About the Size And Power Performance of Test of No Autocorrelation Under Weak Assumption

by

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Abstract

In the presence of statistically dependent but uncorrelated time series, it has been proved that the proposed Lobato (2001) test has a much more accurate control over size than the Box-Pierce test. However, this comes at the expense of power loss compared with the Box-Pierce test. Su (2005) recently proposed two classes of modified Lobato tests based on recent development in heteroskedasticity-autocorrelation (HAC) robust inferences in regression models, seeking to find out a test that not only retains the good size properties of the Lobato test but also dominates the Lobato test in terms of power performance. His Lobato-PSJ test outperforms the Jansson (2004) modification and was recommended by Su (2005). However, the choice of sharp power parameters in Su (2005) seems to be arbitrary. The purpose of this essay is to modify the Lobato-PSJ test in Su (2005) by replacing the power parameter ρ in sharp origin kernels with an optimal data-driven power parameter $\hat{\rho}_T^*$ in Phillips, Sun and Jin (2003). The modified Lobato-PSJ test is optimal on a MSE criterion and has the advantage of not requiring any user-chosen numbers. Monte-Carlo simulation results show that the proposed test exhibits very similar control over size as the Lobato (2001) test meanwhile it is almost as powerful as the Box-Pierce test in finite samples.

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1 Introduction

Box and Pierce (1970; hereafter, BP) proposed using the Q_K test statistics to test the null hypothesis that the first K autocorrelations of a covariance stationary time series are zero. The Q_K test is the sample size times the sum of the squares of the first K sample autocorrelations. Its value is compared with the critical values of a chi-square distribution with K degrees of freedom to decide if the null hypothesis of no autocorrelation will be rejected or not. The procedure of $Q_K$the test is correct if the time series is assumed to be independent and identically distributed, because the asymptotic covariance matrix of the sample autocorrelations is an identity matrix on this assumption. In the presence of uncorrelated and dependent time series such as GARCH, the asymptotic covariance matrix of the sample autocorrelations depends on the data-generating process and the Q_K test can produce misleading inferences (Romano and Thombs, 1996).

To address this problem, Lobato, Nankervis and Savin (2002; hereafter, LNS) proposed a modified Q_K test robust to statistical dependence. Motivated by development in HAC consistent covariance estimates in regression testing, the modified Q_K test employs a kernel-based consistent estimator of the asymptotic covariance matrix instead of the identity matrix. The modified Q_K test has the advantage of asymptotically following the chi-square distribution under the null. To guarantee the consistency of the asymptotic covariance matrix, users must properly select the bandwidth parameter and statistical inference can be sensitive to that.

Due to the fact that arbitrary choices of the bandwidth parameter are likely to happen in practice, Lobato[1] (2001) suggested another version of the modified Q_K test. Following Lobato (2001), the asymptotic null distribution of the test is pivotal, not standard and can be tabulated by means of simulations. Simulation results show that the Lobato test has excellent controls over size. However, it is not as powerful

[1]The Lobato test does not hinge on the consistent estimates of covariance matrix and has the advantage of not requiring choosing any arbitrary number to implement the proposed test.

as other tests requiring the user-chosen number in terms of asymptotic local power and finite sample power.

Recent development on HAC robust inferences in regression models has shed light on improving the Lobato (2001) test in terms of its power performance. Similar to the Lobato test, Kiefer, Volgelsang and Bunzel (2000; hereafter, KVB) suggested an alternative class of regression testing, which does not utilize HAC consistent covariance estimates and is free from the bandwidth selection[2].

Monte-Carlo simulation results reveal that the KVB test is better at controlling the size compared to the conventional HAC robust test, which is at the cost of power loss (KVB, 2000). In order to improve the power performance of the KVB test, two modified KVB tests were generated by Jansson and Phillips. Jansson (2002, 2004) proposed a weighting scheme, close to that used in Anderson and Darling (1952) in the construction of the test statistics that delivers power improvement while retaining their better size properties in finite samples. Phillips, Sun and Jin(2003; hereafter, PSJ) suggested a new class of sharp origin kernels constructed by taking an arbitrary power of the usual Bartlett kernel. PSJ (2003) intended to modify the KVB test using the sharp origin kernels without truncation[3]. On the criterion of minimizing the asymptotic mean squared error (MSE) of the HAC estimate, an optimal choice of the power parameter is obtained (PSJ, 2003). Based on the plug-in approach of Andrews (1991), a data-driven power parameter is developed by PSJ (2003). PSJ (2003) test was proved to have the same good size properties as the KVB test along with significant power improvement over the KVB test.

Motivated by how Jansson (2002, 2004) and PSJ (2003), Su (2005) proposed two versions of modified Lobato test: the Lobato-Jansson test and the Lobato-PSJ test. They are constructed by replacing the HAC estimate of the Lobato test with modified HAC estimates in Jansson (2002) and PSJ (2003) respectively. Same as

[2]Kiefer and Vogelsang l(2002a; hereafter, KV) later proved that the KVB test is equivalent to the conventional HAC robust test using the full bandwidth Bartlett kernel. Likewise, the Lobato test can also be related to the LNS test using the above argument.

[3]the fullbandwidth

the KVB test, limiting distributions under the null of two modified Lobato tests are non-standard and depend on the weight function and power parameter of sharp origin kernels individually. Simulation results in Su (2005) reveal that the PSJ modification appears to enjoy more power improvement than the Jansson one and perform similarly in size as the Lobato test, while the Jansson modification is more vulnerable to size distortion.

In this essay, following Su (2005) and PSJ (2003), we extend the recommended Lobato-PSJ test in terms of the selection of power parameter. It can be seen that Su (2005) randomly consider five power parameters.[4]. These choices may be arbitrary. The use of PSJ (2003)'s data-determined power parameter on improving HAC estimates give us the motivation that it can be applied to the modified Lobato-PSJ test to gain more power improvement. The suggested test in this paper applies the optimal data-determined power parameter to sharp origin kernels in HAC estimates of the Lobato-PSJ test so that there is no need for users to choose any arbitrary numbers to implement the test. Intensive Monte-Carlo experiments show that the Auto-Sharp [2] test[5] and the Lobato test exhibit very similar accurate control over size. Even in the case of bilinear examples, while other tests, except the Lobato test, suffer serious overrejection, the Auto-Sharp [2] test seems to the least problematic. Moreover, except the Box-Pierce test, the Auto-Sharp [2] test exhibits strong evidence of outperforming other considered modified Q_K tests in terms of finite sample power on all the five processes I considered.

The paper is organized as follows. Section 2 reviews the literature on modified Q_K tests and several relevant papers about developing HAC robust inferences in regression testing. Section 3 introduces the modified Lobato-PSJ test and discusses the limitation of using the data-determined power parameter suggested by PSJ (2003). The finite sample performance of the modified Lobato-PSJ test is investigated in Section 4. Section 5 further examines empirical applications of the proposed test to

[4]They are 4, 8, 16, 24, 32.

[5]That is the modified Lobato-PSJ test compared with simulated critical values.

the same two data sets of asset returns as in Su (2005). Section 6 gives the conclusion.

2 Literature Review

Section 2 reviews the relevant literature on several versions of modified Q_K tests robust to statistical dependence and briefly explains the relevant development on heteroscedasticity-autocorrelation robust inferences (HAC) in regression tests, which is important for improving the Q_K test.

2.1 Box and Pierce (1970) test

First some notations and the assumption of weak dependence will be introduced. Notations in this paper follow those in Su (2005). Let y_t denotes a strictly real-valued and covariance stationary time series with mean μ . Define the lag-j population autocovariance by $\gamma_j = E(y_t - \mu)(y_{t-j} - \mu)$ and the lag-j population autocorrelation by $\rho_j = \gamma_j/\gamma_0$. Let $\gamma = (\gamma_1, \ldots, \gamma_K)$ be the vector of population autocovariances and $\rho = (\rho_1, ..., \rho_K)$ be the vector of population autocorrelations. Suppose a sample of $y_t (t = 1, \ldots, T)$ is observed. Define the sample mean and jth-lag sample autocovariance by $\bar{y} = T^{-1} \sum_{t=1}^{T} y_t$ and $\hat{\gamma_j} = T^{-1} \sum_{t=j+1}^{T} (y_t - \bar{y})(y_{t-j} - \bar{y})$. The estimator of ρ_j is $r_j = \hat{\gamma_j}/\hat{\gamma_0}$. Let $\widehat{C_K} = (\hat{\gamma_1}, \ldots, \hat{\gamma_K})$ be the vector of sample autocovariances and $r = (r_1, \ldots, r_K)$ be the vector of sample autocorrelations. The vector $Z_t = (z_{1,t}, \ldots, z_{K,t})'$ has as its jth component $z_{j,t} = (y_t - \bar{y})(y_{t-j} - \bar{y})$ for $t = 1, \ldots, T$ and $\widetilde{Z}_t = (\widetilde{z_{1,t}}, \ldots, \widetilde{z_{k,t}})'$ has as its jth component $\widetilde{z_{j,t}} = (y_t - \mu)(y_{t-j} - \mu)$. Assume that the weak dependence is defined through using the concept of near epoch dependence. Lobato (2001, p.1070) has given the definition of near epoch dependence on a mixing process. Using the same assumption of weak dependence in Lobato (2001), Su (2005) and LNS (2002), we assume that y_t is covariance stationary and the concept of near epoch dependence (NED) on a mixing process is used to characterize weak dependence of y_t. One of typical examples of weakly dependent processes is the GARCH model. (LNS, 2002)

Assumption 1. (1) Let y_t be a covariance stationary process that satisfies $E|y_t|^s < \infty$ for some $s > 4$ and is L_2-NED of size -1/2 on a process V_t where $V_t$is an

α-mixing sequence of size $-s/(s-4)$. (2) The null hypothesis that y_t is uncorrelated up to order K is satisfied.

The reason why the concept of NED on a mixing process is used to characterize weak dependence of a stochastic process is because it includes many types of stochastic processes [6] so that all the weakly dependent processes mentioned later in this paper are assumed to satisfy the concept of NED.

We are interested in testing the null hypothesis that y_t is uncorrelated up to order K,

$$H_0 : \gamma_1 = \cdots = \gamma_K = 0 \tag{2.1}$$

against the alternative hypothesis that some of the first K autocorrelations in y_t are correlated,

$$H_1 : \gamma_j \neq 0 \text{ for some } j \ j = 1,\ldots, K \tag{2.2}$$

With assumption of the underlying process being independently and identically distributed, Box and Pierce (1970; hereafter, BP) proposed the Q_K test. So the Q_K test is computed as

$$Q_K = T\sum_{j=1}^{K}(r_j)^2 \tag{2.3}$$

Q_K test asymptotically follows a chi-square with K degrees of freedom when H_0 is true. Also mentioned by Lobato (2001) and LNS (2002), under assumption (2.1), the following central limit theorem (CLT) can be derived:

$$\sqrt{T}(\widehat{C_K} - \gamma) \Rightarrow N(0, 2\pi f_z^{\sim}(0)) \tag{2.4}$$

where $f_z^{\sim}(0)$ is the spectral density matrix at zero frequency of the vector $\widetilde{Z}_t$. LNS (2002) proves that the application of the delta method to equation (2.4) will lead to a CLT for the sample autocorrelations:

$$\sqrt{T}(r - \rho) \Rightarrow N(0, V) \tag{2.5}$$

[6] Lobato(2001) has pointed out that these stochastic processes include mixing process, moving averages processes of infinite order, and various types of nonlinear processes.

where V is the asymptotic covariance matrix of the sample autocorrelations. Assuming V is known, the null (2.1) can be tested using a test statistic of the form $Tr'V^{-1}r$, which asymptotically follows a $\chi^2(K)$ under the null. Because BP (1970) actually assume that (y_t) is identically and independent distributed, they can replace V with the identity matrix. However, if y_t is only uncorrelated, the Q_K test is not valid and can produce misleading inferences since the asymptotic covariance matrix of sample autocorrelations (V) is possibly not an identity matrix and depends on the data generation process[7] . LNS (2002), Su (2005) and LNS (2001) show that the test possibly suffers from substantial overrejections [8] if the underlying processes are uncorrelated but weakly dependent.

2.2 Lobato, Nankervis and Savin (2002) test

Several alternative testing procedures have been proposed [9] in order to create a test robust to statistical dependence. The first stream of alternative test procedures hinges on finding a consistent estimator of the asymptotic covariance matrix of the sample autocorrelations. Major contributions have been made by LNS (2001) and LNS (2002).As shown in Den Haan and Levin (1997; hereafter, HL), consistent covariance matrix estimation can be classified into two broad categories: non-parametric kernel-based procedures and parametric procedures.

First non-parametric kernel-based procedures and relevant works (e.g. Andrews 1991; Andrews and Monahan 1992; Newey and West 1994) will be discussed. The key step in constructing a consistent HAC covariance matrix is to estimate the spectral density matrix at zero frequency of a vector of $\widetilde{Z}_t$. Each kernel-based procedure uses a weighted sum of the autocovariances to estimate the spectral density at zero frequency, where weights are determined by the kernel and the bandwidth parameter.

[7]See Romano and Thombs(1996) for more examples about V when the process is possibly dependent but uncorrelated

[8]Overrejections mean that the probability of the Q_k test wrongly rejecting the null hypothesis is higher than the nominal level originally set up. For example, it is higher than 5% significance level.

[9]See Lobato(2001), LNS(2002),LNS(2001)and Su(2005) for details

Assuming $\widehat{\Phi}_{K,l}$ satisfying **Assumptions 2-3** in appendix **A.1**, a modified test was suggested by LNS(2002),

$$\widehat{Q}_{K,l}^{LNS} = T\widehat{C_K}'\hat{\Phi}_{K,l}^{-1}\widehat{C_K} \tag{2.6}$$

In the two lemmas following **Assumptions 2-3**, LNS (2002) prove that the modified $\widehat{Q}_{K,l}^{LNS}$ converges in distribution to a $\chi^2(K)$ under **Assumptions 1-3**.

$$\hat{Q}_{K,l}^{LNS} \Rightarrow \chi^2(K) \tag{2.7}$$

$\hat{Q}_{K,l}^{LNS}$ has the advantage of asymptotically following a standard chi-square distribution under the null hypothesis (2.1) and robust to a wide range of dependence. However, as pointed out by Lobato (2001), it also has the disadvantage of requiring the selection of a user-chosen bandwidth parameter and the statistical inference can be sensitive to that choice. In Monte Carlo experiments of LNS (2002), he suggested two automatic-data based covariance matrix estimation procedures, one of which uses the formula (2.2) in Newey and West (1994) to get an optimal automatic data-dependent bandwidth parameter. Following HL (1997), this can be accomplished in five steps. Their five steps are concisely summarized in **A.1** for reader's guidance.

Newey and West (1994) is just one way to construct the data-dependent bandwidth parameter. Under a similar set of technical conditions, Andrews (1991) introduced his data-dependent automatic bandwidth parameters and proved the optimality of the QS kernel through using an asymptotic truncated mean squared error criterion (MSE)[10] . Andrews and Monahan (1992) further extended the technique by applying a prewhitening AR filter of order b and then followed almost the same procedure as Andrews (1991) to obtain the optimal bandwidth parameter [11]. The prewhitening procedure is believed to improve the accuracy of tests. Their regression model applicable to our $Q_K$test is still listed in equation (2.8).

$$Z_t = \sum_{k=1}^{b} \widehat{A_K} Z_{t-k} + \hat{e}_t \quad \text{for } t = b+1, \ldots, T \tag{2.8}$$

[10]For details about the asymptotic truncated MSE criterion, see Andrews(1991) pages 825-828.
[11]Please refer to Andrews and Monahan(1992) for information.

Andrews and Monahan (1992) considered b equal to one or zero in their Monte-Carlo simulations. If b is set equal to zero, then $\hat{e}_t = Z_t$ and the estimator is the same as Andrews (1991)'s estimator.

The LNS test in subsequent tables of sizes and powers in Section 4 does not consider the prewhitening procedure. It follows the usually used Andrews (1991)'s approach and adopts the Bartlett kernel with its relevant optimal bandwidth parameter. As pointed out by HL (1997), the major difference between Newey and West (1994) and Andrews (1991) lies in how they estimate $S^{(q)}$ and S. Andrews (1991) suggest that a parametric model be used to provide initial estimates of $S^{(q)}$ and S, which are then plugged into equation of $\alpha(q)$. Andrews (1991)'s procedure is summarized in **A.2** with three steps.

LNS (2002) also employ the VARHAC to estimate the asymptotic covariance matrix of the sample autocorrelations, which is a parametric estimator of the covariance matrix. VARHAC estimator was first proposed by Den Haan and Levin (1994). The main spirit of VARHAC, as indicated by Lobato (2001), is to estimate the spectral density at zero frequency of $(y_t - \mu)(y_{t-j} - \mu)$ by fitting an autoregression to the process$(y_t - \bar{y})(y_{t-j} - \bar{y})$. HL(1997) also gave a good summary of main steps. It is presented in **A.3**. Because they estimate the long-run variance parametrically and hence are different in nature from the other tests, VAR tests are better not included in simulation tables of section 4 for comparison purpose.

2.3 Lobato (2001)'s test

The second stream of alternative tests for zero autocorrelations does not emphasis on the consistent estimator of the asymptotic covariance matrix of the sample autocorrelations and is asymptotically pivotal under the null (2.1), which means that its asymptotic null distribution does not contain any unknown parameter. Typical papers adopting the second stream methodology are Lobato (2001), Su (2004) and Su (2005).

Lobato (2001) test was actually motivated by how KVB (2000) constructed their new test statistics for testing hypotheses in regression models with serial correlation/heteroskedasticity of unknown form. In the presence of heteroskedastic or autocorrelated error, KVB (2000) did not take the conventional approach of estimating the long-run variance (a spectral density at zero frequency) consistently based on kernel-based robust estimators and constructing standard tests with the asymptotic normality of the OLS estimates. Instead,he used a moment matrix constructed from the data to transform the OLS estimates, which results in the asymptotic distribution of the transformed estimates that does not depend on any unknown parameters. Hence asymptotical null distributions of t^* and F^*, which are new regression test statistics proposed by KVB (2000), do not depend on any unknown parameters so that users do not have to choose a bandwidth parameter to implement regression tests. Limiting distributions of t^* and $F^*$are nonstandard but easily tabulated by simulations [12]. KVB (2000) also have given reasons why they developed new tests. According to KVB (2000), it is because consistent nonparametric estimates of variance-covariance matrices in models have two weaknesses. First, consistent kernel-based estimates of sample autocovariances require the introduction of a bandwidth parameter (l) that satisfies certain conditions [13], which has been mentioned above. Although Andrews (1991) suggest that the bandwidth can be chosen using the automatic data-dependent procedure proposed in his paper using the AR (1) plug-in method, arbitrary choices are possibly be made in practice and statistical inferences are sensitive to that. Second, KVB (2000)'s Monte-Carlo simulation results have shown that the sampling variability of HAC estimators ($\widehat{\Phi}^{HAC}$) can result in size distortions of tests in finite samples, whereas KVB (2000)'s new test statistics have controlled the finite sample size better than HAC estimator tests. This is sharpened by results that new tests have controlled the finite sample size more accurately than HAC based tests in occasions of testing for joint hypotheses and the autoregressive root approaching

[12]See Table I for critical values for t^* and Table II for criticl values for F^* in KVB(2000)

[13]Su(2005) think that consistency of $\widehat{\Phi}_{K,l}$ requires $l \to \infty$ and $(l/T) \to 0$ as $T \to \infty$.

one. However, KVB (2000) new test statistics are not without flaws. According to Figure 2 in KVB (2000), the local asymptotic power and the finite sample power of new tests are close to those consistent HAC estimator tests yet still below consistent HAC estimator tests. Jansson (2002) explained that any size improvements (relative to consistent HAC estimator tests) gained by using inconsistent variance estimators necessarily caused a loss of local asymptotic power.

In spirit of KVB (2000), Lobato (2001) proposed the following modified Q_K test.

$$\widehat{Q}_K^L = T\widehat{C}_K'\widehat{\Omega}_K^{-1}\widehat{C}_K \tag{2.9}$$

where $S_t = \sum_{j=1}^{t}(Z_j - \widehat{C}_K)$ and $\widehat{\Omega}_K = T^{-2}\sum_{t=1}^{T} S_t S_t'$. Under Assumption 1, it follows that

$$\widehat{\Omega}_K \Rightarrow \Lambda_K \Xi_K \Lambda_K' \tag{2.10}$$

where $\Xi_K = \int_0^1 \overline{B}_K(r)\overline{B}_K(r)'dr, \overline{B}_K(r) = B_K(r) - rB_K(1)$ is a K-vector of standard Brownian bridge. Under Assumption 1 and equation (2.10), Su (2005) show that

$$\widehat{Q}_K^L \Rightarrow B_K(1)\Xi_K^{-1}B_K(1)' \equiv U_K^L \tag{2.11}$$

Although $\widehat{\Omega}_K$ is not a consistent estimator of Φ_K in equation (2.7), the asymptotic distribution of the Lobato test statistics does not contain any unknown parameter and has the advantage of free from the selection of a bandwidth parameter. The asymptotic null distribution of $\widehat{Q}_K^L$ is not standard [14] but can be tabulated precisely by simulations in Table I of Lobato (2001)'s paper.

Lobato (2001) also proposed the bootstrap Q_K test following Romano and Thombs (1996). The basic idea of the bootstrap Q_K test, as explained by Lobato (2001) is to compare the value of Q_K test statistic against critical values based on the bootstrap method. Lobato (2001) adopted the single moving block (SMB) bootstrap procedure. For details of how to implement the procedure, see Lobato (2001, pp.1070). As a comparison with $\widehat{Q}_K^L$, Lobato also reported empirical probabilities of LNS (2002)'s

[14] See Lobato(2001,pp.1069) for a description of the distribution.

$\widehat{Q}_K^{LNS}$ using different values of the bandwidth parameter and the vector autoregressive heteroskedasticity and autocorrelation consistent (VARHAC) version of $\widehat{Q}_K^{AR}$ using the Schwartz criterion from LNS (2002). In the case of K=1, Monte-Carlo simulation results show that Lobato test $\widehat{Q}_K^{L}$ accurately control the type I error of all six different uncorrelated processes in finite sample sizes of $T = 100$ and $T = 500$, whereas empirical probabilities of $\widehat{Q}_K^{LNS}$ and bootstrap $\widehat{Q}_K^{L}$ test are sensitive to chosen values of the bandwidth parameter and the block length respectively. $\widehat{Q}_K^{AR}$ precisely controls the type I error in most cases. However, it suffers from over-rejection in cases of the GARCH model at $T = 100$ and the bilinear model. Hence the Lobato(2001) test ($\widehat{Q}_K^{L}$) is very successful at controlling the type I error just as its KVB (2000) counterpart. On the other hand, in terms of asymptotic local power against Pitman's alternative shown in Figure 2 of Lobato (2001), $\widehat{Q}_K^{L}$ is less satisfactory than $\widehat{Q}_K^{LNS}$ and the boostrap $\widehat{Q}_K^{L}$test. Simulation results in Figure 3 and Figure 4 (Lobato, 2001) also suggest that $\widehat{Q}_K^{AR}$is more powerful than $\widehat{Q}_K^{L}$.

Su (2004) stated that $\widehat{Q}_K^{L}$ and $\widehat{Q}_K^{LNS}$ can be related in a special way. This idea also originated from recent works of Kiefer and Voglesang. KV (2002a) suggested that HAC robust tests proposed by KVB (2000) are equivalent to using Bartlett kernel [15] HAC standard errors with the bandwidth parameter (l) equal to sample size (T) [16] . Following the above conclusion, Su (2004) show that $\widehat{\Omega}_K = \widehat{\Phi}_{K,l}/2$ and therefore $\widehat{Q}_K^{L} = 2\widehat{Q}_K^{LNS}$.

KV (2002b) further extended this result to the analysis of the asymptotic covariance matrix estimated by seven different kernels with truncation lag equal to sample size. KV (2002b) pointed out that inconsistent estimates of $\widehat{\Phi}_{K,l=T}$ are resulted by such estimators since $l = T$ violates the rule that $l \to \infty$ and $(l/T) \to 0$ as $T \to \infty$. Nevertheless, KV (2002b) put forward a new idea that consistent testing is possible without consistent estimates of the variance parameters. The asymptotically pivotal test does not hinge on consistent variance estimator and a weaker condition for valid

[15]The Bartlett kernel is defined as $\kappa(x) = 1 - |x|^{-1}$ for $|x| < 1$ and 0, otherwise.

[16]For the proof, see KV(2002a, pp.2093-94).

testing is that the variance estimator be asymptotically proportional to the unknown variance. Hence their new tests are asymptotically pivotal just like KVB (2000)'s new tests. Simulation results show that their new tests better control the type I error regardless of the choice of kernel than tests with conventional HAC estimators. However the improvement in size comes at the expense of power loss, following the pattern of KVB (2000). Noticeably, local asymptotic analysis in KV (2002b) revealed that the Bartlett kernel-based new tests dominate tests based on other kernels in terms of the local asymptotic power.

2.4 Su (2005)'s modified Lobato test

Subsequent papers after KV (2002b) have tried different ways to improve the power performance of the KVB test without losing its good size properties. KV (2003) further suggested a more accurate asymptotic approximation obtained by setting the bandwidth parameter as a fixed proportion of the sample size (i.e. $b = l/T$), where $b \in [0, 1]$) Motivated by KV (2002b, 2003), Su (2004) proposed a version of modified Lobato test ($\widehat{Q}_K^L(b)$):

$$\widehat{Q}_K^L(b) = T\widehat{C}_K'\widehat{\Phi}_{K,l=bT}^{-1}\widehat{C}_K \tag{2.12}$$

where the kernel function $\kappa(\bullet)$ in $\widehat{\Phi}_{K,l=bT}^{-1}$ is the Bartlett kernel defined above. When $b = 1$, $\widehat{Q}_K^L(b)$ is equivalent to the Lobato (2001) test. Furthermore, Su (2004) also proved the asymptotic distribution of $\widehat{Q}_K^L(b)$ under Assumption 1 to be

$$\widehat{Q}_K(b) \Rightarrow B_K(1)\Xi_K^{-1}(b)B_K(1)' \equiv U_K(b) \tag{2.13}$$

where

$$\Xi_K(b) = \frac{2}{b}\int_0^1 \overline{B}_K(r)\overline{B}_K(r)'dr - \frac{1}{b}\int_0^{1-b} \left[\overline{B}_K(r+b)\overline{B}_K(r)' + \overline{B}_K(r)\overline{B}_K(r+b)'\right] dr \tag{2.14}$$

$\overline{B}_K(r)$ is a K-vector of Brownian bridges. From Equation (2.13), we can see that the limiting null distribution of $\widehat{Q}_K^L(b)$ only depends on b, otherwise it is pivotal and non-standard. Actually critical values of $U_K(b)$ have been tabulated using the asymptotic

critical value function in Table I of KVB (2003). Simulation results in Su (2004) show that $\widehat{Q}_K^L(b)$ generally has a good control over size in finite samples regardless of the b chosen [17]. In terms of the finite sample power, Su (2004) pointed out that when $b = 0.3$ or smaller, the modified Lobato test enjoys substantial increase in power compared with the Lobato test in all cases but one case (non-MDS, $T = 100$). In the case of non-MDS, the power of both the modified Lobato test and the Lobato test is lower relative to other cases.

Recently, two modified versions of KVB (2000) tests (Jansson 2003 and 2004, Phillips et al. 2003) were proposed in order to improve power properties of KVB's test without losing its good controls on size. Based on their works, Su (2005) proposed two modified versions of Lobato (2001) tests and compared size and power properties of these two tests. Their contributions will be introduced and discussed in this subsection.

Jansson (2002, 2004) proposed a class of autocorrelation robust tests generalizing the KVB tests by accommodating a weight function $\omega_\varepsilon(\bullet)$ in the definition of the covariance matrix estimator used in the construction of the test statistics. Jansson's modified KVB test matches the KVB procedure in terms of good size properties and it also dominates the KVB test in terms of local asymptotic power. It is possible to control the weight function $\omega_\varepsilon(\bullet)$ in order to improve the power performance of Jansson (2002) new tests. In simulation experiments, Jansson (2002) further showed that if parameter ε of the weight function is sufficiently close to zero, his modified KVB test has better power properties than the KVB (2000) test in finite sample.

Su (2005) introduced the Jansson (2002,2004)'s weight function into the construction of the Lobato (2001)'s test. First $\widehat{\Omega}_K$ is adjusted to $\widehat{\Omega}_{K,\varepsilon}$:

$$\widehat{\Omega}_{K,\varepsilon} = \frac{1}{T^2} \sum_{t=1}^{T} \omega_\varepsilon \left(\frac{t}{T} \right) S_t S_t' \tag{2.15}$$

where $\omega_\varepsilon(\bullet)$ is a weight function satisfying Assumption 2 in Jansson (2002).

[17] The modified test is under-sizes in the case of non-MDS and slightly over-sized at $T = 100$ for GARCH model and bilinear model.

A2.$\kappa : (0,1) \to [0,\infty)$ is Lipschitz continuous; that is, there exists a finite constant M_κ such that $|\kappa(r) - \kappa(s)| \leq M_\kappa |r-s|$ for all $0 \leq r \leq s \leq 1$.

For any $\varepsilon \in (0, 1/2]$, Jansson (2002) considered the following weight function:

$$\omega_\varepsilon\left(\frac{t}{T}\right) = \phi_\varepsilon^{-1} \cdot min\left[\left(\frac{t}{T}\right)^{-2}\left(1-\frac{t}{T}\right)^{-2}, \varepsilon^{-2}(1-\varepsilon)^{-2}\right] \tag{2.16}$$

where ϕ_ε is the scale factor defined as

$$\phi_\varepsilon = \left[\frac{3-2\varepsilon}{3(1-\varepsilon)^2} + 2\ln\left(\frac{1-\varepsilon}{\varepsilon}\right)\right] \tag{2.17}$$

Su (2005) find out that the weight function has the following form in different part of the domain:

$$\omega_\varepsilon(t/T) = \begin{cases} 1/6 & \varepsilon = 1/2 \\ \phi_\varepsilon(t/T) & \varepsilon \in (0,1/2) \text{and} (t/T) \in (\varepsilon, 1-\varepsilon) \\ \phi_\varepsilon \varepsilon & \text{otherwise} \end{cases} \tag{2.18}$$

Su (2005) proposed a modified Lobato (2001) test, the Lobato-Jansson (L-J) test:

$$\widehat{Q}_{K,\varepsilon}^{LJ} = T\widehat{C}_K' \widehat{\Omega}_{K,\varepsilon}^{-1} \widehat{C}_K \tag{2.19}$$

Under Assumption 1, Su(2005) proved the limiting distribution of to be:

$$\widehat{\Omega}_{K,\varepsilon} \Rightarrow \Lambda_K \Xi_{K,\varepsilon} \Lambda_K' \tag{2.20}$$

and therefore,

$$\widehat{Q}_{K,\varepsilon}^{LJ} \Rightarrow B_k(1) \Xi_{K,\varepsilon}^{-1} B_k(1)' \equiv U_{K,\varepsilon} \tag{2.21}$$

where $\Xi_{K,\varepsilon} = \int_0^1 \omega_\varepsilon \overline{B}_K(r) \overline{B}_K(r)' dr$. By equation (2.21), $\widehat{Q}_{K,\varepsilon}^{LJ}$ has a limiting non-standard distribution that only depends on the weight function $\omega_\varepsilon(\bullet)$ and the lag order K. Jansson (2002) suggest that using $\varepsilon = 0.01$ will be an attractive choice. Su (2005) also consider cases where $\varepsilon = 0.1$ and $\varepsilon = 0.5$.

Motivated by works of KVB (2000) and KV (2002a, 2002b), Phillips et.al (2003; hereafter, PSJ) proposed a new family of kernels, called sharp origin kernels, in his construction of regression testing. Sharp origin kernels are constructed by taking

positive powers of the Bartlett kernel and their bandwidth parameters are set equal to the sample size (T). The sharp origin kernels are defined as:

$$\kappa_\rho(x) = \begin{cases} (1-|x|)^\rho & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for} \rho \geq 1 \tag{2.22}$$

When $\rho = 1$, $\kappa_\rho(x)$ is the usual Bartlett kernel. PSJ (2003) show that as ρincreases, $\kappa_\rho(x)$ becomes successively more concentrated at the origin and its peak more pronounced and sharp, which increasingly downweights very high order autocorrelations and serves a role similar to a bandwidth parameter. Figure 1 is from PSJ (2003) and depicts this effect for several values of ρ . It is included in Appendix B for reader's reference.

Stimulated by PSJ (2003) arguments on special size and power properties of sharp origin kernels, Su (2005) proposed another modified Lobato test, the Lobato-PSJ test (L-PSJ test). Define

$$\widehat{\Omega}_{K,\rho} = \sum_{j=-T+1}^{T-1} \kappa_\rho\left(\frac{j}{T}\right)\widehat{\Gamma}(j) \tag{2.23}$$

where $\widehat{\Gamma}(j)$ is defined in equation (A.4). The modified Lobato test therefore has the form:

$$\widehat{Q}_{K,\rho}^{LPSJ} = TC_K'\widehat{\Omega}_{K,\rho}^{-1}C_K \tag{2.24}$$

Su (2005) also deduced the limiting distribution of $\widehat{Q}_{K,\rho}^{LPSJ}$. Following PSJ (2003), it is true that under Assumption 1

$$\widehat{\Omega}_{K,\rho} \Rightarrow \Lambda_K \Xi_{K,\rho} \Lambda_K' \tag{2.25}$$

So it can be inferred that the limiting distribution of $\widehat{Q}_{K,\rho}^{LPSJ}$ under the null (2.1) is

$$\widehat{Q}_{K,\rho}^{LPSJ} \Rightarrow B_K(1)\Xi_{K,\rho}^{-1}B_K(1)' \equiv U_{K,\rho} \tag{2.26}$$

where $\Xi_{K,\rho} = \int_0^1 \int_0^1 \kappa_\rho(r) d\overline{B}_K(r) d\overline{B}_K(s)'$. From equation (2.26), it is clear that the limiting distribution of the L-PSJ test is non-standard and depends on the sharp origin kernels $\kappa_\rho(\bullet)$and lag order K. In Su (2005)'s paper, only $\rho = 4, 8, 16, 24$ and 32 are considered. Critical values of the L-PSJ test and the L-J test are simulated

in Table 1 of Su (2005) by 50000 replications at 10%, 5%, 2.5% and 1% significance levels up to $K = 10$. Simulation results in Su (2005) showed that the L-PSJ test enjoys the same good size properties as the Lobato test, while the size performance of the L-J test is a bit sensitive to the choice of ε. Both the L-PSJ test and the L-J test lead to power improvement. When a large ρ is chosen, power improvement of the L-PSJ is more substantial. The smaller the ε is chosen, the larger power the L-J test has. Noticeably, the PSJ modified Lobato test seems to dominate the L-J test on power improvement.

2.5 Conclusion

Section two addresses the problem that the ordinary Q_K test meets when the time series are uncorrelated but weakly dependent. Along with the discussion of relevant development on HAC estimation in regression testing, this section has also reviewed the development of several modified versions of Q_K test and has analyzed their relative merits and weaknesses regarding the size and power properties. The next section will extend the PSJ modified Lobato test in Su (2005) by applying the PSJ (2003) data-determined power parameter $\hat{\rho}_T^*$.

3 AN EXTENSION FOR THE L-PSJ TEST

When the long run variance $\widehat{\Omega}$ is estimated based on sharp origin kernels without truncation, simulations in PSJ (2003) indicate that this leads to tests with more accurate size properties relative to conventional tests and better power properties than other tests based on simple kernels without truncation (where $\rho = 1$). The analysis of PSJ (2003) actually considered two situations: when $\rho > 1$ is fixed as $T \to \infty$ and when $\rho \to \infty$ with T. His explanation clearly indicates why $\widehat{\Omega}$ based on the sharp origin kernels without truncation delivers accurate size properties along with better power properties in relation to those based on the Bartlett kernel and other kernels without truncation. PSJ (2003) analyzed that when $\rho > 1$ is fixed as $T \to \infty$, HAC estimation based on sharp origin kernels without truncation is still inconsistent just as it is when $\rho = 1$. Nevertheless, compared with the Bartlett kernel, sharp origin kernels put less weight on autocovariances of larger lags and therefore lead to HAC estimates with smaller asymptotic variance. The reduction in asymptotic variance assists in enhancing power. In contrast, tests based on conventional kernel estimates without truncation result in inconsistent HAC estimates and therefore introduce additional variability, which helps better approximating finite sample behavior under the null while compromising power. According to PSJ (2003), sharp origin kernels can even deliver power close to or better than tests based on consistent HAC estimates, while retaining more accurate size than those tests.

When $\rho \to \infty$ with T, sharp origin kernels can provide consistent covariance estimates for Ω under certain rate conditions of ρ . As ρ increases, the variance of $\widehat{\Omega}_{K,\rho}$ estimates declines and bias increases. PSJ (2003) showed that it is likely to find an optimal ρ . Using the criteria of minimizing the asymptotic mean squared error of the HAC estimate, an optimal choice of ρ (ρ_T^*) can be obtained. In practice, because ρ_T^* contains unknown parameter, the AR (1) plug-in approach from Andrews (1991) was employed by PSJ (2003) here, resulting an automated version of the new sharp kernel HAC estimator.

The above argument convince us to revise the Lobato-PSJ test through applying the automatic data-determined value of power parameter $\hat{\rho}_T^*$ to sharp origin kernels in $\widehat{Q}_{K,\rho}^{LPSJ}$.

3.1 Methodology

The reason why it is necessary to amend the L-PSJ test is based on two considerations. First, Su (2005) did not suggest how to choose the value of power parameter in sharp origin kernels and instead he arbitrarily chose five values of power parameter ($\rho = 4, 8, 16, 24, 32$) in Monte-Carol experiments. In simulations of the location model, although PSJ (2003) suggested that using the power parameter $\rho = 16$ provide relatively ideal size and power in regression testing with sample sizes typical in econometric applications, it was found out that $\widehat{Q}_{K,\rho}^{LPSJ}$ with $\rho = 16$ does not necessarily provide the most accurate size or the largest power in all examples from Su (2005). For example, in Table 4 for a class of bilinear models, the L-PSJ test has correct size for $\rho = 4, 8$, but slightly over-rejects for $\rho = 16, 24, 36$. This motivates the study of using an data-driven selection on ρ so that users do not have to make arbitrary choices in practice just like the data-dependent automatic bandwidth parameter in Andrews (1991). Second, the choice of power parameter ρ in sharp origin kernels has important implication for size and power properties of $\widehat{Q}_{K,\rho}^{LPSJ}$. As suggested by PSJ (2003), a power parameter in untruncated sharp origin kernels can serve to control the degree of downweighting, similar to the role of a bandwidth parameter playing in consistent HAC estimates. In the situation of $\rho \to \infty$ with $T \to \infty$, sharp origin kernels can provide consistent estimates for Ω under certain rate conditions on ρ . This consistent HAC estimation brings about the reduction in asymptotic variance and therefore enhances the power performance of $\widehat{Q}_{K,\rho}^{LPSJ}$, meanwhile possibly compromises the size performance of $\widehat{Q}_{K,\rho}^{LPSJ}$. It is believed that there is an opportunity of finding an optimal choice of ρ that not only helps to deliver the largest power but also retains the accurate size. The section 4 of PSJ (2003) gives the formula of the optimal

choice of ρ based on minimizing the asymptotic mean squared error of estimation. Through using the AR (1) plug-in procedure, the formula of data-determined choice of the power parameter $\hat{\rho}_T^*$ is also given.

Next PSJ (2003) procedure about data-determined power parameters is adapted to $\widehat{Q}_{K,\rho}^{LPSJ}$. According to PSJ (2003), the criterion MSE_h can be used to determine an optimal value of ρ that minimizes the asymptotic truncated MSE for some given sequence of weight matrices W_T that converge in probability to a positive semi-definite limit matrix W. W_T is a (possibly random) $K^2 \times K^2$ weight matrix that is positive semi-definite. The MSE_h has the form:

$$MSE_h = E \min \left\{ \rho vec\left(\widehat{\Omega}_{k_\rho} - \Omega\right)' W_T vec\left(\widehat{\Omega}_{k_\rho} - \Omega\right), h \right\} \tag{3.1}$$

The asymptotic form of MSE_h when $T \to \infty$ and $h \to \infty$ was given in part (b) of Theorem 3:

$$\begin{aligned} &\lim_{h\to\infty}\lim_{T\to\infty} MSE_h(\rho, \hat{\Omega}_{k_\rho}, W) \\ =&\lim_{h\to\infty}\lim_{T\to\infty} MSE_h(\rho, \tilde{\Omega}_{k_\rho}, W) \\ =&\lim_{T\to\infty} MSE(tilde\Omega_{k_\rho}, W) \\ =&\vartheta vec(\Omega^{(1)})' W vec(\Omega^{(1)}) + tr\{W(I + K_{KK})\Omega \otimes \Omega\} \end{aligned} \tag{3.2}$$

Let

$$\delta = \delta(\Omega, \Omega^{(1)}) := \frac{tr[W(I + K_{KK})\Omega \otimes \Omega]}{2vec(\Omega^{(1)})' W vec(\Omega^{(1)})} \tag{3.3}$$

PSJ (2003) deduced that the optimal ρ is obtained using equation (3.2):

$$\begin{aligned} \rho_T^* =& argmin \left\{ \frac{\rho^2}{T^2} vec(\Omega^{(1)})' W vec(\Omega^{(1)}) + \frac{1}{\rho} tr[W(I + K_{KK})\Omega \otimes \Omega] \right\} \\ =&\delta^{1/3} T^{2/3} \end{aligned} \tag{3.4}$$

When $\rho = \rho_T^*$, the truncated MSE of $\widehat{\Omega}_{k_\rho}$ is given by equation (3.5)

$$E \min \left\{ vec\left(\widehat{\Omega}_{k_\rho} - \Omega\right)' W_T vec\left(\widehat{\Omega}_{k_\rho} - \Omega\right), h \right\} \tag{3.5}$$

PSJ (2003) mentioned that MSE with ρ_T^* produces the rate of expansion $T^{2/3}$, which is the same as that of MSE of the Bartlett kernel estimate of Ω with optimal bandwidth

chosen to minimize MSE. Therefore, PSJ (2003) conjectured that $\widehat{\Omega}_{k_{\rho_T^*}}$ has asymptotic performance similar to those of conventional consistent HAC estimates with optimal bandwidth choices.

However, equation (3.4) cannot be implemented in practice, since δ contains unknown parameters such as Ω and $\Omega^{(1)}$. To solve this problem, PSJ (2003) further suggested using the plug-in method from Andrews (1991). Thus the unknown parameters in equation (3.3) will be estimated using a parametric model for Ω . PSJ (2003) advised that model selection criteria such as BIC and AIC can be used to find a proper model for Ω. Estimates obtained from the parametric model are used to compute $\widehat{\Omega}$and $\widehat{\Omega}^{(1)}$, which are then plugged into equation (3.4). The resulting $\hat{\rho}_T^* = \hat{\delta}^{1/3}T^{2/3}$ is the data-determined power parameter, where $\hat{\delta} = \delta(\widehat{\Omega}, \widehat{\Omega}^{(1)})$.

PSJ (2003) recommended the AR (1) plug-in method in applications. They argued that even when the approximating model may not be AR (1), $\hat{\rho}_T^*$ estimated from the AR (1) model still produce the optimal rate of expansion $T^{2/3}$ of the power parameter. Moreover, PSJ (2003) revealed that the flat behavior of the MSE function in the neighborhood of ρ_T^* makes the power parameter ρ near ρ_T^* still able to provide good performance in HAC estimation.

In the context of $\widehat{Q}_{K,\rho}^{LPSJ}$, the assumed models are K univariate AR (1) processes and W_T gives weight w_i only to the diagonal elements of $\widehat{\Omega}_{k_\rho}$. δ is estimated by equation (3.6). Note this is actually the inverse of equation (A.21).

$$\hat{\delta} = \frac{\sum_{i=1}^{K} w_i \frac{\hat{\sigma}_i^4}{(1-\hat{\alpha})^4}}{\sum_{i=1}^{K} w_i \frac{4\hat{\alpha}_i^2 \hat{\sigma}_i^4}{(1-\hat{\alpha})^6(1+\hat{\alpha}_i)^2}} \tag{3.6}$$

where

$$\hat{\alpha}_i = \frac{\sum_{t=2}^{T} z_{i,t} z_{i,t-1}}{\sum_{t=2}^{T} z_{i,t-1}^2}, and \quad \hat{\sigma}_i^2 = \frac{\sum_{t=2}^{T} (z_{i,t} - \hat{\alpha}_i z_{i,t-1})^2}{T-1} \tag{3.7}$$

Substituting $\hat{\rho}_T^* = \hat{\delta}^{1/3}T^{2/3}$ back into equation (2.55) gives the automatic data-based $\widehat{\Omega}_{k_{\rho_T^*}}$, which is continuously substituted into equation (2.56). This produces the modified L-PSJ test $\widehat{Q}_{K,\hat{\rho}_T^*}^{LPSJ}$ based on untruncated sharp origin kernels with an automatic data-determined power parameter $\hat{\rho}_T^*$. It can be expected that the limit distribution

of this new modified L-PSJ test still follows that in equation (2.58)

$$\widehat{Q}^{LPSJ}_{K,\hat{\rho}^*_T} \Rightarrow B_K(1)\Xi^{-1}_{K,\hat{\rho}^*_T}B_K(1)' \equiv U_{K,\hat{\rho}^*_T} \tag{3.8}$$

where $\Xi_{K,\hat{\rho}^*_T} = \int_0^1\int_0^1 \kappa_{\hat{\rho}^*_T}(r)d\overline{B}_K(r)d\overline{B}_K(s)'$. The modified L-PSJ test is denoted as Auto-sharp test in Monte-Carol experiments of section 4. There are two testing procedures of $\widehat{Q}^{LPSJ}_{K,\hat{\rho}^*_T}$ considered in section 4. Since sharp origin kernels can provide consistent $\widehat{\Omega}_{k_\rho}$ for Ω when ρ approaches infinity, we also consider comparing the value of $\widehat{Q}^{LPSJ}_{K,\hat{\rho}^*_T}$ with critical values of chi-square distribution with K degrees of freedom. The first test, denoted as the Auto-sharp [1] test, compares the value of with critical values of chi-square distribution with K degrees of freedom. The second test is denoted as the Auto-sharp [2] test in section 4. It compares the value of $\widehat{Q}^{LPSJ}_{K,\hat{\rho}^*_T}$ with simulated critical values computed from equation (3.8).

Analysis of PSJ (2003) advised caution in using the data-determined power parameter $\hat{\rho}^*_T$ in some case. PSJ (2003) mentioned that the power parameter ρ can be expressed as $\rho = aT^b$ for some $a > 0$ and $0 < b < 1$. It is assumed that the true underlying model for the vector Z_t is an ARMA (1,1) model

$$Z_t = aZ_{t-1} + \varepsilon_t + b\varepsilon_{t-1}, \quad \varepsilon_t \sim iid(0, \sigma^2) \tag{3.9}$$

, Suggested by PSJ (2003), a function for the mean of δ can be obtained.

$$\bar{\delta} = \frac{(1-\bar{\alpha}^2)^2}{4\bar{\alpha}^2} \tag{3.10}$$

where

$$\bar{\alpha} = \frac{(a+b)(1+ab)}{1+b^2+2ab} \tag{3.11}$$

Through assuming the ARMA (1,1) model, the true function for δ can be written as:

$$\delta = \left(\frac{\Omega}{\Omega^{(1)}}\right)^2 = \left(\frac{(1-a^2)(1+b)^2}{2(1+ab)(a+b)}\right)^2 \tag{3.12}$$

PSJ (2003) further suggested that the difference between the optimal choice of ρ , ρ^*_T and the data-determined value $\hat{\rho}^*_T$ depends essentially on the difference between δ

and $\bar{\delta}$. Taking a ratio of δ to $\bar{\delta}$, we get

$$\frac{\delta}{\bar{\delta}} = \left(\frac{(1-a^2)(1+b)^2\bar{\alpha}}{(1+ab)(a+b)(1-\bar{\alpha}^2)} \right)^2 = \left(\frac{(1-a^2)(1+b)^2(1+b^2+2ab)}{(1+b^2+2ab)^2-(a+b)^2(1+ab)^2} \right)^2 \quad (3.13)$$

PSJ (2003) presented the surface plot of the ratio $(\frac{\delta}{\bar{\delta}})^{1/3}$ against a and b in Figure 5 of their paper. From Figure 5, PSJ (2003) pointed out that when $b = 0$,$\frac{\delta}{\bar{\delta}} = 1$ for all a. That is to say, if there are no moving average effects and only autoregressive effects in the data generating process of Z_t , the data-determined $\hat{\rho}_T^*$ is exactly equal to the true optimal power parameter ρ_T^*. When $b < 0$ and there are negative moving average effects in equation (3.9), $\frac{\delta}{\bar{\delta}} < 1$ and is increasing in b but does not change its value much for various $a \geq 0$. In this case, the data-determined $\hat{\rho}_T^*$ overestimates the true optimal power parameter ρ_T^* . When $b > 0$ and there are positive moving average effects in equation (3.9), $\frac{\delta}{\bar{\delta}} > 1$ and is increasing with b but increasing at a decreasing rate with large values of a. In this situation, the data-determined $\hat{\rho}_T^*$ underestimates the true optimal power parameter ρ_T^* . However, we needs to be cautious when $b \to -1$ the long run variance $\Omega \to 0$ and, correspondingly, $\frac{\delta}{\bar{\delta}} \to 0$. In this case, the optimal rate for ρ_T^* is no longer $O(T^{2/3})$ when $\Omega = 0$. Hence it becomes less and less ideal to use $\hat{\rho}_T^*$ as $b \to -1$. The final column of Table 2 in PSJ (2003), which gives the ratio of $\frac{MSE_{\hat{\rho}_T^*}}{MSE_{\rho=1}}$, also illustrates this point. The ratio is small over a wide range of values of b, indicating that the use of a sharp origin kernel with data-determined power parameter does minimize the MSE and therefore improve the HAC estimation over the use of the Bartlett kernel. When b is large and negative ($b \leq -0.5$), the ratio of $\frac{MSE_{\hat{\rho}_T^*}}{MSE_{\rho=1}}$ is greater than one, indicating that use of a sharp origin kernel with data-determined power parameter increases MSE and thus does not improve the HAC estimation. In some of the cases in section 4, we actually encounter the problem of $b \to -1$. The data-driven method tends to select a large $\hat{\rho}_T^*$ that is not $O(T^{2/3})$. In the case of large and negative moving average effects, PSJ (2003) advised using model selection procedures in finding a proper ARMA model for use in the plug-in rule rather than the use of an AR (1) model may lead to improvements. However, we restrict our attention to the mechanical use of an AR (1) model in this

paper.

3.2 Conclusion

Section 3 first discussed the motivation behind our proposed test statistics- the modified L-PSJ test and then presented the amended test along with relevant equations. Finally, it reasoned the disadvantage of the data-determined power parameter $\hat{\rho}_T^*$. When moving average effect on the underlying ARMA process is large and negative ($b \leq 0.5$), data-driven $\hat{\rho}_T^*$ tends to select a larger value that does not minimize MSE and hence does not improve estimation of the long-run variance. Monte-Carlo experiments in next section will implemented to study finite sample sizes and powers of Q_K test and seven modified Q_K tests for different DGP processes.

4 Size and Power in Finite Samples

This section is designed to investigate size and power of the modified L-PSJ test in finite samples. For the purpose of comparison, we also report the finite sample size and power of the Box-Pierce test, the LNS (2002) test, the Lobato (2001) test, L-PSJ(8)and L-PSJ(32) tests. Two versions of automatic modified L-PSJ tests (Auto-sharp [1] and Auto-sharp [2]) are considered here, the difference of which has been discussed in section 3. The LNS test utilizes the Bartlett kernel with the optimal data-determined bandwidth in equation (A.14) from Andrews (1991). In Su (2005), the power improvement on L-PSJ test is substantial with a large ρ. The smaller ρ exhibits more stability at size control, especially in bilinear processes. Hence , $\rho = 32$ and $\rho = 8$ stand out as an ideal contrast with Auto-Sharp tests, so as to see how much progress Auto-sharp tests have made. Monte-Carlo simulations are performed with programs written by Dr Jen-Je Su in GAUSS. The only considered null hypothesis here is $H_0 : \gamma_1 = 0$, which is tested at the nominal level of $\alpha = 0.05$. Simulation results are computed using 10 000 iterations for sample size 100 and 500.

Following Lobato (2001) and Su (2005), we consider two sets of examples in the simulation study: the usually used textbook examples and examples frequently met in financial literature. The first set of examples consists of five different uncorrelated processes. The first two of them are i.i.d. sequences with innovations drawn from $N(0, 1)$ and student t distribution with six degrees of freedom. So both of them are independent and uncorrelated over time. The next three processes are uncorrelated yet not independent over time. The first case of three processes is a 1-dependent process from example 2.1 in Romano and Thomb (1996) and is therefore denoted as 'RT'. The RT equation is defined as $y_t = z_t z_{t-1}$, where z_t represent an i.i.d. sequence of standard normal variates. The second example of three processes is an All-Pass ARMA (1,1) model suggested by LNS (2002). Following LNS (2002), the equation of All-Pass ARMA (1,1) is $(y_t - \mu) - \phi(y_{t-1} - \mu) = (z_t - \phi^{-1} z_{t-1})$, where z_t are drawn from i.i.d. $T(10)$ and where $\mu = 0$ and $\phi = 0.8$. As mentioned by LNS

(2002), a special feature of the All-Pass ARMA (1,1) model is that the asymptotic covariance matrix of the sample autocorrelations is an identity matrix. For more details on the all-pass model, see Breidt et al. (1999). The third example of three processes is a none-martingale difference sequence (Non-MDs), which is defined as $y_t = z_{t-2}z_{t-1}(z_{t-2} + z_t + 1)$. Here $z_t$is a sequence of i.i.d. $N(0,1)$ random variables.

Following Su (2005), we then consider two empirical relevant models -GARCH (1,1) and bilinear models. The GARCH (1,1) model is $y_t = z_t\sigma_t$ where z_t is an i.i.d. $N(0,1)$ sequence and $\sigma_t^2 = 0.001 + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$. According to LNS (2002), the GARCH (1,1) model is uncorrelated but not independent over time and is covariance stationary provided $\alpha+\beta < 1$. We considered five different sets of (β, α) suggested by Su (2005): GARCH (A): (0.8,0.02), GARCH (B): (0.8,0.05), GARCH (C): (0.8,0.15), GARCH (D): (0.9,0.02), and GARCH (E): (0.9,0.05). Obviously, they all satisfy the condition of covariance stationary such that $\alpha + \beta < 1$. The bilinear model is given by $y_t = z_t + bz_{t-1}y_{t-2}$, where z_t is a sequence of i.i.d. $N(0, \sigma^2)$ random variables. Granger and Terisvirta (1993) point out that the y_t process is uncorrelated but not independent and is covariance stationary if $b^2\sigma^2 < 1$. We also follow LNS (2002) and Lobato (2001) to set $\sigma^2 = 1$and $b = 0.35$ for Bilinear (A), $b = 0.5$ for Bilinear (B) and $b = 0.65$ for Bilinear (C).

4.1 Experiments Under the Null

Empirical rejection probabilities for the $N(0,1)$, $T(6)$, RT, All-Pass ARMA (1,1) and Non-MDs models at $\alpha = 0.05$ nominal level are reported in Table 1 for sample size 100 and 500. The main conclusions from Table 1 are given as follows. The BP test has a good control over size for $N(0,1)$, $T(6)$ and All-Pass. The excellent performance of the BP test on the All-Pass model can be explained by the fact that covariance matrix of the sample autocorrelations is an identity matrix. However, it over-rejects by a large margin for RT and Non-MDs. Coherent with conclusion from LNS (2002), the LNS test slightly over-rejects for $N(0,1)$, $T(6)$ and All-Pass at $T = 100$ but

accurately controls the type I error for $T = 500$ for all cases. The Lobato test works satisfactorily for $N(0,1)$, $T(6)$ and All-Pass, while it is a bit conservative in the cases of RT and Non-MDs. The Auto-Sharp [1] test performs similarly to the Lobato test in all cases except in the cases of $N(0,1)$, $T(6)$ and All-Pass at $T = 100$. In these cases, the Auto-sharp [1] test tends to over-reject somewhat. Compared with the Auto-Sharp [1] test, the Auto-Sharp [2] test has the most analogous size control as that of the Lobato test in all cases, except more conservative in cases of RT, All-Pass and Non-MDs at $T = 100$. Table 2 presents the estimates of the RP's for five different GARCH (1,1) models. The BP test mostly over-rejects at $T = 500$. Noticeably, the BP test suffers from substantial over-rejection for GARCH (C) model, especially for $T = 500$.

Similar to Su (2005), we also find out that the LNS test tends to over-reject at $T = 100$. The Lobato test works satisfactorily at $T = 500$, but tends to under-reject for GARCH (C) model. At $T = 100$, the Auto-sharp [1] test tends to marginally over-reject. It works excellently at $T = 500$. The Auto-sharp [2] test outperforms the Auto-Sharp [1] test with regards to the control over size, since it only slightly under-rejects for GARCH (C) model similar to the Lobato test. Comparing with Auto-Sharp tests, LPSJ tests are a bit conservative in all cases, especially for the LPSJ(8)test. In GARCH (C) model, LPSJ(8) under-rejects. This is based on the evidence that its RP is only 0.031 at $t = 100$ and 0.033 at $T = 500$. Meanwhile, size control of LPSJ(32) is close to the Lobato test and Auto-Sharp[2]test.

The size of all tests for three bilinear models is given in Table 3. Only the Lobato test accurately controls the type I error in all cases. Except the Lobato test and the LPSJ(8)test, all the other tests over-reject in all cases. The degree of over-rejection is lower for the LNS test, two versions of Auto-Sharp tests and LPSJ (32)at $T = 500$. Among all the tests that have the problem of over-rejection, the Auto-Sharp [2] test and LPSJ(32)test suffer the least serious size distortion. My conclusion confirms with that of Su(2005).

4.2 Experiments Under the Alternative

This section reports the empirical powers of the tests in a Monte-Carlo study. Following Su (2005), the data are generated following an AR (1) process $y_t = u_t + \phi u_t - 1$, whose AR coefficients (ϕ) takes values of $0.05, 0.10, 0.15$ and 0.20. The AR (1) innovations (u_t) follow the $N(0,1)$, GARCH (A, C) and Bilinear (B) models respectively. The power and the size-adjusted power (in the parentheses) for each AR (1) model are reported in Table 4-7. It can be seen that in the previous experiments under the null, some of the candidate tests suffer from size distortion to some extent in some cases, which causes increases/decreases in the type II error and thus decreases/increases in the power of them. Therefore, it is not appropriate to compare the power of these tests with the power of other tests with correct sizes. So the size-adjusted powers are calculated from critical values obtained from the previous experiments under the null and are purposely set up to keep the type I error of all tests to be 0.05.

The RP's under the alternative for AR (1) model with $N(0,1)$ innovations are reported in Table 4. The main facts emerging from the table are the following: (i) The adjusted powers are the same as the unadjusted powers of the Box-Pierce test, since it correctly controls the type I error. As a comparison, the adjusted powers are generally lower than the unadjusted powers of the LNS test, the Auto-Sharp [1] test, the Auto-Sharp [2] test, the Lobato test, the LPSJ(8)test and the LPSJ (32)test, which reflects the fact that these tests tends to be over-sized to some extent. (ii) As the sample size and the AR coefficient increase, the power of each test becomes higher. (iii) Based on size-adjusted power, the Box-Pierce test generally has the highest power in all cases. On the contrary, the Lobato test has the disadvantage of the lowest power among all the tests in the Table 4, no matter the comparison is based on unadjusted powers or based on size-adjusted powers. For instance, the power of the Lobato test is only 0.284 with $\phi = 0.20$ and $T = 100$, whereas the power of most other tests are all more than 0.4. (iv) Similar to what Su (2005) found out in the L-PSJ test, the power improvement of two Auto-Sharp tests is substantial compared

with the Lobato test. The power of the Auto-Sharp tests is slightly lower than that of the Box-Pierce test, particularly in the situation of and a large AR coefficient. The power of the two Auto-Sharp tests is only slightly 0.001 below that of the Box-Pierce test, when $T = 500$ and $\phi = 0.20$ is considered. (v) The Auto-Sharp [1] test and the Auto-Sharp [2] test have the same size-adjusted power, while the unadjusted power of the Auto-Sharp [1] test is generally a bit higher than that of the Auto-Sharp [2] test. (vi) Size-adjusted powers of the Auto-Sharp [1] test, the Auto-Sharp [2] test and the LNS test are quite similar, whereas the adjusted powers of the LPSJ (8) test and the LPSJ(32) test tend to be slightly lower compared with that of the above three tests in small sample size $T = 100$. Based on size-adjusted powers, the LPSJ (32) test seems to be more powerful than the LPSJ(8) test. Hence the L-PSJ test with a large ρ exhibits competitive power.

Table 5 and 6 report empirical powers of all the tests in GARCH (A, C) models respectively. Similar conclusions to those obtained from Table 4 can be reached by examining Table 5. The Box-Pierce test is the test having the highest power, whereas the Lobato test is the one having the lowest power. The size-adjusted power of two Auto-Sharp tests is slightly lower than those of the Box-Pierce test and fairly close to that of the LNS test. The LPSJ(32) test tends to have slightly higher power than the LPSJ(8) test. Interestingly, LPSJ (32) is generally more powerful than Auto-Sharp tests at $T = 100$ and less powerful than them at larger sample size $T = 500$. In Table 6, the story is a bit different from those of Table 4 and 5. Because the Lobato test, the Auto-Sharp [2] test, two LPSJ tests, the LNS test at $T = 500$ and the Auto-Sharp [1] test at $T = 500$ are conservative in the previous experiments, their adjusted power is higher than the unadjusted power. Since the Box-Pierce test is seriously over-sized, the size- adjusted power of it is much lower than the unadjusted power. The conclusion from Table 4 and 5 that the Box-Pierce has the highest size-adjusted power is not completely right for Table 6, because two Auto-Sharp tests dominate the Box-Pierce test in terms of the adjusted power in some cases. Taking $\phi = 0.05$

and $T = 500$ for an example, the size-adjusted power of two Auto-Sharp tests is 0.141, 14.6% higher than the adjusted power of the Box-Pierce test. The evidence of which version of LPSJ test is more powerful exhibit mixed patterns. In some cases, Auto-Sharp [2] test has higher power than LPSJ (32)test. Taking ϕ and $T = 100$ for example, Auto-Sharp [2] test beats LPSJ (32) test. Converse results happen in other settings.

In the end, Table 8 exhibits empirical powers for the Bilinear (B) model. It can be expected that the size-adjusted power of all the tests is lower than the unadjusted power, since all the tests suffer from size distortion. Since the RP's of the Lobato test is closest to the nominal level 0.05, its adjusted power is almost the same as the unadjusted power. In the case of Bilinear (B) model, all the conclusions from Table 4 and 5 still hold. Based on the size-adjusted power, the Box-Pierce test is still the most powerful in all cases. RP's under the alternative of two Auto-Sharp tests are close to that of the Box-Pierce test, especially in the situation of large AR coefficient and $T = 500$. Noticeably, LPSJ(32) test is a bit more powerful than Auto-Sharp tests at $T = 100$, while it is less powerful at $T = 500$.

4.3 Conclusion

Simulation results in Chapter four show that compared with the Lobato test, the Auto-Sharp [2] test and the LPSJ(32) test also perform satisfactorily at controlling the type I error, meanwhile, they have the advantage of retaining considerable power. The Auto-Sharp [2] test seems to have better control in size in small sample size ($T = 100$) than the LPSJ(32) test. As for improvement on power, Auto-Sharp[2] test wins at $T = 500$ in most processes considered, whereas the LPSJ (32) test gains more power in small sample size. In section 5, except for two LPSJ test which have been used to real data in Su(2005), autocorrelation tests considered in section 4 are applied to two data sets of asset returns used in Su (2005).

5 Empirical Applications

The same two data sets of asset returns as in Su (2005) are applied to all the considered tests for no autocorrelation. We restrict our attention to the case of testing $K = 1$ at 5% significance level. Two monthly stock returns are first employed-the value-weighted index returns (VW) and the equal-weighted index (EW) ranging from January 1926 to December 1997. There are 864 observations, taking from Center for Research in Security Prices (CRSP) of the University of Chicago. Then monthly bond returns at five different maturities are considered-one-year (B1), five-year (B5), ten-year (B10), twenty-year (B20) and thirty-year (B30)-from January 1942 to December 1999 for 696 observations. The second data set are taken from the website for the textbook *Analysis of Financial Time Series* by Tsay (2002)[18] .

5.1 Results

Table 8 reports empirical results for the two CRSP returns. Our result is consistent with those in Su (2005). The Box-Pierce test rejects the null of no autocorrelation for these two returns at 5%. The LNS test only rejects the null for the EW returns and the VW returns is significant at $K = 1$. All the other tests cannot reject the null of no autocorrelation at 5% for these two returns.

Empirical results for five bond returns are present in Table 9. It appears that the Box-Pierce test still over rejects the null of no autocorrelation in all five-bond returns, reflecting the fact that the Box-Pierce test can produce misleading inference in the case of statistical dependent but uncorrelated series. The Lobato test only rejects the null hypothesis for the B1 bond return. The LNS test only rejects the null of no autocorrelation for the B1 return and the B5 return. It can't reject the null for other bond returns. Su(2005) found out that the LNS test can reject B10 and B30 at 5 percent level. The Auto-Sharp [2] test rejects the null hypothesis in the case of the B1, B5 and B10 returns. The result of two LPSJ tests is ambiguous: B1 return

[18]Website: http://www.gsb.uchicago.edu/fac/ruey.tsay/teaching/fts/

is autocorrelated by LPSJ(8) test. LPSJ rejects the null at 5 percent level for B10.

5.2 Conclusion

In this section, we applied almost the same data sets as those in Su (2005) to all the tests for no autocorrelation considered. Our results are mostly consistent with what Su (2005) found out in his paper. Two monthly stock returns are not autocorrelated using tests other than the Box-Pierce test, while the B1, B5 and B10 returns are found out to be autocorrelated using the Auto-Sharp [2] test. Contrary to what Su (2005) found out utilizing the L-PSJ test with $\rho = 24$ or 32, we do not find the evidence that the B30 returns are autocorrelated using the Auto-Sharp [2] test.

6 Conclusion

In this paper, optimal data-dependent power parameter is introduced to sharp origin kernels in the recommended Lobato-PSJ test from Su (2005). So the data-driven modified Lobato-PSJ test has the advantage of being implemented without any arbitrary user-chosen parameters. However, we must be cautious when Z_t have large and negative moving average effects, where it is possibly not optimal for the Lobato-PSJ test to use the formula of PSJ (2003) data-dependent power parameter. Monte-Carlo simulation results in section 4 reveal that the Auto-Sharp [2] test is second to the Lobato test at controlling the type I error accurately, meanwhile it can deliver power fairly close to or sometimes better than the Box-Pierce test that has the highest empirical power at larger sample size. The LPSJ(32) test performs better on the empirical power in small sample size like $T = 100$. In practice, we may want to report them all as a reference to each other.

The Auto-Sharp [2] test does not perform perfectly in all cases. From Table 3, we notice that the Auto-Sharp [2] still has the problem of over-rejection in the bilinear examples. This poses a question if the modified Lobato-PSJ test can be further improved. Lobato (2001) suggested that if the test statistics is asymptotically pivotal under the null, application of the bootstrap should provide an asymptotic refinement. Hence we expect that the bootstrap method applied to the null distribution of our proposed test in equation (3.8) will provide more precise simulated critical values for the Auto-Sharp [2] test, and therefore better the performance of Auto-Sharp [2] test. Applying the bootstrap method requires the choice of some bootstrap process and the correspondingly block length. All these questions need careful considerations by researchers.

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A Non-parametric kernel based procedures

A.1 Assumptions from Lobato(2001)

Under **Assumption 1**, as shown by Lobato (2001),

$$T^{-1/2}\sum_{t=1}^{\lceil rT\rceil} Z_t \Rightarrow \Lambda_K B_K(r) \tag{A.1}$$

for any $r \in (0,1]$. $\Rightarrow$ denotes weak convergence and $[x]$ is the integer part of x. Λ_K is the lower triangular $K \times K$ satisfying $\Lambda_K \Lambda_K' = \Phi_K$. Φ_K is a long-run variance of $\widetilde{Z}_t$ and $B_K(r)$ is a K-dimensional vector of independent Brownian motions. Based on Equation (A.1) and under Assumption 1, Su (2005) inferred that

$$T^{1/2}\widehat{C_K} \Rightarrow N(0, \Phi_K) \tag{A.2}$$

Since Φ_K is unknown, LNS (2002) propose that Φ_K can be estimated by a kernel-based estimator of $\Phi_K(\widehat{\Phi_{K,l}})$.

$$\widehat{\Phi}_{K,l} = \sum_{j=-T+1}^{T-1} \kappa\left(\frac{j}{l}\right)\hat{\Gamma}(j) \tag{A.3}$$

where

$$\hat{\Gamma}(j) = \begin{cases} T^{-1}\sum_{t=1}^{T-j} Z_{t+j}Z_t' & \text{for} \quad j \geq 0 \\ T^{-1}\sum_{t=-j+1}^{T} Z_{t+j}Z_t' & \text{for} \quad j < 0 \end{cases} \tag{A.4}$$

are the sample autocovariances of Z_t at $\pm j$ lags. In Equation (A.3), $\kappa(\cdot)$ is a kernel function and l is a bandwidth parameter. As long as the kernel $\kappa(\cdot)$ and the bandwidth satisfy two assumptions stated in LNS (2002, pp.734), the consistency and positive definiteness of $\widehat{\Phi}_{K,l}$ can be guaranteed.

Assumption 2. The kernel $\kappa(\cdot)$ belongs to K where K is the class of functions $K = \{\kappa(\cdot) : \Re; \rightarrow [-1,1]\}$ that is symmetric around zero, continuous at zero at all but a finite number of points, and satisfies $\kappa(0) = 1, \int_{-\infty}^{\infty}|\kappa(x)|dx < \infty, \int_{-\infty}^{\infty}|\psi(\xi)|d\xi$, where $\psi(\xi) = (2\pi)^{-1}\int_{-\infty}^{\infty}\kappa(x)e^{i\xi x}dx$

Assumption 3. The bandwidth sequence satisfies

$$\lim_{n\rightarrow\infty}\left(\frac{1}{l}+\frac{1}{n}\right) = 0$$

.

A.2 Procedure from Newey and West(1994)

Step 1: Obtain estimates for the " prewhitened" residuals by employing AR (1) [19] *on each series of* Z_t . The following AR (1) model is estimated with least squares:

$$Z_t = \widehat{A_1}Z_{t-1} + \widehat{e_t} \quad \text{for } t = 2,\ldots,T \tag{A.5}$$

[19] Actually it can be a AR(b) prefilering. However, we stick to the usual case of AR(1) prefilering.

where $\widehat{A_1}$ is the $(K \times K)$ matrix of AR(1) regression coefficients from equation (A.5).

Step 2: Choose a weighting matrix. Let the weight vector be

$$w = (0 \quad 1 \quad 1 \ldots 1)' \tag{A.6}$$

Step 3: Calculate the data-dependent bandwidth parameter.

$$\alpha(q) = \left[\frac{w' S^{(q)} w}{w' S w} \right]^2 \tag{A.7}$$

Newey and West (1994) propose that $\alpha(q)$ be estimated non-parametrically as

$$\hat{\alpha}(q) = \left[\frac{w' \widehat{S}^{(q)} w}{w' \widehat{S}^{(0)} w} \right] \quad q = 0, 1, 2 \quad where \tag{A.8}$$

$$\widehat{S}^{(q)} = \sum_{j=-n}^{n} |j|^q \hat{\Gamma}_{e,j} \tag{A.9}$$

$$n = \beta_1 \left(\frac{T}{100} \right)^{2/9} \quad \text{for the Bartlett kernel} \tag{A.10}$$

$$n = \beta_2 \left(\frac{T}{100} \right)^{2/25} \quad \text{for the QS kernel} \tag{A.11}$$

$$\hat{\Gamma}_{e,j} = T^{-1} \sum_{t=1}^{T-j} \hat{e}_t \hat{e}'_{t+j} \quad \text{for } j \geq 0 \text{ and,} \tag{A.12}$$

$$\hat{\Gamma}_{e,j} = \hat{\Gamma}'_{e,-j} \quad \text{for } j < 0 \tag{A.13}$$

where the QS kernel indicates the quadratic Spectral kernel. LNS (2002) actually adopts the formula (2.2) from Newey and West (1994) which specifies the kernel to be the Bartlett kernel and therefore $q = 1$. β_1 is set equal to 2 in equation (A.10) for the lag truncation and $n = \beta_1 \left(\frac{T}{100}\right)^{2/9}$. Using the estimate of $\alpha(q)$ given in equation (A.8), the data-dependent bandwidth parameter is determined by

$$l = 1.1447 \left[\hat{\alpha}(1) T \right]^{1/3} \quad \text{for the Bartlett kernel} \tag{A.14}$$

Step 4: Calculate the spectral density of the "prewhitened" residuals.

$$\widehat{\Sigma}_T^{NW-PW} = \sum_{j=-T+1}^{T-1} \kappa \left(\frac{j}{l} \right) \hat{\Gamma}_{e,j} \tag{A.15}$$

where $\hat{\Gamma}_{e,j}$ is defined in equation (A.12).

Step 5: Calculate the HAC estimate of the spectral density. Using the results of Step 4:

$$\widehat{\Phi}_{K,l}^{NW-PW} = \left[I_K - \widehat{A_1} \right]^{-1} \widehat{\Sigma}_T^{NW-PW} \left[I_K - \widehat{A_1} \right]^{-1} \tag{A.16}$$

A.3 Andrews (1991) approach

Step 1: Choose a weighting matrix.
HL(1997) argue that the growth rate of the bandwidth parameter can be controlled to minimize the asymptotic MSE of the spectral estimator and therefore helps to find the asymptotically optimal bandwidth parameter. HL (1997) then conclude that the optimal bandwidth parameter depends on a $K^2 \times K^2$ weighting matrix W and on the q, which characterize the smoothness properties of the kernel. Andrews (1991) indicates that q is 1 for the Bartlett kernel and q is 2 for the QS kernel. For a give kernel with q, the asymptotically optimal bandwidth parameter sequence is given by:

$$l = c(q)\left[\alpha(q)T\right]^{1/(2q+1)} \tag{A.17}$$

Here

$$\alpha(q) = \frac{2vec(S^{(q)})'Wvec(S^{(q)})}{tr(W(I+K)(S\otimes S))} \tag{A.18}$$

$$c(q) = \begin{cases} 1.1447 & \text{for the Bartlett kernel} \\ 1.3221 & \text{for the QS kernel} \end{cases} \tag{A.19}$$

where K is the $K^2 \times K^2$ commutation matrix that transforms $vec(B)$ into $vec(B)'$. $S(q)$ is the qth generalized derivative of the spectral density at frequency zero, which is given as follows:

$$S^{(q)} = \sum_{j=-\infty}^{\infty} |j|^q C_j \tag{A.20}$$

Andrews (1991) actually assigns one to all the elements corresponding to the slope coefficients and zero corresponding to an intercept in weighting matrix W. Denote the kth weight by ω_k.

Step 2: Calculate the data-dependent bandwidth parameter.
Andrews (1991) and Andrews and Monahan (1992) propose that a parametric model be used to provide initial estimates of S and $S^{(q)}$, which are then plugged into equation (2.25) to provide estimates of $\alpha(q)$. Following simulation experiments from Andrews (1991), AR (1) representations for each series of Z_t are estimated to provide estimates of $(\widehat{\rho_k}, \widehat{\sigma_k}^2), k = 1, \ldots, K$, where $\widehat{\rho_k}$ denotes estimates of autoregressive parameters and $\widehat{\sigma_k}^2$ denotes estimates of innovation variance parameters. Therefore, estimates of $\alpha(q)$are given below:

$$\hat{\alpha}(1) = \frac{\sum_{k=1}^{K} \omega_k \frac{4\hat{\rho}_k^2 \hat{\sigma}_k^4}{(1-\hat{\rho}_k)^6(1+\hat{\rho}_k)^2}}{\sum_{k=1}^{K} \omega_k \frac{\hat{\sigma}_k^4}{(1-\hat{\rho}_k)^4}} \tag{A.21}$$

$$\hat{\alpha}(2) = \frac{\sum_{k=1}^{K} \omega_k \frac{4\hat{\rho}_k^2 \hat{\sigma}_k^4}{(1-\hat{\rho}_k)^8}}{\sum_{k=1}^{K} \omega_k \frac{\hat{\sigma}_k^4}{(1-\hat{\rho}_k)^4}} \tag{A.22}$$

Then we obtain the following data-dependent bandwidth parameter for the Bartlett kernel same as that in equation (A.14).For the QS kernel, the data-dependent bandwidth parameter is given by:

$$l = 1.3221 \left[\hat{\alpha}(2)T\right]^{1/5} \tag{A.23}$$

In this paper, we restrict our attention to the case of Bartlett kernel, so we will use equation (A.21) and equation (A.14) to obtain the optimal bandwidth parameter.

Step 3: Calculate the spectral density of Z_t . The result of equation (A.14) is substituted into equation (A.1). From equation (A.1) and (A.2), we obtain the spectral density of Z_t .

A.4 HL(1997) summary on VAR-AIC VAR-BIC

Step 1: Lag order selection for each VAR equation. For the nth element $z_{n,t}$ of the vector $Z_t (n = 1, \ldots, K)$ and for each lag order $\kappa = 1, \ldots, \overline{K}$, the following model is estimated by least squares:

$$z_{nt} = \sum_{j=1}^{K} \sum_{k=1}^{\kappa} \hat{\alpha}_{njk}(\kappa) z_{j,t-k} + \hat{e}_{nt}(\kappa) \quad \text{for } t = \overline{K} + 1, \ldots, T \tag{A.24}$$

Equation (A.24) is intended to regress $z_{n,t}$ on its own lags and lags of other components from Z_t . The reason why HL (1997) put this restriction on equation (A.24) is to avoid the computational cost, when the dimension of Z_t is large. Hence the only specifications considered here are the ones in which all elements of Z_t enter with the same number of lags in the regression for z_{nt}. Next, we will select the lag order κ according to model selection criterion. The model selection criterion is calculated for each possible lag order $\kappa = 0, \ldots, \overline{K}$ The Akaike's (1973) information criterion (AIC) is given by:

$$AIC(\kappa, n) = log\left(\frac{\sum_{t=\overline{K}+1}^{T} \hat{e}_{nt}^2(\kappa)}{T}\right) + \frac{2\kappa N}{T} \tag{A.25}$$

Schwartz' (1978) Bayesian information criterion is given by:

$$BIC(\kappa, n) = log\left(\frac{\sum_{t=\overline{K}+1}^{T} \hat{e}_{nt}^2(\kappa)}{T}\right) + log(T)\frac{\kappa N}{T} \tag{A.26}$$

For each element of Z_t , the optimal lag order κ_n is chosen as the value of κ that minimizes $AIC(\kappa, n)$ or $BIC(\kappa, n)$. LNS (2002) set three different maximum lag lengths $(3, 10, and 15)$ for three sample sizes $(200, 1000, and 5000)$.

Step 2: Calculate the spectral density of the prewhitened residuals. Let $\widehat{K}_T$ be the largest lag-order chosen by the model selection criterion for the K elements of Z_t . From step 1, the restricted VAR can be expressed as:

$$Z_t = \sum_{k=1}^{\widehat{K}_T} \widehat{A}_k^{VAR} Z_{t-k} + \hat{e}_t \tag{A.27}$$

where $\hat{e}_t$ is an $K \times 1$ vector with typical element $\hat{e}_{nt}(\kappa_n)$. The covariance matrix of $\hat{e}_t$ is estimated by equation (A.28):

$$\widehat{\Sigma}_T^{VARHAC} = \frac{\sum_{t=\overline{K}+1}^{T} \hat{e}_t \hat{e}_t'}{T} \tag{A.28}$$

Step 3: Calculate the HAC estimate of the spectral density. The spectral density matrix at frequency zero is estimated by:

$$\widehat{\Phi}^{VAHAC} = \left[I_K - \sum_{k=1}^{\widehat{K}_T} \widehat{A}_k^{VAR} \right]^{-1} \widehat{\Sigma}_T^{VARHAC} \left[I_K - \sum_{k=1}^{\widehat{K_T}} \widehat{A}_k^{VAR} \right]^{-1} \tag{A.29}$$

$$\widehat{\Phi}^{VAHAC} = \left[I_K - \sum_{k=1}^{\widehat{K}_T} \widehat{A}_k^{VAR} \right]^{-1} \widehat{\Sigma}_T^{VARHAC} \left[I_K - \sum_{k=1}^{\widehat{K_T}} \widehat{A}_k^{VAR} \right]^{-1} \tag{A.30}$$

Then $\widehat{\Phi}^{VARHAC}$ is substituted into equation (2.10) to compute $\widehat{Q}_{K,l}^{LNS}$. The obtained test statistics $\widehat{Q}_{K,l}^{LNS}$ based on AIC and BIC model selection criteria are called VAR-AIC test and VAR-BIC test individually. As a kind of $\widehat{Q}_{K,l}^{LNS}$ test, their values are also compared to critical values of chi-square distribution with K degrees of freedom. There are other estimators of Φ_K proposed by authors such as Lee and Phillips (1994), where Z_t is assumed to be a scalar process and they use the Hannan-Rissanen recursion to determine the order and estimate coefficients of an ARMA representation of the data. For more details, see Lee and Phillips (1994). Robison (1995) proposed a non-parametric estimator and it does not require the use of kernel. For discussions regarding asymptotic properties and relative merits of using these estimators discussed before, see HL (1997) section 3, 4 and 5 for more details.

B Figures and Tables

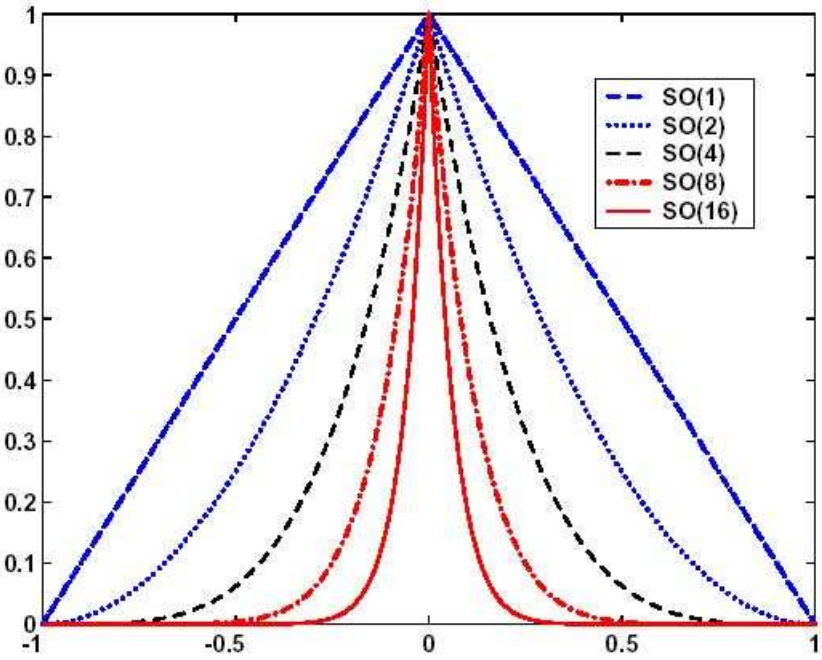

Figure 1: Sharp origin(SO) Kernels $k_\rho(x)$ for $\rho \in [1, 16]$

Table 1: Size(I)

	Normal		T(6)		RT		All-Pass		Non-MDs	
Test	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.050	0.050	0.049	0.045	0.215	0.240	0.052	0.056	0.277	0.391
LNS	0.060	0.054	0.058	0.048	0.039	0.046	0.060	0.055	0.038	0.045
Lobato	0.052	0.053	0.048	0.046	0.036	0.046	0.049	0.049	0.030	0.038
Auto-Sharp[1]	0.062	0.054	0.059	0.049	0.041	0.046	0.062	0.055	0.041	0.046
Auto-Sharp[2]	0.053	0.052	0.050	0.047	0.030	0.045	0.053	0.054	0.030	0.043
LPSJ(8)	0.050	0.050	0.045	0.046	0.030	0.046	0.045	0.046	0.027	0.038
LPSJ(32)	0.051	0.049	0.047	0.049	0.029	0.043	0.043	0.044	0.027	0.041

Notes: Empirical RP's at 5% level. T=sample size. Number of replications is 10 000.

Table 2: Size(II)

	GARCH(A)		GARCH(B)		GARCH(C)		GARCH(D)		GARCH(D)	
Test	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.054	0.053	0.065	0.061	0.088	0.128	0.051	0.056	0.060	0.069
LNS	0.059	0.049	0.059	0.050	0.051	0.045	0.056	0.052	0.058	0.052
Lobato	0.048	0.049	0.047	0.047	0.031	0.036	0.045	0.051	0.044	0.045
Auto-Sharp[1]	0.060	0.049	0.060	0.050	0.051	0.043	0.058	0.053	0.059	0.053
Auto-Sharp[2]	0.051	0.048	0.052	0.049	0.042	0.042	0.049	0.050	0.050	0.051
LPSJ(8)	0.045	0.051	0.044	0.049	0.031	0.033	0.049	0.048	0.046	0.048
LPSJ(32)	0.044	0.050	0.045	0.048	0.041	0.037	0.048	0.050	0.049	0.048

Notes: Empirical RP's at 5% level. T=sample size. Number of replications is 10 000.

Table 3: Size(III)

	Bilinear(A)		Bilinear(B)		Bilinear(C)	
Test	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.093	0.100	0.125	0.138	0.146	0.184
LNS	0.078	0.069	0.084	0.070	0.083	0.070
Lobato	0.050	0.053	0.051	0.052	0.046	0.044
Auto-Sharp[1]	0.077	0.067	0.082	0.069	0.080	0.065
Auto-Sharp[2]	0.065	0.065	0.070	0.064	0.064	0.062
LPSJ(8)	0.052	0.047	0.051	0.052	0.049	0.047
LPSJ(32)	0.060	0.051	0.060	0.057	0.061	0.050

Notes: Empirical RP's at 5% level. T=sample size. Number of replications is 10 000.

Table 4: Power (I) normal

AR	$\phi = 0.05$		$\phi = 0.10$		$\phi = 0.15$		$\phi = 0.20$	
Test	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.066	0.188	0.139	0.586	0.271	0.909	0.456	0.993
	(0.060)	(0.188)	(0.139)	(0.585)	(0.272)	(0.909)	(0.456)	(0.993)
LNS	0.076	0.189	0.148	0.583	0.278	0.906	0.444	0.992
	(0.065)	(0.181)	(0.130)	(0.572)	(0.252)	(0.901)	(0.417)	(0.992)
Lobato	0.058	0.149	0.105	0.418	0.185	0.695	0.284	0.869
	(0.057)	(0.143)	(0.103)	(0.409)	(0.181)	(0.684)	(0.279)	(0.862)
Auto-Sharp[1]	0.079	0.191	0.151	0.586	0.282	0.907	0.452	0.992
	(0.065)	(0.180)	(0.132)	(0.572)	(0.254)	(0.901)	(0.419)	(0.992)
Auto-Sharp[2]	0.067	0.188	0.135	0.580	0.255	0.905	0.421	0.992
	(0.065)	(0.180)	(0.132)	(0.572)	(0.254)	(0.901)	(0.419)	(0.992)
LPSJ(8)	0.061	0.164	0.118	0.507	0.232	0.836	0.366	0.976
	(0.061)	(0.164)	(0.119)	(0.507)	(0.232)	(0.836)	(0.367)	(0.976)
LPSJ(32)	0.064	0.181	0.127	0.561	0.259	0.892	0.425	0.988
	(0.062)	(0.183)	(0.122)	(0.564)	(0.252)	(0.894)	(0.416)	(0.990)

Notes: Empirical RP's under the alternative. The size-adjusted RP are in the parentheses. The nominal level is 5%. Number of replications is 10 000.

Table 5: Power (II) GARCH(A)

AR	$\phi = 0.05$		$\phi = 0.10$		$\phi = 0.15$		$\phi = 0.20$	
Test	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.070	0.192	0.145	0.587	0.276	0.904	0.460	0.993
	(0.064)	(0.185)	(0.135)	(0.580)	(0.263)	(0.900)	(0.446)	(0.993)
LNS	0.075	0.185	0.145	0.571	0.273	0.893	0.442	0.992
	(0.064)	(0.187)	(0.127)	(0.575)	(0.250)	(0.898)	(0.411)	(0.992)
Lobato	0.059	0.141	0.103	0.402	0.179	0.675	0.288	0.863
	(0.063)	(0.142)	(0.108)	(0.405)	(0.188)	(0.678)	(0.298)	(0.865)
Auto-Sharp[1]	0.076	0.185	0.147	0.572	0.278	0.894	0.450	0.992
	(0.065)	(0.187)	(0.128)	(0.574)	(0.249)	(0.895)	(0.415)	(0.992)
Auto-Sharp[2]	0.066	0.181	0.131	0.566	0.254	0.891	0.417	0.992
	(0.065)	(0.187)	(0.128)	(0.574)	(0.249)	(0.895)	(0.415)	(0.992)
LPSJ(8)	0.061	0.169	0.113	0.489	0.202	0.827	0.352	0.970
	(0.069)	(0.167)	(0.125)	(0.486)	(0.217)	(0.826)	(0.376)	(0.974)
LPSJ(32)	0.065	0.180	0.125	0.536	0.233	0.879	0.403	0.989
	(0.074)	(0.179)	(0.142)	(0.535)	(0.254)	(0.877)	(0.430)	(0.989)

Notes: Empirical RP's under the alternative. The size-adjusted RP are in the parentheses. The nominal level is 5%. Number of replications is 10 000.

Table 6: Power (III) GARCH(C)

AR	$\phi = 0.05$		$\phi = 0.10$		$\phi = 0.15$		$\phi = 0.20$	
Test	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.111	0.249	0.182	0.565	0.297	0.848	0.457	0.969
	(0.066)	(0.123)	(0.122)	(0.381)	(0.213)	(0.711)	(0.356)	(0.924)
LNS	0.069	0.129	0.121	0.396	0.213	0.721	0.344	0.914
	(0.068)	(0.055)	(0.118)	(0.236)	(0.209)	(0.552)	(0.340)	(0.821)
Lobato	0.039	0.096	0.073	0.256	0.112	0.462	0.186	0.638
	(0.065)	(0.117)	(0.109)	(0.301)	(0.166)	(0.521)	(0.254)	(0.694)
Auto-Sharp[1]	0.069	0.128	0.122	0.397	0.214	0.723	0.348	0.916
	(0.069)	(0.141)	(0.120)	(0.417)	(0.212)	(0.738)	(0.345)	(0.924)
Auto-Sharp[2]	0.057	0.123	0.105	0.389	0.187	0.713	0.313	0.911
	(0.069)	(0.141)	(0.120)	(0.417)	(0.212)	(0.738)	(0.345)	(0.924)
LPSJ(8)	0.040	0.098	0.079	0.324	0.149	0.602	0.253	0.816
	(0.061)	(0.131)	(0.111)	(0.388)	(0.201)	(0.678)	(0.324)	(0.863)
LPSJ(32)	0.048	0.114	0.103	0.372	0.180	0.688	0.322	0.887
	(0.056)	(0.146)	(0.114)	(0.429)	(0.203)	(0.734)	(0.352)	(0.911)

Notes: Empirical RP's under the alternative. The size-adjusted RP are in the parentheses. The nominal level is 5%. Number of replications is 10 000.

Table 7: Power (IV) Bilinear(B)

AR	$\phi = 0.05$		$\phi = 0.10$		$\phi = 0.15$		$\phi = 0.20$	
Test	T=100	T=500	T=100	T=500	T=100	T=500	T=100	T=500
BP	0.148	0.262	0.204	0.557	0.315	0.835	0.446	0.962
	(0.066)	(0.134)	(0.107)	(0.373)	(0.186)	(0.686)	(0.289)	(0.904)
LNS	0.090	0.150	0.131	0.420	0.219	0.735	0.336	0.923
	(0.054)	(0.116)	(0.082)	(0.359)	(0.154)	(0.682)	(0.246)	(0.899)
Lobato	0.054	0.091	0.075	0.247	0.126	0.464	0.191	0.680
	(0.053)	(0.089)	(0.074)	(0.242)	(0.125)	(0.458)	(0.189)	(0.673)
Auto-Sharp[1]	0.090	0.147	0.132	0.415	0.221	0.732	0.341	0.923
	(0.056)	(0.117)	(0.085)	(0.360)	(0.159)	(0.684)	(0.256)	(0.900)
Auto-Sharp[2]	0.076	0.142	0.111	0.407	0.193	0.723	0.304	0.919
	(0.056)	(0.117)	(0.085)	(0.360)	(0.159)	(0.684)	(0.256)	(0.900)
LPSJ(8)	0.058	0.098	0.092	0.300	0.147	0.591	0.242	0.834
	(0.058)	(0.094)	(0.091)	(0.288)	(0.145)	(0.576)	(0.239)	(0.825)
LPSJ(32)	0.070	0.112	0.114	0.347	0.183	0.666	0.290	0.894
	(0.058)	(0.100)	(0.097)	(0.323)	(0.160)	(0.644)	(0.264)	(0.882)

Notes: Empirical RP's under the alternative. The size-adjusted RP are in the parentheses. The nominal level is 5%. Number of replications is 10 000.

Table 8: Test for no autocorrelations (I) monthly stock returns

	c.v. at 5% of EW	EW	c.v. at 5% of VW	VW
Test	K=1	K=1	K=1	K=1
BP	3.8416	37.1563**	3.8416	9.6675**
LNS	3.8416	3.9031**	3.8416	2.3860
Lobato	45.4000	12.7861	45.4000	6.3698
Auto-Sharp[2]	3.9215	3.5196	3.8745	2.2446

Notes: ** denotes rejection of null at 5% level.

Table 9: Test for no autocorrelations (II) monthly bond returns

	c.v. at 5% of B1	B1	c.v. at 5% of B5	B5
Test	K=1	K=1	K=1	K=1
BP	3.8416	110.2065**	3.8416	14.3393**
LNS	3.8416	14.9894**	3.8416	5.9383**
Lobato	45.4000	70.5567**	45.4000	17.8452
Auto-Sharp[2]	3.9527	14.8541**	3.8995	6.0709**

Notes: ** denotes rejection of null at 5% level.

Table 10: Test for no autocorrelations (III) monthly bond returns

	c.v. at 5% of B10	B10	c.v. at 5% of B20	B20	c.v. at 5% of B30	B30
Test	K=1	K=1	K=1	K=1	K=1	K=1
BP	3.8416	6.1945**	3.8416	4.2944**	3.8416	6.8408**
LNS	3.8416	3.6482	3.8416	2.1322	3.8416	3.4843
Lobato	45.4000	15.5213	45.4000	8.4064	45.4000	9.3932
Auto-Sharp[2]	3.9041	3.9527**	3.9026	2.2264	3.8696	3.4636

Notes: ** denotes rejection of null at 5% level.