Tying and Product Differentiation With Quadratic Costs and Possible Entry into the Primary Market

by

Kristin McMahon

An essay submitted to the Department of Economics
in partial fulfillment of the requirements for
the degree of Master of Arts

Queen's University
Kingston, Ontario, Canada

September 2008

Abstract

Tying is prominent in many industries yet its effects on competition or its strategic possibilities are not well understood. What is particularly interesting about tie-in sales and its effect on competition and social welfare is that there has been no general rule found or established to deal with these scenarios. This paper aims to contribute to the understanding of the effects of tying when product differentiation is present. We modify the model in Egli (2007) to use quadratic transportation costs as opposed to the linear transportation costs it was originally modeled with. Egli finds minimal differentiation for the subgame without tie in sales and we find maximal differentiation. We also extend Egli's decision game to allow firm 2 the ability to enter both the primary and secondary market in response to firm 1's tying. We find that firm one can then strategically *not* tie in order to preserve monopoly profits in the primary market.

Acknowledgements

I would like to thank Professor Roger Ware for his unwaivering support and guidance as both my professor and advisor this year. I would also like to thank my family, friends, and the entire QED MA class of 2008 for their invaluable emotional support throughout the year.

Contents

Abstract **i**

Acknowledgements **ii**

List of Figures **iv**

List of Tables **iv**

1 Introduction **1**

2 Literature Review **2**

3 The Model **8**

4 The Equilibrium **9**

4.1 Demand Specification 9

4.2 The Firm's Behaviour 14

5 Tying Decision and the Effect on Competition with Quadratic Costs **16**

6 Decision Game Extended for Entry in Market A **21**

7 Conclusions **26**

References **29**

List of Figures

1 Demand Regions with Quadratic Transportation Costs 11

2 Timing of the Decision Game 17

3 Timing of the Extended Decision Game 22

List of Tables

1 Differentiation Findings Summary 27

1 Introduction

Tying is prominent in many industries yet its effects on competition or its strategic possibilities are not well understood. Products are said to be tied when a firm requires the sale of good A (the tying good) to include the sale of good B (the tied good). There are many examples of this in the market but examples in the computer and technology industries have inspired much debate and are most prominent in the court cases. For example, in the civil case *United States v. Microsoft (1998)* one of the main issues was Microsoft technologically tying its Internet browser to its operating system and one of the concerns addressed was whether or not this tying was an anti-competitive action. Another example in the technology industry is IBM and its tabulating machines. To use these machines the consumers also had to purchase punch cards and IBM tied the sales of their punch cards to the sale of their machines.

What is particularly interesting about tie-in sales and its effect on competition and social welfare is that there has been no general rule found or established to deal with these scenarios. Modeling and analyzing one scenario will lead to the conclusion that tying is anti-competitive and decreases social welfare. While changing just one aspect of the model to reflect a new factor in the relevant market will change the findings and often times the effect on social welfare is found to be ambiguous. Choi and Stefanadis (2001, p. 70) adeptly explain that, "The debate about tying cannot be conclusive unless formal models incorporate the aspects of the world that practitioners consider important." For that reason the adaptation and continuation of these models is important in expanding and refining our understanding of tying and its consequences.

This paper will focus on tying decisions combined with product differentiation decisions and is very closely related to a model by Alain Egli (2005, 2007). In the sections to follow we present a brief overview of the literature, test the robustness of Egli's model by evaluating it with quadratic costs, determine the quadratic cost outcomes of Egli's decision game, and extend his model to allow entry in both the tying and tied goods markets. We find that using quadratic transportation costs instead of linear results in maximal product differentiation in the case where firm 1 does not tie it's primary and secondary good whereas Egli finds minimal differentiation in the same scenario with linear costs.

2 Literature Review

The two aforementioned technology examples have something in common. They are both examples of a monopolist firm in market A tying to good B which faces competition. This is the main concern of "leverage theory". Leverage theory in the tying law literature informally claims that a firm with monopoly power in one market has the ability and incentive to extend their monopoly power to a competitive market. It claims that monopolies can do this by tying (or bundling[1]) their monopoly good to the competitive good, thereby foreclosing sales in the competitive market. This concept has been met with much criticism in the economic literature. The toughest critics of this theory were members of the Chicago School of Economics. They argued that when a monopolized good is tied to a competitive good the monopoly power can only be leveraged once in that market.[2] They conclude then that there is no additional incentive for monopolies to tie their products. So why do we see so many examples of such tying?

Whinston (1990) claims that the Chicago argument, (that there is no such leverage possibility), breaks down when the assumption that there is constant returns to scale in the competitive market is relaxed. If the competitive market exhibits constant returns to scale then there is no possibility of leverage by the monopoly firm. If, however, the competitive market does not have constant returns to scale, then tying can in fact be used to extend monopoly power to the tied good market. Whinston

[1] Bundling is often used to specify that the tie is of two goods in a one to one ratio. Our models do not use tying for more than a one to one ratio so we use the term tying and bundling interchangeably throughout.

[2] See Whinston (1990, p. 837) or Tirole (1989, p. 334) for a simple theoretical example.

explains with a simple model that without constant returns to scale it is possible for the monopoly firm, through the use of tying, to augment the market for the competitive good by inducing exit from the competitive market. Tying literature that focuses strictly on price discrimination as a benefit of tying misses the bigger picture. Whinston (1990, p. 839) finds that, "tying can lead to a monopolization of the tied good market. Most interestingly, the mechanism through which this exclusion occurs is foreclosure; by tying, the monopolist reduces the sales of its tied good market competitor, thereby lowering his profits below the level that would justify continued operation." Tying is often profitable for the monopolist when precommitment to tying is feasible and he finds that the welfare effects on consumers and efficiency are ambiguous.

Whinstons work has been expanded on by others to investigate alternative market situations where tying can influence the structure of related markets. Choi and Stefanadis (2001) model tying by a monopolist as a barrier to entry in the competitive market[3]. Their model includes "risky upfront R&D[4] investment". They model a situation where success in the R&D investment is necessary for entry into the competitive market to be profitable. The probability of successful innovation from that investment is a function of the level of investment made. When the monopolist ties this means that the entrants must be successful in both markets to be successful at all. The entrants become dependent on each others success and this makes the investment more independently risky and entry less likely.

Like Whinston, Choi and Stefanadis require that a commitment to tying can be made. They explain that if the monopolist does not tie, and only one of the entrants successfully invests, then the incumbent (monopolist) can use a price squeeze to extract the profits of that innovation. However, if the monopolist does not tie and both entrants are successful at innovating, the monopolist will be forced out of the market. The monopolist now faces a trade-off decision. It profits the monopolist to

[3]As opposed to Whinston's paper which models forced exit in the competitive market.

[4]research and development

tie if the risk of both entrants succeeding outweighs the benefits of a price squeeze if only one succeeds. The result is reduced incentives for innovation which can mean reductions in consumer and overall economic welfare. However, this is not to say that tying is always anticompetitive.

As mentioned in the introduction, tying results can not be conclusive unless they include the factors of the market that are considered important. Carlton and Waldman (2002) also extend Whinston (1990) to show how tying can be used to establish and protect monopoly power in the primary (tying) market and emerging complementary (competitive) markets. Most tying papers before Carlton and Waldman focus on the ability of the monopoly to extend their primary market power into the complementary (tied) market and do not look at the ability to preserve monopoly power in their primary market. Their model examines the effects of tying with two time periods and complementary market entry costs. In period one the monopolist has a secure monopoly in primary market A and there can be entry in the complementary market B with complementary entry costs. In period two there is threat of entry into the primary and secondary markets by new firms with primary and secondary entry costs. It is assumed that the primary goods produced by the different firms are the same quality but that the complementary good supplied by the alternative firm in market B is superior.

Carlton and Waldman find that the monopolist only has incentive to tie its goods in the first period when there is a threat of entry in the primary market in the second period. Essentially they conclude that tying in the first period can make entry in the complementary product unprofitable. Profits in the primary market in the second period depend on increased sales in the complementary market. When tying deters entry in the complementary market entry in the primary market is no longer profitable and therefore deterred. Entry into the primary market is no longer a possibility or threat. In summary, they find that firms can use tying, strategically, to deter efficient entry in the primary market and in evolving industries, like the technology industry, and in doing so protect their monopoly positions. This finding is very interesting and important to policy makers and enforcement agencies in protecting competition in the marketplace.

Momentarily we digress from the tying literature to focus on the other main body of work pertinent to this paper which is the product differentiation literature. Product differentiation is found in nearly any market you can think of and should therefore be included in the evaluation of tying effects. Products are said to be differentiated if they differ from each other in either quality, style, location, or any number of factors. The first important paper considering the effect of product differentiation in competition was by Harold Hotelling in 1929. In this paper he pioneered a simple mathematical model for differentiation using a spatial diagram, a market line of length l. The competing firms location on that line represents their amount of differentiation from each other in the factor(s) considered, for example, location. Consumers are uniformly distributed on that line representing each consumers preferred location. Hotelling finds that the optimal response of the firms result in what he calls the principle of minimal differentiation. This means that there is a tendency for firms to locate as close to each other as possible to maximize their profits. Therefore, the firms will both locate at the same location which is the middle of the line or the center of the market.

There is a problem with Hotelling's conclusion however. D'Aspremont, Gabszewicz, and Thisse (1979) point out that there is a discontinuity in the profit functions when the firms are located too close to each other. When the firms are located together at the center of the market there is no stable price equilibrium. This is because if either firms marginally undercut the price of their competitor they will acquire the entire demand for that market. When marginal costs are assumed to be zero this price undercutting will continue until equilibrium (best response) prices are zero. They propose in their paper two solutions to the problem of having no stable equilibrium. Firstly, there will be a stable Cournot equilibrium if the firms are "located outside the quartiles[5]" of the market line. This effectively restricts the strategy space for the competing firms to choose their locations. Secondly, there will always be an equilibrium in prices, for any locations of the two firms, if the transportation costs are quadratic with respect to the distance[6].

Both Tirole (1988) and D'Aspremont, Gabszewicz, and Thisse (1979) work out Hotelling's model with quadratic costs. Interestingly, they find that the principal of minimal differentiation is not the result. Just the opposite is found. Hotelling's differentiation model, now with quadratic costs, results in maximal product differentiation. Meaning that now, instead of locating together in the middle of the market, the competing firms will locate at the very ends of the market line, as far from each other as possible. This result is explained by Tirole as being the result of two conflicting effects. The *market share (or demand) effect* entices the firms to move towards their competitor in the hopes of gaining more of the market. However, the firm recognizes that in moving towards its competitor, (decreasing differentiation), their competitor is forced to lower their prices which in turn lowers the market price. Tirole calls this the *strategic effect* and it is shown to dominate. With quadratic transportation costs firms locate at the ends of the market in an effort to soften competition and avoid this price reduction caused by the strategic effect.

A few of the tying papers mentioned thus far allow for some kind of product differentiation but the level of differentiation is specified in the model as a predetermined amount. Egli (2005) develops a model that combines tying and product differentiation and endogenizes the amount of product differentiation choice in the competitive market. Therefore, whatever level of product differentiation maximizes profits is the level that will emerge in the model. Egli sets up his model in the common form we outlined earlier. There is market A and market B. Firm 1 (the tying firm) has a monopoly in market A and competes with firm 2 in market B. Market B is modeled like Hotellings'

[5]D'Aspremont, Gabszewicz, and Thisse (1979, p. 1147)

[6]See also, Tirole (1988, p. 280) for a quadratic mathematical example

model using linear transportation costs. In the pure tying[7] case, when firm 1 ties, (requires the sale of its good B to accompany the sale of any good A), Egli finds that minimal differentiation is the result in the two stage game where firms first choose their locations and then their profit maximizing prices. Unlike Hotelling's model however, there is no discontinuity in the profit function because of the asymmetry of the market. This asymmetry causes the price undercutting problem to disappear because neither firm 1 nor firm 2's marginal price reductions will enable them to gain the entire market. This is due to consumers varying preferences for good A. A price cut by firm 1 will never entice all consumers to buy good A and conversely, a price cut by firm 2 will never entice all consumers to give up good A.

Egli also looks at the tying decision game and its' effect on competition. To do this the model is extended to four stages[8];

> Stage 1: Firm 1 once and for all chooses to offer its goods in a bundle or separately.
>
> Stage 2: Firm 2 once and for all decides whether to be active in market B or not.
>
> Stage 3: Active firms choose their locations.
>
> Stage 4: Firms simultaneously set prices.

He first analyzes this game's location-then-price subgames and then compares the profits to analyze the first two stages; firm 1's bundling decision and firm 2's entry decision. To deal with the discontinuity in the profit function due to linear transportation costs Egli uses the first solution for equilibrium, mentioned above, by D'Aspremont, Gabszewicz, and Thisse. Egli restricts the strategy space for the two firms to locate outside of the quartiles. This means the firms are restricted to locate no closer to each other than 1/2 for a pure strategy equilibrium to exist. From this

[7] "Pure tying" is Egli's term for the case where firm 1 only ties its two goods in a ratio of one to one.

[8] Egli (2005, p. 15)

decision game Egli finds that tying by firm 1 will always foreclose firm 2's sales. For fixed costs that are high enough this can lead to firm 2 not entering the market. However, when fixed costs are small enough, firm 2 will be active in the market and it may or may not be more profitable for firm 1 to bundle depending on the size of the transportation costs.

In the rest of this paper we will first look at the robustness of Egli's linear transportation cost findings by presenting his model and decision game with quadratic costs. Using quadratic costs, as mentioned above, slightly augments the model so that with any combination of locations chosen by firm 1 and 2 there will be a pure strategy price equilibrium. This eliminates the need for a restricted strategy space in the decision game and leads to some differences in the locations chosen by the firms in the subgames. We also extend Egli's model to present the decision game where firm 2 has the option, in response to firm 1's tying, to enter market A as well as market B and tie it's goods.

3 The Model

Following Egli's method and notation, the model is comprised of two firms; firm 1 and firm 2, and two markets; good A and good B. Firm 1 is a monopolist in market A and also competes in the duopoly market B. Firm 2 competes in market B with firm 1. Good A is non-differentiable and good B is differentiated along a line of length 1 which represents the possible values for good B's differentiable aspect (ex. location). Consumers location, denoted by β, along this line represents their most preferred good B. Let q_1 and q_2 be firm 1 and firm 2's respective locations. We assume that q_1 is always located to the left of q_2 therefore $q_1 < q_2$. We assume that each consumer demands at least good B so every consumer either buys from firm 1 or firm 2. Different from Egli, we assume quadratic transportation costs.

Consumers have the same gross valuation for good B supplied by either firm which we denote r_B. Valuations for good A, r_A, differ between consumers but are distributed uniformly on the interval [0, 1]. We assume that producers pass all transportation costs on to the consumer and those unit transportation costs are represented by t. The consumer pays the quadratic transportation cost for the distance between the firm and the consumers locations, $t|q - \beta|^2$. Consumers purchase at most one of each good so, in the pure tying case, consumers have the option of buying the bundle of good A and B from firm 1, or buying just good B from firm 2. Firm 1 sells its bundle for the retail price of p_1 and firm 2 sells its good B for the retail price of p_2. Consumers are concerned about their generalized price, (their utility of consumption), which is their valuation for the good or bundle minus the costs of acquiring that good or bundle. When the consumer purchases the bundle of goods from firm 1 their utility is

$$v(\beta, q_1, p_1) = r_A + r_B - t|q_1 - \beta|^2 - p_1. \tag{1}$$

If the consumer buys only good B from firm 2 their utility is

$$v(\beta, q_2, p_2) = r_B - t|q_2 - \beta|^2 - p_2. \tag{2}$$

Following Egli (2005) we look at the two stage decision game in the pure tying case where firm 1 ties one of its monopoly good A to one of its competitive good B. In the first stage firms choose their locations, (q_1 and q_2), and in the second stage they simultaneously choose optimal prices, (p_1 and p_2). To solve this game we first need to find the demand equations.

4 The Equilibrium

4.1 Demand Specification

To solve for equilibrium locations and prices in this pure tying version of the model we need to find the demand equations for firm 1 and firm 2. As we explained above, consumers are concerned with their generalized price when deciding whether to purchase firm 1's bundle or firm 2's good B. In a standard Hotelling model, without tying, there are three regions of consumers. Consumers located to the left of firm 1

($\beta \leq q_1$), consumers located between firm 1 and firm 2 ($q_1 < \beta < q_2$), and consumers located to the right of firm 2 ($q_2 \leq \beta$). In this standard version consumers in the hinterlands[9] all demand the good from the firm closest to them because they are only considering transportation costs and their most preferred location (β). For consumers located in the hinterlands, choosing the closest firm yields the highest utility in this case. The only consumers that have a choice to make are the consumers between the two firms. The demand for each firm is found by identifying the indifferent consumer between them. Consumers to the left of that indifferent location demand firm 1's good and consumers to the right of that location demand firm 2's good.

In our model however, with varying valuations for good A (r_A), there is a new dimension to consider to find the indifferent consumer and the corresponding demand for each firm. In this model the demand in the hinterland does not solely belong to the closest firm. Some people in the left hinterland, though closest to firm 1 and purchasing the bundle, will have such a low valuation for good A that the benefits to them of having the bundle (i.e. lesser transportation costs) do not outweigh the costs even though they are closest to the bundle. In the region left of firm 1, ($\beta < q_1$), the consumer will purchase the bundle if their generalized price for it is greater than their generalized price for good B from firm 2, that is if:

$$r_B + r_A - t(q_1 - \beta)^2 - p_1 \geq r_B - t(q_2 - \beta)^2 - p_2$$

We can see that r_B is not a factor because in either case the consumer gets good B so they cancel out. Solving the equality for r_A gives us the indifferent consumers in that region and the following must be satisfied for the consumer to purchase the bundle from firm 1:

$$r_{A_X} \geq p_1 - p_2 + t(q_1 - \beta)^2 - t(q_2 - \beta)^2$$

To find the demand equations we break it down into three regions following Egli's linear model. Region X contains all consumers with $\beta \leq q_1$, region Y contains all

[9]These are the two outer regions, i.e. to the left of firm 1 and to the right of firm 2.

consumers with $q_1 < \beta < q_2$, and region Z contains all consumers with $\beta \geq q_2$. The indifferent consumers are given by the equality for r_A in each region and firm i handles the demand D_{iR} in the respective regions, R = X, Y, Z[10].

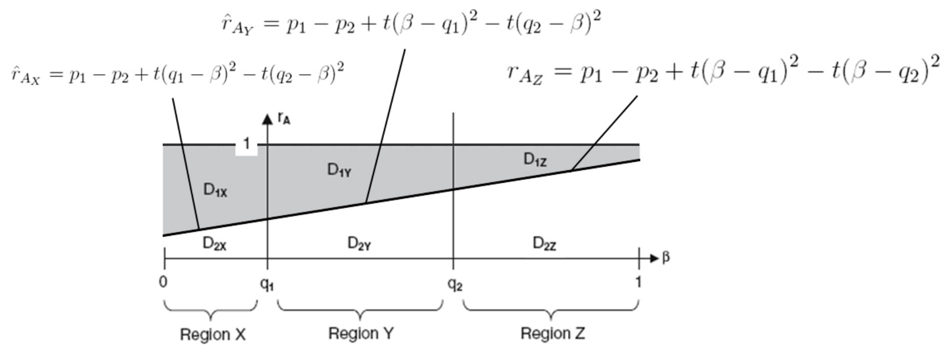

Figure 1: Demand Regions with Quadratic Transportation Costs
Source: Modified from Egli (2007, p. 33)

The line r_A in figure 1 is a straight line with a positive slope because in all three regions the derivative of the segment with respect to β, $(\partial r_A/\partial\beta)$, is the same and equals $2tq_2 - 2tq_1$. This is greater than zero because $q_2 > q_1$.

Demand Region X (D_{iX}): As we noted above, the indifferent voter in each region is where the generalized prices are equalized:

$$r_B + r_A - t(q_1 - \beta)^2 - p_1 = r_B - t(q_2 - \beta)^2 - p_2$$

When the above condition on the generalized prices is satisfied then the consumer is indifferent between purchasing the bundle from firm 1 and good B from firm 2. If the left side is greater than the right, the consumer demands the bundle from firm 1. Conversely if the right side is greater than the left, the consumer demands only good B from firm 2. Solving for r_A as an equality gives us the equation for indifferent consumers in region X. For Egli's linear version the demand in this region only depends on r_A but for the quadratic case we get:

$$\hat{r}_{A_X} = p_1 - p_2 + t(q_1 - \beta)^2 - t(q_2 - \beta)^2 \tag{3}$$

[10]Egli (2005, p.8)

Equation (3) represents the minimum value of r_A in region X required for the consumer to purchase the bundle. The area above the line $\hat{r}_{A_X}$ is therefore demand for firm 1 in that region and the area below it is demand for firm 2. The demand functions for region X are:

$$D_{1X} = q_1 - 0 - \int_0^{q_1} \hat{r}_{A_X}(\beta)d\beta \tag{4}$$

$$D_{2X} = \int_0^{q_1} \hat{r}_{A_X}(\beta)d\beta \tag{5}$$

And evaluating the integrals these become:

$$D_{1X} = q_1[1 - p_1 + p_2 + t(q_2^2 - q_1q_2)] \tag{6}$$

$$D_{2X} = q_1[p_1 - p_2 - t(q_2^2 - q_1q_2)] \tag{7}$$

These are different demand equations for this region than with linear costs. Egli found that $D_{1X} = q_1[1 - p_1 + p_2 + t(q_2 - q_1)]$ and $D_{2X} = q_1[p_1 - p_2 - t(q_2 - q_1)]$. We can see by comparing these that using quadratic costs effects the demand equations for this hinterland region X.

Demand Region Y (D_{iY}): The analogous reasoning and methodology for region X holds for region Y. The only difference is the transportation costs are now evaluated for consumers who's location, (β), is between q_1 and q_2. Hence the indifferent voter is now given by:

$$\begin{aligned} r_B + r_A - t(\beta - q_1)^2 - p_1 &\geq r_B - t(q_2 - \beta)^2 - p_2 \\ \rightarrow \hat{r}_{A_Y} &= p_1 - p_2 + t(\beta - q_1)^2 - t(q_2 - \beta)^2 \end{aligned} \tag{8}$$

Same as for region X, $\hat{r}_{A_Y}$ is the minimum valuation for A required in region Y for the consumer to demand the bundle from firm 1. The resulting demand equations for solving for these areas are:

$$D_{1Y} = q_2 - q_1 - \int_{q_1}^{q_2} \hat{r}_{A_Y}(\beta)d\beta \tag{9}$$

$$D_{2Y} = \int_{q_1}^{q_2} \hat{r}_{A_Y}(\beta)d\beta \tag{10}$$

The demand equations for region Y are equal to:

$$D_{1Y} = (1 - p_1 + p_2)(q_2 - q_1) \tag{11}$$

$$D_{2Y} = (p_1 - p_2)(q_2 - q_1) \tag{12}$$

What is interesting about these demand equations, (11) and (12), is that they are the same as Egli finds for region Y demand equations in his paper with linear costs[11]. Switching to quadratic costs in region Y does not change the demand for firm 1 or 2 in that region.

Demand Region Z (D_{iZ}): Again, all the same reasoning and methodology for region X, and Y holds for region Z. The only difference is the transportation costs are now set up for consumers who's location, (β), is between q_2 and 1. Hence the indifferent voter is now given by:

$$r_B + r_A - t(\beta - q_1)^2 - p_1 \geq r_B - t(\beta - q_2)^2 - p_2$$
$$\rightarrow \hat{r}_{A_Z} = p_1 - p_2 + t(\beta - q_1)^2 - t(\beta - q_2)^2 \tag{13}$$

The demand equations for region Z are:

$$D_{1Z} = 1 - q_2 - \int_{q_2}^{1} \hat{r}_{A_Z}(\beta) d\beta \tag{14}$$

$$D_{2Z} = \int_{q_2}^{1} \hat{r}_{A_Z}(\beta) d\beta \tag{15}$$

Evaluated these regions become:

$$D_{1Z} = (1 - q_2)[1 - p_1 + p_2 - t(q_2 - q_1 - q_1 q_2 + q_1^2)] \tag{16}$$

$$D_{2Z} = (1 - q_2)[p_1 - p_2 + t(q_2 - q_1 - q_1 q_2 + q_1^2)] \tag{17}$$

As was the case for hinterland region X, in this hinterland region Y the demand equations when quadratic costs are used produce a different demand region than Egli found with linear costs. The demand functions with quadratic costs, (equations 16

[11] Egli (2005, p. 10)

and 17), have two more terms than Egli's demand equations; $D_{1Z} = (1-q_2)[1-p_1+p_2-t(q_2-q_1)]$ and $D_{2Z} = (1-q_2)[p_1-p_2+t(q_2-q_1)]$.[12]

Now we can state the total demand for firm 1 and firm 2. Their total demand functions are the sum of their respective fraction of each region. The total demand equations for firm 1 and firm 2 using Egli's framework but with quadratic costs are:

$$D1 = D_{1X} + D_{1Y} + D_{1Z}$$

$$D1 = 1 - p_1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2) \tag{18}$$

$$D2 = D_{2X} + D_{2Y} + D_{2Z}$$

$$D2 = p_1 - p_2 + t(q_2 - q_1)(1 - q_1 - q_2) \tag{19}$$

Interestingly, we find that our quadratic cost model produces the same total demand equations as does the linear cost model. Total demand for firm 1 and total demand for firm 2 are unchanged despite the differences in the demand equations for the hinterland regions.

4.2 The Firm's Behaviour

Next we look at the two stage game where firms, facing the above calculated demand equations, first chose their optimal locations and then simultaneously set their optimal prices. We look at the pricing decision first to solve this decision game by backward induction[13]. First, firms maximize profits

$$\Pi_1 = p_1 D_1 = p_1[1 - p_1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2)],$$

$$\Pi_2 = p_2 D_2 = p_2[p_1 - p_2 + t(q_2 - q_1)(1 - q_1 - q_2)],$$

with respect to their prices. We get reaction functions:

$$p_1(p_2) = (1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2))/2,$$

[12] Egli (2005, p.9)

[13] The equations in this section are identical to Egli's in section 3.2 because we find that the quadratic transportation costs demand equations are found to be identical to Egli's linear cost demand equations (Egli 2005, p. 11)

$$p_2(p_1) = (p_1 + t(q_2 - q_1)(1 - q_1 - q_2))/2.$$

Subbing these reaction functions into each other gives us optimal prices as functions of the locations:

$$p_1^*(q_1, q_2) = (2 - t(q_2 - q_1)(1 - q_1 - q_2))/3$$

$$p_2^*(q_1, q_2) = (1 + t(q_2 - q_1)(1 - q_1 - q_2))/3$$

Next, to look at the location decision made in the first stage, we sub these optimal prices, (p_1^* and p_2^*), into the profit equations:

$$\Pi_1 = [2 - t(q_2 - q_1)(1 - q_1 - q_2)]^2/9$$

$$\Pi_2 = [1 + t(q_2 - q_1)(1 - q_1 - q_2)]^2/9$$

In the first stage firms maximize the above profit equations with respect to their location to find their profit maximizing location. The first order conditions for these profit equations with respect to locations are:

$$\partial\Pi_1/\partial q_1 = 2t(2 - t(q_2 - q_1)(1 - q_1 - q_2))(1 - 2q_1)/9 = 0$$

$$\partial\Pi_2/\partial q_2 = 2t(1 + t(q_2 - q_1)(1 - q_1 - q_2))(1 - 2q_2)/9 = 0$$

As Egli notes in his paper, we can see in the above equations that we see the optimal price equations here and can sub them in. Substituting in the price functions to see the partials more clearly yields:

$$\partial\Pi_1/\partial q_1 = 2tp_1^*(q_1, q_2)(1 - 2q_1)/3 = 0$$

$$\partial\Pi_2/\partial q_2 = 2tp_2^*(q_1, q_2)(1 - 2q_2)/3 = 0$$

To find the locations that satisfy the above first order conditions, i.e. maximize profits, there are two possible solutions. Either q_1 and q_2 equal $1/2$ or p_1^* and p_2^* equal zero. Firms are profit maximizing. Choosing prices equal to zero yields zero profits. Therefore we know that both firms locate at $1/2$ and are therefore exhibiting minimal differentiation. Since we have the same demand equations as Egli finds for linear costs, his proposition 1 holds for the quadratic transportation costs case: **Proposition 1**[14] *In the Hotelling game with tie-in sales firms set equilibrium prices* $p_1^* = 2/3$ *and* $p_2^* = 1/3$. *Both firms locate at* $q = 1/2$. *Equilibrium profits are* $\Pi_1^* = 4/9$ *and* $\Pi_2^* = 1/9$.

As explained in the literature review, there was a discontinuity found in the profit equations for Hotelling's principal of minimum differentiation. It was found that minimal differentiation is not a stable equilibrium because each firm would have the incentive to slightly undercut the price of the competing firm to gain the entire market. This is not a problem for the equilibrium found here because of the asymmetry of the markets. Because firm one ties good A to their good B, the consumers preferences between purchasing from firm 1 or purchasing from firm 2 are differentiated. Firm 2 can not decrease it's price for good B to gain the entire market because there will always be some number of consumers who's value for good A (r_A) is so high that they will never be compensated enough to give up good A. To gain the entire B market, firm one would have to price less than firm 2 ($p_1 < 1/3$) and at these prices could earn no more profits than 1/9 which is less than the profit of 4/9 it would earn if it did not lower it's price[15]. Likewise, whatever amount firm 2 lowers it's price for good B is the amount that firm 2 loses in profit so firm 2 will never undercut the price[16].

5 Tying Decision and the Effect on Competition with Quadratic Costs

Egli (2005) next works out the extended game, (the "decision game"), to look at the tying decision and the effects on competition. He assumes supplying good B involves fixed costs K and assumes that if firm 1 does not bundle both firms are active in market B. As mentioned earlier, he extends the game to four stages[17]:

[14]Egli (2007, p. 35)
[15]Egli (2005, p.13)
[16]Egli, (2005, p.13)
[17]Egli (2005, p. 15)

Stage 1: Firm 1 once and for all chooses to offer its goods in a bundle or separately.

Stage 2: Firm 2 once and for all decides whether to be active in market B or not.

Stage 3: Active firms choose their locations.

Stage 4: Firms simultaneously set prices.

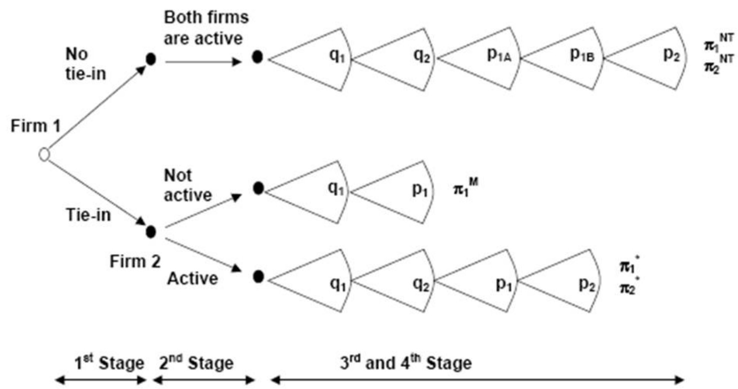

Figure 2: Timing of the Decision Game
Source: Egli (2005, p. 15)

This decision game and its timing is represented by the decision game tree in Figure 2 above. Egli works out the equilibrium outcomes for the three location-then-price subgames of this decision game; without tie-in sales by firm 1, inactive firm 2 with tie-in sales by firm 1, and active firm 2 with tie-in sales by firm 1. In this section Egli restricts the strategy space so that firms 1 and 2 can only locate outside of the quartiles, $(q_1 \leq 1/4, q_2 \geq 3/4)$[18]. He does this because this is one of the ways proposed by D'Aspremont, Gabszewicz, and Thisse to solve the problem, mentioned already, of the discontinuity in the profit functions that result from using linear transportation costs. Imposing this restriction allows a pure strategy equilibrium to exist[19].

[18]Egli (2005, p. 16)
[19]D'Aspremont, Gabszewicz, and Thisse (1979, p. 1137)

This strategy space restriction is not necessary in our model with quadratic transportation costs[20]. A pure strategy equilibrium does exist and the level of differentiation turns out to be different than what Egli finds with linear transportation costs. The next three subsections will summarize the three subgames in turn.

The Subgame without Tie-in Sales: When firm 1 does not tie its good A to its good B we have two separate markets. We have the B market with firm 1 and firm 2 competing, and, we have the market for good A where firm 1 has a monopoly. The pricing and differentiation[21] decisions for market B become a simple Hotelling model with quadratic costs now. This results in maximal differentiation[22]. This means that firm 1 will locate at $q_1 = 0$ and firm 2 will locate at $q_2 = 1$.

With linear costs and the strategy space restrictions imposed Egli finds the opposite. He finds that firm 1 and firm 2 minimally differentiate and, due to the strategy space restriction, locate at 1/4 and 3/4 respectively. Despite the differences in the level of differentiation arrived at, the price and profit findings for the quadratic case turn out to be the same as for Egli's linear case. The firms both set the same price for good B equal to t. And the profits for firm 1 and firm 2 in market B are $(1/2)t - K$ each. In this subgame, without tying, firm 1 also earns monopoly profits from market A. In market A firm 1 maximizes profits at $\Pi_A^M = 1/4$ by setting monopoly price $p_A = 1/2$[23].

Lemma 1[24] *In the subgame without tie-in sales firm 1 sets price $p_A = 1/2$ in market A. Both firms set the same prices $p_{1B} = p_{2B} = t$ in market B. Firm 1 locates at $q_1 = 0$ and firm 2 at $q_2 = 1$ in market B. Overall profits are $\Pi_1^{NT} = (1 + 2t)/4 - K$ and $\Pi_2^{NT} = (1/2)t - K$.*

[20] Tirole (1988, p. 280) notes that "the quadratic-cost model allows us to sidestep these technical issues" when explaining that a pure-strategy price equilibrium exists in this situation.

[21] The differentiated aspect in this model being location.

[22] For a mathematical example of this result see Tirole (1988, p. 280-281). It is proven that in this scenario $\partial\Pi^1/\partial q_1 < 0$ and $\partial\Pi^2/\partial q_2 > 0$, which results in maximal differentiation.

[23] Egli, (2005, p. 17).

[24] Egli (2005, p. 18) with locations modified for quadratic case findings.

The Subgame with Inactive Firm 2 and Tie-in Sales: By the specification of this model, if firm 2 is inactive in market B then firm 1 is a monopoly in both markets. Without competition demand for firm 1's products will inevitably increase and firm 1 will also be able to charge greater than competitive prices. This means that regardless of whether firm 1 ties or not, monopoly profits for firm 1 will be greater than firm 1 profits when firm 2 is active. Therefore Π_1^M is always greater than firm 1 profits in any subgame with competition. This is the same reasoning and findings for quadratic transportation costs as Egli's linear transportation costs.

The subgame with Active Firm 2 and Tie-in Sales: This is the same situation as the equilibrium analyzed in section 4 and Proposition 1 in section 4.2 holds here:

Lemma 2 *In the Hotelling game with tie-in sales firms set equilibrium prices* $p_1^* = 2/3$ *and* $p_2^* = 1/3$. *Both firms locate at* $q = 1/2$. *Equilibrium profits are* $\Pi_1^* = 4/9 - K$ *and* $\Pi_2^* = 1/9 - K$.

Egli finds different equilibrium locations in this subgame because of using restricted strategy space to solve the problem of the profit functions being discontinuous with linear transportation costs. Our model using quadratic transportation costs sidesteps this technical problem and the assymettry of this market leads to price undercutting being unprofitable as explained above.

The above analysis solves the third and fourth stages, (choosing location and price), of this decision game for each subgame. To complete the analysis we first look at the second stage which is firm 2's entry decision and lastly, we examine the first stage which is firm 1's tying decision. Egli's analysis of these first two stages relies only on the profit equations. Since the profit equations remain the same for each subgame with our quadratic cost modification, (only equilibrium locations q_1 and q_2 changed), the rest of this section will be a summary of Egli's reasoning and findings for the rest of this decision game.

Egli next looks at the second stage decision, firm B's entry decision. He assumes that fixed costs are such that when firm 1 does not tie, firm 2 is always active. If firm 1 does tie then firm 2 will do what is most profitable in response. Firm 2 enters market B as long as it earns positive profits. When firm 1 ties and firm 2 enters the B market, firm 2's profit function is $\Pi_2^* = 1/9 - K$. Therefore, firm 2 will enter the market as long as $K \leq 1/9$; otherwise entering would earn firm 2 negative profits[25].

Given firm 2's behaviour in stage 2, Egli looks at firm 1's decision to tie which is stage 1 of this decision game. Firm 1's decision is based on maximizing its' profits so he compares the profit equations of the subgames with firm 1 tying and not tying. Firm 1's decision to tie is dependent on firm 2's decision to enter market B which is itself dependent on fixed costs K. If fixed costs are less than 1/9, firm 2 will not enter market B. Firm 1 will always tie in this situation to forclose any sales by firm 2 and secure monopoly profits for itself. Egli finds the following:

Proposition 2[26] *In the decision game, firm 1 bundles*

- *if $K \leq 1/9$ (i.e., firm 2 is active) and transportation costs per unit distance are small enough (i.e., $t \leq 7/18$). The firms set prices and earn profits given by lemma 2.*

- *if $K > 1/9$ (i.e., firm 2 is inactive). Firm 1 earns monopoly profits Π_1^M.*

- *Otherwise, firm 1 does not bundle and the firms' prices and profits are given by lemma 1.*

Egli shows that firm 1 tying will always foreclose firm 2's sales and can do so to the point where entry by firm 2 is not profitable, (when $K > 1/9$), thereby altering the structure of the market. However, firm 1 tying will not always keep firm 2 out of

[25] Egli (2005, p. 19)
[26] Egli (2005, p. 20)

the tied market if transportation costs are low enough ($t < 7/18$). Egli (2005, p.21) breaks down this result; "As lemma 1 shows, non-bundling prices are lower in the competitive market the lower t is. A decreasing t increases substitutability between goods because transportation costs are lower. Higher substitutability intensifies price competition. Firm 1 circumvents such intense competition by using tie-in sales. Its demand increases and prices become independent of t." What is also interesting is that firm 1's bundling has the ability to actually increase firm 2's profits despite the decrease in sales. When firm 1 ties in scenario one of proposition 2 it makes firm 2's profit equal to $1/9 - K$. Without firm 1 tying firm 2 would have earned $(1/2)t - K$. For $t < 2/9$ this is actually an increase in profits for firm 2.

6 Decision Game Extended for Entry in Market A

As we learned in the previous section, firm 1 can foreclose firm 2's sales by tying. Now we want to see if and how this outcome will be changed by giving firm 2 the option of entering market A. We make a simple extension to stage 2 of Egli's decision game in section 5 so that it is now:

> Stage 2: Firm 2 once and for all decides whether to be active in market B or not, or to enter market A and B with a bundle.

We simplify this extension by making 2 important assumptions on top of the assumptions Egli already makes for the decision game. Firstly, we assume that firm 2 *only* enters market A as a response to firm 1 tying and cannot enter A without also entering B. And secondly, we assume that if firm 2 enters market A and B then they must bundle their two goods[27]. To see the effects of this extension on the tying and entry decisions we solve backwardly as we did in section 5 and, again, we begin with the location-then-price subgames. We are using our same model from section 3 with

[27]These simplifying assumptions are made to avoid the complex issue of mixing and matching between the two firms goods. This is an elaborate problem that goes beyond the scope of this paper. For a good introduction to this literature see Matutes and Regibeau (1988).

quadratic transportation costs. This extension of Egli's game gives us the new branch to the decision tree "Firm 2 Enters A B Tied" as shown in figure 3:

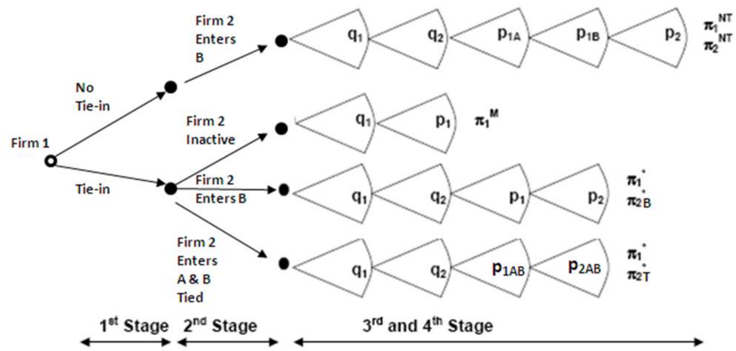

Figure 3: Timing of the Extended Decision Game
Source: Modified from Egli (2005, p. 15)

The Subgame without Tie-in Sales: Since we are keeping the same assumptions as for Egli's decision game we assume that when firm 1 does not tie fixed costs are such that firm 2 enters market B. The fixed costs we are referring to here are the fixed costs we have assumed are associated to producing good B. This subgame of the decision game remains unchanged from section 5 due to our specification that firm 2 will only enter both markets and tie as a response to firm 1 tying. Therefore lemma 1 holds for this extended decision game and is reiterated here:

Lemma 1[28] *In the subgame without tie-in sales firm 1 sets price* $p_A = 1/2$ *in market A. Both firms set the same prices* $p_{1B} = p_{2B} = t$ *in market B. Firm 1 locates at* $q_1 = 0$ *and firm 2 at* $q_2 = 1$ *in market B. Overall profits are* $\Pi_1^{NT} = (1+2t)/4 - K$ *and* $\Pi_2^{NT} = t/2 - K$.

The Subgame with Inactive Firm 2 and Tie-in Sales: This subgame is unchanged from section 5 as well. Firm 1 will earn monopoly profits (Π_1^M) and charge monopoly prices. These profits will intuitively be greater than any competitive scenario for firm 1 irrespective of whether or not they tie due to the increase in demand.

[28]Egli (2005, p. 18) with locations modified for quadratic case findings.

This is firm 1's most profitable and therefore most preferred outcome.

The Subgame with Firm 2 active in market B and Tie-in Sales: This is the last subgame that is the same as in section 5 and therefore lemma 2 holds here again for this extended game:

Lemma 2 *In the Hotelling game with tie-in sales firms set equilibrium prices* $p_1^* = 2/3$ *and* $p_2^* = 1/3$. *Both firms locate at* $q = 1/2$. *Equilibrium profits are* $\Pi_1^* = 4/9 - K$ *and* $\Pi_2^* = 1/9 - K$.

The Subgame with Firm 2 active in Market A and B and Tie-in Sales by Both Firms: This is the subgame unique to our extended game. In this subgame, firm 1 has tied their good A and good B and so essentially is selling one bundled good. Firm 2 has entered both markets A and B as well and tied their products in response to firm 1. Firm 2 is therefore also, essentially, selling one bundled good. This becomes a straight Hotelling game now. We are left with two firms, (each selling one bundled good), facing quadratic transportation costs and hence we arrive at the Hotelling quadratic cost equilibrium of maximal differentiation[29] given by lemma 3:

Lemma 3 *In the Hotelling game with tie-in sales by both firms we get equilibrium prices* $p_{1AB}^* = p_{2AB}^* = t$. *Firm 1 locates at* $q_1 = 0$ *and firm 2 locates at* $q_2 = 1$. *Equilibrium profits are* $\Pi_1^* = \Pi_{2T}^* = (1/2)t - K$.

Now we look at firm 2's entry decisions for stage 2 of our game. When firm 1 does not tie, we have assumed that fixed costs are such that firm 2 will always enter market B. However, when firm 1 ties, firm 2 now has three options to consider; remain inactive, enter market B, or enter both markets A and B with a bundle. Firm

[29] Again, as mentioned in section 5, for a mathematical example of this result see Tirole (1988, p. 280-281). It is proven that in this scenario $\partial\Pi^1/\partial q_1 < 0$ and $\partial\Pi^2/\partial q_2 > 0$ which results in maximal differentiation.

2 still requires that K be less than 1/9 for it to be profitable to enter only market B. To enter both markets firm 2 requires positive profit ($\Pi^*_{2T} \geq 0$) from Lemma 3, $(1/2)t - K \geq 0$. This implies that K need only be less than or equal to $(1/2)t$.

For firm 2 to enter both markets and bundle instead of entering only market B, profits for entering both (from lemma 3) must be greater than profits from entering B (from lemma 2). Therefore, for firm 2 to enter both markets it is required that:

$$\Pi^*_{2T} \geq \Pi^*_{2B}$$

$$\rightarrow (1/2)t - K \geq 1/9 - K$$

$$\rightarrow t \geq 2/9$$

Given these findings we present proposition 3 for firm 2's entry decisions:

Lemma 4 *In the second stage of the extended decision game, firm 2 always enters market B when firm 1 does not tie. When firm 1 does tie firm 2,*

- *remains inactive in either market if* $K \geq 1/9$ *and* $t < 2/9$.
- *enters market B and earns profits given by lemma 2 if* $K \leq 1/9$ *and* $t < 2/9$.
- *enters both the A and B market and ties it's products if* $t \geq 2/9$ *as long as* $K \leq (1/2)t$ *and earns profits given by lemma 3. This shows that it is still possible for firm 2 to be active if* $K > 1/9$.

Lastly, we look at firm 1's tying decision in stage 1 given firm 2's entry decisions in stage 2, (outlined above by proposition 3). As we specified for this decision game, firm 2 can only enter market A and B with a bundle in response to firm 1 bundling. Therefore, if firm 1 does not bundle it prevents firm 2 from having the option of entering both markets and profits are given by lemma 1 ($\Pi^{NT}_1 = (1+2t)/4 - K, \Pi^{NT}_2 = (1/2)t - K$). Firm 1's tying decision is therefore based on the no tying profits from lemma 1 compared to the profits they would earn if they tied. Firm w's profits if they tie must consider firm 2's reaction to that tie. Firm 2's reaction will be one of the three scenarios from proposition 3 depending on the factors of the market at that time.

Firm 1 will always tie if $K \geq 1/9$ and $t < 2/9$ to keep firm 2 inactive in any market and secure monopoly profits for itself. This is the situation where firm 1 can use tying to strategically maintain it's monopoly position and foreclose firm 2's sales.

If however, $K \leq 1/9$ with $t < 2/9$ firm 2 will enter market B if firm 1 ties. In this situation, firm 1's tying decision is based on the comparison of it's no tying profits from lemma 1 (Π_1^{NT}) and it's tying profits from lemma 2 (Π_1^*). So, firm 1 will tie if

$$\Pi_1^* \geq \Pi_1^{NT}$$

$$\rightarrow 4/9 - K \geq (1/2)t + 1/4 - K$$

$$\rightarrow t \leq 7/18$$

Since, for this situation, we have already specified that $t < 2/9$ and since $2/9 < 7/18$, firm 1 will always tie when facing these fixed and transportation cost levels.

Lastly, if $t \geq 2/9$ and $K \leq (1/2)t$ firm 2 enters both markets in response to firm 1 tying. Therefore, firm 1's tying decision is based on the comparison of it's no tying profits from lemma 1 (Π_1^{NT}) and it's tying profits from lemma 3 (Π_1^*). Firm 1 will tie in this situation if,

$$\Pi_1^* \geq \Pi_1^{NT}$$

$$\rightarrow (1/2)t - K \geq (1/2)t + 1/4 - K$$

As we can see the above statement can never be satisfied so firm 1 will never tie if firm 2's response will be to enter both markets. Again, firm 1 is able to strategically protect it's monopoly of market A because, as we specified, firm 2 can only enter both markets in response to firm 1 tying. Firm 1 only needs to be aware of the values of t and K to make this decision. This is information we expect the firms in the market to have so this finding is very interesting. We summarize firm 1's tying decision in the following proposition,

Proposition 3 *In our extended decision game, firm 1*

- *bundles if $K \geq 1/9$ and $t < 2/9$ (i.e., firm 2 is inactive). Firm 1 earns monopoly profits Π_1^M.*

- *bundles if $K \leq 1/9$ and $t < 2/9$ (i.e., firm 2 is active in market B). The firms set prices and earn profits given by lemma 2.*

- *does not bundle if $K \leq (1/2)t$ and $t \geq 2/9$ (i.e., firm 2 would have entered A and B if firm 1 had tied). The firms set prices and earn profits given by lemma 1, the no tying case.*

- *bundles if $K \geq (1/2)t$ and $t \geq 2/9$ (i.e., firm 2 is inactive). Firm 1 earns monopoly profits Π_1^M.*

Again, tying by firm 1 can foreclose firm 2's sales to the point where operation is unprofitable shown in the first and last bullet of the above proposition. This result is also found in the decision game of section 5. Also, same as section 5 finds, when $K \leq 1/9$ and $t < 2/9$, firm 2's profits increase by firm 1 tying. This is again because firm 2's profits without tying depend on t and when t is low enough, ($< 2/9$), profits are greater when they do not depend on t, which is the case when firm 1 ties. What is unique about our extension is that now, when $K \leq (1/2)t$ and $t \geq 2/9$ firm 2 would choose to be active in both markets if they had the option. However, *not* tying by firm 1 will protect its monopoly position and profits in market A. In this situation (point 3 of proposition 3) firm 1 will never tie and firm 2 will never be able to enter market A in response to that tying. It turns out though, in this case, that firm 1 not tying does not affect firm 2's profits. In the tying and no tying case here firm 2 would earn $\Pi_2 = (1/2)t - K$.

7 Conclusions

The purpose of this paper was to add to the understanding of the monopoly tying decision when we endogenize the product differentiation choice. To do this we first evaluate the robustness of Egli's linear cost model of tying with product differentiation by setting up and evaluating the quadratic cost equivalent. We find that the quadratic costs give us different demand equations for the hinterland regions but despite this difference we arrive at the same total demand equations as does Egli with linear costs. So we conclude that the linear cost pure tying demand equations are robust and we, like Egli, get the result of minimal differentiation in the pure tying case.

Next, in evaluating his decision game, Egli restricts his strategy space so that the minimal distance between the locations firms can choose is 1/2. This is not necessary when the model is evaluated with quadratic costs as we have done. This discontinuity of the profit function and the solutions to get around this dicontinuity, (quadratic costs or restricted strategy space), were found by D'Aspremont, Gabszewicz, and Thisse (1979). Using quadratic costs slightly augments the model to sidestep the problem linear costs present. We find that when using quadratic costs instead of restricting the strategy space for linear costs, different equilibrium locations for two of the subgames than Egli are found. We find in the no tying subgame that firms maximally differentiate and locate at $q_1 = 0$ and $q_2 = 1$ where Egli concludes that $q_1 = 1/4$ and $q_2 = 3/4$. For the "active firm 2 with tie-in sales" subgame we, with quadratic costs, do not restrict our strategy space and find that equilibrium q_1 and q_2 both equal 1/2 as opposed to Egli's $q_1 = 1/4$ and $q_2 = 3/4$. Table 1 summarizes the differences in the differentiation findings between Egli (linear transportation costs) and McMahon (quadratic transportation costs):

Table 1: Differentiation Findings Summary

Tying and Entry Case	Egli	McMahon
A. Tying firm 1, firm 2 in market B only	Minimal Diff.	Minimal Diff.
B. No tie firm 1, firm 2 in market B only	Minimal Diff.	Maximal Diff.
C. Both firms tie, both firms in market A and B	——	Maximal Diff.

Despite our differences in locations from Egli we find that his profit equations for each subgame still hold with the new locations. We conclude that his findings for firm 1's tying decision and firm 2's entry decision are robust. Therefore, all of Egli's decision game findings for the linear model hold for the quadratic model. He finds that tying always forecloses firm 2's sales. When $K \geq 1/9$ this foreclosure makes operation unprofitable for firm 2 and by tying, firm 1 gains monopoly power in the market. However, there is a scenario where firm 1 tying foreclose's firm 2's sales but at the same time increases firm 2's profits. This is when $K \leq 1/9$ and $t < 2/9$.

Lastly, we extend Egli's decision game to allow firm 2 to enter market A and B and tie it's goods in response to firm 1 tying. For our extended decision game we find that the three subgames and resulting lemmas from the original quadratic decision game still hold. We find that our extension of this decision game gives us an additional subgame which is the "both firms enter both markets and tie" subgame. In this subgame we evaluate it as an original Hotelling model with two firms and quadratic costs and arrive at maximum differentiation as the equilibrium. We then solve the decision game for firm 1 tying and find that, in this extended game, firm 1 still has the power to foreclose firm 2's sales. Firm 1 has the ability to keep firm 2 inactive and earn monopoly profits for the same situations as the original decision game mentioned above. Firm 1 tying can still increase firm 2's profits if, as explained already, $K \leq 1/9$ and $t < 2/9$. The difference we find now is that firm 1 has the option of not tying to prohibit firm 2 from entering the primary market A. It is profitable for firm 2 to do this if $K \leq (1/2)t$ and $t \geq 2/9$. In this case firm 1 will never tie because its tying profits $((1/2)t - K)$ would be less than its non-tying profits $((1/2)t + 1/4 - K)$.

References

[1] Carbajo, Jose., de Meza, David., and Daniel J. Seidmann (1990) "A Strategic Motivation for Commodity Bundling." *Journal of Industrial Economics.* 38(3), pp. 283-98.

[2] Carlton, Dennis W., and Michael Waldman (2002) "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries." *RAND Journal of Economics.* 33(2), pp. 194-220.

[3] Choi, Jay Pil., Lee, Gwanghoon., and Christodoulos Stefanadis (2003) "The Effects of Integration on RD Incentives in Systems Markets." *Netnomics.* 5, pp. 21-32.

[4] Choi, Jay Pil., and Christodoulos Stefanadis (2001) "Tying, Investment, and the Dynamic Leverage Theory." *RAND Journal of Economics.* 32(1), pp. 52-71.

[5] d'Aspremont, C., Gabszewicz, Jean Jaskold., and J-F Thisse (1979) "On Hotelling's "Stability in Competition"." *Econometrica, Econometric Society.* 47(5), pp. 1145-50.

[6] Economides, Nicholas (1986). "Minimal and Maximal Product Differentiation in Hotelling's Duopoly." *Economics Letters.* 21, pp. 67-71.

[7] Egli, Alain (2005). "On Stability in Competition: Tying and Horizontal Product Differentiation." Universitaet Bern, Departement Volkswirtschaft Working Paper.

[8] Egli, Alain (2007). "On Stability in Competition: Tying and Horizontal Product Differentiation." *The Review of Industrial Organization.* 30(1), pp. 29-38.

[9] Farrell, Joseph., and Michael L. Katz (2000) "Innovation, Rent Extraction, and Integration in Systems Markets." University of California at Berkeley Working Paper.

[10] Fisher, Franklin M. (2000). "The IBM and Microsoft Cases: What's the Difference?" *The American Economic Review.* 90(2), pp. 180-183.

[11] Gilbert, Richard J., and Michael L. Katz (2001) "An Economist's Guide to U.S. v. Microsoft." *The Journal of Economic Perspectives.* 15(2), pp. 25-44.

[12] Hotelling, Harold (1929). "Stability in Competition" *The Economic Journal.* 39(153), pp. 41-57.

[13] Klein, Benjamin (2001). "The Microsoft Case: What Can a Dominant Firm Do to Defend Its Market Position?" *The Journal of Economic Perspectives.* 15(2), pp. 45-62.

[14] Mathewson, Frank., and Ralph Winter (1997) "Tying as a Response to Demand Uncertainty." *RAND Journal of Economics.* 28(3), pp. 566-583.

[15] Matutes, Carmen., and Pierre Regibeau (1988) ““Mix and Match”: Product Compatibility without Network Externalities” *RAND Journal of Economics.* 19(2), pp. 221-234.

[16] Matutes, Carmen., and Pierre Regibeau (1992) “Compatibility and Bundling of Complementary Goods in a Duopoly” *The Journal of Industrial Economics.* 40(1), pp. 37-54.

[17] Peitz, Martin (2008). “Bundling May Blockade Entry” *International Journal of Industrial Organization.* 26, pp. 41-58.

[18] Tirole, Jean (1988). *The Theory of Industrial Organization.*

[19] Whinston, Michael D (1990). “Tying, Foreclosure, and Exclusion.” *American Economic Review.* 80(4), pp. 837-59.

[20] Whinston, Michael D (2001). “Exclusivity and Tying in U.S. v. Microsoft: What We Know, and Don’t Know.” *The Journal of Economic Perspectives.* 15(2), pp. 63-80.