Competition with Product Differentiation: Simulations of Pioneer Behaviour in a Linear City

by

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Obviously, only I can be held accountable for the opinions expresses in this essay and for any possible shortcomings.

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1 Introduction

In today's industries, we observe product differentiation of different degrees of importance. Sometimes, the product variety goes from an extreme to the other while in other cases, all products are the same. These two phenomena are known in the economic literature as the principles of maximum differentiation and minimum differentiation. Between these two opposed results, there is the possibility that firms do differentiate, but not in a maximum way. This is what we generally observe since in most markets, firms try to differentiate their brand from their competitors' brands, but they all want to offer a good that respond to consumers preferences.

An industry starts to exist when a first brand is created. At that moment, the firm that created the new product is in a monopoly position. This firm can be referred to as the pioneer firm and will enjoy its monopoly position until a single or multiple competitors enter the industry. These entrants might want to introduce a different product, may it be extremely or slightly different. On the other hand, it is also possible that these entrants will want to imitate the pioneer's brand. The pioneer will react to the entry of competitors maybe by changing its price and changing the characteristics of its product, if it is possible. The behaviour of the pioneer and of the entrants will therefore explain the degree of differentiation in a specific industry and these behaviours will vary depending on the industry's properties.

In this paper, the behaviour of the pioneer and of a possible entrant will be analyzed by simulating it in different industries characterized by a set of parameters. A pioneer firm is in a monopoly position for one period and competes with an entrant in the second period. The product can be horizontally differentiated in a linear city where the consumers' preferences are single-peaked. The product space is of infinite size since the consumers location has no bound on the right side, but firms are bounded and locate in a finite region. The model allows marginal cost, fixed costs, and consumers' willingness to pay to be a function of the location of the product. Furthermore, the model also allows for differences between the entrant and the pioneer such as a difference in their cost structure. The presence of returns to scale is also analyzed. We simulate the behaviour of the pioneer and the entrant in three different scenarios: when the pioneer isn't forward looking, locates in the monopolist location and can't change location between the two periods, when the pioneer is forward looking and locates in the "best" location for the two periods, and finally when the pioneer is free to change location between the two periods. Through the simulations, we find the best reaction of the entrant to the price and location of the pioneer and the pioneer's behaviour is therefore a result of how the entrant will behave. We find that the pioneer has an important advantage from first entering the market because it earns a lot of profits in the first period compared to the period where there is competition. The entrant also has an advantage from the fact that it enters the market knowing the price and location of the pioneer and can earn more profits in the second period than the pioneer does. We observe that the firms differentiate and each locate on one side of the peak of the consumer's preferences when the pioneer is forward looking and that the entrant will imitate the pioneer if the pioneer locates in the monopolist position while the entrant has a sufficient cost advantage. We go through the parameters to characterized the ideal setup for the pioneer and among them, we observe that the pioneer prefers a low transportation cost and increasing returns to scale because with those parameters, it can extract more profit in the first period.

The model developed is of a certain level of complexity and approach many issues discussed in the literature on product differentiation. For this reason and since the literature is wide on product differentiation, it is important to first proceed with a decent literature review. It is also an objective of this paper to carefully define product differentiation and present a complete explanation of the subject. After this, the model will be presented and its results will be analysed. Finally, some extensions to the presented model will be proposed.

2 The Theory

The theory[1] of product differentiation is a topic that is approached both in economics and in marketing.[2] It is therefore natural to explore the marketing literature. It is to be understood that some economists might have made really important contributions to the topic but that they won't be cited here since their contribution is not directly linked to the issues covered in the present paper.

2.1 Defining the Terms

First, we define the principle agents that are involved in the model presented in this paper to have a clear understanding of the important terms. Firms are agents producing and selling their products directly to the consumers. There will be no distributors or retailers involved. The firms are assumed to be profit maximizing. On the other hand, the consumers are assumed to maximize their utility, which means that they consume products only when they gain a benefit from it and that when facing a choice between two products, they will choose the one that generate the biggest benefit. A brand is produced by a specific firm and is characterized by a set of characteristics chosen by that firms. It can be a physical object or a service. The term brand will be used to express the product of a specific firm. An industry is a group of firms competing in a specific market, where a market is define by a set of products that are highly substitutable. In the marketing literature, a market can be viewed as a generic market or a product market. They are both characterized by a set of consumers having similar needs and firms offering products to satisfy those needs, but they are different since in a generic market, firms offer a wide selection of

[1]The core economic theory covered in this paper that is not the subject of a reference can all be retrieved in these textbooks: Church and Ware (2000), Shy (1995) and Tirole (1988). A reader who is interested in reading more about product differentiation should consider reading Beath and Katsoulacos (1991). The core marketing theory in this paper that is not the subject of a reference can all be retrieved in these textbooks: Kotler, Armstrong and Cunningham (1999) and Shapiro, Wong, Perreault and McCarthy (2002).

[2]Marketing is a process by which consumers, individuals or groups, obtain products they need and want by creating and exchanging them with others.

ways to satisfy the need while in a product market, firms offer products that are close substitutes. An example of a generic market is transportation while a product market could be the one of cars. In this paper, the term market correspond to a product market which is equivalent to the definition of a market in the antitrust literature.

2.2 The Basics of Competition

One of the first approaches to model competition among firms was from Cournot. He proposed that firms competed in quantities, which means that firms were producing a certain quantity and that the price in a market was giving by the demand function given the total quantity produced by all firms. When there are two firms in the market, Cournot demonstrated that a pair of quantities defines the equilibrium in the market. Therefore, he concluded that firms choose a production level such that they maximise their profit knowing the behaviour of the other firm.

Forty-five years later, Bertrand proposed an objection based on the fact that either firm could simply decrease its price by a small amount to take all the market and nearly double its profits. This would be followed by the other firm to also decrease its price such that an equilibrium is only reached when firms produce at marginal cost. This result is called the Bertrand Paradox. According to Bertrand, firms are choosing prices and not quantities. This means that with Bertrand competition, in a two indentical firm industry, firms will have no market power and these two firms will enjoy no profits.

This outcome is in contrast with what we can observe in real oligopoly markets. The Edgeworth solution to the Bertrand paradox state that when the firms of an industry first need to choose a capacity, a limitation to the quantity they can produce, the Bertrand paradox is eliminated. The other characteristic of an industry that will eliminate the Bertrand Paradox is when firms can use product differentiation. An industry where product differentiation is observed is characterized by the fact that products in this industry aren't perfect substitutes. The outcomes of Bertrand and

Cournot are only very different when the products are homogeneous and when there is no capacity constraint.

2.3 Product Differentiation

Products are said to be differentiated in a market when there exist a significant way to distinguish the product of a firm from the one of the other firms. According to Chamberlin (1962), the way firms differentiate their products can be "real or fancied", as long as consumers see a difference and that it leads them to have a preference for one product over another. "Where such differentiation exists, even though it be slight, buyers will be paired with sellers, not by chance and at random (as under pure competition), but according to their preferences". Even though there are differences between products in a market, a market is defined such that the products in it are all highly substitutable but not necessarily perfect substitutes. We can refer to those products as imperfect substitutes. The study of product differentiation assumes that the characteristics of products are selected by the firm producing it.

Without product differentiation, the firm with the smallest price controls the entire market. In reality, the quantity sold by a firm is a function of its price and the price of its competitors. Therefore, a small decrease in price by a firm will steal only a few consumers to its rivals. The opposite is also true, since a small increase in the price will cause the firm to lose some business to its competitors, but not all. Many reasons will cause consumers to have preferences for products over others and this is what causes this stabilising effect. Assuming that consumers simply buy the cheapest product in the market causes the instability described by the Bertrand Paradox.

Product differentiation can be divided in two different types: horizontal or vertical. Horizontal product differentiation is when consumers do not agree which brand is preferable compared to others. Vertical product differentiation occurs when consumers unanimously agree on the ordering of brands. Philips and Thisse (1982) therefore observe that with horizontal differentiation, when a change in the product characteristics occurs, there is a consumer that lose satisfaction and one that gain satisfaction while with vertical differentiation, consumers would all gain or all lose satisfaction. A classic case of vertical product differentiation is when the characteristic on which firms differentiate is quality.

The concept of product differentiation is close to the marketing concept of segmentation. Segmentation can be based on demographic, geographic, psychographic or behavioural characteristics of the consumers such that depending on those characteristics, they would order the products in a different way. Market segmentation is the activity that consist of dividing the market into different groups of consumers who might require different products. After the segmentation of a market, firms proceed to market targeting which is the evaluation of the segments profitability and the selection of which segment to enter.

Product differentiation become of a special interest since the theories of perfect competition and pure monopoly became inadequate to explain the business scene. Smith (1956) observes that perfect competition uses the assumption of homogeneity in products while heterogeneity was the rule rather than the exception. Product differentiation had become a strategy to secure a segment of the market through advertising or promoting the differences between a product and products of the competitors. For a firm, it is a way to protect its business against price competition. According to Smith (1956), product differentiation is a promotional strategy while market segmentation is the division of a heterogeneous market in many homogeneous markets which consists of a merchandising strategy. The term product differentiation seemed to be used differently by different people and its definition needed to be clarified. According to Root (1972), the concept of product differentiation in marketing generally means a level of competition that is "in addition to, and not a substitute for, price competition." On the other hand, economists see it as a force reducing price competition. In a review of 16 "contemporary marketing textbooks", Dickson and Ginter (1987) observed confusion concerning the term. Five textbooks described product differentiation and market segmentation as being alternatives and eleven described product differentiation as a complement or a way to implement market segmentation. Therefore, they defined the terms so that the confusion could be ended. They propose that market segmentation should be seen as a way to disaggregate demand into segments with distinct demand functions while product differentiation results in the consumers perception of differences in the products offered in the market. They also define a product differentiation strategy as the alteration of perceptions to create a state of product differentiation. Therefore, the act of advertising the characteristic of a brand to make sure the consumers do perceive a difference is considered a product differentiation strategy.

Horizontal Product Differentiation

There are two major different approaches to horizontal product differentiation. These are address models also known as location models and monopolistic competition. A third approach is called the characteristic approach. The model presented in this paper is a location model and therefore more weight will be put on the literature concerning this approach. The key element that differentiates location models and monopolistic competition is that competition is localized in location models while monopolistic competition assumes that all brands are equally good substitutes and that competition is market wide. As the number of characteristics on which firms can differentiate increases, there are more direct competitors to a firm in a location model and it approaches monopolistic competition.

Location Models

The pioneer of product differentiation is Harold Hotelling, an American mathematical statistician who's famous economic paper "Stability in Competition" was published in 1929. He proposed the idea of a Linear City Model first developed as a model of location and now used as a basic model of horizontal product differentiation. It has been adopted in several areas in economics since its publication. Location models assume that consumers have preferences defined over the characteristics of a product. The set of all possible products is called the product space. There can be one characteristic like in the linear city model that will be detailed later or more. That number is called the dimension of the product space. In a location model, consumer's preferences are distributed in the product space and the model needs the distribution to be specified. The location of consumers defines their most preferred product. Since consumers are located at different locations, their tastes are said to be asymmetric. Consumers have completely inelastic demands because they will purchase a single unit of only one brand. If a consumer is located at the location of a firm, he can consume its favourite product. If not, there is a transportation cost that is assumed to be strictly increasing in distance. Generally, the cost function is of the form t times the distance between the consumer and the firm $(t \cdot d)$ or t times the square of the distance $(t \cdot d^2)$. It is therefore possible that a consumer prefers to consume a product that is more distant from him than another if the price of the first one is enough lower than the second one. The important implication in location models is that competition is localized. A consumer would consider switching brand only for a product that also has a location close to him.

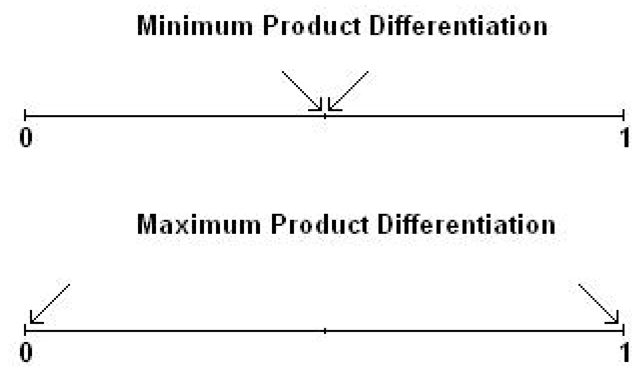
The Linear City Model

The main advantage to study Hotelling's model today is that it is a basic model that requires simple calculation and that generates intuitive and easy to express solutions. The consumers are uniformly distributed on a linear city of length one [0,1]. All the consumers have a single unit of demand and are all identical except for their location. Two firms are located in the city and they produce the exact same good. The only difference is their location. All the firms have the same unit cost and it is independent of the location. At an equal price, consumers choose the product of the firm that is closer to them. We can interpret the location of the firm as the type of product it produces, the characteristics setting of the product, and the location of the consumers as their most desired type of product. When consumers aren't purchasing their most desired product, they face a psychological cost that is represented in the model by a linear transportion cost. The socially optimal location for the duopoly case is the one that minimize these transportation costs. One firm must be located at 0.25 and the other one at 0.75. Hotelling assumes that the price is fixed in the city and that all consumers have a high enough willingness to pay such that they all consume one unit independently of where firms locate. With these assumptions, consumers simply buy at the closest firm. When firms choose location simultaneously, the result is that both firms choose 0.5. This is called the principle of minimum differentiation or Hotelling's law. The conclusion of the paper is that competition will lead to minimal product differentiation.

At that time, Lerner and Signer (1937) observed that there was a considerable interest for the problem of space in the market and for definition of a product and that these two issues are the same problem. It is convenient to use spatial differentiation to approach identical products located differently and also approach differentiated product located at the same place. They also observed that it is important that the transportation cost in Hotelling (1929) are being paid by the consumers. If it wasn't the case, firms could charge different prices to the different consumers. Without this assumption, because of the extreme inelasticity of demand of each buyer, firms could exercise perfect discrimination. Also, with the assumption that consumers buy one unit irrespective of the price, the location of a monopolist is indeterminate as at any location, it would sell to all consumers and if it could set the price, it would set it to infinity. To be realistic, it is therefore necessary to assume an upper limit to the willingness to pay of consumers.

D'Aspremont, Gabszewicz and Thisse (1979) made an importation contribution to the product differentiation literature with the publication of "On Hotelling's "Stability in Competition"" that stated: "The purpose of this note is to show that the so-called Principle of Minimum Differentiation, as based on Hotelling's 1929 celebrated paper, is invalid." This is because no equilibrium price solution will exist when both sellers are not far enough from each other. They propose a counter example where they assume that the transportation costs are quadratic with respect to the distance. Under their slightly modified linear city model, there is a price equilibrium whatever are the locations. The profit functions of the two firms are such that at any given pairs of locations, each firm gains an advantage from moving away as far as possible from the other. This result is considered as the principle of maximum product differentiation.

Figure 1: Minimum Differentiation vs Maximum Differentiation



A Different Version of The Linear City

If you let the firms choose simultaneously their price but with a given location in the city and then consumers decide where to buy still assuming their willingness to pay is high enough such that in equilibrium everyone is consuming one unit, the firms will choose prices as a function of their marginal cost, the transportation cost, their location and the location of the other firm. When the two firms are located at the same place, we obtain Bertrand price competition with homogenous good. If the locations are different, the two firms make profits which is a sign that product differentiation reduce competition. If there is no transportation cost, then consumers do not differentiate between the two goods and whatever are the locations, consumers will choose the lowest price which will conduct to the price equal marginal cost outcome.

The Hotelling-Bertrand model is a two stage game where firms choose position and then select prices simultaneously. Using quadratic transportation cost, the firms profits are increasing in the degree of product differentiation. Therefore, the strictly dominant strategy for the two firms is to locate at the two ends of the linear city. This means that in the two stage game, the equilibrium is maximal product differentiation and the prices are set to a mark-up of the transportation cost(t) over the marginal cost(mc); $t + mc$. The underlying cause of this result is the effect of price competition. As firms get closer to each other, price competition gets stronger and this gives them the incentive to locate away from the other. The choice of location is actually also affected by another effect called the demand effect or the Hotelling effect. This effect gives an incentive to the firm to move toward the middle to capture more demand. When transportation costs are quadratic, the price effect dominates the demand effect and the firms engage in maximum product differentiation. Under certain circumstances, the demand effect may dominate. The mechanisms that reduce the strength of the price competition effect include product differentiation over more then one dimension, competition over quantities or the insertion of a first stage where firms select a capacity constraint, the presence of positive externalities between the firms, collusion agreement between the firms in the market, and limitations of price competition due to government intervention. Another major characteristic of an industry that reduces the degree of differentiation is the non-uniformity of the consumer's distribution in the linear city.

When there are three firms in the market, the peripheral firm, which are the two ones on the sides, can increase their market share by moving closer to the interior competitor. They gain sales in the interior without losing sales on the periphery. With a number of firms higher than two, there are some issues with the existence of an equilibrium. Shaked (1982) explains that the case of 3 firms is peculiar in the sense that no pure strategy Nash equilibrium exists. He demonstrates that the three firm location problem "has a unique doubly symmetric mixed strategy equilibrium." This equilibrium is that firms stay away from the extreme quartiles and choose locations in the remaining half with equal probability.

An interesting extension of the linear city model is the circular city model developed by Salop (1979). It is mainly used as a model to evaluate entry in an industry. The firms will make zero profits but will still charge over marginal cost to cover the fixed costs. When fixed costs converge to zero, the number of entering firms tends to infinity and the market price tends toward marginal cost. In such a case, the market will generate too many firms. Salop observed that the results derived in his paper are simply a result of how the model is presented. The excessive number of firms in equilibrium is not a robust result. It appears to be robust as the number of dimension in the product space increases but the existence of an equilibrium is unlikely with too many dimensions.

Novshek (1980) observes that in a simple and standard location model, an equilibrium with free entry and exit does not always exist. It is necessary that the fixed costs are sufficiently small compared to the other parameters of the industry to obtain an equilibrium. Also, when there is an equilibrium with free entry and exit, it turns out to be approximately competitive.

Economides (1984) observed that when the maximum willingness to pay is not-too-high such that there could be consumers on the extremes of the linear city that prefers not to buy any product, the region where an equilibrium exists is extended in a price game. For low willingness to pay, there exists what Economides call a "touching" equilibrium, which is when all consumers between the firms choose to buy and the indifferent consumers between the two firms surrounding them are indifferent between buying or not. With even lower willingness to pay, we could observe what he calls a "local monopolistic" equilibrium which is when some consumers between firms are not buying.

Economides (1989) suggests the a three stage game where firms decide to enter or not in the first stage, choose location in the second stage and choose price in the third and last stage is a natural game structure since firms can decide "on price in the short run, on variety in the long run, and on entry in the very long run." He finds that there is a non-cooperative equilibrium in each stage and that at the resulting equilibrium, products are located at equidistant in the linear city. It is a result that is different from the principle of minimal product differentiation and also different from the principle of maximum product differentiation. At the equilibrium, there are more firms entering than the surplus maximizing number of firms creating a product diversity that is higher than the optimal product diversity.

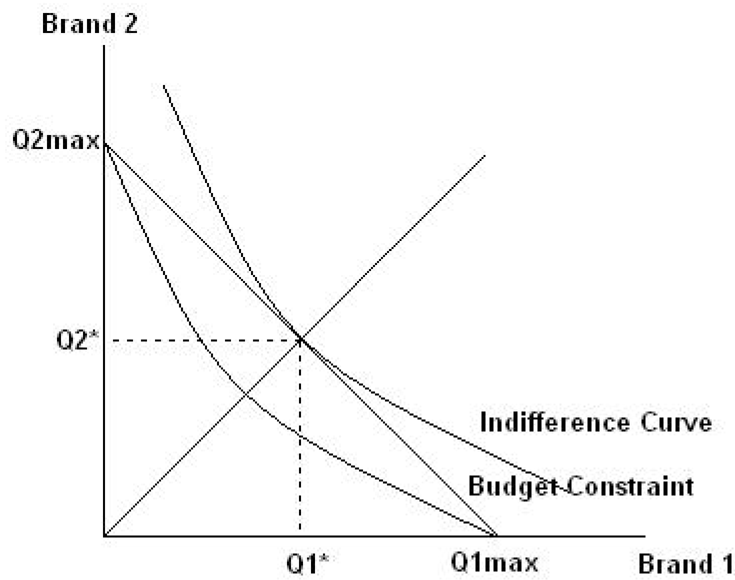
Economides (1987) allows for price and variety to be played simultaneously. At the equilibrium of it's model, firms are local monopolists. There are consumers between firms which are indifferent between buying or not or that will simply not buy. In the long run, when entry and exit are possible, the number of firms may be smaller or bigger than the number of firms maximizing the total surplus, it depends on the fixed costs.

Monopolistic competition

In monopolistic competition, it is assume that the consumers' preferences over the possible differentiated brands are symmetric. Therefore, the use of a representative consumer to represent all the consumers is common in a monopolistic competition model. The approach is based on the fact that consumers have a taste for variety. Figure 2 clearly explains monopolistic competition. The taste for variety is represented by the convex indifference curve which indicates that consumers have diminishing marginal benefit from the consumption of a brand. Note that when the elasticity of substitution is constant, the indifference curves are all symmetric around the 45 degree line.

A really important feature of the monopolistic competition is that every brand is in competition with all the other brands. According to the theory, as the number of firms increases in an industry, all existing firms should see a discrete reduction in

Figure 2: Monopolistic Competition



their demand. This is because every new product is an equally good substitute for every existing product and because consumers desire product variety.

Spence (1976b) “investigate the effects of fixed costs and monopolistic competition on the selection of products and product characteristics in a set of interacting markets.” The fixed costs are forcing industries to choose from all the conceivable products. Since firms are assumed to be profit maximising, the products that are produced are the ones who generate sufficient revenues to cover the fixed and variables costs. Therefore firms enter an industry when there is a conceivable product that would generate positive profit. The concept of deterring entry can then simply be define as the foreclosing of such opportunities through the selection of the characteristics of products.

In Perloff and Salop (1985), competition isn’t localized so that every brand competes with every other brand. They observe that when preferences are more intense, the equilibrium price increases. They also observe that if willingness to pay from all brands is bounded, the possibility of free entry completely eliminates monopoly markups. There can be no single-price equilibrium when many consumers have identical tastes such that the distribution of consumer preferences has one or more modes.

The Characteristic Approach

Lancaster (1966) explained product differentiation in a non-traditional approach based on the idea that utility comes from the characteristics of the products instead of the product itself. The approach also consider that products possess more than one characteristic and that those characteristic can be shared between more than one product. Finally, the characteristic approach also considers that two goods combined may possess different characteristics from those found in the two products separately.

Vertical Product Differentiation

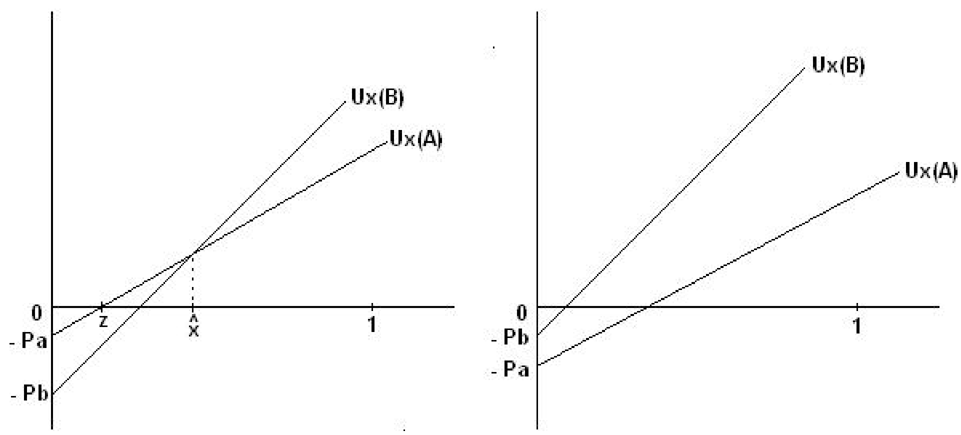
In vertical product differentiation, all consumers agree on the ordering of the products. Quality is frequently used in the literature of vertical product differentiation and generally has the property that consumers have higher willingness to pay for higher quality. When this is the case, a high quality producer would generate greater profits than a lower quality one if the variable and fixed costs are independent of quality. If a monopolist producing a single product faces increasing willingness to pay and increasing production cost in quality, it will set quality to a point where the marginal revenue is equal to the marginal cost.

When using a similar model to one of horizontal product differentiation where the principle of maximal differentiation applies, it still applies with vertical product differentiation.

A fare size of the literature on vertical product differentiation is concerned by issues with experience goods which are goods for which consumers do not observe the quality before they consume the product. From this issue, many interesting model approach the concept of the reputation of a brand but neither this or the experience goods will be discussed in the model.

If we consider a modified linear city model for vertical differentiation, we first need to observe that all consumers have their preferred brand located at 1. Assume consumers are uniformly distributed on the interval [0,1] and that there are two firms, A and B. The utility of the consumers will depend of the quality of the goods and of their prices. As shown on Figure 3, the consumer located at $\hat{x}$ is indifferent between the low quality brand A and the high quality brand B. This is because brand A is offered at a lower price. If brand B was offered at a lower or equal price than brand A, no one would prefer brand A to brand B. On the first graph we can also observe that consumers located from 0 to z would preferred not to buy since both brands would bring them a negative utility.

Figure 3: Representation of Vertical Differentiation

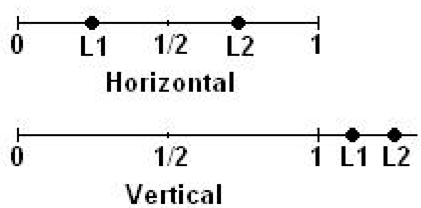


In that type of model, the firm producing the high quality product charges a higher price and both firms want to locate as far as possible from each other.

Shaked and Sutton (1982) propose a three stage game of vertical differentiation where in the first stage, firms decide to enter or not, in the second stage, they decide their level of quality, and in the third stage, they choose their price. Firms therefore have the strategy not to enter, or to enter, choose quality depending on the number of firms, and choose price depending on the number of firms and their quality. Assuming that consumers are identical in every aspects except that they are uniformly distributed by their income level, they observe in the duopoly case that the high quality firm enjoy more profits than it's competitor with lower quality and that the revenues of both firms are increasing in the quality of the high quality firm. In equilibrium, the duopolists earn positive revenues. When there are three firms or more, they all choose to produce the maximum quality level such that price competition brings profit to zero.

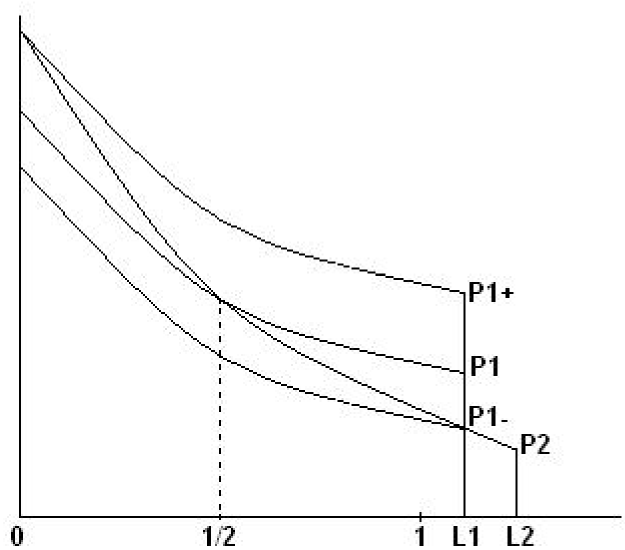
Another good way to present vertical product differentiation can be found in Beath and Katsoulacos (1991). Instead of considering the firms location inside the market, they place the two firms outside the market. Therefore, on on figure 4, all consumers would prefer a firm located at L1 to the firm located at L2 if they had the same price.

Figure 4: Horizontal Differentiation Comapred to Vertical Differentiation



Since all consumers agree that the firm located at L1 offers a better product than the firm located at L2, the firm located at L2 need to have a price lower than the other firm to capture some market. In this model of vertical differentiation, it is necessary that marginal transportation costs are increasing with distance. A quadratic transportation cost function is a good and common example. On Figure 5, we observe that given P2, the price of the firm located at L2, firm's 1 market share will depend on its price. If it sets P1 lower or equal than P1-, it will have all the market and if it sets P1 higher or equal to P1+, it will have no market share.

Figure 5: Other Representation of Vertical Differentiation



Product differentiation does not always has a fundamental influence on a market structure but when vertical differentiation is present and that variable production costs are increasing faster than the willingness to pay as quality increase, the industry can't be perfectly competitive. Horizontal product differentiation does not predict the structure of an industry and the welfare costs of it's outcome are likely to be small while with vertical product differentiation, the welfare cost are likely to be important and so will be the need for policy implications.

Wauthy (1996) propose a model where firms simultaneously choose quality and then compete in prices. He doesn't assume that the market will or won't be covered and shows that "covered or uncovered market are endogenous outcome of the quality game."

Model of Vertical and Horizontal Product Differentiation

Shaked and Sutton (1987) propose a model where a product is differentiated with respect to a vertical and a horizontal characteristic. Consumers have a preferred horizontal characteristic and they have different incomes. The free entry equilibrium is that at least one firm has a positive market share and therefore enjoys profits.

Other

Anderson, Palma and Thisse (1992) consider product differentiation in the context of probabilistic discrete choice models. They consider that a discrete choice approach provides an ideal framework to explain product differentiation since we deal with heterogeneous consumers making choices form a discrete set of substitutable products. They consider that the discrete approach is a link between the representative consumer and the location models. The key element of their modeling is that consumers aren't perfectly rational in their product selection. They use a probabilistic process to explain why consumers do not always select the same product when they must make repeated choices.

Schmitt (1995) considers the presence of tariffs and international transport costs to explain firms' behaviour in term of product differentiation. The model suggests that since tariffs and international transport costs increase the price of traded goods, it is natural for a local firm to imitate the traded good. Therefore, product imitation is an outcome of trade protection and international transport cost.

2.4 Welfare

As with every topic in industrial organization, welfare is an important aspect of the product differentiation literature. The common approach with product differentiation model is to look for the conditions that maximize the total surplus. When it is assume that all consumers have high enough willingness to pay so that all consumers buy one product, the pricing doesn't affect total surplus since it is just allocating the surplus between the firms and the consumers. What is important is the choice of characteristics since it is responsible for the transportation costs. The other important issue is the optimal number of firms. There is a trade between the increase in fixed costs from introducing new brands and the decrease in transportation or disutility costs from adding another brand. What has to be understood is that there is an optimal degree of product differentiation.

Lancaster (1975), observe that in the presence of constant or decreasing returns to scale, it is socially optimal that all consumers have their preferred product so that there is no transportation costs. With some degree of increasing returns to scale, it becomes socially optimal that the market is only served by a monopolist, but with increasing returns to scale and a uniform distribution of consumers, monopolistic competition will generate too many products than the socially desired number. With perfect competition and increasing returns to scale, the firms won't be able to take advantage of the scale economies and the optimal degree of product differentiation wouldn't be reached.

As stated in Spence (1976a), the market is responsible for the selection of the products that are produced and sold. Welfare is affected by the fact that firms are profit maximizing and not total surplus maximizing. Since profits do not capture the entire net surplus, the market tends to eliminate products that should be made available. In a monopolistically competitive industry characterized by falling average costs and no entry barriers, prices are set above marginal costs and firms enter until profits are zero. This is when revenues simply cover the fixed costs and therefore when prices are equal to average costs. Also, when products in an industry are complements, they tend to be undersupplied in a non-competitive industries.

Anderson, Palma and Thisse (1997) consider a market where there is a public firm competing private ones and evaluate the impact of privatizing the public firm. In the short run, the privatization has a negative impact on welfare since prices rise because the disciplinary role of the public firm disappears. In the long run, the privatization will lead to more firms entering the industry and their will a positive net effect if consumers have enough desire of product variety. When the public firm is making negative profits, the long run effect will always be positive. The interesting interpretation of this result is that it suggests that a profitable public firm should not necessarily be privatized.

Dixit and Stiglitz (1977) are interested by scale economies. In a way they consider there is a trade-off between quantity and diversity. This is the case since by producing fewer products but in greater quantity, resources can be saved while this creates less variety which causes a welfare loss. The optimality of the outcome depends on the details of the costs and demand functions.

Antitrust Issues

One antitrust case related to product differentiation is famous and therefore is worth mentioning. In 1972, the U.S. Federal Trade Commission investigated the ready-to-eat cereal industry because the major firms of this industry were accused of making it a non-competitive one through product differentiation related actions. The critics mainly concerned the fact that the proliferation of brands wasn't a result of consumer's preferences but a strategy of the firms to deter entry in the industry. Church and Ware (2000) state that the complaint was due to brand proliferation and pre-emption strategies followed by the 4 firms representing more than 85% of the market share. They were accused of flooding the market to fill market niches that could have supported an entrant. During the investigation, Root (1972) mentioned that the charges were based on the argument that product differentiation "achieved through advertising and/or styling changes" was responsible for the non-competitiveness of the industry. Also according to Root (1972), "the basic issue in the FTC charge is whether successful product differentiation resulting in increased market share is unfair competition".

There are some strategic behaviours related to product differentiation by an incumbent firm that can profitably deter the entry of competitors. Three strategies are commonly discussed in the antitrust literature, they are: brand proliferation, brand specification, and brand pre-emption. Brand proliferation is the action of locating its multiple brands such that no niches or locations are available that will support profitable entry, brand specification is the action of deterring entry by strategic choice of its product, and brand pre-emption is the action of introducing a brand before an entrant could enter to eliminate the possibility of a profitable entry.

Moving Toward the Model

In the following section, we combine the linear city model of Hotelling (1929) with a non-uniform distribution of the consumers preferences used by many including Perloff and Salop (1985) with the idea of a two-stage game where a pioneer firm enters in the first period while an entrant enters in the second period. We don't assume the market is covered and therefore use a low enough willingness to pay as suggested by Economides (1984) and others. We use discrete choices of location and price like Anderson, Palma, and Thisse (1992) to build a model of horizontal product differentiation that explain the degree of product differentiation in today's industries. From Lancaster (1975), we get the idea of allowing the possibility of increasing or decreasing returns to scale. Finally, the use of the two-stage game gives the possibility to observe the behaviour of the pioneer when there is the threat of a possible entrant to complete the discussions on the antitrust issues related to product differentiation.

3 The Model

Here, a model is proposed that is partially a new approach to horizontal product differentiation but strongly influenced by the theory covered previously. After carefully going through the theory, the purpose of this model is to discuss the behaviour of a pioneer firm and an entrant in a linear city model, making sure the model is consistent with the reality. A location model will be considered where consumers aren't uniformly distributed and can be located in a region where the firm can't locate. Furthermore, the assumption that consumers have high enough willingness to pay so that the market is covered will not be used. As the model is of a certain degree of complexity and that in real life, pricing strategy and the selection of a product characteristics are both discrete choices, the analysis will be made through computer simulations. With this process, it is possible to consider many parameters and therefore carefully approach horizontal product differentiation for any type of industry.

Characterizing an Industry

We will consider that in an industry, firms choose only one characteristic of a product. Therefore, the dimension of the product space in the model will be one. A really important characteristic of an industry is the presence or not of transportation cost. When transportation cost are present and are high, only the consumers close to a firm would buy its brand. As the transportation cost gets closer to zero, more and more consumers consider buying the brand. With a transportation cost set at zero, that would mean that all consumers, where ever they are would buy the product that generates the greatest positive benefit in the market. It doesn't mean that all the products are the same to the consumers, but that all consumers are symmetric in their preferences. It could still be the case that consumers have a higher willingness to pay for a product compared to another. Therefore, to characterize an industry, it is important to consider how the willingness to pay for products varies with the characteristic. On the other hand, the costs of producing a product may also be influenced by the location choice of the firm. Some products might require higher variable cost than others while in some industries, the fixed costs of a firm could also depend on the location of the firm in the product space. To carefully describe an industry, we must consider the presence of returns to scale. Finally the distribution of the consumers along the city corresponds to another important point that need to be specified. Since industries can be so different, it is interesting to have a model that has parameters such that all those characteristics can be acknowledge.

3.1 Consumers

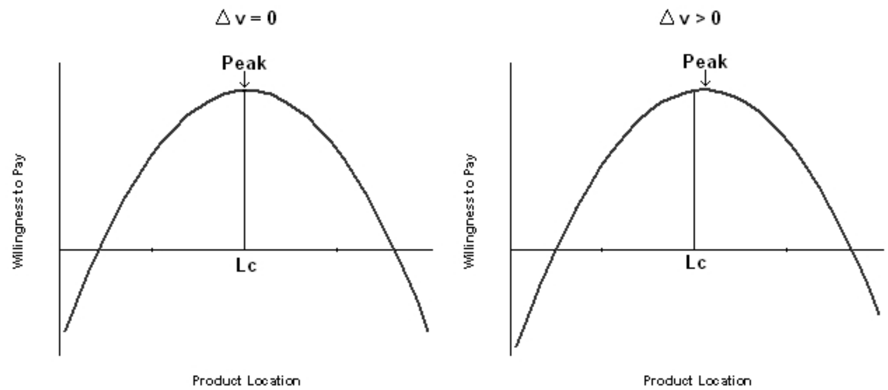
First, consumers do perfectly perceive differences in products so that no firms use a product differentiation strategy. All the differences between products are real and the products do not need to be experienced. All consumers agree that a product has an intrinsic value that is independent of their preferences. Therefore all consumers consider that a product has a value V that can vary with the product characteristic. We consider the core product as the product with the lowest level of the characteristic that the product can be differentiated on. The core product is located at zero in the linear city and has a value of βv. In the marketing theory, a core product is referred to as the service or benefice why the consumers really buy the product for while here it is simply that the characteristic on which firms can differentiate the product on is at its minimum value. For some products, the value will be constant with regard to its location, but for others, we can expect the value to depend on the characteristic of the product. For simplicity, we assume a linear form where Δv is the change in value per unit of characteristic. Therefore, a positive Δv means that consumers are giving a higher value to a product as its location (L) moves to the right.

$$V(L) = \beta v + \Delta v \cdot L \tag{1}$$

Consumers face quadratic transportation costs when consuming a product that isn't located at the same place than they are. The willingness to pay of a specific consumer for a specific good will be the value (V) that all consumer attribute to this good minus the transportation cost that the specific consumer faces to reach the specific good. Consumers have a single peaked preference function with the peak located at their location when Δv is equal to zero. When Δv is not zero, the peak can be located on the right or the left of the location of the consumer. If Δv is greater than zero, the peak is slightly to the right. These different scenarios are shown in Figure 6 for a consumer located at Lc.

The consumer decision process starts by the recognition of a need and is followed by the information search. We assume that the consumers have access to perfect information. Next, the consumer evaluate the possible alternatives which are the products present in the market or the decision not to buy. The consumers have a unit demand and will buy the product that generates the biggest positive benefit for them. The benefit a consumer enjoys is defined as the difference between its

Figure 6: Willingness to Pay for a Specific Consumer



willingness to pay and the price paid. Once the product is purchase, there is no post-purchase behaviour of any kind.

In the product space, we do not assume here that the consumers are uniformly distributed. They are distributed according to a Chi-Square distribution. This distribution has many interesting properties that justify its use in this model. A χ^2 distribution differs depending on its degree of freedom (df). All χ^2 distribution are single peaked with mean df and variance $2 \cdot df$. The mode of the distribution is at df-2 for all χ^2 with $df \geq 2$. The density of the distribution between zero and infinity is one and therefore we can easily observe the proportion of consumers buying the good. It is a right skewed distribution function and its skewness can be calculated as $\sqrt{8/df}$. At the mode, the location where the PDF is at its highest value, we can find the most demanded product. With a χ^2 distribution of degree of freedom of 2, the mode is at zero, at the core product. The simulations will use the chi-square distribution with degree of freedom of two(2), four(4), six(6), and eight(8). The probability distribution function (PDF) and the cumulative distribution function (CDF) are shown in Figure 7 and Figure 8.

Figure 7: PDF of the Chi-Square Distribution

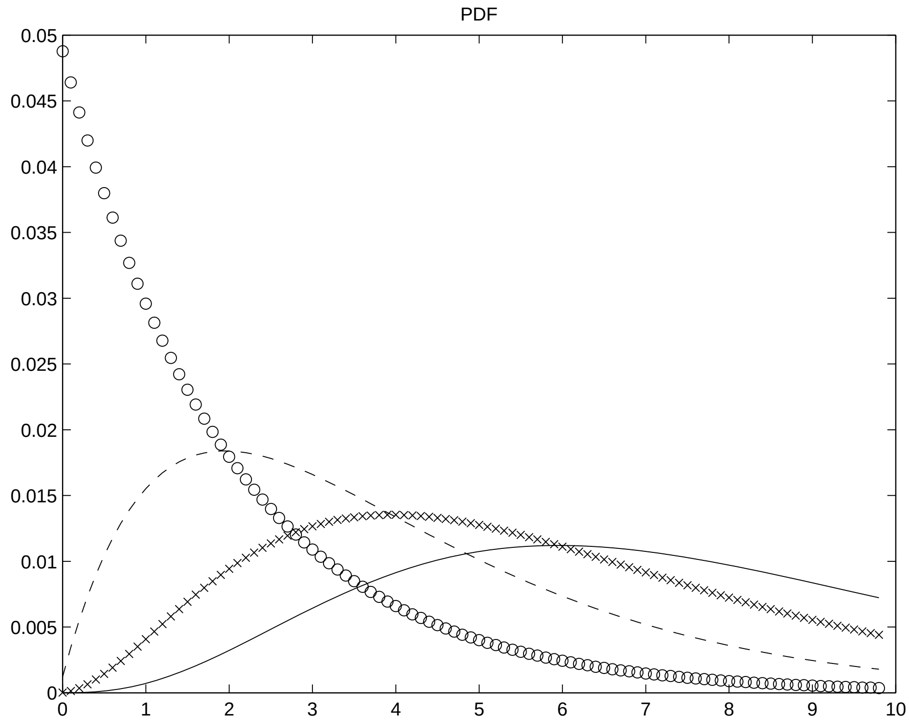
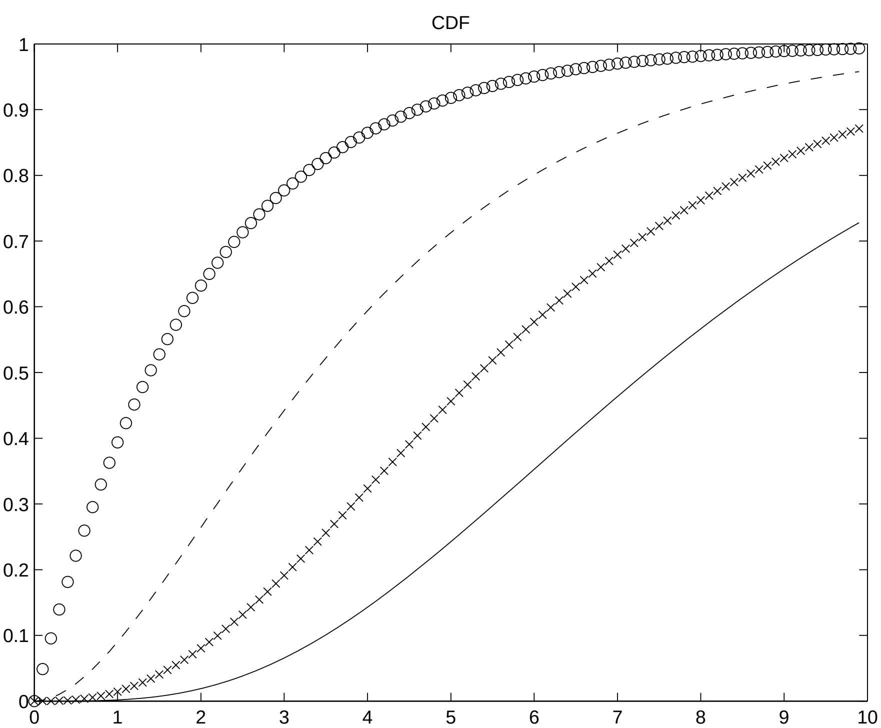


Figure 8: CDF of the Chi-Square Distribution



The CDF of a χ^2 distribution with degree of freedom df at a location L is given by:

$$CDF(df, L) = \frac{\gamma(df/2, L/2)}{\Gamma(df/2)} = P(df/2, L/2) \tag{2}$$

Where γ(k,x) is the lower incomplete gamma function, Γ(k,x) is the gamma function and P(k,x) is the regularized gamma function. The exact expressions for the CDF of the χ^2 distribution with degree of freedom 2, 4, 6, and 8 are given in the appendix. The major implication is that the linear city is of infinite length since there is no limit on the right side for the location of consumers. Eaton and Lipsey (1975) proposed a model where they proved that with a density function that is not rectangular over any finite range of the product space, it is necessary for equilibrium that the number of firms is smaller or equal to twice the number of modes.

3.2 Firms

The firms locate in the linear city between zero and $\bar{L}$. $\bar{L}$ is considered as the maximum level of the characteristic that the current technology can support. One could simply imagine that location after $\bar{L}$ is considered as a residential zone or a suburb of the city. The firms are assume to be profit maximizing and will choose location and price to do so. The action of selecting a location in the linear city is really similar to the process where a firm first does market segmentation, than market targeting, and finally market positioning. This is because the firm first considers the set of possible location that it could select and evaluate the resulting outcome. The theory of the marketing mix suggests that a firm has a set of controllable marketing tools that it can use to maximize its profit. These tools are referred to as the four Ps. They are the product, the price, the place and, promotion. The product is defined as the good or service that the firm offers. This first P corresponds to the location in the linear city model. The price is what is paid by the consumers and is selected by the firm. The place is the firm's activities that make the product available to the consumers and the promotion is the set of activities used to persuade the consumers to buy the product. We assume in this model that all firms have a identical place and that no brands benefit of a better accessibility. Also, the firms do not use any promoting activities.

They face fixed costs that can be different depending of the location of the firm. When located at zero or when the firm produce the core product, the fixed cost are βf and we assume a linear function such that for a location L, the fixed costs are:

$$F(L) = \beta f + \Delta f \cdot L \tag{3}$$

The marginal cost of production can also vary with the location of the firm. At zero, the firm faces a marginal cost of βmc and once again we assume a linear function such that Δmc represents the change in marginal costs per unit of location. An industry can be characterized by the presence of constant, increasing or decreasing returns to scale and therefore we allow the marginal cost to vary with the quantity produced. Since the population on the linear city is of 1 and they have unit demand, the quantity (Q) is between 0 and 1. We can interpret it as the percentage of the population buying the brand of the firm. The presence of returns to scale is represented by the parameter α. When α is negative, the marginal cost is decreased and we observe increasing returns to scale and when α is positive, the marginal cost in increased and we observe decreasing returns to scale. If α is set at 0, the returns to scale are said to be constant. The quantity for a firm will depend of all the prices and locations occupied by a firm in the market. We use $\vec{P}$ as the vector of all prices in the market and $\vec{L}$ as the vector of all locations in the market.

$$MC(\vec{P}, \vec{L}) = \beta mc + \Delta mc \cdot L + \alpha \cdot Q(\vec{P}, \vec{L}) \tag{4}$$

The quantity sold by a monopolist located at L and selling at price P can be found by the density between the two indifferent consumers. The left indifferent consumer is located at x where:

$$\beta v + \Delta v \cdot L = P + t \cdot (L - x)^2 \tag{5}$$

Therefore,

$$x = L - \sqrt{\frac{V(L) - P}{t}} \tag{6}$$

The location x can't be negative so the possible value of x has an inferior bound at zero. The right indifferent consumer is located at y where:

$$\beta v + \Delta v \cdot L = P + t \cdot (y - L)^2 \tag{7}$$

Therefore,

$$y = L + \sqrt{\frac{V(L) - P}{t}} \tag{8}$$

When there are two firms, they can both be serving different areas of the city such that the right indifferent consumer of the firm located on the left of the other is on the left of the left indifferent consumer of the firm located on the right side. This was referred to as a monopolistic equilibrium by Economides(1984). When this isn't the case and that there is an indifferent consumer between buying from firm A located at a and firm B located at b, the location z of the indifferent consumer is such that:

$$V(a) - Pa - t \cdot (z - a)^2 = V(b) - Pb - t \cdot (b - z)^2 \tag{9}$$

Therefore,

$$z = \frac{(b^2 - a^2) - \frac{(V(b) - V(a) - Pb + Pa)}{t}}{2(b - a)} \tag{10}$$

In this case, the y of firm A and the x of firm B would now be z. The demand for a firm is therefore:

$$Q(\vec{P}, \vec{L}) = CDF(k, y) - CDF(k, x) \tag{11}$$

It is normally assumed that a firm selling with a price equal to the consumer willingness to pay will have one consumer demanding its product. Since here x and y would be the same and therefore have the same CDF, the demand is 0. This is equivalent to assume that consumers only buy a product if they gain something form it. The firm acting as a monopoly has a potential reach on each of its side of the same distance but that doesn't mean that it reaches the same number of consumers on each side. This is because the consumers distribution function isn't uniform. When t is set to zero, all consumers will buy from a firm if a product that generates a positive benefit is available.

The firms locate to obtain a competitive advantage over their competitors. The location of the product defined how the consumers order the brand relative to competing brands and a competitive advantage is referred to as the offer of greater value to the consumers that justify higher prices. Therefore, the competitive advantage of a firm in this model is the proximity to certain consumers.

A firm locating at L and selling at price P has a profit function of the following form:

$$\Pi(\vec{P},\vec{L}) = (P - MC(\vec{P},\vec{L})) \cdot Q(\vec{P},\vec{L}) - F(L) \tag{12}$$

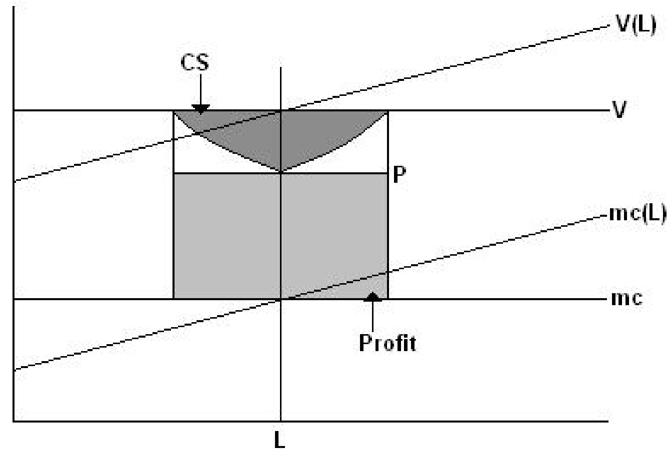
The profit of a monopolist and the consumer surplus are represented in Figure 9 for a price P and a location L. On this figure, α is set to zero so the marginal cost of the monopolist is only changing in its location (L). If the monopolist increase its price, the quantity will be reduced since the new indifferent consumers will now be closer to its location. To evaluate the consumer surplus and the profit of the monopolist in the figure, it is necessary to consider the distribution of the consumers on the linear city.

3.3 The Timing

The major difference between this approach and the literature lies in the timing of the model. We consider a firm first entering the market and being a monopolist for one period. This firm is referred to as the pioneer firm. In the second period, there is a possible entrant that sets its location and price after the pioneer did. We will consider many different industries by allowing the parameters to changes and observe the pioneer's behaviour in those industries.

When a new product is introduced, it is expected to go through what is called the product life cycle. The first stage is the product development where a pioneer firm

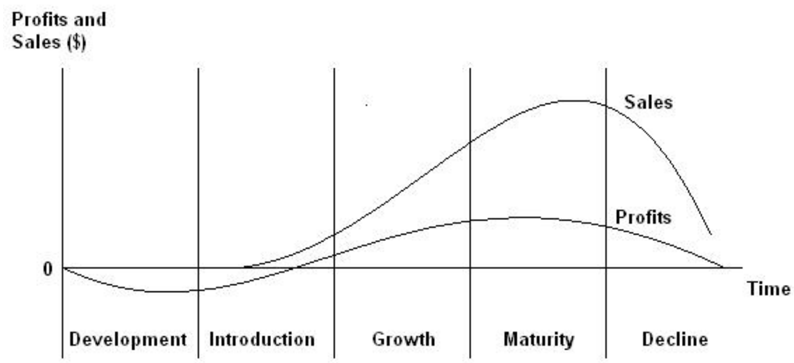
Figure 9: Representation of The Model



creates the product. The pioneer than introduce its brand in the market in the stage called market introduction. The next stage is market growth where the industry's sales grow fast. The pioneer's profits rise but entrants will start to enter the industry and the sales and profits will be shared. The entrants might copy the pioneer or might try to differentiate from it. Therefore, the pioneer loses its monopoly position and now has to compete in the next stage called the market maturity. Finally the last stage is when sales decline, leading to the end of an industry. A graph representing the life cycle of a typical product is shown in Figure 10. When being a pioneer in a market, some firms make important strategic mistakes by not realizing that entrants will soon be at the door to enter the industry. The pioneer enjoys increasing sales and profits and is sometimes blind to the upcoming competition. This might result in the pioneer being forced to leave the industry since entrants have adapted their self more adequately to the consumers' preferences.

One could see the role of the pioneer firms has really important since they are the one taking the risk of developing new products. This role could become harder to play as time goes on since it is now a fact in the marketing literature that the product life cycles tend to get shorter. The importance of entering a market in the early stage

Figure 10: Product Life Cycle of a Typical Product



becomes even more important; the pioneers are the ones making the profits.

Carpenter and Nakamoto (1989) observe that the market shares tend to be larger for brands that entered the market in the beginning of the product life cycle. The pioneers appear to have larger shares than entrants. They suggest that somehow, the mechanism where a firm first enters a market reduces competition, making it more difficult for entrants to compete the pioneer. They refer to this phenomenon as the pioneer advantage. They consider a location model where an early entrant can selects the preferable market location leaving less attractive locations for the possible future entrants. They also insist on the fact that the consumers preference are influenced by what the pioneer first offer and this is the source of the pioneer advantage. By first entering a market, a pioneer influences the way consumers evaluate the brands. In their model, there is a shift of the preferences toward the location of the pioneer.

Schmalensee (1982) presents a model where the pioneer benefits of an advantage from the imperfection of information concerning product quality. Consumers get to know the quality of the pioneer brand and it gets hard for en entrant to convince a consumer to switch to its brand. It consists of an advantage of early entry when there is product differentiation in an industry that is not explained by advertising or consumer irrationality. He insists at the end that there are no general public policy prescriptions from this result since pioneering brands deserves to be rewarded for their innovation and the risk taken.

After evaluating the behaviour of a monopolist with one brand or a product line of two brands, we first consider the scenario where the pioneer chooses the location without being forward looking. It chooses the monopoly location and is trapped their in the second period when facing a possible entrant. After this, we consider the scenario where the pioneer selects a position that it will need to keep for the two periods. Finally we consider the scenario where the pioneer can change location between the two periods. In all these scenarios, the pioneer can always adjust its price. It is also to be noted that the pioneer uses a discount rate (dr) to calculate its overall profit of the two periods.

For every simulation, for any set of parameters, the profit of a monopolist is evaluated for all the possible combinations of price and location. A monopolist would therefore set the location and the price to the values that generate the maximum profit. When a monopolist has two brands in the market, the profit for all the combinations of the price and location of the first and second brand is calculated. The monopolist would obviously set the locaions and the prices to maximize its profit. When there is a possibility of an entrant, the profit of the entrant for all the combinations of its price and its location are calculated for every price and location of the pioneer. For a given price and a given location of the pioneer, we obtain the price and location of the entrant that maximize its profit. This correspond to the best respond of the entrant to the pionner choice of price and location. From that best responce of the entrant, we can easily obtain the resulting profit of the pioneer. When for a price and a location of the pioneer, the best response of the entrant generates a negative profit for the entrant, it doesn't enter and the pioneer benefit of the profit generated by that price and location as if it was alone in the market. Now that the pioneer knows its resulting profit for every combination of price and location, it will select the one that maximizes its profit. In the first scenario, when the pioneer is trapped in the monopoly location, it must choose for the best combination of that location and of any price for the second period. When the pioneer is foward looking but has to stay in the same location for the two periods, it selects the location to maximze the discounted sum of the maximum profits that can be generated with that location in period one and two. With the third scenario, when the pioneer can change location between the two periods, it selects the monopoly price and location in period one and selects the best combination of price and location in period two. The discount rate therefore only has an impact on the location selection in the second scenario.

It is normally assumed that in the long run, the location can be changed so the use of those different scenarios can be first interpreted as if the second period is the long run, or second as if that in this industry, firms can easily change the characteristics of their brand.

Lerner and Signer (1937) observed that when a monopolist need to react to the possible entry of a competitor, the presence of costs of movement will have an important stabilizing effect since the monopolist will prefer not to move unless its gain from moving is higher than the cost of movement.

Setting the Parameters

In the simulations presented in the following section, $\bar{L}$ is set at 9.9 and firms can locate at every interval of 0.1 in the product space. Therefore, there is 100 possible possibility of location from 0 to 9.9. The firms can price at every interval of 0.1[3]. By considering different values for the parameters, we quickly obtain millions of possible combinations. Since computing time is always an issue for simulations, 80 different combinations have been simulated for which the data is in the appendix.[4]The first 32 simulations [5] are the different combinations of: transportation cost or not, constant

[3]For readers who strongly desire to make it realistic, they should consider a price of 5.1, as an example, as $0.51 such that the discrete choice of price really correspond to reality.

[4]The simulation of the 80 scenarios with price and location interval possibility set at 0.1 took around 10 hours of computing time on a powerful computer.

[5]see Table A1 to A2 in the appendix

value or not, and constant production cost or not. When there are transportation costs, t is set at 1. βv is se to 6 and when the value isn't constant, Δv is set to 0.1. βmc is set to 1 for the pioneer and the entrant and when the production costs are increasing in location, Δmc is set to 0.1 for both firms. βF is equal to 0.25 and ΔF is 0 for the two firms. α is 0 and dr is 1.1, a discount rate of 10%. From those 32 simulations, we consider a basic case. It is with transportation costs and when both production costs and the value do not change with location. For this case, we consider the impact of a change in t, α, dr and βmc. We than have a look to cases where the two firms have different βF and also different Δmc and βmc.

4 The Results

In this section, we go over the results for the monopoly and the duopoly, we have a quick look on the no transportation cost case, and we finally consider the impact of a change in the parameters.

4.1 Monopoly

We first observe the behaviour of the pioneer when there is no threat of a possible entrant. With the basic case, the monopolist locates close to the mode. Since the distribution of the consumers' preference is right-skewed, the firm prefers to locate slightly to the right of the mode. With the degree of freedom set to two, the firm locates such that its indifferent consumer on its left is located at zero. For a better understanding of the profits evolution with price and location, see Figures 12 to 17 in the appendix. For the basic case with degree of freedom 2 and 6, they show the profit for all the combinations of price and location, the profit and location for a given price, and the profit and price for a given location.

Proposition 1 *More the consumers' preferences are concentrated around the mode, the more the monopolist will make profits, the higher the price will be, and bigger the quantity will be.*

By looking at Table A-1, we can easily observe that the monopolist enjoys bigger profits as the degree of freedom is smaller. Actually, as the degree of freedom is smaller, the firm sells to a bigger proportion of the population and sets its price higher. This is consistent in the simulations but is slightly sensible to cases where $\Delta v > 0$. This is caused by the properties of the χ^2 distribution. As the degree of freedom gets higher, the demand gets more spread but also moves to the right. Proposition 1 is robust as long as Δv, Δmc, and Δf are nul. Actually, when $\Delta v > 0$, it gives an incentive to the firm to move to the right as it can charge a higher price. When $\Delta mc > 0$, this gives an incentive to the firm to move to the left as the production costs increase with the location. When they are both positive and of similar value, the behaviour of the firm is similar to the one where they are both equal to zero.

Welfare

We consider the first-best outcome, where price and location are selected to maximize the welfare. This is when price is set to marginal cost[6]. Since the firm faces fixed costs, this is an outcome that can't be reached without some kind of a governmental transfer to the firm. We need to consider the second-best outcome where the welfare is maximized with the constraint that the firm breaks even.

Proposition 2 *More the consumers' preferences are concentrated around the mode, the more the welfare is important.*

Still from Table A-1, we observe that the price for the second-best case is significantly lower than the price set by the monopolist and the location is close but slightly to the right. The price increases with the degree of freedom since a higher price is required to break even. Proposition 2 is mainly explained by the fact that the firm can reach a bigger proportion of the population when the preferences are concentrated around the mode. As with proposition 1, proposition 2 is also robust as

[6]In the simulations, the value of the welfare is calculated through an estimation. Therefore, the price shown in the tables is a little off for the first-best. On the other hand, the price for the second-best case is valid

long as Δv, Δmc, and Δf are nul.

Product Line of Two Brands

When the monopolist can use two brands to serve the market, it will prefer to do so if the fixed costs per brand are low enough.

Proposition 3 *The profit the monopolist earn when using two brands is smaller than twice the profit it earns when using only one brand*

This is because we use a single peaked distribution of the consumers' preferences. When using only one brand, the monopolist selects the location and the price that maximize its profit. It is therefore impossible that the profit using two brands can be higher than twice the profit using one brand. Proposition 3 is robust for all cases where the two brands have an identical cost structure. Under the use of a product line of two brands, we still observe that the profits are higher for smaller degree of freedom. The two brands are located on each side of the location used when there is only one brand. In a typical model of product differentiation where the consumers are uniformly distributed, the sum of the profit of two brands is maximized when the products are located at each end of the product space. Without the assumption of sufficient willingness to pay to cover the market and with a non-uniform distribution of the consumers' preferences, the monopolist locates the two products to cover the area around the mode.

Proposition 4 *When the monopolist owns two brands in the market, it is possible to observe some cannibalization between those two.*

The presence of cannibalization is explained by the fact that the choices of price and of locations are discrete. It is possible that a consumer would have a positive utility form the consumption of the two brands. This is because the movement of one brand away from the other would create a gap between the reach of the two brands where the demand is more important than the new demand captured by the movement. The presence of cannibalization isn't robust; it is simply that it sometimes happen. As an exemple, in Table A-1 with $\Delta v = 0$, $\Delta mc = 0$, and $df = 6$, brand 1 reaches up to 4.5247 ($\sqrt{6-4.5}+3.3$) on its right while brand 2 reaches up to 4.5168 ($5.7-\sqrt{6-4.6}$) on its left.

Cannibalization has been discussed by Moorthy (1984) who proposed a theory of market segmentation where consumers use self-selection instead of using a model where consumers can be isolated by the firms. The self-selection assumption creates competition among a firm's own product line. This phenomenon is referred to as cannibalization and comes into play in his model since the monopolist can't determine the optimal location and its price for each segment separately. In the present model, cannibalization is marginal and is present only because consumers locate continuously on the linear city while the firm must make discrete choices.

The distance between the two brands gets larger for a bigger degree of freedom. When $\Delta v > 0$, the two brands are moved to the right and when $\Delta mc > 0$, the two brands are moved to the left. Even though it is not consistent, the prices of the brands tend to be higher than the price of the brand when the monopolist only has one brand. The prices for the two brands are frequently the same, but this isn't always the case.

If the pioneer would use two brands as a strategic behaviour when there is the possibility of an entrant, the action could be considered as brand proliferation. It is the action of using multiple brands to eliminate locations that could support the entry of a competitor.

4.2 Duopoly

When there is a possible entrant in the second period, we consider the three scenarios described earlier. The entrant tends to enter since we did set the fixed costs low enough such that there is place for two or more firms in the market. With no fixed costs, we should actually expect that there would be firms entering in all the 100 possible locations. Here, we only consider that there is one possible entrant. When the pioneer is in the monopolist location, it is where the demand is and need to lower its price a lot so that the entrant prefers developing a product in a niche than cutting the pioneer's price.

Proposition 5 *When the pioneer and the entrant are identical and the pioneer is in the monopolist location, the pioneer enjoys bigger profit than the entrant.*
($\Pi p2 > \Pi E$)

This can be observed in all the tables in the appendix where the pioneer and the entrant are indentical and can therefore be considered as robust. The entrant enters on the right side of the pioneer and sets its price higher than the pioneer. The entrant has a really smaller demand than the pioneer but it prefers locating in a niche.

If the pioneer is forward looking it will choose the best location to maximize its discounted profit of the two periods. When the demand is really concentrated in an area, as with degree of freedom 2, the pioneer tends to choose the monopolist location to extract the maximum profit it can in first period and defend its place with a low price in second period. When the demand is more spread in the product space, the pioneer can gain from choosing to locate on one side of the mode. This reduces its profit in period 1 but will considerably increase its profit in the second period such that its discounted profit of the two periods is higher. When the demand is really spread, as with degree of freedom 8, we can even observe that the firm isn't even affected by the entrant; the pioneer has the same price and enjoys the same profits in the two periods. If the pioneer is free to change location between the two periods, it selects the monopolist location in the first period and chooses to move on one side of the mode for period 2 to minimize the impact of price competition on its profit.

Proposition 6 *The profit in period 2 of the pioneer in the free location scenario is always bigger or equal than the one in the best location scenario, and the one in the best location scenario is always bigger or equal than the one in the monopolist location* *scenario.*

(Monopolist location $\Pi p2 \leq$ *Best location* $\Pi p2 \leq$ *Free location* $\Pi p2$*)*

Proposition 7 *The discounted profit of the pioneer in the free location scenario is always bigger or equal than the one in the best location scenario, and the one in the best location scenario is always bigger or equal than the one in the monopolist location scenario.*

(Monopolist location $D\Pi \leq$ *Best location* $D\Pi \leq$ *Free location* $D\Pi$*)*

Propositions 6 and 7 are consistent in all simulations and their explanations belongs to simple logic. Starting from the monopolist location scenario where by definition, the profit is maximized in period 1, the pioneer will choose a different location in the best location scenario if the discounted profit is higher and that can only be the case if the profit in period 2 is higher since any change in the location will reduce the profit in period 1. In the free location scenario, the pioneer maximizes its profit in both periods and therefore achieves the maximum discounted profit.

Proposition 8 *When the pioneer choose a different location than the monopolist location, the entrant can enjoy a bigger profit than the pioneer.*

($\Pi p2 \leq \Pi E$*)*

Proposition 8 refers to what we can define as the entrant's advantage. Because of the setup of the model, where the entrant benefits of selecting its price and location after the pioneer, the pioneer is forced to select a location for which the entrant is better of locating elsewhere than just cutting the price of the pioneer. The entrant gains in locating away from the mode to reduce price competition but since the entrant selects its price and location after the pioneer, the entrant benefits of an advantage. This is a robust result. It can be observed in the simulations when the two firms have an identical cost structure. We also observe that the pioneer only has a bigger profit than the entrant in period 2 in the best location scenario if it chooses the monopolist location.

Mergers and Patents

Even though the entrant has an advantage and can benefits from higher profits in period 2, the pioneer's profit in period 1 in the free location scenario are always bigger than the entrant's profit in period 2 and the pioneer's profit in period 1 in the best location scenario tends to be bigger than the entrant's profit in period 2.

Proposition 9 *In the free location scenario, if we assume that when the pioneer buys the entrant, it can choose its location, the pioneer will always do so and locate the two brands in the second period just like in the product line of two brands.*

We covered earlier the two products case and establish that the monopolist choose the two combinations of price and location to maximize the total profit. Therefore the profit of the entrant in period 2, which is the price to pay to buy the entrant, is smaller or equal to the difference between the profit a monopolist achieve with two brands and the profit of the pioneer in period 2. From equation 13 and the previous explanation, we can conclude that proposition 9 is robust.

$$\Pi E \leq \Pi 2brands - \Pi p2 \tag{13}$$

If the pioneer could use a patent to stop the entry of a competitor in the second period, the maximum price it would be willing to pay is the difference between the monopolist profit and the profit in period 2 when there is the possibility of an entrant.

Movement Cost

Imagine there is a fixed cost that the pioneer faces if it wants to change location. As an example, this cost could be the cost of a necessary marketing campaign to change the brand image or the costs of changing the machinery in the factory. The firm would change location only if the cost is smaller than the difference between the free location discounted profit and the best location discounted profit.

4.3 No Transportation Cost, t=0

When there is no transportation cost[7], the monopolist sells to all the population by setting the price just under V. Since everyone is consuming the good, the price has no impact on the welfare since it is just a transfer between the firm and the consumers. When $\Delta v > 0$, the monopolist will prefer to locate to the extreme right where it can charge a higher price. It sees no difference between locating at the extreme right or really close, as long as the willingness to pay is higher than the maximum discrete price it can charge. To maximize the welfare, the monopolist needs to locate to the extreme right, at $\bar{L}$, since the consumer surplus will be higher. When $\Delta mc > 0$, than the monopolist wants to locate to the extreme left. When both $\Delta v = 0.1$ and $\Delta mc = 0.1$, the firm will choose to locate where the willingness to pay is just higher to the maximum price it can charge. Since the monopolist can reach all the consumers with one brand, there is no need to use a second brand in the product space.

When there is an entrant in the second period, the firms engage Bertrand price competition and profits are reduced to zero. In this case the pioneer advantage is of a real importance; all the profits are made by the pioneer before the entry of the competitor.

4.4 Impact of the Different Parameters

Here, we analyze the impact of a change in some parameters. We have a look at the isolated impact of the change in the parameter on the basic case and therefore consider the three scenarios and the four different degrees of freedom to evaluate the degree of robustness of the propositions.

Parameter t

In Table A-3, we observe the basic case with a value of t set to 0.5, 1, and 2. As the value of t increases, the profit of a monopolist decreases and it locates closer to

[7]see Table A-2 for the results with no transportation cost

the mode. The monopolist reaches a smaller proportion of the population and the welfare is also decrease. With two brands, there is a bigger impact on the profit when t is big since the first brand couldn't capture well the region where the consumers' preferences are concentrated. The distance between the two brands of the monopolist decreases as t increases.

When there is the entry of a competitor, a higher t also decreases the distance between the two brands. The impact on the profits of the pioneer and the entrant depends on the distribution of the consumers' preferences. When the demand is highly concentrated, with a low degree of freedom, a bigger t reduces the impact of price competition and the pioneer enjoys bigger profit in period 2 as t increases. When the demand is spread, with a high degree of freedom, the profit in period 2 decreases since there was place for two firms and that with a high t, they both lose a lot of demand.

Proposition 10 *It is preferable for the pioneer firm to face low transportation cost.*

This is because even if for a concentrated demand, the pioneer can benefit from higher profit in period 2 with a higher t, the gain won't cover the lost in period 1. In all the three scenarios, the discounted profit of the pioneer is decreasing in t. This is the case for all the different df and is therefore a robust result. Concerning the entrant, it depends on the scenario and the distribution of the consumers' preferences. In the monopolist location and best location scenarios, the entrant is better off with a high t when the demand is concentrated and with a low t when the demand is spread. In the free location scenario, the entrant is always better off with a low t.

Parameter α

In Table A-4, we observe the basic case with a value of α set to -1, 0, and 1. When $\alpha = -1$, there are increasing returns to scale, when $\alpha = 1$, there are decreasing returns to scale, and when $\alpha = 0$, the returns to scale are said to be constant. In the monopoly case, the firm makes more profit with $\alpha = -1$ since the marginal cost is decreased.

The impact is more important for a low degree of freedom since the firm reaches a higher proportion of the population. Also under increasing returns, the firm sets its price lower to increase its quantity and benefit more from the returns. With $\alpha= 1$, the monopolist has lower profit and will charge a higher price. The use of two brands is more interesting for the monopolist when α is bigger since it prefers having two brands both with small quantity produced. When $\alpha= -1$, the monopolist using two brands is still better off than with only one brand, but it faces higher marginal cost.

When there is the entry of a competitor, the pioneer enjoys bigger profits in period 2 with decreasing returns to scale when the demand is concentrated around the mode and with increasing returns to scale for a more spread demand. The entrant's profit varies the same way when the pioneer is in the monopolist location and in the best location, but when the pioneer is selecting a free location in period two, the entrant always prefers increasing returns to scale.

Proposition 11 *It is preferable for the pioneer firm to face increasing returns to scale.*

This is because even for a concentrated demand, the discounted profit of the pioneer for the two periods is always bigger with increasing returns to scale. The gains obtain in the first period from increasing return to scales are more important. In Table A-4, for the three scenarios and all the df, we can observe that proposition 11 is robust.

Parameter dr

In Table A-5, we observe the basic case with a value of dr set to 1, 1.1, and 2, where a value of 1 means a discount rate of 0%, a value of 1.1 means a discount rate of 10% and a value of 2 means a discount rate of 100%. The discount rate has no impact on the results for the monopolist, since it occurs in the first period and therefore there is nothing to discount.

For the duopoly, the parameter dr has no impact on the behaviour of the pioneer and of the entrant in the monopolist location and the free location scenarios, they will behave the same way in period 2. The discount rate will only change the discounted profits. When the pioneer chooses a location for the best location scenario, as the discount rate is higher, the pioneer will put more weight on the first period and will tend to locate closer to the monopolist location. This is the case for any type of distribution of consumers' preferences. As the pioneer locates closer to the monopolist location, it makes price competition more important and the entrant is worse off.

Proposition 12 *It is preferable for the pioneer firm to face a low discount rate.*

This is simply because the profit earned in the second period will be worth more to the pioneer in the first period. The discounted profits are always more important with a small discount rate if the profits in period 2 are positive. If the pioneer would be forced to have a loss in the second period, it would prefer a high discount rate, but simply by assuming free exit, the pioneer would leave the market and therefore a negative profit is impossible. Without the possibility of a negative profit in period 2, proposition 12 is robust and we should expect it to be robust to any other change in the basic case.

Parameter βf vs βf2

In Table A-6, we observe the basic case with different values of βf and βf2. We approach the case where one of the firms has lower fixed costs than the other. We set one firm with a low fixed cost of 0.25 and the other one with 0.75. Higher fixed costs don't change the behaviour of the monopolist. When βf is 0.75, the monopolist lose 0.5 in profits and the welfare is lower of 0.5. When approaching the monopolist with a product line of two brands, we consider βf as the fixed cost for the first brand and βf2 as the one for the second brand. With those values, it is still preferable for the monopolist to carry two brands. This is especially the case when the second brand has a lower fixed cost than the first one. As both βf and βf2 are fixed costs, the monopolist doesn't behave differently when carrying two brands depending on which one has which fixed cost.

For the duopoly, the pioneer's profits are obviously lower in period 2 when its fixed costs are more important. With the pioneer trapped in the monopolist location when the entrant faces lower fixed cost than the pioneer, it is possible that the pioneer is forced to leave the market. When the entrant has a higher fixed cost than the pioneer, the pioneer will have similar profits than when they both have the same fixed costs unless the entrant prefers not to enter the market. When the entrant enters the market, whatever are its fixed costs, it will behave the same way. If βf2 would be higher than 0.75, we would observe more simulations where the entrant prefers not to enter.

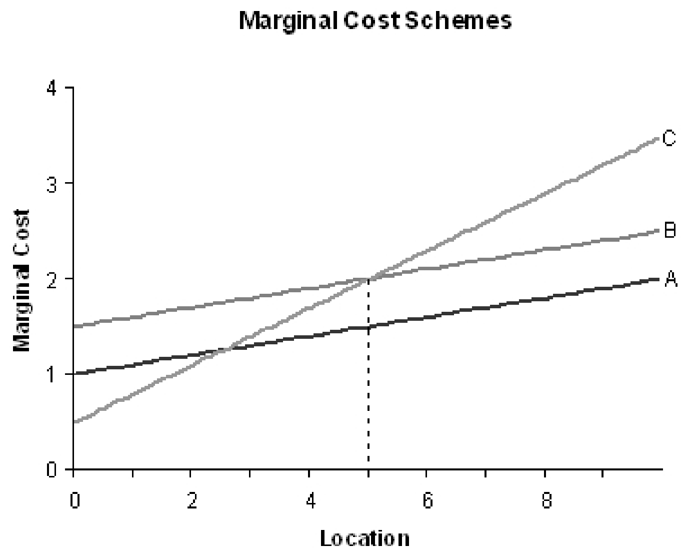
Proposition 13 *It is preferable for the pioneer firm to have lower fixed costs than the entrant and the pioneer has incentives to make the fixed costs of the entrant bigger.*

The discounted profit of the pioneer for the two periods can only be bigger when the fixed costs of the entrant are increased. This robust result is observed in Table A-6 and is expected to be robust to any other change in the basic case. If the pioneer could make the fixed costs of a possible entrant bigger, it could stop an entrant from entering the market and be better off.

Parameter Δmc vs Δmc2

In Table A-7, we observe the basic case with different values of Δmc and Δmc2. The different schemes of marginal cost used are represented in the Figure 11 where scheme A has βmc = 1 and Δmc = 0.1, scheme B has βmc = 1.5 and Δmc = 0.1, and scheme C has βmc = 0.5 and Δmc = 0.3. We first consider an industry where all products face scheme A. Then we consider an industry where the first product, the one of the monopolist and therefore the pioneer in period 2, follows scheme B and the second product, the second brand of a monopolist or the brand of the entrant, follows scheme C. Finally, we consider an industry where the first product follows scheme C and the second one follows scheme B. The schemes B and C are such that a firm with scheme C has a lower marginal cost in the first half of the product space than a firm with scheme B and a higher one in the second half.

Figure 11: Different Schemes of Marginal Cost Used for Simulations



For the monopolist, the three schemes have $\Delta mc > 0$ and therefore they all give an incentive to the monopolist to locate a little more to the left than they would if $\Delta mc = 0$. The monopolist is just worst off with B compared to A since for any location, its marginal cost will be higher. When comparing A with C and B with C, the monopolist is better off with the scheme where the marginal cost is lower where the mode of the distribution of the consumers' preferences is located. For degree of freedom 2 and 4, the modes are at 0 and 2, and the monopolist prefers C over A and C over B for both cases. For degree of freedom 6, the mode is at 4 and the monopolist still prefers C over B but now prefers and A over C. Finally, for degree of freedom 8, the mode is at 6 and the monopolist prefers A over C and B over C. The welfare is also maximized with the scheme where the marginal cost is the lowest at the mode. When carrying two brands in the market, the monopolist prefers having one of its brand with B and the other with C than the two with A when the demand is really concentrated (degree of freedom 2), but prefers to have both with A when the demand is more spread and more to the right. This is because with degree of freedom

2, the monopolist will sell more of the good with scheme C. For all the distributions of consumers' preferences, the monopolist with B and C will always locate the brand under B on the right of the brand under C.

For the duopoly, when both firms are under A, things are really similar to the basic case where $\Delta mc = 0$, since both firms are identical. When the pioneer is under B and the entrant is under C, the entrant has an advantage over the pioneer when the mode is located in the first half. The entrant will make more profit than the pioneer when the pioneer is in the monopolist location and can even force the pioneer to exit. In the best location scenario, the pioneer will locate to the right of the mode even if marginal cost are higher there since the entrant has an advantage on the left side. In the free location scenario, the pioneer still locate on the right of the mode while the entrant takes the left. When the degree of freedom is 8, the pioneer enjoys bigger profit than the entrant in period 2 in all three scenarios. This is because it can be located close to the mode where it has a lower marginal cost and force the entrant to locate away.

When the pioneer is under C and the entrant is under B, the pioneer has an advantage over the entrant when the mode is located in the first half. If this is the case, the pioneer will enjoy higher profits than the entrant in the monopolist location since it can charge a higher price for which the entrant can't go below. In the best location and free location scenarios, the pioneer will now enjoys bigger profit than the entrant by always locating on the left side of the mode while the entrant locates on the right side of the mode. We should expect that for a higher degree of freedom where the mode would be even more to the right, that the pioneer would be worst off since locating on the left of the mode would now mean locating on the second half of the product space.

Proposition 14 *It is preferable for the pioneer firm to have a lower marginal cost than the entrant at the mode.*

The major impact on the discounted profit comes from the first period where the profits are more important. When the firms are not identical, the pioneer is better off when it is under C and the entrant is under B if the demand is more concentrated in the first half and under B while the entrant is under C when the demand is concentrated in the second half. In the three scenarios, the pioneer makes more profit in the first period when it has the advantage of marginal cost where the mode is. In the second period, when the pioneer chooses the best location or can change location before period 2, it is still the case. Therefore, we observe that a forward looking pioneer will enjoy higher discounted profits when it has a lower marginal cost than the entrant at the mode. Even if it is generally observed in the simulations, proposition 14 is not robust to when the pioneer is in the monopolist location and that the mode is just slightly to the right of the midle of the product space.

Parameter βmc

In Table A-8, we observe the basic case with a value of βmc and βmc2 set to 0, 1, and 2. In the monopoly case, as the marginal cost are higher, the profit will decrease while the monopolist charges a higher price, locates closer to the mode, and reaches a smaller proportion of the population. The welfare is also decreased by an increase of the marginal cost. The monopolist is still better off to carry two brands in the market. As the marginal cost increases, the monopolist increases the price of both brands and the distance between them is smaller. In the duopoly case, the increase in marginal cost for the two firms does not change the behaviour of the entrant and of the pioneer. In the monopolist location scenario, the pioneer reduces its price and the entrant locates on the right side. In the best location scenario, the pioneer will locate more far away from the mode when the marginal costs are low and the entrant still enters on the other side. In the free location scenario, the pioneer still locates on one side while the entrant takes the other side.

Proposition 15 *It is preferable for the pioneer firm that both firms have lower marginal costs.*

Since the behaviour of the two agents doesn't change in the second period and that in both periods, the profit of the pioneer is decreasing in βmc, the discounted profit of the pioneer is also decreasing in βmc. This robust result is also expected to be robust to any other change in the basic case.

5 Possible Extensions

Many extensions of the previously discussed model could bring interesting conclusions. Introducing new parameters, changing the setup of the model, and changing the timing of the model could contribute to the comprehension of product differentiation. One interesting change of the setup of the model would be to investigate vertical product differentiation. This could be done in a similar model. One could consider the consumers location on the linear city as the minimum level of desired quality. This would mean a consumer would face transportation cost for a lower quality product than its location but no transport cost on its right.

One could propose a similar model but with a consumer probability distribution function that isn't single peaked. This would change the nature of things in many ways. It would also be interesting to consider the possibility that the pioneer uses brand proliferation just before period two to see the impact on the entry of a competitor.

Finally, the actual model could bring other interesting conclusions just by simulating different combinations of parameters. It would give the possibility to evaluate the robustness of the propositions more rigorously. This simulations could include different $\bar{L}$ such that a $\bar{L}$ is on the left of the desired location of a firm. It would also be interesting to observe the combined effect of two changes of parameters to observe if one dominates the other.

6 Conclusion

We first covered the literature review of product differentiation and in addition, a model with many parameters was presented for which, via simulations, we described the behaviour of a pioneer firm that evolves in an industry where products can be horizontally differentiated. In a two period game, we observed that there is such a thing as the pioneer advantage. By first entering the market, a pioneer benefits of really important profits before there is the possibility of the entry of a competitor. When an entrant observes the price and location of the pioneer in period 2 and then enters, its profit can be higher than the pioneer's profit in period 2. Therefore, there is such a thing as the entrant advantage.

When the pioneer locates where the demand is, it has to reduce its price up to a point where the entrant will choose to locate in a niche market. If the pioneer can change location or first locate elsewhere in the product space, it will move away from the mode of the single-peaked distribution of consumers' preferences to make price competition softer but has to leave an interesting location to the entrant. When the pioneer moves away from the mode to increase its profit, the profit of the entrant also increase and this is when we observe the entrant advantage. This is an interesting result in the sense that as presented in the literature review, the concept of brand specification says that a firm can, by selecting a specific location, enjoy bigger profits by deterring the entry to the market while we observed that the pioneer can enjoy higher profits by leaving a better location to the entrant.

We observed that if the pioneer firm isn't forward looking and makes the mistake of locating in the monopolist location when it can't change location, it will have a smaller or equal discounted profit than if it would locate in the best location. The discounted profit can be considerably smaller. We also observed that the pioneer could use its profit of the first period to buy the entrant and maximize its profit. From the analysis of the parameters, we observed that the discounted profits enjoyed by the pioneer are highly influenced by the distribution of the consumers' preferences.

We observed no maximum differentiation. The generally observed outcome of the game was generally what Economides (1987) defines as a touching equilibrium. We observed minimal product differentiation in the sense that it is possible that the entrant chooses to imitate the pioneer. Product imitation happens in the monopolist location scenario when the entrant has a lower fixed cost than the pioneer.

We have observed many similarities with the theory covered in the literature review but some differences. The differences such as the fact that the pioneer prefers low transportation cost while the theory says firms profits are increasing in transportation costs can be explained by the setup of the model.

The assumption in this paper that creates the most striking difference with reality is the one setting the dimension of the product space to one. Beath and Katsoulacos (1991) insist on the fact that it "might be argued that there is a wide range of products for which a scalar representation of quality is unsuitable. More precisely, that products are differentiated by a multiplicity of characteristics and that any good can only be represented by a vector w of these". Introducing a second dimension would make the model closer to reality, but even more would be needed to make it realistic. We hope that the model presented here and the results of the simulations in addition of other studies cited in this paper offer a good explanation of product differentiation.

References

Anderson, S. P., A. Palma, and J.-F. Thisse (1992) *Discrete Choice Theory of Product Differentiation* (The MIT Press)

—— (1997) 'Privatization and efficiency in a differentiated industry.' *European Economic Review* 41, 1635–1654

Beath, J., and Y. Katsoulacos (1991) *The Economic Theory of Product Differentiation* (Cambridge University Press)

Carpenter, G. S., and K. Nakamoto (1989) 'Consumer preference formation and pioneering advantage.' *Journal of Marketing Research* 26(3), 285–298

Chamberlin, E. H. (1962) *The Theory of Monopolistic Competition,* eight edition ed. (Harvard University Press)

Church, J., and R. Ware (2000) *Industrial Organization: A Strategic Approach* (McGraw Hill)

DAspremont, C., J. J. Gabszewicz, and J.-F. Thisse (1979) 'On hotellings "stability in competition".' *Econometrica* 47(5), 1145–1150

Dickson, P. R., and J. L. Ginter (1987) 'Market segmentation, product differentiation, and marketing strategy.' *Journal of Marketing* 51(2), 1–10

Dixit, A. K., and J. E. Stiglitz (1977) 'Monopolistic competition and optimum product diversity.' *The American Economic Review* 67(3), 297–308

Eaton, B. C., and R. G. Lipsey (1975) 'The principle of minimum differentiation reconsidered: Some new developments in the theory of spatial competition.' *The Review of Economic Studies* 42(1), 27–49

Economides, N. (1984) 'The principle of minimum differentiation revisited.' *European Economic Review* 24, 345–368

—— (1987) 'On nash equilibrium existence and optimality in oligopolistic competition in prices and varieties.' *Greek Economic Review* 9(2), 198–209

—— (1989) 'Symmetric equilibrium existence and optimality in differentiated product markets.' *Journal of Economic Theory* 47, 178–194

Hotelling, H. (1929) 'Stability in competition.' *The Economic Journal* 39(153), 41–57

Kotler, P., G. Armstrong, and P. H. Cunningham (1999) *Principles of Marketing,* fourth canadian edition ed. (Prentice Hall)

Lancaster, K. J. (1966) 'A new approach to consumer theory.' *The Journal of Political Economy* 74(2), 132–157

—— (1975) 'Socially optimal product differentiation.' *The American Economic Review* 65(4), 567–585

Lerner, A. P., and H. W. Singer (1937) 'Some notes on duopoly and spatial competition.' *The Journal of Political Economy* 45(2), 145–186

Moorthy, K. S. (1984) 'Market segmentation, self-selection, and product line design.' *Marketing Science* 3(4), 288–307

Novshek, W. (1980) 'Equilibrium in simple spatial (or differentiated product) models.' *Journal of Economic Theory* 22, 313–326

Perloff, J. M., and S. C. Salop (1985) 'Equilibrium with product differentiation.' *The Review of Economic Studies* 52(1), 107–120

Root, H. P. (1972) 'Should product differentiation be restricted?' *Journal of Marketing* 36(3), 3–9.

Salop, S. C. (1979) 'Monopolistic competition with outside goods.' *The Bell Journal of Economics* 10(1), 141–156

Schmalensee, R. (1982) 'Product differentiation advantages of pioneering brands.' *The American Economic Review* 72(3), 349–365

Schmitt, N. (1995) 'Product differentiation and international trade.' *International Economic Review* 36(3), 583–608

Shaked, A. (1982) 'Existence and computation of mixed strategy nash equilibrium for 3-firms location problem.' *The Journal of Industrial Economics* 31(), 93–96. Symposium on Spatial Competition and the Theory of Differentiated Markets

Shaked, A., and J. Sutton (1982) 'Product differentiation and industrial structure.' *The Journal of Industrial Economics* 36(2), 131–146

—— (1987) 'Relaxing price competition through product differentiation.' *The Review of Economic Studies* 349(1), 3–13

Shapiro, S.J., K. Wong W.D. Perreault, and E. J. McCarthy (2002) *Basic Marketing: A Global-Managerial Approach,* tenth canadian edition ed. (McGraw-Hill Ryerson)

Shy, O. (1995) *Industrial Organization: Theory and Applications* (The MIT Press)

Smith, W. R. (1956) 'Product differentiation and market segmentation as alternative marketing strategies.' *Journal of Marketing* 21(1), 3–8

Spence, M. (1976a) 'Product differentiation and welfare.' *The American Economic Review* 66(2), 407–414. Papers and Proceedings of the Eighty-eight Annual Meeting of the American Economic Association

—— (1976b) 'Product selection, fixed costs, and monopolistic competition.' *The Review of Economic Studies* 43(2), 217–235

Tirole, J. (1988) *The Theory of Industrial Organization* (The MIT Press)

Wauthy, X. (1996) 'Quality choice in models of vertical differentiation.' *The Journal of Industrial Economics* 44(3), 345–353

CDF of the χ^2 distribution

$$CDF(2,L) = 1 - e^{-\frac{L}{2}}$$

$$CDF(4,L) = 1 - e^{-\frac{L}{2}} - \left(\frac{L}{2} \cdot e^{-\frac{L}{2}}\right)$$

$$CDF(6,L) = \frac{2 \cdot \left[1 - e^{-\frac{L}{2}} - (\frac{L}{2} \cdot e^{-\frac{L}{2}})\right] - \left[(\frac{L}{2})^2 \cdot e^{-\frac{L}{2}}\right]}{2}$$

$$CDF(8,L) = \frac{3 \cdot \left(2 \cdot \left[1 - e^{-\frac{L}{2}} - (\frac{L}{2} \cdot e^{-\frac{L}{2}})\right] - \left[(\frac{L}{2})^2 \cdot e^{-\frac{L}{2}}\right]\right) - \left((\frac{L}{2})^3 \cdot e^{-\frac{L}{2}}\right)}{6}$$

Figure 12

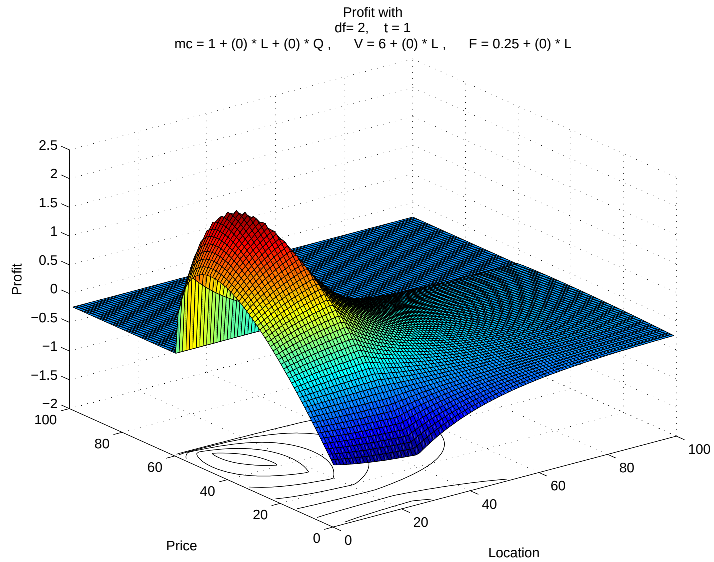


Figure 13

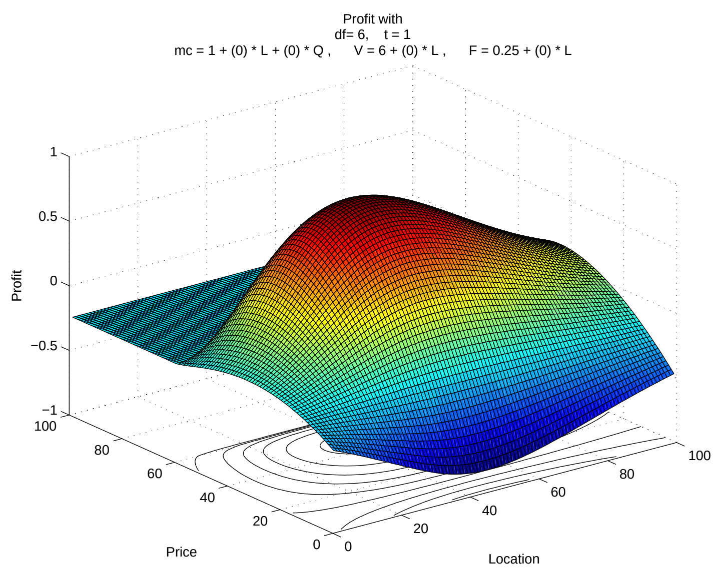


Figure 14

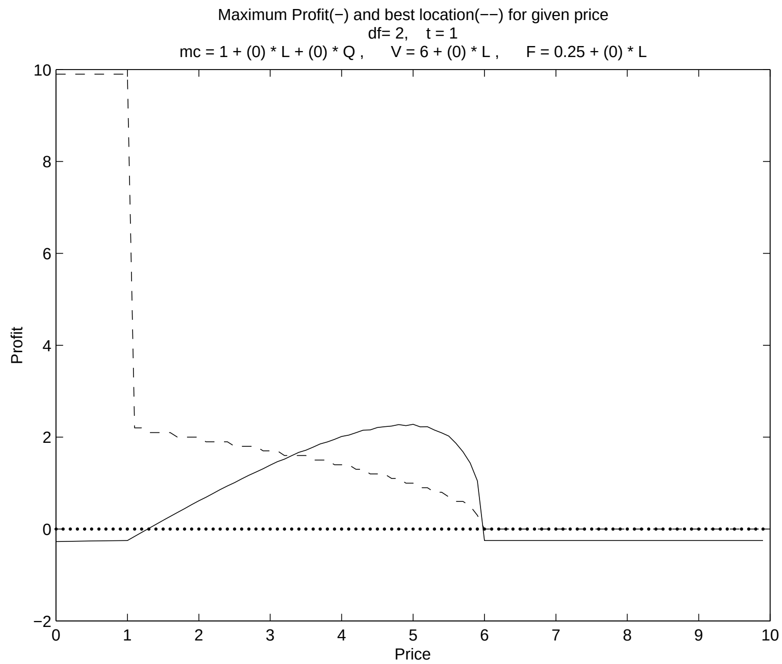


Figure 15

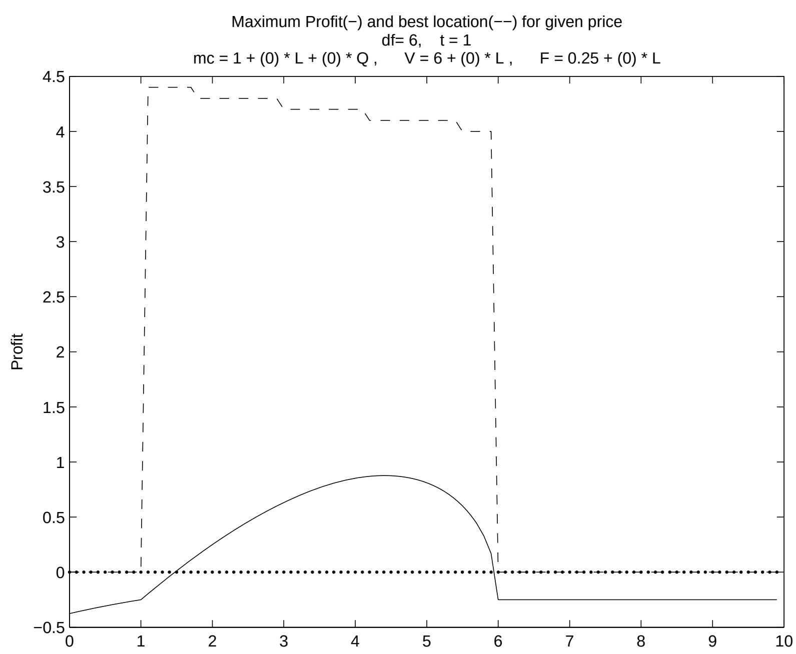


Figure 16

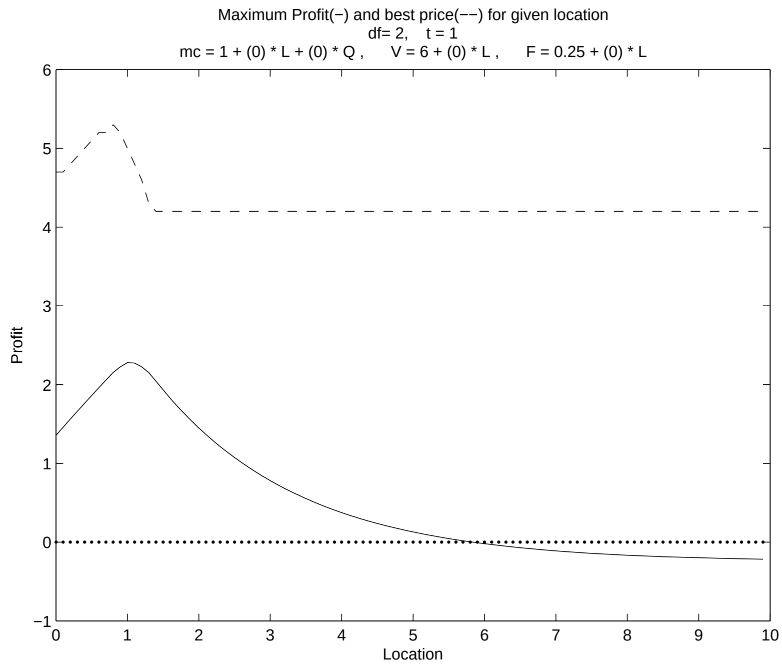


Figure 17

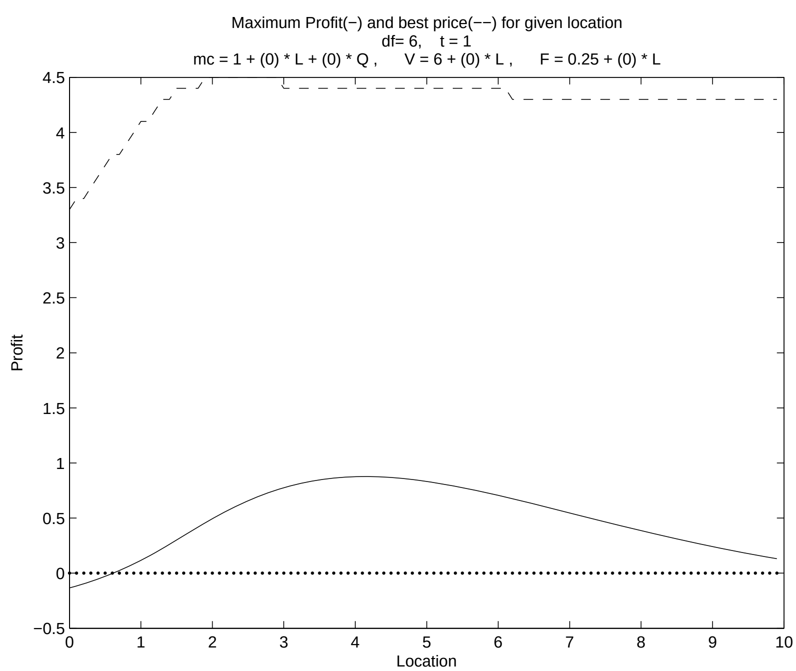


Table A-1 (t=1, Δv and Δmc)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
1	X	X	Δmc	1.0	1.0	0.25	0.25	0.0	0.0	6.0	0.0	1.1	X

			Monopoly																			
			Monopoly							First Best			Second Best			2 Products				
Δv	Δmc	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
0.0	0.0	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
0.0	0.0	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
0.0	0.0	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
0.0	0.0	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6
0.1	0.0	2	2.35	4.9	1.1	0.0	2.2	0.67	2.90	3.41	0.0	1.2	3.37	1.4	1.2	3.15	5.1	0.8	5.0	2.9
0.1	0.0	4	1.33	4.6	2.4	1.1	3.7	0.44	1.85	2.46	0.0	2.5	2.44	1.4	2.6	2.25	4.9	1.7	4.8	4.1
0.1	0.0	6	1.02	4.7	4.4	3.1	5.7	0.34	1.45	1.96	0.5	4.4	1.95	1.5	4.5	1.83	4.9	3.4	4.9	5.9
0.1	0.0	8	0.87	4.8	6.4	5.0	7.8	0.30	1.26	1.72	0.7	6.5	1.71	1.6	6.5	1.61	4.9	5.4	5.1	8.0
0.0	0.1	2	2.22	5.0	1.0	0.0	2.0	0.63	2.65	3.22	0.0	1.1	3.18	1.5	1.1	2.91	5.0	0.7	5.0	2.7
0.0	0.1	4	1.14	4.5	2.1	0.9	3.3	0.42	1.60	2.12	0.1	2.3	2.10	1.7	2.3	1.88	4.8	1.6	4.8	3.8
0.0	0.1	6	0.75	4.5	3.9	2.7	5.1	0.32	1.09	1.50	1.0	3.9	1.48	1.9	3.9	1.32	4.7	3.1	4.7	5.4
0.0	0.1	8	0.53	4.6	5.7	4.5	6.9	0.26	0.79	1.14	1.4	5.7	1.12	2.2	5.7	0.97	4.7	4.8	4.7	7.1
0.1	0.1	2	2.28	5.1	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.5	1.2	3.02	5.1	0.8	5.2	2.8
0.1	0.1	4	1.23	4.7	2.2	1.0	3.4	0.43	1.70	2.28	0.3	2.4	2.26	1.7	2.4	2.06	4.9	1.6	5.0	3.9
0.1	0.1	6	0.88	4.8	4.1	2.8	5.4	0.33	1.26	1.72	1.0	4.2	1.70	1.9	4.2	1.56	4.9	3.2	5.1	5.6
0.1	0.1	8	0.69	5.0	6.1	4.8	7.4	0.28	1.01	1.41	1.2	6.1	1.39	2.2	6.1	1.27	5.1	5.2	5.3	7.6

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.38	1.8	2.62	0.30	4.9	4.1	2.28	0.38	2.62	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.17	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.76	0.56	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.14	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.36	0.52	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.58	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.12	0.46	4.7	9.0	0.63	0.63	1.21	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.27	4.4	4.9	0.63	4.5	7.4
0.38	1.8	2.70	0.32	5.2	4.2	2.35	0.38	2.70	4.9	1.8	1.1	0.32	5.2	4.2	0.97	3.23	4.7	2.7	2.09	5.4	0.8
0.67	2.5	1.94	0.62	5.2	5.5	1.11	1.09	2.10	4.6	4.8	3.8	1.13	5.0	1.5	1.09	2.33	4.8	3.8	1.13	5.0	1.5
0.61	2.8	1.58	0.61	5.1	7.6	0.91	0.90	1.73	4.7	4.5	3.2	0.90	5.0	5.8	0.90	1.84	4.5	3.2	0.90	5.0	5.8
0.62	3.2	1.44	0.60	5.4	9.5	0.80	0.79	1.52	4.8	4.5	5.2	0.79	4.9	8.0	0.80	1.60	4.8	7.9	0.80	4.9	5.2
0.30	1.8	2.48	0.24	5.0	4.0	2.22	0.30	2.48	5.0	1.8	1.0	0.24	5.0	4.0	0.92	3.06	4.6	2.5	1.92	5.5	0.7
0.50	2.5	1.60	0.48	4.9	5.0	0.91	0.90	1.73	4.5	4.5	3.5	0.94	5.0	1.3	0.90	1.96	4.5	3.5	0.94	5.0	1.3
0.42	2.9	1.13	0.41	4.9	6.7	0.65	0.65	1.24	4.5	4.3	2.8	0.64	4.8	5.2	0.65	1.34	4.4	5.2	0.65	4.7	2.8
0.36	3.3	0.86	0.34	4.9	8.4	0.49	0.48	0.92	4.5	4.3	4.6	0.47	4.8	7.0	0.48	0.97	4.7	7.0	0.49	4.7	4.7
0.38	1.9	2.62	0.30	5.3	4.1	2.28	0.38	2.62	5.1	1.9	1.0	0.30	5.3	4.1	0.91	3.11	5.1	2.5	2.04	5.4	0.8
0.57	2.6	1.76	0.56	5.3	5.2	1.02	0.97	1.90	4.7	5.2	3.6	1.08	5.1	1.5	0.99	2.13	4.7	3.7	1.03	5.1	1.4
0.53	3.1	1.36	0.52	5.3	7.1	0.78	0.78	1.48	4.9	5.0	5.5	0.78	5.0	3.1	0.78	1.58	5.0	5.5	0.78	5.0	3.1
0.47	3.5	1.12	0.46	5.4	9.1	0.63	0.63	1.21	4.9	4.9	4.9	0.64	5.2	7.4	0.63	1.27	4.9	4.9	0.64	5.2	7.4

Table A-2 (t=0, Δv and Δmc)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
0	X	X	Δmc	1.0	1.0	0.25	0.25	0.0	0.0	6.0	0.0	1.1	X

			Monopoly																	
			Monopoly							First Best			Second Best			2 Products				
Δv	Δmc	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
0.0	0.0	2	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.0	0.0	4	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.0	0.0	6	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.0	0.0	8	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.1	0.0	2	5.65	6.9	9.5	0.0	80.2	1.00	5.70	5.74	0.0	9.9	5.74	1.3	9.9	5.40	5.9	0.0	6.9	9.9
0.1	0.0	4	5.65	6.9	9.6	0.0	87.1	1.00	5.71	5.74	0.0	9.9	5.74	1.3	9.9	5.40	5.9	0.0	6.9	9.9
0.1	0.0	6	5.65	6.9	9.7	0.0	93.4	1.00	5.72	5.74	0.0	9.9	5.74	1.3	9.9	5.40	5.9	0.0	6.9	9.9
0.1	0.0	8	5.65	6.9	9.8	0.0	99.2	1.00	5.73	5.74	0.0	9.9	5.74	1.3	9.9	5.40	5.9	0.0	6.9	9.9
0.0	0.1	2	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.0	0.1	4	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.0	0.1	6	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.0	0.1	8	4.65	5.9	0.0	0.0	100.0	1.00	4.75	4.75	0.0	0.0	4.75	1.3	0.0	4.40	5.9	0.0	5.9	0.0
0.1	0.1	2	4.74	6.9	9.1	0.0	40.7	1.00	4.75	4.75	0.0	0.7	4.75	1.6	0.7	4.49	6.0	0.0	6.9	9.1
0.1	0.1	4	4.74	6.9	9.1	0.0	40.7	1.00	4.75	4.75	0.0	0.7	4.75	1.6	0.7	4.49	6.0	0.0	6.9	9.1
0.1	0.1	6	4.74	6.9	9.1	0.0	40.7	1.00	4.75	4.75	0.0	0.7	4.75	1.6	0.7	4.49	6.0	0.0	6.9	9.1
0.1	0.1	8	4.74	6.9	9.1	0.0	40.7	1.00	4.75	4.75	0.0	0.7	4.75	1.6	0.7	4.49	6.0	0.0	6.9	9.1

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
-0.15	1.1	4.51	-0.05	1.2	7.2	4.65	0.05	4.70	5.9	1.3	9.9	-0.05	1.2	2.7	0.05	4.70	1.3	9.9	-0.05	1.2	2.7
-0.15	1.1	4.51	-0.05	1.2	7.2	4.65	0.05	4.70	5.9	1.3	9.9	-0.05	1.2	2.7	0.05	4.70	1.3	9.9	-0.05	1.2	2.7
-0.15	1.1	4.51	-0.05	1.2	7.2	4.65	0.05	4.70	5.9	1.3	9.9	-0.05	1.2	2.7	0.05	4.70	1.3	9.9	-0.05	1.2	2.7
-0.15	1.1	4.51	-0.05	1.2	7.2	4.65	0.05	4.70	5.9	1.3	9.9	-0.05	1.2	2.7	0.05	4.70	1.3	9.9	-0.05	1.2	2.7
-0.05	1.2	5.60	-0.05	1.2	9.6	5.65	0.05	5.70	6.9	1.3	9.9	-0.05	1.2	9.1	0.05	5.70	1.3	9.9	-0.05	1.2	9.1
-0.05	1.2	5.60	-0.05	1.2	9.7	5.65	0.05	5.70	6.9	1.3	9.9	-0.05	1.2	9.1	0.05	5.70	1.3	9.9	-0.05	1.2	9.1
-0.05	1.2	5.60	-0.05	1.2	9.8	5.65	0.05	5.70	6.9	1.3	9.9	-0.05	1.2	9.1	0.05	5.70	1.3	9.9	-0.05	1.2	9.1
-0.05	1.2	5.60	-0.05	1.2	9.9	5.65	0.05	5.70	6.9	1.3	9.9	-0.05	1.2	9.1	0.05	5.70	1.3	9.9	-0.05	1.2	9.1
-0.05	1.2	4.60	-0.06	1.2	0.1	4.65	-0.05	4.60	5.9	1.2	0.0	-0.06	1.2	0.1	0.00	4.65	1.3	0.5	-0.01	1.3	0.6
-0.05	1.2	4.60	-0.06	1.2	0.1	4.65	-0.05	4.60	5.9	1.2	0.0	-0.06	1.2	0.1	0.00	4.65	1.3	0.5	-0.01	1.3	0.6
-0.05	1.2	4.60	-0.06	1.2	0.1	4.65	-0.05	4.60	5.9	1.2	0.0	-0.06	1.2	0.1	0.00	4.65	1.3	0.5	-0.01	1.3	0.6
-0.05	1.2	4.60	-0.06	1.2	0.1	4.65	-0.05	4.60	5.9	1.2	0.0	-0.06	1.2	0.1	0.00	4.65	1.3	0.5	-0.01	1.3	0.6
-0.06	2.1	4.69	-0.07	2.1	9.2	4.70	0.00	4.70	6.9	2.2	9.5	-0.01	2.2	9.6	0.00	4.74	2.2	9.5	-0.01	2.2	9.6
-0.06	2.1	4.69	-0.07	2.1	9.2	4.70	0.00	4.70	6.9	2.2	9.5	-0.01	2.2	9.6	0.00	4.74	2.2	9.5	-0.01	2.2	9.6
-0.06	2.1	4.69	-0.07	2.1	9.2	4.70	0.00	4.70	6.9	2.2	9.5	-0.01	2.2	9.6	0.00	4.74	2.2	9.5	-0.01	2.2	9.6
-0.06	2.1	4.69	-0.07	2.1	9.2	4.70	0.00	4.70	6.9	2.2	9.5	-0.01	2.2	9.6	0.00	4.74	2.2	9.5	-0.01	2.2	9.6

Table A-3 (parameter t)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
X	0.0	0.0	0.0	1.0	1.0	0.25	0.25	0.0	0.0	6.0	0.0	1.1	X

		Monopoly																	
		Monopoly							First Best			Second Best			2 Products				
t	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
0.5	2	2.77	5.0	1.4	0.0	2.8	0.76	3.27	3.77	0.0	1.4	3.74	1.3	1.4	3.45	5.0	1.0	5.0	3.8
0.5	4	1.74	4.6	2.4	0.7	4.1	0.55	2.30	2.94	0.1	2.8	2.92	1.4	2.8	2.66	4.9	1.8	4.7	4.9
0.5	6	1.29	4.5	4.2	2.5	5.9	0.44	1.77	2.36	0.5	4.4	2.34	1.4	4.4	2.16	4.8	3.2	4.8	6.3
0.5	8	1.05	4.4	6.2	4.4	8.0	0.38	1.49	1.99	0.8	6.3	1.97	1.5	6.3	1.83	4.6	4.9	4.7	8.2
1.0	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
1.0	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
1.0	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
1.0	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6
2.0	2	1.79	4.7	0.8	0.0	1.6	0.55	2.30	2.81	0.0	0.9	2.77	1.4	0.9	2.54	4.7	0.6	4.7	2.2
2.0	4	0.83	4.4	2.1	1.2	3.0	0.32	1.20	1.67	0.6	2.2	1.65	1.5	2.2	1.49	4.5	1.6	4.6	3.3
2.0	6	0.56	4.4	4.1	3.2	5.0	0.24	0.84	1.20	0.6	4.1	1.18	1.8	4.1	1.04	4.4	3.4	4.4	5.2
2.0	8	0.42	4.4	6.0	5.1	6.9	0.20	0.65	0.97	0.6	6.0	0.94	1.8	6.0	0.80	4.4	5.3	4.4	7.1

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.10	1.4	2.86	0.09	4.5	5.0	2.76	0.18	2.92	5.1	1.5	1.3	0.11	4.4	4.8	0.92	3.61	4.8	3.3	2.46	5.5	1.0
0.48	2.0	2.18	0.45	4.9	6.7	1.68	0.64	2.26	4.7	2.3	2.0	0.58	4.9	6.2	1.09	2.73	4.4	4.8	1.52	5.1	1.7
0.57	2.3	1.81	0.53	4.9	8.4	1.05	1.04	2.00	4.4	4.3	6.2	1.07	4.9	2.9	1.04	2.24	4.3	6.2	1.07	4.9	2.9
0.49	2.4	1.50	0.48	4.7	9.9	0.90	0.90	1.73	4.4	4.4	8.1	0.92	4.7	4.7	0.90	1.88	4.4	8.1	0.92	4.7	4.7
0.38	1.8	2.62	0.30	4.9	4.1	2.28	0.38	2.62	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.17	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.76	0.56	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.14	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.36	0.52	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.58	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.12	0.46	4.7	9.0	0.63	0.63	1.21	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.27	4.4	4.9	0.63	4.5	7.4
0.48	2.1	2.23	0.43	4.7	3.0	1.77	0.52	2.25	5.0	2.2	0.7	0.47	4.6	2.9	0.90	2.61	4.0	2.0	1.52	5.5	0.5
0.53	2.8	1.32	0.52	4.6	4.2	0.74	0.72	1.39	4.5	4.1	1.4	0.71	4.6	3.2	0.72	1.49	4.3	3.2	0.75	4.8	1.5
0.41	3.1	0.94	0.41	4.4	6.2	0.53	0.51	0.99	4.4	4.0	3.3	0.51	4.4	5.2	0.52	1.03	4.4	3.2	0.52	4.4	5.0
0.35	3.4	0.74	0.34	4.5	8.0	0.41	0.40	0.77	4.4	4.0	5.3	0.39	4.4	7.2	0.40	0.79	4.4	7.0	0.40	4.4	5.2

Table A-4 (parameter α)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
1	0.0	0.0	0.0	1.0	1.0	0.25	0.25	0.0	0.0	6.0	X	1.1	X

		Monopoly																		
		Monopoly							First Best			Second Best			2 Products					
α	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2	
-1.0	2	2.71	4.8	1.1	0.0	2.2	0.66	3.25	4.01	0.0	1.3	3.99	0.5	1.3	3.42	4.8	0.9	4.8	3.1	
-1.0	4	1.43	4.2	2.3	1.0	3.6	0.46	2.03	2.78	0.0	2.6	2.75	0.7	2.5	2.31	4.5	1.7	4.6	4.1	
-1.0	6	0.99	4.2	4.1	2.8	5.4	0.35	1.44	2.06	0.0	4.2	2.02	1.0	4.2	1.74	4.4	3.3	4.5	5.8	
-1.0	8	0.77	4.2	6.1	4.8	7.4	0.29	1.15	1.66	0.0	6.1	1.62	1.1	6.1	1.40	4.4	5.1	4.5	7.6	
0.0	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8	
0.0	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9	
0.0	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7	
0.0	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6	
1.0	2	1.88	5.0	1.0	0.0	2.0	0.63	2.31	2.63	1.0	1.0	2.62	2.1	1.0	2.67	5.0	0.7	5.0	2.7	
1.0	4	1.06	4.7	2.2	1.1	3.3	0.40	1.44	1.84	1.8	2.3	1.84	2.2	2.3	1.84	4.8	1.6	4.8	3.8	
1.0	6	0.77	4.6	4.1	2.9	5.3	0.31	1.08	1.43	1.8	4.1	1.42	2.2	4.1	1.40	4.7	3.3	4.7	5.6	
1.0	8	0.62	4.5	6.1	4.9	7.3	0.27	0.91	1.20	1.5	6.1	1.19	2.1	6.1	1.15	4.7	5.1	4.7	7.4	

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.25	0.8	2.94	0.23	4.9	4.4	2.68	0.32	2.96	5.0	0.9	1.0	0.26	4.9	4.3	0.91	3.55	4.2	2.5	2.12	5.0	1.0
0.54	1.6	1.92	0.53	4.7	5.5	1.11	1.09	2.10	4.1	4.0	3.8	1.13	4.9	1.4	1.09	2.42	4.0	3.8	1.13	4.9	1.4
0.57	2.2	1.51	0.54	4.4	7.3	0.86	0.86	1.64	4.2	4.1	5.6	0.86	4.5	3.0	0.86	1.77	4.1	5.6	0.86	4.5	3.0
0.51	2.5	1.24	0.49	4.5	9.2	0.70	0.69	1.33	4.2	4.5	7.5	0.71	4.4	5.0	0.69	1.40	4.1	7.6	0.71	4.5	5.0
0.38	1.8	2.62	0.30	4.9	4.1	2.28	0.38	2.62	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.17	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.76	0.56	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.14	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.36	0.52	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.58	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.12	0.46	4.7	9.0	0.63	0.63	1.21	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.27	4.4	4.9	0.63	4.5	7.4
0.39	2.6	2.23	0.34	4.9	3.8	1.84	0.46	2.26	5.3	2.7	0.8	0.40	4.9	3.6	0.94	2.73	4.7	2.3	1.63	5.5	0.7
0.61	3.1	1.62	0.56	4.8	5.0	0.91	0.87	1.70	4.6	4.8	3.5	0.93	5.0	1.5	0.88	1.86	4.5	3.6	0.92	5.0	1.4
0.51	3.2	1.23	0.50	4.7	6.9	0.70	0.70	1.33	4.6	4.6	5.4	0.70	4.8	3.1	0.70	1.41	4.6	5.4	0.70	4.8	3.1
0.44	3.3	1.02	0.43	4.7	8.9	0.59	0.55	1.09	4.6	4.0	5.2	0.54	4.6	7.8	0.57	1.14	4.5	7.4	0.58	4.6	5.0

Table A-5 (parameter dr)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
1	0.0	0.0	0.0	1.0	1.0	0.25	0.25	0.0	0.0	6.0	0.0	X	X

		Monopoly																	
		Monopoly							First Best			Second Best			2 Products				
dr	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
1.0	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
1.0	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
1.0	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
1.0	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6
1.1	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
1.1	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
1.1	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
1.1	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6
2.0	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
2.0	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
2.0	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
2.0	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.38	1.8	2.65	0.30	4.9	4.1	2.28	0.38	2.65	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.26	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.82	0.56	4.7	5.2	1.02	0.97	1.98	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.23	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.41	0.52	4.6	7.1	0.78	0.77	1.55	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.65	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.16	0.46	4.7	9.0	0.63	0.63	1.27	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.33	4.4	4.9	0.63	4.5	7.4
0.38	1.8	2.62	0.30	4.9	4.1	2.28	0.38	2.62	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.17	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.76	0.56	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.14	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.36	0.52	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.58	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.12	0.46	4.7	9.0	0.63	0.63	1.21	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.27	4.4	4.9	0.63	4.5	7.4
0.38	1.8	2.47	0.30	4.9	4.1	2.28	0.38	2.47	5.0	1.8	1.0	0.30	4.9	4.1	0.98	2.77	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.53	0.56	4.7	5.2	1.21	0.69	1.56	4.5	2.7	1.9	0.66	4.6	4.9	0.99	1.73	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.14	0.52	4.6	7.1	0.85	0.66	1.18	4.4	3.2	3.5	0.64	4.5	6.4	0.77	1.26	4.4	5.5	0.78	4.7	3.1
0.47	2.9	0.93	0.46	4.7	9.0	0.65	0.61	0.96	4.4	3.8	5.1	0.60	4.5	7.8	0.63	1.01	4.4	4.9	0.63	4.5	7.4

Table A-6 (parameter βf vs βf2)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
1	0.0	0.0	0.0	1.0	1.0	X	X	0.0	0.0	6.0	0.0	1.1	X

			Monopoly																	
			Monopoly							First Best			Second Best			2 Products				
βf	βf2	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
0.25	0.25	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
0.25	0.25	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
0.25	0.25	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
0.25	0.25	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6
0.25	0.75	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	2.52	4.8	0.7	5.0	2.8
0.25	0.75	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	1.56	4.7	1.6	4.7	3.9
0.25	0.75	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.06	4.5	3.3	4.6	5.7
0.25	0.75	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	0.77	4.4	5.1	4.5	7.6
0.75	0.25	2	1.78	5.0	1.0	0.0	2.0	0.63	2.21	2.81	0.0	1.2	2.74	2.0	1.2	2.52	4.8	0.7	5.0	2.8
0.75	0.25	4	0.73	4.5	2.2	1.0	3.4	0.42	1.20	1.78	0.0	2.4	1.71	2.3	2.4	1.56	4.7	1.6	4.7	3.9
0.75	0.25	6	0.38	4.4	4.1	2.8	5.4	0.33	0.75	1.22	0.6	4.2	1.11	2.7	4.2	1.06	4.5	3.3	4.6	5.7
0.75	0.25	8	0.19	4.4	6.1	4.8	7.4	0.28	0.51	0.91	0.6	6.1	0.77	3.1	6.1	0.77	4.4	5.1	4.5	7.6

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.38	1.8	2.62	0.30	4.9	4.1	2.28	0.38	2.62	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.17	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.76	0.56	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.14	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.36	0.52	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.58	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.12	0.46	4.7	9.0	0.63	0.63	1.21	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.27	4.4	4.9	0.63	4.5	7.4
0.53	2.0	2.76	-0.05	1.9	1.0	2.27	0.54	2.76	4.8	2.0	1.1	-0.04	1.9	1.1	0.98	3.17	4.5	2.6	1.50	5.5	0.7
0.58	2.4	1.76	0.06	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	0.57	4.9	1.5	0.99	2.14	4.3	3.7	0.53	5.0	1.4
0.53	2.7	1.36	0.02	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.28	4.7	3.1	0.77	1.58	4.4	5.5	0.28	4.7	3.1
0.52	3.1	1.17	-0.01	3.0	6.1	0.63	0.63	1.21	4.4	4.4	4.9	0.13	4.5	7.4	0.63	1.27	4.4	4.9	0.13	4.5	7.4
-0.12	1.8	1.67	0.30	4.9	4.1	1.78	-0.12	1.67	5.0	1.8	1.0	0.30	4.9	4.1	0.48	2.22	4.5	2.6	2.00	5.5	0.7
0.08	2.4	0.81	0.56	4.7	5.2	0.52	0.47	0.94	4.4	4.8	3.6	1.07	4.9	1.5	0.49	1.18	4.3	3.7	1.03	5.0	1.4
0.03	2.7	0.40	0.52	4.6	7.1	0.28	0.27	0.53	4.4	4.4	5.5	0.78	4.7	3.1	0.27	0.63	4.4	5.5	0.78	4.7	3.1
-0.03	2.9	0.16	0.46	4.7	9.0	0.13	0.13	0.25	4.4	4.4	4.9	0.63	4.5	7.4	0.13	0.31	4.4	4.9	0.63	4.5	7.4

Table A-7 (parameter Δmc vs $\Delta mc2$)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
1	0.0	X	X	X*	X*	0.25	0.25	0.0	0.0	6.0	0.0	1.1	X

			Monopoly																	
			Monopoly							First Best			Second Best			2 Products				
Δmc	Δmc2	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
0.10	0.10	2	2.22	5.0	1.0	0.0	2.0	0.63	2.65	3.22	0.0	1.1	3.18	1.5	1.1	2.91	5.0	0.7	5.0	2.7
0.10	0.10	4	1.14	4.5	2.1	0.9	3.3	0.42	1.60	2.12	0.1	2.3	2.10	1.7	2.3	1.88	4.8	1.6	4.8	3.8
0.10	0.10	6	0.75	4.5	3.9	2.7	5.1	0.32	1.09	1.50	1.0	3.9	1.48	1.9	3.9	1.32	4.7	3.1	4.7	5.4
0.10	0.10	8	0.53	4.6	5.7	4.5	6.9	0.26	0.79	1.14	1.4	5.7	1.12	2.2	5.7	0.97	4.7	4.8	4.7	7.1
0.10	0.30	2	1.90	5.0	1.0	0.0	2.0	0.63	2.33	2.80	0.0	1.0	2.76	2.0	1.1	2.94	5.1	2.6	5.3	0.8
0.10	0.30	4	0.94	4.7	2.1	1.0	3.2	0.40	1.31	1.79	1.0	2.2	1.77	2.2	2.2	1.79	4.9	3.6	4.9	1.5
0.10	0.30	6	0.59	4.7	3.8	2.7	4.9	0.30	0.87	1.23	1.5	3.9	1.21	2.5	3.9	1.16	4.9	5.1	4.7	2.9
0.10	0.30	8	0.41	4.7	5.6	4.5	6.7	0.25	0.64	0.92	1.8	5.6	0.89	2.7	5.4	0.76	4.8	6.6	4.8	4.4
0.30	0.10	2	2.40	5.0	1.0	0.0	2.0	0.63	2.84	3.46	0.0	1.0	3.43	1.1	0.9	2.94	5.3	0.8	5.1	2.6
0.30	0.10	4	1.18	4.5	1.9	0.7	3.1	0.42	1.64	2.16	0.1	2.1	2.14	1.6	2.1	1.79	4.9	1.5	4.9	3.6
0.30	0.10	6	0.68	4.6	3.4	2.2	4.6	0.30	0.98	1.37	1.0	3.5	1.35	2.1	3.5	1.16	4.7	2.9	4.9	5.1
0.30	0.10	8	0.39	4.7	5.0	3.9	6.1	0.24	0.62	0.89	1.8	5.0	0.87	2.7	5.0	0.76	4.8	4.4	4.8	6.6

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.30	1.8	2.48	0.24	5.0	4.0	2.22	0.30	2.48	5.0	1.8	1.0	0.24	5.0	4.0	0.92	3.06	4.6	2.5	1.92	5.5	0.7
0.50	2.5	1.60	0.48	4.9	5.0	0.91	0.90	1.73	4.5	4.5	3.5	0.94	5.0	1.3	0.90	1.96	4.5	3.5	0.94	5.0	1.3
0.42	2.9	1.13	0.41	4.9	6.7	0.65	0.65	1.24	4.5	4.3	2.8	0.64	4.8	5.2	0.65	1.34	4.4	5.2	0.65	4.7	2.8
0.36	3.3	0.86	0.34	4.9	8.4	0.49	0.48	0.92	4.5	4.3	4.6	0.47	4.8	7.0	0.48	0.97	4.7	7.0	0.49	4.7	4.7
-0.25	1.5	1.67	0.23	1.4	1.0	1.90	-0.25	1.67	5.0	1.5	1.0	0.23	1.4	1.0	0.78	2.61	4.9	2.4	2.13	5.5	0.7
0.04	2.2	0.98	0.36	5.1	5.0	0.75	0.75	1.43	4.6	4.8	3.4	1.02	5.0	1.3	0.75	1.62	4.8	3.4	1.02	5.0	1.3
0.16	2.8	0.74	0.26	5.1	6.5	0.55	0.55	1.06	4.7	4.8	4.7	0.58	4.8	2.5	0.55	1.09	4.8	4.7	0.58	4.8	2.5
0.26	3.6	0.64	0.20	4.9	3.0	0.40	0.40	0.76	4.7	4.7	6.2	0.34	4.9	4.0	0.40	0.77	4.7	6.2	0.34	4.9	4.0
0.83	2.2	3.16	0.20	5.1	3.9	2.36	0.90	3.18	5.1	2.3	0.9	0.23	5.1	3.7	1.13	3.43	2.9	0.0	0.60	5.0	2.6
0.79	3.0	1.90	0.45	5.0	4.6	1.12	0.91	1.95	4.6	3.5	1.5	0.57	4.9	4.1	0.98	2.07	4.4	1.2	0.71	4.9	3.5
0.55	3.6	1.17	0.40	4.9	6.0	0.65	0.63	1.22	4.6	4.2	2.9	0.49	4.9	5.3	0.64	1.26	4.6	2.8	0.52	4.8	5.1
0.33	4.0	0.69	0.32	4.8	7.5	0.39	0.38	0.74	4.7	4.5	4.6	0.36	4.9	6.9	0.38	0.74	4.7	4.5	0.37	4.9	6.7

* When: $\Delta mc = \Delta mc2$, $\beta mc = \beta mc2 = 1$
$\Delta mc < \Delta mc2$, $\beta mc = 1.5$ and $\beta mc2 = 0.5$
$\Delta mc > \Delta mc2$, $\beta mc = 0.5$ and $\beta mc2 = 1.5$

Table A-8 (parameter βmc)

Constant Parameters													
t	Δv	Δmc	Δmc2	βmc	βmc2	βf	βf2	Δf	Δf2	βv	α	dr	df
1	0.0	0.0	0.0	X	βmc	0.25	0.25	0.0	0.0	6.0	0.0	1.1	X

		Monopoly																		
		Monopoly							First Best			Second Best			2 Products					
βmc	df	π	P	L	x	y	Q	W	W	P	L	W	P	L	π	P1	L1	P2	L2
0.0	2	2.94	4.8	1.1	0.0	2.2	0.66	3.48	4.15	0.0	1.2	4.14	0.3	1.2	3.86	4.6	0.8	4.7	3.1
0.0	4	1.68	4.2	2.3	1.0	3.6	0.46	2.28	2.99	0.0	2.5	2.97	0.4	2.5	2.76	4.4	1.6	4.5	4.1
0.0	6	1.22	4.1	4.2	2.8	5.6	0.36	1.71	2.30	0.0	4.2	2.29	0.5	4.2	2.16	4.2	3.2	4.4	5.8
0.0	8	0.98	4.1	6.1	4.7	7.5	0.30	1.39	1.91	0.0	6.1	1.89	0.6	6.1	1.79	4.2	5.0	4.2	7.7
1.0	2	2.28	5.0	1.0	0.0	2.0	0.63	2.71	3.31	0.0	1.2	3.27	1.4	1.2	3.02	4.8	0.7	5.0	2.8
1.0	4	1.23	4.5	2.2	1.0	3.4	0.42	1.70	2.28	0.0	2.4	2.26	1.4	2.4	2.06	4.7	1.6	4.7	3.9
1.0	6	0.88	4.4	4.1	2.8	5.4	0.33	1.25	1.72	0.6	4.2	1.70	1.5	4.2	1.56	4.5	3.3	4.6	5.7
1.0	8	0.69	4.4	6.1	4.8	7.4	0.28	1.01	1.41	0.6	6.1	1.39	1.6	6.1	1.27	4.4	5.1	4.5	7.6
2.0	2	1.65	5.0	1.0	0.0	2.0	0.63	2.08	2.48	0.1	1.1	2.44	2.4	1.1	2.20	5.0	0.7	5.0	2.7
2.0	4	0.82	4.8	2.2	1.1	3.3	0.38	1.16	1.61	1.4	2.3	1.59	2.5	2.3	1.40	5.0	1.6	5.0	3.6
2.0	6	0.56	4.7	4.1	3.0	5.2	0.30	0.84	1.18	1.8	4.1	1.16	2.6	4.1	1.01	4.8	3.3	4.8	5.5
2.0	8	0.43	4.7	6.1	5.0	7.2	0.25	0.66	0.95	1.8	6.1	0.93	2.7	6.1	0.79	4.8	5.2	4.8	7.4

Duopoly																					
Monopolist Location						Best Location									Free Location						
π p2	P p2	D π	E π	E P	E L	π p1	π p2	D π	P p1	Pp2	L	E π	E P	E L	π p2	D π	P p2	L p2	E π	E P	E L
0.40	0.8	3.30	0.34	4.6	4.4	2.94	0.40	3.30	4.8	0.8	1.1	0.34	4.6	4.4	1.15	3.98	4.1	2.5	2.33	5.0	1.0
0.72	1.5	2.33	0.69	4.6	5.6	1.35	1.28	2.51	4.0	4.3	3.8	1.41	4.7	1.5	1.30	2.86	4.3	3.9	1.45	4.8	1.5
0.71	1.9	1.87	0.67	4.4	7.5	1.06	1.06	2.02	4.0	4.0	5.7	1.08	4.6	3.1	1.06	2.19	4.0	5.7	1.08	4.6	3.1
0.66	2.2	1.58	0.64	4.4	9.3	0.89	0.89	1.70	4.0	4.1	7.6	0.90	4.3	4.9	0.89	1.79	4.1	7.6	0.90	4.3	4.9
0.38	1.8	2.62	0.30	4.9	4.1	2.28	0.38	2.62	5.0	1.8	1.0	0.30	4.9	4.1	0.98	3.17	4.5	2.6	2.00	5.5	0.7
0.58	2.4	1.76	0.56	4.7	5.2	1.02	0.97	1.90	4.4	4.8	3.6	1.07	4.9	1.5	0.99	2.14	4.3	3.7	1.03	5.0	1.4
0.53	2.7	1.36	0.52	4.6	7.1	0.78	0.77	1.48	4.4	4.4	5.5	0.78	4.7	3.1	0.77	1.58	4.4	5.5	0.78	4.7	3.1
0.47	2.9	1.12	0.46	4.7	9.0	0.63	0.63	1.21	4.4	4.4	4.9	0.63	4.5	7.4	0.63	1.27	4.4	4.9	0.63	4.5	7.4
0.28	2.7	1.90	0.21	5.0	3.8	1.65	0.28	1.90	5.0	2.7	1.0	0.21	5.0	3.8	0.71	2.29	4.8	2.3	1.41	5.5	0.7
0.41	3.2	1.19	0.39	4.9	4.9	0.68	0.68	1.29	4.7	4.7	3.5	0.70	5.1	1.4	0.68	1.44	4.7	3.5	0.70	5.1	1.4
0.36	3.5	0.89	0.34	4.9	6.7	0.52	0.49	0.96	4.7	4.3	3.2	0.47	4.8	5.6	0.50	1.01	4.7	5.4	0.51	4.9	3.2
0.31	3.7	0.71	0.29	4.8	8.7	0.40	0.38	0.75	4.7	4.3	5.2	0.37	4.8	7.6	0.39	0.78	4.7	5.0	0.39	4.7	7.3