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More on Phantom Bidding

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Abstract

A *phantom bidding* model is analyzed for a sale auction. The following questions are addressed: the effects of phantom bidding on overall social welfare and buyers' profits. It is shown that social welfare may increase or decrease as the auctioneer switches from the fixed reserve price policy to phantom bidding. The buyers' profits will increase whenever social welfare increases.

Key Words: English auction, phantom bidding, fixed/flexible reserve price
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1 Introduction

In an independent private values model with distributionally heterogeneous bidders but distributional identities of specific bidders unknown to the auctioneer/seller, Graham, Marshall and Richard (1990, 1996) have shown that if an English auction is conducted then the auctioneer will engage in non-constant reserve prices that depend upon the highest observed bid. This practice, known as *phantom bidding* (or lift-lining), is quite intuitive: the auctioneer extracts useful information contained in the bid sequence, thus increasing his ex-ante profit.

Using a conditional independent private value model with known identities, we reexamine the phantom bidding problem and address two related questions that so far have not been investigated in the auctions literature: the effects of phantom bidding on overall social welfare and buyers' (ex-ante) profits. It is well known that reserve price in excess of seller's valuation introduces ex-post inefficiency as the object may not sell even if the maximum buyer valuation realized may exceed the seller's valuation. Does this inefficiency problem worsen because of phantom bidding? If the answer is yes, then surely the buyers will lose because the seller will always gain. If the overall welfare improves then a priori it is not clear whether the buyers will get a share of this improvement. We show that under certain conditions, the social welfare does improve because of phantom bidding, and the buyers' payoffs will improve whenever overall social welfare improves.

In Section 2 we analyze the two bidders problem, which is then generalized to n bidders case in Section 3. Section 4 concludes.

2 Two Bidders Problem

A seller uses the standard English (or ascending-bid) auction to award an object to one of two bidders. The seller may announce a *fixed* reserve price before the auction starts, or alternatively he may enter the auction when one of the bidders has stopped bidding and then announce his own bid (or reserve price) in an attempt to further raise the standing bid, which we call the *flexible* (reserve price) rule. Not announcing any reserve price at the start of the auction means the bidders expect the seller to use the flexible rule. We show that while phantom bidding (or flexible rule) always benefits the seller, the overall social welfare may either decrease or increase.

Assume that two bidders both draw their valuations with probability α from i.i.d. $F(\nu)$ with $\nu \in [a,b] \subset \mathcal{R}_+$ and density $f(\nu)$, and with probability $1-\alpha$ from i.i.d. $G(\nu)$ with $\nu \in [b,c] \subset \mathcal{R}_+$ and density $g(\nu)$; both $F(\cdot)$ and $G(\cdot)$ are continuous distributions. Ex-ante, the seller does not know which distribution the buyers' valuations belong to, but holds the prior belief α. The seller's own valuation of the object is zero which is common knowledge.

Conditional on the true distribution, both bidders' valuations are independently distributed. This conditional independence is different from the independence assumption in Myerson (1981), since the unconditional buyers' valuations are correlated. Unlike Graham, Marshall and Richard (1990, 1996), both bidders in our model are identical. So identity is not an issue here.

The expected revenue to the seller from setting a fixed reserve price r is calculated as follows:

$$\begin{aligned} Y &= \alpha\left\{2rF(r)[1-F(r)] + \int_r^b \nu \cdot 2f(\nu)[1-F(\nu)]d\nu\right\} \cdot \mathbf{I}_{[a\le r<b]} \\ &\quad +(1-\alpha)\left\{2rG(r)[1-G(r)] + \int_r^c \nu \cdot 2g(\nu)[1-G(\nu)]d\nu\right\} \cdot \mathbf{I}_{[a\le r<c]} \\ &\equiv \alpha\Phi(r)\cdot \mathbf{I}_{[a\le r<b]} + (1-\alpha)\Gamma(r)\cdot \mathbf{I}_{[a\le r<c]}\,, \qquad (1) \end{aligned}$$

where $I_{[\cdot]}$ is the indicator function and $\Phi(r)$ and $\Gamma(r)$ denote the expressions in the brackets $\{\cdot\}$ in (1). Note that $g(\nu) = 0$ for $\nu \in [a,b]$. In the special case when $\alpha = 1$, denote the revenue-maximizing reserve price by r_1^* (assuming $r_1^* > a$, i.e., the reserve price is interior) which is obtained by satisfying the following first order condition:

$$-r_1^* f(r_1^*) + 1 - F(r_1^*) = 0,$$

i.e.,

$$r_1^* - \frac{1-F(r_1^*)}{f(r_1^*)} = 0. \qquad (2)$$

(The second order condition is easily met: $\frac{\partial^2 Y}{\partial r^2}|_{r=r_1^*} = F(r_1^*)(-2f(r_1^*)) < 0$.) On the other hand, when $\alpha = 0$, the seller-optimal reserve price r_2^* ($r_2^* > b$) is derived similarly:

$$r_2^* - \frac{1-G(r_2^*)}{g(r_2^*)} = 0. \qquad (3)$$

Assume that $\nu - (1-F(\nu))/f(\nu)$ and $\nu - (1-G(\nu))/g(\nu)$ are strictly increasing in ν, implying r_1^* and r_2^* are unique. (Corner solutions $r_1^* = a$ or $r_2^* = b$ are obtained when (2) or (3) are not satisfied by any interior value.) These functions are called "virtual" functions in Myerson (1981) and the above monotonicity conditions are satisfied by frequently used distributions, such as uniform and normal distributions.

First consider the **flexible reserve price** policy. The seller decides the reserve price after one of the bidders drops out at some value x. It is easy to see that a bidder drops out at his valuation. Clearly, if $a \leq x < b$, the seller updates $\alpha(x) = 1$ and thus optimally chooses $r^*(x) = r_1^*$; on the other hand, if none drop out between a and b (or equivalently, $b < x < c$), the seller updates $\alpha(x) = 0$ and chooses $r^*(x) = r_2^*$.[1] Couple of points to be noted here. If $r_1^* < x < b$, then the seller immediately awards the good to the remaining bidder at the price x. Similarly, if $r_2^* < x < c$, then the seller awards the good to the active bidder at the other bidder's dropout price x. We summarize our discussion in the following lemma.

Lemma 1 *The optimal flexible reserve price policy for the seller is determined as follows:*

$$r^*(x) = \begin{cases} r_1^*, & \text{if } a \leq x < b; \\ r_2^*, & \text{if } b < x < c. \end{cases}$$

Now go back to the derivation of the optimal **fixed reserve price**, r^*, for the seller. Assuming that r^* is "interior" (i.e., distinct from a, b and c), it must satisfy the following first order condition:

$$\begin{aligned} &\alpha F(r^*)\{-r^* f(r^*) + 1 - F(r^*)\} \cdot \mathbf{I}_{[a<r^*<b]} \\ &+(1-\alpha)G(r^*)\{-r^* g(r^*) + 1 - G(r^*)\} \cdot \mathbf{I}_{[b<r^*<c]} = 0. \end{aligned} \tag{4}$$

It is easy to see that there are exactly two solutions to eq. (4), r_1^* and r_2^*. These solutions yield local maxima to the seller's revenue, viz. $Y = \alpha\Phi(r_1^*) + (1-\alpha)\Gamma(b)$ and $Y = (1-\alpha)\Gamma(r_2^*)$: if $\alpha\Phi(r_1^*) + (1-\alpha)\Gamma(b) \geq (1-\alpha)\Gamma(r_2^*)$ then the optimal fixed reserve price $r^* = r_1^*$, otherwise $r^* = r_2^*$. Note that the corner solutions yield similar revenues. Simplifying, we obtain the following lemma:

Lemma 2 *The optimal fixed reserve price for the seller is determined as follows:*

$$r^* = \begin{cases} r_1^*, & \text{if} \quad \alpha \geq \frac{\Gamma(r_2^*)-\Gamma(b)}{\Phi(r_1^*)+\Gamma(r_2^*)-\Gamma(b)}; \\ r_2^*, & \text{otherwise.} \end{cases}$$

[1] The probability of one or both bidders dropping out at the value, b, is zero because $F(\cdot)$ and $G(\cdot)$ are continuous distributions.

Below we report our first important observation.

Proposition 1 *The (ex-ante) social welfares corresponding to the fixed and the flexible reserve price rules can be compared as follows:*

$$\begin{aligned} W_{fix} &> W_{flex} \quad \textit{if} \quad r^* = r_1^* \ ; \\ W_{fix} &< W_{flex} \quad \textit{if} \quad r^* = r_2^*. \end{aligned}$$

Proof. The social welfare is the sum of buyers' surplus and seller's surplus. Since the seller's valuation of the object is zero, there will be welfare loss whenever the object remains unsold and at least one buyer's valuation is strictly positive. The probability that a buyer's valuation is zero is zero.

When $r^* = r_1^*$, the only case the object is not sold is when both valuations are below r_1^* under the fixed reserve price rule; on the other hand, under the flexible reserve price rule the object is not sold when both valuations are below r_1^*, and also when $G(\nu)$ is the true distribution and both valuations are below r_2^*. However, when $r^* = r_2^*$ and the true distribution is $F(\nu)$, the object is not sold under the fixed reserve price rule whereas the object may be sold under the flexible reserve price rule; if the true distribution is $G(\nu)$ then the object is sold under the same circumstances for both fixed and flexible reserve price rules. **Q.E.D.**

Next we compare bidders' (ex-ante) profits under flexible rule and under fixed reserve price rule.

Proposition 2 *The (ex-ante) profits to a bidder corresponding to the fixed and the flexible reserve price rules compare as follows:*

$$\begin{aligned} \Pi_{fix} &> \Pi_{flex} \quad \textit{if} \quad r^* = r_1^* \ ; \\ \Pi_{fix} &< \Pi_{flex} \quad \textit{if} \quad r^* = r_2^*. \end{aligned}$$

Proof. Suppose $r^* = r_1^*$. In this case, when ν_1, ν_2 are drawn from $F([a, b])$, a bidder will be indifferent between fixed reserve price rule and flexible rule because his profit margins in the event of winning the object remain unchanged and the probability of winning is not affected whether the auctioneer employs fixed or flexible rule. On the other hand, if ν_1, ν_2 are drawn from $G([b, c])$, probability of winning is not affected by the fixed or flexible rule, the winner's ex-post profit never increases under the flexible rule while it is reduced whenever $b < \nu_1 < r_2^* < \nu_2$ (ex-post profit under fixed reserve price rule is $\nu_2 - \nu_1$ because $r^* = r_1^*$, whereas ex-post profit under flexible rule is $\nu_2 - r_2^*$).

Suppose now that $r^* = r_2^*$. If ν_1, ν_2 are drawn from $G([b,c])$ then a bidder is indifferent between the fixed reserve price rule and the flexible rule: there is no change in the probability of winning nor is there any change in the profit margin. On the other hand, if ν_1, ν_2 are drawn from $F([a,b])$ then the probability of the object's sale increases; a bidder's ex-post profits, and thus ex-ante profits, will increase sometimes and never decrease: ex-post profits will increase from zero to $\nu_2 - r_1^*$ when $\nu_1 < r_1^* < \nu_2$, as the auctioneer switches from fixed to flexible rule. **Q.E.D.**

The favorable effect of phantom bidding on buyers' profits is a new and somewhat surprising result. Perhaps the conventional wisdom would have been one of unambiguous and adverse effect on buyers' profits. The other interesting aspect is the unidirectional effects on social welfares and buyers' profits, as the auctioneer switches from fixed to flexible reserve price rule. We like to note, however, that this result is due to our assumption that the two alternative supports from which both buyers' valuations are drawn do not overlap. In the case of overlapping supports, while flexible and fixed reserve prices can be calculated following similar procedures as in the non-overlapping support case analyzed in this paper, the comparison of the two reserve price rules in terms of their effects on social welfare and buyers' profits no longer yield clear-cut results, although similar intuitions remain valid.

3 The n Bidders Case

Extending the above result to n bidders case is straightforward. Modified calculations are as follows:

$$Y = \alpha \Phi_n(r) \cdot \mathbf{I}_{[a \le r < b]} + (1-\alpha)\Gamma_n(r) \cdot \mathbf{I}_{[a \le r < c]},$$

where

$$\Phi_n(r) = r \cdot nF(r)^{n-1}[1 - F(r)] + \int_r^b \nu \cdot n(n-1)F(\nu)^{n-2}[1-F(\nu)]f(\nu)d\nu,$$

and

$$\Gamma_n(r) = r \cdot nG(r)^{n-1}[1 - G(r)] + \int_r^c \nu \cdot n(n-1)G(\nu)^{n-2}[1-G(\nu)]g(\nu)d\nu.$$

The first order condition for the determination of r^* is:

$$\begin{aligned} \alpha \cdot nF(r^*)^{n-2}\{-r^*F(r^*)f(r^*) \quad &+ \quad F(r^*)(1-F(r^*))\} \cdot \mathbf{I}_{[a<r^*<b]} \\ +(1-\alpha) \cdot nG(r^*)^{n-2}\{-r^*G(r^*)g(r^*) \quad &+ \quad G(r^*)(1-G(r^*))\} \cdot \mathbf{I}_{[b<r^*<c]} = 0, \end{aligned}$$

which give the same solutions r_1^* and r_2^* as in the 2-bidders' case. There are two local maxima: $Y = \alpha\Phi_n(r_1^*) + (1-\alpha)\Gamma_n(b)$ and $Y = (1-\alpha)\Gamma_n(r_2^*)$. The optimal fixed reserve price is thus derived as:

$$r^* = \begin{cases} r_1^*, & \text{if} \quad \alpha \geq \frac{\Gamma_n(r_2^*)-\Gamma_n(b)}{\Phi_n(r_1^*)+\Gamma_n(r_2^*)-\Gamma_n(b)}; \\ r_2^*, & \text{otherwise.} \end{cases}$$

($\Gamma_n(r_2^*) - \Gamma_n(b) > 0$ by the assumption of the interior solution r_2^*.) Thus, comparing the two bidders and the n bidders cases, the only difference is in the range of α values over which the optimal fixed reserve price is r_1^* or r_2^*.

The comparison of welfares and bidders' profits is exactly the same as in the two bidders' case.

4 Conclusion

While it is generally known that phantom bidding favors the auctioneer, perhaps less anticipated is that it may benefit the buyers as well. In this paper we show that this is indeed possible and phantom bidding can increase the overall social welfare.

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