:

$$var(p_t) \quad = \quad \frac{(1-\gamma_p)(\omega_{pe}^2 var(e_t) + \omega_{pu}^2 var(u_t))}{(1+\gamma_p^2)((1-\gamma_p)^2 - \delta_p^2)}. \tag{50}$$

The results using the Wicksellian rule are summarized in Table 4.

If looking at the determined equilibrium results of the model with the Taylor rule, when the weight of inflation inertia, α, takes the value 0.10, the loss function values are similar to those without inflation inertia but the optimal policy coefficients are smoother. However when the weight of inflation inertia increases both the loss function values and the policy coefficients are higher than without inertia and there are more undetermined equilibria.

Under the simple Wicksellian rule the loss function values and the policy coefficients increase as the weight of inflation inertia increases. However the policy coefficients are lower than the results without inflation inertia except when the weight of the inflation inertia takes its highest value. Also, while the model under a Wicksellian rule had no undetermined equilibrium, with inflation inertia they appear.

If comparing the Taylor rule and the Wicksellian rule results Giannoni's conclusions do not change much in general. That is the Wicksellian rule loss function values and policy coefficient responses are all lower than under the Taylor rule except when the weight of inflation inertia takes its highest value. In this case the Taylor rule loss function's values are lower but only consist of three determined equilibria. Also, if focusing on the case where the weight of inflation inertia, α, takes the value 0.35, which is the mid-point of the range that Nason and Smith find for the weight of the inflation lag, the simple Wicksellian rule systematically achieves better results than the simple Taylor rule.

6 Extension of the policy rules

In this section, I look at how the loss value varies when expectation of inflation in the Taylor rule and expectation of the price-level in the Wicksellian rule are added. This is done in order to reproduce more accurately the decisions made by the policymaker, which now can be forward-looking. The Taylor rule (7) then becomes:

$$i_t = \psi_x x_t + \psi_\pi E_t \pi_{t+1} \tag{51}$$

and the Wicksellian rule becomes:

$$i_t = \psi_x x_t + \psi_p E_t p_{t+1} \tag{52}$$

Using the solution method I am now able to solve the problems with these modified policy rules, (51) and (52). The results are summarized in Tables 5 and 6 for the extension of the Taylor rule and of the Wicksellian rule respectively.

As for the simple Wicksellian rule above, the loss function values increase as the weight of inflation inertia, α, increases. The results with the extension of the Taylor rule follow the same pattern but slightly decrease when the weight of inflation inertia, α, takes its highest value.

If only comparing the extension of the Taylor rule with the extension of the Wicksellian rule when the weight of inflation inertia, α, takes the value 0.35, the Wicksellian rule again is superior to the Taylor rule. This is explained by lower welfare losses and smoother price-level policy coefficients. However the output gap policy coefficients under the Wicksellian rule are higher than under the Taylor rule but they are more stable for different shock persistences. Furthermore, in this case, both policies have the same incidence of undetermined equilibrium.

Finally when comparing the simple rules with their extensions it can be seen that when the weight of the inflation inertia take its highest value most of the loss function values and the output gap coefficients are lower. They also reach less undetermined equilibria. However, most of the price-level and inflation coefficients are higher with the extension rules. On the other hand, for all the other weights of inflation inertia and without inflation inertia, the simple rules achieve better welfare and their policy coefficients are mostly lower.

7 Conclusion

Following the work of Gorodnichenko and Shapiro (2007), Coletti, Lalonde and Muir (2008), and Giannoni (2010) I show that a policy that responds to price-level fluctuation is superior to a policy that responds to inflation. In order to demonstrate that I optimize the policy coefficients of a Taylor rule and a Wicksellian rule subject to a loss function that is composed of the variabilities of the inflation, of the output gap, and of the nominal interest rate under a New Keynesian framework. The Phillips curve I use is an hybrid, which includes the output gap, the expectation of inflation, and the first lag of inflation. I use this type of Phillips curve in order to follow the empirical evidence that a purely forward-looking Phillips curve does not exist and to add inflation persistence, or inflation inertia, to the model. I test the model with differents values of shock persistences and weights on the inflation lag of the Phillips curve in order to see how the policy coefficients respond to them and how the loss function values change. For most of the cases the simple Wicksellian rule is superior to the Taylor rule. The Wicksellian rule achieves a lower welfare loss, its policy coefficient responses to different persistences of shocks are less abrupt, and there is less incidence of undetermined equilibria than under a Taylor rule.

In future research I want to investigate how the outcomes of the models discussed in this paper vary when the parameter of the speed of price adjustment, k, the parameter of the inverse of the intertemporal elasticity of substitution, σ, and the loss function weight parameters, λ_x and λ_i, change. As stated in the calibration subsection, in this paper they are taken as given by Rotemberg and Woodford (1997) and Woodford (2003). I think it could be useful to change the value of those parameters in order to apply the models to a much larger range of countries.

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Appendix

Table 2: Simple Wicksellian rule results

		V(π)	V(x)	V(i)	V(p)	E[L]	ψ_p	ψ_π	ψ_x	ψ_i
My results										
$\rho_r = 0$	$\rho_u = 0$	0.122	13.978	1.228	0.124	1.083	1.996	-	0.180	-
	$\rho_u = 0.35$	0.145	14.808	1.261	0.153	1.154	1.793	-	0.200	-
	$\rho_u = 0.9$	0.167	19.361	1.386	0.181	1.424	1.720	-	0.228	-
$\rho_r = 0.35$	$\rho_u = 0$	0.155	16.311	2.820	0.138	1.604	3.470	-	0.172	-
	$\rho_u = 0.35$	0.165	17.379	2.896	0.150	1.682	3.295	-	0.200	-
	$\rho_u = 0.9$	0.155	21.960	3.147	0.145	1.952	3.260	-	0.248	-
$\rho_r = 0.9$	$\rho_u = 0$	0.227	11.466	8.548	0.195	2.794	5.443	-	0.336	-
	$\rho_u = 0.35$	0.219	12.227	8.713	0.190	2.862	5.620	-	0.376	-
	$\rho_u = 0.9$	0.211	17.185	8.746	0.183	3.099	5.708	-	0.396	-
Giannoni's results										
$\rho_r = 0$	$\rho_u = 0$	0.122	13.923	1.237	0.008	1.087	1.997	-	0.182	-
	$\rho_u = 0.35$	0.146	15.191	1.294	0.015	1.186	1.383	-	0.228	-
	$\rho_u = 0.9$	0.142	24.466	1.391	0.09	1.653	0.853	-	0.274	-
$\rho_r = 0.35$	$\rho_u = 0$	0.149	15.898	2.646	0.013	1.543	2.872	-	0.139	-
	$\rho_u = 0.35$	0.161	17.597	2.778	0.018	1.669	2.338	-	0.201	-
	$\rho_u = 0.9$	0.096	28.79	2.761	0.012	2.14	3.323	-	0.14	-
$\rho_r = 0.9$	$\rho_u = 0$	0.158	7.072	10.459	0.089	2.973	2.613	-	0.134	-
	$\rho_u = 0.35$	0.166	8.239	10.699	0.102	3.093	2.426	-	0.241	-
	$\rho_u = 0.9$	0.110	18.822	10.779	0.094	3.567	2.626	-	0.259	-

Table 3: Simple Taylor rule with inflation inertia

		V(π)	V(x)	V(i)	V(p)	E[L]	ψ_p	ψ_π	ψ_x	ψ_i
$\alpha = 0$										
$\rho_r = 0$	$\rho_u = 0$	0.269	13.495	2.03	1.68	1.401	-	0.641	0.325	-
	$\rho_u = 0.35$	0.391	13.957	2.233	4.224	1.593	-	1.291	0.263	-
	$\rho_u = 0.9$	0.144	30.354	2.159	3.217	2.122	-	3.658	0.038	-
$\rho_r = 0.35$	$\rho_u = 0$	0.358	9.989	6.747	3.631	2.435	-	0.888	0.694	-
	$\rho_u = 0.35$	0.479	10.451	6.949	6.175	2.627	-	1.724	0.572	-
	$\rho_u = 0.9$	0.233	26.848	6.876	5.168	3.156	-	5.041	0.089	-
$\rho_r = 0.9$	$\rho_u = 0$	0.5	0.529	10.437	39.175	2.993	-	-1.743	-3.222	-
	$\rho_u = 0.35$	0.622	0.991	10.64	41.719	3.185	-	-2.575	-4.495	-
	$\rho_u = 0.9$	0.375	17.388	10.566	40.712	3.714	-	-5.108	-0.283	-
$\alpha = 0.10$										
$\rho_r = 0$	$\rho_u = 0$	0.296	13.484	2.203	-	1.463	-	0.264	0.264	-
	$\rho_u = 0.35$	0.385	14.463	2.434	-	1.654	-	0.212	0.212	-
	$\rho_u = 0.9$	0.150	28.950	2.230	-	2.066	-	0.044	0.044	-
$\rho_r = 0.35$	$\rho_u = 0$	0.360	10.056	7.099	-	2.518	-	0.472	0.472	-
	$\rho_u = 0.35$	0.427	11.361	7.285	-	2.692	-	0.408	0.408	-
	$\rho_u = 0.9$	0.210	26.253	6.566	-	3.019	-	0.040	0.040	-
$\rho_r = 0.9$	$\rho_u = 0$	0.673	0.502	9.466	-	2.931	-	-4.640	-4.640	-
	$\rho_u = 0.35$	0.798	0.629	10.098	-	3.212	-	-3.328	-3.328	-
	$\rho_u = 0.9$	1.202	0.419	5.797	-	2.591	-	-0.408	-0.408	-
$\alpha = 0.35$										
$\rho_r = 0$	$\rho_u = 0$	0.298	14.077	2.553	-	1.576	-	2.576	0.416	-
	$\rho_u = 0.35$	0.301	15.935	2.734	-	1.711	-	2.773	0.400	-
	$\rho_u = 0.9$	0.173	24.690	2.425	-	1.931	-	3.403	0.352	-
$\rho_r = 0.35$	$\rho_u = 0$	0.326	13.887	6.151	-	2.444	-	4.022	0.416	-
	$\rho_u = 0.35$	0.304	16.013	6.271	-	2.552	-	4.258	0.448	-
	$\rho_u = 0.9$	0.182	24.429	5.686	-	2.696	-	5.340	0.224	-
$\rho_r = 0.9$	$\rho_u = 0$	1.029	0.404	8.524	-	3.060	-	-1.160	0.960	-
	$\rho_u = 0.35$	0.858	1.347	10.853	-	3.484	-	-1.990	-2.516	-
	$\rho_u = 0.9$	0.365	13.812	10.241	-	3.445	-	10.490	-1.232	-
$\alpha = 0.89$										
$\rho_r = 0$	$\rho_u = 0$	0.248	14.144	2.694	-	1.563	-	3.101	0.672	-
	$\rho_u = 0.35$	0.232	14.971	2.553	-	1.553	-	3.101	0.672	-
	$\rho_u = 0.9$	0.183	17.558	2.412	-	1.595	-	3.128	0.768	-
$\rho_r = 0.35$	$\rho_u = 0$	0.180	0.934	1.501	-	0.579	-	-2.786	-1.440	-
	$\rho_u = 0.35$	0.181	0.970	1.549	-	0.593	-	-2.786	-1.440	-
	$\rho_u = 0.9$	0.179	1.049	1.617	-	0.611	-	-2.786	-1.440	-
$\rho_r = 0.9$	$\rho_u = 0$	0.189	1.005	1.537	-	0.600	-	-2.786	-1.440	-
	$\rho_u = 0.35$	0.192	0.883	1.137	-	0.502	-	-2.398	-1.600	-
	$\rho_u = 0.9$	0.191	0.938	1.195	-	0.518	-	-2.398	-1.600	-

Table 4: Simple Wicksellian rule with inflation inertia

		V(π)	V(x)	V(i)	V(p)	E[L]	ψ_p	ψ_π	ψ_x	ψ_i
$\alpha = 0$										
$\rho_r = 0$	$\rho_u = 0$	0.122	13.978	1.228	0.124	1.083	1.996	-	0.180	-
	$\rho_u = 0.35$	0.145	14.808	1.261	0.153	1.154	1.793	-	0.200	-
	$\rho_u = 0.9$	0.167	19.361	1.386	0.181	1.424	1.720	-	0.228	-
$\rho_r = 0.35$	$\rho_u = 0$	0.155	16.311	2.820	0.138	1.604	3.470	-	0.172	-
	$\rho_u = 0.35$	0.165	17.379	2.896	0.150	1.682	3.295	-	0.200	-
	$\rho_u = 0.9$	0.155	21.960	3.147	0.145	1.952	3.260	-	0.248	-
$\rho_r = 0.9$	$\rho_u = 0$	0.227	11.466	8.548	0.195	2.794	5.443	-	0.336	-
	$\rho_u = 0.35$	0.219	12.227	8.713	0.190	2.862	5.620	-	0.376	-
	$\rho_u = 0.9$	0.211	17.185	8.746	0.183	3.099	5.708	-	0.396	-
$\alpha = 0.10$										
$\rho_r = 0$	$\rho_u = 0$	0.149	14.219	1.149	0.189	1.102	1.310	-	0.204	-
	$\rho_u = 0.35$	0.174	14.933	1.194	0.222	1.173	1.319	-	0.212	-
	$\rho_u = 0.9$	0.202	19.560	1.248	0.259	1.436	1.328	-	0.220	-
$\rho_r = 0.35$	$\rho_u = 0$	0.236	16.853	2.830	0.314	1.713	1.408	-	0.300	-
	$\rho_u = 0.35$	0.267	17.649	2.829	0.356	1.782	1.408	-	0.300	-
	$\rho_u = 0.9$	0.327	22.672	2.681	0.432	2.048	1.388	-	0.280	-
$\rho_r = 0.9$	$\rho_u = 0$	0.536	0.660	10.479	7.655	3.041	-0.118	-	-3.916	-
	$\rho_u = 0.35$	0.714	0.823	10.717	7.613	3.282	-0.186	-	-3.536	-
	$\rho_u = 0.9$	0.658	4.292	13.116	1.178	3.959	3.691	-	2.332	-
$\alpha = 0.35$										
$\rho_r = 0$	$\rho_u = 0$	0.159	11.133	2.238	0.329	1.222	1.434	-	0.388	-
	$\rho_u = 0.35$	0.190	12.127	2.240	0.394	1.301	1.434	-	0.388	-
	$\rho_u = 0.9$	0.252	15.763	2.289	0.522	1.549	1.434	-	0.388	-
$\rho_r = 0.35$	$\rho_u = 0$	0.238	15.172	3.465	0.493	1.784	1.434	-	0.388	-
	$\rho_u = 0.35$	0.270	16.165	3.466	0.558	1.864	1.434	-	0.388	-
	$\rho_u = 0.9$	0.331	19.801	3.516	0.686	2.111	1.434	-	0.388	-
$\rho_r = 0.9$	$\rho_u = 0$	0.412	1.158	12.413	1.795	3.397	-1.507	-	-3.736	-
	$\rho_u = 0.35$	0.389	1.488	13.320	1.504	3.604	-3.110	-	-4.624	-
	$\rho_u = 0.9$	0.626	5.760	14.212	1.821	4.256	-4.290	-	-3.080	-
$\alpha = 0.89$										
$\rho_r = 0$	$\rho_u = 0$	1.355	20.576	9.917	2.740	4.683	5.593	-	3.480	-
	$\rho_u = 0.35$	1.443	21.867	9.917	2.917	4.833	5.593	-	3.480	-
	$\rho_u = 0.9$	1.599	24.176	9.917	3.234	5.100	5.593	-	3.480	-
$\rho_r = 0.35$	$\rho_u = 0$	1.366	20.799	11.069	2.763	4.977	5.593	-	3.480	-
	$\rho_u = 0.35$	1.454	22.090	11.069	2.940	5.126	5.593	-	3.480	-
	$\rho_u = 0.9$	1.610	24.400	11.069	3.256	5.394	5.593	-	3.480	-
$\rho_r = 0.9$	$\rho_u = 0$	0.561	22.245	15.404	1.434	5.264	-6.110	-	-2.492	-
	$\rho_u = 0.35$	0.594	23.598	15.441	1.522	5.371	-6.110	-	-2.484	-
	$\rho_u = 0.9$	0.646	25.995	15.535	1.663	5.560	-6.110	-	-2.460	-

Table 5: Extension of the Taylor rule

		V(π)	V(x)	V(i)	V(p)	E[L]	ψ_p	ψ_π	ψ_x	ψ_i
$\alpha = 0$										
$\rho_r = 0$	$\rho_u = 0$	0.289	13.490	1.989	-	1.406	-	-6.110	0.384	-
	$\rho_u = 0.35$	0.391	14.023	2.219	-	1.587	-	4.190	0.384	-
	$\rho_u = 0.9$	0.143	30.464	2.141	-	2.111	-	15.090	0.384	-
$\rho_r = 0.35$	$\rho_u = 0$	0.378	9.964	6.708	-	2.439	-	0.402	0.800	-
	$\rho_u = 0.35$	0.491	10.510	6.892	-	2.621	-	4.610	0.580	-
	$\rho_u = 0.9$	0.236	26.878	6.859	-	3.145	-	10.810	0.268	-
$\rho_r = 0.9$	$\rho_u = 0$	0.519	0.461	10.414	-	2.998	-	-3.545	-1.964	-
	$\rho_u = 0.35$	0.620	0.994	10.644	-	3.180	-	-3.957	-1.640	-
	$\rho_u = 0.9$	0.374	17.404	10.567	-	3.704	-	-5.685	-0.280	-
$\alpha = 0.10$										
$\rho_r = 0$	$\rho_u = 0$	0.315	13.457	2.146	-	1.467	-	9.110	0.300	-
	$\rho_u = 0.35$	0.432	14.113	2.557	-	1.713	-	4.038	0.360	-
	$\rho_u = 0.9$	0.157	30.381	2.220	-	2.139	-	13.110	0.268	-
$\rho_r = 0.35$	$\rho_u = 0$	0.359	10.023	7.111	-	2.518	-	13.709	-0.080	-
	$\rho_u = 0.35$	0.494	9.757	7.830	-	2.810	-	5.230	0.544	-
	$\rho_u = 0.9$	0.316	35.180	4.527	-	3.073	-	21.110	-0.592	-
$\rho_r = 0.9$	$\rho_u = 0$	0.736	0.588	9.103	-	2.912	-	-4.559	0.304	-
	$\rho_u = 0.35$	0.741	0.967	10.134	-	3.179	-	-2.906	-1.860	-
	$\rho_u = 0.9$	0.400	18.223	8.361	-	3.248	-	-4.231	-0.200	-
$\alpha = 0.35$										
$\rho_r = 0$	$\rho_u = 0$	0.281	14.064	2.636	-	1.578	-	9.890	0.092	-
	$\rho_u = 0.35$	0.331	16.659	3.235	-	1.894	-	5.803	0.204	-
	$\rho_u = 0.9$	0.136	29.570	1.929	-	2.010	-	20.110	-0.052	-
$\rho_r = 0.35$	$\rho_u = 0$	0.325	13.832	6.166	-	2.444	-	9.750	-0.156	-
	$\rho_u = 0.35$	0.324	15.560	7.615	-	2.868	-	7.360	0.092	-
	$\rho_u = 0.9$	0.182	32.267	4.485	-	2.790	-	18.110	-0.476	-
$\rho_r = 0.9$	$\rho_u = 0$	1.013	0.408	8.593	-	3.060	-	0.010	-4.600	-
	$\rho_u = 0.35$	0.817	1.831	10.900	-	3.477	-	-2.358	-1.872	-
	$\rho_u = 0.9$	0.213	12.024	13.396	-	3.952	-	-10.970	-1.636	-
$\alpha = 0.89$										
$\rho_r = 0$	$\rho_u = 0$	0.217	15.751	2.416	-	1.543	-	6.020	0.084	-
	$\rho_u = 0.35$	0.210	17.070	2.517	-	1.624	-	5.015	0.136	-
	$\rho_u = 0.9$	0.174	19.725	2.579	-	1.730	-	4.477	0.240	-
$\rho_r = 0.35$	$\rho_u = 0$	0.198	17.863	4.474	-	2.111	-	6.980	-0.016	-
	$\rho_u = 0.35$	0.190	19.185	4.741	-	2.230	-	6.010	0.072	-
	$\rho_u = 0.9$	0.142	22.278	4.884	-	2.364	-	6.040	0.176	-
$\rho_r = 0.9$	$\rho_u = 0$	0.108	13.269	11.159	-	3.378	-	10.640	-0.064	-
	$\rho_u = 0.35$	0.082	12.524	12.025	-	3.521	-	10.890	0.212	-
	$\rho_u = 0.9$	0.041	15.351	11.841	-	3.572	-	13.195	0.392	-

Table 6: Extension of the Wicksellian rule

		V(π)	V(x)	V(i)	V(p)	E[L]	ψ_p	ψ_π	ψ_x	ψ_i
$\alpha = 0$										
$\rho_r = 0$	$\rho_u = 0$	0.125	14.040	1.204	0.130	1.083	3.610	-	0.184	-
	$\rho_u = 0.35$	0.158	14.948	1.296	0.186	1.181	2.150	-	0.232	-
	$\rho_u = 0.9$	0.185	20.258	1.391	0.236	1.485	1.630	-	0.268	-
$\rho_r = 0.35$	$\rho_u = 0$	0.154	14.515	2.771	0.125	1.505	7.641	-	-0.120	-
	$\rho_u = 0.35$	0.184	17.631	3.069	0.189	1.754	3.770	-	0.192	-
	$\rho_u = 0.9$	0.176	23.128	3.387	0.197	2.085	3.013	-	0.280	-
$\rho_r = 0.9$	$\rho_u = 0$	0.155	7.166	9.210	0.124	2.673	8.284	-	-0.160	-
	$\rho_u = 0.35$	0.049	6.082	7.989	0.029	2.226	-8.080	-	0.320	-
	$\rho_u = 0.9$	0.051	8.221	8.482	0.030	2.448	-8.080	-	0.320	-
$\alpha = 0.10$										
$\rho_r = 0$	$\rho_u = 0$	0.185	14.212	1.243	0.317	1.161	0.778	-	0.260	-
	$\rho_u = 0.35$	0.234	14.889	1.224	0.400	1.238	0.775	-	0.256	-
	$\rho_u = 0.9$	0.314	20.929	0.950	0.523	1.543	0.740	-	0.212	-
$\rho_r = 0.35$	$\rho_u = 0$	0.285	15.780	3.581	0.520	1.887	0.876	-	0.388	-
	$\rho_u = 0.35$	0.349	17.121	3.379	0.629	1.968	0.857	-	0.364	-
	$\rho_u = 0.9$	0.514	22.704	2.984	0.908	2.308	0.823	-	0.320	-
$\rho_r = 0.9$	$\rho_u = 0$	0.530	2.809	7.515	1.329	2.438	-3.991	-	2.116	-
	$\rho_u = 0.35$	0.449	2.229	9.042	0.316	2.690	-5.221	-	2.532	-
	$\rho_u = 0.9$	0.506	2.463	9.305	0.365	2.820	-4.853	-	2.416	-
$\alpha = 0.35$										
$\rho_r = 0$	$\rho_u = 0$	0.149	14.894	1.439	0.276	1.204	2.520	-	0.160	-
	$\rho_u = 0.35$	0.143	16.006	1.719	0.269	1.317	2.591	-	0.200	-
	$\rho_u = 0.9$	0.109	21.653	1.774	0.214	1.567	2.500	-	0.244	-
$\rho_r = 0.35$	$\rho_u = 0$	0.184	16.822	3.244	0.308	1.757	3.290	-	0.124	-
	$\rho_u = 0.35$	0.196	18.604	3.349	0.361	1.879	2.713	-	0.180	-
	$\rho_u = 0.9$	0.156	24.047	3.499	0.297	2.136	2.690	-	0.228	-
$\rho_r = 0.9$	$\rho_u = 0$	0.270	2.835	9.039	0.193	2.540	-4.900	-	2.520	-
	$\rho_u = 0.35$	0.271	2.936	9.415	0.193	2.634	-4.900	-	2.520	-
	$\rho_u = 0.9$	0.273	3.130	10.309	0.195	2.856	-4.900	-	2.520	-
$\alpha = 0.89$										
$\rho_r = 0$	$\rho_u = 0$	0.074	23.611	3.896	0.220	2.127	5.158	-	0.212	-
	$\rho_u = 0.35$	0.061	23.481	3.562	0.183	2.029	5.073	-	0.216	-
	$\rho_u = 0.9$	0.048	25.000	3.086	0.153	1.977	4.634	-	0.256	-
$\rho_r = 0.35$	$\rho_u = 0$	0.067	23.284	5.788	0.186	2.551	6.030	-	0.184	-
	$\rho_u = 0.35$	0.055	23.393	5.408	0.153	2.454	5.980	-	0.188	-
	$\rho_u = 0.9$	0.041	25.142	4.918	0.116	2.408	5.946	-	0.220	-
$\rho_r = 0.9$	$\rho_u = 0$	0.047	19.392	10.063	0.109	3.353	8.840	-	0.160	-
	$\rho_u = 0.35$	0.036	19.785	9.619	0.083	3.256	8.929	-	0.176	-
	$\rho_u = 0.9$	0.030	16.027	9.747	0.084	3.100	8.627	-	-0.016	-