Capital Asset Pricing Model: An Analysis of Model Assumptions and Empirical Evidence

by

Abir Paul

An essay submitted to the Department of Economics in partial fulfillment of the requirements for the degree of Master of Arts

Queen's University
Kingston, Ontario, Canada
August 2009

Acknowledgements

I would first like to thank my supervisor, Prof. Wulin Suo, for his helpful guidance and advice during the planning and writing of the essay. I am also grateful to my classmates, Patrick Egbunonu and Omar Abdi for their helpful suggestions and encouragement. Finally, I thank all of my family members whose support has made anything I have achieved possible.

Contents

1. Introduction

Markowtiz (1952) first laid the ground work for the CAPM (Capital Asset Pricing Model). In this framework, risk of a security is usually measured by the standard deviation of its return. The standard deviations of security returns for any two securities are not additive if they are combined together unless the returns of those two assets are perfectly positively correlated. The standard deviation of security return of a portfolio is less than the sum of the standard deviation of those assets constituted the portfolio. Markowitz developed the efficient frontier of portfolio, the efficient set from where the investors select the portfolio which is most suitable for them. Technically, an investor will hold a mean-variance efficient portfolio which will return the highest payoff to them with a given level of standard deviation.

When we analyze the risk of an individual security, we have to consider the other securities of the portfolio as well. We are interested in the additional risk being added to the portfolio when one additional security is added to the portfolio. Thus the concept of risk share of an individual security to the portfolio is different from the risk of that security itself. An investor faces two kinds of risks. One is called the systematic risk and the other is known as unsystematic risk. Unsystematic risk is a kind of risk which can be minimized or eliminated by increasing the size of the portfolio, namely, by increasing the diversity of the portfolio. The systematic risk is also known as the market risk which depends on the overall movement of the market and the financial condition of the whole economy. By diversifying the portfolio, we cannot eliminate the systematic risk.

The empirical evidence of CAPM is not very encouraging. However, there is a debate whether it is fair to conclude that CAPM is invalid in real world or we have failed to build a comprehensive and valid test model. The estimation strategy of CAPM is not free from the data-snooping bias. Because of the non-experimental nature of economic theory we cannot avoid this problem. A number of investigations have already been done to test the validity of the CAPM. Thus, no attempt has been made in this paper to do that again. In this paper we will make an attempt to see what restrictions and assumptions are required to hold the model to be true. We will do this in two different phases. First we will see what happens theoretically if we relax the assumptions on which CAPM is built and then we will analyze what we observe from the empirical experience.

2. Derivation of the CAPM

We know that in equilibrium the price is set in a way where there is no excess demand which implies that supply for all assets equals the demand for them. Moreover, in equilibrium the market portfolio is comprised of all marketable assets held in proportion to their value weights. The proportion of each asset in equilibrium must be

$$w_i = \frac{\text{Market value of individual asset}}{\text{Market value of all assets}}$$

Now let us assume that the portfolio *a*% is invested in risky asset *I* and (1-*a%*) in the market portfolio has the following mean and standard deviation:

$$E(R'_p) = aE(R'_i) + (1-a)E(R'_m)$$

$$\sigma\left(R'_p\right) = \left[a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}\right]^{\frac{1}{2}}$$

Where

σ_i^2=the variance of the risky asset *I*

σ_m^2=the variance of the market portfolio

σ_{im}=the covariance between asset *I* and the market portfolio

Now, the change in the mean and standard deviation with respect to a change in the portfolio *a* can be determined as follows:

$$\frac{\partial}{\partial a} E\left(R'_p\right) = E\left(R'_i\right) - E\left(R'_m\right)$$

$$\frac{\partial}{\partial a} \sigma\left(R'_p\right) = \frac{1}{2}\left[a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}\right]^{-\frac{1}{2}}\left[2a\sigma_i^2 - 2\sigma_m^2 + 2a\sigma_m^2 + 2\sigma_{im} - 4a\sigma_{im}\right]$$

In equilibrium the market portfolio has the value weight which is invested in the risky asset. Thus the percentage *a* stands for the excess demand for an individual risky asset but in equilibrium this must be zero. To get the equilibrium price of risk we will evaluate the above two partial derivatives at *a*=o. Thus we obtain,

$$\frac{\partial}{\partial a} \mathrm{E}\left(\mathrm{R}'_p\right)\Big|_{a=0} = E\left(R'_i\right) - E\left(R'_m\right)$$

$$\frac{\partial}{\partial a}\sigma(R'_p)\bigg|_{a=0} = \frac{1}{2}\left(\sigma_m^2\right)^{-\frac{1}{2}}\left(-2\sigma_m^2 + 2\sigma_{im}\right) = \frac{\left[\sigma_{im} - \sigma_m^2\right]}{\sigma_m}$$

Therefore the slope of the risk-return trade off evaluated at the equilibrium point is

$$\left.\frac{\dfrac{\partial E(R'_p)}{\partial a}}{\dfrac{\partial \sigma(R'_p)}{\partial a}}\right|_{a=0} = \frac{\left[E(R'_i) - E(R'_m)\right]}{\dfrac{\left(\sigma_{im} - \sigma_m^2\right)}{\sigma_m}}$$

We know that the capital market line also represents the equilibrium relationship. The slope of the capital market

$$\frac{\left[E(R'_m) - R_f\right]}{\sigma_m}$$

where,

σ_m=standard deviation of the market portfolio

Now equating this with slope of the opportunity set evaluated at the equilibrium point, we have

$$\frac{\left[E(R'_m) - R_f\right]}{\sigma_m} = \frac{\left[E(R'_i) - E(R'_m)\right]}{\dfrac{\left(\sigma_{im} - \sigma_m^2\right)}{\sigma_m}}$$

We can rearrange this to solve for $E(R'_i)$

$$E(R'_i) = R_f + \frac{\left[E(R'_m) - R_f\right]\left[\sigma_{im}\right]}{\sigma_m^2} \qquad (2.1)$$

This is known as the capital asset pricing model. The required rate on any asset is equal to the risk free rate of return and the risk premium. The price of the risk represents the slope of the line. The quantity of the risk is known as beta,

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

This is the ratio of the covariance between the return on risky asset *I* and the return of market portfolio to the variance of the market portfolio return. The risk free asset has the zero betas because its covariance is zero with the market portfolio but the market portfolio has the beta of one because the covariance of the market portfolio with itself is identical to the variance of the market portfolio:

$$\beta_m = \frac{Cov(R_m, R_m)}{Var(R_m)} = \frac{Var(R_m)}{Var(R_m)} = 1$$

3. The Problem of Measuring the Performance of the Model

Roll (1977) concluded that the only legitimate test of CAPM is whether or not market portfolio is mean-variance efficient. If the performance is measured relative to an ex post efficient index then no asset will have abnormal performance which is measured as a departure from the security market line. Any ranking of the portfolios is possible, depending on which inefficient index has been chosen. This implies that even if the CAPM is valid and markets work efficiently, the security market line cannot be used to measure the performance of the portfolio selection technique. Moreover, testing the efficiency of the market portfolio and CAPM depends on a joint hypothesis which is almost impossible to test because it is very difficult to measure the market portfolio. Roll argues that there is nothing unique about the market portfolio. We can always pick any portfolio (efficient) as an index then we can select the minimum variance portfolio which is uncorrelated with the efficient market portfolio. However, we have to remember that Roll's critique never implies that the CAPM is an invalid theory. The fact that the portfolio residuals exhibit no departures from linearity implies that market portfolio that was chosen was ex post efficient. So, we can only test directly whether the true market portfolio is ex post efficient. As the market portfolio contains all types of assets, it is impossible to observe properly.

4. Efficient Set of Mathematics

The mathematics of mean-variance efficient set is known as the efficient set of mathematics. To test the validity of the CAPM, one of the most important parts is to test the mean-variance efficiency of the model. Thus, it is very important to understand the underlying mathematics of the model. Here, we will discuss some of the useful results (Roll, 1977) of it.

Here we assume that there are N risky assets with a mean vector μ and a covariance matrix Ω. In addition we also assume that the covariance matrix is of full rank. ω_a is vector of the portfolio weight. This portfolio has the average return; $\mu_a = \omega_a'\mu$ and variance $\sigma_a^2 = \omega_a'\Omega\omega_a$. Portfolio p is the minimum variance portfolio with the mean return μ_p if its portfolio weight vector is the solution to the following constrained optimization:

$$\min \omega'\Omega\omega$$

subject to $\omega'\mu = \mu_p$

and, $\omega'\iota = 1$

We solve this minimization problem by setting the Lagrangian function. Let's define the following:

$$A = \iota'\Omega^{-1}\mu$$

$$B = \mu'\Omega^{-1}\mu$$

$$C = \iota'\Omega^{-1}\iota$$

$$D = BC - A^2$$

The efficient frontier can be generated from any two minimum variance portfolios. Let us assume that p and r be any two minimum variance portfolios. The covariance of these two portfolios is as follows:

$$Cov[R_p, R_r] = \frac{C}{D}\left(\mu_p - \frac{\mu}{C}\right)\left(\mu_r - \frac{A}{C}\right) + \frac{1}{C} \qquad (4.1)$$

For a global minimum-variance portfolio g we have the following:

$$\omega_g = \frac{1}{C}\Omega^{-1}\iota$$

$$\mu_g = \frac{A}{C}$$

$$\sigma_g^2 = \frac{1}{C}$$

The covariance of the asset return of the global minimum portfolio g and any other portfolio *a* is defined as:

$$Cov\left[R_g, R_a\right] = \frac{1}{C} \qquad (4.2)$$

For a multiple regression of the return of an asset or portfolio on any minimum variance portfolio except the global minimum variance portfolio and underlying zero-beta portfolio R_{ap} we have,

$$R_a = \beta_0 + \beta_1 R_{op} + \beta_2 R_p + \epsilon_p$$

$$E\left(\epsilon_p\right) = 0$$

$$\beta_2 = \frac{Cov\left[R_a, R_{op}\right]}{\sigma_p^2} = \beta_{ap}$$

$$\beta_1 = \frac{Cov\left[R_a, R_{op}\right]}{\sigma_{op}^2} = 1 - \beta_{ap}$$

$$\beta_0 = 0$$

The above result deserves some more attention. Here we will prove the result. As $Cov[R_p, R_{op}] = 0$. The result $\beta_2 = \beta_{ap}$ is obvious. So, we just need to show that $\beta_1 = 1 - \beta_{ap}$ and $\beta_0 = 0$. Let us assume that r be the minimum variance portfolio with expected return $\mu_a = \mu_r$. From the minimization problem we can write the following:

$$R_r = (1 - \lambda)R_{op} + \lambda R_p \text{ where } \lambda = \frac{(\mu_r - \mu_{op})}{\mu_p - \mu_{op}}$$

Since, we have $Cov[R_{p,} R_{op}] = 0$ and $Cov[R_{p,} R_{op}] = 0$, we can write the following:

$$\beta_{rop} = \frac{Cov[R_r, R_{op}]}{Var[R_{op}]}$$

$$= \frac{Cov[(1 - \lambda)R_{op} + \lambda R_p, R_{op}]}{Var[R_{op}]}$$

$$= (1 - \lambda)$$

We also have,

$$\beta_{rp} = \frac{Cov[R_r, R_p]}{Var[R_p]}$$

$$= \frac{Cov[(1 - \lambda)R_{op} + \lambda R_p, R_p]}{Var[R_p]}$$

$$= \lambda$$

and,

$\mu_r = \beta_{rop}\mu_{op} + \beta_{rp}\mu_p$

Portfolio *a* can be expressed as a combination of portfolio r and an arbitrage portfolio $a^\star$ which is composed of portfolio *a* minus portfolio $a^\star$. The return of $a^\star$ is expressed as:

$R_{a^\star} = R_a - R_r$

Since $\mu_a = \mu_r$, the expected return of $a^\star$ is zero. Because, as mentioned earlier that it is an arbitrage portfolio with an expected return of zero, for a minimum variance portfolio q. We have the following minimization problem:

$$\textbf{min}\ Var\left[R_q + cR_{a^\star}\right]$$

The solution to the optimization problem is c=0. Any other solution will contradict q from being the minimum variance.

Since, $Var\left[R_q + cR_{a^\star}\right] = Var\left[R_q\right] + 2\,cCov\left[R_q, R_{a^\star}\right] + c^2 Var\left[R_{a^\star}\right]$, thus taking the derivative gives the following expression:

$$\frac{\partial}{\partial c} Var\left[R_q, R_{a^\star}\right] = 2\,Cov\left[R_{q,} R_{a^\star}\right] + 2\,cVar\left[R_{a^\star}\right]$$

Setting the derivative equal to zero and by substituting in the solution that c=0 gives:

$$Cov[R_q, R_{a\star}] = 0 \tag{4.3}$$

Thus the return of $a^{\star}$ is uncorrelated with the return of all other minimum variance portfolios. Using the above results we have,

$$Cov[R_a, R_p] = Cov[R_q + R_{a\star}, R_p]$$
$$= Cov[R_r, R_p]$$

and,

$$Cov[R_a, R_{op}] = Cov[R_r + R_{a\star}, R_{op}]$$
$$= Cov[R_{r,} R_{op}]$$

Therefore, the following results are immediate:

$$\beta_{aop} = \beta_{rop}$$
$$\beta_{ap} = \beta_{rp}$$

So, now we have $\beta_1 = \beta_{aop} = 1 - \beta_{ap}$ and since, $\mu_r = \mu_a$, it follows that $\beta_0 = 0$

Another important assumption of the CAPM is if the market portfolio is the tangency portfolio then the intercept of the excess return market model is zero. To prove this let us consider the following model with the IID assumptions of the error term:

$$Z_t = \alpha + \beta Z_{mt} + \epsilon_t$$

Now by taking the unconditional expectation we get,

$\alpha = \mu - \beta\mu_m$

As we have showed above, the weight vector of the market portfolio is,

$$\omega_m = \frac{1}{\iota'\Omega^{-1}\mu}\Omega^{-1}\mu$$

Using this weight vector, we can calculate the covariance matrix of asset and portfolio returns, the expected excess return and the variance of the market return,

$$Cov\left[Z, Z_m\right] = \Omega\omega_m = \frac{1}{\iota'\Omega^{-1}\mu}\mu$$

$$\mu_m = \omega'_m\Omega\omega_m = \frac{\mu'\Omega^{-1}\mu}{\left(\iota'\Omega^{-1}\mu\right)^2}$$

$$Var\left[Z_m\right] = \omega'_m\Omega\omega_m = \frac{\mu'\Omega^{-1}\mu}{\left(\iota'\Omega^{-1}\mu\right)^2}$$

Combining these results provide,

$$\beta_m = \frac{Cov\left[Z, Z_m\right]}{Var\left[Z_m\right]} = \frac{\iota'\Omega^{-1}\mu}{\mu'\Omega^{-1}\mu}\mu \qquad (4.4)$$

Now, by combining the expression for beta and the expression for the expected excess return give,

$$\beta_m\mu_m = \mu$$

Therefore, the immediate result is $\alpha = 0$

In the framework of CAPM it is assumed that in equilibrium every asset is priced in a way that the risk adjusted required rate of return falls on the security market line. It is also assumed that the risk of an individual asset is linearly additive when the assets are combined into portfolios.

5. Relaxing the Model Assumptions: Some of the assumptions under which the CAPM is developed are violated in the real world. Here we will see whether it is possible to extend the model by relaxing the unrealistic assumptions without drastically changing it and later we will analyze what we experience from the empirical investigations.

5.1 Market with Risky Asset: Here we will see how CAPM is affected if there is no risk free asset that is how the model reacts if the market agents cannot borrow and lend at the risk free rate. Let us consider a portfolio M which lies on the efficient set. Now, suppose that we can identify all the market portfolios which are uncorrelated with the true market portfolio. This implies that they have the same systematic risks. Since, they have the same systematic risks, they must have the same expected return $E(R_z)$.

Let us form a portfolio with *a*% in the market portfolio and (1-*a*%) in the minimum variance zero-beta portfolio. The mean and the standard deviation of this portfolio can be written as follows:

$$E(R_p) = aE(R_m) + (1-a)E(R_z)$$

$$\sigma(R_p) = \left[a^2\sigma_m^2 + (1-a)^2\sigma_z^2 + 2a(1-a)r_{zm}\sigma_z\sigma_m\right]^{\frac{1}{2}}$$

But as the correlation r_{zm} is zero, the last term drops out. The partial derivative of the mean portfolio return is,

$$\frac{\partial}{\partial a} E(R_p) = E(R_m) - E(R_z)$$

and the partial derivative of the standard deviation is,

$$\frac{\partial}{\partial a} \sigma(R_p) = \frac{1}{2}\left[a^2\sigma_m^2 + (1-a)^2\sigma_z^2\right]^{-\frac{1}{2}}\left[2\,a\sigma_m^2 - 2\,\sigma_z^2 + 2\,a\sigma_z^2\right]$$

Now by taking the ratio of these derivatives and evaluating them at point *a*=1, we can obtain the slope of the line $E(R_z)M$

$$\frac{\frac{\partial E(R_p)}{\partial a}}{\frac{\partial \sigma(R_p)}{\partial a}} = \frac{\left[E(R_m) - E(R_z)\right]}{\sigma_m} \qquad (5.1)$$

And, the line must pass through the point $\left[E(R_m), \sigma(R_m)\right]$ and the intercept of the tangent line must be $E(R_z)$. Then we can write the equation of that tangent line as follows:

$$E(R_p) = E(R_z) + \frac{\left[E(R_m) - E(R_z)\right]}{\sigma_m}\sigma_p$$

We also know that in equilibrium the slope of a line which is tangent to a portfolio comprised of the market portfolio and the other assets at the point represented by the market portfolio which is equal to the following equation,

$$\left.\frac{\frac{\partial E(R_p)}{\partial a}}{\frac{\partial \sigma(R_p)}{\partial a}}\right|_{a=0} = \frac{[E(R_i) - E(R_m)]}{\frac{(\sigma_{im} - \sigma_m^2)}{\sigma_m}} \qquad (5.2)$$

If we equate equation (5.1) and (5.2), we have

$$\frac{[E(R_m) - E(R_z)]}{\sigma_m} = \frac{[E(R_i) - E(R_m)]\sigma_m}{\sigma_{im} - \sigma_m^2}$$

Now solving for the required rate of return on asset i, we have

$$E(R_i) = (1 - \beta_i)E(R_z) + \beta_i E(R_m) \qquad (5.3)$$

Equation (5.3) shows that we can write the expected rate of return of any asset as a linear combination of the expected rate of returns of the market portfolio and the minimum variance portfolio. If we rearrange equation (5.3) we will get the CAPM equation. The only exception is the expected rate of return on the zero-beta portfolio has replaced the rate of return on the risk–free asset:

$$E(R_i) = E(R_z) + [E(R_m) - E(R_z)]\beta_i$$

This implies that CAPM necessarily does not require the existence of a pure riskless asset. The above equation is typically known as the two factor model.

5.2 Returns not Jointly Normal

Asset returns cannot be normally distributed, because of the possibility of the largest negative return (100%). However, the assumption of normally distributed asset return implies that there is a finite possibility that returns will be less than minus 100% and that asset price will be negative. But the probability of the asset return being minus 100% can be very small and that may not have any impact on the empirical validity of CAPM. Normality assumption also implies that the mean and variance are needed to describe the distribution completely. Fama (1965) observed that the daily return distribution of NYSE securities are systematically distributed and it does not have finite variance. This kind of distribution has the fat tails. Fama (1965) showed that as long as this distribution is systematic, investors can use standard deviation rather than variance and then the theory of portfolio choice remains valid.

5.3 The Existence of Nonmarketable Assets

We know that if there is no transaction cost and assets are divisible, two different fund separations are obtained. No matter what is the shape of the indifference curve, the investor will hold either the risk free asset or the market portfolio. However, empirical results tell us that this does not happen in reality. People hold portfolios of risky asset and there exits the possibility of the existence of the nonmarketable goods. Mayer (1972) shows that when investors hold the risky nonmarketable assets, the CAPM takes the following form:

$$E(R_j) = R_f + \lambda\left[V_m Cov(R_j, R_m) + Cov(R_j, R_H)\right]$$

where,

$$\lambda = \frac{\left[E(R_m) - R_f\right]}{V_m \sigma_m^2 + Cov(R_m, R_H)}$$

V_m = current market value of all marketable assets

R_H = total return on nonmarketable assets

This has several implications. First, individuals will hold different portfolios containing risky asset because their human capital (for example) has different amount of risks. Second, the equilibrium price can still be determined by the shape of the individual's indifference curve which implies that the separation principal still holds true. Third, we can still measure the risk with covariance. However, now we have two different measure of covariance; one contains the marketable assets and the other contains the nonmarketable goods.

5.4 The Model in Continuous Time

Metron (1973) assumed that trading takes place continuously over time and asset return is log-normally distributed. When the risk free rate of interest is non-stochastic then the equilibrium asset return must have the following form:

$$E(R_i) = r_f + [E(R_m) - r_f]\beta_i$$

This is analogous to the continuous time version of CAPM. Metron shows that when the risk free rate is stochastic then investors hold portfolios chosen from the riskless asset, the market portfolio and the portfolio which has perfect negative correlation with riskless asset. Thus it exhibits three-fund separation. The required rate of return of the jth asset is,

$$E(R_j) = r_f + \gamma_1[E(R_m) - r_f] + \gamma_2[E(R_N) - r_f]$$

Where

$$\gamma_1 = \frac{[\beta_{jm} - \beta_{jN}\beta_{Nm}]}{1 - \rho_{Nm}^2}$$

$$\gamma_2 = \frac{[\beta_{jN} - \beta_{jm}\beta_{Nm}]}{1 - \rho_{Nm}^2}$$

Metron argued that the sign of γ_2 is negative for the high beta assets and positive for the low beta assets. We know that this argument is valid in empirical findings.

6. Empirical Tests of the CAPM

CAPM is expressed in terms of expected return and risks. It is a simple linear model. In the ex ante form we have

$$E(R_j) = R_f + [E(R_m) - R_f]\beta_j \qquad (6.1)$$

However some suggest that it may not be linear. Thus the important question is: how well the model fits the data. We have to transform this ex ante form to a form which can use the observed data. Empirical evidence supports that on average the realized rate of return approaches to the expected return which is known as fair game. By using the notion of the fair game we can write the above equation as follows:

$$R_{jt} = E(R_{jt}) + \beta_j \delta_{mt} + \varepsilon_{jt} \qquad (6.2)$$

Where,

$\delta_{mt} = R_{mt} - E(R_{mt})$

$E(\delta_{mt}) = 0$

$E(\varepsilon_{jt}) = 0$

$Cov(\varepsilon_{jt}, \delta_{mt}) = 0$

Now using the CAPM assumption that asset returns are jointly normal, β_j in the fair game model is defined as it is defined in the CAPM. Now by substituting $E(R_j)$ into equation (6.1), we obtain,

$$R_{jt} = R_{ft} + [E(R_{mt}) - R_{ft}]\beta_j + \beta_j[R_{mt} - E(R_{mt})] + \varepsilon_{jt}$$

$$= R_{ft} + (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt}$$

Now subtracting R_{ft} from both sides, we have

$$R_{jt} - R_{ft} = (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt} \qquad (6.3)$$

This is the ex post version of the model. In the ex ante model the slope can take negative sign whereas, in the ex post model it cannot. We can experience the market rate of return as negative. In that case the slope of the security market line will be downward.

There are three relationships between expected return and market beta which is implied by the model. First, the expected returns of all assets are linearly related to their respective betas. Second, the premium for beta is positive which implies that the expected return on the market portfolio exceeds the expected return on assets. Moreover, the returns of these assets are uncorrelated with the expected return of market portfolio. Third, in the Sharpe-Lintner model we see that the underlying assets those are uncorrelated with the market portfolio have the expected returns which are equal to the risk neutral interest rate. In that model, if we subtract the risk free rate from the expected market return, we get the beta premium. Conventionally the tests of CAPM are based on those three implications mentioned above.

6.1 Tests on Risk Premiums

Most of the previous cross-section regression tests primarily focus on the Sharpe-Lintner model's findings about the concept and the slope term which studies the relationship between expected return and the market beta. In that model, researchers regressed the mean asset returns on the estimated asset betas. The model suggests that the constant term in the cross-section regression stands for the risk free interest rate and the slope term stands for the difference between market interest rate and risk free interest rate.

There are some drawbacks of these studies. First of all, the estimated betas for individual assets are imprecise which creates the measurement error when we use them to explain average returns. Secondly, the error term in the regression has some common sources of variation which produces positive correlation among the residuals. Thus the regression has the downward bias in the usual OLS estimate. Blume (1970) and Black, Scholes and Jensen (1972) worked on overcoming the shortcomings of Sharpe-Lintner model. Instead of working on the individual securities they worked on the portfolios. They combined the expected returns and market betas in a same way that if the CAPM can explain the security return, it can also explain portfolio return. The econometric theory suggests the estimated beta for diversified portfolios are more accurate than the estimated beta for the individual security. Thus, if we use the market portfolio in the regression of average return on betas, it lessens these critical problems. However, grouping shrinks the range of estimated betas and shrinks the statistical power as well. To tackle this researchers sort securities to create two portfolios. The first one contains securities with the lowest beta and it moves up to the highest beta.

We know that when there exists a correlation among the residuals of the regression model, we cannot draw accurate inference from that. Fama and Macbeth (1973) suggested a method to address this inference problem. They ran the regression of returns on beta based on the monthly data rather than estimating a single cross-section regression of the average returns on beta. In this approach the standard error and the time series average returns can be used to check whether the average premium for beta is positive and whether the return on the asset is equal to the average risk free interest rate.

Jensen (1968) noted that Sharpe-Lintner model also implies a time series regression test. According to Sharpe-Lintner model, the average realized CAPM risk premium explains the average value of an asset's excess return. The intercept term in the regression entails that "Jensen's alpha". The time series regression takes the following form:

$$R_{it} - R_{ft} = \alpha_i + \beta_{im}(R_{mt} - R_{ft}) + \varepsilon_{jt}$$

In early studies we reject Sharpe-Lintner model of CAPM. There exists a positive relation between average return and beta, However, this relation is too flat. In Sharpe-Lintner model the intercept stands for the risk free rate and the slope term indicates the expected market return in excess of the risk neutral rate. In that regression model the intercept is greater than the risk neutral rate and the coefficient on beta is less than $E(R_M) - R_f$. In Jensen's study the p value for the thirty years period is 0.02 only which indicates that the null hypothesis is rejected at 5% significance level. The five and ten year sub-period demonstrates the strongest evidence against the restrictions imposed by the model.

In past several studies it has been confirmed that the relationship in between average return and beta is too flat (Blume: 1970 and Stambaugh: 1982). With the low betas the constant term in the time series regression of excess asset return on excess market return is positive and it becomes negative for the high betas of the underlying assets.

In the Sharpe-Linter model, it has been predicted that portfolios are plotted along a straight line where the intercept equals the risk free rate, R_f, and the slope equals to the expected excess return on the market rate $E(R_M) - R_f$. Fama and French (2004) observed that risk premium for beta (per unit) is lower than the Sharpe-Lintner model and the relationship between asset return and beta is linear. The Black version of CAPM also observes the same where it predicts only the beta premium is positive.

6.2 Testing the Ability of Market Betas of Explaining Expected Returns

Both the Sharpe-Lintner and Black model predict that market portfolio is mean-variance efficient. The mean-variance efficiency implies that the difference in market beta explains the difference in expected return of the securities and portfolios. This prediction plays a very important role in testing the validity of the CAPM.

From the study by Fama and Macbeth (1973), we see that we can add pre-determined explanatory variables to the month wise cross section regressions of asset return on the market beta. Provided that all the differences in expected return are explained by the betas, the coefficients of any additional variable should not be dependably different from zero. So, in the cross-section analysis the important thing is to carefully choose the additional variables. In this regard we can take the example of the study by Fama and MacBeth (1973). In that work the additional variables are squared betas. These variables have no impact in explaining the average asset return.

By using the time series regression we can also test the hypothesis that market betas completely explain expected asset return. In the time series regression analysis, the constant term is the difference between the asset's average return and the excess return predicted by the Sharpe-Lintner model. We cannot group assets in portfolios where the constant term is dependably different from zero. For example, for a portfolio, the constant term for a high P/E ratio and low P/E ratio should be zero. Therefore, in order to test the hypothesis that betas are sufficient to explain expected returns, we can estimate the time-series regression for the portfolios and then test the joint hypothesis for the intercepts against zero. In this kind of approach we have to choose the form of the portfolio in a way which will depict any limitation of the CAPM prediction.

In past literatures, researchers tend to follow different kinds of tests to see whether the constant term in the time-series regression is zero. However, it is very debatable to conclude about the best small sample properties of the test. Gibbons, Shanken and Ross (1989) came up with an F-test for the constant term that has the exact-small sample properties and it is asymptotically efficient.

For the tangency portfolio, this F-test builds an entrant by combining the market proxy and the average value of an asset’s excess return. Then we can test if the efficient set and the risk free asset is superior to that one obtained by combining the market proxy and risk free asset alone. From the study of Gibbons, Ross, and Shanken (1989) we can also test whether market betas are sufficient enough to explain the expected returns. The statistical test what is conventionally done is whether the explanatory variables can identify the returns which are not explained by the market betas. We can use the market proxy and can build a test to see if the market proxy lies on the minimum variance frontier.

All these early tests really do not test the CAPM. These tests actually tested whether or not the market proxy is efficient. Its noteworthy here that the time series regression does not include all marketable assets and it is really very difficult to get the true market portfolio data (Roll, 1977). So, many researchers concluded that the prospect of testing the validity of CAPM is not very encouraging.

Based on the early literatures, we can conclude that the market betas are sufficient enough to explain expected returns which we see from the Black version of CAPM. That model also predicts that the respective risk premium for beta is positive. But at the same time the prediction made by Sharpe and Lintner that the risk premium beta is derived from subtracting the risk free interest rate from the expected return is rejected. The attractive part of the Black model is, it is easily tractable and very appealing for empirical testing.

6.3 Recent Tests on CAPM

Recent investigations started in the late 1970s challenged the success of the Black version of the CAPM. In recent empirical literatures we see that there are other sources of variation in expected returns which do not have any significant impact on the market betas. In this regard Basu's (1977) work is very significant. He shows that if we sort the stocks according to P/E ratios, then the future returns on high E/P ratios are significantly higher than the return in CAPM. Instead of sorting the stocks by P/E, if we sort it by market capitalization then the mean returns on small stocks are higher than the one in CAPM (Banz, 1981) and if we do the same by book-to-market equity ratios then the set of stocks with higher ratio gives higher average return (Statman and Rosenberg, 1980).

The ratios have been used by Statman and Rosenberg (1980) associates the stock prices which involve the information about expected returns which are not captured by the market betas. The price of the stock does not solely depend on the cash flows, rather it depends on the present discounted value of the cash flows. So, the different kind of ratios discussed above play a crucial role in analyzing the CAPM. In line with this Fama and

French (1992) empirically analyzed the failure of the CAPM and concluded that these ratios have impact on stock return which is provided by the betas. In a time series regression analysis they also concluded the same thing. They also observed that the relationship between the average return and the beta is even flatter after the sample periods on which early CAPM studies were done. Chan, Hamao, and Lakonishok (1991) observed a strong significant relationship between book-to-market equity and asset return for Japanese data which is consistent with the findings of Fama and French (1992), implies that the contradictions of the CAPM associated with price ratios are not sample specific.

7. Single-Factor CAPM

In practice, to check the validity of the CAPM we test the SML. Although CAPM is a single period ex-ante model, we rely on the realised returns. The reason being the ex ante returns are unobservable. So, the question which is so obvious to ask: does the past security return conform to the theoretical CAPM?

We need to estimate the security characteristic line (SCL) in order to investigate the beta. Here the SCL considers the excess return on a specific security j to the excess return on some efficient market index at time t. The SCL can be written as follows:

$$R_{it} - R_{ft} = \eta_i + b_i(R_{mt} - R_{ft}) + \varepsilon_{it} \qquad (7.1)$$

Here η_i is the constant term which represents the asset return (constant) and b_i is an estimated value of β_i. We use this estimated value as an explanatory variable in the following cross-sectional regression:

$$R_{it} = \gamma_0 + \gamma_1 b_i + u_{it} \qquad (7.2)$$

Conventionally this regression is used to test for a positive risk-return trade off. The coefficient of γ_1 is significantly different from zero and is assumed to be positive in order to hold the CAPM to be true. This also represents the market price of risk. When we test the validity of CAPM we test if b_i is true estimate of β_i. We also test whether the model specification of CAPM is correct.

The CAPM is single period model and they do not have any time dimension into the model. So, it is important to assume that the returns are IID and jointly multivariate normal. The CAPM is very useful in predicting stock return. We also assume that investors can borrow and lend at a risk free rate. In the Black version of CAPM we assume that zero-beta portfolio is unobservable and thus becomes an unknown parameter. In the Black model the unconstrained model is the real-return market model where we also have the IID assumptions and the joint normality return.

Many early studies (e.g. Lintner, 1965; Douglas, 1969) on CAPM focused on individual security returns. The empirical results are off-putting. Miler and Scholes (1972) found some statistical setback faced when using individual securities in analyzing the validity of the CAPM. Although, some of the studies have overcome the problems by using portfolio returns. In the study by Black,Jensen and Scholes (1972) on New York stock exchange data, portfolios had been formed and reported a linear relationship between the beta and average excess portfolio return. The intercept approaches to be negative (Positive) for the beta greater than one (less than one). Thus a zero beta version was developed of the CAPM model. The model was developed in a way where the intercept term is allowed to take different values in different period. Fama and Mcbeth (1973) extended the work of Black et al (1972). They showed the evidence of a larger intercept than the risk neutral rate. They also found that a linear relationship exists between the average returns and the beta. It has also been observed that this linear relation becomes stronger when we work with a dataset for a longer period. However, other subsequent studies provide weak empirical evidence of this zero beta version.

We have mixed findings about the asset return and beta relationship based on the past empirical research. If the portfolio used as a market proxy is inefficient then the single factor CAPM is rejected. This is also true if the proxy portfolio is inefficient by a little margin (Roll: 1977, Ross: 1977). Moreover, there exists survivorship bias in the data used in testing the validity of CAPM (Sloan, 1995). Bos and Newbold (1984) observed that beta is not stable for a period of time. There is an issue of the model specification too. Amihud, Christen and Mendelson (1993) observed that there are errors in variables and these errors have impact on the conclusion of the empirical research.

We experience less favourable evidence for CAPM in the late 1970s in the so called anomalies literature. We can think the anomalies as the farm characteristics which can be used to group assets in order to have a high ex post Sharpe ratio relative to the ratio of the market proxy for the tangency portfolio. These characteristics provide explanatory power of the average mean returns in the cross section beyond the beta of the CAPM which is a contradiction to the prediction of CAPM.

We have already mentioned that the early anomalies include the size effect and P/E ratio. Basu (1977) observed that the portfolio formed on the basis of P/E ratio is more efficient than the portfolio formed according to the mean-variance efficiency. With a lower P/E firms have higher sample average return and with high P/E ratio have lower average return than would be the case if the market portfolio is mean-variance efficient. On the other hand the size effect shows that low market capitalization firms have higher sample return than would be expected if the market portfolio was mean-variance efficient.

Fama and French (1992,1993) observed that beta cannot explain the difference between the portfolio formed based on ratio of book value of equity to the market value of equity. Firm has higher average return for higher book market ratio than originally predicted by the CAPM. However, these results signal economically deviations from CAPM. In these anomalies literatures, there are hardly any motivations to study the farm characteristics. Thus there is a possibility of overstating the evidence against the CAPM since there are sample selection bias problem in estimating the model and also there is a problem of data snooping bias. This a kind of bias refers to the biases in drawing the statistical inference that arises from data to conduct subsequent research with the same or related kind of data. Sample selection bias is rooted if we exclude certain sample of stocks from our analysis.

Sloan (1995) argued that data requirements for the study of book market ratios lead to failing stocks being excluded which results the survivorship bias.

Despite an ample amount of evidences against CAPM, it is still being widely used in the field of finance. There is also the controversy about how we should interpret the evidence against the CAPM. Some researchers often argue that CAPM should be replaced with multifactor model with different sources of risks. In the following section we will analyze the multifactor model.

8. Multifactor Models

So far we have not talked anything about the cross sectional variation. In many studies we have observed that market data alone cannot explain the cross sectional variation in average security returns. In the analysis of CAPM, some variables like, ratio of book-to-market value, price-earning ratio, macroeconomic variables, etc are treated as the fundamental variables. The presence of these variables account for the cross-sectional variation in expected returns.

Fama and French (1995), in their study showed that the difference between the return of small stock and big stock portfolio (SMB) and the difference between high and low book-to-market stock portfolio (HML) become useful factors in cross sectional analysis of the equity returns. Chung, Johnson and Schill (2001) found that the SMB and HML become statistically insignificant if higher order co-moments are included in the cross sectional portfolio return analysis. We can infer from here that the SMB and HML can be considered as good proxies for the higher order co-moments. Ferson and Harvey (1999) made a point that many econometric model specifications are rejected because they have the tendency of ignoring conditioning information.

Now we will show one of the very important results of the multifactor model. Let us consider a regression of portfolio on the returns of any set of portfolios from which the entire minimum variance boundary can be generated. We will show that the intercept of this regression will be zero and that factor regression coefficients for any asset will sum to unity. Let the number of the portfolios in the set be K and R_{Kt} is the (Kx1) vector of time period t of asset returns. For any value of the constant μ, there exists a combination of portfolio and assets. Let us consider μ be the global minimum variance portfolio and we denote the portfolio as *op*. Corresponding to *op* is minimum variance portfolio *p* which is uncorrelated with the return of *op*. As long as *p* and *op* are efficient portfolios in terms of the minimum variance their returns are the linear combinations of the elements of R_{Kt},

$$R_{pt} = R'_{Kt}\omega_p^K$$

$$R_{opt} = R'_{Kt}\omega_{op}^K$$

where ω_p^K and ω_{op}^K are (Kx1) vectors of portfolio weights. As *p* and *op* are minimum variance portfolios, their returns are linear combinations of the elements of R_{Kt},

$$\mu = \iota\mu_{op} + \beta_p\left(\mu_p - \mu_{op}\right) \qquad (8.1)$$

where,

$$\beta_p = \frac{Cov\left[R_t, R_{pt}\right]}{\sigma_p^2}$$

$$= \frac{1}{\sigma_p^2} Cov\left[R_t, R'_{Kt}\omega_p^K\right]$$

$$= \frac{1}{\sigma_p^2} Cov\left[R_t, R'_{Kt}\right]\omega_p^K$$

Substituting the above results into equation (8.1) provide,

$$\mu = \iota\mu_{op} + Cov\left[R_{Kt}, R'_{Kt}\right]\omega_p^K\left(\frac{\left(\mu_p - \mu_{op}\right)}{\sigma_p^2}\right)$$

Then for the K portfolios we have,

$$\mu_K = \iota\mu_{op} + Cov\left[R_{Kt}, R'_{Kt}\right]\omega_p^K\left(\frac{\left(\mu_p - \mu_{op}\right)}{\sigma_p^2}\right)$$

By rearranging, we get the following,

$$\frac{\omega_p^K\left(\mu_p - \mu_{op}\right)}{\sigma_p^2} = Cov\left[R_{Kt,} R'_{Kt}\right]^{-1}\left(\mu_K - \iota\mu_{op}\right)$$

Again by substituting this value into (8.1) returns the following:

$$\mu = \iota\mu_{op} + Cov\left[R_t, R'_{Kt}\right]Cov\left[R_{Kt}, R'_{Kt}\right]^{-1}\left(\mu_K - \iota\mu_{op}\right) \qquad (8.2)$$

Now let us consider a multivariate regression of N assets on K factor portfolios,

$$R_t = a + BR_{Kt} + \epsilon_t$$

where a is the (Nx1) intercept vector, B is (NxK) matrix of the coefficients of the regression. From the econometric theory we have,

$$B = Cov\left[R_{t,} R'_{Kt}\right]Cov\left[R_{Kt,} R'_{Kt}\right]^{-1}$$

$$a = \mu - B\mu_K$$

Again by substituting these expressions into (8.2) give,

$$\mu = \iota\mu_{op} + B\left(\mu_K - \iota\mu_{op}\right)$$

$$= (\iota - B\iota)\mu_{op} + B\mu_K$$

Since the above expression is true for the different values of μ_{op},

then $(\iota - B\iota) = 0$

That is the factor regression coefficients for each asset sum to unity. If $(\iota - B\iota) = 0$ then $\mu = B\mu_K$ and we have *a=o* which indicates that the regression intercept will be zero for all assets including asset *a.*

As we have already proved that the intercept of a multifactor model of expected return is zero. For example, let us consider the following regression,

$$R_{it} - R_{ft} = \alpha_i + \beta_{iM}\left(R_{Mt} - R_{ft}\right) + \beta_{is}SMB_t + B_{ih}HML_t + \epsilon_{it}$$

Fama and French (1993) using the above model observed that the model captures most of the variations of portfolio returns when these portfolios are based on B/M equity and other price ratios that cause problems for CAPM. The estimated intercept term of the above regression is used to calibrate the sensitivity of the stock prices to the new information. From the theoretical perspective, the discouraging part of the three factor model is its empirical motivation. The SMB and HML explanatory returns are not motivated by forecasts on the state variables. The three factor model is not free from the momentum effect either. Stocks that did well in past several months historically tend to do well in future months too. We can easily distinguish this momentum effect from the value effect captured by B/M equity and other ratios. The three factor model cannot explain this momentum effect.

Arbitrage pricing theory (APT) by Ross (1976) shows that we do not need the condition of man-variance optimization for all investors. APT is more general than CAPM since it allows multiple risk factors. Moreover, APT does not require the market portfolio identification as it is required in the CAPM. APT gives us an approximate nexus for expected asset return with some unknown unidentified factors. Unless arbitrage opportunity exists, we just simply cannot rule out the theory. Therefore we need some additional assumptions in order to test the validity of the theory. In APT we assume that markets are competitive and frictionless. The asset return generating process is as follows:

$$R_i = a_t + b_i' f + \epsilon_i$$

$$E(\epsilon_i) = 0$$

$$E(\epsilon_i^2) = \sigma_i^2 \leq \sigma^2$$

Ross (1976) shows that the absence of the arbitrage opportunity in large economy implies that,

$$\mu = \iota\lambda_0 + B\lambda_k$$

Where μ is the (Nx1) vector of the expected asset return, λ_o is zero-beat parameter and λ_k is the (Nx1) vector for the factor risk premium. The coefficient of the zero-beta parameter is the conforming vectors of ones. Again, this is just a specification. To make it testable we need more restrictions on it. To obtain those restrictions we need to have additional structure to make the approximation exact. One of the restrictions is the market portfolio has to be well diversified and the factors have to be pervasive. We can consider the market portfolio be diversified if the proportion of any asset to overall economy asset is very insignificant. The requirements of factors be pervasive allows the investors to diversify their idiosyncratic risk without restricting the choice of factor risk revelation. Dybvig (1985) investigated the influence of the deviations from the exact factor pricing based on preference revealed by an investor. He observed that if the parameters of the economy are reasonably specified then the effect of deviating from the exact factor pricing is negligible.

The multifactor model does not specify the number of factors and the identification of the model. Therefore, to estimate the model we need to determine these factors. Conventionally we consider four types of exact factor pricing model. They are,

1) Factors are the portfolios of the underlying traded assets and there exists a risk free asset

2) Factors are the portfolios of the underlying traded assets and there is no risk free asset

3) Factors are the portfolios of the traded assets

4) Factors are the portfolios of the traded assets where the factor portfolios span the efficient frontier of the risky assets.

There are a number of empirical literatures on multifactor models. Chen, Roll and Ross (1986) observed that the empirical evidence to support the exact factor pricing model is fixed. One of the strongest evidences comes from the testing by using dependent portfolios formed on market value of equity and B/M ratios. In addition, the multifactor model cannot explain the size effect and B/M effect properly. However, the portfolios are based on dividend yield on own variance provide little evidence against exact factor pricing. On the other hand, Fama and French (1993) observed some encouraging results of multifactor model if five factors are using instead of three. They concluded that for stocks we need three factors but for bond portfolios we need to include five factors. Lehmann and Modest (1988) analyzed the sensitivity of the number of the dependent variables included in the model. With fewer portfolios the p-values were lower in the result which is an issue of the power of the test. By reducing the number of portfolios and without deviating from the null hypothesis, we can increase the power of the test. This is true because now we require to test fewer restrictions than before.

The multifactor models provide an alternative to the single-factor CAPM but researchers using these models, have to be aware of two pitfalls arise when factors are chosen to fit the data without considering the economic theory. First of all, because of the data snooping bias the model may over fit the data and the model looses its ability to predict asset return in future. Secondly, the model may capture empirical regularities that arise because of market inefficiency and in this case it may fit the data but they will imply Sharpe ratios for factor portfolios that are too high for being consistent with a reasonable underlying market equilibrium model. However, we have to wait for the sufficient amount of new data become available to test the usefulness of the multifactor models.

9. CAPM with Higher-Order Co-Moments

We know that the unconditional security return distribution is not normal. Moreover, the mean and variance of security returns are not sufficient enough to characterise the distribution completely. Thus it encourages the researchers to look for the higher order co-moments. In practice we estimate the skewness (third moment) and kurtosis (fourth moment). In many studies researchers paid attention to the validity of CAPM in the presence of the higher order co-moments and their effects on the asset pricing. In many studies skewness has been incorporated in the asset pricing models and it provided mixed results.

Harvey and Siddique (2000) investigated an extended version of CAPM. Since the conditional skewness confines the asymmetry in risk, this version of CAPM is usually preferred over the fundamental one. In recent times, this concept of conditional skewness has become very useful in measuring the value at risk. From the study of Harvey and Siddique (2000) we notice that the conditional skewness captures the variation in cross-sectional regression analysis of expected returns significantly. This also holds true when factors based on size and book-to-market ratio are also considered.

In some studies we see that in determining the security valuations, the non-diversified skewness and kurtosis play an important role. Fang and Lai (1997) reported a four-moment CAPM and in their study they showed that systematic variance, systematic skewness and kurtosis contribute to the risk premium of the underlying asset.

10. Conditional Asset Pricing Models

Levy (1974) suggested to estimate different betas for bull and bear markets. Following that suggestion, Fabozzi and Francis (1977) estimated the betas for bull and bear markets. However, they didn't find any evidence of beta instability. However, in another work Fabozzi and Francis (1978) reported that investors need a positive premium in order to accept the downside risk. On the other hand a negative premium corresponds with the up market beta. This up market beta is considered as a more appropriate measure of portfolio risk.

There are few other studies examined the randomness of beta. Kim and Zumwalt (1979) examined the variation in returns on portfolios in both up and down markets. They concluded that the up market comprises the months for which the market returns exceed the average market return, the average risk neutral rate and zero. They specified three measures to identify what make up an up and down market. Those months for which the market returns exceed the average market return and when it is above the risk free rate or greater than zero constitute the up market. They observed that the respective betas of the down market is more accurate measure for the portfolio risk than the single beta that we see in the conventional CAPM. In an investigation on risk-return relationship Chen (1982) allowed the beta to be non-stationary and observed that investor need compensation when they assume downside risk no matter whether the betas are constant or changing. Their findings coincide with findings of Kim (1979). Bhardwaj and Brooks (1993) concluded that the systematic risks are different in bull and bear time periods. The also classified the market as Kim and Zumwalt (1979) did but instead of comparing the market return with mean return they compared it with the median return.

Pettengill, Sundaram and Mathur (1995) observe that if we use the realized return then the beta-expected return relationship becomes conditional on the excess market return. From that study we see that there exists a positive relationship between beta and expected return during a up market. In line with their study, Crombez and Vennet (2000) studied the conditional relationship between asset return and beta. They concluded that beta is a dependable estimator in both bull (upward market) and bear market (downside risk). For different kind of specifications of the up and the down market this beta factor becomes robust and the investors can increase the expected asset return by considering the up and down market separately. Therefore different moments vary and correspond to the up and the down market. Galagedera and Silvapulle (2002) analyzed the asset return and the higher order co-moments in both bull and bear market and suggested that in the skewed market return distribution, the excess return is related to the systematic co-skewness.

11. CAPM: Conditional on Time-Varying Volatility

Engle (1982) introduced the ARCH/GARCH process with which we can test the time varying volatility of stock return. This approach has drawn a considerable attention to the recent CAPM literatures. Although not quite convincing, this approach provides much stronger evidence of risk-return relationship which we find in the conventional model of CAPM.

Fraser, Hoesli, Hamelink and Macgregor (2000) did a cross-sectional regression analysis of risk-return relationship based on unconditional identification of the betas with the betas estimated from the ARCH and GARCH model. When the excess return is negative they allowed a negative relationship of risk and return. They concluded that CAPM works better in a downward moving market than an upward one. They also observed that for a bear market, beta as a proxy for risk measure works more accurately. Braun, Nelson and Sunier (1995) studied the leverage effects. By using EGARCH models they investigated the variation in beta. This kind of model allows the market volatility and asymmetrical response of beta to see the good and bad news impact on the asset or portfolio returns. Galagedera and Faff (2003) did a study on a conditional three-beta model. They modeled that one as GARCH (1,1) process and depending on the size and nature of the volatilities, they defined three state of volatilities. From that study we see that for most of the market portfolios, the betas are not significantly different from zero.

12.Conclusion

The empirical evidences of Sharpe (1964) and Lintner (1965) version of CAPM have never been encouraging. However, allowing for the flatter trade off of average asset return of market beta achieved some success but researches after late 1970s observed that some other important variables such as different kind of price ratios, the momentum effect, size, etc have crucial impact on asset return. Thus, CAPM hold well conditionally. The textbook model often refers to estimate the cost of the equity capital. This kind of model suggests to estimate market beta and to combine with the risk premium to get an estimate of the cost of the equity. But, we have already seen that the beta and average return relationship is much flatter than the Sharpe-Lintner version of CAPM. Therefore the estimates for the high-beta assets are too high and it is too low for the low-beta assets.

The normality assumption of asset return is very important to hold the CAPM but the stock return is non-normal in high frequency data. When the asset return is normal then mean and variance can explain the asset return distribution. To explain the non-normal asset return distribution we need the higher order moments. Again, the empirical evidence in favour of the higher order moments is not unarguable. As the market beta failed to explain the cross-sectional variation in security returns, the multifactor model came in front. As we have already observed, these models incorporated some important variables such as different price ratios, size, etc. Some authors argue that CAPM is overstated because of the problem of market proxy, negligence of conditioning information and data snooping bias and CAPM might hold true in a dynamic equilibrium setting.

According to Roll's critique, CAPM is not testable as long as the composition of the true market portfolio is known. Thus we cannot test the theory properly as long as all the assets are included in the sample. The proxy measure of market portfolio has some difficulties too. The market proxies themselves can be mean variance efficient and they can be highly correlated to each other. Moreover the identification problem creates problem to the testability of the theory. Despite having the simplicity and being the fundamental concept of asset pricing theory, empirical evidences, model testing limitations and the debate over the result interpretation make its use in real life application arguable. Again, one can argue here that no risk based model has the ability to explain the anomalies of stock market behaviour.

Bibliography

Amihud, Y., Christensen, B.J. and Mendelson, H. 1993. “Further evidence on the risk-return relationship”, *Working Pape*r, New York University.

Basu, Sanjay. 1977. “Investment Performance of Common Stocks in Relation to Their Price- Earnings Ratios: A Test of the Efficient Market Hypothesis.” *Journal of Finance*.

Bhardwaj, R.K. and Brooks, L.D. 1993. “*Dual betas from bull and bear markets: reversal of the size effect*”, *The Journal of Financial Research*.

Black, F. 1993. “Beta and return”, *Journal of Portfolio Management*.

Black, Fischer, Michael C. Jensen and Myron Scholes. 1972. “The Capital Asset Pricing Model: Some Empirical Tests,” in *Studies in the Theory of Capital Markets*.

Blume, Marshall. 1970. “Portfolio Theory: A Step Towards its Practical Application.” *Journal of Business*.

Blume, Marshall and Irwin Friend. 1973. “A New Look at the Capital Asset Pricing Model.” *Journal of Finance*.

Campbell, John Y. and Robert J. Shiller. 1989. “The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors.” *Review of Financial Studies*.

Chan, Louis K.C., Yasushi Hamao and Josef Lakonishok. 1991. “Fundamentals and Stock Returns in Japan.” *Journal of Finance*.

Chen, S.N. 1982. “An examination of risk-return relationship in bull and bear markets using time-varying security betas”, *Journal of Financial and Quantitative Analysis.*

Dechow, Patricia M., Amy P. Hutton and Richard G. Sloan. 1999. “An Empirical Assessment of the Residual Income Valuation Model.” *Journal of Accounting and Economics*.

Fabozzi, F.J. and Francis, J.C. 1977. “Stability tests for alphas and betas over bull and bear market conditions”, *Journal of Finance*,

Fabozzi, F.J. and Francis, J.C. 1978. Beta as a random coefficient, *Journal of Financial and Quantitative Analysis*.

Faff, R. and Brooks, R.D. 1998. "Time-varying beta risk for Australian industry portfolios: an exploratory analysis", *Journal of Business Finance and Accounting*.

Faff, R., Ho, Y.K. and Zhang, L. 1998. "A generalised method of moments test of the three-moment capital asset pricing model in the Australian equity market", *Asia Pacific Journal of Finance*.

Faff, R., Lee, J. and Fry, T. 1992. "Time stationarity of systematic risk: some Australian evidence", *Journal of Business Finance and Accounting*.

Fama, Eugene F. 1996. "Multifactor Portfolio Efficiency and Multifactor Asset Pricing." *Journal of Financial and Quantitative Analysis*.

Fama, Eugene F. and Kenneth R. French. 1992. "The Cross-Section of Expected Stock Returns." *Journal of Finance*.

Fama, Eugene F. and Kenneth R. French. 1993. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics*.

Fama, Eugene F. and Kenneth R. French. 1995. "Size and Book-to-Market Factors in Earnings and Returns." *Journal of Finance*.

Friend, Irwin and Marshall Blume. 1970. "Measurement of Portfolio Performance under Uncertainty." *American Economic Review*.

Gibbons, Michael R. 1982. "Multivariate Tests of Financial Models: A New Approach." *Journal of Financial Economics*.

Gibbons, Michael R., Stephen A. Ross and Jay Shanken. 1989. "A Test of the Efficiency of a Given Portfolio." *Econometrica*.

Harvey, C. and Siddique, A. 2000. "Conditional skewness in asset pricing tests", *Journal of Finance*.

Haugen, Robert. 1995. *The New Finance: The Case against Efficient Markets*. Englewood Cliffs, N.J.: Prentice Hall.

Huang, H.R. 2000. "Tests of regimes-switching CAPM", *Applied Financial Economics.*

Jensen, Michael C. 1968. "The Performance of Mutual Funds in the Period 1945–1964." *Journal of Finance*.

J. Fred Weston and Thomas E. Copeland. 1988. "Financial Theory and Corporate Policy" *Addison Wesley*.

Lintner, John. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics*.

Markowitz, Harry. 1952. "Portfolio Selection." *Journal of Finance*.

Markowitz, Harry. 1959. *Portfolio Selection: Efficient Diversification of Investments*. Cowles Foundation Monograph No. 16. New York: John Wiley & Sons, Inc.

Merton, Robert C. 1973. “An Intertemporal Capital Asset Pricing Model.” *Econometrica*.

Miller, Merton and Myron Scholes. 1972. “Rates of Return in Relation to Risk: A Re-examination of Some Recent Findings”.

Pastor, Lubos and Robert F. Stambaugh. 1999. “Costs of Equity Capital and Model Mis-pricing.” *Journal of Finance.*

Roll, Richard. 1977. “A Critique of the Asset Pricing Theory’s Tests’ Part I: On Past and Potential Testability of the Theory.” *Journal of Financial Economics*.

Ross, Stephen A. 1976. “The Arbitrage Theory of Capital Asset Pricing.” *Journal of Economic Theory*.

Sharpe, William F. 1964. “Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk.” *Journal of Finance.*