Distortionary Taxation in a Search Model of Money and Banking

by

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Abstract

There is a large literature on money search. However, very few considered fiscal policy. In my paper, I examine both monetary and fiscal policies in a model of money and banking. I show that the fiscal policy can affect money allocation and improve social welfare.

1. Introduction

The purpose of this paper is studying monetary and fiscal policies in a model with money and banking. There is a large literature on the money search, but very few consider fiscal policy. This is because of the information constraint. “Anonymous trade in goods market” makes taxation difficult. The key difference of my paper with others is other than using monetary policies, I include both monetary and fiscal policies to improve money allocation and social welfare. I address the following question in this paper: How does fiscal policy affect money allocation and social welfare?

To answer this question, I built a monetary model based on Berentsen, Camera and Waller (2007) (call it BCW.) The framework of this model is constructed by using Lagos and Wright’s divisible money model. I include financial intermediaries in the market, and call them banks. I do not allow private banks to issue their own money. So there is only outside money circulates in the model. However, many papers, such as Cavalcanti and Wallace (1999), Williamson (2004) and Sun (2007), indicate the other direction of monetary model, which is they allow banks issue inside money, and agents use inside money as medium of exchange. In this paper I will leave the case with inside money for future study.

I assume government charges a proportional tax from agents, and then uses this revenue to reallocate the money agents hold. Therefore, proportional tax is one of fiscal policies we can use for redistribute money holding. A proportional tax stimulates agents to hold cash for escaping tax. Therefore, I build my model with two cases. In the first case, I suppose agents deposit all cash in banks, so no one escapes tax. I find a positive tax will reduce the welfare, while a negative tax will rise welfare up. Because with the negative tax, government pays subsidies to agents’ deposits, in this case, agents will want to deposit all the cash. Therefore, the result I find matches the assumption I set, which is agents deposit all cash in banks.

In the second case, I assume each agent holds α percent of cash to escape tax. And I find if government charges a small amount of tax, agents still deposit all cash. Since interests from depositing action is more attractive for agents. However, as the tax rate increases, tax payment becomes a problem for agents, and then agents begin to hold cash in order to avoid the gradually increased tax payments.

Two results are showed in this paper. Firstly, if tax rate is negative, government pays subsidies to agents. This fiscal policy therefore would improve money allocation and welfare. Secondly, when tax rate reach high enough, agents would like to hold cash in order to avoid paying tax. In this case, fiscal policy also improves social welfare.

I did the following literature reviews. In Kiyotaki and Wright (1989), they introduce fiat money as a medium of exchange. Later on, in Lagos and Wright (2005), they extend search model with divisible money. The framework of my paper is built based on this divisible money search model. In He, Huang and Wright (2005), they address money and banking in a search model of money. In Kocherlakota (2003), he introduces nominal government bonds or outside bonds into market, so that agents with scarce cash would sell outside bonds. This application of bond market improves the allocation of money. In Berentsen and Waller (2008) claim the issuing of inside bonds can also improve the money allocation. They also show that inside bonds are more flexible than outside bond since inside money allows agents to extend or contract their cash constraint depending on their liquidity needs. Moreover, some authors introduce bank system into market and use it to release agents' cash constraint. In Berentsen, Camera and Waller (2007), financial intermediaries play an important role in credit market. They receive deposit from agents and make a loan to others. Financial intermediaries in that paper work as a big saving box, they collect agents' idle money and reallocate to other agents who meet with cash constraint problem. In Cavalcanti and Wallace (1999), He, Huang and Wright (2005) and Sun (2007), indicate the other direction of monetary model, which is they allow banks issue inside money, and agents use inside money as medium of exchange.

The paper is organized as follows. In section 2, I discuess the environment of this model and build some assumptions. In section 3, I will explain agents' welfare condition. In section 4, I will solve and explain the market equilibrium condition for the behavior of agents deposit all cash in banks; Check the welfare and compare it with the welfare without fiscal policy. In section 5, I will do the exactly same process as section 4, but changes the assumption to agents are going to hold some cash. The last part is the conclusion.

2 The Environment

The purpose of my paper is to investigate the effects of fiscal policy and monetary policy on welfare. Government executes fiscal policy through tax system. Therefore, government needs to know all agents' money balance in order to implement fiscal policy. In BCW model, financial intermediation, "bank", is introduced in money market. It has the right to record all agents' account information. Thus, the amount of collected tax is tractable. There are two markets in each discrete time period. Markets are perfect competitive. The second market opens only after the first market closes, no idle time between the first and the second market. I assume all agents anonymously trade in good market, agents cannot point out their trade partners. Therefore, credit trade between agents does not allow in my model. Fiat money has its own value in the market. Hence, fiat money is insured to be used to trade consumption goods. Agents in the market are continually and infinitely living. All agents are averagely distributed in the region [0, 1]. Each of agent can produces and consumes one kind of perishable good.

The preference shock happens at the beginning of the first market. In this shock, agents have a probability of n to become a producer and a probability of (1 − n) to become a consumer. Producers can only produce the perishable goods at the first market but cannot consume them. On the other hand, consumers can only consume but cannot produce. Therefore, we could simply call the consumers are buyers while producers are sellers in the first market.

There is no preference shock in the second market. Therefore, all agents produce and consume in this market. The different preferences of agents in the first market imply that the money holding of buyers and sellers are not identical at the beginning of the second market. Thus, market decision of buyers and sellers will be different in the second market.

Agents' utility is $u(q_b)$ in the first market. q_b is the unit of goods consumed in the first market. This utility formula is monotonously increasing and concave in terms of q, where $u'(q_b) > 0$, $u''(q_b) < 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. The utility cost $c(q_s)$ is occurred when q_s units of consumption goods are produced. The distribution of $c(q_s)$ follows $c'(q_s) > 0$, $c''(q_s) > 0$, which implies the unit cost increases along with the increase of production. In the second market, agents get utility U(x) form x units of goods consumption. This utility also satisfy that $U'(x) > 0$, $U''(x) < 0$, $U'(0) = +\infty$, and $U'(\infty) = 0$. The disutility of produce one unit of good is 1, which means unit labor cost is one and one unit of labor is required to produce one unit of consumption goods. Finally, we define β as the discount factor across periods. β belongs to the region (0, 1).

Base on Lagos and Wright's divisible money model, bank system is added in my model. By assuming there is one central bank, which is dependent with federal government, government controls the money supply through central bank. The money growth rate, r, is defined as $M_t = rM_{t-1}$, where M_t is money stock per capita in period t and M_{t-1} is money stock per capita in period t-1. The positive value of money stock in each period implies that r must be a positive number. Government uses two variables to control the value of r. One is the money injection rate, τ, which is set by central bank. τM_{t-1} denotes the lump sum transfers delivered from central bank to agents at the period t. This transaction process is finished in two steps. The first step is that agents receive the amount of $\tau_1 M_{t-1}$ transfer at the beginning of the first market. The second step is agents receive the amount of $\tau_2 M_{t-1}$ transfer at the beginning of the second market. Therefore, the total money injection rate we so far got from market 1 and 2 are $\tau_1 + \tau_2 = \tau$. Furthermore, if government would like to transfer different amounts of money to buyers and sellers, we can set $\tau_1 = (1 - n)\tau_b + n\tau_s$, where τ_b and τ_s denote the money transfer rate for buyer and seller. The other variable used to adjust the money growth rate is proportional tax rate t. This tax rate is charged base on agents' money holding at the end of the second market. In this model with the first case assuming agents deposit all cash into banks at the end of the second market, government can get the agents money holding information from private banks. Therefore, proportional tax is tractable. If an agent holds m_{t+1} money at the end of market, he/she will pay tm_{t+1} taxes, where m_{t+1} is the amount of money holding at the end of period t. Tax works as a cost of holding cash. Thus, a raise of tax will lead to a reduction of cash holding and an increase of consumption in current period. Tax revenue will be paid back to the agents as lump-sum transaction. Then we have the equation $\tau M_{t-1} = (r - 1)M_{t-1} + t\, M_{t-1}$ (a) on hold, where the total money injection is equal to the government tax revenue plus the money growth across periods. Moreover, tax rate can be negative, since government might wish to give subsidization to agents.

Beside central bank, other financial intermediations are called private banks. Bank system is free entry. So all private banks are perfectly competitive, they accept nominal deposits and make nominal loans to agents at the beginning of the first market. We assume the deposits and loans take action within one period only. The quasi-linear utility function assures one period loans are optimal for agents. Furthermore, we set an assumption that agents will deposit all their money holding to banks at the end of the second market. Therefore, government could charge proportional tax based on agents' account information at the end of the second market. Using this assumption, the agents' decisions and effects of fiscal policy on welfare are more tractable. Later on, I will release this assumption in the second case, which is agents will not deposit all of their cash holding in banks at the end of second market. The results of case 2 are not analytically informative and need simulation work. So I will focus on case 1 in this paper. Suppose agents' financial histories can only be recorded by banks, credit trade is not doable between buyers and sellers. The cost of this record-keeping is zero for banks. Banks have no right to record the goods trading histories. So agents cannot use consumption goods to repay loans. Issuing inside money is not allowable; it ensures that outside money is the only medium of exchange in goods market.

Default is a serious problem in financial market. I assume banks can perfectly enforce repayment . In this case, default is not going to happen. Therefore, banks can offer agents any required amount of loans. Borrowing constraints have no binding. In the case banks do not have the right to force agents repaying their loans, then banks need to punish the defaulters. The only feasible punishment is abolishment the agents' rights of borrowing and deposit money from banks. We will leave the case of banks with no enforcement of repayment require from agents for future study. In this paper, we will work on the case that banks can force agents to repay their loans.

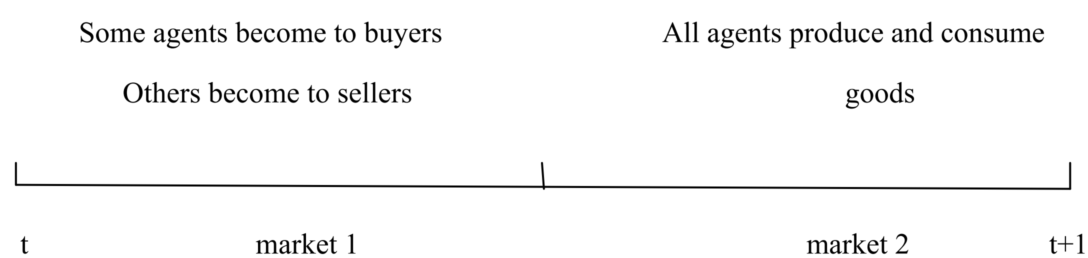
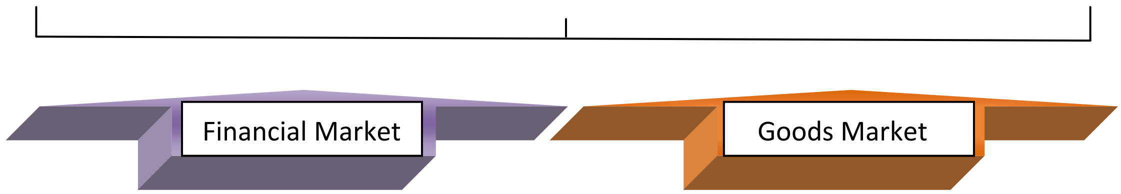


Figure 1

Market 1

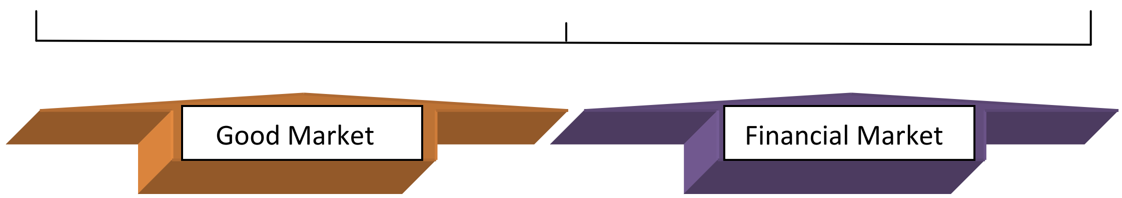


Financial market: Agents borrow and deposit money

Goods market: agents trade consumption Goods

Figure 2

Market 2



Goods market: agents trade consumption goods

Financial market: Agents repay the loans and get interest payment from banks

Figure 3

The time line of our model is showed in Figure 1, 2 and 3. Firstly, the financial market opens, agents can borrow and deposit money there. After the financial market closed, the goods market open. In the goods market, money is the only thing agents can be used to trade consumption goods. Deposit and borrow money is not allowable in goods market. After the second goods market closed, the financial market open again. Agents repay their loan and get interest payment in this market. After these transactions done, agents deposit all idle money into banks, and then government charges tax according to agents' account balance.

3. Welfare

Let us set welfare of each agent in period t as ω_t. It is composed by two parts. The first part is agent's welfare getting from goods market one, the other part is agent's welfare in market two. Thus, ω_t can be represent as $(1 - n)\ u(q_{bt}) - nc(q_{st}) + U(x_t) - x_t$, where q_{bt} and q_{st} are agent's consumption and production in market 1 at time t. Agent's lifetime utility in steady state can be described as

$$\omega = \frac{1}{1-\beta}\omega_t = \frac{1}{1-\beta}[(1 - n)\ u(qb) - nc(qs) + U(x) - x] \qquad (1)$$

The maximized lifetime utility must satisfy the following good market constraint:

$$(1 - n)q_b = nq_s$$

Goods demand equals to goods supply in market 1. First order conditions of equation (1) are

$$\frac{\partial W}{\partial x}: \quad U'(x^*) = 1 \qquad (2)$$

$$\frac{\partial W}{\partial q}: \quad u'(q^*) = c'\left(\frac{1-n}{n}q^*\right) \qquad (3)$$

Where $q^* \equiv q^*_b \equiv \frac{1-n}{n}q^*_s$. Social welfare best decisions, represent by x and q, satisfy (2) and (3). Although government cannot enforce agents to produce and consume at social best level, they will lead agents' decision to social optimal level through monetary and fiscal policy.

4. Case 1: Agents deposit all cash in banks at the end of market 2

4.1 Agent's decision in goods market.

In steady state equilibrium, We set V(m) as expected utility of an agent calculating from market 1. m is agent's money holding at the beginning of market 1. W(m, ℓ, d) is defined as the expected utility calculating from market 2, where m in W is agent's money holding at the beginning of market 2, ℓ and d are agent's loans and deposits at time t. In order to solve the symmetric equilibrium condition, we can follow the Lagos and Wright model, and calculate the equilibrium backward.

1) The second market

All agents produce and consume goods in this market. Agent's consumption and production in turn are x and h. Agents are also need to repay loans, redeem deposit and adjust their money balances for next period. Nominal interest rate for loan and deposit are i and i_d. If ℓ units of money has been borrowed, then they need to pay $(1 + i)\ell$ at the end of the second market. And d unit of deposit will create $i_d d$ units of benefits. Then the question of agents maximize the expected value for the second market can be described as

$$W(m, l, d) = \max_{x,h,m_{t+1}}[U(x) - h + \beta V_{t+1}(m_{t+1})] \tag{4}$$

$$\text{s.t } X + \frac{\phi m_{t+1}}{1-t} = h + \phi(m + \tau_2 M_{t-1}) + \phi(1 + i_d)d - \phi(1 + i)\ell$$

Using constraint to replace h in the value function W, we get

$$W(m, \ell, d) = \max_{x,m_{t+1}}[U(x) - X - \frac{\phi m_{t+1}}{1-t} + \beta V_{t+1}(m_{t+1})]$$

$$+ \phi[m + \tau_2 M_{t-1} + (1 + i_d)d - (1 + i)\ell] \tag{5}$$

Where m_{t+1} is the money of agents hold at time t + 1and ϕ is price level in market 2.The first order conditions are

$$\frac{\partial W}{\partial x}: \quad U'(x) = 1 \tag{6}$$

$$\frac{\partial W}{\partial m_{t+1}}: \quad \frac{\phi}{(1-t)} = \beta V'_{t+1}(m_{t+1}) \tag{7}$$

Where $V'_{t+1}(m_{t+1})$ is the extra utility that agent can get form one additional unit of cash holding in period t + 1, which is independent with m. The right hand side of (7) tells us that the tax rate t is an important element to control and hence affect the choice of m_{t+1} since ϕ and β are uncontrollable. The function (6) shows the marginal utility of an additional unit of goods was consumed in market 2. The identical utility function U(x) and U'(x) = 1 imply that all agents choose the same optimal value of x across time. The envelope conditions are

$$W_m = \phi \tag{8}$$

$$W_\ell = -\phi(1 + i) \tag{9}$$

$$W_d = \phi(1 + id) \tag{10}$$

Equations (8) to (10) are marginal value of money holding, loan and deposit in the market 2.

2) The first market

In the first market, agents get a preference shock before they trade in the goods market. An agent can become a seller with the probability of n, would produces q_s units of goods but consume none goods. On the other hand, an agent can become a buyer with the probability of 1 – n, would consume q_b units of goods, but produce none goods. Nominal price p is a competitive price in the first market. An agent's expected lifetime utility with m money holding at the opening of the first market is

$$V(m) = (1 - n)[u(q_b) + W(m + \tau_b M_{t-1} + \ell_b - d_b - pq_b, \ell_b, d_b)]$$
$$+ n[-c(q_s) + W(m + \tau_s M_{t-1} + \ell_s - d_s + pq_s, \ell_s, d_s)] \tag{11}$$

Where ℓ_b and d_b are buyer's deposit and loan, while ℓ_s and d_s are sell's deposit and loan. Positive interest rate implies that buyer will never deposit money in the bank and seller will never borrow money from banks. Thus, $\ell_s = d_b = 0$. In order to simplify the notation, we can drop the subscript of ℓ and d, and then use ℓ to denote loans borrowed by buyers and use d to denote sellers' deposits. Then equation (11) can be simplified to

$$V(m) = (1 - n)[u(q_b) + W(m + \tau_b M_{t-1} + \ell - pq_b, \ell)]$$
$$+ n[-c(q_s) + W(m + \tau_s M_{t-1} - d + pq_s, d)] \quad (12)$$

Where pq_b is money spent by buyer and pq_s is money earned by seller. The lump sum transfers received by Buyers and sellers are $\tau_b M_{t-1}$ and $\tau_s M_{t-1}$. Note that financial market closes before the goods market opens. Therefore, seller's earnings pq_s cannot be deposited into banks. The value function V(m) is composed by two part. One is seller's value function, the other is buyer's. So we can investigate these two parts separately below.

4.1.1 Seller's decisions and value function:

The seller's problem is

$$\max_{q_{s,d}} V_s = [-c(q_s) + W(m + \tau_s M_{t-1} - d + pq_s, d)]$$

$$\text{s.t. } d \leq m + \tau_s M_{t-1}$$

The first order conditions are

$$\frac{\partial V_s}{\partial q_s}: \quad -c'(q_s) + pW_m = 0 \quad (13)$$

$$\frac{\partial V_s}{\partial d}: \quad -W_m + W_d - \lambda_d = 0 \quad (14)$$

Where λ_d is the Lagrangian multiplier on the deposit constraint. Substitute (8) into (13) we get

$$c'(q_s) = p\phi \quad (15)$$

According to equation (15), the optimal value of seller's production, q_s, is independent of m and d. They will keep producing consumption goods until the ratio of marginal costs of markets equals to the relative price of goods across markets. Therefore, no matter how much money seller holds or deposits, every seller produces the same amount of goods. Furthermore, as the nominal deposit interest rate is larger than zero, sellers would like to deposit all their idle money into banks. Thus, sellers deposit constraint is always binding.

4.1.2 Buyers' decisions and value function:

The buyer's problem is

$$\max_{q_{d},\ \ell} V_b = [u(q_b) + W(m + \tau_b M_{t-1} + \ell - pq_b, \ell)]$$

$$\text{s.t. } m + \tau_b M_{t-1} + \ell \geq pq_b$$

$$\ell \leq \bar{\ell}$$

The first constraint is buyer's budget constraint; Buyers cannot spend more money than they have in the first market. The second constraint is buyer's borrowing constraint. The upper bound of the loan size is $\bar{\ell}$. Banks set this upper bound is because borrowers may fail to repay the loan. So banks may not want lend too much loan to borrowers.

The first order conditions are

$$\frac{\partial V_b}{\partial q_b}: \quad u'(q_b) - pW_m - p = 0 \tag{16}$$

$$\frac{\partial V_b}{\partial \ell}: \quad W_m + W_\ell + \lambda - \lambda_\ell = 0 \tag{17}$$

Using (8), (9) and (15) the equation (16) and (17) are simplified to

$$\frac{u'(q_b)}{c'(q_s)} = 1 + \frac{\lambda}{\phi} \tag{18}$$

$$\lambda = \phi i - \lambda_{\ell} \tag{19}$$

Where λ is the multiplier of budget constraint, and λ_{ℓ} is multiplier of buyer's borrowing constraint. If $\lambda = 0$, buyer's budget constraint may not binding, buyer will have some idle money in account. Then equation (18) reduces to $u'(q_b) = c'(q_s)$, which implies trades are efficient.

If $\lambda > 0$, buyer's budget constraint is binding, buyer uses all available money to buy consumption goods, and then equation (18) changes to

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \frac{\lambda_{\ell}}{\phi} \tag{20}$$

If $\lambda_{\ell} = 0$, buyers are not restricted by borrowing constraint, then

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i \tag{21}$$

In this case the buyer borrows enough money to consume at the optimal level. At this point, buyer's marginal benefit of borrowing ($u'(q_b)$) equals to the marginal cost ($c'(q_s)(1+i)$).The buyer hence spends all his/her money and consumes $q_b = \frac{(m+ \tau_b M_{t-1} + \ell)}{p}$.

In the case of $\lambda_{\ell} > 0$, equation (18) implies

$$\frac{u'(q_b)}{c'(q_s)} > 1 + i \tag{22}$$

Buyer's borrowing constraint is binding. Buyer cannot borrow as much money as he wants. The marginal benefit of borrowing is larger than the marginal cost. So buyer wants to pay a higher interest rate than the nominal loan rate. In this case, buyer borrows at upper bound point and spends all money for consumption, which brings out that q_b equals to $\frac{(m+ \tau_b M_{t-1} + \bar{\ell})}{p}$. Since all buyers are identical and facing the same problem when they enter the market, thus the optimal value of q_b is same for all buyers. The seller's case is same as buyer, thus q_s is same for all sellers. According to goods market clearing condition, we get

$$q_s = \frac{1-n}{n} q_b \tag{23}$$

4.1.3 Bank's decision:

Banks are the last type of agents I want to discuss. Banks accept nominal deposits from producers and lend them to consumers at the beginning of the first market, and then banks closes before goods market opens. At the end of second market, banks open again. They receive nominal interest payment from buyer and pay nominal deposit interest payment to producer at this time. Bank market is free entry, so banks are perfect competitive and take nominal interest rate as given. Moreover, bank's decision must satisfy the market clear condition. We know all agents are identical. So in the equilibrium, all buyers borrow same amount ℓ and all sellers deposit same amount d. Then the market clearing condition and bank's decision must satisfy $(1 - n)\ell = nd$. We assume banks cannot communicate with each other and agents cannot bargain the interest rate with banks. Moreover, bank's reserve rate is zero. So banks are not required to hold any amount of cash. The banks' problem as follows

$$\max_{\alpha,\iota} V_{\mathbf{B}} = (i - i_d)\,\ell$$

$$\text{s.t. } \ell \leq \bar{\ell}$$

$$u(q_b) - (1 + i)\ell\varphi \geq \Gamma$$

Where Γ is the surplus borrower getting from the third bank. The first and second constraints are loan and bank surplus constraints. The first order condition is

$$\frac{\partial V_B}{\partial \ell}: \qquad i - i_d - \lambda_L + \lambda_\Gamma \left[u'(q_b)\frac{dq_b}{d\ell} - (1+i)\phi\right] = 0 \tag{24}$$

Where λ_L and λ_Γ are the Lagrange multipliers of loan and bank surplus constrain.

Case 1: with the assumption banks could enforce agents to repay loans, if $i - i_d > 0$, banks earn profit with loan. Therefore, the banks would allow as much as loan to the borrowers. Then wherever borrowers get more benefits by taking a loan at a bank than other banks, there will be more potential borrowers show up and borrow from this bank than others. Thus, at the equilibrium, borrowers get the same benefits by taking a loan from any bank. This implies that bank surplus constraint is always binding and $\lambda_\Gamma > 0$. As the assumption I made before, bank market is free to entry. Therefore, if banks make any profit, other institutions will enter the bank market to become one of the banks and this action will draw the profit down until the profit meets zero, where $i = i_d$. Since case 1 of $i > i_d$ is not accord with my previous assumption about bank market, I will not investigate more around this case in this paper.

Case 2: If bank market is perfect competitive $i = i_d$, banks earn zero profit according to $\frac{dq_b}{d\ell} = \frac{i}{p}$.

Substitute (15) into $\frac{dq_b}{d\ell}$, we get

$$\frac{dq_b}{d\ell} = \frac{\varphi}{c'(q_s)} \tag{25}$$

Using (25) to replace $\frac{dq_b}{d\ell}$ in (24), we get

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \frac{\lambda_L}{\lambda_\Gamma \phi} \tag{26}$$

If $\lambda_L = 0$, loan constraint is not binding. Equation (26) is converted to (21), the marginal benefit of borrowing is equal to the marginal cost. If $\lambda_L > 0$, loan constraint is binding, banks cannot lend enough money or make enough loans to borrowers. In this case, equation (26) implies (22).

4.2 Marginal value of money:

Taking derivate of (12) with respect to m, I get the marginal value of money as:

$$V'(m) = (1-n)\left[u'(q_b)\frac{\partial q_b}{\partial m} + W_m\left(1 - p\frac{\partial q_b}{\partial m} + \frac{\partial \ell}{\partial m}\right) + W_\ell \frac{\partial \ell}{\partial m}\right]$$

$$+ n[-c'(q_s)\frac{\partial q_s}{\partial m} + W_m\left(1 + p\frac{\partial q_s}{\partial m} - \frac{\partial d}{\partial m}\right) + W_d\frac{\partial d}{\partial m} \qquad (27)$$

As the envelop theorem shows at market two, $W_m = \phi$, $W_\ell = -\phi(1+i)$ and $W_d = \phi(1 + id)$. Moreover, the reasons of $\frac{\partial q_s}{\partial m} = 0$ and $\frac{\partial d}{\partial m} = 1$ for sellers is seller's production is independent with his/her money holdings, and they will only deposit all his/her cash when i > 0. Therefore,

$$V'(m) = (1-n)\left[u'(q_b)\frac{\partial q_b}{\partial m} + \phi\left(1 - p\frac{\partial q_b}{\partial m} + \frac{\partial \ell}{\partial m}\right) - \phi(1+i)\frac{\partial \ell}{\partial m}\right]$$

$$+ n\phi(1 + i_d)$$

Since i > 0, interest rate will act as a tax on cash holding, buyers will not borrow any extra amount of money and put them in pocket. Therefore, buyer's budget constraint is binding. $m + \tau_b M_{t-1} + \ell = pq_b$. It implies that $1 - p\frac{\partial q_b}{\partial m} + \frac{\partial \ell}{\partial m} = 0$. Hence

$$V'(m) = (1-n)\left[u'(q_b)\frac{\partial q_b}{\partial m} - \phi(1+i)\frac{\partial \ell}{\partial m}\right] + n\phi(1 + i_d)$$

Note that $u'(q_b)\frac{\partial q_b}{\partial m} - \phi(1+i)\frac{\partial \ell}{\partial m} = u'(q_b)\frac{\partial q_b}{\partial m} - \phi(1+i)\left[p\frac{\partial q_b}{\partial m} - 1\right] = \frac{\partial q_b}{\partial m}[u'(q_b) -$

ϕ1+ip+ ϕ1+i= ϕ1+i= u′qbp (where u′qb= c′qs1+i= ϕ1+ip). Thus, we get:

$$V'(m) = (1-n)\frac{u'(q_b)}{c'(q_s)} + n\phi(1 + i_d) \qquad (28)$$

The marginal value of money has two components. If the agent is a buyer, he/she receives $\frac{u'(q_b)}{c'(q_s)}$ utility when he/she spends one unit of money. If he/she is a seller he can lend the money and receive$1 + i_d$ utility.

4.3 Market equilibrium

In this part, we will talk about market equilibrium base on the assumptions as banks can force borrowers to repay the loans at no cost, and moreover, agents will deposit all their cash at the end of the second market. Using lagged one period of equation (7) to replace $V'(m)$ in (28). Then use goods market clearing condition (23) to get

$$\frac{r}{\beta(1-t)} - 1 = (1-n)\left[\frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} - 1\right] + ni_d \tag{29}$$

Agents are unconstrained by the borrowing constraint, $\bar{\ell} = \infty$, since banks can force agents to repay their loans, so $\lambda_L = 0$. This implies that (18) holds. Substitute it into (27) yields

$$\frac{r}{\beta(1-t)} - 1 = (1-n)i + ni_d \tag{30}$$

Perfect competitive bank market implies $i = i_d$ at equilibrium. So

$$\frac{r-\beta(1-t)}{\beta(1-t)} = i \tag{31}$$

Using (a) to replace r in (31), we get:

$$\frac{\tau+1-\beta-t(1-\beta)}{\beta(1-t)} = i \tag{32}$$

We can rewrite (32) in terms of q_b using (23):

$$\frac{\tau+1-\beta-t(1-\beta)}{\beta(1-t)} = \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} - 1 \tag{34}$$

$$\Rightarrow \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} = \frac{\tau+1-t}{\beta(1-t)} \tag{35}$$

Definition 1 *When repayment of loans can be enforced and agents deposit all cash into banks at the end of market two, a monetary equilibrium with credit is occur when an interest rate satisfying (32) and a quantity q_b satisfying (35).*

Proposition 1: *When repayment of loans can be enforced and agents deposit all cash into banks at the end of the second market, a unique monetary equilibrium exists. With a constant and positive money injection rate, as tax rate increases, the equilibrium interest rate and demand of consumption also increases.*

Tax works as the cost of agents holding money and interest rate works as subsidization of agents deposit money into banks. When tax rate increases, in order to keep bank account balance unchanged, the interest rate has to increases. The high interest rate can stimulate agents to deposit more. Thus, agents spend less on consumption goods. The proof of Proposition 1 is provided in appendix.

Let's see how the consumption of good changes in an economy without tax. We set $\bar{q}_b$ as equilibrium consumption when there is no tax in the market. From the paper "Money, Credit and Banking"(page 15), we know $\bar{q}_b$ can be solved through

$$\frac{r}{\beta} = \frac{u'(\bar{q}_b)}{c'\left(\frac{1-n}{n}\bar{q}_b\right)} \tag{38}$$

Comparing (38) and (35), we get Proposition 2

Proposition 2: *With a constant and positive money injection rate, τ, if agents deposit all money in banks, government's subsidy will increase the demand of consumption goods and welfare.*

Government subsidy makes people feel rich in current period. Thus, people will spend more in current market. Until now, we proved the welfare will increase when government pay subsidies to agents. Next, I will show the exactly change of welfare according to one unit change of tax rate.

4.4 The change of welfare:

Before we calculate welfare, we should know agents' production in the second market. Let's set m $=M_{t-1}$, m is money holding at the beginning of the first market. We know buyer's budget constrain is holding in the first market. Therefore, buyer has zero money holding at the beginning of the second market. The production of a buyer in market two is

$$h_b = x^* + \phi[\frac{m_{t+1}}{1-t} + (1+i)\ell - \tau_2 M_{t-1}] \quad (40)$$

x^* is buyer's consumption in market two, which equals to $U'^{-1}(1)$. Equation (40) indicates buyer's production earning in market two, which equals to his/her consumption cost in market two plus the money he/she saves for next period and the loan he/she needs to pay, then, minus the money received from government. In equilibrium we have

$$\frac{m_{t+1}}{1-t} = M = M_{t-1} + (\tau_1 + \tau_2)M_{t-1} \quad (42)$$

$$c'(q_s)q_b = \phi p q_b = \phi[(1+\tau_1)M_{t-1} + \ell] \quad (43)$$

$$\phi\ell = nc'(q_s)q_b \quad (44)$$

Using (42), (43) and (44), equation (40) is converted to

$$h_b = x^* + c'(q_s)q_b + inc'(q_s)q_b \quad (45)$$

For sellers, they share $(1+\tau_1)M_{t-1}$ amount of money at the beginning of the second market. So each of them holds $\frac{1}{n}(1+\tau_1)M_{t-1}$. The production of seller in market two is

$$h_s = x^* + \phi\{\frac{m_{t+1}}{1-t} - [pq_s + (1+\tau_1)M_{t-1} + i_d d + \tau_2 M_{t-1}]\}$$

$$\Rightarrow h_s = x^* - c'(q_s)q_s - \phi i_d d \quad (46)$$

Seller's production should cover the consumption minus the earnings from deposit in second market. From (45) and (46) we get

$$\frac{\partial h_b}{\partial t} = 0 + (1+in)\left[c''(q_s)\frac{\partial \frac{(1-n)}{n} q_b}{\partial t} q_b + c'(q_s)\frac{\partial q_b}{\partial t}\right] + \frac{\partial i}{\partial t} nc'c'(q_s)q_b \quad (47)$$

$$\frac{\partial h_s}{\partial t} = 0 - c''(q_s)\frac{\partial \frac{(1-n)}{n} q_b}{\partial t} q_b - \phi(\frac{\partial i}{\partial t} d + \frac{\partial d}{\partial t} i) \quad (48)$$

$\frac{\partial i}{\partial t}$ can be solved from equation (32). We know sellers will deposit all their money into banks when i > 0. Then sellers' deposit constraint is binding.

$$\frac{\partial d}{\partial t} = -M_{t-1} \quad (49)$$

The value function of an agent can be described as

$$W(t) = \max_t\{(1-n)[U(x^*) - h_b + \beta V_{t+1}(m_{t+1}(t))]$$

$$+ n[U(x^*) - h_s + \beta V_{t+1}(m_{t+1}(t))]\} \quad (50)$$

The first order condition of (50) is

$$\frac{dW}{dt} = (1-n)\left(-\frac{\partial h_b}{\partial t} + \beta V'_{m_{t+1}}\frac{\partial m_{t+1}}{\partial t}\right) + n\left(-\frac{\partial h_s}{\partial t} + \beta V'_{m_{t+1}}\frac{\partial m_{t+1}}{\partial t}\right) \quad (51)$$

Note $x^* = u'^{-1}(1)$. Therefore the value of x^* is independent with t. Using (47), (48) and (7) to simplify (51). Hence

$$\frac{dW}{dt} = (1-n)\left(-(1+in)\left[c''(q_s)\frac{\partial q_b}{\partial t} q_b + c'(q_s)\frac{\partial q_b}{\partial t}\right] - \frac{\partial i}{\partial t} nc'c'(q_s)q_b\right)$$

$$+n[c''(q_s)\frac{\partial q_b}{\partial t}q_b + \phi(\frac{\partial i}{\partial t}d - -M_{t-1}i) + \frac{\phi}{(1-t)}\frac{\partial m_{t+1}}{\partial t}] \quad (52)$$

From (42), we get

$$m_{t+1} = [M_{t-1} + (\tau_1 + \tau_2)M_{t-1}][1 - t]$$

$$\frac{\partial m_{t+1}}{\partial t} = -[M_{t-1} + (\tau_1 + \tau_2)M_{t-1}] \quad (53)$$

Substitute (53) and (37) into (52), we get $\frac{dW}{dt}$ in terms of n, t ,α and q_b. If we know the formula of $c(q_s)$, we can simulate the value of W using data of n, t, α and q_b. The simulation part I will leave it for future investigation.

5. Case 2: Agents do not deposit all cash in banks.

α percent of money is held on cash at the end of market 2. Agents decide cash holding rate α before the preference shock. If an agent is buyer in the first market, agent will withdraw all money deposited in the bank and spend them for consumption goods. Therefore, all money deposited by buyers at the end of period t will be withdrawn at the beginning of the period t + 1. Thus buyer's deposited money will not earn any interest rate, but pay tax for period t. Hence, buyers will prefer hold cash. For sellers, the amount of money does not deposit in banks at the end of the period t, cannot deposit at the beginning of this period. Since if sellers have any amount of cash in banks, government will realize these sellers escape tax in period t. And then government will punish these sellers. We assume the punishment is much higher than the interest rate earnings by depositing cash. Thus, if a seller holds any amount of cash at the end of period t, they will hold it until the end of period t + 1. Then we conclude that buyers' optimal choice is holding all cash at the end of the second market and sellers' optimal choice is depositing all cash into banks.

If αis the cash holding rate, for buyers, they pay $(1 - \alpha)m_{t-1}t$ tax, which they are actually not willing to pay. For sellers, they lose $\alpha m_{t-1}i_d$ amount of interest payment. We know agent has a probability of n to become a seller and a probability of 1-n to be a buyer in the first market. Hence, in the equilibrium we must have

$$(1-n)(1-\alpha)m_{t-1}t = n\alpha m_{t-1}i_d \tag{54}$$

Government monitors financial market though bank system. Government cannot charge tax on the part of money, which is not deposited in bank. Therefore, the value function of market W is changing to:

$$W(m, l, d) = \max_{x,h,m_{t+1}}[U(x) - h + \beta V_{t+1}(m_{t+1})] \tag{55}$$

$$\text{s.t } X + \phi\widetilde{m_{t+1}} = h + \phi(m + \tau_2 M_{t-1}) + \phi(1 + i_d)d - \phi(1 + i)\ell$$

Where m_{t+1} is money holding after tax at the beginning of t + 1, $\widetilde{m_{t+1}}$ is money holding before tax at the beginning of t + 1. In this case, we have $m_{t+1} = (1-\alpha)(1-t)\widetilde{m_{t+1}} + \alpha\widetilde{m_{t+1}}$. Then the constraint changes to:

$$X + \phi\frac{m_{t+1}}{\alpha+(1-\alpha)(1-t)} = h + \phi(m + \tau_2 M_{t-1}) + \phi(1 + i_d)d - \phi(1 + i)\ell \tag{56}$$

Substitute h from equation (55) into equation (56), we get

$$W(m, \ell, d) = \max_{x,m_{t+1}}[U(x) - X - \phi\frac{m_{t+1}}{\alpha + (1-\alpha)(1-t)} + \beta V_{t+1}(m_{t+1})]$$

$$+ \phi[m + \tau_2 M_{t-1} + (1 + i_d)d - (1 + i)\ell] \tag{57}$$

The first conditions are

$$\frac{\partial W}{\partial x}: \quad U'(x) = 1 \tag{58}$$

$$\frac{\partial W}{\partial m_{t+1}}: \quad \frac{\phi}{\alpha+(1-\alpha)(1-t)} = \beta V'_{t+1}(m_{t+1}) \tag{59}$$

Where $V'_{t+1}(m_{t+1})$ is independent with m. Therefore, the optimal value of m_{t+1} does not lie on m. From equation (59), we get m_{t+1} is not only depend on tax rate t, but also the cash holding rate α. Equation (58) is identical with (2), which implies the optimal value of agents' consumption in market two is same as the value x* in case 1. The envelope conditions in case 2 are

$$W_m = \phi \tag{60}$$

$$W_\ell = -\phi(1 + i) \tag{61}$$

$$W_d = \phi(1 + id) \tag{62}$$

The envelope conditions in this case are same as the envelope conditions in the first case.

In the first market, seller's problem is different from the first case. Since the money, which is not deposited at the end of the second market, cannot be deposited after preference shock. Therefore, the seller's constraint is changed to $d \le \frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha} m + \tau_s M_{t-1}$, which is money deposit is less than the money used for deposit after tax plus the money received from government.

5.1 Sellers' decisions:

$$\max_{q_{s,d}} V_s = [-c(q_s) + W(m + \tau_s M_{t-1} - d + pq_s, d)]$$

$$\text{s.t. } d \le \frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha} m + \tau_s M_{t-1}$$

From the first order conditions we get:

$$c'(q_s) = p\phi \tag{63}$$

The optimal value of seller's production q_s is independent with m and d. Seller will not stop producing consumption goods until the ratio of marginal costs of markets equals to the relative price of goods across markets. Therefore, no matter how much money seller hold or deposit, every seller produces the same amount of goods. Furthermore, when the nominal deposit interest rate is larger than zero, sellers deposit all their idle money into banks

5.2 Buyers' decisions:

For the buyers, they will use all money in the first market. Therefore, the cash holding rate α has no influence on buyers' decision.

$$\max_{q_{d},\ \ell} V_{b} = [u(q_{b}) + W(m + \tau_{b}M_{t-1} + \ell - pq_{b}, \ell)]$$

$$\text{s.t. } m + \tau_{b}M_{t-1} + \ell \geq pq_{b}$$

$$\ell \leq \bar{\ell}$$

The first order conditions imply that

$$\frac{u'(q_{b})}{c'(q_{s})} = 1 + i + \frac{\lambda_{\ell}}{\phi} \tag{64}$$

If $\lambda_{\ell} = 0$, borrowing constraint is not binding and equation (64) equal to (18). If $\lambda_{\ell} > 0$, borrowing constraint is binding and implies (19).

5.3 Banks

In this case, agents deposit rate α is independent with bank's decision. Therefore, the bank's problem is same as the first case. So I will not repeat the process here.

5.4 Marginal value of money:

The marginal value of money is

$$V'(m) = (1-n)\left[u'(q_{b})\frac{\partial q_{b}}{\partial m} + W_{m}\left(1 - p\frac{\partial q_{b}}{\partial m} + \frac{\partial \ell}{\partial m}\right) + W_{\ell}\frac{\partial \ell}{\partial m}\right]$$

$$+ n[-c'(q_s)\frac{\partial q_s}{\partial m} + W_m\left(1 + p\frac{\partial q_s}{\partial m} - \frac{\partial d}{\partial m}\right) + W_d\frac{\partial d}{\partial m} \tag{65}$$

which equals to the marginal value of money in the first case. In the first case, sellers deposit all idle money into banks. Thus, $\frac{\partial d}{\partial m} = 1$. But in the current case, agents decide the deposit rate before they know they are buyers or sellers. Therefore, $\frac{\partial d}{\partial m} = 1 - \alpha$. using it in (25), after simplify we get

$$V'(m) = \phi[(1-n)\frac{u'(q_b)}{c'(q_s)} + n(1 + i_d)\frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha}] \tag{66}$$

Using lagged one period of equation (59) to replace $V'(m)$ in (66). Then substitutes goods market clearing condition (23) into (66), we get

$$\frac{\phi r}{\alpha + (1-\alpha)(1-t)} = \phi\beta[(1-n)\frac{u'(q_b)}{c'(q_s)} + n(1 + i_d)\frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha}]$$

$$\Rightarrow \frac{r}{\beta[\alpha + (1-\alpha)(1-t)]} - \frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha} = (1-n)\left[\frac{u'(q_b)}{c'(q_s)} - \frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha}\right] + ni_d\frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha} \tag{67}$$

As I discussed in the first case, banks can force agents to repay their loans. Therefore, loan constraint will never binding. So $\lambda_L = 0$. This implies that (21) is on hold. Substitute equation (21) into (67) yields

$$\frac{r}{\beta[\alpha + (1-\alpha)(1-t)]} - \frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha} = (1-n)(1 + i - \frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha}) + ni_d\frac{(1-\alpha)(1-t)}{(1-\alpha)(1-t)+\alpha} \tag{68}$$

Zero profit of bank implies $i = i_d$, so

$$\frac{r}{\beta(1-n)[\alpha + (1-\alpha)(1-t)] + \beta n(1-\alpha)(1-t)} - 1 = i \tag{69}$$

Using (a) to replace r in (53), we get

$$\frac{\tau - t + 1}{\beta(1-n)[\alpha + (1-\alpha)(1-t)] + \beta n(1-\alpha)(1-t)} - 1 = i \tag{70}$$

If $\alpha = 0$, agents deposit all money into bank after the second market closed. Then equation (70) converts to (32). We can rewrite this in terms of q_b using (23):

$$\frac{\tau - t + 1}{\beta(1-n)[\alpha + (1-\alpha)(1-t)] + \beta n(1-\alpha)(1-t)} = \frac{u'(q_b)}{c'\left(\frac{1-n}{n} q_b\right)} \tag{71}$$

Definition 2: *When repayment of loans is enforced, a monetary equilibrium with credit occurs when interest rate satisfies equation (70) and a quantity q_b satisfies equation (71).*

Proposition 3: *Assume repayment of loans can be enforced. A unique monetary equilibrium exists. With constant monetary and fiscal policy, if $t > n$, as cash holding rate αincreases, the demand of consumption goods also increases, but the interest rate at the equilibrium goes down. If $t < n$, the increase of cash holding rate α will lead to a raise of interest rate and a decline of demand of consumption goods at the equilibrium level.*

As I explained at the beginning of this case, buyers do not want to deposit any money in banks, but sellers would like to deposit all money in banks. With t as the tax rate on money deposit and n as the probability of becomes a seller. In order to understand proposition 3, we could consider t as the benefit of holding cash and n as the benefit of deposit money into banks. If t is greater than n, it implies that government would try to reduce agents' deposit in order to increase social benefit. In this condition, as α increases, government would not like to raise interest rate to stimulate deposit, but cut interest to boost agents' consumption. By contrast, if n is greater than t, as α increases, government would like to increase interest rate in order to reduce cash holding. When interest rate is increased, agents will reduce consumption and deposit idle money into banks. The proof of proposition 3 is provided in appendix.

The proposition 3 describes the situation based on the assumption that agents choose cash holding rate α after the tax rate was set. If agents choose cash holding rate before government sets the tax rate, how does interest rate and demand change while the tax rate increases? This question is discussed in proposition 4.

Proposition 4: *Assume repayment of loans can be enforced. A unique monetary equilibrium exists. Government sets tax rate after agents' decision of cash holding rate, 1) If* $(1-\alpha)\tau > \alpha(1-n)$*, both* $\frac{\partial i}{\partial t}$ *and* $\frac{\partial \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}}{\partial \alpha}$ *are positive. Therefore, as tax rate t increases, interest rate also increase, but demand of consumption goods declines. 2) If* $(1-\alpha)\tau < \alpha(1-n)$*, a higher value of tax rate will reduce the interest rate, but arise the demand of consumption goods at equilibrium level.*

Later on I will talk about welfare base on the assumption that agents choose cash holding rate α after tax rate t was set, which is when $\frac{\partial t}{\partial \alpha}$ is zero, but $\frac{\partial \alpha}{\partial t}$ is not zero.

Proposition 5: *If* $t > n$*, a positive value of α increases the demand of consumption at the equilibrium level and improves money allocation and social welfare.*

In proposition 5, we compare welfares in case 1 and case 2.We found if t is greater than n, a positive rate of cash holding will increase welfare. Both cases are built on the assumption that tax rate t is positive. From proposition 5, another question comes out relatively, which is whether a positive tax rate can improve allocation and welfare with a positive cash holding rate?

According to proposition 2 and 5, if we can show $\frac{u'(\bar{q}_b)}{c'\left(\frac{1-n}{n}\bar{q}_b\right)} > \frac{u'(\hat{q}_b)}{c'\left(\frac{1-n}{n}\hat{q}_b\right)}$, then $\bar{q}_b < \hat{q}_b$. Using (39) and (71), we get

$$\frac{\tau+1}{\beta} > \frac{\tau - t + 1}{\beta(1-n)[\alpha + (1-\alpha)(1-t)] + \beta n(1-\alpha)(1-t)}$$

$$\Rightarrow n(\tau+1)(t-\alpha) > t\tau \tag{90}$$

$$\Rightarrow t\left(1 - \frac{\tau}{n(\tau+1)}\right) > \alpha$$

Under the condition of $t\left(1-\frac{\tau}{n(\tau+1)}\right) > \alpha$ and tax rate lager than probability of becomes a seller, fiscal policy can improve welfare and money allocation, which clearly answered the above question. Note that the values of α and t also need to satisfy equation (74). Let us consider a special case. If τ is a very small number, then equation (90) is reduced to t > α. Therefore, if the money injection rate is approaching zero and agents hold a positive amount of cash and meanwhile under the condition of tax rate is larger than both n and α, then we could conclude that a positive proportional tax can improve money allocation and welfare.

At equilibrium $i = i_d$. Substitute (70) into (54), we have

$$(1-n)(1-\alpha)m_{t-1}t = n\alpha m_{t-1}\left[\frac{\tau-t+1}{\beta(1-n)[\alpha+(1-\alpha)(1-t)]+\beta n(1-\alpha)(1-t)}-1\right] \quad (74)$$

Take derivative with respect to t on both sides:

$$\begin{aligned}
&-\frac{\partial\alpha}{\partial t}(1-n)m_{t-1}t+(1-n)(1-\alpha)m_{t-1} \\
&\qquad=\frac{\partial\alpha}{\partial t}nm_{t-1}\left[\frac{\tau-t+1}{\beta(1-n)[\alpha+(1-\alpha)(1-t)]+\beta n(1-\alpha)(1-t)}-1\right] \\
&\qquad+\frac{n\alpha m_{t-1}(-1)}{\beta(1-n)[\alpha+(1-\alpha)(1-t)]+\beta n(1-\alpha)(1-t)} \\
&\qquad+\frac{n\alpha m_{t-1}(\tau-t+1)}{\beta\{(1-n)[\alpha+(1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2}(-1)\{(1-n)\left(-1+\alpha+t\frac{\partial\alpha}{\partial t}\right) \\
&\qquad+n\left[-\frac{\partial\alpha}{\partial t}(1-t)-1+\alpha\right]\}
\end{aligned}$$

$$\Rightarrow\frac{\partial\alpha}{\partial t}=$$

$$\frac{n\alpha(\tau-t+1)(t-n)-n(\tau-t+1)\{(1-n)[\alpha+(1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}-(1-n)t\beta\{(1-n)[\alpha+(1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2}{n\alpha(\tau-t+1)(1-\alpha)-n\alpha(1-n)[\alpha+(1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}-(1-n)(1-\alpha)\beta\{(1-n)[\alpha+(1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2} \quad (75)$$

5.5 The change of the welfare:

Before we calculate welfare, we should know agents' production in the second market. Let's set m =M_{t-1}, m is money holding at the beginning of the first market. We know buyer's budget constraint is binding in the first market. Therefore, buyers hold zero amount of money at the beginning of the second market. On the other hand, sellers have $\frac{1}{n}(1+\tau_1)M_{t-1}$ amount of money holding on hand. The productions of buyer and seller in market two in turn are

$$h_b = x^* + \phi[\frac{m_{t+1}}{\alpha+(1-\alpha)(1-t)} + (1+i)\ell - \tau_2 M_{t-1}] \tag{76}$$

$$h_s = x^* + \phi\{\frac{m_{t+1}}{\alpha+(1-\alpha)(1-t)} - [pq_s + (1+\tau_1)M_{t-1} + i_d d + \tau_2 M_{t-1}]\} \tag{77}$$

Since in equilibrium, we have:

$$\frac{m_{t+1}}{\alpha+(1-\alpha)(1-t)} = M = M_{t-1} + (\tau_1+\tau_2)M_{t-1} \tag{78}$$

$$c'(q_s)q_b = \phi p q_b = \phi[(1+\tau_1)M_{t-1} + \ell] \tag{79}$$

$$\phi\ell = nc'(q_s)q_b \tag{80}$$

Then (76) and (77) convert to

$$h_b = x^* + c'(q_s)q_b + nc'(q_s)q_b \tag{81}$$

$$h_s = x^* - c'(q_s)q_s - \phi i_d d \tag{82}$$

From equations (81) and (82), we get

$$\frac{\partial h_b}{\partial t} = 0 + (1+in)\left[c''(q_s)\frac{\partial\frac{(1-n)}{n}q_b}{\partial t}q_b + c'(q_s)\frac{\partial q_b}{\partial t}\right] + \frac{\partial i}{\partial t}nc'c'(q_s)q_b \tag{83}$$

$$\frac{\partial h_s}{\partial t} = 0 - c''(q_s)\frac{\partial\frac{(1-n)}{n}q_b}{\partial t}q_b - \phi(\frac{\partial i}{\partial t}d + \frac{\partial d}{\partial t}i) \tag{84}$$

$\frac{\partial i}{\partial t}$ can be solved from equation (70). As we know agents will deposit (1- α) percent of cash into banks. Hence,

$$\frac{\partial d}{\partial t} = -(1 - \alpha)M_{t-1} \tag{85}$$

The value function of an agent can be described as

$$W(t) = \max_{t}\{(1 - n)[U(x^*) - h_b + \beta V_{t+1}(m_{t+1}(t))]$$
$$+ n[U(x^*) - h_s + \beta V_{t+1}(m_{t+1}(t))]\} \tag{86}$$

The first order condition of (86) is

$$\frac{dW}{dt} = (1 - n)\left(-\frac{\partial h_b}{\partial t} + \beta V'_{m_{t+1}}\frac{\partial m_{t+1}}{\partial t}\right) + n\left(-\frac{\partial h_s}{\partial t} + \beta V'_{m_{t+1}}\frac{\partial m_{t+1}}{\partial t}\right) \tag{87}$$

$\frac{dW}{dt}$ is the marginal welfare of tax rate t. If (87) is positive, an increase in tax rate will improve welfare. By contrast, a negative value of $\frac{dW}{dt}$ implies that welfare and tax rate have a negative relationship. Equation (78) tells us

$$m_{t+1} = [M_{t-1} + (\tau_1 + \tau_2)M_{t-1}][\alpha + (1 - \alpha)(1 - t)]$$

$$\Rightarrow \frac{\partial m_{t+1}}{\partial t} = [M_{t-1} + (\tau_1 + \tau_2)M_{t-1}][\frac{\partial \alpha}{\partial t} - \frac{\partial \alpha}{\partial t}(1 - t) - (1 - \alpha)] \tag{89}$$

Using (83), (84), (76) and (89) we could solve (87) in terms of n, t, αand q_b. If we know the formula of c(q_s), we can simulate the value of W using data of n, t, α and q_b. The simulation part I will leave it for future investigation.

6. Conclusion:

In this paper, I introduced fiscal policy into money and banking. Government charges a proportional tax on agents' account balance at the end of the second market. The main purpose of this paper is to show the effect of monetary and fiscal policies on the money allocation and welfare. I found the new additive fiscal policy provided different results on welfare in different periods. If tax rate is less than zero, which means government pay subsidies to agents, then agents prefer to deposit all money into banks at the end of the second market. This fiscal policy therefore would improve money allocation and welfare. On the other hand, when tax rate turns to be positive, the fiscal policy effect became more complicated. Whenever tax rate is lower than the probability of becoming a seller, agents would deposit all cash at the end of the second market. In this case, welfare is reduced by fiscal policy. An agent's decision changes as the tax rate changes. If tax rate high enough, agents would like to hold cash in order to avoid paying tax. In this case, fiscal policy improves social welfare. In a special case, if there is deflation, a higher tax rate can increase consumption and improve welfare. This model can be extended into many different directions. For example, we can try to allow private banks to issue bank notes.

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Appendix

Proof of Proposition 1:

Because $u(q_b)$ is a strictly concave function, there is a unique value of q_b that satisfies equation (18). Therefore, monetary equilibrium is unique in my model.

Taking derivate of (32) and (35) with respect to t, we get

$$i_t = -\frac{(1-\beta)}{\beta(1-t)} + \frac{1}{\beta}\frac{\tau+1-\beta-t(1-\beta)}{(1-t)^2} \tag{33}$$

$$\frac{\partial \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}}{\partial t} = -\frac{1}{\beta(1-t)} + \frac{1}{\beta}\frac{\tau+1-t}{(1-t)^2} \tag{36}$$

After simplify (33), we get

$$i_t = \frac{\tau}{\beta(1-t)^2} \geq 0$$

The money injection rate τ is always non-negative. Therefore, as tax rate increases, equilibrium interest rate i will also increase.

From (34), we have

$$\frac{\partial \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}}{\partial t} = \frac{\tau}{\beta(1-t)^2} \geq 0 \tag{37}$$

The sign of right hand side of (35) depends on the value of τ. $\frac{\partial \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}}{\partial t}$ will always no less then zero because τ is always considered as non-negative in my model. As t increases, $\frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}$ also increases. Since $u(q_b)$ is strictly concave and $c(q_b)$ is strictly convex, a raise of value $\frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}$

implies an decline of q_b. If money injection is positive, an increasing of tax rate will lead to an increase on consumption goods demand.

Proof of Proposition 2:

If the tax rate is zero, equation (a) changes to $\tau = r - 1$(b), using (b) to replace r in (38), we get

$$\frac{\tau+1}{\beta} = \frac{u'(\bar{q}_b)}{c'\left(\frac{1-n}{n}\bar{q}_b\right)} \tag{39}$$

We want to show $q_b > \bar{q}_b$, which requires $\frac{u'(\bar{q}_b)}{c'\left(\frac{1-n}{n}\bar{q}_b\right)} > \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}$. Using (39) and (35), we have v

$$\frac{\tau+1}{\beta} > \frac{\tau+1-t}{\beta(1-t)}$$

$$\Rightarrow \tau t < 0$$

We have $\tau > 0$. Therefore, if $t < 0$, then we have $q_b > \bar{q}_b$. A negative t means government pays a subsidy to agents. Hence government's subsidy will increase the demand of consumption goods in market one. In the second market, we have $U'(x) = 1$, which tells us the consumption in market two is not affected by tax rate. Therefore, as tax rate increases, total goods consumption will increase in any period.

Proof of proposition 3:

From (70) and (71) we get

$$\frac{\partial i}{\partial \alpha} = \frac{(\tau-t+1)}{\beta} \frac{(-1)\beta(t-n)}{\{(1-n)[\alpha + (1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2} = \frac{\partial \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}}{\partial \alpha} \tag{72}$$

Therefore, as t changes, interest rate and demand of consumption goods moved to opposite way. We know $\frac{(\tau-t+1)}{\beta\{(1-n)[\alpha + (1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2}$ is positive. Thus the sign of equation (72) depends on the sign of $(t-n)$.

Case 1, if $t > n$, both $\frac{\partial i}{\partial \alpha}$ and $\frac{\partial \frac{u'(q_b)}{c'(\frac{1-n}{n}q_b)}}{\partial \alpha}$ are negative. Thus as cash holding rate αincreases, the equilibrium interest rate will decrease, but consumption goods' demand will go up.

Case 2, if $t < n$, both $\frac{\partial i}{\partial \alpha}$ and $\frac{\partial \frac{u'(q_b)}{c'(\frac{1-n}{n}q_b)}}{\partial \alpha}$ are positive. Therefore, we conclude that if $t < n$, the increase of cash holding rate α will lead to a raise of interest rate and a decline of demand of consumption goods at equilibrium level.

Proof of proposition 4:

Taking derivate of (70) and (71) with respect to t, we get

$$\frac{\partial i}{\partial t} = \frac{-1}{\beta(1-n)[1-t+\alpha t]+\beta n(1-\alpha)(1-t)} + \frac{(\tau-t+1)}{\beta}\frac{(1-\alpha)}{\{(1-n)[\alpha + (1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2} \tag{72}$$

$$\frac{\partial \frac{u'(q_b)}{c'(\frac{1-n}{n}q_b)}}{\partial \alpha} = \frac{-1}{\beta(1-n)[1-t+\alpha t]+\beta n(1-\alpha)(1-t)} + \frac{(\tau-t+1)}{\beta}\frac{(1-\alpha)}{\{(1-n)[\alpha + (1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2} \tag{73}$$

From (72) and (73), we get

$$\frac{\partial i}{\partial t} = \frac{-1}{\beta(1-n)[1-t+\alpha t]+\beta n(1-\alpha)(1-t)} + \frac{(\tau-t+1)}{\beta}\frac{(1-\alpha)}{\{(1-n)[\alpha + (1-\alpha)(1-t)]+n(1-\alpha)(1-t)\}^2} = \frac{\partial \frac{u'(q_b)}{c'(\frac{1-n}{n}q_b)}}{\partial \alpha}$$

After simplify

$$\frac{\partial i}{\partial t} = \frac{(1-\alpha)\tau - \alpha(1-n)}{\beta\{(1-n)[\alpha + (1-\alpha)(1-t)] + n(1-\alpha)(1-t)\}^2} = \frac{\partial \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)}}{\partial \alpha}$$

So the signs of $\frac{\partial i}{\partial t}$ and $\frac{\partial \frac{u'(q_b)}{c'(\frac{1-n}{n}q_b)}}{\partial \alpha}$ depend on the sign of $[(1-\alpha)\tau - \alpha(1-n)]$. If $(1-\alpha)\tau > \alpha(1-n)$, both of $\frac{\partial i}{\partial t}$ and $\frac{\partial \frac{u'(q_b)}{c'(\frac{1-n}{n}q_b)}}{\partial \alpha}$ are positive. Therefore as tax rate t increases, interest rate will also increase while demand of consumption goods will decline. On the other hand, if $(1-\alpha)\tau < \alpha(1-n)$, a higher value of tax rate will reduce the interest rate but arise the demand of consumption goods at equilibrium level.

Proof of Proposition 5:

Let's compare the equilibrium value of q_b in case 1 and case 2. Assume the equilibrium value of q_b in the case 2 is $\tilde{q}_b$ and it satisfies the equation (70). The equilibrium value of q_b in the case 1 is $\hat{q}_b$. In order to show the positive value of α will increase the equilibrium value of q_b, we must have

$$\frac{\tau+1-t}{\beta(1-t)} > \frac{\tau-t+1}{\beta(1-n)[\alpha + (1-\alpha)(1-t)] + \beta n(1-\alpha)(1-t)}$$

$$\Rightarrow \alpha t - \alpha n > 0$$

If α is positive, we have

$$\Rightarrow t > n$$

Therefore, if $t > n$, a positive value of α increases equilibrium demand of consumption in market one. Also according to U'(x) = 1, the equilibrium consumption is independent with α in second market. Thus if $t > n$, a positive value of α will increase agent's consumption and improve allocation and welfare.

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