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Industry Dynamics over the Business Cycles

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Abstract

We develop a theoretical model of the dynamics of an industry over the business cycle. In the economy, both aggregate demand and the productivity of a firm's technology evolve stochastically. Each period, firms must choose whether to produce or to exit and attempt to sell off their resources to an entrant, so there is a non–trivial opportunity cost of production. We characterize the intertemporal evolution of the distribution of firms, where firms are distinguished by their capital in place and the productivity of their technology. We characterize exit rates by age, size and productivity. A useful social planner's characterization of the competitive equilibrium is provided. Predictions of our theoretical model are broadly consistent with observed cyclical patterns.

Keywords: stochastic heterogeneity, aggregate shocks, social planner, exit, capital in place, thin markets.

JEL: E32, L16

1 Introduction.

In this paper we develop a theoretical model of the dynamics of an industry over the business cycle. The purpose is to characterize the intertemporal evolution of the distribution of firms, where firms are distinguished both by their capital in place and their productivity. We characterize exit and investment decisions of firms over the cycle by their age, size and productivity, and obtain theoretical findings that are broadly consistent with those in the empirical literature.

To highlight our contribution, we first describe the key modeling features of our economy, and then discuss their empirical importance and how they compare with the modeling assumptions made in related papers. These key features are:

A. Firms differ in innate productivity, and the productivity of a firm's technology evolves stochastically over time.

B. Both random aggregate demand shocks and aggregate output combine to determine equilibrium output prices.

C. Firms choose both capital and labor, making capital choices prior to learning their productivities and the aggregate demand shock, and making labor and production/exit decisions after learning both their productivity and market conditions.

D. Exiting firms sell their resources to entering firms. Exiting firms cannot produce: Production takes the time that would go to the search necessary to find a purchaser for their resources and the time for the purchaser to 'retool' those resources for its own purposes.

E. We characterize industry dynamics in three ways: contrasting outcomes *across* different demand shock histories, contrasting outcomes *along* a particular demand shock history, and detailing how *anticipation* of future demand shocks affects outcomes.

A. Hopenhayn (1990, 1992a, 1992b) was the first to stress the importance of (stochastic) heterogeneity across firms within a rich theoretical equilibrium model in order to explain empirical regularities regarding the individual actions of firms.[1] For example, entering and exiting firms account for about 40 percent of manufacturing firms (Dunne *et al.* 1989a). Entering and exiting firms differ substantially from continuing enterprises. Both entering and exiting firms are substantially smaller than continuing firms[2] , exiting firms have higher costs and the conditional probability of exit declines with both age and size (Dunne *et al.* 1989b, Evans 1987). Surviving entering firms grow more quickly than their older counterparts. Cabellero, Engel and Haltiwanger (1995) found

[1] Other models of industry dynamics with individual firm heterogeneity, but no business cycle components, include Jovanovic (1982) and Ericson and Pakes (1989).

[2] Lieberman (1990) found that 76 percent of exiting firms were in the bottom half of the size distribution the previous year.

that most of U.S. aggregate manufacturing employment fluctuations are due to fluctuations in the cross–sectional distribution of employment deviations.

The magnitudes of these fluctuations are enormous: about 60 percent of all firms exit within 5 years of entry and are replaced by new firms (Davis *et al.* 1995); rates of job creation and destruction average about 10 percent a year, but are highly concentrated — only 23 percent of job destruction is accounted for by establishments that shrink by less than 20 percent over a span of one year (Davis and Haltiwanger 1992). These findings emphasize the importance of firm specific sources of uncertainty for firm survival and investment dynamics. Hopenhayn's (1992) model without aggregate uncertainty generated predictions consistent with many of these empirical observations.

B. However, firm specific uncertainty is but one component affecting industry dynamics. Exit and investment decisions of firms also vary significantly over the business cycle, and the business cycle affect different sizes of firms in different ways. Campbell (1995) summarized the empirical findings that exit rates are counter–cyclical, but correlations of future exit rates with future GDP growth are positive large, persistent and statistically significant. Davis and Haltiwanger (1992) found that downsizing by large firms is strongly counter–cyclical. Davis *et al.* (1995) found that small firms exit more quickly in response to downturns than larger firms, which downsize first. Lieberman (1990) found that in industry downturns, small firms exit and large firms downsize causing a strong trend toward size convergence. In industry upturns, small exiting firms are replaced by new entrants, large producers expand and mean firm size and cross–sectional variance in firm sizes gradually increase. Our theoretical model generates predictions broadly consistent with these empirical regularities characterizing these industry dynamics over the business cycle.

C. As is standard to most macroeconomic business cycle models, we assume that capital investments take time and must be made prior to learning demand and technology productivity shocks. This requires that we track the dynamic evolution of the joint distribution of capital and firm productivities over the business cycle, but it allows us to generate predictions consistent with the empirical regularities discussed above. In contrast, Hopenhayn (1992a, 1992b) assumed that all inputs, including capital, are chosen *after* firms learn demand and productivity shocks, so that a firm's technology uniquely determines its inputs and outputs, and distinctions cannot be drawn between firm size and technology. Campbell (1995) numerically analyzed a general equilibrium real business cycle model of industry dynamics, melding a version of Hopenhayn's (1992a, 1992b) model with a simple vintage capital model that embodies aggregate uncertainty (only) through innovations to the mean technology quality of new entrants. Campbell assumed that once capital stocks are chosen by entrants, capital stocks cannot be adjusted to reflect the productivity of a firm's technology, so that capital levels of a given generation are best interpreted as the number of entrants at that date. We first explore how the economy behaves when all inputs are freely adjustable; that is, we first adopt the Hopenhayn timing for inputs. We then characterize the ways in which capital in place affects outcomes.

D. Most models of industry dynamics (*e.g.* Hopenhayn 1992a, 1992b, Campbell 1995) allow for entry (entry has an invariant cost), but have no opportunity cost of production and simplify exit by assuming a time and state invariant value of exit. In contrast, we fix the number of firms in the economy, enrich exit decisions and explore the consequences. We endogenize the value of exit in a non–trivial way that reflects current market conditions: each period, the opportunity cost to a firm of using its specialized resources in production is that it could exit the market and sell its resources to a potentially more efficient entrepreneur. That is, production takes the time that would go to finding a buyer who can better employ the resources, and the time the buyer must expend in order to re–tool the resources for its own use. Such an entrepreneur lacks other access to input resources. This formulation takes seriously resource constraints on new entrants: entrants must obtain "storefront space" from existing firms before entering the market, as entering from scratch is prohibitively costly.

The key features of this exit formulation are that there is a non–trivial opportunity cost of continued production and that the endogenous value both of remaining in production and of exiting the market vary over the business cycle. That is, when demand is higher, the foregone cost of production is higher, but persistence in demand implies that the value of a more productive future technology is also higher. We detail conditions under which the opportunity cost of foregone production varies more with current market conditions than does the sale price of resources. As a consequence, downturns in demand cause increased exit of less productive firms, so that the future distributions of firm productivities (ordered by stochastic dominance) and hence output are increased at each future date and state. Thus, our theoretical findings are consistent with the empirical regularities summarized by Campbell (1995). So too, the fluctuations in exit associated with demand fluctuations combined with a constant positive productivity growth rate generate the asymmetric features of business cycles: recessions are sharper and shorter–lived than booms.

We allow for the possibility that the resources of a bankrupt firm are sufficiently specialized that their resale market may be thin. That is, there may be insufficient competition, so that a bidder for a bankrupt firm's resources may win with an offer that is less than their expected value. This limited competition drives a wedge between the social and private opportunity cost of the resources. A bad firm recognizing that it may not receive the social value of its resources, may continue to operate rather than fold. We illustrate, by way of an example, that when resale markets are thin, a downturn in demand can actually *enhance* welfare because it narrows the wedge between the social and private opportunity cost of the resources, increasing exit. Thus, the Darwinian, cleansing effect of a downturn on exit can raise future expected welfare by more than the reduction in welfare caused by the immediate downturn. We also detail the cross–industry implications of varying thinness of markets for exit, capital stocks and firm productivities.

E. We provide three different types of characterizations of business cycle dynamics. We first provide sufficient conditions that enable us to compare outcomes *across* different demand realization paths, contrasting investment and exit decisions and their consequences for aggregate output, profits and productivity distributions when one history of demand realizations is uniformly better than another. We also detail the consequences for outcomes *along* a demand history path, comparing outcomes at date t with those at date $t+1$, so that we can detail the effect of continued good or bad market conditions on outcomes. Lastly, we characterize the consequences of an improvement in *anticipated future* market conditions on current and interim firm decisions and the consequences for the aggregate economy. For each of these three types of characterizations, we show that our theoretical predictions match qualitatively the empirical regularities in the data.

Campbell (1995) and Caballero and Hammour (1994) both consider the consequences of anticipated future changes in the economy. Caballero and Hammour develop a deterministic model in which the productivity of new technologies grows at a constant rate; the productivity of a firm's technology is determined by its date of entry. Anticipation of a cyclical downturn in demand leads both to less entry of more productive firms and to more exit of older, less productive, technologies. Equilibrium entry–exit behavior depends critically on the structure of entry costs. If a firm's cost to entry is independent of the number of entrants, then fluctuations in demand are accommodated entirely by variation in the number of entrants and the age at which firms exit is constant, implying that *average* firm productivity would fall in a downturn. Only if there are sufficient entry cost externalities, so adjustment is primarily via exit (of unproductive firms), can downturns lead to a higher *average* technology quality.

In the next section we present the model. Section 3 develops the analysis when all inputs are variable, as in Hopenhayn (1992a,1992b). Section 4 considers the case where capital stocks are determined prior to the realizations of demand and technology productivity shocks. We extend the social planner insights of Hopenhayn (1990), detailing conditions under which the outcomes in the competitive equilibrium correspond to those obtained by a social planner who maximizes expected total discounted surplus. We provide explicit derivative characterizations of the solution to the social planner's problem that enable us to determine the consequences of different demand shocks for current output and exit, and for future distributions of firm productivities and output at each date and state. Section 5 considers the possibility that the market for an exiting firm's specialized resources are thin. Section 6 concludes. All proofs are in the appendix.

2 The Model

In each period, demand conditions are represented by a demand function, $p(Y, \theta)$, a function of aggregate output, Y, and a random demand shock, $\theta \in \Theta$. It is assumed that $\frac{\partial p}{\partial Y} \leq 0$, that for all θ, $p(0, \theta) > 0$, and that $p(\cdot, \theta) \geq 0$. The aggregate demand shock, θ, follows a Markov process: given the current value of θ, next period's θ shock is drawn according to $\Theta(\cdot \mid \theta)$. We assume that $\Theta(\cdot \mid \theta)$ is continuous in θ.

The output of a firm is given by $f(\ell, \alpha, k)$, where $\alpha \in [0,1]$ captures the quality or productivity of its technology, $k \in [0, \infty)$ is the level of its capital stock, and $\ell \geq 0$ is its input of labor. The production function is strictly monotone increasing in its arguments, strictly concave in k and ℓ, features strictly complementary inputs, and $f(\ell, \alpha, k) = 0$ if either ℓ or k is 0. At the end of the period an operating firm with technology α receives a new realization of its technology, drawn from a distribution $P(\cdot \mid \alpha)$, that is assumed continuous in α.

Depending on context, a firm is characterized either by its technology, or by both its technology *and* its capital stock. Given its type, each firm in existence decides whether to produce that period, or to exit the market and attempt to find a purchaser for its resources. A firm that exits in the current period does not produce that period. The only way a new firm can enter is by purchasing the resources of a firm that exits the market. Implicit is an assumption that it is less costly for a new firm to purchase resources of an exiting firm than it is to start from scratch.

For most of the analysis, we assume that the market for a firm's resources is competitive. That is, we assume that there are always multiple potential buyers who bid the purchase price up to the market value of the resources, the present discounted expected value of the firm going into production next period. However, in section 5, we consider the possibility of thin markets for the firm's resources.

There is a significant opportunity cost to continued production: production takes both the time that would go to finding a buyer that can better employ the resources, as well as any time the buyer requires to re–tool the resources for its own use. One can interpret our model both in a manufacturing context and a retail context. In the retail setting, an operating firm employs storefront space that could be sold to a new entrant who potentially could better exploit those resources. Since the value of storefront space varies over the business cycle, so too do the respective values of producing and exiting. When demand is higher, the foregone cost of production is higher, but persistence in demand implies that the value of a more productive future technology is also higher.

2.1 The Timing of a Firm's Decisions

We study two timing formulations for the input choices of firms. In the first, the firm chooses its capital stock and labor each period after demand and technology productivity shocks have been realized: both capital and labor are freely adjustable. In the second formulation, firms must make their capital investments prior to knowing the demand and technology realizations. This second timing formulation captures the time–to–build feature of investment. In the first formulation, the only identifying feature of a firm is the productivity of its technology because, given any demand, a firm's technology quality uniquely determines its input levels. We use μ to denote the distribution of firm technology productivity parameters in the economy. In contrast, in the second timing formulation, firms are distinguished by both their technologies *and* their capital stocks. This is because after capital stocks have been chosen, firms will have different technology productivity realizations. We again use μ to denote the distribution over firm technology productivity parameters, α, and let τ denote the joint distribution over firm productivities and capital stocks. We refer to the first model as featuring "variable capital" and to the second as featuring "capital in place".

Variable capital: The timing within a period in the case of variable capital is as follows. The aggregate shock, θ, determines demand, $p(Y,\theta)$. Given a market price p, each firm, characterized by technology, α, decides whether to remain in the market or exit. If the firm remains in the market, it chooses capital and labor to maximize profit. Otherwise, it exits and searches for a buyer. Thus, if the prevailing market price is p (a function of the measure on firms supplying the market and the shock θ), firm α solves: $\max_{\ell,k} pf(\ell,k,\alpha) - w\ell - rk$, where $w > 0$ is the wage rate and $r > 0$ is the price of a unit of capital. The solution to this program gives input demands, $\ell(p,\alpha)$ and $k(p,\alpha)$, and supply function $y(p,\alpha) = f(\ell(p,\alpha), k(p,\alpha), \alpha)$. With measure μ on technologies, given some exit rule α^*, if μ_{α^*} denotes the measure on firms that remain in the market, then aggregate supply is $Y = \int_C y(p,\alpha)\mu_{\alpha^*}(d\alpha)$. Thus, in equilibrium, profit for firm α in the period is determined by the aggregate shock, the measure of firms continuing to operate, and the productivity of its technology: $\pi(\theta, \mu_{\alpha^*}, \alpha)$. The time–line for decisions is illustrated below:

$\longrightarrow$	θ_t, α_t realized	$\longrightarrow$	Exit decisions made	$\longrightarrow$	Capital and Labor choices	$\longrightarrow$	Production and Sale	$\longrightarrow$	θ_{t+1}, α_{t+1} realized	$\longrightarrow$

Capital in place: With capital in place, given the aggregate demand shock, θ, and the joint distribution, τ, over firm technologies and capital stocks, each firm, fully characterized by $s = (k,\alpha)$, decides whether to produce or sell its resources. Firms that decide to continue operating then choose labor to maximize profits. At output price $p > 0$ and wage $w > 0$, the operating profit of firm s is given by $\pi(p,\ell,s) = pf(\ell,s) - w\ell$. Let $\ell(p,s)$ be the associated unique profit–maximizing level of labor employed, generating associated profits, $\pi(p,s) = \max_\ell \pi(p,\ell,s) = \pi(p,\ell(p,s),s)$. Given p, the output of firm s is uniquely determined by $y(p,s) = f(\ell(p,s),s)$. If τ is the distribution of firm types ((α,k) pairs), and τ_ψ is the distribution following exit under exit rule ψ, then aggregate output when the market–clearing price is p is $Y = \int_T y(p,s)\tau_\psi(ds)$. The price when aggregate output equals Y and the demand shock is θ is $p(Y,\theta)$. Equilibrium requires that given θ, $Y = \int_T y(p(Y,\theta),s)\tau_\psi(ds)$. Since $p(Y,\theta)$ is non–increasing in Y and f is strictly concave in ℓ, there is a unique solution $Y^* = \int_T y(p(Y^*,\theta),s)\tau_\psi(ds)$. Let equilibrium output at (τ_ψ,θ) be $Y(\tau_\psi,\theta)$. Thus, given τ and θ, there is a unique equilibrium output $Y(\tau_\psi,\theta)$ and the operating profit to firm s is $\pi(\theta,\tau_\psi,s)$. Finally, given θ, the distribution of active firm productivities and the measure of exiting firms, both active firms and new entering firms choose investments, $\imath$, to maximize future expected profits, where firms take expectations over future demand and technology shocks. Then firms realize their productivity shocks and next period's demand shock is realized. A firm's capital stock is assumed to depreciate at the rate $\rho \geq 0$, so that if a firm with current capital stock k invests in $\imath$ additional units of capital at price r, its capital stock the next period becomes $\rho k + \imath \geq 0$. We allow the possibility that firms can dis–invest, *i.e.* $\imath < 0$. For capital in place, the time–line is:

$\longrightarrow$ θ_t, α_t realized $\longrightarrow$ Exit decisions made $\longrightarrow$ Labor choice $\longrightarrow$ Production and Sale $\longrightarrow$ Investment i_t Chosen $\longrightarrow$ θ_{t+1}, α_{t+1} realized $\longrightarrow$

Although the "capital–in–place" or "time–to–build" timing assumption is standard for dynamic real business cycle models — capital investments take time and must be made prior to learning demand and technology productivity shocks, while firms can adjust their labor demands after learning these realizations — it is not the timing assumption that Hopenhayn (1992a,b) or Campbell (1995) consider. Hopenhayn assumes that all inputs are chosen *after* firms learn the demand and productivity shocks, so that, given the aggregate state, a firm's technology uniquely determines its inputs and output. Campbell assumes that once capital stocks are chosen by entrants, capital stocks cannot be adjusted to reflect the productivity of a firm's technology.

It is more difficult to analyze the economy in which some inputs are determined prior to learning demand and technology productivity realizations and some inputs are determined after learning the shocks because it requires that the *joint distribution* of capital and technology qualities be tracked over time. However, tracking the joint distribution allows us to distinguish the very different effects that firm size (capital) and technology have on production and exit decisions, and to match the associated empirical regularities.[3]

2.2 The Evolution of Technologies

We assume that the transition distribution on a firm's technology, $P(\cdot \mid \alpha)$, is such that firms with better existing technologies tend to remain better:

A1. If $\alpha' \geq \alpha$, $P(\cdot \mid \alpha') \succeq P(\cdot \mid \alpha)$. [4]

If a firm exits and has its assets purchased by an entering firm, we assume that the technology of the new entrant reflects that of a firm with an intermediate level of technology quality, $\bar{\alpha}$:

A2. The technology quality of a new firm is drawn from the distribution $P(\cdot \mid \bar{\alpha})$, $\bar{\alpha} \in (0, 1)$.

Hence, it may be profit–maximizing for a low α firm to exit and sell its resources to a new entrant which, given **A1**, will have a stochastically better technology the next period. That entrants tend

[3] Downsizing by large firms is strongly counter–cyclical; in downturns, small firms exit more quickly than large firms, which downsize first, reducing the cross–sectional variation in firm sizes, etc.

[4] Given two probability measures ψ and ψ' on $[0, 1]$, ψ (first order) dominates ψ', written $\psi \succeq \psi'$ if for each $x \in [0, 1]$, $\psi((x, 1]) \geq \psi'((x, 1])$.

to be smaller than continuing firms (see *e.g.* Dunne *et al.* (1989a)) is consistent with $\bar{\alpha}$ being smaller than the mean α of a continuing firm. A firm with technology $\alpha \geq \bar{\alpha}$ will not exit. Not only would exit require foregoing the profits of production, but any purchaser will draw a technology from a worse distribution next period. Consequently, the amount a purchaser would be willing to pay could never compensate α for the revenues it expects from remaining in the industry. Only firms with technologies sufficiently below $\bar{\alpha}$ (below some (state dependent) α^*) will exit, balancing lost profit against future increased value. Thus, if technologies below α^* exit, if μ is the technology distribution, then the distribution on remaining technologies is $\mu_{\alpha^*}(X) = \mu(X \cap [\alpha^*, 1])$. The technology distribution next period becomes:

$$\int_{[\alpha^*,1]} P(\cdot \mid \alpha)\mu(d\alpha) + \int_{[0,\alpha^*)} P(\cdot \mid \bar{\alpha})\mu(d\alpha).$$

A somewhat more complicated expression holds for the capital in place model (see Section 4). This expression assumes that all exiting firms re–enter the market next period.[5] In Section 5, we consider the case where there is positive probability that a firm which exits may not find a bidder for its resources and hence may not re–enter the market next period.

As an aside, we note that it is straightforward to extend the analysis (see Bergin and Bernhardt 1993, 1995) so that the transition distribution on the productivity of a firm's technology depends not only on the firm's existing technology, but also its level of research and development, d: $P(\cdot \mid \alpha, d)$. Many of the qualitative insights obtained with R&D are similar to those obtained with capital in place; others depend on the particular structure that one places on the R&D process. Perhaps as important, we could relax the time independence of the transition distributions on demand and technology qualities: $\Theta_t(\cdot \mid \theta)$ and $P_t(\cdot \mid \alpha)$. Then one can allow for technological growth and for trends in demand by assuming that $\Theta_t(\cdot \mid \theta) \succeq \Theta_{t-1}(\cdot \mid \theta)$ and $P_t(\cdot \mid \alpha) \succeq P_{t-1}(\cdot \mid \alpha)$. Where allowing for growth affects our qualitative findings, we will discuss the issues in more detail.

2.3 Existence of Equilibrium

Under mild assumptions on the transition functions and payoffs, it can be shown that an equilibrium exists, even if the θ and α processes are nonstationary. In principle, this may involve strategies that condition at time t on the past history of aggregate shocks. When only the current distribution over firms and the current shock θ affect agents' decisions, the process evolves as a Markov process. These issues are discussed at length in Bergin and Bernhardt (1995).

For the most part, we concern ourselves with the situation where the market for an exiting firm's resources is competitive. In this case, we show that the unique competitive equilibrium may be

[5] When there is positive probability that an exiting firm will not find a buyer, the second integral must be scaled down accordingly.

expressed as the solution to a social planner problem. That is, the decision rules of individual firms in the competitive equilibrium correspond to those of a social planner who maximizes the expected discounted sum of consumer and producer surplus (see, for example, Lucas and Prescott (1971), Jovanovic (1982), Hopenhayn (1990, 1992a,b), or Campbell (1995)). Here we make explicit use of our derivative characterization of the solution to the social planner's problem, a characterization that enables us to determine the consequences of a marginal increase in exit on future output, investment, prices and profits in all future dates and states of the world. In section 5, we consider less competitive markets for a firm's resources, in which a bidder for a bankrupt firm can sometimes extract rents. In this case, firms are more reluctant to exit in the competitive equilibrium than is optimal from the perspective of a social planner who seeks to maximize total discounted expected surplus.

3 Variable Capital

Suppose the current aggregate shock is θ, and the current distribution over technology qualities is μ. An exit rule defines, by technology, a set of firms that exit. Since $P(\cdot \mid \alpha)$ is increasing in α and the value of exit is independent of α, any equilibrium rule will have the property that if some α continues to operate, then so do all firms with technologies better than α. Consequently, in considering the distribution of operating firms, we need only consider truncated distributions from the set

$$\mathcal{C}(\mu) = \{\hat{\mu} \mid \hat{\mu}(X) = \mu(X \cap [\alpha', 1]), 0 \leq \alpha' \leq 1\}.$$

Let μ_α denote the distribution obtained when all firms with technologies that are less productive than α exit: $\mu_\alpha(X) = \mu(X \cap [\alpha, 1])$. Let

$$\mu^\alpha(\cdot) = \int_{[0,\alpha)} P(\cdot \mid \bar{\alpha})\mu(d\alpha) + \int_{[\alpha,1]} P(\cdot \mid \alpha)\mu(d\alpha)$$

denote the distribution of next period's technology qualities when firms exit if and only if their technologies are worse than α.

3.1 Equilibrium Exit

In this section we determine the value of continuing in the industry and producing, v^c, the value of exiting to sell the resources of the firm, v^e, and then solve for the marginal exiting firm. Finally, we show how the exit decisions in the competitive equilibrium correspond to those made by a social planner who seeks to maximize expected discounted social surplus.

Suppose we fix an exit rule associating an α–threshold for exit as a function of (μ, θ): $\alpha(\mu, \theta)$. This fully determines the evolution of the aggregate distribution μ over time (given that the aggregate shock evolves according to $\Theta(\cdot \mid \theta)$). Given this, valuations for individual firms may be computed. Let $\alpha^* = \alpha(\theta, \mu)$, so that the expected value to a firm α from operating is

$$v^c(\theta, \mu, \alpha) = \pi(\theta, \mu_{\alpha^*}, \alpha) + \beta \int\int v(\tilde{\theta}, \mu^{\alpha^*}, \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)P(d\tilde{\alpha} \mid \alpha),$$

where $v(\theta, \mu, \alpha) = \max\{v^c(\theta, \mu, \alpha), v^e(\theta, \mu, \alpha)\}$ reflects that the firm will make optimal exit decisions in the future, "v^c" denoting the value of continuing to operate, and "v^e" denoting the value of exiting. That is, the value of an operating firm α facing market conditions (θ, μ) is equal to the sum of the maximized operating profits it receives plus the discounted expected value of continuing to operate given that it chooses inputs optimally and makes future operation–exit decisions optimally. Hence, the expected value to exit is

$$v^e(\theta, \mu, \alpha) = \beta \int\int v(\tilde{\theta}, \mu^{\alpha^*}, \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)P(d\tilde{\alpha} \mid \bar{\alpha}),$$

reflecting that an exiting firm can sell its resources for their full discounted expected value to a new firm whose technology quality is drawn according to $\bar{\alpha}$.

Note that v^e is independent of α while v^c is increasing in α so that there is a unique value, $\hat{\alpha}$, at which v^c and v^e are equal. Firms with technologies above $\hat{\alpha}$ wish to continue and firms with technologies below $\hat{\alpha}$ wish to exit. A necessary condition for equilibrium is that $\alpha^* = \hat{\alpha}$. Thus, a necessary condition for equilibrium is that the exit rule $\alpha(\theta, \mu)$ satisfy: $v^c(\theta, \mu, \alpha(\theta, \mu)) = v^e(\theta, \mu, \alpha(\theta, \mu))$ for all (θ, μ).

For this economy, one can relate the exit decisions in the competitive equilibrium to the exit rules of a social planner who seeks to maximize discounted social surplus. The period social surplus can be represented as the area between the demand and supply curves. Let $P_s(Q, \mu')$ denote the aggregate supply curve when the distribution of firms in operation is μ'. Then if total output is Q^*, the social surplus is $S(Q^*, \theta, \mu') = \int_{[0,Q^*]}[P(Q, \theta) - P_s(Q, \mu')]dQ$. The social planner program is the optimization of the present value of the social surplus stream, by choice of continuation (and hence exit) distribution. The functional equation for the social planner's problem is:

$$V(\theta, \mu) = \max_{\alpha}\{\max_{Q} \int_0^Q [P(Q, \theta) - P_s(Q, \mu_\alpha)]dQ + \beta \int_\Theta V(\tilde{\theta}, \mu^\alpha)\Theta(d\tilde{\theta} \mid \theta)\}.$$

Here, $P_s(Q, \mu_\alpha)$ is the aggregate supply function, reflecting that the labor choices of operating firms correspond to those made by the social planner. (If at price P, firm α supplies $y(P, \alpha)$, then total output is $Q(P, \mu_\alpha) = \int y(P, \tilde{\alpha})\mu_\alpha(d\tilde{\alpha})$. Inverting for P gives $P_s(Q, \mu_\alpha)$.) The solution to this program gives at each (θ, μ), an exit rule, $\alpha(\theta, \mu)$, and determines the evolution of the aggregate distribution over time.

Proposition 1 *Given* **A1** *and* **A2**, *the exit rule characterizing the solution to the social planner problem, α^*, satisfies for each (μ, θ)*

$$v^c(\theta, \mu, \alpha^*) = v^e(\theta, \mu, \alpha^*).$$

This exit rule corresponds to that in the unique competitive equilibrium. Further, $\alpha^* < \bar{\alpha}$.

Proposition 2 *If* **A1** *and* **A2** *hold, then, for each* (μ, θ)*, there exists a unique exit threshold* $\alpha^*(\mu, \theta)$*, such that firms with* $\alpha > \alpha^*(\mu, \theta)$ *remain in the market and firms with* $\alpha < \alpha^*(\mu, \theta)$ *exit.*

It is the less productive firms that find it optimal to exit so that their resources can be reallocated to better uses. Dunne *et al.* (1989b) find that higher cost plants exit first. **A2** is a stronger assumption than is needed for these two propositions to hold. However, it proves useful for expositional and analytic reasons for the evolution of the type of a new entrant not to depend on the type of firm it replaces.

3.2 Efficiency, Profitability, Age, and Exit Behavior

In this section we consider how different firms evolve along a business cycle according to various criteria (profitability, age–exit behavior, etc.). We first place additional structure on the transition distribution, $P(\cdot \mid \alpha)$. We first define a notion of conditional stochastic dominance: Given two distributions, F and G on $[0,1]$, F conditionally dominates G, written $F \succeq_c G$ if $F(\cdot \mid \alpha \geq \hat{\alpha}) \succeq G(\cdot \mid \alpha \geq \hat{\alpha})$ for each $\hat{\alpha} \in [0,1)$.

C1. If $\alpha > \alpha'$ then $P(\cdot \mid \alpha) \succeq_c P(\cdot \mid \alpha')$.

C2. $\int P(\cdot, \alpha) P(d\alpha, \bar{\alpha}) \succeq_c P(\cdot, \bar{\alpha})$.

According to **C1**, given two firms that choose to stay in the market, if one firm has a better technology than the other, then given that both firms "draw" a technology better than $\hat{\alpha} \in [0,1)$, the firm with the better current technology is more likely to draw a better technology in $(\hat{\alpha}, 1]$ than the firm with the lower current technology. Assumption **C1** just extends our stochastic dominance assumption for transition distributions, **A1**, to hold when conditioned on any future exit decision. We use the notation $P(X \mid \tilde{\alpha} > \hat{\alpha}, \alpha)$ to denote the conditional probability of event X (*i.e.*, that $\tilde{\alpha}$ is in X) given that $\tilde{\alpha}$ exceeds $\hat{\alpha}$, under the distribution $P(\cdot, \alpha)$.[6] In words, condition **C2** says that a firm, $\hat{\alpha}$, which comes into business after bankruptcy, has a better distribution (in the conditional dominance sense) one period later, having remained in operation, than the initial distribution governing the technology on coming into operation. **C2** says that firms tend to improve after their first period of production. Together, **C1** and **C2**, imply that the productivity of a firm's technology tends to increase with age.

E1. $\exists \epsilon > 0, \delta > 0$, and some integer N, such that $P^N((0, \epsilon], 1) > \delta$.

[6] Note that for any α, $P(X \mid \tilde{\alpha} > \hat{\alpha}, \alpha)$ is a distribution with support in $[\hat{\alpha}, 1]$. Then, **C1** is the requirement that when $\alpha > \alpha'$ then for any $\hat{\alpha} \in [0,1)$, $P(\cdot \mid \tilde{\alpha} > \hat{\alpha}, \alpha) \succeq P(\cdot \mid \tilde{\alpha} > \hat{\alpha}, \alpha')$, or equivalently $P((x,1] \mid \tilde{\alpha} > \hat{\alpha}, \alpha) \geq P((x,1] \mid \tilde{\alpha} > \hat{\alpha}, \alpha')$ for all $x \in [\hat{\alpha}, 1]$, with strict inequality for some x.

Here, P^N is the N^{th} iterate on the transition function P. **E1** ensures that even the firms with the most productive technologies eventually become bad. **E1** is a regeneration assumption, which ensures that if exit occurs with positive probability, then with probability one even the best firms eventually have bad technology realizations, and hence exit in finite time.

Theorem 1

1. *Better (higher α) firms are more profitable and produce more output.*
2. *Furthermore, if* **E1** *and* **C1** *hold and exit occurs with positive probability, then better firms are less likely to exit, expect to produce more in the future, earn higher profits and have longer expected lifetimes.*
3. *Furthermore, if* **C1** *and* **C2** *hold then the older an active firm is, the greater is its expected productivity and the longer its expected lifetime (i.e. the conditional probability of exiting into bankruptcy falls with age).*

Hopenhayn (1992a) derives very similar results in an economy that features no demand uncertainty and the formulations of entry and exit discussed earlier. Dunne, Roberts and Samuelson (1989) and Evans (1987) document that the hazard rate for exit declines with age for both plants and firms.

3.3 Cyclical Fluctuations

In this section we study the impact of cyclical variations in θ on exit decisions, output, and profitability of firms. This, in turn, allows us to characterize the consequences of demand shocks for current and future output, and current and future aggregate productivity. We characterize business cycle dynamics in three different ways. We first provide sufficient conditions that enable us to compare outcomes *across* different demand realization paths. We then detail the consequences for outcomes *along* a demand history path, comparing outcomes at date t with those at date $t+1$. Lastly, we characterize the consequences of an improvement in *anticipated future* market conditions on current and interim firm decisions, and the consequences for the aggregate economy.

One way to characterize the distribution of technology qualities is through the distribution's implications for output: distribution $\bar{\mu}$ is better than distribution $\hat{\mu}$ if and only if for all θ, output is greater in the bar economy than the hat economy. Our social planner program permits such a characterization over the business cycle (see *e.g.* lemma 5).

Using the social planner characterization directly, one can also show that, *ceteris paribus*, increasing exit increases output at each future date and state. As a consequence, future prices and firm profits for both continuing and exiting firms are lower when there is more exit. The impact of improving the distribution of technologies is similar: again, output is increased at each future date and state. As a consequence, the better is the distribution of firm qualities (higher μ), *i.e.*

the "more competitive" is the economy, then the lower are both the expected values of continuing to operate and of exiting:

Lemma 1 *The functions $v^c(\theta, \mu, \alpha)$ and $v^e(\theta, \mu, \alpha)$ are continuously decreasing in μ.*

Next, let $Y(\theta, \mu_{\alpha(\theta,\mu)})$ be the equilibrium level of output when the aggregate demand shock is θ, and the distribution of firms in the market is $\mu_{\alpha(\theta,\mu)}$. The following result states that, the better is the distribution of firms, *ceteris paribus*, the greater is aggregate output, and, hence, the lower is price and consequently the lower is the profit of any given firm, α, that produces:

Proposition 3 *Suppose $\bar{\mu} \succ_c \hat{\mu}$. Then $Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}) > Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})})$. Hence, $p(Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}), \theta) < p(Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})}), \theta)$, and $\pi(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}, \alpha) < \pi(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})}, \alpha)$.*

An alternative approach is to characterize the distribution of technology qualities not just by their output consequences over the business cycle, but also by stochastic dominance. We now consider conditions under which, ceteris paribus, downturns in demand induce more (unproductive) firms to exit (worsening the immediate effect of the downturn), to be replaced by firms that are stochastically better. We then show that worse current demand conditions always imply that future distributions of firm technologies are more productive in the conditional stochastic dominance sense. That is, not only do recessions have the cleansing effect of weeding out more firms with unproductive technologies, but past recessions also reduce the number of unproductive technologies at each date in the future. Exit decisions thus mitigate the impact of a long–run downturn in demand.

If there is positive persistence in aggregate demand shocks, then the consequences of a high demand shock on the relative values of production and exit are ambiguous. With persistence, a high demand shock raises both the value of producing in the current period as well as the value of having a better technology in the future to better exploit higher expected future demand shocks. Clearly, if there is sufficiently little persistence in the aggregate demand shock process, then a high demand shock raises the immediate cost of not producing by more than the expected cost of having an inferior technology in the future, so the higher is θ, the less exit there is. Indeed, if demand shocks are independently distributed, continuation payoffs do not depend directly on the current demand realization, but current profits do.

To proceed, we strengthen the assumption on the transition process for technology qualities, $P(\cdot \mid \alpha)$, so that we can always order distributions according to conditional stochastic dominance:

S1. The transition process for technology qualities is of the form: $P(\cdot \mid \alpha) = w(\alpha)F(\cdot) + [1 - w(\alpha)]G(\cdot)$, where $F \succ_c G$ and where $w(\alpha) \in [0, 1]$ is continuous and strictly increasing in α.

Note that assumption **S1** implies assumption **C1**. The role of **S1** merits discussion. If an exit rule, $\alpha^* < \bar{\alpha}$, is given and the current distribution is μ, then the distribution next period is $\hat{\mu}(\cdot) = \int_{[0,\alpha^*)} P(\cdot \mid \bar{\alpha}) d\mu + \int_{[\alpha^*,1]} P(\cdot \mid \alpha) d\mu$. If λ is a measure on technologies and f a real–valued measurable function on technologies, write $\lambda(f) \stackrel{\text{def}}{=} \int f d\lambda$. Also, let $g_f(\alpha) = P(f \mid \bar{\alpha})$, $\alpha < \alpha^*$ and $g_f(\alpha) = P(f \mid \alpha)$, $\alpha \geq \alpha^*$. Thus, $\hat{\mu}(f) = \int g_f d\mu$. Note however that even when f is monotonically increasing, the function g_f is not monotonic (it has a downward jump at α^*). See figure 1. Thus, for a given exit rule, if $\mu' \succ \mu$, it may not be the case that $\hat{\mu}'(f) = \int g_f d\mu' \succ \hat{\mu}$. For example, let $\mu(\{0\}) = 1$, $\mu'(\{\frac{4}{10}\}) = 1$, $\bar{\alpha} = \frac{5}{10}$ and $\alpha^* = \frac{3}{10}$. Then $\hat{\mu}(\cdot) = P(\cdot \mid \frac{5}{10}) \succ P(\cdot \mid \frac{4}{10}) = \hat{\mu}'(\cdot)$. In the appendix, we show that with this **S1** assumption, distributions over characteristics are totally ordered in terms of $\succeq_c$. In addition, increasing the measure of exiting firms, *i.e.* increasing the exit threshold, α^*, or improving the current distribution over firm technologies, μ, improves the distribution of firm technology qualities the next period.

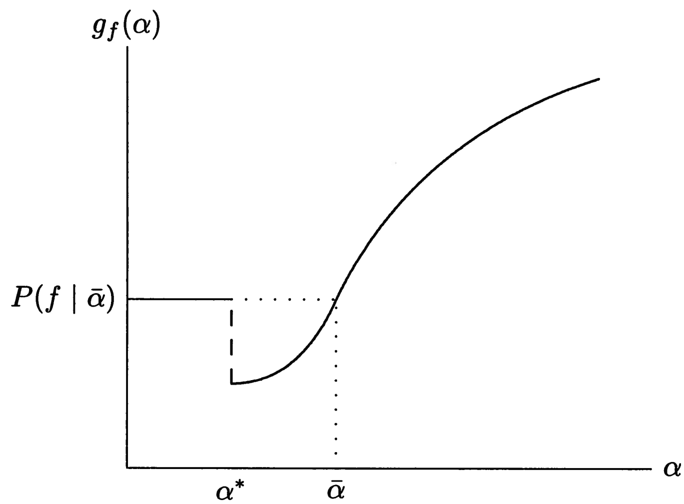

Figure 1

We now demonstrate is that if the profit function is multiplicatively separable between prices and technology qualities (*e.g.*, $f(k,\ell,\alpha) = h(\alpha,k)\ell^a, f(k,\ell,\alpha) = h(\alpha,k)\ell - a\ell^2$) and the demand shock can take on only one of two values then there is less exit when demand is high than when demand is low, no matter how great the level of persistence:

Lemma 2 *Given* **S1**, *suppose that the support for θ is $\{\underline{\theta},\bar{\theta}\}$, $\underline{\theta} < \bar{\theta}$ and that profits are multiplicatively separable: $\pi(\alpha,p(\mu,\theta)) = g(p)h(\alpha)$ for some functions g and h. Then for any $\mu \in [\underline{\mu},\bar{\mu}]$, $\alpha(\mu,\underline{\theta}) > \alpha(\mu,\bar{\theta})$.*

Here, $\bar{\mu}$ and $\underline{\mu}$ are, respectively, the distributions of firm qualities induced by an arbitrarily long sequence of $\underline{\theta}$ and $\theta = \bar{\theta}$ realizations starting from μ_∞, the unique stationary distribution when demand is independent of θ.

The key feature of both the two–state and *i.i.d.* demand shock environments is that they feature 'sufficient' mean reversion in the θ process, so that high θ's imply that present conditions tend to be better than future conditions and low θ's imply that conditions are likely to improve. For the next result, we do not directly place structure on the θ process, but rather just assume that there is 'sufficient' mean reversion in the θ process so that there is less exit in good times:

D1: If $\theta < \theta'$ then $\alpha(\mu, \theta) > \alpha(\mu, \theta')$.

Proposition 4 *Suppose assumptions* **S1** *and* **D1** *hold. Let* $\hat{\mu}_0 = \bar{\mu}_0$. *Consider two aggregate shock histories,* $\hat{\theta}^t, \bar{\theta}^t$ *where* $\hat{\theta}_0 > \bar{\theta}_0$, $\hat{\theta}_\tau \geq \bar{\theta}_\tau, 0 \leq \tau \leq t$. *Then*

1. $\bar{\mu}_{\tau+1} \succ_c \hat{\mu}_{\tau+1}, \quad 0 \leq \tau \leq t$.
2. $\exists \delta > 0$ *such that if* $\bar{\theta}_{\tau+1} \geq \hat{\theta}_{\tau+1} - \delta$ *then* $Y_{\tau+1}(\bar{\theta}_{\tau+1}, \bar{\mu}_{\tau+1\bar{\alpha}_{\tau+1}}) > Y_{\tau+1}(\hat{\theta}_{\tau+1}, \hat{\mu}_{\tau+1\hat{\alpha}_{\tau+1}}), \quad 0 \leq \tau \leq t$. $Y_{t+\tau}((\bar{\theta}^t, \theta_{t+1}, ..., \theta_{t+\tau}), \bar{\mu}_{t+\tau\bar{\alpha}_{t+\tau}}) > Y_{t+\tau}((\hat{\theta}_{\tau+1}, \theta_{t+1}, ..., \theta_{t+\tau}), \hat{\mu}_{t+\tau\hat{\alpha}_{t+\tau}}), \quad \tau \geq 0$.
3. $\bar{p}_\tau < \hat{p}_\tau, \quad 0 \leq \tau \leq t$.
4. $\pi(\hat{\theta}_\tau, \hat{\mu}_{\tau\hat{\alpha}_\tau}, \alpha) > \pi(\bar{\theta}_\tau, \bar{\mu}_{\tau\bar{\alpha}_\tau}, \alpha), \quad 0 \leq \tau \leq t$.

Here, for example, $\bar{\mu}_\tau$ is the distribution on technology along the $\bar{\theta}$ sequence, and $\bar{\mu}_{\tau+1\bar{\alpha}_{\tau+1}}$ is the $\bar{\mu}_{\tau+1}$ distribution truncated by the exit rule, $\bar{\alpha}_\tau = \alpha_\tau(\bar{\theta}, \bar{\mu}_\tau)$. Characterizations 2 through 4 do not require assumption **S1**. The role of **S1** is discussed further in the appendix, in results 1 and 2.

The immediate effect of a lower demand realization is to induce more inefficient firms to exit. This leads to a better future distributions of firm qualities and this higher productivity persists at all future dates (4.1). The higher productivity caused by a past downturn implies greater future output once demand 'recovers' sufficiently (4.2), illustrating the cleansing effect of recessions for future output. Indeed, if there is sufficiently little persistence in demand, then a lower current demand shock *raises* expected future discounted total surplus.[7] In turn, this increased future competition resulting from the demand demand turn implies lower future prices (4.3) and hence lower profits for a firm of a given technology quality (4.4). The greater the stress of a downturn of a business cycle (*i.e.* the lower are the demand realizations and the longer the lower demand realizations persist), the more fit are the survivors, and hence the more productive is the entire industry. Campbell (1995) summarizes findings in the empirical literature that exit rates are counter–cyclical, but correlations of exit rates with future GDP growth are positive, large, persistent and statistically significant.

These results can also be stated in terms of their converse — more prolonged recessions lead to greater output when demand finally improves, and greater booms lead to steeper declines to lower levels of output when demand finally falls.

These predictions need not arise in a model such as Hopenhayn's (1992a,b) (extended to allow for aggregate demand shocks) in which exiting firms can produce prior to exiting, because there is *no* opportunity cost to production. Indeed, in Hopenhayn, were demand shocks *i.i.d.*, the distribution of firm qualities would not vary over time in the limiting economy, because neither the value of entry nor exit would fluctuate. So, too, if firms in our model were able to produce prior to exiting,

[7] These features are not shared by Caballero and Hammour (1993), where with sufficient entry externalities, anticipation of a fall in demand leads to fewer firms (with higher average productivities).

and if there were a fixed exit (*'re–tooling'*) cost, then again these predictions would not arise.

A direct implication of proposition 4 is that the economy grows faster coming out of a recession:

Corollary 1 *Suppose that* **D1** *holds. If* $\bar{\mu}_t = \hat{\mu}_t$, $\bar{\theta}_t < \hat{\theta}_t$, *then* $\exists \delta > 0$ *such that if* $\bar{\theta}_{t+1} \geq \hat{\theta}_{t+1} - \delta$ *then* $\bar{Y}_{t+1} - \bar{Y}_t > \hat{Y}_{t+1} - \hat{Y}_t$.

Since the number of firms in our economy is fixed, it precludes a rich analysis of entry, since firms can only enter by taking over exiters. A particular consequence is that exit trivially leads entry (as is found in the data). Still, if entry is interpreted as the number of firms in their first period of production, then entry is greater when the economy leaves a recession both because there are more exiting firms due to the past downturn and because the increase in demand implies that more firms find it optimal to produce.

These findings illustrate the subtle, persistent effects of an increase in demand on future output. Because upturns encourage bad firms to continue to operate, an upturn has a long–run negative effect on future output because the resulting distribution of firm types, ordered by stochastic dominance, is worse. It also suggests that upturns in business cycles tend to peter out on their own as a prolonged upturn leads to increasingly inefficient distributions of firms. Conversely, downturns are sharp and short: the immediate consequences of a downturn in demand are reinforced by the loss of output due to the increase in the exit rate. However, this increased exit leads to better future distributions of firm technologies. This stochastically better distribution of technology productivities tends to mitigate any effects of a prolonged downturn in demand. Indeed, in our economy, in a two–state environment, the peaks and troughs of a business cycle are reached almost immediately:

Corollary 2 *Suppose that* $\theta \in \{\bar{\theta}, \underline{\theta}\}$, *that the conditions for lemma 2 hold, and that* $\mu_{t-1} \in [\underline{\mu}, \bar{\mu}]$. *Then*

1. *Aggregate output is higher in good times than bad:* $Y_t(\bar{\theta}, \mu) > Y_t(\underline{\theta}, \mu), \forall \mu$.
2. *Suppose* $\theta_{t-1} = \underline{\theta}, \theta_t = \bar{\theta}, \theta_{t+1} = \bar{\theta}, ..., \theta_{t+\tau} = \bar{\theta}, \theta_{t+\tau+1} = \underline{\theta}$. *Then in a prolonged demand boom, aggregate output evolves according to* $Y_{t-1} < Y_t > Y_{t+1} > \cdots > Y_{t+\tau} > Y_{t+\tau+1}$, *so that prices rise:* $p_t < p_{t+1} < \cdots < p_{t+\tau}$. *Hence, the output and profits of any given firm,* α, *rise as the demand boom continues:* $y_{t-1}(\alpha) < y_t(\alpha) < y_{t+1}(\alpha) < \cdots < y_{t+\tau}(\alpha)$ *and* $\pi_{t-1}(\alpha) < \pi_t(\alpha) < \pi_{t+1}(\alpha) < \cdots < \pi_{t+\tau}(\alpha)$.
3. *Suppose* $\theta_{t-1} = \bar{\theta}, \theta_t = \underline{\theta}, \theta_{t+1} = \underline{\theta}, ..., \theta_{t+\tau} = \underline{\theta}, \theta_{t+\tau+1} = \bar{\theta}$. *Then in a prolonged downturn in demand, aggregate output evolves according to* $Y_{t-1} > Y_t < Y_{t+1} < ... < Y_{t+\tau} < Y_{t+\tau+1}$, *so that prices fall:* $p_{t-1} > p_t > p_{t+1} > \cdots > p_{t+\tau}$. *Hence, individual output and profits of any given* α *fall as demand continues to be low:* $y_{t-1}(\alpha) > y_t(\alpha) > y_{t+1}(\alpha) > \cdots > y_{t+\tau}(\alpha)$ *and* $\pi_{t-1}(\alpha) > \pi_t(\alpha) > \pi_{t+1}(\alpha) > \cdots > \pi_{t+\tau}(\alpha)$.

Here, $\{Y_j\}_{j=t-1}^{t+\tau+1}$ is aggregate output along the θ sequence. The associated profit and output of a firm with technology α at time j are denoted $\pi_j(\alpha)$ and $y_j(\alpha)$. Note that the increasingly inefficient distribution of firms caused by a prolonged demand boom, while decreasing aggregate output, raises the individual output of any given α.

Were there exogenous systematic growth in technological opportunities, then our theoretical model generates predictions consistent with the empirical regularity that recessions are sharper and more asymmetric than booms. Output gains continue after long periods of high demand if the exogenous growth in productivities is enough to offset the increasingly inefficient (endogenous) distribution of firm productivities. Conversely, any systematic growth sharply reinforces the endogenous improvement in firm productivities caused by a downturn in demand.

Our last characterization of industry dynamics concerns the impact of an anticipated future increase in demand on current exit decisions and hence future distributions of firm productivities:

Proposition 5 *Consider a one–time anticipated improvement in T–th period ahead demand, so that:*

$$\bar{\Theta}_T(\theta_T \mid \theta_{T-1}) \succ \hat{\Theta}_T(\theta_T \mid \theta_{T-1}), \forall \theta_{T-1}.$$

Then for any given demand shock history, $\theta_0, \ldots, \theta_{t-1}$, an anticipated increase in future demand improves the distribution of firm productivities at all earlier dates: $\bar{\mu}_\tau \succ \hat{\mu}_\tau, t < \tau \leq T$.

The key to this finding is that an anticipated improvement in future demand does not directly affect current profits; it only raises the future value of having a better technology, and hence the value of exit. If the anticipated improvement in demand is only one period ahead, then the future value of being productive is increased, raising current exit and lowering current output, thereby indirectly raising current prices and profits. When the anticipated demand increase is further in the future, its anticipation raises the distribution of productivities at all earlier dates in the stochastic dominance sense. This is because to increase future productivity, exit must be raised, increasing the distribution of prices prior to the productivity improvement. In particular, suppose to the contrary that exit first increased at the last date prior to the improvement in demand, $T-1$. But this increased exit reduces period $T-1$ output, raising period $T-1$ prices and hence the value of exit at date $T-2$. Thus, anticipation of increased future exit in advance of the demand increase, drives exit and productivity increases at earlier dates.

This finding is related to that of Campbell (1995). Campbell assumes aggregate uncertainty only in the realization of an aggregate cutting edge technology innovation. Entry takes 5 quarters, but the 5–quarter ahead cutting edge technology innovation is revealed immediately. Campbell shows that an anticipated productivity increase raises exit through the general equilibrium effects on current consumption. When the representative agent learns of a positive future productivity shock, the general equilibrium consequence is that the consumer delays gratification, *consuming less* and working more. This reduced consumption drives increased exit prior to the entry of the more productive firms and greater output, so that Campbell can mimic numerically the empirical finding that exit leads increases in productivity. Were the aggregate technology shocks instead revealed at production (*i.e.* when individual firms learn their idiosyncratic productivities), then Campbell would find that increases in productivity would *lead* exit.

4 Capital in Place.

We now turn to capital in place, so that a firm's type is given by $s = (k, \alpha)$, and both the productivity of a firm's technology and its capital stock affect its production decision. With capital in place, in equilibrium, the less capital, k, that a firm employs or the more inefficient it is (lower α), the relatively more attractive is the option to leave the market and attempt to sell its resources. We exploit this equilibrium feature and represent the rule characterizing exit as a measurable function, $\psi : [0,1] \to R_+$. Thus, at α, if $s = (\alpha, k)$ is such that $k \geq \psi(\alpha)$, then the firm remains in operation; and if $k < \psi(\alpha)$, then the firm exits. Let $\psi^< = \{(\alpha, k) \mid k < \psi(\alpha)\}$ and $\psi^\geq = \{(\alpha, k) \mid k \geq \psi(\alpha)\}$. Under exit rule ψ, $\psi^<$ is the subset of firms in $T = [0,1] \times [0, \infty)$ that exits and $\psi^\geq$ is the subset of T that continues to produce. If τ is the distribution on T prior to the exit decision, define $\tau_\psi(X) = \tau(X \cap \psi^\geq)$ to be the distribution over firms that continue in operation. If the current aggregate shock is θ and the exit rule is ψ, then the profit of firm s is $\pi(\theta, \tau_\psi, s)$. The value to a firm s of continuing to operate is given by

$$v^c(\theta, \tau, s) = \pi(\theta, \tau_\psi, s) + \beta \max_i \{ \int v(\tilde{\theta}, \tau^{\psi \imath}, \rho k + i, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) P(d\tilde{\alpha} \mid \alpha) - ri \},$$

where $\tau^{\psi \imath}$ is the joint distribution over capital and technology productivities induced by the exit and investment rules, and $v(\theta, \tau, s) = \max\{v^c(\theta, \tau, s), v^e(\theta, \tau, s)\}$ reflects that the firm will make optimal exit decisions in the future, "v^c" denoting the value to continuing to operate, and "v^e" denoting the value of exiting. That is, the value of an operating firm s facing market conditions (θ, τ) is equal to the sum of the maximized operating profits it receives plus the discounted expected value of continuing to operate given that it chooses investment optimally and makes future operation–exit decisions optimally. The value to a firm s of exit is given by:

$$v^e(\theta, \tau, s) = \beta \max_\imath \{ \int v(\tilde{\theta}, \tau^{\psi \imath}, \rho k + \imath, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) P(d\tilde{\alpha} \mid \bar{\alpha}) - r\imath \}.$$

The equilibrium condition is then that each firm on the margin between exit and continuation should be indifferent at each θ:

$$v^c(\theta, \tau, (k, \alpha)) = v^e(\theta, \tau, (k, \alpha)) \qquad \forall (k, \alpha) \in \{(k, \alpha) \mid k = \psi(\alpha)\}.$$

4.1 Equilibrium.

In this section we formulate the social planner problem with investment and show that the solution to the social planner's problem determines equilibrium behavior for individual firms. Denote the expected profit–maximizing investments for continuing and exiting firms by $\imath(\theta, \tau^{\psi \imath}, k, \alpha)$

and $\imath(\theta, \tau^{\psi \imath}, k, \bar{\alpha})$, respectively. If θ and τ are, respectively, the current aggregate shock and the current characteristics distribution, then next period's characteristics distribution over technologies is given by:

$$\mu'(\cdot) = \int_{\psi\geq} P(\cdot \mid \alpha)\tau(ds) + \int_{\psi<} P(\cdot \mid \bar{\alpha})\tau(ds).$$

For a firm $s = (k, \alpha)$ that continues in existence, the new distribution over capital and technology is a distribution $G^c(\cdot \mid t) = \delta_{\rho k + \imath(\theta, \tau^{\psi \imath}, k, \alpha)}(\cdot) \otimes P(\cdot \mid \alpha)$, while for a firm s that exits the market and re–enters under new management, the distribution is denoted $G^e(\cdot \mid s) = \delta_{\rho k + \imath(\theta, \tau^{\psi \imath}, k, \bar{\alpha})}(\cdot) \otimes P(\cdot \mid \bar{\alpha})$. Next period's joint distribution on T is given by

$$\tau^{\psi i}(\cdot) = \int_{\psi\geq} G^c(\cdot \mid s)\tau(ds) + \int_{\psi<} G^e(\cdot \mid s)\tau(ds).$$

Individual valuation functions are then:

$$v^c(\theta, \tau, k, \alpha) = \pi(\theta, \tau_\psi, k, \alpha) + \beta\{\int v(\tilde{\theta}, \mu^{\psi \imath}, \rho k + \imath(\theta, \tau^{\psi \imath}, k, \alpha), \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)P(d\tilde{\alpha} \mid \alpha) - r\imath(\theta, \tau^{\psi \imath}, k, \alpha)\}$$

and

$$v^e(\theta, \tau, k, \alpha) = \beta\{\int v(\tilde{\theta}, \tau^{\psi \imath}, \rho k + \imath(\theta, \tau^{\psi \imath}, k, \bar{\alpha}), \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)P(d\tilde{\alpha} \mid \bar{\alpha}) - r\imath(\theta, \tau^{\psi \imath}, k, \bar{\alpha})\}.$$

The associated social planner problem is:

$$\begin{aligned} V(\theta, \tau) = sup_\psi\{\max_Y \int_{[0,Y)} [P_d(Y, \theta) - P_s(Y, \tau_\psi)]dY + \\ \beta \sup_{\imath = (\imath^c, \imath^e)} [\int_\Theta V(\tau^{\psi \imath}, \tilde{\theta})\Theta(d\tilde{\theta} \mid \theta) - r\int_{\psi\geq} \imath^c(k, \alpha)\tau(dk, d\alpha) - r\int_{\psi<} \imath^e(k, \alpha)\tau(dk, d\alpha)]\}, \end{aligned}$$

where ψ is the exit rule and τ_ψ is the measure determined in the current period following exit: $\tau_\psi(X) = \tau(X \cap \{(\alpha, k) \mid k \geq \psi(\alpha)\}$.

The exit rule, ψ and the investment rule $\imath = (\imath^c, \imath^e)$ determine a joint distribution over (α, k) given θ. The conditional distribution over future $(\tilde{\alpha}, \tilde{k})$ given current (α, k) depends on whether the firm exits or not: $G(\cdot \mid s)$ for a continuing firm and $\bar{G}(\cdot \mid s)$ for a firm that exits. In equilibrium, these will depend on θ, but, for ease of notation this dependence is not made explicit.

Theorem 2 *The solution to the social planner problem determines the same entry–exit rule and investment rule as the equilibrium condition.*

The central ideas of this theorem can easily be explained. Consider figure 2, in which we illustrate the current period surplus, $\int_{[0,Y)}[P_d(Y, \theta) - P_s(Y, \tau_\psi)]dY$, (where $P_s(Y, \tau_\psi)$ is the inverse supply function when the mass of firms in the market is τ_ψ) and the impact of a reduction in the exit rate.

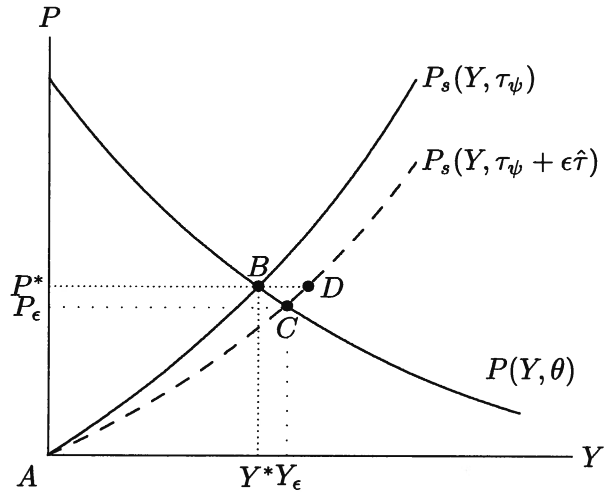

Figure 2

There are two first order effects on profits from a reduction in exit. First, there is a reduction in firm surplus of (approximately) $P^*P_\epsilon B$ due to the lower price, which is transferred to consumers and hence netted out in the total surplus calculations, and which competitive firms also do not internalize. Second, there is the area ABC, which represents the increase in surplus resulting from the increase in the mass of firms in the market: $\tau_\psi + \epsilon\hat{\tau}$. This is approximated by the area ABD since $ABD = ABC + BCD$, where BCD is of smaller order of magnitude than ABC (ϵ^2 vs. ϵ). Now $ABD = AP^*D - AP^*B$, $AP^*B = \int_{[0,P^*)} Y(P,\tau_\psi)dP$, and $AP^*D = \int_{[0,P^*)} Y(P,\tau_\psi + \epsilon\hat{\tau})dP$, where, for example, $Y(P,\tau_\psi) = \int_T q(P,s)d\tau_\psi$ is aggregate output at price P and $q(P,s)$ the supply of firm s at price P. ($Y(P,\tau_\psi)$ is the aggregate supply function with inverse supply function $P_s(Y,\tau_\psi)$.) Thus, $ABC \approx ABD$ and

$$ABD = \int_{[0,P^*)} Y(P,\tau_\psi + \epsilon\hat{\tau})dP - \int_{[0,P^*)} Y(P,\tau_\psi)dP = \int_T \pi(s)d[\tau_\psi + \epsilon\hat{\tau}] - \int_T \pi(s)d\tau_\psi = \epsilon\int_T \pi(s)d\hat{\tau}.$$

(Observe that $\int_{[0,P^*)} Y(P,\tau_\psi)dP = \int_{[0,P^*)}\int_T q(P,s)d\tau_\psi dP = \int_T\int_{[0,P^*)} q(P,s)dPd\tau_\psi = \int_T \pi(s)d\tau_\psi$, where $\pi(s)$ is the profit of firm s at price P^*.) Hence, the change in current surplus from the change in exit corresponds to the profits of the marginal firms. So, too, the sole first order effect of the change in exit on future surplus corresponds to the difference between the discounted future expected profits of the marginal firm that exits and the discounted expected profits if it does not. The marginal firm trades off entry and exit in the same way as the social planner. (For example, were the model two periods, then in the second period one would have a figure similar to Figure 2, where, as a result of fewer firms exiting, average efficiency would be lower in the second period and the supply curve would move in, with loss in surplus approximately equal to the loss in profit incurred by those firms that failed to exit.) Similarly, with investment, the first order effect on prices that is not internalized by firms represents a transfer from producers to consumers and hence is netted out in the total surplus calculation, leaving only the direct first order effect on firm profits.

Corollary 3 *There is a unique competitive equilibrium. The competitive equilibrium corresponds to the solution to the social planner's problem.*

4.2 Firm Behavior.

In this section we characterize the actions of a firm along a business cycle. The next result characterizes each firm's investment and follows immediately from the first order conditions for firm optimization. It reflects the result that the optimal level of next period's capital stock depends only on the firm's current type, the current demand shock, and the next period's distribution of firm types:

Lemma 3 *Investment, $\imath(\theta, \tau^{\psi_\imath}, k, \alpha)$, satisfies $\imath(\theta, \tau^{\psi_\imath}, k, \alpha) = k^*(\theta, \tau^{\psi_\imath}, \alpha) - \rho k$, where $k^*(\theta, \tau^{\psi_\imath}, \alpha)$ solves*

$$(1-\beta\rho)r = \beta Pr(\tilde{\alpha} \geq \alpha^*(k^*, \tilde{\theta}, \tau^{\psi_\imath})|\alpha) E\{\pi_k(\theta, \tau^{\psi_\imath}, k^*, \tilde{\alpha})|\alpha\}.$$

Better firms invest so that their capital stock is greater:

Lemma 4 *Suppose assumption* **C1** *holds. Then $k^*(\theta, \tau^{\psi_\imath}, \alpha)$ is monotonically increasing in α.*

The marginal exiting firm, $\psi(\alpha, \theta, \tau_\psi)$, is given by the solution to

$$\begin{aligned}&\pi(\theta, \tau_\psi, \psi(\alpha, \theta, \tau_\psi), \alpha)\\ &\quad = \beta\{\int v(\tilde{\theta}, \mu^{\psi_\imath}, \rho k^*(\theta, \tau^{\psi_\imath}, \bar{\alpha}))\Theta(d\tilde{\theta} \mid \theta)[P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha)] - r[k^*(\theta, \tau^{\psi_\imath}, \bar{\alpha}) - k^*(\theta, \tau^{\psi_\imath}, \alpha)]\}.\end{aligned}$$

Inspection reveals that the continuation payoffs do not depend on the current capital stock, but current operating profits are an increasing function of the current capital stock. Hence, holding productivity, α, constant, larger firms are less ready to exit. This is consistent with Lieberman (1990), who finds that 76 percent of firms that exit are in the bottom half of the size distribution the previous year. Evans (1987) and Dunne *et al.* (1989) also find that smaller firms are more likely to exit.

With variable capital, Theorem 1 demonstrated that better firms were likely to remain better and hence less likely to exit in the future. Because firms with better technologies are more likely to continue to have better technologies, they will invest so as to have more capital. Since a greater capital stock makes exit even less attractive, introducing capital reinforces the results of Theorem 1:

Corollary 4

1. *On average, larger (greater k) or better (higher α) firms are more profitable, and produce more output.*

2. *Further, if* **E1** *and* **C1** *hold and exit occurs with positive probability, then better or larger firms are less likely to exit, expect to operate better technologies on a larger and more profitable scale and have longer expected lifetimes.*

3. *Further if* **C1** *and* **C2** *hold then the older is an active firm, the greater is its expected productivity, the greater is its capital stock and profit, and the longer is its expected lifetime (i.e. exit hazard rates fall with age).*

Corollary 4 offers qualitatively identical characterizations to Theorem 1 of the relationship between firm productivity, α, and exit, profits, and age when capital is in place. *In addition*, it characterizes the relationship between capital stocks and these variables.

There is considerable evidence that large firms are more profitable, less likely to exit and have longer expected lifetimes. Dunne *et al.* (1989b) find that plant failure rates decline with size and age. (See also Evans (1987), Pakes and Ericson (1995)). In models such as that in section 3 or in Hopenhayn (1992a,b), a firm's technology quality determines all inputs, so distinctions cannot be drawn between firm productivity and firm size. With capital in place, capital stocks are determined *prior* to the productivity shock, so that a firm's size can be distinguished from the productivity of its technology. *Ex post*, holding realized technology productivity constant, the reason larger firms exit less is that larger firms can more profitably employ any given level of labor. Hence, the larger is the firm the greater is the opportunity cost of exit. That larger firms are also more productive, on average, than smaller firms only reinforces the lower likelihood of exit by large firms than small.

Campbell (1995) generates greater exit of "young" firms than "old" by assuming variation in entry quality. Since this uncertainty vanishes after the first quarter, to obtain greater exit of "young" firms, Campbell must aggregate so that "young" firms represent plants in their first four quarters of production.

4.3 Cyclical Fluctuations.

We now characterize the economy's behavior across different paths of demand realizations. The introduction of capital complicates characterizations because both the productivity of a firm's technology and its capital affect exit decisions and hence the future distribution of productivities. While a higher demand shock leads to less exit, thereby improving the next period's distribution of firm productivities, investments in capital (by at least some firms) are also increased, both because the reduced competition raises prices and because persistence in demand means higher current demand makes higher future demand more likely. Both being larger and being better lead to more production, so it is possible that future prices will be lower in the economy that had a lower demand shock. In turn, future exit rates might go in the 'wrong' direction for some θ realizations.

What we first show is that the reduced exit and resulting worse distribution of firms caused by higher demand has only a second–order effect on investment. We show that the first order effect on output of an increase in exit on next period's distribution of firm qualities always dominates the second order effect on investment. In particular, any marginal increase in exit, *ceteris paribus*, leads to greater output (and hence lower prices) at all future dates along any given future θ path:

Lemma 5 *Let ψ be the equilibrium exit rule in the current period and τ the distribution over (α, k). Let $\psi'(\alpha) \leq \psi(\alpha), \forall \alpha$. Thus, ψ' corresponds to more exit at any technology level. Define the measure $\hat{\tau}(X) = \tau\{(\alpha, k) \in X \mid \psi(\alpha) \geq k > \psi'(\alpha)\}$. Let the current period be t, let $\theta_t^j = (\theta_t, \theta_{t+1}, \ldots, \theta_{t+j})$ and let $Y_{t+l}(\theta_t^l, \tau)$ be equilibrium aggregate output at period $t+l$, following aggregate shock history θ_t^l given the current distribution τ. Then any increase in exit, ceteris paribus, increases output:*

$$\lim_{\epsilon \to 0} \frac{Y_{t+l}(\theta_t^l, \tau + \epsilon\hat{\tau}) - Y_{t+l}(\theta_t^l, \tau)}{\epsilon} < 0.$$

Increased exit lowers current surplus, but raises future total surplus. Similar social planner arguments with variable capital would imply that the increased exit caused by lower current demand shocks leads to more output in the future at each date and state. In particular, for two different demand shock histories, one uniformly higher than the other, higher demand shocks lead to stochastically worse distributions of firm productivities at all future dates and states. With capital in place, we can apply lemma 5 directly to the case when demand is independently distributed over time and then use a continuity argument to extend the analysis to the case where there is sufficiently little persistence in demand.

Proposition 6 *Suppose demand is independently distributed. Let $\hat{\tau}_0 = \bar{\tau}_0$. Consider two aggregate shock histories, $\hat{\theta}^t, \bar{\theta}^t$ where $\hat{\theta}_0 > \bar{\theta}_0$, $\hat{\theta}_{t'} \geq \bar{\theta}_{t'}, \forall t', 0 \leq t' \leq t$. Then*

1. $\bar{\mu}_{t'+1} \succ_c \hat{\mu}_{t'+1}, \quad 0 < t' \leq t.$
2. $\bar{p}_{t'} < \hat{p}_{t'}, \quad 0 < t' \leq t.$
3. $\bar{k}^*_{t'}(\alpha) > \hat{k}^*_{t'}(\alpha), \quad 0 < t' \leq t.$
4. $\hat{\pi}_{t'}(\alpha) > \bar{\pi}_{t'}(\alpha), \quad 0 < t' \leq t.$
5. *$\exists \delta > 0$ such that if $\bar{\theta}_{t+1} \geq \hat{\theta}_{t+1} - \delta$ then $Y_{t+1}(\bar{\theta}_{t+1}, \bar{\tau}_{t\bar{\psi}_t}) > Y_{t+1}(\hat{\theta}_{t+1}, \hat{\tau}_{t\hat{\psi}_t})$.*

Here, $\bar{\tau}_t$ is the aggregate distribution over states (capital and technology) at time t along the $\bar{\theta}$ sequence, $\bar{\mu}_t$ is its marginal distribution on technologies and $\bar{\psi}_t$ is the equilibrium exit rule. Also, $\bar{\tau}_{t\bar{\psi}_t}$ is the period t distribution on firms' states at time t under the exit rule $\bar{\psi}_t$ given the $\bar{\theta}$ sequence. The optimal level of capital along the $\bar{\theta}$ sequence at time t and technology α is given by $\bar{k}^*(\alpha)$ and the associated profit is $\bar{\pi}_t(\alpha) \equiv \bar{\pi}_t(\bar{k}^*(\alpha), \alpha)$. The "hat" variables are defined similarly.

Worse initial demand conditions lead to more exit and hence more productive future distributions of firms (6.1). Along any identical future demand shock history, lemma 5 implies that this increased exit in the economy with the worse initial demand shock leads to greater output (6.5), and hence lower prices (6.2) and profits (6.4) for any given α. In turn, the lower prices imply that investment is less profitable for a firm with any given technology quality α (6.3). The reduced investment further increases the attractiveness of future exit. Continued lower demand shocks reinforce this result. Worse demand conditions lead to a distribution of more productive, but smaller, firms. Upturns lead to distributions of firms that are unproductive, but large. The greater the magnitude or stress of a fall in demand, the more attractive is exit to inefficient firms and hence the more productive is the industry in the future.

Again continuity arguments imply that the results in Proposition 6 extend as long as there is *sufficiently little* positive persistence in demand. Higher current demand leads to larger firms both because firms anticipate that future demand shocks will be larger and because the distribution of technology qualities is worse. Both better demand shocks and worse technology distributions imply better distributions over future prices, making larger capital stocks more attractive. Since greater capital stocks make exit less attractive, this reasoning implies that there are two stages to the onset of a recession in the capital–in–place economy. At the first stage, firms that are both unproductive *and* small tend to exit. Larger, but less productive, firms find the opportunity cost of foregone production too great, so they remain in the industry, produce and downsize. Then if the downturn continues and the productivities of their technologies continue to be low, they exit. Davis *et al.* (1995) find that small firms exit more quickly in response to downturns than larger firms, which downsize first.

The cross–sectional variance of firm sizes tends to shrink in downturns, both because the bottom tail is truncated by increased exit and because larger firms downsize by more than smaller firms in response to lower prices. This is consistent with Lieberman (1990) who finds that in industry downturns, small firms exit and large firms downsize causing a strong trend toward size convergence. Conversely, in industry upturns, small exiting firms are replaced by new entrants, large producers expand and mean firm size and cross–sectional variance in firm sizes increase.

Again, the economy with capital in place exhibits most of the empirical features of the variable capital economy: the stress of a downturn in demand increases exit, further reducing immediate output, but increasing future productivity growth and output. The permanent replacement of less productive firms by more productive firms means that these effects persist.

The previous proposition characterized outcomes across two histories of demand shock realizations, one better than the other. The next result characterizes outcomes over time along a single path, and is presented for the two–state case: $\theta \in \{\underline{\theta}, \bar{\theta}\}, \underline{\theta} < \bar{\theta}$. *Ceteris paribus*, a greater demand shock is associated with greater aggregate output. An increase in demand first leads to an increase in output, and indeed output may increase the next period if there is a successive high demand shock due to the lagged investment effect. However, output will subsequently fall even if demand shocks remain high because the distribution of firm qualities worsens. Conversely, a fall in demand first causes a fall in output, and the lagged investment effect may cause output to continue to fall if demand stays low. Thereafter, aggregate output rises as the distribution of firm qualities improves in the face of worse demand shocks.

Without loss of generality let $Pr(\theta_{t+1} = \bar{\theta} \mid \theta_t = \bar{\theta}) = \bar{\Theta}(1-\gamma)+\gamma$ and $Pr(\theta_{t+1} = \bar{\theta} \mid \theta_t = \underline{\theta}) = \bar{\Theta}(1-\gamma)$, where $\bar{\Theta}$ is the unconditional probability that $\theta_{t+1} = \bar{\theta}$ and γ is the level of persistence in demand.

Proposition 7 *Suppose* **C1**, **C2**, **S1**, *and* $\theta \in \{\underline{\theta}, \bar{\theta}\}$. *Let date 0 capital stocks be chosen optimally by firms at date* -1 *given* μ_{-1} *and* θ_{-1}, *and suppose that both* $\exists \mu \preceq \mu_0$ *and* $\exists \mu' \succeq \mu_0$ *that can be realized along some equilibrium paths. Then there exists a* γ^*, *such that for* $0 \leq \gamma \leq \gamma^*$, $Y(\bar{\theta}, \cdot) > Y(\underline{\theta}, \cdot)$. *Suppose* $\theta_{-1} = \underline{\theta}, \theta_0 = \bar{\theta}, \theta_1 = \bar{\theta}, ..., \theta_t = \bar{\theta}, \theta_{t+1} = \underline{\theta}$, *for some* t. *Then as the boom continues:*

1. *Aggregate output evolves according to:* $Y_{-1} < Y_0; Y_1 > \cdots > Y_t > Y_{t+1}$.
2. *The distribution of firm productivities worsens:* $\mu_{-1} \succ \mu_0 \succ \cdots \succ \mu_t$.
3. *Optimal capital stocks increase:* $k_0(\alpha) < k_1(\alpha) < \cdots < k_t(\alpha)$.
   Suppose $\theta_{-1} = \bar{\theta}, \theta_0 = \underline{\theta}, \theta_1 = \underline{\theta}, ..., \theta_t = \underline{\theta}, \theta_{t+1} = \bar{\theta}$. *Then as the recession continues:*
4. *Aggregate output evolves according to:* $Y_{-1} > Y_0; Y_1 < \cdots < Y_t < Y_{t+1}$.
5. *The distribution of firm productivities improves:* $\mu_t \succ \mu_{t-1} \succ \cdots \succ \mu_{-1}$.
6. *Optimal capital stocks fall:* $k_0(\alpha) > k_1(\alpha) > \cdots > k_t(\alpha)$.

The longer the boom in demand, the worse is the resulting future distribution of firm productivities (7.2). Any given firm responds to the less productive competition by increasing its capital stock (7.3), but this increased capital stock only partially offsets the less productive distribution of firms (7.1). In turn, the greater capital stock and higher prices reduce the attractiveness of exit. Since firms invest more when demand is high and both high investment and greater demand reduce exit, the distribution of firm productivities must worsen as demand stays high (7.2). Thus, in prolonged periods of high demand, aggregate output initially increases, first due to less exit and then to greater investment, but then falls, as the distribution of firms worsens (7.1). The longer a demand boom lasts, the less productive is the distribution of firms that continue to produce and the worse the subsequent recession.

A recession initiated by a fall in demand leads first to exit of smaller firms and a reduction in capital stocks by all firms. Relative to the economy with adjustable capital, capital in place smooths the response by firms to fluctuations in demand. Without capital in place, unproductive firms respond to downturns in demand by exiting immediately. With capital in place, inefficient large firms first downsize and then exit. Firms exit only once the opportunity cost of foregone production has fallen sufficiently. Hence, output may fall the first two periods of a recession as it takes longer for the industry to react to the lower demand than when capital is freely adjustable. Then the distribution of more productive firms kicks in to increase output, even while demand stays low (7.4). Similarly, when demand first increases, it takes time for the firms to respond with greater investments in capital stocks so that it takes time for output to peak. To the extent that capital is putty–clay, large firms take even longer to downsize, so that they are even slower to exit, smoothing the initial onset of a recession, but lengthening the time it takes for the economy to recover.

Again, if we were to introduce an exogenous constant growth rate in average firm productivity then one could match the asymmetric features of business cycles. That is, downturns would tend to be shorter and sharper than booms. Downturns are sharp because the first effect of a reduction in demand is increased exit; short because the consequence of exit is a future improved distribution of firm productivities. Recessions would last only as long as it takes for firms first to adjust their capital stocks to reflect the worsened demand conditions and then exit to be replaced by more efficient new firms. Conversely, during prolonged periods of high demand, increases in output tend to peter out, as the failure of inefficient firms to exit leads to worsening distributions of technologies, so that the sole source of increased output is the exogenous trend in productivity.

5 Thin Resale Markets

So far we have assumed that the market for the perhaps very specialized resources of an exiting firm are deep. Phrased differently, the exiting firm extracts all of the surplus in any bargaining game with firms purchasing its resources. The deep market assumption may be reasonable for a retail shop setting, or a small plant, but may be less reasonable for larger manufacturing plants. When resource markets are thin, we show by construction that not only can a recession weed out 'bad' firms, but that a downturn in demand can actually *raise* welfare (total surplus) by inducing more efficient exit.

We capture the possibility that some matching or search is required to find a purchaser that can well–employ the particular storefront space by assuming that with positive probability an exiting firm may find only one bidder, or even none. The probability of finding j potential buyers is denoted $b_j, j = 0, 1, 2$. If there is no buyer, the firm waits until the next period to continue searching for a buyer. A single bidder can extract rents from the lack of competition: we assume that in this case, the firm is sold for its reservation value — the expected value from waiting an additional period in the hope of more competition for the resources. That is, the single bidder makes a take–it–or–leave–it offer that leaves the existing firm indifferent between accepting the offer and waiting in hope that it will receive more bidders in the future for its resources. If the firm finds a buyer, a sale occurs and the firm's resources can be employed in the market the next period. An exiting firm cannot return until its resources have been purchased by another firm.[8]

[8] Equivalently, a continuing bankrupt firm is assumed to be so unproductive that it never re–enters.

The value to a firm s of exit is given by:

$$v^e(\theta, \tau, s) = \beta \max_{\imath} b_2 \{ \int v(\tilde{\theta}, \tau^{\psi_\imath}, \rho k + \imath, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) P(d\tilde{\alpha} \mid \bar{\alpha}) - r\imath \}$$
$$+ \beta(b_0 + b_1) \{ \int v^e(\tilde{\theta}, \tau^{\psi_\imath}, 0, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) P(d\tilde{\alpha} \mid \bar{\alpha}) + r\rho k \}.$$

Implicit in $v^e(\theta, \tau, s)$ is that it is optimal for a firm that fails to find a bidder to completely disinvest: $\imath = -\rho k$. The qualitative findings are robust to considering bargaining solutions that offer an exiting firm a greater share of its value when it finds only a single bidder. When there is a single bidder for an exiting firm, that bidder chooses investment, $\imath(\theta, \tau^{\psi_\imath}, k, \bar{\alpha})$, optimally, even though its offer reflects the optimal investment opportunity available to an exiting firm without a bidder.

The analysis extends straightforwardly as long as the exiting firm extracts all of the surplus from exit. This is the case if the probability of one bidder is zero, even when there is a probability that the exiting firm finds no bidders (*i.e.* $b_0 > 0, b_1 = 0$). In this case, it is easy to show that the unique competitive equilibrium still corresponds to the solution to the social planner's discounted expected total surplus maximization problem, where the social planner's program reflects that only fraction b_2 of firms that are out of the market will re–enter the next period.

Qualitatively, the effect of increasing b_0 is to reduce the value of exit. For any given capital stock and market conditions, a firm has to have a lower productivity realization for exit to be optimal. If $b_0 = 1$, then firms never exit. For any given history of demand shocks, the larger is b_0, the worse is the distribution of firm technology qualities. As well, since firms are less likely to exit, they are more likely to exploit any given capital stock: the value of any given capital stock is increased. In turn, greater capital stocks further reduce exit. If b_0 is interpreted as reflecting the degree to which inputs are specialized in an industry, inter–industry comparisons can be made. Industries with more specialized inputs will tend to have more unproductive, but larger, firms and lower rates of exit. This is consistent with the finding of Dunne *et al.* (1989a) that substantial and persistent differences in entry and exit rates across industries exist, and that industries with higher entry rates also have higher exit rates.

The analysis is more complicated when $b_1 > 0$, *i.e.* when a bidder for a bankrupt firm's resources can extract rents. Social and private incentives to exit no longer coincide. When a bidder for a bankrupt firm can extract rents, firms are more reluctant to exit in the competitive equilibrium than is optimal from the perspective of a social planner who seeks to maximize total discounted expected surplus. While we can prove that an equilibrium exists using the techniques of Bergin and Bernhardt (1995), we have no proof that the equilibrium is unique.

When $b_1 > 0$, equilibrium corresponds to the solution to a modified social planner's optimization problem only if aggregate demand is perfectly elastic, $p(Y, \theta) = p(\theta)$. In that case, exit decisions depend only on b_2 and $b_0 + b_1$, because if there is a single bidder, that bidder will offer $\beta \int v^e(\tilde{\theta}) \Theta(d\tilde{\theta}|\theta)$, leaving the exiting firm indifferent between accepting and rejecting the offer.

That is, the marginal exiting firm equates

$$v^e(\theta,k) = \pi(\theta,k,\alpha^*(\theta,k)) + \beta\int v(\tilde{\theta},k^*(\theta,\bar{\alpha}),\tilde{\alpha}) + r\rho k - rk^*(\theta,\alpha^*)\Theta(d\tilde{\theta}|\theta)P(d\tilde{\alpha}|\alpha^*),$$

where the value of exit is given by

$$v^e(\theta,k) = \beta[r\rho k + \int[b_2[v(\tilde{\theta},k^*(\theta,\bar{\alpha}),\tilde{\alpha}) - rk^*(\theta,\bar{\alpha})] + [b_0+b_1]v^e(\tilde{\theta},0)]\Theta(d\tilde{\theta}|\theta)P(d\tilde{\alpha}|\bar{\alpha}).$$

The smaller is b_2, the lower is v^e; v is reduced only due to the reduction in the value of exiting, so that $\alpha^*(\theta,k;b_2)$ is an increasing function of b_2. In turn, the smaller is b_2, the *more* any given (α,θ) invests, because it is less likely to exit:

$$(1-\beta\rho)r = \beta Pr(\tilde{\alpha} \geq \alpha^*(k^*,\tilde{\theta})|\alpha)E\{\pi_k(\theta,k^*,\tilde{\alpha})|\alpha\}.$$

In turn, the greater investment in capital further reduces exit, since $\alpha^*(\theta,k;b_2)$ declines in k.

While private incentives depend only on b_0+b_1, aggregate output is a decreasing function of the fraction of exiting firms that are left inactive, b_0. Since individual firms exit according to their private incentives, $1-b_0-b_1$, rather than the social incentives, $1-b_0$, there is too little exit from a total surplus maximization perspective. The greater is b_1, the greater is the wedge between equilibrium exit and efficient exit.

In fact, when $b_1 > 0$, recessions may induce inefficient firms to act more in accordance with maximizing social surplus, by increasing the attraction of exit, so that privately optimal actions are more closely aligned with socially optimal actions. When the exiting firm does not extract all the surplus from its decision, so that firms are too reluctant from a surplus–maximizing standpoint to exit, *it may be that recessions can actually increase welfare* by inducing particularly inefficient firms to bite the bullet and sell their resources to a firm that could use them more productively. That is, the loss in current surplus caused by the immediate reduction in demand is less than the increase in future surplus generated by the future distribution of more efficient firms. We illustrate this with a simple example.

Example: Let production technologies be given by $\alpha f(\ell)$, where $\alpha \in \{\underline{\alpha},\hat{\alpha}\}, \underline{\alpha} < \hat{\alpha}$. Suppose that the productivity of a firm's technology does not vary with time, $\alpha_t = \alpha_{t-1}$, and that entrants are equally likely to draw each technology: $Pr(\alpha = \hat{\alpha}|entry) = .5$. Demand is perfectly elastic: $p(Y,\theta) = \theta, \theta \in \{\underline{\theta},\bar{\theta}\}, \underline{\theta} < \bar{\theta}$, and demand shocks are independently and identically distributed. Markets for the resources of a bankrupt firm are thin: $1 > b_2 = 1-b_1 > 0$. The discount factor is $\beta > .5$. There are as many good firms as bad.

Suppose that $\underline{\alpha}$ is sufficiently small relative to $\hat{\alpha}$ that total surplus is maximized when $\underline{\alpha}$ firms exit. Let b_1 be such that a $\underline{\alpha}$ firm is just indifferent between continuing to operate and exiting. Such a b_1 exists because the value of exiting varies continuously with b_1 from 0 when $b_1 = 1$ to the expected value of the new entrant when $b_1 = 0$. Consider the consequence for total surplus of a one–time marginal reduction in $\underline{\theta}$. This drop in demand induces all unproductive, $\underline{\alpha}$, firms to exit. The marginal loss in profits to a good firm from this reduced consumer demand is $\hat{\alpha}f(\ell^*(\hat{\alpha},\underline{\theta}))$. The marginal loss in expected discounted profits to a bad firm is zero, since it now exits and previously was indifferent between exiting and not. If we let $z = \frac{\beta E[\pi(\theta,\alpha)]}{(1-\beta)}$, then the value of exit is $\frac{(1-b_1)z}{(1-b_1\beta)}$. The social gain from inducing exit of inefficient $\underline{\alpha}$ firms is therefore $\frac{b_1(1-\beta)z}{(1-\beta b_1)}$. Since there are just as many good firms as bad firms, the one–time reduction in θ from $\underline{\theta}$ increases social surplus if and only if $\frac{b_1(1-\beta)z}{(1-\beta b_1)} > \hat{\alpha}f(\ell^*(\hat{\alpha},\underline{\theta}))$. But note that $\hat{\alpha}f(\ell^*(\hat{\alpha},\underline{\theta}))$ does not depend on $\bar{\theta}$. Let $b_1(\bar{\theta})$ be the value of b_1 that leaves $\underline{\alpha}$ just indifferent to exit when $\theta = \underline{\theta}$: $\frac{db_1}{d\theta} > 0$. As $\bar{\theta}$ is increased, it must be the case that $\frac{b_1(1-\beta)z}{(1-\beta b_1)}$ eventually exceeds $\hat{\alpha}f(\ell^*(\hat{\alpha},\underline{\theta}))$, so the one–time recession of a lower θ increases total surplus. ■

There is nothing special about this example: if b_1 is sufficiently large, bad firms fail to internalize the social cost of their failure to exit and thereby have their technologies replaced by ones that are stochastically better. Downturns cause more bad firms to interalize these social costs and exit. If this gain from increased exit outweighs the foregone period surplus from lower consumer demand then total surplus is raised.

6 Conclusion

In this paper we develop a model to study the dynamics of an industry over the business cycle. We allow for both aggregate demand uncertainty and for idiosyncratic uncertainty about the evolution of the productivity of an individual firm's technology. We endogenize the value of exit by assuming that firms must find a willing entrant to whom they can sell off their resources. We characterize the intertemporal evolution of the distribution of firms, where firms are distinguished by their capital in place and technology. We analyze industry dynamics in three ways: contrasting outcomes *across* different demand shock histories, *along* a particular demand shock history, and detailing how *anticipation* of future demand shocks affects outcomes. We characterize exit rates by age, size and productivity. Our theoretical predictions about cyclical patterns in exit and productivity are broadly consistent with those cyclical patterns exhibited in the data. We show how downturns in demand induce larger firms to downsize, and lead to increased exit, especially by smaller firms, causing future reductions in the cross–sectional variance in firm sizes and improving future distributions of firm productivities.

The next stage in the research is to see whether this model can match quantitatively the features of the economy at a disaggregated plant level. This will require additional fine–tuning of the theoretical model. This is because most firms do not adjust their factors of production in any given period, and when they do, the adjustments are large. Thus, to do a good job matching the plant–level data numerically, it is almost certainly necessary to modify the model to allow for costs of adjusting factors of production.

Fortunately, in our continuum setting our social planner argument (Theorem 2) can be extended to allow for fixed costs of adjusting input levels. So, too, with an appropriate modification of the social planner's objective, the analysis can allow for taxes. The key feature is that the first order effect on firm surplus due to price changes is transferred back and forth between producers and consumers and is netted out in total surplus calculations. This social planner's formulation will represent the core of our future numerical work.

7 Appendix

Proposition 1 *Given* **A1** *and* **A2**, *the exit rule characterizing the solution to the social planner problem, α^*, satisfies*

$$v^c(\theta, \mu, \alpha^*) = v^e(\theta, \mu, \alpha^*).$$

Proof: This is a special case of when firms are characterized by both capital and technology: we provide the proof for the general case later. ■

Proposition 2 *If* **A1** *and* **A2** *hold then given μ, θ there exists an exit threshold α^*, such that firms with $\alpha > \alpha^*$ remain in the market and firms with $\alpha < \alpha^*$ exit.*

Proof: From the definitions of v^c and v^e, v^c is strictly increasing in α while v^e is independent of α. Therefore, there is a unique $\hat{\alpha}$ with $v^c(\theta, \mu, \alpha) > v^e(\theta, \mu, \alpha), \forall \alpha > \hat{\alpha}$ and $v^c(\theta, \mu, \alpha) < v^e(\theta, \mu, \alpha), \forall \alpha < \hat{\alpha}$. ■

Theorem 1

1. *Better (higher α) firms are more profitable and produce more output.*
2. *Furthermore, if* **E1** *and* **C1** *hold and exit occurs with positive probability, then better firms are less likely to exit, expect to produce more in the future, earn higher profits and have longer expected lifetimes.*
3. *Furthermore, if* **C1** *and* **C2** *hold then the older an active firm is, the greater is its expected productivity and the longer its expected lifetime (i.e. hazard rates into bankruptcy fall with age).*

Proof : *1.* $d\pi(\theta, \mu, \alpha)/d\alpha \geq 0$ and $dq(\theta, \mu, \alpha)/d\alpha \geq 0$ (both strict if $q_\alpha > 0$): Firms with better technologies are more profitable and produce more output.

2. Proposition 1 demonstrates that good firms will not exit before firms with inferior technologies. If $\alpha > \alpha'$ then by **C1**,

$$P^N(\cdot, \alpha) \succeq_c P^N(\cdot, \alpha'), N = 1, 2, \ldots$$

which given **E1** (firms exit in finite time with probability one) ensures that better firms have longer expected lifetimes.

3. Let **C1** and **C2** hold. Fix an aggregate shock realization θ^∞. Let (μ_t, θ_t) be the aggregate distribution and aggregate shock at time t, and $\alpha(\mu_t, \theta_t)$ the exit threshold. Then the normalized distribution of two year old firms at time $t+1$ is given by $\int P(\cdot, \alpha) P(d\alpha \mid \alpha \geq \alpha(\mu_t, \theta_t), \bar{\alpha})$. Similarly, the normalized distribution of one year old firms is $P(\cdot, \bar{\alpha})$. In the discussion to follow we make use of the following three observations:

(a) Given a probability measure $\mu(\cdot)$, the measure $\mu(\cdot \mid \alpha \geq \bar{\alpha})$ satisfies

$$\mu(\cdot \mid \alpha \geq \bar{\alpha}) \succeq_c \mu(\cdot)$$

(b) If $P(\cdot, \alpha)$ satisfies $P(\cdot, \alpha) \succeq_c P(\cdot, \alpha')$ whenever $\alpha > \alpha'$ and μ, ν are two probability measures such that $\mu \succeq_c \nu$, then

$$\hat{\mu}(\cdot) = \int P(\cdot, \alpha)\mu(d\alpha) \succeq_c \int P(\cdot, \alpha)\nu(d\alpha) = \hat{\nu}(\cdot).$$

(c) If μ, ν are two probability measures such that $\mu \succeq_c \nu$, then $\varphi(\cdot) = \mu(\cdot \mid \alpha \geq \bar{\alpha})$ and $\beta(\cdot) = \nu(\cdot \mid \alpha \geq \bar{\alpha})$ satisfy $\varphi(\cdot) \succeq_c \beta(\cdot)$

From **C1** and **C2**,

$$\int P(\cdot, \alpha)P(d\alpha \mid \alpha \geq \alpha(\mu_t, \theta_t)) \succeq_c \int P(\cdot, \alpha)P(d\alpha, \bar{\alpha}) \succeq_c P(\cdot, \bar{\alpha}).$$

(The first comparison follows from observations a and b, using assumption **C1** — that $P(\cdot, \alpha) \succeq_c P(\cdot, \alpha')$ whenever $\alpha > \alpha'$. The second comparison follows from **C2**). Thus, the distribution of two year old firms (normalized to be a probability) dominates that of new firms. Assume now that the (normalized) distribution (at time $t + k - 1$) of k year old firms, μ_k, conditionally dominates that of $k - 1$ year old firms, $\mu_{k-1} : \mu_k \succeq_c \mu_{k-1}$. Then the distribution of $k + 1$ year old firms (at time $t + k$), $\hat{\mu}_{k+1}$, is related to the distribution of k year old firms (at time $t + k$), $\hat{\mu}_k$ according to

$$\hat{\mu}_{k+1}(\cdot) = \int P(\cdot, \alpha)\mu_k(d\alpha \mid \alpha \geq \alpha(\mu_{t+k-1}, \theta_{t+k-1})\ ) \succeq_c$$
$$\int P(\cdot, \alpha)\mu_{k-1}(d\alpha \mid \alpha \geq \alpha(\mu_{t+k-1}, \theta_{t+k-1})\ ) = \hat{\mu}_k(\cdot),$$

(using **C1** with observations b and c). Thus, the distribution of $k+1$ year old firms dominates that of k year old firms. Hence the hazard into bankruptcy falls with the age of a firm. ■

Proposition 3 *Suppose* $\bar{\mu} \succ_c \hat{\mu}$. *Then* $Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}) > Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})})$. *This implies* $p(Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}), \theta) < p(Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})}), \theta)$, *and* $\pi(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}, \alpha) < \pi(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})}, \alpha)$.

Proof: Suppose not and $Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}) \leq Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})})$. Then since greater output implies lower price, $p(Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}), \theta) \geq p(Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})}), \theta)$. Also, since aggregate output is increasing in μ, $Y(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}) \leq Y(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})})$ and $\bar{\mu} \succ_c \hat{\mu}$ imply that $\alpha(\theta, \bar{\mu}) > \alpha(\theta, \hat{\mu})$. Since a firms profit depends on aggregate output through price, $\bar{\pi} = \pi(\theta, \bar{\mu}_{\alpha(\theta,\bar{\mu})}, \alpha(\theta, \bar{\mu})) \geq \pi(\theta, \hat{\mu}_{\alpha(\theta,\hat{\mu})}, \alpha(\theta, \hat{\mu})) = \hat{\pi}$. Since $\bar{\mu} \succ_c \hat{\mu}$ and $\alpha(\theta, \bar{\mu}) > \alpha(\theta, \hat{\mu})$, this implies that the next period's distribution of firms satisfies, $\bar{\mu}^{\alpha(\theta,\bar{\mu})} \succ_c \hat{\mu}^{\alpha(\theta,\hat{\mu})}$. Then,

$$\hat{\pi} = \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}(\theta,\hat{\mu})}, \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)[P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}(\theta, \hat{\mu}))]$$
$$> \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v(\tilde{\theta}, \bar{\mu}^{\bar{\alpha}(\theta,\bar{\mu})}, \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)[P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \bar{\alpha}(\theta, \bar{\mu}))] = \bar{\pi},$$

where the equalities are the equilibrium exit rules and the inequality follows from monotonicity in μ. However, this contradicts $\bar{\pi} > \hat{\pi}$.

■

Lemma 1 *The functions $v^c(\theta, \mu, \alpha)$ and $v^e(\theta, \mu, \alpha)$ are continuously decreasing in μ.*

Proof: In lemma 5, below, we show that for the capital in place model, lowering the exit threshold increases output at each future date and state — because the distribution of technologies is improved at each future date and state. Essentially the same argument applies here — improving the current distribution has a "knock–on" effect, improving the distribution in subsequent states and dates. ■

The next two results clarify the role of **S1**.

Result 1 *Let $\mu = \beta F + (1-\beta)G$ and $\mu' = \beta' F + (1-\beta')G$, where $\beta' > \beta$. Then $\mu' \succ_c \mu$.*

Proof: To see this, it is sufficient to show that $\phi_\beta(\alpha)$ is increasing in β for each $\alpha \geq \alpha^*$, where:

$$\phi_\beta(\alpha) = \mu([\alpha, 1] \mid \alpha \geq \alpha^*) = \frac{\mu([\alpha, 1])}{\mu([\alpha^*, 1])}.$$

Now,

$$\begin{aligned}\phi_\beta(\alpha) = \mu([\alpha, 1] \mid \alpha \geq \alpha^*) =& \frac{\beta F([\alpha, 1]) + (1-\beta)G([\alpha, 1])}{\beta F([\alpha^*, 1]) + (1-\beta)G([\alpha^*, 1])} \\ =& \frac{\beta[F([\alpha, 1]) - G([\alpha, 1])] + G([\alpha, 1])}{\beta[F([\alpha^*, 1]) - G([\alpha^*, 1])] + G([\alpha^*, 1]} \\ =& \frac{\beta k(\alpha) + d(\alpha)}{\beta k(\alpha^*) + d(\alpha^*)},\end{aligned}$$

where $k(\tilde{\alpha}) = [F([\tilde{\alpha}, 1]) - G([\tilde{\alpha}, 1])]$ and $d(\tilde{\alpha}) = G([\tilde{\alpha}, 1])$. Therefore,

$$\begin{aligned}\frac{\partial \phi_\beta}{\partial \beta} =& \frac{[\beta k(\alpha^*) + d(\alpha^*)]k(\alpha) - [\beta k(\alpha) + d(\alpha)]k(\alpha^*)]}{[\beta k(\alpha^*) + d(\alpha^*)]^2} \\ =& \frac{[d(\alpha^*)k(\alpha) - d(\alpha)k(\alpha^*)]}{[\beta k(\alpha^*) + d(\alpha^*)]^2}\end{aligned}$$

The numerator is

$$F([\alpha, 1])G([\alpha^*, 1]) - G([\alpha, 1])G([\alpha^*, 1]) - [F([\alpha^*, 1])G([\alpha, 1]) - G([\alpha^*, 1])G([\alpha, 1])].$$

Canceling $G([\alpha, 1])G([\alpha^*, 1])$ gives the numerator as

$$F([\alpha, 1])G([\alpha^*, 1]) - F([\alpha^*, 1])G([\alpha, 1]) = F([\alpha^*, 1])G([\alpha^*, 1]) \left[\frac{F([\alpha, 1])}{F([\alpha^*, 1])} - \frac{G([\alpha, 1])}{G([\alpha^*, 1])}\right].$$

Thus, $\mu' \succeq_c \mu$.

■

Given an exit rule α^*, $\alpha^* < \bar{\alpha}$, consider the measure of firm technology qualities the succeeding period:

$$\mu^{\alpha^*}(\cdot) = \int_{[0,\alpha^*)} P(\cdot \mid \bar{\alpha})\mu(d\alpha) + \int_{[\alpha^*,1]} P(\cdot \mid \alpha)\mu(d\alpha).$$

The next lemma asserts that either increasing the measure of exiting firms, *i.e.* increasing the exit threshold, α^*, or improving the current distribution over firm technologies, μ, improves the distribution of firm technology qualities the next period. Let $\mathcal{W}(F,G) = \{\mu \mid \exists \beta \in [0,1], \mu = \beta F + (1-\beta)G\}$:

Result 2 *Suppose that* **C2** *and* **S1** *hold. Then the measure $\mu^{\alpha^*}(\cdot)$, viewed as a function of μ and α^*, satisfies:*

1. *$\hat{\alpha}^* \geq \alpha^*$ implies $\mu^{\hat{\alpha}^*}(\cdot) \succ_c \mu^{\alpha^*}(\cdot)$.*
2. *If $\hat{\mu}, \mu \in \mathcal{W}(F,g)$, $\hat{\mu} \succ \mu$ implies $\hat{\mu}^{\alpha^*}(\cdot) \succ_c \mu^{\alpha^*}(\cdot)$.*

Proof :

$$\begin{aligned}
\mu^{\alpha^*}(\cdot) &= \int_{[0,\alpha^*)} P(\cdot \mid \bar{\alpha})\mu(d\alpha) + \int_{[\alpha^*,1]} P(\cdot \mid \alpha)\mu(d\alpha) \\
&= F(\cdot) \int_{[\alpha^*,1]} w(\alpha)\mu(d\alpha) + G(\cdot) \int_{[\alpha^*,1]} (1 - w(\alpha))\mu(d\alpha) \\
&\qquad + \mu([0,\alpha^*))w(\bar{\alpha})F(\cdot) + \mu([0,\alpha^*))(1 - w(\bar{\alpha})G(\cdot) \\
&= [\int_{[\alpha^*,1]} w(\alpha)\mu(d\alpha) + \mu([0,\alpha^*))w(\bar{\alpha})]F(\cdot) + \\
&\qquad [\int_{[\alpha^*,1]} (1 - w(\alpha))\mu(d\alpha) + \mu([0,\alpha^*))(1 - w(\bar{\alpha}))]G(\cdot).
\end{aligned}$$

Let

$$x_{\alpha^*}(\alpha) = \chi_{[0,\alpha^*)}(\alpha)w(\bar{\alpha}) + \chi_{[\alpha^*,1]}(\alpha)w(\alpha),$$

so that $E_\mu\{x_{\alpha^*}\} = \int x_{\alpha^*}(\alpha)\mu(d\alpha)$ gives the weight on $F(\cdot)$ in the distribution $\mu^{\alpha^*}(\cdot)$:

$$\mu^{\alpha^*}(\cdot) = E_\mu\{x_{\alpha^*}\}F(\cdot) + [1 - E_\mu\{x_{\alpha^*}\}]G(\cdot).$$

To prove 1, note that $\alpha^* < \bar{\alpha}$, and since w is monotone increasing, the function x_{α^*} is constant and equal to $w(\bar{\alpha})$ on $[0,\alpha^*)$, has a "downward jump" at α^* (from $w(\bar{\alpha})$ to $w(\alpha^*)$), and is monotone increasing on $[\alpha^*,1]$. If $\bar{\alpha} \geq \hat{\alpha}^* \geq \alpha^*$, then $x_{\hat{\alpha}^*} \geq x_{\alpha^*}$, so that

$$E_\mu\{x_{\hat{\alpha}^*}\} \geq E_\mu\{x_{\alpha^*}\}.$$

Thus 1 holds. Now we prove 2.

Let $\mu = \beta F(\cdot) + (1-\beta)G(\cdot)$. We need to show that as β increases, $E_\mu(x_{\alpha^*})$ increases. Note that

$$E_\mu(x_{\alpha^*}) = \mu([0,\alpha^*))w(\bar{\alpha}) + \mu([\alpha^*,1])E_\mu\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}.$$

Since

$$\int_{[\alpha^*,1]} w(\alpha)\mu(d\alpha) = \beta \int_{[\alpha^*,1]} w(\alpha)F(d\alpha) + (1-\beta)\int_{[\alpha^*,1]} w(\alpha)G(d\alpha) =$$
$$\beta F([\alpha^*,1])E_F\{w(\alpha) \mid \alpha \geq \alpha^*\} + (1-\beta)G([\alpha^*,1])E_G\{w(\alpha) \mid \alpha \geq \alpha^*\}.$$

Thus,

$$E_\mu\{x_{\alpha^*}\} = [\beta F([0,\alpha^*)) + (1-\beta)G([0,\alpha^*))]w(\bar{\alpha}) +$$
$$\beta F([\alpha^*,1])E_F\{w(\alpha) \mid \alpha \geq \alpha^*\} + (1-\beta)G([\alpha^*,1])E_G\{w(\alpha) \mid \alpha \geq \alpha^*\},$$

or

$$E_\mu\{x_{\alpha^*}\} = \beta[F([0,\alpha^*))w(\bar{\alpha}) + F([\alpha^*,1])E_F\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}] +$$
$$(1-\beta)[G([0,\alpha^*))w(\bar{\alpha}) + G([\alpha^*,1])E_G\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}].$$

Using conditional stochastic dominance (x_{α^*} is increasing on $[\alpha^*,1]$),

$$E_F\{x_{\alpha^*} \mid \alpha \geq \alpha^*\} \geq E_G\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}.$$

Provided $E_F\{x_{\alpha^*} \mid \alpha \geq \alpha^*\} \geq w(\bar{\alpha})$, $E_\mu\{x_{\alpha^*}\}$ is increasing in β. A sufficient condition for this to be so is that $E_F\{w\} \geq w(\bar{\alpha})$. But **C2** implies that $E_F\{w\} \geq w(\bar{\alpha})$. Under **S1**, **C2** holds if and only if

$$[\int w(\alpha)P(d\alpha,\bar{\alpha})]F(\cdot) + [1 - \int w(\alpha)P(d\alpha,\bar{\alpha})]G(\cdot) \succeq_c P(\cdot \mid \bar{\alpha}) = w(\bar{\alpha})F(\cdot) + [1 - w(\bar{\alpha})]G(\cdot).$$

Since $\int w(\alpha)F(d\alpha) \geq \int w(\alpha)P(d\alpha,\bar{\alpha})$,

$$\int w(\alpha)F(d\alpha)]F(\cdot) + [1 - \int w(\alpha)F(d\alpha)]G(\cdot) \succ_c \int w(\alpha)P(d\alpha,\bar{\alpha})]F(\cdot) + [1 - \int w(\alpha)P(d\alpha,\bar{\alpha})]G(\cdot),$$

so that $\int w(\alpha)F(d\alpha) = E_F\{w\} \geq w(\bar{\alpha})$.

■

Lemma 2 *Given* **S1**, *suppose that* $\theta \in \{\underline{\theta},\bar{\theta}\}$, $\underline{\theta} < \bar{\theta}$ *and that profits are multiplicatively separable:* $\pi(\alpha, p(\mu,\theta)) = g(p)h(\alpha)$ *for some functions* g *and* h. *Then for any* $\mu \in [\underline{\mu},\bar{\mu}]$, $\alpha(\mu,\underline{\theta}) > \alpha(\mu,\bar{\theta})$.

Proof: Let $Pr(\theta = \bar{\theta}|\bar{\theta}) = \rho$; $Pr(\theta = \underline{\theta}|\underline{\theta}) = \phi$.

Suppose that $\rho = \phi = 1$, so that for some $\theta \in \{\underline{\theta}, \bar{\theta}\}$, $\theta_t = \theta_0 = \theta$, and the marginal exiting firm, $\alpha^*(\theta, \mu)$, is determined by the solution to

$$\pi(\theta, \mu_{\alpha^*}, \alpha^*) = \delta \int_{\tilde{\alpha}} v(\theta, \mu_{\alpha^*}, \tilde{\alpha})[P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha^*)].$$

With $\rho = \phi = 1$ the state is constant over time and equal to θ. Suppose that the aggregate distribution converges to some distribution $\mu_\infty(\theta)$: $lim_{t\to\infty}\mu_t \to \mu_\infty(\theta)$, with associated price sequence, $p(\mu_t, \theta) \to p(\mu_\infty, \theta)$. From the multiplicative decomposition of profits, asymptotically, the value functions are multiplicative functions of price, so that the exit rule, α^*, is asymptotically independent of θ (and only relative prices matter). Hence, $\mu_\infty(\underline{\theta}) = \mu_\infty(\bar{\theta}) = \mu_\infty$ with associated exit rule α_∞.

Let $\alpha^{\rho\phi}(\theta, \mu)$ be the equilibrium exit threshold at state θ when the aggregate distribution is μ and the transition probabilities on θ given by ρ and ϕ. Note that $\alpha^{1\phi}(\bar{\theta}, \mu_\infty) = \alpha_\infty = \alpha^{\rho 1}(\underline{\theta}, \mu_\infty)$ because at $\rho = 1$ ($\phi = 1$) the exit rule at $(\bar{\theta}, \mu_\infty)$ ($(\underline{\theta}, \mu_\infty)$) is independent of ϕ (ρ).

For any (θ, μ), observe that $\alpha^{\rho\phi}(\theta, \mu)$ is (a) increasing in ρ and (b) decreasing in ϕ. (Raising ρ raises the future payoff leaving current payoff unchanged and therefore leads to more exit, increasing ϕ reduces the future payoff leaving current payoff unchanged and therefore leads to less exit.) Thus,

$$\alpha^{1\phi}(\underline{\theta}, \mu_\infty) > \alpha^{\rho\phi}(\underline{\theta}, \mu_\infty) > \alpha^{\rho 1}(\underline{\theta}, \mu_\infty) = \alpha_\infty = \alpha^{1\phi}(\bar{\theta}, \mu_\infty) > \alpha^{\rho\phi}(\bar{\theta}, \mu_\infty) > \alpha^{\rho 1}(\bar{\theta}, \mu_\infty).$$

Next, fix some ρ, ϕ pair. Since $\alpha^{\rho\phi}(\underline{\theta}, \mu_\infty) > \alpha_\infty > \alpha^{\rho\phi}(\bar{\theta}, \mu_\infty)$, at the distribution μ_∞, more exit occurs at $\underline{\theta}$ than at $\bar{\theta}$. Starting at μ_∞, denote the next periods distribution following $\bar{\theta}$ by $\mu_1^{\bar{\theta}}$ and following $\underline{\theta}$ by $\mu_1^{\underline{\theta}}$. Noting that $\mu_\infty = \mu_\infty^\infty$, $\mu_1^{\bar{\theta}} \prec \mu_\infty \prec \mu_1^{\underline{\theta}}$.

To make dependence on μ_∞ explicit, write $\mu_1^{\bar{\theta}}(\mu_\infty) \prec \mu_\infty \prec \mu_1^{\underline{\theta}}(\mu_\infty)$. Since for fixed θ, improving (worsening) the current distribution improves (worsens) next periods distribution, if the aggregate shock at $\mu_1^{\bar{\theta}}(\mu_\infty)$ is $\bar{\theta}$, then next periods distribution $\mu_2^{\bar{\theta},\bar{\theta}}(\mu_\infty) \prec \mu_1^{\bar{\theta}}(\mu_\infty)$. Letting $\bar{\theta}(t)$ denote t realizations of $\bar{\theta}$, and $\mu_t^{\bar{\theta}(t)}(\mu_\infty)$ the associated distribution in t periods, the distribution satisfies: $\mu_{t+1}^{\bar{\theta}(t+1)}(\mu_\infty) \prec \mu_t^{\bar{\theta}(t)}(\mu_\infty)$. Let $\underline{\mu}$ denote the limit. Similar reasoning gives $\mu_{t+1}^{\underline{\theta}(t+1)}(\mu_\infty) \succ \mu_t^{\underline{\theta}(t)}(\mu_\infty)$, where $\mu_t^{\underline{\theta}(t)}(\mu_\infty)$ is the distribution t periods later following a history of t-$\underline{\theta}$ shocks. let $\bar{\mu}$ denote the limit.

Now, fix some $\mu \in (\underline{\mu}, \mu_\infty)$. Given $\bar{\theta}$, let $\mu^{\bar{\theta}}(\mu)$ be next periods distribution. From the previous reasoning, $\mu^{\bar{\theta}}(\mu) \prec \mu$. Next, consider μ and the shock $\underline{\theta}$. In this case, $\mu^{\underline{\theta}}(\mu) \succ \mu$. To see this, note that as ϕ increases, the future payoff is lower and this reduces exit. However, at $\underline{\theta}$ with $\phi = 1$, the system stays in state $\underline{\theta}$ and the aggregate distribution increases to μ_∞ asymptotically. In particular, $\mu^{\underline{\theta}}(\mu) \succ \mu$ when $\phi = 1$. However, when $\phi < 1$ the future payoff increases and this increases exit (recall $\alpha^{\rho\phi}(\theta, \mu)$ is decreasing in ϕ so exit increases as ϕ falls). But more exit improves next periods distribution, so that for the given $\phi\rho$ pair, $\mu^{\underline{\theta}}(\mu) \succ \mu$. Thus, $\mu^{\underline{\theta}}(\mu) \succ \mu \succ \mu^{\bar{\theta}}(\mu)$. This implies that exit at $(\underline{\theta}, \mu))$ is greater than exit at $\bar{\theta}, \mu)$, or $\alpha^{\rho\phi}(\underline{\theta}, \mu) > \alpha^{\rho\phi}(\bar{\theta}, \mu)$. This completes the argument. ■

Proposition 4 *Suppose assumptions* **S1** *and* **D1** *hold. Let $\hat{\mu}_0 = \bar{\mu}_0$. Consider two aggregate shock histories, $\hat{\theta}^t, \bar{\theta}^t$ where $\hat{\theta}_0 > \bar{\theta}_0$, $\hat{\theta}_\tau \geq \bar{\theta}_\tau, \forall \tau, 0 \leq \tau \leq t$. Then*

1. $\bar{\mu}_{\tau+1} \succ_c \hat{\mu}_{\tau+1}, \quad 0 \leq \tau \leq t.$
2. $\exists \delta > 0$ *such that if* $\bar{\theta}_{t+1} \geq \hat{\theta}_{t+1} - \delta$ *then* $Y_{t+1}(\bar{\theta}_{t+1}, \bar{\mu}^{\alpha_{t+1}(\bar{\theta}_{t+1},\bar{\mu})}) > Y_{t+1}(\hat{\theta}_{t+1}, \hat{\mu}^{\alpha_{t+1}(\hat{\theta}_{t+1},\hat{\mu})}), \quad 0 \leq \tau \leq t.$
3. $\bar{p}_\tau < \hat{p}_\tau, \quad 0 \leq \tau \leq t.$
4. $\pi(\hat{\theta}_\tau, \hat{\mu}_\tau, \alpha) > \pi(\bar{\theta}_\tau, \bar{\mu}_\tau, \alpha), \quad 0 \leq \tau \leq t.$

Proof:

1. Follows immediately from **D1** and lemma 1. Consider the first period τ in which $\hat{\theta}_\tau > \bar{\theta}_\tau$. Then, **D1** ensures the higher demand shock implies less exit, and hence a worse distribution of firms. From lemma 1, even if subsequent demand shocks are identical, the worse distribution is preserved, and subsequent higher demand shocks only reinforce the result.
2. Follows from 1., **D1** and continuity of Y in θ.
3. Suppose not. Then there must be less exit in the hat economy: $\bar{\alpha}_t > \hat{\alpha}_t$. But then,

$$\begin{aligned}\pi(\hat{\theta}_t, \hat{\mu}_t^{\hat{\alpha}_t}, \hat{\alpha}_t) < \pi(\hat{\theta}_t, \hat{\mu}^{\hat{\alpha}_t}, \bar{\alpha}_t) < \pi(\bar{\theta}_t, \bar{\mu}_t^{\bar{\alpha}_t}, \bar{\alpha}_t) \\ = \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v^*_{t+1}(\tilde{\theta}, \bar{\mu}_{\bar{\alpha}_{t+1}}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \bar{\theta})[P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \bar{\alpha}_{t+1})] \\ < \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v^*_{t+1}(\tilde{\theta}, \hat{\mu}_{\hat{\alpha}_{t+1}}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \hat{\theta})[bP(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}_{t+1})],\end{aligned}$$

   a contradiction.
4. Follows from 3. ■

Corollary 1 *Suppose that* **D1** *holds. If $\bar{\mu}_t = \hat{\mu}_t$, $\bar{\theta}_t < \hat{\theta}_t$, then $\exists \delta > 0$ such that if $\bar{\theta}_{t+1} \geq \hat{\theta}_{t+1} - \delta$ then $\bar{Y}_{t+1} - \bar{Y}_t > \hat{Y}_{t+1} - \hat{Y}_t$.*

Proof: Follows immediately from proposition 4, and $\bar{Y}_t < \hat{Y}_t$. ■

Corollary 2 *Suppose the conditions for lemma 2 hold, $\theta \in \{\bar{\theta}, \underline{\theta}\}$, and that $\mu_{t-1} \in [\underline{\mu}, \bar{\mu}]$. Then*

1. $Y_t(\bar{\theta}, \mu) > Y_t(\underline{\theta}, \mu), \forall \mu.$
2. *Suppose $\theta_{t-1} = \underline{\theta}, \theta_t = \bar{\theta}, \theta_{t+1} = \bar{\theta}, ..., \theta_{t+\tau} = \bar{\theta}, \theta_{t+\tau+1} = \underline{\theta}$. Then $Y_{t-1} < Y_t > Y_{t+1} > \cdots > Y_{t+\tau} > Y_{t+\tau+1}$, so that $p_t < p_{t+1} < \cdots < p_{t+\tau}$. Hence, $y_t(\alpha) < y_{t+1}(\alpha) < \cdots < y_{t+\tau}(\alpha)$ and $\pi_t(\alpha) < \pi_{t+1}(\alpha) < \cdots < \pi_{t+\tau}(\alpha)$.*
3. *Suppose $\theta_{t-1} = \bar{\theta}, \theta_t = \underline{\theta}, \theta_{t+1} = \underline{\theta}, ..., \theta_{t+\tau} = \underline{\theta}, \theta_{t+\tau+1} = \bar{\theta}$. Then $Y_{t-1} > Y_t < Y_{t+1} < ... < Y_{t+\tau} < Y_{t+\tau+1}$, so that $p_t > p_{t+1} > \cdots > p_{t+\tau}$. Hence, $y_t(\alpha) > y_{t+1}(\alpha) > \cdots > y_{t+\tau}(\alpha)$ and $\pi_t(\alpha) > \pi_{t+1}(\alpha) > \cdots > \pi_{t+\tau}(\alpha)$.*

Proof: Follows from the proof to proposition 4: a high demand shock leads to a worse distribution of firm types the next period. ■

Proposition 5 *Consider a one–time anticipated improvement in T–th period ahead demand, so that:*

$$\bar{\Theta}_T(\theta_T \mid \theta_{T-1}) \succ \hat{\Theta}_T(\theta_T \mid \theta_{T-1}), \forall \theta_{T-1}.$$

Then for any given demand shock history, $\theta_0, \ldots, \theta_{t-1}$, the anticipated increase in future demand improves the distribution of firm productivities at all earlier dates: $\bar{\mu}_\tau \succ \hat{\mu}_\tau, t < \tau \leq T$.

Proof:

$$\int\int \bar{v}_T(\tilde{\theta}_T, \mu, \tilde{\alpha})\bar{\Theta}_T(d\tilde{\theta}_T \mid \theta_{T-1})P(d\tilde{\alpha} \mid \alpha) > \int\int \hat{v}_T(\tilde{\theta}_T, \mu, \tilde{\alpha})\hat{\Theta}_T(d\tilde{\theta}_T \mid \theta_{T-1})P(d\tilde{\alpha} \mid \alpha).$$

For any given μ_{T-1}, current profits for any marginal exiter $\pi(\theta_{T-1}, \mu_{T-1\alpha^*}, \alpha^*)$ are the same in both the hat and bar economies, but the value to exit is greater in the bar economy. Hence, for any given μ_{T-1}, there must be more exit in the bar economy than the hat economy, so that $\bar{\mu}_T(\mu_{T-1}) \succ \hat{\mu}_T(\mu_{T-1})$. In turn, this increased exit in the bar economy implies that for any given μ_{T-1} and θ_{T-1}, date $T-1$ prices are higher in the bar economy than the hat economy. In turn, this implies that

$$\int\int \bar{v}_{T-1}(\tilde{\theta}_{T-1}, \mu, \tilde{\alpha})\Theta_{T-1}(d\tilde{\theta}_T \mid \theta_{T-2})P(d\tilde{\alpha} \mid \alpha) > \int\int \hat{v}_{T-1}(\tilde{\theta}_{T-1}, \mu, \tilde{\alpha})\Theta_{T-1}(d\tilde{\theta}_T \mid \theta_{T-2})P(d\tilde{\alpha} \mid \alpha).$$

Hence, for any given μ_{T-2}, there is more exit in the bar economy than the hat economy, so that $\bar{\mu}_{T-1} \succ \hat{\mu}_{T-1}$. Monotonicity of μ_T in μ_{T-1} then implies that $\bar{\mu}_T(\bar{\mu}_{T-1}(\mu_{T-2}), \theta) \succ \bar{\mu}_T(\hat{\mu}_{T-1}(\mu_{T-2}), \theta)$.

Repeating the argument, observe that since for any given μ_{T-2} there is more exit in the bar than hat economy, the distribution of prices must be higher in the bar than hat economy at date $T-2$, so that date $T-2$ continuation payoffs must be greater for any given μ_{T-2} in the bar economy than the hat economy. Hence, for any given μ_{T-3}, there must be more exit in the bar economy than the hat economy. Repeating this argument, it follows that $\bar{\mu}_t \succ \hat{\mu}_t, 0 < t \leq T$. ■

Theorem 2 *The solution to the social planner problem determines the same entry–exit rule and investment rule as the equilibrium condition.*

Proof: There are three components in the social planner problem:

1. Current Surplus:

$$\max_Y \int_{[0,Y)} [P_d(Y, \theta) - P_s(Y, \tau_\psi)]dY.$$

2. Continuation valuation:

$$\int_\Theta V(\tau^{\psi_1}, \tilde{\theta})\Theta(d\tilde{\theta} \mid \theta).$$

3. Investment:
$$r[\int_{k\geq\psi(\alpha)} \imath^c(k,\alpha)\tau(dk,d\alpha) + \int_{k<\psi(\alpha)} \imath^e(k,\alpha)\tau(dk,d\alpha)].$$

First, we consider the impact of a perturbation in the exit rule on each of these separately. Then, we consider the impact of a perturbation in the investment rule. (At the end of the proof, we provide a simple motivation for these calculations.)

Current Surplus. With ψ be the equilibrium exit rule, the current surplus is

$$\begin{aligned}
\int_{[0,Y^*)} [P_d(Y,\theta) - P_s(Y,\tau_\psi)]dY &= \int_{[0,Y^*)} P_d(Y,\theta)dY - [P^*Y^* - \int_{[0,P^*)} Y(P,\tau_\psi)dP] \\
&= \int_{[0,Y^*)} P_d(Y,\theta)dY - P^*Y^* + \int_{[0,P^*)} Y(P,\tau_\psi)dP \\
&= \int_{[0,Y^*)} P_d(Y,\theta)dY - P^*Y^* + \int_{[0,P^*)}\int_T q(P,s)d\tau_\psi dP \\
&= \int_{[0,Y^*)} P_d(Y,\theta)dY - P^*Y^* + \int_T\int_{[0,P^*)} q(P,s)dP d\tau_\psi \\
&= \int_{[0,Y^*)} P_d(Y,\theta)dY - P^*Y^* + \int_T \pi(s)d\tau_\psi.
\end{aligned}$$

The approximate impact of a small perturbation of the exit rule, from τ_ψ to $\tau_\psi + \epsilon\hat{\tau}$ is thus

$$\int_T \pi(s)d[\tau_\psi + \epsilon\hat{\tau}] - \int_T \pi(s)d\tau_\psi.$$

Dividing by ϵ and passing to the limit gives $\int_T \pi(s)d\hat{\tau}$. If we fix a region, R, along the boundary ψ and define $\hat{\tau}(X) = \tau(X\cap R)$, then this limit is $\int_R \pi(s)d\tau$. We give some additional discussion on these calculations at the end of the proof.

Continuation Valuation. Next, consider the future distribution over T. Let $G(\cdot\mid s)$ be the distribution over T of a firm which continues in existence T and $\bar{G}(\cdot\mid s)$ the distribution over T of a firm which exits. Write $\psi^< = \{(\alpha,k)\in T\mid k<\alpha\}$ and $\psi^\geq = \{(\alpha,k)\in T\mid k\geq\alpha\}$. Thus, next period's distribution over T is given by:

$$\tau^{\psi_1}(\cdot) = \int_{\psi^<} \bar{G}(\cdot\mid s)d\tau + \int_{\psi^\geq} G(\cdot\mid s)d\tau.$$

Fix a region in $\psi^<$, $R\subset\psi^<$, and let the fraction ϵ remain in the market with the remainder $(1-\epsilon)$ in the region continuing to exit. Denote the complement of R by R^c. In this case next period's distribution is given by:

$$\begin{aligned}
\tau^{\psi^\epsilon_1}(\cdot) =& \int_{\psi^<\cap R^c} \bar{G}(\cdot\mid s)d\tau + (1-\epsilon)\int_R \bar{G}(\cdot\mid s)d\tau + \epsilon\int_R G(\cdot\mid s)d\tau + \int_{\psi^\geq} G(\cdot\mid s)d\tau \\
=& \int_{\psi^<} \bar{G}(\cdot\mid s)d\tau + \int_{\psi^\geq} G(\cdot\mid s)d\tau + \epsilon[\int_R G(\cdot\mid s)d\tau - \int_R \bar{G}(\cdot\mid s)d\tau] \\
=& \tau^{\psi^\epsilon_1}(\cdot) + \epsilon[\int_R G(\cdot\mid s)d\tau - \int_R \bar{G}(\cdot\mid s)d\tau].
\end{aligned}$$

We establish in Result 3 below that the social valuation function satisfies the following derivative property:

$$\lim_{\epsilon \downarrow 0} \frac{[V(\tau^{\psi^{\epsilon}\imath}, \theta) - V(\tau^{\psi \imath}, \theta)]}{\epsilon} = \int_R \int_T v(\theta, \tau^{\psi \imath}, \tilde{s}) G(d\tilde{s} \mid s) d\tau - \int_R \int_T v(\theta, \tau^{\psi \imath}, \tilde{s}) \bar{G}(d\tilde{s} \mid s) d\tau.$$

Investment. The impact of the perturbation on investment is given by:

$$-r\{\int_{\psi^< \cap R^c} \imath^e(s) d\tau + (1-\epsilon) \int_R \imath^e(s) d\tau + \epsilon \int_R \imath^c(s) d\tau + \int_{\psi^\geq} \imath^c(s) d\tau\} =$$
$$-r\{\int_{\psi^<} \imath^e(s) d\tau + \int_{\psi^\geq} \imath^c(s) d\tau + \epsilon[\int_R \imath^c(s) d\tau - \int_R \imath^e(s) d\tau]\}.$$

Thus, the investment variation is $\epsilon r[\int_R \imath^e(s) d\tau - \int_R \imath^c(s) d\tau]$. Dividing by ϵ and taking the limit gives $r[\int_R \imath^e(s) d\tau - \int_R \imath^c(s) d\tau]$.

Total effect. Perturbing the equilibrium exit rule as described, letting the perturbation go to 0 and combining the three components gives:

$$\int_R \pi(\alpha, s) d\tau + \beta r[\int_R \imath^e(s) d\tau - \int_R \imath^c(s) d\tau] +$$
$$\beta \int_R \int_T \int_\Theta v(\tilde{\theta}, \tau^{\psi \imath}, \tilde{s}) \Theta(\tilde{\theta} \mid \theta) G(d\tilde{s} \mid s) d\tau - \beta \int_R \int_T \int_\Theta \Theta(\tilde{\theta} \mid \theta) v(\tilde{\theta}, \tau^{\psi \imath}, \tilde{s}) \bar{G}(d\tilde{s} \mid s) d\tau.$$

Rearranging:

$$\int_R \{[\pi(\alpha, s) + \beta\{\int_T \int_\Theta v(\tilde{\theta}, \tau^{\psi \imath}, \tilde{s}) \Theta(\tilde{\theta} \mid \theta) G(d\tilde{s} \mid s) - r\imath^c(s)] - \beta[\int_T \int_\Theta v(\tilde{\theta}, \tau^{\psi \imath}, \tilde{s}) \Theta(\tilde{\theta} \mid \theta) \bar{G}(d\tilde{s} \mid s) - r\imath^e(s)]\} d\tau.$$

Thus, the perturbation is the integral over R of the value of continuing less the value of exit. This must be non-positive for all R.

Investment decisions. Suppose that an investment strategy λ is optimal. Fix some region R in (k, α) space of small τ measure and let $\hat{\lambda}$ be a measure with $\hat{\lambda}(\cdot \mid k, \alpha)$ with $\hat{\lambda}(\cdot \mid k, \alpha) = \lambda(\cdot \mid k, \alpha)$ outside R. Define a new conditional measure which equals $\lambda(\cdot \mid k, \alpha)$ outside R and equals $(1-\varepsilon)\lambda(\cdot \mid k, \alpha) + \varepsilon\hat{\lambda}(\cdot \mid k, \alpha)$ on R. For any measurable function, h, on (i, k, α) the integral with respect to this measure is $\int h\lambda(di \mid k, \alpha)\tau(dk \times d\alpha) + \varepsilon[\int h\hat{\lambda}(di \mid k, \alpha)\tau(dk \times d\alpha) - \int h\lambda(di \mid k, \alpha)\tau(dk \times d\alpha)]$ or $\int h\lambda(di \times dk \times d\alpha) + \varepsilon[\int h\hat{\lambda}(di \times dk \times d\alpha) - \int h\lambda(di \times dk \times d\alpha)]$. Thus, the impact on cost is:

$$\varepsilon[\int_R \int_\imath r\hat{\lambda}(di \times dk \times d\alpha) - \int_R \int_\imath r\lambda(di \times dk \times d\alpha)] = \varepsilon \int_R \int_\imath [\int r\hat{\lambda}^\imath(d\imath \mid k, \alpha) - \int_\imath r\lambda^\imath(d\imath \mid k, \alpha)] d\tau.$$

From the earlier discussion, when the continuation aggregate distribution, $\tau^{\psi \imath}(\cdot)$, is perturbed to a measure $\tau^{\psi^{\epsilon}\imath}(\cdot)$, equal to $\tau^{\psi \imath}(\cdot) + \varepsilon[\int_R G(\cdot \mid s) d\tau - \int_R \bar{G}(\cdot \mid s)] d\tau$, we obtain

$$\lim_{\epsilon \downarrow 0} \frac{[V(\tau^{\psi^{\epsilon}\imath}, \theta) - V(\tau^{\psi \imath}, \theta)]}{\epsilon} = \int_R \int_T v(\theta, \tau^{\psi \imath}, \tilde{s}) G(d\tilde{s} \mid s) d\tau - \int_R \int_T v(\theta, \tau^{\psi \imath}, \tilde{s}) \bar{G}(d\tilde{s} \mid s) d\tau.$$

In the present context, this yields:

$$\int_R \int_T v(\theta, \tau^\lambda, \tilde{k}, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \hat{\lambda}^k (d\tilde{k} \mid k, \alpha) d\tau - \int_R \int_T v(\theta, \tau^\lambda, \tilde{k}, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \lambda^k (d\tilde{k} \mid k, \alpha)] d\tau$$

$$= \int_R [\int_T v(\theta, \tau^\lambda, \rho k + \imath, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \hat{\lambda}^i (d\imath \mid k, \alpha) - \int_T v(\theta, \tau^\lambda, \rho k + \imath, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \lambda^i (d\imath \mid k, \alpha)] d\tau.$$

Dividing the cost variation by ε and combining both terms gives

$$\int_R \Big[r\hat{\lambda}^\imath (d\imath \mid k, \alpha) - r\lambda^\imath (d\imath \mid k, \alpha)$$
$$+ \beta (\int_T v(\theta, \tau^\lambda, \rho k + \imath, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \hat{\lambda}^i (d\imath \mid k, \alpha) - \int_T v(\theta, \tau^\lambda, \rho k + \imath, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \lambda^i (d\imath \mid k, \alpha)) \Big] d\tau.$$

Rearranging the above expression:

$$\int_R \Big[\{ r\hat{\lambda}^\imath (d\imath \mid k, \alpha) + \beta \int_T v(\theta, \tau^\lambda, \rho k + \imath, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \hat{\lambda}^i (d\imath \mid k, \alpha) \}$$
$$- \{ r\lambda^\imath (d\imath \mid k, \alpha) + \beta \int_T v(\theta, \tau^\lambda, \rho k + \imath, \tilde{\alpha}) P(d\tilde{\alpha} \mid \alpha) \lambda^i (d\imath \mid k, \alpha)) \} \Big] d\tau.$$

This is the variation in social surplus when the aggregate distributional strategy is changed on a small mass of firms (the (k, α)'s in R). The term inside the integral is the gain in value for firm (k, α) from the perturbation in the investment strategy from $\lambda^i(d\imath \mid k, \alpha)$ to $\hat{\lambda}^i(d\imath \mid k, \alpha)$. The deviation does not raise social surplus only when there is no R set of positive measure where (k, α)'s in R gain from the deviation.

This completes the proof. ■

We next show that the derivative property of the valuation function is preserved under iteration.

Result 3

$$\lim_{\epsilon \downarrow 0} \frac{[V(\tau^{\psi^{\epsilon}\imath}, \theta) - V(\tau^{\psi\imath}, \theta)]}{\epsilon} = \int_R \int_T v(\theta, \tau^{\psi\imath}, \tilde{s}) G(d\tilde{s} \mid s) d\tau - \int_R \int_T v(\theta, \tau^{\psi\imath}, \tilde{s}) \bar{G}(d\tilde{s} \mid s) d\tau.$$

Proof: Let $G^*(\cdot \mid s, \theta)$ and $\bar{G}^*(\cdot \mid s, \theta)$ be the subsequent period's conditional distributions. These depend on next period's θ shock since this affects investment and the decision to exit. Let γ be the next period exit rule and ψ the current period exit rule. Then the two step ahead distribution is given by:

$$\int_{\gamma<} \bar{G}^*(\cdot \mid s, \theta) d\tau^{\psi\imath} + \int_{\gamma\geq} G^*(\cdot \mid s, \theta) d\tau^{\psi\imath}.$$

Expanding:

$$\int_{\gamma<} \bar{G}^*(\cdot \mid \hat{s}, \theta) \{ \int_{\psi<} \bar{G}(d\hat{s} \mid s) d\tau + \int_{\psi\geq} G(d\hat{s} \mid s) d\tau \} +$$
$$\int_{\gamma\geq} G^*(\cdot \mid \hat{s}, \theta) \{ \int_{\psi<} \bar{G}(d\hat{s} \mid s) d\tau + \int_{\psi\geq} G(d\hat{s} \mid s) d\tau \}.$$

Rewrite this as:

$$\int_{\psi<}\{\int_{\gamma<}\bar{G}^*(\cdot \mid \hat{s},\theta)\bar{G}(d\hat{s}\mid s)+\int_{\gamma\geq}G^*(\cdot\mid\hat{s},\theta)\bar{G}(d\hat{s}\mid s)\}d\tau+$$
$$\int_{\psi\geq}\{\int_{\gamma<}\bar{G}^*(\cdot\mid\hat{s},\theta)G(d\hat{s}\mid s)+\int_{\gamma\geq}G^*(\cdot\mid\hat{s},\theta)G(d\hat{s}\mid s)\}d\tau.$$

We may write the measures in brackets as $H(\cdot \mid s,\theta)$ and $\bar{H}(\cdot \mid s,\theta)$ to give the measure as:

$$\phi(\tau^{\psi_\imath})(\cdot)=\int_{\psi<}\bar{H}(\cdot\mid s,\theta)d\tau+\int_{\psi\geq}H(\cdot\mid s,\theta)d\tau$$

If the exit region is perturbed in the current period, leaving the investment rules and next period's exit rule γ unchanged, we obtain

$$\phi(\tau^{\psi^\epsilon{}_\imath})(\cdot)=\phi(\tau^{\psi_\imath})(\cdot)+\epsilon\{\int_R H(\cdot\mid s,\theta)d\tau-\int_R\bar{H}(\cdot\mid s,\theta)d\tau.\}$$

Consider next period's profit where the exit rule is γ. This has the form:

$$\int_{\gamma\geq}\pi(\hat{s},\theta)\{\int_{\psi<}\bar{G}(d\hat{s}\mid s)d\tau+\int_{\psi\geq}G(d\hat{s}\mid s)d\tau\}$$
$$=\int_{\psi<}\int_{\gamma\geq}\{\pi(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)d\tau+\int_{\psi\geq}\int_{\gamma\geq}\pi(\hat{s},\theta)G(d\hat{s}\mid s)d\tau\ \}.$$

Under the perturbation, this gives:

$$\epsilon[\int_R\int_{\gamma\geq}\pi(\hat{s},\theta)G(d\hat{s}\mid s)d\tau-\int_R\int_{\gamma\geq}\pi(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)d\tau]$$

For the continuation valuation, the perturbation yields

$$\beta\epsilon\{\int_R\int_T\int_\Theta v(\tilde{\theta},\phi(\tau^{\psi_\imath}),\tilde{s})\Theta(d\tilde{\theta}\mid\theta)H(d\tilde{s}\mid s,\theta)d\tau-\int_R\int_T\int_\Theta v(\tilde{\theta},\phi(\tau^{\psi_\imath}),\tilde{s})\Theta(d\tilde{\theta}\mid\theta)\bar{H}(d\tilde{s}\mid s,\theta)d\tau.\}$$

For investment we obtain

$$\int_{\gamma\geq}\imath^c(\hat{s},\theta)\{\int_{\psi<}\bar{G}(d\hat{s}\mid s)d\tau+\int_{\psi\geq}G(d\hat{s}\mid s)d\tau\}+\int_{\gamma<}\imath^e(\hat{s},\theta)\{\int_{\psi<}\bar{G}(d\hat{s}\mid s)d\tau+\int_{\psi\geq}G(d\hat{s}\mid s)d\tau\}$$

Rearranging:

$$\int_{\psi<}\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)\int_{\psi<}\bar{G}(d\hat{s}\mid s)d\tau+\int_{\gamma<}\imath^e(\hat{s},\theta)\int_{\psi<}\bar{G}(d\hat{s}\mid s)d\tau\}+$$
$$\int_{\psi\geq}\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)G(d\hat{s}\mid s)d\tau+\int_{\gamma<}\imath^e(\hat{s},\theta)G(d\hat{s}\mid s)d\tau\}.$$

The variation on this expression is:

$$-\epsilon\int_R\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)d\tau+\int_{\gamma<}\imath^e(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)d\tau\}$$

$$+\epsilon\int_R\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)G(d\hat{s}\mid s)d\tau+\int_{\gamma<}\imath^e(\hat{s},\theta)G(d\hat{s}\mid s)d\tau\}.$$

Combining these terms:

$$\int_R\Big\{\int_{\gamma\geq}\pi(\hat{s},\theta)G(d\hat{s}\mid s)-\int_{\gamma\geq}\pi(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)$$

$$+\beta[\int_T\int_\Theta v(\tilde{\theta},\phi(\tau^{\psi_\imath}),\tilde{s})\Theta(d\tilde{\theta}\mid\theta)H(d\tilde{s}\mid s,\theta)-\int_T\int_\Theta v(\tilde{\theta},\phi(\tau^{\psi_\imath}),\tilde{s})\Theta(d\tilde{\theta}\mid\theta)\bar{H}(d\tilde{s}\mid s,\theta)]$$

$$+\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)G(d\hat{s}\mid s)+\int_{\gamma<}\imath^e(\hat{s},\theta)G(d\hat{s}\mid s)\}-\int_{\gamma\geq}\imath^c(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)d\tau+\int_{\gamma<}\imath^e(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)\}\Big\}d\tau.$$

Rearranging the components:

$$\Big[\int_R\Big\{\int_{\gamma\geq}\pi(\hat{s},\theta)G(d\hat{s}\mid s)+\beta\int_T\int_\Theta v(\tilde{\theta},\phi(\tau^{\psi_\imath}),\tilde{s})\Theta(d\tilde{\theta}\mid\theta)H(d\tilde{s}\mid s,\theta)$$

$$+\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)G(d\hat{s}\mid s)+\int_{\gamma<}\imath^e(\hat{s},\theta)G(d\hat{s}\mid s)\}\Big\}d\tau\Big]$$

$$-\Big[\int_R\Big\{\int_{\gamma\geq}\pi(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)+\beta\int_T\int_\Theta v(\tilde{\theta},\phi(\tau^{\psi_\imath}),\tilde{s})\Theta(d\tilde{\theta}\mid\theta)\bar{H}(d\tilde{s}\mid s,\theta)]$$

$$+\{\int_{\gamma\geq}\imath^c(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)d\tau+\int_{\gamma<}\imath^e(\hat{s},\theta)\bar{G}(d\hat{s}\mid s)\}\Big\}d\tau\Big].$$

■

Corollary 3 *There is a unique competitive equilibrium. The competitive equilibrium corresponds to the solution to the social planner's problem.*

Proof: Immediate.

Lemma 3 *Investment, $\imath(\theta,\tau^{\psi_\imath},k,\alpha)$, satisfies $\imath(\theta,\tau^{\psi_\imath},k,\alpha)=k^*(\theta,\tau^{\psi_\imath},\alpha)-\rho k$, where $k^*(\theta,\tau^{\psi_\imath},\alpha)$ solves*

$$(1-\beta\rho)r=\beta Pr(\tilde{\alpha}\geq\alpha^*(k^*,\tilde{\theta},\tau^{\psi_\imath})|\alpha)E\{\pi_k(\theta,\tau^{\psi_\imath},k^*,\tilde{\alpha})|\alpha\}.$$

Proof: Investment maximizes

$$\int v(\tilde{\theta},\mu^{\psi_\imath},\rho k+\imath\tilde{\alpha})\Theta(d\tilde{\theta}\mid\theta)P(d\tilde{\alpha}\mid\alpha)-r\imath.$$

Rewriting, next period's capital stock, $k^*(\cdot)$, maximizes

$$\int v(\tilde{\theta},\mu^{\psi_\imath},k^*,\tilde{\alpha})\Theta(d\tilde{\theta}\mid\theta)P(d\tilde{\alpha}\mid\alpha)-rk^*+r\rho k$$

$$=\int_\theta\int_{\tilde{\alpha}\geq\alpha^*(\cdot)}\pi(\tilde{\theta},\mu^{\psi_\imath},k^*,\tilde{\alpha})+\beta[E[v(\cdot)|\tilde{\alpha},\tilde{\theta}]+r\rho k^*]\Theta(d\tilde{\theta}\mid\theta)P(d\tilde{\alpha}\mid\alpha)$$

$$+\int_\theta\int_{\tilde{\alpha}<\alpha^*(\cdot)}\beta[E[v(\cdot)|\tilde{\alpha},\tilde{\theta}]+r\rho k^*]\Theta(d\tilde{\theta}\mid\theta)P(d\tilde{\alpha}\mid\alpha)-rk^*+r\rho k.$$

The first order condition for k^* is

$$0 = \int_\theta \int_{\tilde{\alpha} \geq \alpha^*(\cdot)} [\pi_k(\tilde{\theta}, \mu^{\psi_t}, k^*, \tilde{\alpha}) + \beta r \rho] \Theta(d\tilde{\theta} \mid \theta) P(d\tilde{\alpha} \mid \alpha) + \int_\theta \int_{\tilde{\alpha} < \alpha^*(\cdot)} \beta r \rho \Theta(d\tilde{\theta} \mid \theta) P(d\tilde{\alpha} \mid \alpha) - r.$$

Re–arranging yields the result. ∎

Lemma 4 *Suppose assumption* **C1** *holds. Then $k^*(\theta, \tau^{\psi_t}, \alpha)$ is monotonically increasing in α.*

Proof: From **C1**, $\alpha > \alpha'$ implies $P(\cdot \mid \alpha) \succeq_c P(\cdot \mid \alpha')$, so that α is both less likely to exit than α' and is more likely to have a better operating technology. Since α and k are complements, $\pi_{\alpha k}(p, \alpha, k) > 0$, the result follows from inspection of the first order conditions for k^*. ∎

Corollary 4

1. *On average, larger (greater k) or better (higher α) firms are more profitable, and produce more output.*
2. *Further, if* **E1** *and* **C1** *hold and exit occurs with positive probability, then better or larger firms are less likely to exit, expect to operate better technologies on a larger and more profitable scale and have longer expected lifetimes.*
3. *Further if* **C1** *and* **C2** *hold then the older is an active firm, the greater is its expected productivity, its capital stock and profit, and the longer is its expected lifetime (i.e. exit hazard rates fall with age).*

Proof : Follows from the previous lemma and the proof to theorem 1. ∎

Lemma 5 *Let ψ be the equilibrium exit rule in the current period and τ the distribution over (α, k). Let $\psi'(\alpha) \leq \psi(\alpha), \forall \alpha$. Thus, ψ' corresponds to more exit at any technology level. Define the measure $\hat{\tau}(X) = \tau\{(\alpha, k) \in X \mid \psi(\alpha) \geq k > \psi'(\alpha)\}$. Let the current period be t, let $\theta_t^j = (\theta_t, \theta_{t+1}, \ldots, \theta_{t+j})$ and let $Y_{t+l}(\theta_t^l, \tau)$ be equilibrium aggregate output at period $t+l$, following aggregate shock history θ_t^l given the current distribution τ. Then:*

$$\lim_{\epsilon \to 0} \frac{Y_{t+l}(\theta_t^l, \tau + \epsilon\hat{\tau}) - Y_{t+l}(\theta_t^l, \tau)}{\epsilon} < 0.$$

Proof: From theorem 2, if the current exit rule is ψ, then next period's distribution is given by:

$$\int_{\psi <} \bar{G}(d\hat{s} \mid s) d\tau + \int_{\psi \geq} G(d\hat{s} \mid s) d\tau.$$

Under the perturbation of the exit rule, this becomes:

$$\int_{\psi <} \bar{G}(\cdot \mid s) d\tau + \int_{\psi \geq} G(\cdot \mid s) d\tau + \epsilon\Big[\int_R G(\cdot \mid s) d\tau - \int_R \bar{G}(\cdot \mid s) d\tau\Big].$$

If γ gives next period's exit rule, then the variation in next period's output is approximately

$$\epsilon[\int_{\gamma>} Y(P_{t+1}, \tilde{s})[\int_R G(d\tilde{s} \mid s)d\tau - \int_R \bar{G}(d\tilde{s} \mid s)d\tau].$$

Now, on the region R agents exit so that $\alpha < \bar{\alpha}$ on this region the conditional distribution over α is better under $\bar{\alpha}$. By lemma 6, higher α's choose higher levels of capital k next period. Thus, the distribution $\bar{G}(d\tilde{s} \mid s)$ dominates $G(d\tilde{s} \mid s)$ so that

$$\int_R [\int_{\gamma>} q(P_{t+1}, \tilde{s})G(d\tilde{s} \mid s) - \int_{\gamma>} q(P_{t+1}, \tilde{s})\bar{G}(d\tilde{s} \mid s)]d\tau < 0,$$

where $q(P_{t+1}, \tilde{s})$ is the supply of firm $\tilde{s}$ at price P_{t+1}. For the next period, the distribution evolves according to:

$$\int_{\psi<} \{\int_{\gamma<} \bar{G}^*(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s) + \int_{\gamma\geq} G^*(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)\}d\tau +$$
$$\int_{\psi\geq} \{\int_{\gamma<} \bar{G}^*(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s) + \int_{\gamma\geq} G^*(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s)\}d\tau,$$

(see theorem 2) or with the measures in brackets written as $H(\cdot \mid s, \theta)$ and $\bar{H}(\cdot \mid s, \theta)$ to give the measure as:

$$\phi(\tau^{\psi_t})(\cdot) = \int_{\psi<} \bar{H}(\cdot \mid s, \theta)d\tau + \int_{\psi\geq} H(\cdot \mid s, \theta)d\tau.$$

Let $F(\cdot \mid s, \theta) = \bar{G}^*(\cdot \mid \hat{s}, \theta)\chi_{\gamma<}(\hat{s}) + \int_{\gamma\geq} G^*(\cdot \mid \hat{s}, \theta)\chi_{\gamma\geq}(\hat{s})$ so that this may be written $\int F(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)$ and the second term as $\int F(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s)$. Thus, next period's distribution may be written:

$$\int_{\psi<} [\int F(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)]d\tau + \int_{\psi\geq} [\int F(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s)]d\tau.$$

Thus, the perturbation gives:

$$\int_{\psi<} [\int F(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)]d\tau + \int_{\psi\geq} [\int F(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s)]d\tau +$$
$$\epsilon \int_R [\int F(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s) - \int F(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)]d\tau.$$

Hence, the limiting change in supply at price P_{t+2} two periods ahead is

$$\int_R [\int_{\hat{s}} \int_{s'} q(P_{t+2}, s')F(ds' \mid \hat{s}, \theta)G(d\hat{s} \mid s) - \int_{\hat{s}} \int_{s'} q(P_{t+2}, s')F(ds' \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)]d\tau < 0.$$

For the next period, let G' and $\bar{G}'$ be the transition functions. Then the three period ahead distribution is given by (with exit rule ϕ):

$$\int_{\phi<} \bar{G}'(\cdot \mid \tilde{s}, \tilde{\theta})(\int_{\psi<} [\int F(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)]d\tau + \int_{\psi\geq} [\int F(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s)]d\tau)$$
$$\int_{\phi\geq} \bar{G}'(\cdot \mid \tilde{s}, \tilde{\theta})(\int_{\psi<} [\int F(\cdot \mid \hat{s}, \theta)\bar{G}(d\hat{s} \mid s)]d\tau + \int_{\psi\geq} [\int F(\cdot \mid \hat{s}, \theta)G(d\hat{s} \mid s)]d\tau).$$

This rearranges to:

$$\int_{\psi<}[\int_{\phi<}\int \bar{G}'(\cdot \mid \tilde{s},\tilde{\theta})F(d\tilde{s} \mid \hat{s},\theta)\bar{G}(d\hat{s} \mid s) + \int_{\phi\geq}\int G'(\cdot \mid \tilde{s},\tilde{\theta})F(d\tilde{s} \mid \hat{s},\theta)\bar{G}(d\hat{s} \mid s)]d\tau$$

$$+ \int_{\phi\geq}[\int_{\phi<}\int \bar{G}'(\cdot \mid \tilde{s},\tilde{\theta})F(d\tilde{s} \mid \hat{s},\theta)G(d\hat{s} \mid s) + \int_{\phi\geq}\int G'(\cdot \mid \tilde{s},\tilde{\theta})F(d\tilde{s} \mid \hat{s},\theta)G(d\hat{s} \mid s)]d\tau.$$

Let

$$J(\cdot \mid \tilde{\theta},\theta,\hat{s}) = \chi_{\phi<}(\hat{s})\int \bar{G}'(\cdot \mid \tilde{s},\tilde{\theta})F(d\tilde{s} \mid \hat{s},\theta) + \chi_{\phi\geq}(\hat{s})\int \bar{G}'(\cdot \mid \tilde{s},\tilde{\theta})F(d\tilde{s} \mid \hat{s},\theta).$$

The three step ahead distribution (along $\theta, \tilde{\theta}$) is given by

$$\int_{\phi<}\int J(\cdot \mid \tilde{\theta},\theta,\hat{s})\bar{G}(d\hat{s} \mid s)d\tau + \int_{\phi\geq}\int J(\cdot \mid \tilde{\theta},\theta,\hat{s})G(d\hat{s} \mid s)d\tau.$$

As before, the perturbation on the region R gives us:

$$\epsilon\int_R[\int J(\cdot \mid \tilde{\theta},\theta,\hat{s})G(d\hat{s} \mid s) - \int J(\cdot \mid \tilde{\theta},\theta,\hat{s})\bar{G}(d\hat{s} \mid s)]d\tau.$$

Hence, the limiting change in supply at price P_{t+3} two periods ahead is

$$\int_R[\int_{\hat{s}}\int_{s'} q(P_{t+3},s')J(ds' \mid \tilde{\theta},\theta,\hat{s})G(d\hat{s} \mid s) - \int_{\hat{s}}\int_{s'} q(P_{t+3},s')J(ds' \mid \tilde{\theta},\theta,\hat{s})\bar{G}(d\hat{s} \mid s)]d\tau < 0.$$

Proceeding inductively gives the result. ■

Proposition 6 *Suppose demand is independently distributed. Let $\hat{\tau}_0 = \bar{\tau}_0$. Consider two aggregate shock histories, $\hat{\theta}^t, \bar{\theta}^t$ where $\hat{\theta}_0 > \bar{\theta}_0$, $\hat{\theta}_{t'} \geq \bar{\theta}_{t'}, \forall t', 0 \leq t' \leq t$. Then*

1. $\bar{\mu}_{t'+1} \succ_c \hat{\mu}_{t'+1}, \quad 0 < t' \leq t.$
2. $\bar{p}_{t'} < \hat{p}_{t'}, \quad 0 < t' \leq t.$
3. $\bar{k}^*_{t'}(\alpha) > \hat{k}^*_{t'}(\alpha), \quad 0 < t' \leq t.$
4. $\hat{\pi}_{t'}(\alpha) > \bar{\pi}_{t'}(\alpha), \quad 0 < t' \leq t.$
5. *$\exists\delta > 0$ such that if $\bar{\theta}_{t+1} \geq \hat{\theta}_{t+1} - \delta$ then* $Y_{t+1}(\bar{\theta}_{t+1}, \bar{\tau}^{\alpha_{t+1}(\bar{\theta}_{t+1},\bar{\tau})}) > Y_{t+1}(\hat{\theta}_{t+1}, \hat{\tau}^{\alpha_{t+1}(\hat{\theta}_{t+1},\hat{\tau})})$.

Proof: In period 0, since current demand conditions are better in the hat economy than the bar economy, but future prospects are identical, there must be more exit in the bar economy than the hat economy: $\alpha^*(k,\hat{\theta}_0,\tau_0) < \alpha^*(k,\bar{\theta}_0,\tau_0)$. Hence, $\bar{\mu}_1 \succ_c \hat{\mu}_1$. Suppose now that demand realizations in the hat and bar economies are identical in periods 1 to t. Lemma 5 implies then that output in the bar economy must exceed that in the hat economy in period 1 to t. Since this is true for all identical demand histories, the time 0 conditional distribution of prices in the hat economy stochastically dominates that in the bar economy. Consequently, $\bar{k}^*_{t'}(\alpha) < \hat{k}^*_{t'}(\alpha), t' \leq t$. Both higher prices 1 in the hat economy and $\bar{k}^*_1(\alpha) < \hat{k}^*_1(\alpha)$ imply that operating profits of any given

α firm are higher in the hat economy, $\hat{\pi}_1(\alpha) > \bar{\pi}_1(\alpha)$. Exit, $\alpha^*(k_1, \theta_1, \mu_1)$, is decreasing in both θ_1 and k_1, and monotonicity of period two's distribution in μ_1 implies that $\bar{\mu}_2 \succ_c \hat{\mu}_2$. Continuing, this argument, it must be that if $\bar{\theta}_{t'} = \hat{\theta}_{t'}$, $0 < t' \leq t$, then properties 1–4 hold at t'. To complete the argument, observe that an increase in $\hat{\theta}'_t$, so that $\hat{\theta}_{t'} > \bar{\theta}_{t'}$, just further reduces exit in the hat economy, increasing future prices, optimal capital stocks and profits. Item 5 just follows from continuity of period $t+1$ aggregate output in θ_{t+1}. ■

Proposition 7 *Suppose* **C1**, **C2**, **S1**, *and* $\theta \in \{\underline{\theta}, \bar{\theta}\}$. *Let date 0 capital stocks be chosen optimally by firms at date* -1 *given* μ_{-1} *and* θ_{-1}, *and suppose that both* $\exists \mu \preceq \mu_0$ *and* $\exists \mu' \succeq \mu_0$ *that can be realized along some equilibrium paths. Then there exists a* γ^*, *such that for* $0 \leq \gamma \leq \gamma^*$, $Y(\bar{\theta}, \cdot) > Y(\underline{\theta}, \cdot)$. *Suppose* $\theta_{-1} = \underline{\theta}, \theta_0 = \bar{\theta}, \theta_1 = \bar{\theta}, ..., \theta_t = \bar{\theta}, \theta_{t+1} = \underline{\theta}$, *for some* t. *Then*

1. $Y_{-1} < Y_0; Y_1 > \cdots > Y_t > Y_{t+1}$.
2. *The distribution of firm productivities worsens with time:* $\mu_{-1} \succ \mu_0 \succ \cdots \mu_t$.
3. *Optimal capital stocks increase:* $k_0(\alpha) < k_1(\alpha) < \cdots < k_t(\alpha)$.

Suppose $\theta_{-1} = \bar{\theta}, \theta_0 = \underline{\theta}, \theta_1 = \underline{\theta}, ..., \theta_t = \underline{\theta}, \theta_{t+1} = \bar{\theta}$. *Then*

4. $Y_{-1} > Y_0; Y_1 < \cdots < Y_t < Y_{t+1}$.
5. *The distribution of firm productivities improves with time:* $\mu_t \succ \mu_{t-1} \succ \cdots \mu_{-1}$.
6. *Optimal capital stocks shrink:* $k_0(\alpha) > k_1(\alpha) > \cdots > k_t(\alpha)$.

Proof: Suppose first that $\gamma = 0$ so that demand is *i.i.d.* Then there is less exit if $\theta = \bar{\theta}$ than if $\theta = \underline{\theta}$, so that $Y(\bar{\theta}, \tau_{\alpha^*(\bar{\theta})}) > Y(\underline{\theta}, \tau_{\alpha^*(\bar{\theta})}) > Y(\underline{\theta}, \tau_{\alpha^*(\underline{\theta})})$. $\alpha(k, \tau, \bar{\theta}) < \alpha(k, \tau, \underline{\theta}) \rightarrow \mu_1(\tau_0, \bar{\theta}) < \mu_1(\tau_0, \underline{\theta})$. From proposition 6, $Y_1(\tau_1(\tau_0, \bar{\theta}), \theta) < Y_1(\tau_1(\tau_0, \underline{\theta}), \theta)$ so that $p_1(\tau_1(\tau_0, \bar{\theta}), \theta) > p_1(\tau_1(\tau_0, \underline{\theta}), \theta)$ and hence any given α invests more if the initial demand realization is $\bar{\theta}$, reducing the probability of exit, than that α would invest if the initial demand realization is $\underline{\theta}$.

Suppose that $\mu_0 \succ \mu_1(\tau_0, \underline{\theta}) \succ \mu_1(\tau_0, \bar{\theta})$. But then by monotonicity in μ, $\mu_0 \succ \mu_t$ for all possible demand shock paths (note that $\mu_0 \succ \mu_1 \rightarrow$ less investment for date 0 and hence more exit.) But this is inconsistent with the premise of the proposition. A similar contradiction precludes $\mu_1(\tau_0, \underline{\theta}) \succ \mu_1(\tau_0, \bar{\theta}) \succ \mu_0$. Hence, $\mu_1(\tau_0, \underline{\theta}) \succ \mu_0 \mu_1(\tau_0, \bar{\theta})$. Monotonicity in μ then implies that $\mu_1(\tau_0, \bar{\theta}) \succ \mu_2(\tau_1(\tau_0, \bar{\theta}), \bar{\theta})$, and this argument extends to arbitrary t. In turn, proposition 6 then ensures that if $\mu_t \succ \mu_{t+1}$ then $Y_{t+1}(\tau_{t+1}, \theta) < Y_t(\tau_t, \theta)$, so that $p_{t+1}(\tau_{t+1}, \theta) > p_t(\tau_t, \theta)$ and hence any given α invests more in $t+1$ than t reducing the probability of exit in $t+1$, relative to t. A similar argument holds for a series of $\underline{\theta}$ demand realizations.

Finally, observe that a continuity argument implies that these results continue to hold if the persistence parameter in demand, γ, is sufficiently small. ■

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