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Efficiency and the Fiscal Gap in Federal Systems

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EFFICIENCY AND THE FISCAL GAP IN FEDERAL SYSTEMS

Robin Boadway* and Michael Keen†

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Abstract: This paper investigates the efficiency argument for a vertical fiscal gap in a federation using a simple model of a central government and several identical states. Each level provides a public good to residents within its jurisdiction and finances it by taxing labour income and rents. If labour supply is fixed, there need not be a fiscal gap even if households are perfectly mobile. With variable labour supply, however, decentralised decision-making by the states will generally be inefficient because states' tax policies will affect not only their own revenues but also those of the federal government. If the federal government chooses its budgetary policy first and the states take this policy as given, federal policies can be chosen to replicate the second-best optimum. Moreover, with or without mobile households, second-best optimal federal policy involves negative federal labour tax rates and can plausibly also require a *negative* fiscal gap, with transfers going from the states to the federal government. Thus, on efficiency grounds, there can be no presumption that inter-governmental transfers should go from higher levels of government to lower.

JEL: H73, H77, H41

Key Words: Fiscal Gap, Fiscal Federalism

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I. Introduction

A common feature of federations is that higher levels of government collect more revenues than they need for their own expenditure requirements and transfer funds to lower levels; that is, a (positive) *fiscal gap* exists.[1] Indeed, this occurs (between the national government and municipalities) even in unitary nations. Yet, despite the extensive literature on the role and structure of intergovernmental grants, there is very little analysis of the optimal size — or, indeed, sign — of the fiscal gap in federal systems.[2] Our purpose in this paper is to take a first step in the direction of a theory of the optimal fiscal gap by analysing the efficiency arguments for a mismatch between expenditure responsibilities and revenue-raising in a federation.

The conventional wisdom in the fiscal federalism literature is that a positive fiscal gap is desirable, reflecting the notion that the case for decentralising expenditure responsibilities is stronger than that for decentralising revenue-raising. The decentralised provision of public goods and services is said to improve efficiency by ensuring that public service provision is better suited to local tastes and needs, by inducing greater political accountability, by enhancing cost effectiveness and innovation through inter-jurisdictional competition, and by reducing administrative and agency costs associated with a central bureaucracy. Devolving taxation responsibilities can also be beneficial in improving fiscal accountability and allowing tax structures to respond to local characteristics. On the other hand, decentralised taxation is alleged to detract from the equity and efficiency of the tax system. Equity is compromised to the extent that the central government is no longer able to ensure a common standard of vertical and horizontal equity across the nation. Furthermore, local jurisdictions are predicted to compete away redistributive elements of their tax systems. Efficiency is said to be violated to the extent that tax bases are more mobile from the perspective of lower levels of government, deterring them from raising tax rates through the induced loss of tax base. This can lead to wasteful tax competition to attract mobile factors of production resulting in the non-optimal provision of public goods and services by lower-level jurisdictions. The conventional proposed policy response to all this is to retain relatively more taxing responsibility than expenditure responsibility at high

[1] Notable exceptions include China, as well as the United States under the original articles of federation. Looser organisations of sovereign nations, such as the European Union and the United Nations, also usually finance their central budgets principally by transfers from member nations.

[2] A recent exception is Brennan and Pincus (1994), who develop a theory of conditional grants based on considerations similar to ours (that is, differences in the excess burden of raising revenues at two levels of government) and use it primarily to evaluate the positive effects of grants (such as the flypaper effect). However, in their model the marginal costs of public funds from raising additional amounts of revenue at the two levels of government are independent of one another, whereas in our analysis their interdependence is crucial.

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APPENDICES

A. Proof of Proposition 7

Substituting (38) for n_τ in the state's first-order condition (43) yields:

$$\frac{b'}{v'} = \left(\frac{2g_t}{1-w_\tau} + \frac{g_n}{w_n}\right)^{-1} . \tag{A.1}$$

Noting from (13) and (16) that $g_t = nl - (g_n nl'/l)$, this becomes:

$$\frac{b'}{v'} = \left(\frac{g_t}{1-w_\tau} + \frac{g_n + nlw_n}{w_n(1-w_\tau)}\right)^{-1} \tag{A.2}$$

or, using (4) and (13),

$$\frac{nb'}{u_x} = \left(1 - \frac{tl'}{l} - \theta f'' nl' + \frac{g_n + nlw_n}{nlw_n(1-w_\tau)}\right)^{-1} . \tag{A.3}$$

From (7), (10) and (16) we obtain:

$$g_n + nlw_n = (1-w_\tau)(tl + \theta f'' nl^2) . \tag{A.4}$$

Proposition 7 follows from (A.3) and (A.4) by recalling the definition of r in (8).

B. Proof That (44) is Satisfied at the Second-Best Optimum

By (A.4), (43) will be satisfied if:

$$tl + \theta f'' nl^2 > 0 . \tag{A.5}$$

At the second-best optimum, this may be written, using (45), as:

$$tl + Tl - \frac{\tau^* l}{1+w_\tau} > 0 . \tag{A.6}$$

Since $t + T = \tau^*$, this becomes:

$$\tau^* l \left(1 - \frac{1}{1+w_\tau}\right) > 0 , \tag{A.7}$$

which is ensured by (6).

levels of government.

Analysing the case for a fiscal gap necessarily requires explicitly modeling the decisions of more than one level of government and the interactions between them; indeed, this is the essence of a federal system of government. However, such models are rare in the federalism literature. One apparent reason for this is that even relatively simple models of this kind rapidly become very complicated. The classic analysis in Gordon (1983), for example, identifies many externalities arising from non-cooperative behaviour by lower-level governments, and discusses possible responses at the federal level; but that federal layer is not explicitly modeled.[3] By concentrating solely on efficiency aspects and including only a minimal number of budgetary decisions by the different levels of government, we are able to obtain some insight into the consequences of intergovernmental interaction for the fiscal gap, and to evaluate that part of the conventional wisdom that is concerned with efficiency. As will be seen, incorporation of both levels of government brings to light potential externalities between them that have previously been generally neglected. A recent paper by Dahlby (1994) also recognises some of the same forces as in our paper, especially differences in the perceived efficiency costs of raising revenues at different levels of government using concurrent revenue sources. Of course, a full theoretical account would also need to take into consideration equity issues arising from the heterogeneity of lower-level jurisdictions.

The model used is a simple one of a federal system with two levels of government: a *federal* level with one government and a *state* level with two or more. Each government provides a public good to the residents of its own jurisdiction, and each obtains tax revenues through a distortionary tax on labour. Thus, they both occupy the same tax base. Intergovernmental transfers may occur between the federal government and the states, and it is on these that we focus. What is of crucial importance is the way in which the governments interact with one another in their decision-making. We assume throughout that states behave as Nash competitors both with respect to one another and with respect to the federal government; that is, they take as given the budgetary decisions taken by other governments. However, the federal government, which sets both its own tax rates and the level of grants to the states, is assumed to be able to act as first mover (or Stackelberg leader), committing to its policies before the states and anticipating their effect on states' decisions. This is an extreme assumption, motivated by the fact that there is always only one federal government but usually several state governments. We consider briefly the consequences of the federal government not having first-mover advantage, but being a Nash competitor with respect to the states. Given the structure of the problem, and the desire to focus solely on efficiency issues, states are assumed to be identical and only symmetric Nash equilibria among the states are considered.

Our analysis is concerned with the size (and sign) of the fiscal gap – that is, the dif-

[3]See also Wildasin (1986) for a comprehensive analysis of decision-making in a federation.

ference between aggregate state expenditures and revenues from own sources – rather than with the role of intergovernmental transfers *per se.* It is important to recognise the distinction between the two. Many of the most common objectives of intergovernmental transfers require little, if any, fiscal gap, and for those that do the size of the fiscal gap is largely irrelevant. For example, one argument for transfers is to internalise the externality arising from the existence of interjurisdictional spillovers.[4] But it is sufficient for this purpose to implement a pure relative price incentive on lower levels of government; an income transfer is unnecessary. Another key role of intergovernmental transfers involves correcting for fiscal inefficiencies and/or fiscal (horizontal) inequities arising in a decentralised federation when lower-level jurisdictions have different fiscal capacities or needs.[5] For this purpose, purely redistributive transfers from jurisdictions with higher-than-average fiscal capacities to those with lower-than-average fiscal capacities would suffice; no net fiscal gap between the two levels is required. To abstract from such familiar considerations, we exclude from the model both interjurisdictional spillovers and differences in fiscal capacity across jurisdictions. Incorporating them would add little to our analysis; and by the same token our analysis could add little of substance to that literature. Any intergovernmental transfers that emerge from our analysis result solely from the need for a fiscal gap on efficiency grounds.

The paper proceeds as follows. Following the description of the model in Section II, Section III considers the benchmark case of a 'unitary' state, in which the federal government is able to choose not only its own tax rate and expenditures but also those of the states. In this case, the issue of transfers is not relevant, but the rules for deciding on tax rates and public goods provision at both levels characterise a second-best optimum (given the need to use distorting wage taxes). The next two parts characterise optimal state and federal policies for two cases. In Section IV, the population is assumed to be immobile among states. Though unrealistic, this case is instructive in isolating an important component of the fiscal gap, one that works in a surprising direction. In Section V, households can move costlessly among states, and state governments take this into account in deciding their policies. Section VI briefly considers the consequences of some alternative assumptions on mobility and government behaviour. Section VII concludes.

II. The Structure of the Model

The nation comprises k identical states and nk identical households. In the next two sections, households are assumed to be, respectively, completely immobile and costlessly mobile among states, respectively. Mobility is not an important issue for describing the model: we concentrate on symmetric allocations, so that mobility does

[4]See Oates (1972), who develops the classic arguments of Musgrave (1959).

[5]See Boadway and Wildasin (1984). Indeed, in the case of inefficiencies arising from fiscal externalities, voluntary transfers among the states suffice, as Myers (1990) has shown.

government to establish a fiscal gap in order to finance transfers for equity reasons has also been ruled out. In this simple framework, some striking results emerge. In particular, we have shown that, contrary to the conventional wisdom, efficiency arguments call for a *negative* fiscal gap in perfectly reasonable circumstances.

In addition to focusing only on efficiency issues, the analysis has been carried out in the simplest of models in which the efficiency costs of taxation arise solely from a labour market distortion. More complete analyses would take account of the myriad of other distortions imposed by public sector decision making, especially capital market distortions and inter-commodity distortions (including cross-border shopping).

Moreover, we have modelled the interaction between levels of government in a very rudimentary way. Both levels co-occupy the same tax base. In the confines of our model, this is not a serious shortcoming since there is ultimately only one tax base. (For example, income and consumption are the same thing, so direct and indirect taxes are equivalent.) In more complicated models with many commodities, distinctions between commodity taxes and income taxes become blurred, especially since commodity taxes can be imposed at differential rates. This gives rise to more avenues for interaction between the two levels of government, as well as raising issues of the appropriate assignment of taxes by level of government. Nonetheless, it seems likely that the main thrust of the present results would continue to hold in models of this sort, since the overlap of tax bases will inevitably be substantial. For example, if states are allocated indirect taxes and the federal government direct taxes, the interactive effects at the heart of this paper will be very much at work. Of course, once the model is complicated by many commodities and the assignment of different taxes to different levels of government, the exact correspondence between direct and indirect taxes falls part. The consequence is that the federal government acting as a Stackelberg leader may no longer be able to choose its policies so as to be able to induce the second-best outcome.

Attention has been restricted to simple forms of strategic behaviour by the federal government and the states. Given the relatively small number of member governments in many federations, other types of interactive decision-making might be considered. For example, state and federal governments in some federations may better be modelled as choosing policies in a bargaining framework, particularly in relation to transfers. Moreover, state governments may have the ability in some federations to discriminate in their tax policies between initial residents and migrants, another possibility ignored here. As Bewley (1981) shows, such an ability to dscriminate gives rise to problems of non-existence of equilibria alongside those of inefficiency.

Despite these (and other) qualifications, however, it seems safe to conclude that co-occupancy of the tax base by different levels of government – which is in a real sense an inherent feature of all federal systems – may have more profound implications than has commonly been realised.

federation.[27] Effectively, g and G are perfect substitutes. These results on grants can be summarised as follows:

Proposition 10: *When all governments behave as Nash competitors, the MCPF will be highest at whichever level of government exerts the more damaging effect on the revenues of the other. If preferences are linear in both state and federal public services, the optimal intergovernmental grant should go from the level of government with the lowest MCPF to that with the highest, and should be sufficient to crowd out entirely public services provided by the former.*

In the more general case in which preferences for public services are strictly concave, this simple result of transferring funds to the level of government with the highest MCPF may no longer apply. To see this, rewrite (57) as follows:

$$\frac{n}{u_x}\left(\frac{dV}{dS}\right) = \frac{nb'}{u_x}\left(1 - g_T\frac{d\bar{T}}{dS}\right) + \frac{nB'}{u_x}\left(-k + G_t\frac{d\bar{t}}{dS}\right) . \tag{59}$$

We expect that $g_T, G_t < 0$; and, with downward-sloping reaction curves, it can be readily shown that $d\bar{T}/dS > 0$ and $d\bar{t}/dS < 0$. Therefore, the exogenous shift in the state's revenue function $g(\cdot)$, and thus the term multiplying the state's marginal cost of public funds, will be less than one. Similarly, the term multiplying the federal marginal cost of public funds will be more than $-k$. Thus, the rule of transferring from the level with the lowest MCPF to that with the highest may not increase welfare. It depends on the relative magnitude of the indirect effects, and the determinants of that are rather complex.

VII. Concluding Remarks

It is a surprising feature of the literature on fiscal federalism that the interaction between the decisions of higher and lower levels of government is rarely modelled explicitly. This paper has set out a model that does so, and turned it to the analysis of a key issue in federal relations: the optimal size and sign of the fiscal gap. To this end, we have assumed away all reasons for a fiscal gap other than those arising from pure efficiency reasons. In particular, administrative and harmonisation advantages of centralised tax collection have been neglected. The possible need for the federal

[27]Another way to see this is by considering the special case in which $\theta = 1/2$, so that both levels share equally in rents. Then the condition $G_T > kg_t$ reduces to $T > t$. In this case, transfers should be increased to the jurisdiction with the lowest tax rate, thereby increasing the requirement for the higher-tax jurisdiction to raise revenues. This makes sense because the higher-tax jurisdiction will have the highest perceived marginal cost of public funds, and therefore be closer to the socially optimal level of revenues.

not affect the allocation of population among states (though it may affect the decision rules followed by the state governments). Given symmetry, subscripts denoting states of residence will typically be omitted and only introduced when necessary. For simplicity, the utility of each household takes the separable form:

$$u(x,l) + b(g) + B(G) , \tag{1}$$

where x is a private good (and numeraire), l is labour supplied, g is the state public good, and G is the federal public good. The benefits of state public goods accrue only to residents of the state, while those of the federal public good accrue to all members of the nation regardless of where the reside. The functions $b(g)$ and $B(G)$ are both increasing and concave, while $u(x,l)$ is increasing in x, decreasing in l and quasi-concave. The budget constraint faced by each household is:

$$x = (w - \tau)l , \tag{2}$$

where w is the wage rate and τ is the per unit tax on labour. The latter, in turn, is given by $\tau = t + T$, where t is the state tax on labour and T the federal tax.[6]

The household maximises utility (1) subject to the budget constraint (2), which yields the first-order condition:[7]

$$(w - \tau)u_x + u_l = 0 . \tag{3}$$

The solution to (3) gives labour supply $l(w - \tau)$; for concreteness of interpretation, we assume $l'(w - \tau) > 0$. Substituting $l(w - \tau)$ into (1) yields indirect utility $v(w - \tau) + b(g) + B(G)$. Differentiating (1) with respect to w and using (2) gives:

$$v'(w - \tau) = u_x l(w - \tau) . \tag{4}$$

The production side of the economy is similarly simple. Each state is endowed with the same amount of some fixed factor.[8] Output is produced by applying labour services to the fixed factor according to an increasing and strictly concave production function $f[nl(w - \tau)]$, where n is the population of the state and therefore the number of workers. The output can be used interchangeably for private consumption x, state public good provision g or federal public good provision G. Thus, the marginal rate of transformation between public goods of each type and private goods is unity. The private sector is perfectly competitive, so the market wage is given by the usual marginal productivity condition:

$$w = f'[nl(w - \tau)] . \tag{5}$$

[6]Deductibility of state taxes against federal taxes complicates matters somewhat without yielding substantive additional insights. Basing taxes on labour rather than on income makes no significant difference either.

[7]For functions of several variables, derivatives are indicated by subscripts; for functions of a single variable, differentiation is indicated by a prime.

[8]We give them equal endowments of the fixed factor to avoid the possibility of grants being used for horizontal redistribution.

This yields a wage function $w(\tau,n)$ with the following properties:

$$w_\tau = \frac{-f''nl'}{1-f''nl'} \quad \in (0,1) \tag{6}$$

$$w_n = \frac{f''l}{1-f''nl'} = -\frac{w_\tau l}{nl'} \quad < 0\ . \tag{7}$$

(Notice that, because of separable utility, the wage is independent of g and G, which simplifies the analysis considerably).

Since the production function is strictly concave, rents will be generated. We assume that they all accrue to the public sector (since they represent an efficient source of tax revenues). A proportion θ accrues to the federal government and the remaining $(1-\theta)$ accrues to the state in which the rents are generated. The rent-sharing parameter θ is taken as given throughout the analysis (and to lie in $[0,1]$). The rents generated in the typical state are:

$$r(\tau,n) = f\big[nl(w(\tau,n)-\tau)\big] - nl(w(\tau,n)-\tau)f'\big[nl(w(\tau,n)-\tau)\big]\ . \tag{8}$$

Differentiating (8) and using (6) and (7) yields:

$$r_\tau = \frac{n^2lf''l'}{1-f''nl'} = -nlw_\tau \quad < 0 \tag{9}$$

$$r_n = \frac{-nl^2f''}{1-f''nl'} = -\frac{r_\tau l}{nl'} \quad > 0\ . \tag{10}$$

Given the wage function $w(\tau,n)$ and the fiscal variables τ $(=t+T)$, g and G, household indirect utility is given by:

$$v\big[w(\tau,n)-\tau\big] + b(g) + B(G) \tag{11}$$

which, when migration is costless, will be equalised across all states. The migration equilibrium determines the value of population n in each state. We return to some of the features of the migration equilibrium later. For now, we simply note that in a symmetric equilibrium with all states pursuing the same policies, both the population n and the wage rate w will be identical in all states.

State governments provide the pure local public good g and finance it by a labour tax t plus any transfer S they receive from the federal government.[9] The state budget constraint is thus:

$$g(t,T,S,\theta,n) = ntl\big[w(\tau,n)-\tau\big] + S + (1-\theta)r(\tau,n)\ . \tag{12}$$

[9] It would make no significant difference to the results if — reflecting, perhaps, congestion costs — the benefits accruing to state residents from g depended upon state population, or even if g were a publicly provided private good.

the welfare effect of a small change in S can be seen, using the federal and state government first-order conditions (24) and (47) as well as $g_S = 1$ and $G_S = -k$, to be given by:

$$\frac{n}{u_x}\left(\frac{dV}{dS}\right) = \left(\frac{nb'}{u_x} - \frac{knB'}{u_x}\right) + \frac{1}{k}\left(\frac{knB'}{u_x}\right)G_t\frac{d\bar{t}}{dS} - \left(\frac{nb'}{u_x}\right)g_T\frac{d\bar{T}}{dS}\ , \tag{57}$$

where, from (48):

$$\frac{d\bar{t}}{dS} = \frac{t_S + T_St_T}{1-T_tt_T}\ ; \qquad \frac{d\bar{T}}{dS} = \frac{T_S + t_ST_t}{1-T_tt_T}\ . \tag{58}$$

The interpretation, once again, follows from that for the case where the federal government is first mover. The first effect in (57) shows that, other things being equal, a transfer from the level with the lower MCPF to that with the higher is desirable; against this, however, must be borne in mind the effect of the transfer on the tax rates that the two levels are induced to set.

Given the complexity of (58), the sign of dV/dS is difficult to evaluate in general. Consider, however, the special case in which preferences for public goods are linear ($b'' = B'' = 0$). In this case, $d\bar{t}/dS = d\bar{T}/dS = 0$, so any change in transfers is reflected fully in expenditures on public goods.[26] The last two terms involving changes in $\bar{t}$ and $\bar{T}$ drop out of (57) implying that the transfer should be made towards whichever level of government has the higher MCPF in the Nash equilibrium. This result is easily explained when one recalls that the MCPF perceived by a Nash government will typically be less than the second-best value for the federation, since the Nash government will neglect the effect of its tax increases on the revenues of the other governments. Thus, both the federal government and the states will choose public service levels according to MCPFs that are too low, and both will have an incentive to over-spend. However, if $G_t < kg_T$, that incentive will be higher for the federal government than for the states, and therefore G will deviate more from the second-best level than g will. Transferring funds from the federal government to the states will cause G to fall and g to rise and thereby move the economy closer to the second-best optimum.

Another feature of this linear case is that, since b' and B' are (by assumption) constant, it would be welfare-improving to continue changing S in the direction indicated by the sign of dV/dS until some bound is reached. Since changes in S cause the donor government to reduce public services, this bound will occur when their public service level falls to zero. This is also easy to explain. If a marginal unit of resources spent on g has greater value than that spent on G (taking account that the resources spent on G will have benefits over all k jurisdictions), only g should be provided in the

[26] This follows from (30) and the expression for T_S in the previous footnote.

Proposition 3 above: the MCPF is lower for the level of government whose tax rate exerts the most powerful effect on the tax base of the other.

Consider now the role of grants in such a federation. The following shows – in answer to the first of the questions above, and in contrast to the case in which the federal government is the first mover – that grants do not suffice to take the federation to a second-best optimum:

Proposition 9: *If both state governments and the federal government behave as Nash competitors in choosing their own tax rates and expenditure levels and treat the level of grants as given, a grants commission cannot replicate the second-best optimum.*

Proof: This proceeds by contradiction. The second-best outcome can be obtained as the solution to:

$$\max_{t,T} \quad v[w(t+T)-t-T]+b[g(t,T)]+B[G(t,T)] \tag{50}$$

which has the first-order conditions:

$$(w_\tau - 1)v' + b'g_t + B'G_t = 0 \tag{51}$$
$$(w_\tau - 1)v' + b'g_T + B'G_T = 0\,. \tag{52}$$

If both the states and the federal government behave as Nash competitors, (24) and (47) will be satisfied. Therefore, by (51) and (52), $G_t = g_T = 0$, or:

$$G_t + kg_T = 0 \tag{53}$$

From (14) and (18), we have:

$$G_t + kg_T = (w_\tau - 1)(T+t)knl' + k\theta r_\tau + k(1-\theta)r_\tau \tag{54}$$
$$= (w_\tau - 1)\tau^* knl' + kr_\tau < 0\,, \tag{55}$$

at the second-best optimum, which contradicts (53). Therefore, the second-best is unobtainable. □

The intuition is straightforward: with both the states and the federal government behaving non-optimally, a single instrument is an insufficient corrective device.

Next, consider the effect on per capita utility of an incremental change in S. Defining:

$$V(S) = v[w(\bar{t}(S)+\bar{T}(S),n) - \bar{t}(S) - \bar{T}(S)] + b[g(\bar{t}(S),\bar{T}(S),S,\theta)]$$
$$+B[G(\bar{T}(S),\bar{t}(S),S,\theta)]\,, \tag{56}$$

We do not impose any restrictions on the sign of S. In particular, we do not rule out the possibility that $S < 0$, in which case the federal government imposes on the states a requirement to make a transfer to it: we refer to such cases as involving a 'negative' fiscal gap. Note from (12), for later use, that:

$$g_t = (w_\tau - 1)ntl' + nl + (1-\theta)r_\tau \tag{13}$$
$$g_T = (w_\tau - 1)ntl' + (1-\theta)r_\tau = g_t - nl \tag{14}$$
$$g_S = 1 \tag{15}$$
$$g_n = tl + ntl'w_n + (1-\theta)r_n\,. \tag{16}$$

The state selects its policy variables t and g to maximise the per capita utility of its residents, taking as given the policies of the federal government and all other state governments.[10] The presumption is that there are enough states for each to ignore the effects of its policies on other states, as well as on the federal government. The solution to the state problem is a policy set (t,g) that depends upon federal policies (T,G,S), the allocation of rents θ and, in the immobile population case, state population n. The implications of mobility for state behaviour are spelled out in Section V.

The federal government is (until Section VI) the first mover and recognises the effect of its policies on the behaviour of the states. Since we dealing with symmetric equilibria in which populations are identical in all states, federal policies, being uniform across states, will have no effect on the allocation of population; thus we need not be concerned with analysing any migration response to federal policies. The federal budget constraint is given by:

$$G(T,t,S,\theta) = knTl\big[w(\tau,n)-\tau\big] - kS + k\theta r(\tau,n)\,. \tag{17}$$

Again for later use, note that:

$$G_T = (w_\tau - 1)knTl' + knl + k\theta r_\tau \tag{18}$$
$$G_t = (w_\tau - 1)knTl' + k\theta r_\tau = G_T - knl \tag{19}$$
$$G_S = -k\,. \tag{20}$$

The federal government chooses its policies (T,G,S), taking as given its share of the rents θ and bearing in mind the implications for (t,g), to maximise per capita utility.

III. Optimal Policies in the 'Unitary' Nation

As noted at the outset, the usual argument for a positive fiscal gap is that it is necessary to avoid outcomes resulting from state government behaviour that, while

[10]The assumption that states maximise per capita rather than total utility is one of substance: it is well-known that the two criteria can give different results (Wildasin (1986)).

optimal from the state's perspective, are inefficient from the national one. For exactly this reason, this section considers the benchmark case in which states are given no taxing or spending powers. We do this by consolidating the budget constraints (12) and (17) – thereby eliminating inter-governmental transfers S and the rent-sharing parameter θ from the problem – and allowing the federal government to choose the total tax on labour τ, g and G.

Before characterising the unitary optimum, note that the model developed in the preceding section has the feature, like most others in the literature, that there is indeed no loss from adopting a unitary government structure, since we have deliberately suppressed all of the possible gains from decentralisation outlined in the Introduction. An implication is that the decentralised outcomes with which we are ultimately concerned cannot be welfare superior to the best unitary one; the best decentralised regime will be that which mimics most closely the unitary optimum. It may seem strange (though it is certainly not unusual) to analyse issues of fiscal federalism in a model with this feature. But our purpose, it should be emphasised, is not to model the reasons for decentralising fiscal responsibilities to the states; only the consequences of doing so. For this purpose, the intrinsic superiority of the unitary form provides a useful clarity of focus.

Consider then the unitary optimum. Assuming it to be symmetric, it will involve all states having the same numbers of residents. Thus we can suppress n from the problem, which can consequently be written as:

$$\max_{\{\tau,g,G\}} \quad v\big[w(\tau)-\tau\big]+b(g)+B(G) \tag{21}$$

subject to

$$G+kg=kn\tau l\big[w(\tau)-\tau\big]+kr(\tau)\ . \tag{22}$$

The first order conditions for this readily reduce to:

$$\frac{nb'(g)}{u_x}=\frac{knB'(G)}{u_x}=\frac{1}{1-\frac{\tau l'}{l}}\ . \tag{23}$$

Equations (23), together with the budget constraint (22), can be solved for the second-best optimal values of the choice variables, which we denote by τ^*, g^* and G^*. We assume them to be unique.

The unitary government, it should be noted, will deploy the distorting labour tax only if the federation is under-populated, in the sense that aggregate rents $kr(0,n)$ are insufficient to finance the first-best level of aggregate expenditure kg^*+G^*. In particular — and this is a useful special case that we shall make use of below — it is straightforward to show that if n is optimal, rents exactly suffice to finance first-best levels of public expenditure.[11]

[11]These claims, which are simple variants of the usual Henry George Theorem (Hartwick (1980),

them. There may be some institution responsible for overseeing the choice of grants, akin to the grants commissions that exist in some federations (such as Australia and India).

We assume then that S is taken as given by both levels of government and investigate the welfare consequences of changing it, given the equilibrium responses by the governments (and θ). For simplicity, we analyse the case in which households are immobile across states and consider only symmetric equilibria within the federation. The issues to be addressed are twofold: Can a grants commission replicate the second-best optimum? And, to which level of government should it make grants?

The problem of the representative state government is exactly as in Section IV, and yields the solutions $t(T,S)$ and $g(T,S)$ described there.[24] The problem of the federal government is to choose T to maximise per capita utility $v[w(\tau,n)-\tau]+b(g)+B[G(T,t,S,\theta)]$, now taking t and g as given. The first-order condition is:

$$(w_\tau-1)v'+B'G_T=0\ , \tag{47}$$

which yields the federal government's reaction function $T(t,S)$.[25] We assume there to be a unique and stable symmetric Nash equilibrium, described by a pair $(\bar{t},\bar{T})$ satisfying:

$$\bar{t}=t(\bar{T},S)\ ; \qquad \bar{T}=T(\bar{t},S)\ , \tag{48}$$

with stability requiring that $t_T T_t<1$.

Conditions (24) and (47) implicitly describe the MCPF at the state and federal levels in the Nash equilibrium. Combining the two and using (14) and (19), they are related as:

$$\frac{nkB'}{u_x}=\frac{nb'}{u_x}\left(\frac{kg_T+knl}{G_t+knl}\right)\ . \tag{49}$$

The sign of the difference between the two MCPFs thus turns on the difference between the two 'cross-effects', G_t and g_T: the MCPF at federal level will exceed that at state level if and only if $G_t<kg_T$. The intuition for this follows from that given for

[24]We omit θ and n as arguments of the states' best response functions, since they are treated as fixed in this exercise.

[25]Differentiating (47) gives:

$$T_t=-1+\frac{(w_\tau-1)B'knl'+B''G_tknl}{(w_\tau-1)^2v''+v'w_{\tau\tau}+B'G_{Tt}+B''G_TG_t}\ ,$$

which, like t_T in (29), is ambiguous in sign, and:

$$T_S=\frac{kB''G_T}{(w_\tau-1)^2v''+v'w_{\tau\tau}+B'G_{Tt}+B''G_TG_t}\geq 0\ .$$

Using (7), this simplifies to:

$$T = \theta f''nl + \frac{t + \theta f''nl}{w_\tau} \, . \tag{45}$$

Since at the second-best optimum, $T = \tau^* - t$, we obtain:

$$T = \theta f''nl + \frac{\tau^*}{(1+w_\tau)} = T^* + \frac{\tau^*}{(1+w_\tau)} \tag{46}$$

where T^* is the federal tax rate when households are fully mobile. Since $\tau^*/(1+w_\tau) > 0$, the result follows. □

The resolution between the ambiguity of Proposition 7 and the certainty of Proposition 8 is that while the former holds for arbitrary federal policies, optimisation by the federal government implies that (43) will indeed be satisfied when T and S are set so as to implement the second-best (a direct proof of this being given in Appendix B).

Note that since τ^* and total revenue are the same here as in the earlier cases of immobile and perfectly mobile labour, the state tax will be lower and S larger when labour is mobile but residency fixed. This tends to confirm the conventional wisdom outlined in the Introduction that interstate competition for mobile factors provides a argument for increasing the fiscal gap; the optimal fiscal gap may still be negative – since the considerations identified earlier continue to apply – but it is more likely to be positive.

B. *Optimal Grants with Nash Behaviour by All Governments*

It has been assumed so far that the federal government has first-mover advantage with respect to the states. This seems a natural assumption for most federations, given the dominance of the federal government in public sector budgets, the fact that there are typically many states but only one federal government, and institutional settings in which the federal government determines the level of intergovernmental grants. However, as an alternative it is useful to consider the case in which the federal government has no advantage over the states in terms of controlling tax policies. We therefore now consider the case in which the federal government and the states both behave as Nash competitors with respect to tax and expenditure policies of each other.

Adopting the Nash assumption for both levels of government immediately raises a problem for the analysis of inter-governmental grants. It would be implausible for the federal government both to be able to set grant levels to the states and to assume that state fiscal policies are exogenously given. It is more reasonable to suppose that grants are set independently from tax and expenditure policies of the federal and state governments, preferably in full knowledge of how these governments will respond to

Equations (23) are standard optimality conditions for public goods supply in a distorted economy, being simplified versions of the well-known Atkinson and Stern (1974) rule.[12] They indicate that both the sum of the marginal rates of substitution of each state public good g for the private good x and the sum, across the entire nation, of the marginal rates of substitution of G for x should equal $1/(1-\tau l'/l)$. Following common practice, we refer to this term as the ***marginal cost of public funds*** (MCPF). As long as the federation is under-populated, so that $\tau > 0$, the MCPF is greater than unity.[13]

IV. Optimal Federal Policies with Immobile Households

We now reintroduce state governments and consider optimal federal policies first in a federation in which state populations are fixed. This may seem unrealistic, but has the advantage of allowing one to study the fiscal gap in a setting in which issues of inter-state tax competiton do not arise, enabling an evaluation of the conventional wisdom according to which the main efficiency argument for a positive fiscal gap arises from mobility of state governments' tax bases. As before, state populations are identical and we concentrate on symmetric equilibria in which all states follow the same policies.

As discussed above, we assume that the federal government moves first, committing itself to its tax, expenditure and transfer policies. The states move second, taking federal policies as given. Since the federal government can anticipate the effects of its actions on state behaviour, we proceed in the conventional way to analyse decisions in reverse order.

A. *The Representative State's Problem*

State governments choose t and g to maximise the per capita utility of their residents (11) taking as given state population n, federal policy variables (T, G, S) and the

Wildasin (1986)) are established by introducing the tax on rents as a choice variable and varying n (holding k fixed).

[12]The Atkinson-Stern rule includes a term in the numerator involving the effect of changes in g on tax revenues. This is absent from (23) as a consequence of additively separable utility.

[13]The MCPF gives the net cost to society of raising an additional dollar of tax revenue. To see this, rewrite (23), using $l' = -\partial l/\partial\tau$, as:

$$\frac{1}{1-\frac{\tau l'}{l}} = 1 - \frac{\tau\frac{\partial l}{\partial \tau}}{l + \tau\frac{\partial l}{\partial \tau}}$$

Thus, the net cost of raising an extra dollar of tax revenue is one dollar plus $-\tau\partial l/\partial\tau/(l+\tau\partial l/\partial\tau)$. The numerator of this term is the conventional expression for the change in deadweight loss from a change in the labour tax rate, while the denominator is the change in tax revenue. Thus, the entire term is the change in deadweight loss per dollar of tax revenue raised. When added to the one dollar of tax revenue collected, it gives the cost to society of raising an additional dollar of tax revenue.

state's share of rents θ, and subject to the state budget constraint (12). The first-order condition is simply:

$$(w_\tau - 1)v' + b'g_t = 0 . \tag{24}$$

Substituting for g_t from (13), and then from (4) for v', from (9) for r_τ and from (6) for w_τ, we obtain:

Proposition 1: *With households immobile across states, the MCPF of the state government is given by:*

$$\frac{nb'}{u_x} = \frac{1}{1 - \frac{tl'}{l} - \theta f'' nl'} . \tag{25}$$

Together with the state budget constraint (12), this policy rule yields the state policy variables as functions $t(T, S, \theta, n)$ and $g(T, S, \theta, n)$.[14] To interpret the state government rule in (25), note that the denominator can be written as:

$$1 - \frac{tl'}{l} - \theta f'' nl' = 1 - \frac{\tau l'}{l} + \left(\frac{Tl'}{l} - \theta f'' nl'\right) \tag{26}$$

$$= 1 - \frac{\tau l'}{l} + \frac{G_t}{(w_\tau - 1)knl} , \tag{27}$$

the second equality being from (19). Equation (26) shows that the MCPF at state level differs from the second-best optimal decision rule for g in two ways; and (27) then shows that the sign of this divergence depends simply on that of the impact on federal revenues of an increase in state taxation. If this externality is negative, then (recalling from (6) that $w_\tau < 1$) the state MCPF is lower than that corresponding to the unitary rule: state taxes, loosely speaking, would then be expected to be higher than in the unitary second best, and state expenditures correspondingly higher.[15]

The two components of the external effect on the right-hand side of (26) have straightforward interpretations. First, the state government neglects the effect that the labour supply response to its tax decisions has on federal tax revenues: in (25), the proportionate change in labour supply is multiplied by t rather than by τ as in (23). Given our assumption that $l' > 0$, this labour supply effect will diminish federal revenues so long as $T > 0$.

The second term in the external effect, $-\theta f'' nl'$, reflects the change in the federal government's receipt of the rents as a result of the induced labour supply (and wage

[14]Separability again removes effects through G.

[15]The fact that the MCPF for state governments within a federation tends to be lower than the second-best optimal value was first noted by Dahlby (1994), who goes on to suggest that this is also true of the federal MCPF. This implicitly assumes that the federal government also behaves as a Nash competitor; we return to this issue in Section VI.

that given by rule (27) if and only if

$$tl > \theta \frac{\partial r}{\partial n}\Big|_l \tag{43}$$

and is below if the reverse equality applies.

Proof: See Appendix A.

The implication, loosely speaking, is that – for arbitrary federal policies – the effect on the level of state taxes of this kind of labour mobility is ambiguous. To see why, note that attracting an additional worker affects both the level and the distribution of income within the state. Output rises by wl, the gross wage paid to that worker; of which the state captures tl in taxes. At the same time, the reduction in the gross wage leads to a redistribution from labour to the fixed factor; and with $\theta > 0$ this redistribution has the undesirable effect, from the state's perspective, of transferring income to the federal government. A state will thus wish to attract additional workers only if the first effect dominates the latter.

Consider now the problem of the federal government. Since the two states will respond identically to a change in the federal tax, such a change will have no effect on the allocation of workers. The federal problem thus has exactly the same structure as in Section IV (though the precise form of the states' response t_T will generally be different). Once again, by choosing T and S appropriately, the federal government has enough policy instruments to replicate the unitary second best. Note, however, that the size and sign of the fiscal gap may be very different from that in the case where households are fully mobile. Suppose for instance that $\theta = 0$, so that all rents go to the states. In the cases analysed in Sections IV and V, where residency and place of work coincide, it is then optimal for the federal government to set $T = 0$ and finance itself by a negative fiscal gap. In the present case, however, Proposition 7 indicates that a positive federal tax would be needed to stymie the downward pressure on tax rates from inter-state competition for workers. Thus — as the traditional arguments cited in the Introduction woud lead one to expect — it seems more likely that transfers should go from the states to the centre. This is confirmed in:

Proposition 8: *The federal tax required to replicate the second-best optimum is higher when labour is mobile but residency fixed than when households are fully mobile.*

Proof: From condition (A.3) of Appendix A (describing state behaviour), the value of T needed to achieve the second-best outcome satisfies:

$$\frac{Tl'}{l} - \theta f'' nl' + \frac{tl + \theta f'' nl^2}{nlw_n} = 0 . \tag{44}$$

A. *Tax Base Mobility and the Possibility of Tax Competition*

We have assumed so far that mobility applies to households. While this captures whatever inter-state competition for population may exist, it does not capture the other form of inter-state competition that has been prominent in the literature, *viz.*, competition for factors of production. Although previous analyses have typically treated this as competition for capital, we analyse the case of mobility of labour; the two are much the same.[23]

Suppose then that while households are free to work in any state, they remain resident in their initial location. They receive the going wage in the state in which they work, and pay labour taxes there as well. But they benefit from public expenditure g in their state of residency. As above, we treat the special case in which there are just two states, A and B. To maintain symmetry, the number of residents in each state is fixed at $\bar{n}/2$. Given free mobility of labour, equilibrium now requires that the after-tax wage rate be equated across the two states:

$$w(\tau_A, n_A) - \tau_A = w(\tau_B, \bar{n} - n_A) - \tau_B \; . \tag{41}$$

This yields a worker allocation function $n_A(\tau_A, \tau_B)$; expenditures of the state governments no longer enter the mobility condition, since workers benefit from g in their home state. Differentiating (41) gives the same value for n_τ in a symmetric equilibrium as in (37) above.

As usual, the analysis proceeds backwards, starting with the state's problem. This is to choose t to maximise:

$$v[w(\tau, n(\tau)) - \tau] + b[g(t, T, n(\tau), S, \theta)] \, ,$$

The first-order condition for which is:

$$(w_\tau + w_n n_\tau - 1)v' + (g_t + g_n n_\tau)b' = 0 \; . \tag{42}$$

Routine manipulations then yield:

Proposition 7: *When labour is mobile but residency fixed, the state MCPF is above*

[23]A standard model of capital tax competition is that of Zodrow and Mieszkowski (1986) which considers local public goods economies with fixed immobile populations and a given stock of capital that is perfectly mobile across local jurisdictions. Our model, in which capital has been suppressed as a factor of production, could be interpreted as applying to the case where the nation as a whole is small in the world economy and faces a fixed rate of return on capital. In such a model, adding capital would be of no consequence; taxing capital would be analogous to taxing local factors of production. Some aspects of capital income taxation within a federation are analysed by Kotsogiannis (1994). An alternative form of tax competition would be that involving commodity taxation as in Mintz and Tulkens (1986) or Kanbur and Keen (1993). Similar types of strategic interaction effects are captured in our case of labour tax competition.

rate) response to increases in state expenditures and taxes. From the social point of view, this change in rents is a pure transfer, and so does not appear in the second-best MCPF expression for the unitary nation, (23). From the state's perspective, however, it represents a redistribution of income between workers in the state and the federal government, and the state cares only about the first of these. Again, given $l' > 0$, since $f'' < 0$ this term is positive for $\theta > 0$; an increase in the state tax rate reduces labour supply causing a reduction in rents, a proportion θ being at the expense of the federal government (and hence not recognised by the state.) This effect also tends to reduce the magnitude of the right-hand side of (26) and to cause the state MCPF to be lower than the second-best optimal value.[16]

It will prove useful below to consider the way in which the federal government's decisions impact the states' choice of tax rate $t(T, S, \theta, n)$. Differentiating (24), the slope of the state's reaction function with respect to the federal tax rate is found to be

$$t_T = -1 + \frac{(w_\tau - 1)b'nl' + b''g_t nl}{(w_\tau - 1)^2 v'' + v'w_{\tau\tau} + b'g_{tt} + b''g_t^2} \; , \tag{28}$$

the denominator of the second term being negative by the second-order condition on the states' problem. In general t_T can thus have either sign. Note though that (28) implies:

$$1 + t_T > 0 \, , \tag{29}$$

so that, although the state tax may fall in response to an increase in the federal tax, it cannot fall by so much that the combined rate $\tau = t + T$ also falls. For the effect of the intergovernmental transfer, one finds that

$$t_S = -\left(\frac{b''g_t}{(w_\tau - 1)^2 v'' + v'w_{\tau\tau} + b'g_{tt} + b''g_t^2} \right) \leq 0 \; . \tag{30}$$

As one would expect, an increase in the transfer S received by the state can only reduce the state tax rate. Note that if $b(\cdot)$ is linear, so that income effects are concentrated entirely on the state public good, then the state tax is independent of the transfer.

B. *The Federal Government's Problem*

Consider now the policy response of the federal government, given the behaviour of the states as summarised in (25). Assume that the federal government also takes θ as given and chooses its policy variables T, G, and S. Furthermore, it can choose T and/or S

[16]In terms of the interpretation of the MCPF for the unitary nation in footnote 13, from the point of view of the states the change in deadweight loss attributable to changes in state taxes (the numerator) is $(t + \theta f''nl)\partial l/\partial t$ rather than $\tau \partial l/\partial t$, reflecting the fact that from the point of view of the state, the distortion in the labour market is the state's marginal tax rate t (rather than τ) less the incremental change in federal rents from an increase in state taxes. Similarly, the revenue cost of raising t is $l + (t + \theta f''n)\partial l/\partial t$.

to be negative, in the latter case thereby imposing on the states the requirement to finance part of G by transfers to the federal government. This allows the fiscal gap to be determined completely endogenously from an efficiency perspective. The problem of the federal government is then to maximise:

$$V(T,S;\theta) \equiv v[w(\tau)-\tau] + b[g(t,T,S,\theta)] + B[G(T,t,S,\theta)] \qquad (31)$$

subject to $t = t(T,S,\theta,n)$, as given by the solution to the state problem described above.

As discussed above, the federal government will choose its policies so as to replicate as closely as possible – given its lack of direct control over the states' decisions – the second-best optimal allocation of resources in the unitary nation, characterised in Section III: having a seemingly more constrained decision problem than the unitary government, it can certainly do no better than the unitary optimum. And it is easily seen that, when it moves first, the federal government is able to replicate the unitary second-best exactly. To see this, note that it has three policy instruments (T,S,G) with which to achieve the three second-best policy variables (τ^*,G^*,g^*), given that, from the solution to the representative state's problem $\tau = T + t(T,S,G)$ and $g = g(T,S,G)$. If the federal government sets $G = G^*$, it can also induce the optimal values for τ and g by choosing T and S such that $\tau^* = T + t(T,S)$ and $g^* = g(T,S)$. Thus:

Proposition 2: *With households immobile across states, the federal government acting as a first mover with respect to the states can choose policies (G,T,S) to achieve the second-best optimum values for τ, g and G.*

Taking θ as given is clearly no real restriction in this context (assuming there are no other restrictions on federal policy variables, such as non-negativity requirements): it would be a redundant policy instrument. That is not to say, however, that the value of θ is irrelevant to the federal government's choice of policy variables. As will be seen shortly, the optimal values of both T and S depend upon the federal government's share of rents θ.

It remains to characterise the federal government's choice of T and S to implement the second-best optimum. Consider first the choice of T in maximising (31). The necessary condition is:

$$(w_\tau - 1)(1+t_T)v' + (g_t t_T + g_T)b' + (G_t t_T + G_T)B' = 0 \,. \qquad (32)$$

Using the first-order condition (24) for the state problem together with (14) and (19), the MCPFs at state and federal level are related according to:

$$\frac{knB'}{u_x} = \frac{nb'}{u_x}\left(\frac{1}{1+(1+t_T)G_t/knl}\right) \,. \qquad (33)$$

rules apply with or without mobile households. Moreover, the same federal policy prescriptions also apply. To summarise:

Proposition 6: *If states take migration responses from their own actions fully into account, their behaviour when labour is perfectly mobile is exactly the same as when labour is completely immobile. Federal policies to achieve the second-best optimum are also identical.*

This result that state behaviour is essentially unaffected by migration of households has its counterpart in the literature. A standard result in existing models of federalism is that, in a world with perfect labour mobility and non-distorting taxes, if state governments maximise per capita utilities taking into account migration responses to their own policy actions and taking the policies of other states as fixed, their behaviour will be perfectly is efficient in the sense that it will follow first-best (Samuelson) rules for expenditures and taxation.[22] The result continues to apply in the model used here with distortionary taxation. The reason for this is that,with states maximising their own per capita utilities, and with these constrained by migration equilibrium to be the same across states, states are effectively maximising national per capita utility.

An important implication of Proposition 6 is that the efficiency argument for a fiscal gap is independent of the degree of mobility of labour. Rather, it derives from the distortionary nature of taxation. To see this, consider the special case in which the labour supply is completely inelastic ($l' = 0$). In this case, with or without population mobility, state behaviour follows the Samuelson rule for g by (25). Moreover, since (by (19) and (9)) $G_t = 0$, the federal government will also follow the Samuelson rule for G (by (33)). Therefore, from (34), $dV/dS = 0$ at any value of S; that is, the size of the fiscal gap is completely irrelevant. Thus, at least for the case of labour taxation, mobility of the tax base is not a satisfactory reason for the existence of a fiscal gap.

VI. Consequences of Alternative Assumptions

This section explores the robustness of the stark and somewhat unexpected results above – with the federal government choosing non-positive labour tax rates, and quite possibly a negative fiscal gap – to alternative assumptions on the way in which the economy operates and governments interact. The treatment cannot be exhaustive within the space available, so concentrates on two of the more important assumptions: the nature of tax base mobility and the strategic relationship between the two levels of government.

[22]See Boadway (1982), Wildasin (1986) and Krelove (1992). Note that this result, like that in Proposition 6, is contingent on states maximising per capita rather than total utilities.

equilibrium.[21]

Differentiating (36) yields, at a symmetric equilibrium, the following expressions for the effect of each state's tax and expenditures on its own population, taking as given the policies of the other state:

$$n_\tau = \frac{1-w_\tau}{2w_n} = \frac{(w_\tau - 1)nl'}{2lw_\tau} \tag{37}$$

$$n_g = \frac{-b'}{2v'w_n} = \frac{b'nl'}{2v'lw_\tau} \tag{38}$$

where we have used (7) for w_n and suppressed the state subscripts.

As above, the representative state government is assumed to maximise the per capita utility of its residents, now taking account of both its budget constraint and the migration response $n_A(\cdot)$. Writing the typical state's population function as $n(\tau, g)$ (suppressing arguments referring to the other state), per capita utility is:

$$v\big[w(\tau, n(\tau, g)) - \tau\big] + b(g) + B(G)\ . \tag{39}$$

and the state budget constraint:

$$g[t, T, n(\tau, g), S, \theta]g = n(\tau, g)tl\big[w(\tau, n(\tau, g)) - \tau\big] + S + (1-\theta)r(\tau, n(\tau, g))\ , \tag{40}$$

where we have distinguished between t as it enters through the migration function $n(\tau, g)$ and as it enters otherwise. The state's problem is to choose t and g to maximise (39) subject to (40). Using (37), (38) and (7), the first-order condition for this problem evaluated at a symmetric equilibrium reduces to precisely the same condition, (24), as found for the case in which individuals are immobile. Thus, the same state decision

[21] In fact, migration equilibria in federal models of this sort are particularly prone to problems of instability and multiple equilibria (see Stiglitz, 1977). In local public goods models with no federal public good, stability of equilibrium requires that the population of the nation as a whole be at least as large as the sum of the 'optimal' population levels (those that maximise per capita utilities) in all states (Boadway and Flatters, 1982); that is, the nation must be 'over-populated'. In the present model, with both federal and state public goods, matters are not quite so simple. For given federal policies, the level of utility achieved in a symmetric equilibrium when states are maximising the per capita utilities of their residents (as outlined below) can be denoted $V(n, S, T, \theta)$, where n is each state's population. Stability requires $V_n < 0$, which can be shown to be equivalent to:

$$\theta f'' + \frac{g - S - (1-\theta)r}{n^2} < 0\ .$$

For there to be positive labour taxation τ on the other hand, we require – by the Henry George principle mentioned earlier – that $kg^* + G^* > kr^*$, or that the nation be under-populated. Clearly under-population in the sense required for $\tau > 0$ is consistent with over-population in the sense required for stability.

Recalling from (29) that $1 + t_T > 0$ we thus have:

Proposition 3: *With households immobile across states, the MCPF for the federal government exceeds that of the state government if and only if an increase in the state tax rate reduces federal revenues (i.e. $G_t < 0$).*

The explanation is straightforward. For whilst the federal government takes full account of the impact of its policies on the revenues of the state governments, the latter ignore their effect on federal revenues: if $G_t < 0$, this asymmetry reduces the MCPF perceived at the state level relative to that at the federal level.

Consider now the federal government's choice of transfer S. Differentiating (31) and using (15), (20) and (24) one finds the welfare effect of a small change in S to be:

$$\frac{n}{u_x}\left(\frac{dV}{dS}\right) = \left(\frac{nb'}{u_x} - \frac{knB'}{u_x}\right) + \frac{1}{k}\left(\frac{knB'}{u_x}\right)G_t t_S\ . \tag{34}$$

There are thus two considerations in determining the optimal intergovernmental transfer. The first is a gain from transferring funds from whichever jurisdiction has the lower MCPF to whichever has the higher. If $G_t < 0$, for example — for which (from (19)) it is sufficient that the federal tax rate T be strictly positive — then by (33) it will be desirable, on this account, to transfer funds *away* from the states and towards the centre: the exact opposite, that is, of the usual presumption. Recalling from (30) that $t_S < 0$ if $b'' < 0$, the second consideration is that a transfer towards the states — inducing a reduction in the state tax rate — will be desirable so long as $G_t < 0$. These two considerations thus point in opposite directions, and will do so for any value of $G_t \neq 0$. In one special case, however, (34) gives a very sharp result:

Proposition 4: *Suppose that the federal tax rate is positive, and that $b(\cdot)$ is linear. Then, with households immobile and the federal government behaving as first mover, welfare is increased by a small transfer from the states to the federal government.*

At an optimum, of course, the transfer S will be chosen so that $dV/dS = 0$. Combining this condition with (33) gives $G_t = 0$. Thus, as one would expect, federal and state MCPFs are equated; and (from (27)) their common value is then exactly as given by the second-best rule for the unitary state. By (19) and (9), the federal tax rate that satisfies $G_t = 0$ at the second-best optimum, T^*, is given by:

$$T^* = \theta f''\big[nl(w(\tau^*, n) - \tau^*)\big]nl(w(\tau^*, n) - \tau^*) \leq 0\ . \tag{35}$$

Note that, since $f'' < 0$, $T^* < 0$ for $\theta > 0$. That is, the federal government should subsidize labour.[17] To summarise:

Proposition 5: *With households immobile across states, the federal government acting as first mover should subsidise labour at the rate $T = \theta f''nl$.*

Intuitively, since the states have an incentive to set their tax rate t too high as a result of neglecting the adverse revenue effect their tax increases have on federal government revenues, the federal government should provide an offsetting labour subsidy.

The sign of the optimal intergovernmental transfer S, however — the sign, that is, of the optimal fiscal gap — is ambiguous. Recalling the federal budget constraint (17), it is clear that this will depend, *inter alia*, on the strength of preferences towards the federal public good and on the size and distribution of rents. To fix ideas, it is useful to consider three special cases.

Case 1: $\theta = 0$. Suppose that all rents are allocated to the states. In this case, Proposition 5 implies that $T = 0$. The federal government then has no receipts of its own, so that federal expenditure on G must be financed entirely by transfers from the states. Thus $S^* < 0$ and, contrary to the usual presumption, the optimal fiscal gap is negative.[18] Note, moreover, that if θ is thought of as a choice variable, this case points to a particularly simple recipe for replicating the unitary second best in the myopic case: simply allocate all rents and tax powers to the states, and finance federal expenditures by transfers to the centre.

Case 2: Optimal National Population. A second instructive special case is that in which the total population of the federation is optimal, so that rents exactly suffice to finance first-best levels of expenditure; that is, $kg^* + G^* = kr(0, n)$. Suppose, too, that θ happens to be such that each level of government receives exactly enough rents to finance its first-best expenditure. This situation might seem a happy one, with no need for either level to deploy distorting taxes, and no need for transfers betweeen them. Non-cooperative behaviour, however, prevents this outcome. For, just as discussed above, the states will still perceive a gain from taxing labour as a device for transferring rents from the central government, which again calls for offsetting behaviour by the federal government. As a consequence of needing to subsidise labour whilst still spending G^*, the federal government will find itself having to extract revenues from the state governments. That is, the optimal fiscal gap is again negative.

[17]The solution to (35), given τ^*, will be a unique value of T^*. Therefore, the second-best optimal value of S will also be unique since it must satisfy $g^* = g(T^*, S, G^*)$ given that $G_S = 1$ by (15).

[18]It should be pointed out that this negative fiscal gap is not an inevitable consequence of the states having all the rents. The federal government could have chosen to finance its expenditures by its own labour tax T rather than by a transfer from the states. What Proposition 5 shows is that the federal government would never use $T > 0$ to raise revenues, regardless of the value of θ.

Case 3: $G^* = 0$. Finally, take the extreme case in which federal expenditure is worthless, so that $G^* = 0$. In this case, one might expect to reach the orthodox conclusion that transfers should go from the centre to the states. But even in this case the optimal fiscal gap may be negative. For Proposition 5 implies that the federal government will still need to finance a labour subsidy, and its revenue from rents may not be sufficient for it to do so. To see this, note that setting $G = 0$ in the federal budget constraint and using (35) gives: $S = \theta r(1 - E_r)$, where $E_r \equiv = -f''(nl)^2/r > 0$ denotes the elasticity of rents with respect to employment nl. If that elasticity exceeds one – as is certainly possible[19] – the optimal fiscal gap is thus negative even when there is no need for any public expenditure at the federal level: intuitively, a high responsiveness of rents implies that the reduction in rents consequent upon the states' taxation of labour has a powerful effect in diminishing the federal government's ability to finance the corrective subsidy from its own resources. Even in the simple case considered in this section – with no mobility of factors between states, and consequently no inter-state tax competition – the nature of the optimal fiscal gap is thus more complex than might have been expected. In particular, it can quite plausibly be negative.

V. Optimal Federal Policies with Perfectly Mobile Households

Suppose now that households can relocate costlessly between states, and therefore do so until utilities are equalised. State governments are fully aware of this, and take account of the effect of their own actions on the allocation of population among states. In all other respects, the model remains the same. It is convenient to consider the special case in which there are only two[20] states (identical, as before), indexed A and B.

To determine the effect of state policies on population allocation, we need to characterize the migration equilibrium. Denoting by $\bar{n}$ the total population, of whom n_A reside in state A and the remaining $\bar{n} - n_A$ in state B, free mobility implies that in equilibrium:

$$v\left[w(\tau_A, n_A) - \tau_A\right] + b(g_A) = v\left[w(\tau_B, \bar{n} - n_A) - \tau_B\right] + b(g_B) \,. \qquad (36)$$

This determines n_A as a function of the tax rates and public expenditures of the two states, $n_A(\tau_A, \tau_B, g_A, g_B)$. We assume it yields a unique and stable symmetric

[19]The value of E_r is related to the elasticity of substitution between labour and the fixed factor in the production function. For example, if the production function in nl and the fixed factor is CES, it can be shown that the elasticity of rents varies inversely with the elasticity of substitution, σ. Moreover, $E_r < 1$ for $\sigma \geq 1$, while $E_r > 1$ for $\sigma = 0$. Therefore, there will be some value of σ sufficiently less than one such that $E_r > 1$, so a reverse fiscal gap exists when the elasticity of substitution is at least that low.

[20]This restriction is inessential to the results.