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Optimal Nonlinear Income Taxation for Reduction of Envy

Yukihiro Nishimura
Queen's University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Yukihiro Nishimura

Department of Economics, Queen's University, Kingston, Ontario K7L 3N6, Canada.[*][†]

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Abstract

This paper examines the optimal nonlinear income taxation problem based on Chaudhuri (1986) and Diamantaras and Thomson's (1990) λ-equitability in a two-class economy. An allocation is λ-equitable if no agent envies a proportion λ of the bundle of any other agent. We examine the properties of Pareto undominated allocations for various λ-equitability requirements. When there is one output, the marginal income tax rate can increase only if (but not if) leisure is a luxury. In a multi-commodity model with commodity taxes, the goods preferred by the low skilled agent and/or of high Hicksian elasticities are taxed more heavily.

Key Words: Income Taxation, Envy

JEL Classification: D63, H21

1 Introduction

The optimal taxation literature typically uses a welfaristic social welfare function as the government's objective function. A value judgment is welfaristic if the evaluation of social states is based solely on the welfare (utility) of individuals. Sen (1979) pointed out that a problem of welfarism is its informational parsimony; namely, its disregard of non-utility information, such as rights, freedom and opportunities. This paper incorporates information on the level of envy, in addition to the utility information, into the evaluation of alternative social states, and derives optimal tax rules on the basis of an equity criterion.

We define envy with respect to agents' bundle holdings. Agent i envies agent j if the former perceives the latter's bundle to be better than his own. As in the real world, when sophisticated tax-transfer policies based on one's ability are infeasible, there arises a hierarchy of envy: for any allocation implementable by income and commodity taxation, the high skilled agent never envies the low skilled agent, whereas the low skilled always envies the high skilled, except for some trivial allocations (Bös and Tillmann (1985)). The celebrated equity concept of no-envy faces difficulty in this situation, and some cardinal measure to evaluate the intensity of envy is required when the government's objective is the reduction of envy.

Chaudhuri (1986) and Diamantaras and Thomson (1990) considered the following measure of envy. An allocation is λ-equitable if no agent envies a proportion λ of the bundle

of any other agent. The value of λ measures the intensity of envy. If λ is unity, it is the usual no-envy. Any feasible allocation satisfies zero-equitability. On the basis of this envy measure, this paper studies how given λ-equitability requirements affect the qualitative properties of the tax implementable allocations, by looking at the efficient allocations among the λ-equitable allocations. Consider the maximal welfare combinations if the government is constrained by a given λ-equitability requirement. Call this *the λ-constrained second-best frontier*. Varying λ, we can depict the set of the second-best welfare frontiers with various envy constraints.[1]

We consider a two-class economy with identical preferences, commonly examined in the optimal taxation literature, in which there are agents with high and low productivity. The following two issues are central to this paper. First, we examine how the second-best utility possibility frontier contracts with the additional requirement of λ-equitability. A natural prediction is that, the more favorable is the allocation to the low skilled agent, the less is the amount of envy. In environments where the above natural conjecture holds, the contraction of the utility possibility frontiers occurs in the area which is favorable to the high skilled agent; some of these allocations are judged inequitable (Lemmas 2 and 3). Second, we characterize the tax rules at points which are *not* second-best Pareto efficient. The tax rule along the λ-constrained second-best frontier departs from the usual second-best Pareto

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[1] Chaudhuri (1986) and Diamantaras and Thomson (1990) originally propose to choose the Pareto efficient allocations which maximize λ. The tax policy implication of their approach is examined in a companion paper (Nishimura (1999)). A social choice theoretic foundation of the λ-constrained second-best frontier is given in Nishimura (1998).

optimal solutions (Stiglitz (1982), Edwards *et al.* (1994) and Nava *et al.* (1996)) only if the allocation is not second-best Pareto efficient. Our standpoint is that there is a positive reason to consider this 'third-best' allocation if the government imposes λ-equitability as a prerequisite.

Our results regarding the second issue are as follows. In the standard one-output economy, in contrast with the standard finding that progressive taxation in the sense of an increasing marginal income tax rate is never second-best Pareto efficient (Phelps (1973), Sadka (1976) and Seade (1977)), the following can be shown. When leisure is a luxury, the marginal tax rate can increase, if anywhere, only at allocations where the amount of redistribution to the low skilled agent is less than any second-best Pareto efficient and λ-equitable allocations. When leisure is a necessity, the marginal income tax rate never increases (Proposition 1).

In the multi-commodity setting, we consider the case where the demand for goods is separable with leisure, referred to as weak separability by Atkinson and Stiglitz (1976). It is known that the optimal commodity taxes are zero when we consider only efficiency. In contrast, the optimal commodity tax formula along the λ-constrained second-best frontier suggests that, if a good is preferred by the low skilled (envying) agent, and its Hicksian elasticities are high for the high skilled agent, then it is taxed more heavily (Proposition 2). The proposition is worth comparing with the traditional Ramsey (1927) inverse elasticity rule and its extension by Diamond (1975). The optimal commodity taxes are zero if, in

4

addition to weak separability, sub-utility of the goods is homothetic (Corollary 1).

The rest of the paper proceeds as follows. Section 2 introduces the model and the class of feasible policies. In Section 3 we introduce the equity objective under consideration. In Section 4 we characterize tax rules along the λ-constrained second-best frontier in a one-output economy. In Section 5 we extend the analysis to the case with many commodities. Section 6 concludes. Proof of a proposition is given in an appendix.

2 The Model

Consider a two-agent economy with one consumption good, c, and labor, l. Each agent i ($i = 1, 2$) has preferences represented by the common utility function $u(c_i, l_i)$, where $c_i \in I\!\!R_+$ and $l_i \in [0, \bar{l}]$. The utility function is strictly quasi-concave and twice continuously differentiable with $u_c > 0$ and $u_l < 0$, where the subscript denotes the partial derivative. We also assume that both consumption and leisure are normal goods. Each agent i is endowed with an exogenous skill level w_i, with $w_2 > w_1 > 0$. The pre-tax income $w_i l_i$ is denoted by y_i. The set of feasible allocations in an economy is $\{((c_1, l_1), (c_2, l_2)) \in (I\!\!R_+ \times [0, \bar{l}])^2 \,|\, c_1 + c_2 \leq w_1 l_1 + w_2 l_2\}$. A feasible allocation is often denoted by $x = (x_1, x_2)$. This economy is called *a one-output economy*. We consider a multi-commodity economy in Section 5.

A nonlinear income tax is used for pure redistribution. The government can observe the pre-tax income, $y_i = w_i l_i$, of each agent i, but cannot observe either w_i or l_i. Implementabil-

ity requires the allocation to satisfy *self-selection*. An allocation satisfies self-selection iff:

$$u(c_i, l_i) \geq u(c_j, \frac{w_j}{w_i}l_j) \ \text{ if } \ \frac{w_j}{w_i}l_j \leq \bar{l}, \ \forall i, \ j, \tag{1}$$

that is, agent i cannot be better off by choosing the pre-tax and post-tax income of agent j by working $w_j l_j / w_i$ hours. An allocation is *second-best Pareto efficient* iff it is Pareto efficient among those allocations that satisfy (1).

3 The Equity Objective

We consider envy with respect to agents' consumption-leisure bundles. Agent i envies agent j in an allocation x iff $u(x_j) > u(x_i)$.[2] In the class of income tax implementable allocations, we can observe the hierarchical structure of envy. The higher skilled agent *never* envies the lower skilled agent, whereas the lower skilled envies the final consumption-leisure bundle of the higher skilled, except when the former's labor supply is zero (Bös and Tillmann (1985)). This can be easily verified by checking the consequence of (1):

$$u(c_i, l_i) \geq u(c_j, \frac{w_j}{w_i}l_j) \geq u(c_j, l_j) \text{ if } w_i \geq w_j, \tag{2}$$

with the inequalities strict if $w_i > w_j$ and $l_j > 0$. This observation shows that the classic concept of no-envy, which requires that no agent envies any other agent, is not a useful concept in examining equitable and efficient taxation, since *in any no-envy and income tax*

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[2]Readers might be tempted to ask whether it is possible for agents to *observe* the other agents' labor supply. A reasonable supposition is that the agents, as well as the government, know the demographic data in the economy, i.e., preferences and the ability level of the other agent(s), hence they correctly *infer* the other agents' labor supply as a best response to the given tax schedule.

implementable state, at most one agent supplies labor.[3] [4] If we attach a negative weight to envy, and moreover efficiency considerations are required, we are faced with reconstructing an alternative concept of no-envy.

In the social choice literature, the principal approach to evaluate a social state based on envy is to introduce a cardinal measure for the intensity of envy. Among these, we will adopt a radial contraction measure advocated by Chaudhuri (1986) and Diamantaras and Thomson (1990). Let $\lambda \in I\!R_+$ be a nonnegative real number, and let $\circ$ be an operation such that $\lambda \circ (c_i, l_i) \equiv (\lambda c_i, \bar{l} - \lambda(\bar{l} - l_i))$, which represents a proportional contraction of agent i's consumption-leisure bundle. Given an allocation x, let $\lambda_{ij}^x \in I\!R_+$ be a value such that $u(x_i) = u(\lambda_{ij}^x \circ x_j)$ when $u(x_i) \leq u(x_j)$, and $\lambda_{ij}^x \equiv 1$ when $u(x_i) > u(x_j)$.[5] When agent i envies agent j, the value of λ_{ij}^x measures the amount by which one would have to shrink j's bundle in order for i to stop envying j, and thus indicates the intensity of envy which agent i has against agent j in allocation x. Figure 1 depicts the idea. Given an allocation x, agent i envies agent j. The straight line connecting the point $(0, \bar{l})$ and $x_j = (c_j, l_j)$ crosses the indifference curve of agent i passing through x_i exactly once, due to continuity and

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[3]Bös and Tillmann (1985) originally showed that, if the ability distribution is variable (the true economy is drawn from an ability distribution whose upper bound is infinity), the only income tax implementable no-envy allocation is 'no production'. We will consider the case at which the statistical properties of the true economy are known, typically assumed in the normative analysis of optimal taxation.

[4]Notice that no-envy and self-selection are disjoint conditions if productivities are unequal. There are no-envy allocations which violate (1), and most of the self-selective states invite envy of the lower skilled as shown in (2).

[5]Here, we modify the definition of Diamantaras and Thomson (1990). In the current environment, it may happen that the λ-expansion of the leisure of the opponent meets the upper bound $\bar{l}$, so that a value of λ_{ij}^x such that $u(x_i) = u(\lambda_{ij}^x \circ x_j)$ may not exist when $u(x_i) > u(x_j)$.

monotonicity. The intersection is denoted by $(\tilde{c}, \tilde{l})$, and thus $u(x_i) = u(\tilde{c}, \tilde{l})$. The amount of envy that agent i has against agent j at allocation x is regarded to be negatively correlated with $\lambda_{ij}^x = \dfrac{\bar{l} - \tilde{l}}{\bar{l} - l_j} = \dfrac{\tilde{c}}{c_j}$, the ratio between the hypothetical leisure-consumption bundle and agent j's actual bundle.

<center>Figure 1 around here.</center>

Let $\lambda^x \equiv \min_{i,j} \lambda_{ij}^x$. An allocation x is λ-*equitable* iff $\lambda^x \geq \lambda$, or equivalently, $u(x_i) \geq u(\lambda \circ x_j)$ $\forall i, j$. In this paper, we consider the maximal welfare combinations when the government is constrained by a given λ-equitability. They can be derived by solving the following restricted Pareto optimization problem,

$$\max u(x_1)$$

$$\text{s.t.} \quad u(x_2) \geq \bar{v}_2 \tag{3}$$

$$u(x_1) \geq u(\lambda \circ x_2)$$

$$x \text{ is feasible and satisfies } (1),$$

with varying $\bar{v}_2 \in I\!\!R$ (notice that, by (2), only the low skill's envy constraint is relevant, i.e., the envy constraint is that $\lambda_{12}^x \geq \lambda$). Denote the pairs of the maximal utility level of agent 1 (where it exists) and the corresponding utility level $\bar{v}_2$ *the λ-constrained second-best frontier.* Varying λ, we can depict the set of the welfare frontiers with various envy constraints.

<center>8</center>

A tax policy implication of the solution is that, from the allocation there is no direction of tax changes which is Pareto improving and maintains λ-equitability. Some second-best Pareto inefficient allocations are included in the solutions, since our standpoint is that global Pareto efficiency is not necessarily appealing when there are additional equity requirements. However, the Pareto criterion is used as a secondary criterion, to avoid allocations such as no production. The approach to require λ-equitability as a prerequisite and give efficiency consideration second is an extension of Tadenuma's (1998) *Equity-first Principle*. The formalization and discussions are given in Nishimura (1998).

4 One Output Case

4.1 Preliminary: Characterization of the Second-best Pareto Optima

Before exploring the tax rules along the λ-constrained second-best frontier, in this section we report the characterization of the second-best Pareto efficient allocations as a benchmark. Let $u_r^i \equiv \partial u / \partial r |_{(c,l)=x_i}$ $(i = 1,\ 2,\ r = c,\ l)$ and $MRS_i(y, c) \equiv -\dfrac{1}{w_i}\dfrac{u_l^i}{u_c^i}$ be the marginal rate of substitution between gross income and consumption of agent i evaluated at the optimum. Following Stiglitz (1982), we regard $t_i \equiv 1 - MRS_i(y, c)$, the difference between the marginal rate of transformation and the marginal rate of substitution, as the marginal income tax rate. 'The self-selection constraint of agent i against agent j $(i \neq j)$' is abbreviated to SS_i.

Lemma 1 *(Stiglitz (1982)) Consider a one-output economy. Among the second-best Pareto efficient allocations, there are three possibilities on the marginal income tax rate. Each case*

corresponds to the degree of redistribution to agent 1, i.e., case 1 is the most redistributive to agent 1.

1. $t_1 > 0$, $t_2 = 0$. *'Redistributive area': The tax is redistributive to agent 1, SS_2 is binding, and the allocation of agent 1 is distorted.*

2. $t_1 = t_2 = 0$. *'Non-distortionary area': No self-selection constraint is binding, and the allocation is first-best Pareto efficient.*

3. $t_1 = 0$, $t_2 < 0$. *'Regressive area': The tax is redistributive to agent 2, SS_1 is binding, and the allocation of agent 2 is distorted.*

We can observe the classic finding of optimal nonlinear income taxation in this lemma: progressive taxation in the sense of an increasing marginal income tax rate is never second-best Pareto efficient (Phelps (1973), Sadka (1976) and Seade (1977)).

4.2 Variations in λ^x along the Second-best Pareto Efficient Allocations

In this section we will investigate the variation in λ^x, the maximal value at which the allocation x satisfies λ-equitability, along the second-best frontier, and derive its implication for the contraction of the λ-constrained second-best frontier from the second-best frontier.

The following results are shown in the companion paper (Nishimura (1999)):

Lemma 2 *(Nishimura (1999)) Consider a one-output economy.*

a. Among the second-best Pareto efficient allocations where cases 1 and 2 in Lemma 1 apply, the envy measure λ^x monotonically increases as we increase the amount of redistribution to the low skilled agent.

b. If leisure is a luxury (the income elasticity of leisure is greater than unity), among the second-best Pareto efficient allocations, the envy measure λ^x monotonically increases as we increase the amount of redistribution to the low skilled agent.

The contraction of the λ-constrained second-best frontier from the second-best Pareto frontier is illustrated in Figure 2.

Figure 2 around here.

The outer frontier ABL is the second-best Pareto frontier (the second-best welfare frontier with a zero-equitability requirement). Point L corresponds to the allocation which maximizes the utility of the low-skilled agent along the second-best frontier (hereafter *the leximin allocation*). Point E is an allocation with equal utility (i.e., $\lambda^x = 1$), which is in general not second-best Pareto efficient. Consider the λ-constrained second-best frontier with the value of $\lambda = \lambda^B$ corresponding to λ^x of point B in Figure 2. The λ^B-constrained second-best frontier can be depicted as $A'BL$ in the figure when leisure is a luxury. First, any second-best Pareto efficient allocation x where the amount of redistribution to agent 1 (and thus his utility level) is higher than B has greater λ^x than λ^B. By definition, these

11

allocations belong to the λ^B-constrained second-best frontier. Second, the contraction of the λ^B-constrained second-best frontier from the original second-best Pareto frontier occurs to the north-west of B, since allocations along AB have lower λ^x than λ^B.[6] As λ increases, the constrained second-best frontier shrinks further. The frontier $A''C$ is an extreme case where no Pareto efficient allocation is regarded as λ-equitable.

The lemma also has an importance in characterizing the marginal income tax schedule along the λ-constrained second-best frontier. By solving the following Kuhn-Tucker problem corresponding to (3),

$$\mathcal{L} \equiv \zeta_1 u(c_1, l_1) + \zeta_2(u(c_2, l_2) - \bar{v}_2) + \mu(u(c_1, l_1) - u(\lambda \circ (c_2, l_2))) +$$

$$\eta_1(u(c_1, l_1) - u(c_2, \frac{w_2}{w_1} l_2)) + \eta_2(u(c_2, l_2) - u(c_1, \frac{w_1}{w_2} l_1)) + \gamma(\sum_{i=1,2} [w_i l_i - c_i]), \quad (4)$$

the Lagrange multiplier of the envy constraint, μ, is positive only if the allocation is not second-best Pareto efficient. Then, in the case of the λ^B-constrained second-best frontier discussed above, for example, we know that $\mu = 0$ at the leximin allocation, and μ would become positive when $\bar{v}_2$ is sufficiently high. The characterization of the tax schedule is given in the next section.

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[6]The same explanation applies when leisure is a necessity, except for some ambiguity in the regressive area: the contracted possibility frontier may reach the outer frontier again after the departure from the outer frontier at some regressive allocations.

4.3   Marginal Tax Schedule along the λ-Constrained Second-Best Frontier

From the first order conditions of (4), the marginal income tax rate on the low skilled agent is derived as follows:

$$t_1 = \frac{\eta_2}{\gamma}\hat{u}_c^2 (MRS_1(y,c) - \widehat{MRS}_2(y,c)) \tag{5}$$

where $\hat{u}_r^i \equiv \partial u/\partial r\big|_{(c,l)=(c_j, \frac{w_j}{w_i}l_j)}$ $(i=1,2,\ i\neq j,\ r=c,\ l)$, and $\widehat{MRS}_i(y,c) \equiv -\frac{1}{w_i}\frac{\hat{u}_l^i}{\hat{u}_c^i}$. The shadow price of the resource constraint, γ, is shown to be positive.

This is the same as the usual case. Unless $\bar{v}_2$ is high enough, SS_2 becomes binding and $\eta_2 > 0$. The standard result based on the welfaristic approach — the marginal tax rate on the low skilled agent is positive — follows from the so-called agent monotonicity condition (Seade (1982)), implied by normality.

On the other hand, the marginal income tax rate of the high skilled agent is derived as follows:

$$t_2 = \lambda\frac{\mu}{\gamma w_2}\bar{u}_c^1(MRS_2(l,c) - \overline{MRS}_1(l,c)) + \frac{\eta_1}{\gamma}\hat{u}_c^1(MRS_2(y,c) - \widehat{MRS}_1(y,c)) \tag{6}$$

where $MRS_2(l,c) \equiv -\frac{u_l^2}{u_c^2}$, $\bar{u}_r^1 \equiv \partial u/\partial r\big|_{(c,l)=\lambda\circ x_2}$ $(r=c,\ l)$, and $\overline{MRS}_1(l,c) \equiv -\frac{\bar{u}_l^1}{\bar{u}_c^1}$.

The first term is new. The implication of the envy constraint is similar to that of self-selection, but it is the difference in the marginal rate of substitution between consumption and labor, not efficiency-unit labor, between the envying and the envied agent. Since individuals differ in their incomes and corresponding welfare levels, the income elasticity of demand

13

matters, as in the previous section. This is because λ-equitability considers a proportional shrinkage of the envied agent's bundle. If leisure is a luxury, $MRS_2(l,c) > \overline{MRS}_1(l,c)$, because the agents prefer leisure more relative to consumption as they become richer (and their corresponding utility level becomes higher). In this case, the first term is positive, if μ is positive. On the other hand, if the allocation is redistributive to agent 2, at which $\eta_1 > 0$, λ^x is low at the second-best Pareto efficient allocation, and thus a departure from second-best Pareto efficiency is required under sufficiently high λ-equitability. Hence, when both μ and η_1 are positive, there are offsetting effects in determining the marginal income tax rate, which should be clarified.

For various λ, we can draw the λ-constrained second-best frontier. The following proposition characterizes the frontier when λ is such that some second-best Pareto efficient allocations which are redistributive to the low skilled agent are judged inequitable. The proof is given in the appendix.

Proposition 1 *Consider the λ-constrained second-best frontier in a one-output economy where λ is given such that some second-best Pareto efficient allocations in the redistributive area in Lemma 1 is excluded because of inequitability. When leisure is a luxury, the signs of the marginal tax rates along the λ-constrained second-best frontier can be the following: 1. $t_1 > 0$, $t_2 = 0$. 2. $t_1 > 0$, $t_2 > 0$. 3. $t_1 = 0$, $t_2 > 0$. When leisure is a necessity, there are three possibilities: 1. $t_1 > 0$, $t_2 = 0$. 2. $t_1 > 0$, $t_2 < 0$. 3. $t_1 = 0$, $t_2 < 0$. Each case*

corresponds to the degree of redistribution to agent 1, i.e., case 1 is the most redistributive

to agent 1.[7]

Figure 3 around here.

Figure 3.a depicts the case where leisure is a luxury. As shown in Lemma 2, once a point on the second-best Pareto frontier is regarded as inequitable, any point located north-west of that point on the second-best utility possibility frontier is also inequitable, and is therefore excluded from the λ-constrained second-best frontier. Figure 3.b represents the case where leisure is a necessity. Contrary to the left panel, the λ-constrained second-best frontier approaches the original second-best Pareto frontier again in the north-western (regressive) area. The illustration is to emphasize the possibility that λ^x may increase in a region that is redistributive to the high skilled agent.

Several comments on Proposition 1 are in order. First, with a high λ-equitability requirement, the regressive area is excluded from the λ-constrained second-best frontier, if leisure is a luxury. This is not necessarily so when leisure is a necessity. Second, it is worth noting that a pooling equilibrium[8] is not desirable on both equity and efficiency grounds

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[7] When λ is such that every allocation in case 1 of Lemma 1 is regarded as equitable, case 2 in the proposition is replaced by $t_1 = t_2 = 0$ (the non-distortionary area in Lemma 1). With a lower λ such that every allocation in case 2 of Lemma 1 is regarded as equitable, in addition to the above replacement, case 3 in the proposition is replaced by $t_1 = 0$, $t_2 \leq 0$, with strict inequality when leisure is a necessity. On the other hand, with high λ such that every allocation in cases 1 and 2 of Lemma 1 is judged inequitable, the conclusion of Proposition 1 holds as well.

[8] Notice that, since we assume a purely redistributive tax, at a pooling equilibrium, no redistribution occurs. Therefore, the equilibrium is Pareto inferior to the laissez-faire equilibrium.

when leisure is a necessity. Finally, in contrast with the standard finding of the progressivity of income taxation, we know the following: *When leisure is a luxury, an increasing marginal tax rate applies, if anywhere, only in cases 2 and 3, where the amount of redistribution to the low skilled agent is less than any second-best Pareto efficient and λ-equitable allocations (case 1). When leisure is a necessity, the increasing marginal income tax rate never applies.*

5   Multi Commodity Economy

In this section we consider a k-output economy. Let $c_i \equiv (c_i^m)_{m=1}^k \in I\!\!R_+^k$ be the consumption goods by agent i ($i = 1, 2$). Utility $u(c_i, l_i)$ is increasing in the first k elements and decreasing in the last element. Quasi-concavity, differentiability and normality continue to be assumed. Production exhibits constant marginal costs, and firms are competitive, so that taxes do not change pre-tax prices. Let $p = (p^m)_{m=1}^k$ be the marginal cost of producing consumption goods. The production possibility is now $\{((c_1, l_1), (c_2, l_2)) \in (I\!\!R_+^k \times [0, \bar{l}])^2 | p \cdot (c_1 + c_2) \leq w_1 l_1 + w_2 l_2\}$, with $w_2 > w_1 > 0$.

In addition to the nonlinear income tax, commodity taxes are used as a policy tool. Commodity tax $\tau = (\tau^m)_{m=1}^k$ is a wedge between the consumer price and the producer price. Tax revenue $\tau \cdot \sum_{i=1,2} c_i$ is used for redistribution.[9]

5.1   Variations in λ^x in a Special Case

For simplicity, we assume that the preferences are *weakly separable*:

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[9]The extension considering public good provision from tax revenue is straightforward. An optimal public good provision rule along the λ-constrained second-best frontier is available upon request to the author.

$$u(c_i, l_i) = u(f(c_i), l_i). \tag{7}$$

It is known that second-best Pareto efficiency implies zero commodity taxes in this environment (Atkinson and Stiglitz (1976)). As a consequence, this serves as a benchmark case. Lemma 2.a can be readily extended to this economy.

Lemma 3 *Consider a k-output economy, in which preferences are weakly separable. Among the second-best Pareto efficient allocations that are redistributive to the low skilled agent, the envy measure λ^x monotonically increases as we increase the amount of redistribution to the low skilled agent.*[10]

Below we show that along the λ-constrained second-best frontier where the envy constraint is binding, the optimal commodity tax is *not* zero. Rather, distortionary commodity taxes serve to relax the envy constraint.

5.2 Optimal Commodity Taxes

We can apply Edwards *et al.* (1994) and Nava *et al.* (1996) for the basic setup and derivation of the first order conditions of the Lagrangean. We use the same notation for the Lagrange multipliers as (4): μ is that of the envy constraint, η_i is that of SS_i, γ is that of the resource constraint. As in the previous section, γ can be shown to be positive.

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[10]The consequence of Lemma 2.b holds when $f(\cdot)$ is homothetic and moreover leisure is a luxury.

The optimal commodity tax rule along the λ-constrained second-best frontier for each good $m = 1, ..., k$ is the following:

$$\sum_{i=1,2} \tau \cdot \frac{\partial \tilde{c}_i}{\partial \tau^m} = \lambda \frac{\mu}{\gamma} \frac{\partial u}{\partial (\lambda \circ x_2)} \cdot \left(\frac{\partial \tilde{c}_2}{\partial \tau^m}, 0 \right) \qquad (8)$$

where $\tilde{c}_i = \tilde{c}_i(\tau, l_i, u) = \arg \min_{c_i} \{ (p + \tau) \cdot c_i | u(c_i, l_i) \geq u \}$ is the compensated demand with stationary labor supply (so that the last element of the constrained Hicksian vector which corresponds to labor supply is zero), $\partial u / \partial (\lambda \circ x_2) = \partial u / \partial (c, l) |_{(c,l) = \lambda \circ x_2}$.

The left hand side of (8) is the so-called *index of discouragement*. The smaller is this value, the more demand is discouraged, which implies that the good and its complements tend to be taxed more. The right hand side determines which goods should be taxed heavily. It is the inner product of the marginal utility of the low skilled agent and the substitution effect of the compensated demand, which reflects the reduction of envy through discouragement of consumption by the high skilled agent due to taxation. The term is nonzero only if $\mu > 0$, which holds if the allocation is second-best Pareto inefficient.

Consider a) the product of the own substitution effect of the envied agent (here, the high skilled) and its marginal utility to the envying agent (here, the low skilled), and b) the product of the cross substitution effect of a complement of the taxed good and its marginal utility to the envying agent. These terms are negative. The smaller (larger in absolute value) these values are, the more demand for the good is discouraged. We interpret high values of $\partial u / \partial c^m |_{(c,l) = \lambda \circ x_2}$ as 'agent 1 prefers good m'.[11] Then we can conclude the following:

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[11]If readers care that the definition depends on cardinal representation, normalize the marginal utility

18

Proposition 2 *Consider a k-output economy, in which preferences are weakly separable. In any λ-constrained second-best allocation at which the envy constraint is binding, if good m or its Hicksian complement in terms of the high skilled agent's preference is preferred by the low skilled agent, and/or their Hicksian derivatives with respect to τ^m of the high skilled agent are small, ceteris paribus, demand is more discouraged. That is, it and its complements are taxed more heavily.*

It is justified for the reduction of envy to tax more heavily (in the sense of having a larger reduction in compensated demand) the goods that the low skilled agent prefers and whose Hicksian elasticities are high. This observation, although a natural consequence of the envy-reducing taxation problem, contrasts with the traditional Ramsey (1927) inverse elasticity rule and Diamond's (1975) extension to the many person economy. The former says that, under the linear tax schedule which maximizes the welfare of a representative individual, the tax rate is proportional to the reciprocal of the commodity's Hicksian own elasticity. The latter says that, under a linear tax schedule which maximizes a Bergson-Samuelson social welfare function, the goods which the low utility agent, whose weight in the social welfare function is higher under concavity, prefers should not be taxed. [12]

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by that of some good, say labor, in the form of the marginal rate of substitution between good m and the numeraire. The author acknowledges professor Yongsheng Xu for the comment.

[12] Without assuming weak separability, the tax formula is the following:

$$\sum_{i=1,2} \tau \cdot \frac{\partial \tilde{c}_i}{\partial \tau^m} = \lambda \frac{\mu}{\gamma} \frac{\partial u}{\partial (\lambda \circ x_2)} \cdot \left(\frac{\partial \tilde{c}_2}{\partial \tau^m}, 0 \right) + \sum_{i=1,2, i \neq j} \frac{\eta_j}{\gamma} \hat{\alpha}_j (c_i^m - \hat{c}_j^m)$$

where $\hat{\alpha}_j$, $\hat{c}_j^m$ are the marginal utility of income and consumption of the mimicker of the self-selection problem, respectively. The first term of the RHS may conflict with the second term in the following way. Suppose

A sufficient condition for the commodity tax to be zero is that $f(c_i)$ is homothetic. In this case, the λ-contraction of the path of the envied agent's compensated demand (the shift of the consumption-leisure bundle along the indifference curve) with the price change moves along some indifference curve of the envying agent. Therefore, $\dfrac{\partial u}{\partial(\lambda \circ x_2)} \cdot \left(\dfrac{\lambda \partial \tilde{c}_2}{\partial \tau^m}, 0\right)$ is zero for all m.[13]

Corollary 1 *Consider a k-output economy, in which preferences are weakly separable. If sub-utility on the goods, $f(\cdot)$ in equation (7), is homothetic, then the optimal commodity tax is zero.*

A sufficient condition to establish that non-distortionary public policies (other than the income tax) are desirable is in fact more restrictive than weak separability. This is because a non-Paretian value judgment requires distortionary public policies in general.

6   Concluding Remarks

Ever since the introduction of the Ramsey (1927) inverse elasticity rule, the optimal taxation literature has emphasized that the tax rule is heavily dependent on the structure of demand,

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that the tax system is redistributive to the unskilled agent, and thus $\eta_1 = 0$ and $\eta_2 > 0$. The interpretation of the term $c_i^m - \hat{c}_j^m$ is that, if the good is consumed more by agent i than by the potential mimicker who is masquerading as agent i, the consumption is encouraged (Edwards *et al.* (1994) and Nava *et al.* (1996)). Roughly speaking, the good which agent i prefers should be encouraged. However, the opposite is implied by the first term. An open problem remains in the general case in that we are not sure whether Lemma 2 can be extended.

[13]With homotheticity but without additive separability, the first term of the RHS of the formula in footnote 12 is zero, and thus the optimal formula coincides with the second-best Pareto efficient formula. This point is discussed in the concluding remarks.

such as Hicksian elasticity, income elasticity, and complementarity with leisure. This carries through to the analysis of envy-reducing optimal taxation, although of course the tax rules derived are in general different from the welfaristic optimal taxation. In the one-output case, we showed that the income elasticity of leisure determines the marginal income tax schedule. Our second main proposition is an 'inverse Ramsey rule', Proposition 2. The envy reduction effect in equation (6) and (8) requires the examination of the demand structure of individuals in determining the optimal taxation for reduction of envy.

The typically assumed demand structure in quantitative analysis is *homotheticity*. That is, denoting $x_i^e \equiv (c_i, \bar{l} - l_i)$ and u^e for the corresponding utility function,

$$u^e(x_i^e) = g(h(x_i^e)) \ (i = 1, 2), \ g \text{ is an increasing function}, h \text{ is homogeneous of degree 1.}$$

We can show that λ^x *is minimized at the leximin allocation among the second-best Pareto efficient allocations in two-class economies* (Nishimura, 1999) and *the Pareto undominated λ-equitability does not require distortionary taxation by the envy requirement itself* (see foot-note 13 as well as the explanation before Corollary 1).[14] With homotheticity, there is no conflict between envy reduction and welfare enhancement of the envying agent. The second proposition is an application of taxation based on sorting: since the λ-contraction of the envied agent's indifference curve coincides with the envying agent's indifference curve, there is no need for encouragement/discouragement of some goods through distortionary taxes

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[14]Derivation of the first claim in n-agent economies is not straightforward. Nishimura (1999) has verified the claim among a limited class of allocations. The second claim follows in n-agent economies as well.

when we are concerned with Pareto-restricted envy-minimization: it merely invites Pareto worsening without reducing envy.

This paper examines tax policy implications on the basis of a non-welfaristic value judgment. No-envy and its alternatives have been one of the most celebrated concepts in the social choice literature, but, needless to say, there are other criteria to be considered. The analysis of tax policy implications based on non-welfaristic value judgments[15] will not only deepen the discussion of the normative analysis of taxation, but also shed light on the theory of distributive justice, from the viewpoint of practical implementability.

Appendix

Proof of Proposition 1: Suppose that leisure is a luxury. Along the λ-constrained second-best frontier, starting from the leximin allocation, there is a range which coincides with the second-best Pareto efficient frontier. This is area 1. Decreasing the transfer to agent 1 sufficiently, the envy constraint becomes binding at some allocation, from which the allowable allocations depart from the second-best Pareto efficient frontier. The situation in area 2 is as follows. On the one hand, as the transfer to agent 1 is still high, self-selection constraint of agent 2 against agent 1 is binding, so that $\eta_2 > 0$. On the other hand, there are two cases to consider concerning t_2. First, only the envy effect is at work: $\eta_1 = 0$, $\mu > 0$. Second, $\eta_1 > 0$ and $\mu > 0$. Under agent monotonicity, the latter happens only at a pooling

equilibrium. Decreasing the transfer to agent 1 further, the allocations belong to area 3, where $\eta_2 = 0$, $\mu > 0$. η_1 might be positive, but the reasoning below shows that the envy effect dominates in determining the marginal tax rate of agent 2, and thus $t_2 > 0$.

We now show that $t_2 < 0$ is excluded when λ satisfies the requirement of the proposition. Let $I(a) \equiv \{b|u(b) = u(a)\}$ be an indifference curve passing through a, and $\tilde{I}((c, l), \lambda) \equiv \{(\hat{c}, \hat{l})|\lambda\hat{c} = \tilde{c}, \ \lambda(\bar{l} - \hat{l}) = \bar{l} - \tilde{l} \text{ for } (\tilde{c}, \tilde{l}) \in I((c, l))\}$ be a homothetic expansion of $I((c, l))$ with proportion $\dfrac{1}{\lambda}$. Let x be a critical allocation where the envy constraint is binding but $\mu = 0$, i.e., the allocation is second-best Pareto efficient. If leisure is a luxury, $I(x_2)$ is steeper than $\tilde{I}(x_1, \lambda^x_{12})$ in (c, l)-space. Consider another allocation z where the redistribution to agent 1 is less than x. To maintain λ-equitability, $z_2 \equiv (c_2^*, l_2^*)$ must be below $\tilde{I}(x_1, \lambda^x_{12})$, i.e., there exists $(\hat{c}, \hat{l}) \in \tilde{I}(x_1, \lambda^x_{12})$ such that $c_2^* \leq \hat{c}$, $l_2^* \geq \hat{l}$, since $u(z_1) < u(x_1)$. Also, for an allocation to be constrained efficient, $z_2 \in \{a|u(a) > u(x_2)\}$ is required. The intersection of these areas is in the south-western area of the income-consumption curve where $MRS_2(y, c) = 1$.[16] In that area, $MRS_2(y, c) < 1$ under normality. By the same argument, further discouragement of agent 2's bundle continues as we increase $\bar{v}_2$ in (3). Therefore, the marginal income tax rate of agent 2 is positive when the allocations are in areas 2 and 3.

When leisure is a necessity, we know that t_2 is negative unless both μ and η_1 are zero. The pattern of the sign of the marginal income tax rates when leisure is a necessity is left to the readers. $Q.E.D.$

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[16] Let $(c_2(T), y_2(T))$ be the utility-maximizing bundle of agent 2 subject to the budget constraint $c = y - T$. The income-consumption curve where $MRS_2(y, c) = 1$ is the collection of $(c_2(T), y_2(T))$, $T \in \mathbb{R}$.

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Figure 1: λ_{ij}^x

Figure 2: the λ-constrained second-best frontier (when leisure is a luxury)
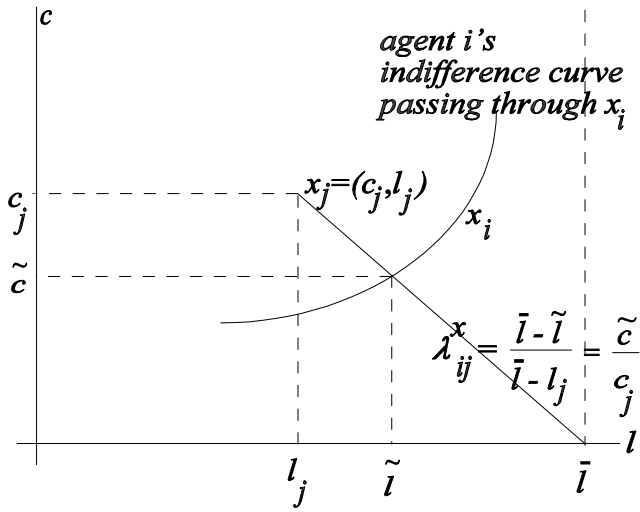
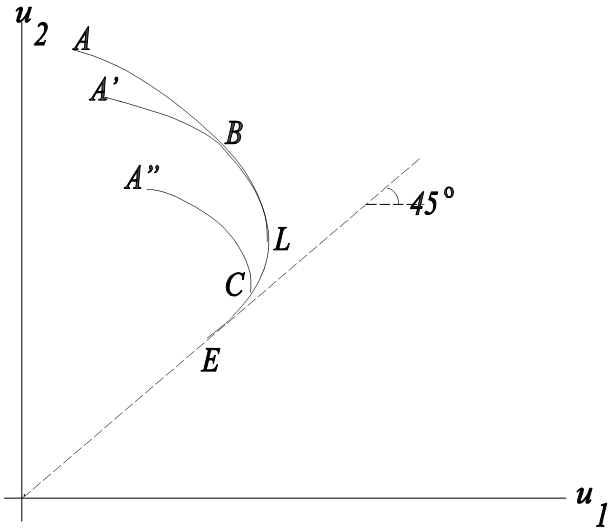Figure 3: Characterization of marginal tax rates

Figure 1: λ^x_{ij}

Figure 2: the λ-constrained second-best frontier (when leisure is a luxury)
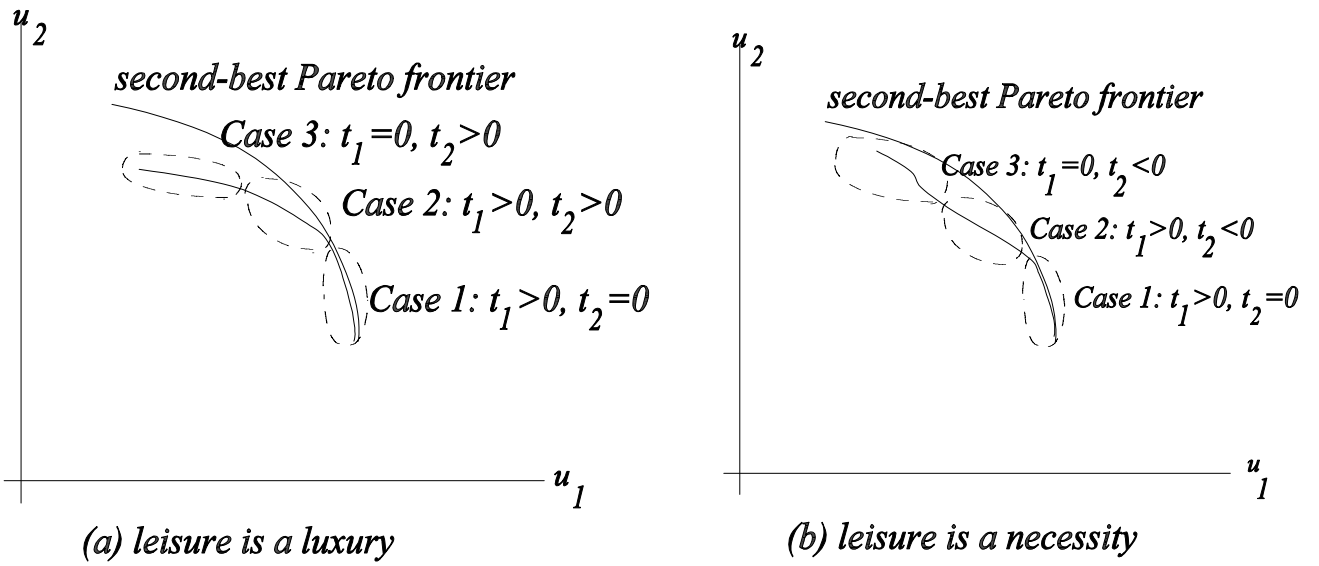
(a) leisure is a luxury

(b) leisure is a necessity

Figure 3: Characterization of the marginal income tax rate