



Queen's Economics Department Working Paper No. 891

Providing Public School Education in Developing Countries: A Theoretical Analysis

Bagala P. Biswal

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

12-1993

Discussion Paper #891

**Providing Public School Education in
Developing Countries: A Theoretical Analysis**

by

**Bagala P. Biswal
Queen's University**

December 1993

**PROVIDING PUBLIC SCHOOL EDUCATION
IN DEVELOPING COUNTRIES: A THEORETICAL ANALYSIS**

Bagala P Biswal*

Queen's University

ABSTRACT

Provision of universal free public education has been argued for in the literature on equity ground. This paper develops a new model of public school education and demonstrates how the presence of private tutoring in developing countries compromises the above argument. The teachers, by shirking at school and supplying private tutoring to the students at a cost, divert the benefits of free public education towards themselves. This model also conforms with the merit-cum-means principle adopted in developing countries to subsidize the education of the poor and high ability students when it is extended to an heterogeneous environment.

JEL Classification: H42, H52, I22

Key Words: Private Tutoring, Club, Imperfect Monitoring

* Address for correspondence to the author: Department of Economics, Queen's University at Kingston, Ontario, Canada K7L 3N6. I am grateful to Professors Robin Boadway, Frank Flatters and Raghendra Jha, and participants in the Canadian Economic Association Annual Meeting, 1992, and Queen's University for their helpful comments. All remaining errors are mine.

1. INTRODUCTION

Motivation

The coexistence of public and private school systems¹ is observed in both developed and developing countries. In most of the developing countries, while the students in public schools acquire most of their education from private tutoring, the students in private schools acquire all of their education from school teaching.² One can easily imagine its coverage considering the fact that the public system provides school education to a large subset of the total population in most of the developing countries.³ Even if education at public school is provided free of charge, the cost of private tutoring constitutes a sizable proportion of family expenditure on education and eventually makes the education of their children extremely expensive and hence a choice for parents.

In developing countries, despite a spectacular increase of participation in pre-primary school programmes over the last two decades, it is still the case in some regions that a substantial number of children never get to school at all. Over 300 million children are out of the reach of primary and secondary school. Moreover, large numbers of children who do get to school do not participate long enough to acquire literacy and other basic skills. Nearly one-third of the children who start

¹ In India, for example, the public schools are commonly known as the government schools and the private schools are commonly known as the public schools. However, for the purpose of this paper, we define the schools funded by the government as the public schools and the schools funded by the private persons or institutions as the private schools. We also interpret the word "school" as referring to primary and secondary only.

² A small percentage of students in private schools are also observed to acquire some of their education through private tutoring but the reasons for the presence of private tutoring in private schools are entirely different from that of the public schools. This point will become more clear as we go further into the analysis.

³ A recent study conducted by the United Kingdom's Department of Education and Science reported in the World Education Report (1991) show that the proportion of pre-primary enrollment in private institutions in Southern Asia, and East Asia and Oceania are as low as 9 and 11 percents respectively.

the first grade are estimated to drop out before completing grade four. The drop out problem is more serious in Latin America and the Caribbean where nearly half the children who start grade one never finish grade four. ⁴

The problems extend beyond student attendance: “Behind the difficulty of maintaining enrollment growth in primary education in Sub-Saharan Africa during the 1980s lay even greater difficulties in maintaining the quality of educational process. In many developing countries the pressures of increasing enrollments led to overcrowding of classes and over-burdening of teachers, while at the same time declining real incomes, undermined teachers’ morale, encouraging absenteeism and the search for supplementary sources of earnings and support. Widely prevailing shortages of text books and learning materials got worse as funds available for their purchase dried up. In addition, in many countries the whole infrastructure of support services deteriorated: school inspection and supervision, in-service teacher education, curriculum development, school health services, and maintenance of schools’ furniture, equipment and physical facilities.” (Source: World Education Report (1991), UNESCO.)

The basic purpose of the above discussion is to point out the fact that the developing countries are facing innumerable problems in fulfilling their targets of providing basic education to all and eradicating illiteracy. The low enrollment ratio and the high illiteracy rate indicate that poor people are not using the services, even when they are supplied freely by the public authorities. There are many reasons for these phenomena in LDCs but they are not the focus of the analysis.

We focus in our analysis on the mechanism of private tutoring and how it affects the education of the students in the public school systems and the cost of

⁴ The sources of the data used in this section are Statistical Year Book, UNESCO (1992), Human Development Report, UNDP 1991), and World Development Report, World Bank Staff (1986,1987).

education provided publicly in developing countries. There has been no explanation in the economics literature of any aspect of the practice of private tutoring, especially its causes, and its effects on human capital formation and the cost of education. Filling this gap is the primary contribution of this paper.

Literature Survey

The case for the provision of basic education via public expenditure has been argued in the literature in various different ways. Scholars contributing to the human capital literature (see e.g. Becker (1975), Butts (1978), Stiglitz (1974), etc.) have long stressed the fact that the main purpose of a public school system is to provide society with an educated and skillful work force. The seminal works of Romer (1986) and Lucas (1988) opened up the literature on endogenous growth which emphasizes human capital investment as the engine of long-run growth. Johnson (1984) and Creedy and Francois (1990) argue that the education of skilled workers exerts a positive externality on the earning power of uneducated, unskilled workers. To the extent that formal schooling is a significant component of human capital investment, the involvement of public sector in these investment decisions is important.

Bowles and Gintis (1976) and Lotts (1990) provide an alternative argument for public provision of education which can be summarized as indoctrination-socialization theories. The government has a direct interest in the content of education and so, of course, should support and control it. In the Bowles and Gintis version, children need to be trained in the hierarchical discipline of the capitalist industry which is imitated, in their view, by the hierarchical discipline of schools. In Lott's view, government workers seek to maximize their rents, a goal that tends to be resisted by the rest of the nation. Through government schooling, citizens are conditioned to accept more willingly certain kinds of rent-seeking by government workers.

Another argument for public provision of education is based on its redis-

tributive effects. This argument centers on the quality of education supplied by the government. Besley and Coate (1991) argue that public expenditure on education can be viewed as an excellent instrument for redistribution of income from the rich to the poor, even when financed by a head tax. An appropriately chosen quality will separate consumers according to income: low-income consumers will choose public education and high-income consumers will purchase a higher quality private education. Since taxes to support public education fall on everyone either equally (Besley and Coate (1991)) or in some progressive pattern (Ireland (1990)), the resulting redistribution favors those with low incomes. Usher (1977) showed that when the decisions are taken by majority rule, a commodity is more likely to be socialized the greater the inequality of income in the community and less diverse the tastes of individuals for that commodity.

Finally, Flatters and Macleod (1991) considered the mechanism of tax collection services in the presence of corruption in the context of developing countries. The government has a target of tax revenue, and tax collectors are in-charge of collecting this from the tax payers. They showed that some corruption might be condoned as a necessary part of tax administration, as the total cost of tax collection is less with corruption than without it. Our analysis considers a similar issue relating to the provision of education in developing countries. We propose to provide explanations under which private tutoring can be considered as a necessary part of the education system and contributes to accumulating human capital.

The plan of the rest of the paper is the following. Section two is devoted to explaining the stylized facts and general description of the economy. We develop a simple three-agent club theoretic model in section three examining the behavior of the students, the teachers and the government, and analyze the market for private tutoring and its policy implications. This model is extended in section four to a heterogeneous students' environment where the students are differentiated by their income and ability. Section five is devoted to summarizing the results, extensions and their implications from a policy point of view.

2. General Description of the Economy

The economy consists of three sets of agents: the *Students*, the *Teachers* and a *Government*. The main objective of the government is to provide a given level of education to all students. The students are initially assumed to be identical; subsequently, we consider the case when they are differentiated by their incomes and abilities. All teachers are assumed to be identical.⁵ They are hired by the government for the purpose of teaching. It is observed that the salaries of the teachers are lower than the salaries in other comparable professions. This wage differential⁶ is assumed to create a disincentive to work fully, or reduce their efforts at school. The teachers' disinclination to work fully at school is assumed to generate demand for private tutoring, since the students have to do well in their examinations and compete for good jobs in future.

The demand for private tutoring in the economy arises for various reasons. A student may need private tutoring if the teacher does not teach properly in the class. Alternatively a student may demand private tutoring if he is academically weak and needs special attention of the teacher. This occurs when the students are differentiated by their quality or ability. This paper further explores both these reasons for the demand for private tutoring and argues that in the presence of imperfect monitoring, the teachers supply less than their full effort at school, thereby generating a demand for private tutoring outside the school. They supplement their low official salary through the supply of private tutoring. If the former is the cause

⁵ In actual practice, we may observe quality differences in teachers from school to school or in the same school from teacher to teacher. We intend to incorporate this feature in our future research.

⁶ Table I gives a comparison of salaries of the teachers with those of other occupations requiring comparable training. Like teaching, those other professions demand a fairly long period of training and certain degree of responsibility. These data were collected from towns in 15 developing countries in an ILO survey (1991) on "Teachers' salaries" for the years 1982 and 1985. The general impression which emerges from these comparisons is that, in the wage hierarchy, teachers do not occupy the place to which their qualifications and responsibilities should entitle them.

of private tutoring, then some students may drop out of school if they cannot afford to buy this service. The quality of public education will therefore decline due to low effort of the teacher at school. If the latter is the cause of private tutoring, then the high ability students may get their required education from school and the low ability students may seek private tutoring to make up their loss.

It seems appropriate to quote Rasmusen (1990) in the context which says, "While agency theory can be used to explain and perhaps improve government policy, it also is useful to explain the development of many curious private institutions." Following a principal-agent approach, the government in this model can be regarded as the principal who hires an agent; the teacher, for teaching. It is easier for the government to observe the teachers' output than their effort, so it offers a contract to pay the teachers based on output. The output of a teacher in this case not only depends on his effort inside the class, but also on his effort outside the class. The teacher charges a price for his effort outside the class to those who demand it. The supply of effort of a teacher outside the class is more commonly known in the context as private tutoring. The possibility of private tutoring by a teacher outside the class distorts his actual output at school. The government has no way to distinguish between the education acquired from school and education acquired from private tutoring by a student.⁷

The revenue of the government is assumed to be exogenous. There are two ways the government can spend money for the achievement of its objectives: on the salaries of teachers and on monitoring the performance of the teachers at school. We assume that the government has the necessary educational infrastructure (building etc) and need not have to spend any money on the fixed cost. The monitoring expenditure greatly depends on the technology of monitoring⁸ which is assumed

⁷ See Mirrlees (1975), Holmstrom (1979,1982) for an interesting discussion on the literature on moral hazard with hidden action.

⁸ The quality of monitoring technology can be determined by the level of proper infrastructure (road, transport, communication, telephone, computer, etc.) in the economy, expertise and training of the monitors, credibility of the monitors' threat to carry out

to be exogenous to the model. The salaries of the teachers and the monitoring expenditure are endogenous to our models and constitute the choice variables of the government.

The purpose of monitoring is to evaluate the quality of education in public schools and the performance of the teachers. Usually, an inspection team visits the school at random intervals. The team interviews the students, asks questions and uses other methods to test the knowledge of the students about their courses. It must be kept in mind that the status of the teachers in the society is such that the students have no incentive to reveal the true performance of the teachers at school to the monitors. Since the students learn their course materials from private tutoring, they do not have any problem in answering questions from the monitors. The team then makes its evaluation about the performance of the teacher and submits its report to the government, recommending sanctions⁹ when necessary against the teacher. The recommendations of the team are accepted by the government.

We assume that the monitors can only observe the output, here the total education of the students.¹⁰ Though it will be more accurate to consider the effort of the teachers at school, we should realize the difficulty involved in finding it out. Alternatively, the monitors may consider the amount of education acquired from a school to judge the teachers' performance at school instead of the total education of the students. We should also realize the difficulty involved in distinguishing between

against the teachers, etc. If the monitoring technology in an economy is better, then a government needs to spend less on monitoring to achieve the same level of efficiency or output compared to the situation when this technology is not as good.

⁹ Sanctions can take various forms starting from warnings to punishments which may seriously affect remuneration, career advancement and even employment. It may be extreme to assume that the teacher is fired if he is caught shirking, but it does not change substantially the results nor the analysis.

¹⁰ There are several ways to measure the quality of education. See Hanushek (1986) and Card and Krueger (1992) for evidence on the impact of school quality on the rate of return to education, test scores, graduation rates etc.

the education acquired from school and education acquired from private tutoring. Considering all these difficulties, we assumed that the monitors only observe the total level of education of the students.

To simplify the analysis, we formally model the monitoring and enforcement process in the following way. The probability of a teacher being caught shirking and being fired is $q(E, m)$. This probability depends on E , which represents the total level of education acquired by a student both from school and from private tutoring, and m represents the cost of monitoring for a given level of technology in the economy. We assume that the probability function $q(E, m)$ satisfies the following properties: $q_E \leq 0$ if $E < \bar{E}$, $q_m \geq 0$ if $E < \bar{E}$, $q_{EE} < 0$, $q_{mm} < 0$, $q_{Em} = q_{mE} \leq 0$ and $q(E, 0) = 0$, where subscripts denote partial derivatives. $q_E \leq 0$ if $E < \bar{E}$ means that the probability that the teacher is fired decreases as the level of education of a student increases. Similarly, $q_m \geq 0$ if $E < \bar{E}$ means that the probability of a teacher being fired increases with spending on monitoring if the education level of the student is less than a minimum level of education, \bar{E} . In both the cases, if $E > \bar{E}$, then this probability goes to zero. This is because, the teacher has accomplished the objective of the government by achieving the given level of education in the economy. The second-order conditions of the firing technology are assumed to obey the diminishing returns to scale properties. Finally, $q(E, 0) = 0$ means that the teacher is never fired, regardless of educational output achieved, if there is no monitoring by the government.

The next section has been devoted to developing a three-agent model involving the students, the teachers and a planner. It is therefore important to explain the timing of decisions of the agents in the model.

Figure 1 goes here

The timing of decisions of all the agents takes place in one period but in three different stages as shown in Figure 1. In stage 0, the government hires the teachers and decides on their salaries and the money to be spent on monitoring. In stage

1, the teachers, after observing the decisions of the government but before any monitoring is done, make a decision on the level of effort at school. In stage 2, the students observe their education level achieved in school and make a decision on how much private tutoring to buy. If the teacher is not fired, he becomes a monopoly¹¹ supplier of private tutoring and sets the price of the private tutoring he provides to the students. The monopoly power of a teacher to provide private tutoring enables him to extract most of the benefits¹² of free public education meant for the students. In the next section, we solve the model recursively by a backward induction procedure.

3. Private Tutoring: A Club Theory Approach

We observe in developing countries that the teachers generally offer tutoring to students in “batches” like a club. A systematic theory of club goods¹³ started with a seminal paper by Buchanan (1965). A club good is one whose benefits can be jointly consumed by a number of users simultaneously in the sense as a pure public good, but is distinguished from the latter by two essential characteristics: (i) there is congestion in consumption; and (ii) exclusion is economically feasible. The presence of congestion implies that it will be optimal to restrict utilization to a finite group of users, while the existence of an exclusion mechanism means that utilization restrictions can be implemented. We, therefore, adopt a club good

¹¹ We assume that the the students go only to their class teacher for private tutoring. This is due to the obvious reasons of familiarity and ready accessibility. Otherwise, they incur additional transaction costs of obtaining private tutoring from other teachers. This transaction cost may also include certain emotional costs which are due to possible discrimination by the class teacher in the class room or in the class examinations for not going to him for private tutoring. All these together forces a student to go to his class teacher for private tutoring.

¹² See Tullock (1967), Kruger (1974), and Posner (1975) for an excellent discussion on the related subject of rent seeking.

¹³ See Berglas (1976, 1981), Boadway (1980), Cornes and Sandler (1986), Kennedy (1988), Oakland (1972), inter alias. on the theory of club goods.

theoretic approach to model this market.¹⁴

The students benefit from the provision of the number of hours of tutoring services in a club. Their utility, however, declines if the number of students admitted to a club increases. This is due to the congestion from the new members. The teacher is assumed not to discriminate between the students as they are all identical. The teacher makes an offer to the students regarding the club services. The package contains information on the hours of tutoring, number of members, number of clubs and club fee. The students assess the offer and decide whether to accept the offer or reject it. The students can choose only whether they want to become a member of the club or not.¹⁵

The Model

We shall introduce the model by explaining the features of the club. If the student chooses to be a member of a club, he gets $g(T, n)$ where g represents the club service, T represents the hours of tutoring available from a club, and n represents the number of students in a club. It is assumed that $g_1 > 0$ and $g_2 < 0$. The first property indicates that the benefit from a club increases with the number of hours of private tutoring, and the second indicates that the benefit from a club decreases as its size increases. This is due to the congestion¹⁶ effect which increases with the number of students in a club. The teacher offers a package $\{T, p, n, b\}$ to the students where p represents the club price of the tutoring services available from a club and b represents the number of clubs. Other notations carry their usual meanings.

¹⁴ Biswal (1993) modeled the provision of such tutoring as a purely private good, sold on a one-to-one basis by the teacher to each student.

¹⁵ In Biswal (1993), student's have more power as they choose the amount of tutoring and the teacher as a monopolist fixes a price.

¹⁶ Congestion in private tutoring could arise due to decrease in the average time devoted to each student with the increasing number of students in the club. So, as the number of students in a club increases, it brings in more disutility to the existing members.

The rest of this section is devoted to characterizing the role of the students, the teacher and the planner.

3.1 The Students

The students are interested in their present and future consumption. Education acquired in the present period increases their future earning power and hence their future consumption. Investment in education can be interpreted as the only form of intertemporal savings for students. Like Barham (1991), we also assume that there is no capital market and hence the students can not save and borrow against the future income through the capital market. This is a reasonable assumption as the existence of capital market to finance primary or secondary education is known to be not present anywhere in the world and especially in a developing world. The students receive an exogenous income from their parents which they spend on their present consumption and education. Thus there is a trade off between spending on present consumption and on acquiring education. Although they get free education¹⁷ from the school, in the event of shirking by the teachers in the classroom, they must purchase private tutoring if they want to acquire 'full' education.

The students have the choice of accepting the offer and becoming a member of the club, or, rejecting it and not becoming a member. In the latter case, the students depend on school education only. We do not consider the possibility of getting education from any source other than the school and the private tutoring by the class teacher. The students receive the same amount of income from their parents and they have the same ability. They make a choice between their present consumption and their education from private tutoring.¹⁸ If the student takes

¹⁷ In India, for example, there is no entrance fee to a public school. However, there is a school tuition fee in the range of rupees (Rs) 5-10 per month (i.e. approximately \$ 0.20 - 0.40 per month) at the primary and secondary school levels. For many poor students, this fee is exempted by the government. So, we assume that the cost of entering a public school is almost equal to zero.

¹⁸ We can also consider a household utility function of Stiglitz (1974) which depends on the consumption of the family and the education of the children. In either case, it will neither

tutoring then the utility attained is:

$$u(C, \gamma E)$$

where

$$C + p = Y \tag{i}$$

$$E = \{S \cdot e + g(T, n)\} \tag{ii}$$

$$C \geq 0, E \geq 0, \text{ and } 0 \leq e \leq 1$$

where C is the amount of present consumption, E is the amount of total education which includes education from school and from tutoring, Y is the income of a student, S is the amount of education from school, e is the effort of the teacher at school, γ is the return to education, and other notations carry their usual meaning. Equation (i) represents the student's budget constraint. He spends money on his present consumption and on private tutoring. Equation (ii) represents the total education of a student from school and from private tutoring.

Assumptions:

- (i) $u_1 \geq 0, u_2 \geq 0,$
- (ii) $u_{11} \leq 0, u_{22} \leq 0$ and $u_{12} = u_{21} \geq 0$
- (iii) $u(C, 0) < u(C, E) \forall E > 0,$
- (iv) For $E \geq 0, \lim_{C \rightarrow \infty} u_C(C, E) = 0$ and for $E \geq 0, \lim_{C \rightarrow 0} u_C(C, E) = \infty.$

Assumption (i) and (ii) state that utility is increasing and concave in both the arguments. (iii) states that education is a desired commodity. Finally, assumption (iv) ensures that consumption is neither infinite nor zero.

We can rewrite the student's unconstrained utility function as:

$$u(Y - p, \gamma\{S \cdot e + g(T, n)\}). \tag{1}$$

change the results nor the analysis.

The condition which guarantees the student to join a club is the following:

$$u(Y - p, \gamma\{S \cdot e + g(T, n)\}) \geq \bar{u}(Y, \gamma \cdot S \cdot e). \quad (2)$$

Equation (2) implies that a student chooses to become a member of a club if he derives at least as much utility from a club as he derives from not joining a club. We know from the assumption (*iii*) that education is a desirable commodity and the student's education as a member is always greater than his education as a non-member. It is, therefore, necessary for the teacher to offer a package to the students which increases their utility by overcompensating the loss from decrease in private consumption by increasing their education. We know that the teacher is a monopolist as far as the provision of the club services is concerned. Since there are no competing firms, the lower bound on the utility the monopolist offers is the reservation utility of the student. Equation (2), therefore, holds with equality which we can write as:

$$u(Y - p, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}(Y, \gamma \cdot S \cdot e) = 0. \quad (2')$$

Equation (2') implies that the utility loss to the students from the payment of club fee (or the loss of private consumption) is equal to the utility gain to them from additional education through tutoring.

3.2 The Teacher

The objective of the teacher is to maximize income from the private tutoring and he may be offering tutoring in one or more than one club of optimum size. We assume that there are N students in a class and n students per club given that they are all homogeneous, and there are b clubs. If $b = 1$, then $N = n$. More generally,

$$N - n \cdot b \geq 0. \quad (3)$$

Equation (3) implies that the total number of students in a class is at least as great as the number students in all the clubs. If $N = n^* \cdot b^*$ (where $*$ implies optimum

size), then there is no integer problem.¹⁹ We can, therefore, rewrite equation (3) as:

$$N - n \cdot b = 0. \quad (3')$$

The teacher also faces a time constraint. He has to split the total time available for private tutoring optimally among all the clubs. The total time available to a teacher has been normalized to unity. He spends S hours of time at school and devotes $1 - S$ hours to supply private tutoring to the b clubs. We can represent this as:

$$1 - S - b \cdot T = 0. \quad (4)$$

Equation (4) implies that the teacher uses all his after-school hour time on private tutoring. This is mainly due to the following assumptions: (i) the more time the teacher allocates to a club, the more revenue he receives, and (ii) the teacher derives no disutility from his labor supply. So, this constraint holds with equality.

The teacher makes his decisions in two stages. In stage 1, he decides how much effort, e , to be supplied at school. In stage 2, he decides the optimal package $\{T, p, n, b\}$ to offer to the students in order to attract them for private tutoring. Since these decisions come in sequence, variables $\{T, p, n, b\}$ depend on his effort, e , at school. We solve this problem by backward induction in a two stage approach.

3.2.1 Stage 2

We assumed that the teacher's utility in this stage depends only on his total income which he earns from two sources; the salary from the government and the

¹⁹ See Kennedy (1988) on integer problem. In this case, we do not face an integer problem due to the single ownership of all the clubs. The teacher owns all the clubs and all students in the class go to him for private tutoring. If there are some residual students, then the teacher can easily allocate them in such a way that the congestion in all the clubs are equal and the students receive the same hours of tutoring.

income from private tutoring. The teacher is assumed to derive no disutility from his labor supply.²⁰

We summarize the teacher's problem in the following way:

$$\max_{\{T,p,n,b\}} V(\bar{w} + n \cdot b \cdot p) \quad (5)$$

subject to the constraints:

$$u(Y - p, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}(Y, \gamma \cdot S \cdot e) = 0 \quad (2')$$

$$N - n \cdot b = 0 \quad (3')$$

$$1 - S - b \cdot T = 0 \quad (4)$$

$$T > 0, p > 0, n > 0, \text{ and } b > 0$$

where \bar{w} is the teacher's salary from the government. All other notations carry their usual meaning. The teacher's utility function has the usual concavity properties: $V' > 0$ and $V'' < 0$. We have already explained the constraints of the teacher previously. Finally, T , p , n , and b are strictly positive. If any one of these variables is not positive, then the teacher can never earn positive income from private tutoring.

We eliminate b by substituting $b = \frac{N}{n}$ from the constraint (3') into the teacher's problem. So the teacher is left with only three choice variables, $\{T, p, n\}$. We can also rewrite the constraint (4) as:

$$1 - S - \frac{N}{n} \cdot T = 0 \quad (4')$$

We write the full Lagrangean with the constraints (2') and (4') as:

$$\begin{aligned} \max_{\{T,p,n\}} & V(\bar{w} + N \cdot p) + \lambda_1 [u(Y - p, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}(Y, \gamma \cdot S \cdot e)] \\ & + \lambda_2 [1 - S - \frac{N}{n} \cdot T] \end{aligned} \quad (6)$$

²⁰ This does not change the basic results and the analysis of the model. See Appendix 1 for the case where the teacher has a labor-leisure choice.

λ_1 and λ_2 are the Lagrangean multipliers to the constraints (2') and (4'). λ_1 can be interpreted as the shadow value of the student's utility, and λ_2 as the shadow value of the teacher's time which he devotes to private tutoring.

The first order conditions are:

$$\lambda_1 \cdot u_2 \cdot \gamma \cdot g_1 - \lambda_2 \cdot \frac{N}{n} = 0 \quad (7.a)$$

$$V' \cdot N - \lambda_1 \cdot u_1 = 0 \quad (7.b)$$

$$\lambda_1 \cdot u_2 \cdot \gamma \cdot g_2 + \lambda_2 \cdot \frac{N}{n^2} \cdot T = 0. \quad (7.c)$$

Using equations (7.a) and (7.b), and equations (7.b) and (7.c), we can derive the following optimal tutoring rule (OTR) and optimal congestion rule (OCR):

$$V' n \frac{u_2}{u_1} \gamma g_1 = \lambda_2 \quad (OTR)$$

$$V' n \frac{u_2}{u_1} \gamma g_2 = -\lambda_2 \frac{T}{n}. \quad (OCR)$$

OTR states that the marginal benefit to the teacher from providing tutoring to all the students in a club must be equal to his shadow cost of time. In other words, the teacher allocates time to a club in such way that the marginal benefit of time must be equal to its shadow cost to the teacher. Similarly, OCR states that the loss of utility to the teacher due to congestion from adding one more student to a club must be equal to the teacher's shadow value of time multiplied by the congestion factor represented by average time devoted to each student in a club.

We can rearrange to write the optimal conditions as:

$$V' \cdot N \cdot \frac{u_2}{u_1} \cdot \gamma \cdot (g_1 \cdot T + g_2 \cdot n) = 0. \quad (8)$$

By assumption on the properties of the teacher's and the students' utility functions, $V' N \frac{u_2}{u_1} \gamma$ is not equal to zero. We can write (8) as:

$$g_1 \cdot T + g_2 \cdot n = 0. \quad (8')$$

Equation (8') can be interpreted as the optimal club rule and is instrumental in determining the optimal values of T and n . This says that the marginal benefit to the teacher from increasing tutoring time by one unit must be equal to the marginal loss to the teacher due to the congestion from admitting one more student to the club.

We have three equations (2'), (4') and (8') to solve for three unknowns T , p and n . However, it can be seen that the equations (4') and (8') involve only the variables T and n . We can, therefore, solve for T and n from the equations (4') and (8'). We then substitute the value of n into the equation (3') to solve for b .

The optimum number of students in a club, n , the optimum number of hours of tutoring, T , and optimum number of clubs, b , depend only on the total number of students, N , and the time available after the school hour, $(1 - S)$. These do not depend on the effort, e , of the teacher at school.²¹ We can substitute the value of T and n into the equation (2') to solve for p which depends on the effort, e , of the teacher. We can write the solutions as:

$$T^* = T(S, N) \quad (9.a)$$

$$n^* = n(S, N) \quad (9.b)$$

$$b^* = b(S, N) \quad (9.c)$$

$$p^* = p(e, Y, \gamma, S, N). \quad (9.d)$$

We can see from the equations (9.a), (9.b) and (9.c) that the variables T , n and b do not depend on the effort of the teacher, or the income and the ability of the students. These variables only depend²² on S (or, $1 - S$) and N which are exogenous

²¹ This result can change if leisure is introduced into the model and the teacher derives more utility from leisure with the increasing effort at school.

²² If $g = T^\alpha - n^\beta$ and $1 - S - \frac{N}{n}T = 0$, then we can solve: $T = \left[\left(\frac{1-S}{N} \right) \left(\frac{\alpha}{\beta} \right)^{\frac{1}{\beta}} \right]^{\frac{\beta}{\beta-\alpha}}$, $n = \left[\frac{\alpha}{\beta} T^\alpha \right]^{\frac{1}{\beta}}$ and $b = \frac{N}{n}$. We can check that $T_N < 0$, $T_{(1-S)} > 0$, $n_N < 0$, $n_{(1-S)} > 0$, $b_N > 0$ and $b_{(1-S)} > 0$ with the restriction that $(\beta - \alpha) > 0$.

to the model. This is mainly due the fact that the teacher maximizes his income by devoting all his after school time to tutoring and he does not discriminate between the students. However, the membership fees of a club, p , depends on e , Y , and γ . We check that p depends on these variables in the following ways:

$$\frac{dp}{de} = \frac{\gamma \cdot S(u_2 - \bar{u}_2)}{u_1} < 0 \quad (10.a)$$

$$\frac{dp}{dY} = \frac{(u_1 - \bar{u}_1)}{u_1} > 0 \quad (10.b)$$

$$\frac{dp}{d\gamma} = \frac{(u_2(Se + T) - \bar{u}_2Se)}{u_1} < 0. \quad (10.c)$$

Equation (10.a) implies that the effect of effort on club fee is negative. This is because $(u_2 - \bar{u}_2)$ is negative. This follows from the decreasing marginal utility assumption. This means that the marginal utility of a student with education who does not get private tutoring is higher than the one who gets it when the amount of school education is same the for both. Thus the teacher has to reduce the price of tutoring when he supplies high effort at school in order to attract students for private tutoring.

We can also similarly interpret the equation (10.b). The teacher charges a higher price for the private tutoring when the income of the students increase. This uses the fact that $(u_1 - \bar{u}_1)$ is positive. We know that the income of the students are identical. The student who demands tutoring is left with less money for his private consumption and hence his marginal utility from private consumption is higher than the one who does not demand tutoring. This result is the same as the result we derived in chapter three.

Equation (10.c) implies that the effect of ability on club fees is negative. This uses the fact that $(u_2(Se + T) - \bar{u}_2Se)$ is negative. This is due to the fact that the marginal utility of a student with education who does not get private tutoring is higher than the one who gets it. And as the ability of a student increases, his marginal utility from school education also decreases. Thus the teacher charges a lower price to a higher ability student in order to attract him to the club.

3.2.2 Stage 1

The teacher in this stage makes his choice of effort, e , at school. He observes his salary from the government, \bar{w} , income from the private tutoring if not fired, Np , alternative income if fired, \hat{w} , and the monitoring, m . He then chooses his effort, e , to maximize his expected utility, EV . We formulate the problem in the following way:

$$\max_{\{e\}} EV = \max_{\{e\}} [1 - q(E, m)]V(\bar{w} + N \cdot p(e)) + q(E, m)V(\hat{w}) \quad (11)$$

subject to

$$E = g(\cdot) + S \cdot e \quad (i)$$

$$\hat{w} > 0 \quad (ii)$$

where \hat{w} is the alternative wage of the teacher if fired and is assumed to be non-negative. All other notations carry their usual meaning. Equation (11) implies that the expected utility of the teacher is equal to his utility from his total income, which includes his salary and income from tutoring, with the probability $[1 - q(E, m)]$ that he is not fired and his utility from the alternative wage which he gets with the probability $q(E, m)$ that he is fired. The expected utility function has the following properties: $EV' > 0$ and $EV'' < 0$.

The first order condition is:

$$[1 - q(\cdot)] \left(V' \cdot N \cdot p_e \right) - q_E \cdot S \left(V(1) - V(2) \right) = 0. \quad (12)$$

This implies that if the teacher increases his effort in the school, the utility loss from the decrease in income from tutoring must be equal to the utility gain due to the fall in the probability of the teacher being fired, and vice-versa. We can rearrange equation (12) and derive e as:

$$e^* = e(\bar{w}, \hat{w}, m). \quad (13)$$

The properties of the effort function are: $e_{\bar{w}} > 0$, $e_{\hat{w}} < 0$ and $e_m > 0$. The first inequality implies that as the salary of the teacher increases, effort at school increases. An increase in salary increases the cost of getting fired and hence induces the teacher to reduce its probability by increasing effort. The second inequality implies that as the alternative wage increases, the supply of effort at school decreases. In other words an increase in the alternative wage reduces the cost of getting fired and hence induces the teacher to supply less effort at school. The third inequality implies that as the monitoring by the government increases, the effort of the teacher at school increases. This is due to the fact that the increase in monitoring increases the cost of getting fired and hence it induces the teacher to supply higher effort at school to reduce the probability.

We substitute $e(\bar{w}, \hat{w}, m)$ into the teacher's expected utility function in equation (11) and write his indirect expected utility function (*IEV*) as:

$$IEV = v(\bar{w}, \hat{w}, m). \quad (14)$$

We differentiate *IEV* with respect to \bar{w} , \hat{w} and m using the envelope rule and obtain the following results: $v_{\bar{w}} > 0$, $v_{\hat{w}} < 0$, and $v_m > 0$. These results are quite intuitive. Increase in salary and in alternate wage increases the indirect utility of the teacher and monitoring reduces it.

Proposition:1 *In the absence of monitoring by the government, the teachers will supply zero effort in the school, i.e., $e(\bar{w}, 0) = 0$.*²³

3.3 The Government

The government is interested in the education of the students and is provided by the government free of cost to everybody. The government tries to achieve this objective with the least possible social cost. We follow a cost minimization approach

²³ Proof of Proposition 1 can be seen from Appendix 2.

similar to Flatters and MacLeod (1991) in solving the government's problem. The direct cost of education, which is borne by the government, includes the salary of the teachers and the cost of monitoring. In addition to these, there is also an indirect cost of education which is borne by the students themselves to pay for the private tutoring.

The precise policy problem we characterize is the one in which the government chooses an optimal combination of the salary of the teachers, \bar{w} , and the monitoring cost, m , which guarantees every student a minimum level of education. The government is constrained as well by the need to guarantee the teachers an expected utility at least equal to their reservation utility, in order to ensure that they will be prepared to work as teachers rather than in some alternative employment.

The government's problem can be formalized in the following way:

$$\min_{\{\bar{w}, m\}} c(\bar{w}, m) = \bar{w} + m + N \cdot p(\bar{w}, m) \quad (15)$$

subject to the constraints:

$$g(\cdot) + S \cdot e(\bar{w}, m) \geq \bar{E} \quad (16.a)$$

$$v(\bar{w}, m) \geq RU \quad (16.b)$$

$$\bar{w} \geq 0, \text{ and } m \geq 0.$$

Equation (15) is the total cost of achieving a level of education through both school and private tutoring. The government spends $\bar{w} + m$ and the students spend $N \cdot p(\bar{w}, m)$ to get $g + S \cdot e$ level of education. If the government can observe perfectly the cost of tutoring then the above cost function is valid. Otherwise the cost function will include only that portion of the cost of education which is borne by the government. We can accordingly characterize the government's problem.

Equation (16.a) is an iso-education constraint which guarantees that the actual education level of a student is at least as great as the minimum level of

education, i.e., \bar{E} . It can be seen here that the government's choice variables do not affect the amount of education from private tutoring. The government's choice variables, however, affect the cost of private tutoring and hence can influence the aggregate cost of education to the society. Equation (16.b) is the participation constraint of the teacher. It guarantees that the indirect expected utility of a teacher should be no less than his reservation utility. \bar{w} and m are assumed to satisfy the non-negativity requirements. μ_1 and μ_2 are the Lagrangean multipliers for the constraints (16.a) and (16.b) respectively. μ_1 can be interpreted as the shadow cost of the minimum level of education in the economy. μ_2 can be interpreted as the shadow cost of the reservation utility of a teacher.

We can solve for four unknowns \bar{w} , m , μ_1 and μ_2 from the above problem. The optimum solutions \bar{w}^* and m^* minimize the total cost of education. Every student gets their minimum education and the teachers get, at least, their reservation utility.

We follow a diagrammatic approach to analyze the government's problem. The iso-education constraint can be represented by a downward sloping line, E_0 , since $\frac{d\bar{w}}{dm}|_E < 0$. The incentive constraint can be represented by an upward sloping line, v_0 , as $\frac{d\bar{w}}{dm}|_v > 0$. If the government cannot observe the cost of tutoring, then the cost function can be represented as a downward sloping 45^o line since $\frac{d\bar{w}}{dm}|_c = -1$. If the government can observe the amount of expenditure on tutoring, then the shape of the cost function can be shown as: $\frac{d\bar{w}}{dm}|_c = -\frac{1+N \cdot p_m}{1+N \cdot p_w} > 0$. This says that the slope of the cost function could be negative or positive. This ambiguity arises due to the trade off between the government's cost and the students' tutoring cost.

Considering the two cases of perfect observability and non-observability of the tutoring cost by the government, and using the first order conditions, we analyze the problem in the following three possible ways:

Case I: $\mu_1 > 0$ and $\mu_2 > 0$ (both constraints satisfied)

Case II: $\mu_1 > 0$ and $\mu_2 = 0$ (only the iso-education constraint satisfy)

Case III: $\mu_1 = 0$ and $\mu_2 > 0$ (only the incentive constraint satisfy)

Case I

This case can be analyzed in two different diagrams, Figure 2 and Figure 3, due to the possibility of having cost functions with different slopes.

Figure 2 goes here

Figure 3 goes here

In these diagrams, we have an interior solution at point x_0 where both the constraints satisfy. We can see that as long as the slope of cost function, whether downward sloping as the case in Figure 2 or upward sloping as the case in Figure 3, lies in between the slope of the iso-education constraint and the incentive constraint. x_0 is an interior solution and c_0 is the minimum cost for the planner. The solution to this problem is $\{\bar{w}^*, m^*\}$. In this case, since both constraints bind, the values of both the multipliers are positive, i.e., $\mu_1 > 0$ and $\mu_2 > 0$.

The students receive the minimum level of education set by the planner and the teachers receive their reservation utility. If the cost function is tangent to the iso-education line below the x_0 point when downward sloping or tangent to the incentive line below the x_0 point when upward sloping, then these points cannot represent the solutions. This is due to the fact that these solutions cannot satisfy the incentive constraint or the iso-education constraint respectively. The cost function has to rise until it passes through x_0 point. We again get back to the cases explained in Figure 2 and 3 respectively.

Case II

Case II has been analyzed in Figure 4.

Figure 4 goes here

In this case, the slope of the cost function is greater than the slope of iso-education constraint at the point x_0 . This implies that the opportunity cost of providing education is higher than the shadow value of the minimum education. In this case, c_0 will shift backward to c_1 until it is tangent to iso-education line at point x_1 which is a solution to the problem. This satisfies the iso-education constraint. However, the incentive constraint is not binding. The teachers are better off in this case since they are on the utility level v_1 which is higher than v_0 . The utility level of the teachers can be reduced only by increasing the cost which is surely not attractive for the government since the education target has already been achieved at a lower cost.

The optimal solution is represented by \bar{w}^* and m^* , which correspond to the point of equilibrium x_1 . In this case, the values of the multipliers are: $\mu_1 > 0$ and $\mu_2 = 0$. We know that the monitors observe only the total education level of the students and in actual practice more education comes from private tutoring due to low monitoring at the point x_1 compared to x_0 . We can check that if the iso-education curve is linear, then we have a corner solution and the students receive their full education from private tutoring. This explains no monitoring equilibrium.

Case III

Case III has been analyzed in Figure 5. In this case, the government observes the cost of tutoring.

Figure 5 goes here

If the cost of tutoring is observable and we still get a downward sloping cost function, then the analysis does not differ from what is already done in Case II. If the cost function is upward sloping, then our analysis changes. We can see that the slope of the cost function is higher than the slope of the incentive constraint at the point x_0 . However, it can be verified that the cost function c_0 cannot be the lowest cost to achieve the minimum education \bar{E} . In this case, c_0 will shift to c_1 until it is tangent to the incentive constraint at point x_1 which is the solution to the problem.

This satisfies the incentive constraint. In this case, the iso-education constraint is not binding. The students are better off since they are on a higher education level, E_1 .

The optimal solution is represented by \bar{w}^* and m^* , which correspond to the point of equilibrium x_1 . This is an example of a monitoring policy. The students receive a higher level of education from school compared to other cases as the teachers supply more effort due to higher monitoring. The students benefit by acquiring a higher level of education from school, while the education from private tutoring remains the same. So, the students are on a higher level of education E_1 , which is higher than their minimum education, E_0 . In this case, the teacher's incentive constraint only binds as he gets his reservation utility. The values of both the multipliers are: $\mu_1 = 0$ and $\mu_2 > 0$. We can see from Case III that the teachers get utility equivalent to their reservation utility even though they receive a higher salary. This is due to the fact that the government resorts to higher monitoring and it reduces the utility of the teachers. In order to guarantee the reservation utility to the teachers, the planner must give a higher salary to the teachers under a monitoring regime.

Proposition 2: *In a monitoring regime, increasing reliance on monitoring causes the government to pay a higher salary to the teacher, i.e., $\frac{d\bar{w}}{dm}|_v > 0$.*²⁴

Proposition 2 proves that as the monitoring in the economy increases, the salary of the teachers also increases. This is required to guarantee teachers their reservation utility in order to ensure that they will be prepared to work as teachers rather than in some alternative employment. Thus, in a monitoring regime, teachers' official salaries are higher.

In most of the developing countries, due to lack of proper infrastructure and instruments, monitoring is a costly undertaking. The cost of tutoring in most

²⁴ Proof of Proposition 2 can be seen from Appendix 3.

instances is not observable by the government. An important policy problem that emerges is the trade off faced by the government between granting funds for teachers' salaries and expenses on monitoring teachers' effort in school. Since monitoring is expensive, the natural outcome is the emergence of private tutoring. The teachers provide tutoring and the students make up their loss from school. The teachers by providing tutoring make up their low salary in most of the developing countries. We now move to analyze the case of heterogeneous students in the next section.

4. Heterogeneous Students

We now generalize the case where the students are heterogeneous due to differences in their income, and due to differences in their ability. We assume that the teachers can observe²⁵ the income and the ability of the students. The students, whether homogeneous or heterogeneous, come to the same club. When the students are heterogeneous, to design a package which will be optimal to all the students in the club is difficult. However, the assumption of perfect observability of the students' characteristics enables the teacher to overcome this problem.

4.1 Income Heterogeneity

4.1.1 *The Students*

We assume that the students are differentiated only by their income. The income of the students are represented by Y^i where $i = 1$ and 2 . 1 stands for poor and 2 stands for rich. There are N students of which N^1 are poor and N^2 are rich. We assume that the teacher offers all the students a similar package, $\{T, p, n\}$. The purpose of this exercise is to determine whether there are any condition under which a uniform offer by the teacher to a group of heterogeneous students is optimal. If

²⁵ The teacher can observe the income and the ability of the students due to the following reasons. (i) The teachers are mostly from the same locality so that they have some idea about the student's family status. (ii) The parents provide family information to the teachers at the time of admission of their children. (iii) The teacher can also easily observe the ability of the students from their class room or examination performance.

the teacher offers a uniform package, we call this a uniform club service. If the teacher offers a nonuniform package, we call this a discriminating club service. We shall consider both the cases separately.

We can write the student's membership constraint in the following way:

$$u^i(Y^i - p, \gamma\{S \cdot e + g(T, n)\}) \geq \bar{u}^i(Y^i, \gamma \cdot S \cdot e). \quad (17)$$

It is useful to check at this stage the effect of the teacher's offer on the utility of a student when his income changes.

Proposition:3 *If for a given value of $T > 0$, $p > 0$, $n > 0$, and $Y_j > Y_i$,*

$$u^i(Y^i - p, \gamma\{S \cdot e + g(T, n)\}) = \bar{u}^i(Y^i, \gamma \cdot S \cdot e),$$

then

$$u^j(Y^j - p, \gamma\{S \cdot e + g(T, n)\}) > \bar{u}^j(Y^j, \gamma \cdot S \cdot e).$$

Proposition 3 proves²⁶ that if a student with a given income derives the same utility from a club as his reservation utility, then his utility from the membership of a club will be strictly greater than his reservation utility as his income increases.

4.1.2 The Teacher

a. Uniform Club Service

We assume that the teacher offers a uniform club package $\{T, p, n\}$ to both types of students. The teacher's problem, when he provides a uniform club service to both types, can be written as:

$$\max_{\{T, p, n, b\}} V(\bar{w} + n \cdot b \cdot p) \quad (18)$$

²⁶ Proof of Proposition 3 can be seen from Appendix 4.

subject to the constraints:

$$u^1 (Y^1 - p, \gamma\{S \cdot e + g(T, n)\}) \geq \bar{u}^1 (Y^1, \gamma \cdot S \cdot e) \quad (19.a)$$

$$u^2 (Y^2 - p, \gamma\{S \cdot e + g(T, n)\}) \geq \bar{u}^2 (Y^2, \gamma \cdot S \cdot e) \quad (19.b)$$

$$(N^1 + N^2) - n \cdot b = 0 \quad (19.c)$$

$$1 - S - b \cdot T = 0 \quad (19.d)$$

$$N = N^1 + N^2 \quad (19.e)$$

$$T > 0, p > 0, n > 0, \text{ and } b > 0.$$

All notations carry their usual meaning.²⁷ Equations (19.a) and (19.b) are the membership constraints of the poor and the rich students respectively. These imply that the student is attracted to a club if his utility as a member is at least as great as his utility as a non-member. Equations (19.c) and (19.d) have similar interpretations as already discussed in the homogeneous section. Equation (19.e) implies that the total number of students is equal to the total number of rich and the total number of poor students. Finally, T , p , n , and b are strictly positive. This guarantees positive revenue to the teacher from tutoring.

We can substitute the constraints (19.c) for b and (19.e) for $(N^1 + N^2)$ into the objective function and the other constraints, and rewrite the full Lagrangean as:

$$\begin{aligned} \max_{\{T, p, n, \}} \mathbf{L} = & V(\bar{w} + N \cdot p) \\ & + \lambda_1 [u^1 (Y^1 - p, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}^1 (Y^1, \gamma \cdot S \cdot e)] \\ & + \lambda_2 [u^2 (Y^2 - p, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}^2 (Y^2, \gamma \cdot S \cdot e)] \\ & + \lambda_3 [1 - S - \frac{N}{n} \cdot T]. \end{aligned} \quad (20)$$

²⁷ We use superscripts to denote the types of student and subscripts to denote the derivatives for notational convenience. This notational rule is not applicable to the lagrangeans.

λ_1 and λ_2 are the Lagrangean multipliers to the constraints (19.a) and (19.b), and can be interpreted as the shadow value of the poor and the rich student's reservation utility. λ_3 is the Lagrangean to the time constraint (19.d) and can be interpreted as the shadow value of the teacher's time devoted to private tutoring. We can derive and rearrange the first order conditions to write the following optimal rules:

$$V' n \frac{u_2^1}{u_1^1} \gamma g_1 + \lambda_2 \frac{n}{N} \left(\frac{u_2^2}{u_1^2} - \frac{u_2^1}{u_1^1} \right) u_1^2 \gamma g_1 = \lambda_3 \quad (OTR)$$

$$V' n \frac{u_2^1}{u_1^1} \gamma g_2 + \lambda_2 n \left(\frac{u_2^2}{u_1^2} - \frac{u_2^1}{u_1^1} \right) u_1^2 \gamma g_2 = -\lambda_3 \frac{T}{n}. \quad (OCR)$$

We can see that these two optimal rules are different from the one we already derived in the homogeneous section, by the extra second term in the left hand side. These can be interpreted as the shadow value of the net utility gain to the rich students due to uniform club service. Thus, there is a net loss to the teacher as he is not able to extract the full consumer surplus from the high income students. This takes us to the case where the teacher adopts a discriminatory policy in providing club services to different types of students.

b. Discriminatory Club Service

In this section, the teacher discriminates between the rich and the poor students. We assume first that the only form of discrimination is from charging different prices for the same service. We then consider discrimination in the form of providing a completely different service package to both types of students. We call the first type of discrimination *partial discrimination* and the latter type *full discrimination*. We retain the assumption that the teacher has no problem in distinguishing between the types, and that one type of student can not mimic another type. In other words, we encounter no self selection problem in the sense of Stiglitz (1982).

i. Partial Discrimination

We can write the full Lagrangean of the teacher's problem in the case of partial discrimination as:

$$\begin{aligned}
\max_{\{T, p^1, p^2, n\}} & V(\bar{w} + N^1 p^1 + N^2 p^2) \\
& + \lambda_1 [u^1 (Y^1 - p^1, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}^1 (Y^1, \gamma \cdot S \cdot e)] \\
& + \lambda_2 [u^2 (Y^2 - p^2, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}^2 (Y^2, \gamma \cdot S \cdot e)] \\
& + \lambda_3 [1 - S - \frac{N^1 + N^2}{n} \cdot T]. \tag{21}
\end{aligned}$$

p^1 and p^2 are the club prices charged to the poor and the rich students respectively. All other notations carry their usual meaning.

We can solve the above problem similarly and write the following optimal conditions as:

$$V' \left(\frac{N^1}{N^1 + N^2} \right) n \frac{u_2^1}{u_1^1} \gamma g_1 + V' \left(\frac{N^2}{N^1 + N^2} \right) n \frac{u_2^2}{u_1^2} \gamma g_1 = \lambda_3 \tag{OTR}$$

$$V' \left(\frac{N^1}{N^1 + N^2} \right) n \frac{u_2^1}{u_1^1} \gamma g_2 + V' \left(\frac{N^2}{N^1 + N^2} \right) n \frac{u_2^2}{u_1^2} \gamma g_2 = -\lambda_3 \frac{T}{n}. \tag{OCR}$$

We have been able to derive the modified optimal rules. In this case, since both types of students come to the same club, the teacher's marginal benefit is derived from offering tutoring services to both types weighted by their proportion in the total number of students. We also derive the optimal club rule in this case as:

$$g_1 \cdot T + g_2 \cdot n = 0. \tag{22}$$

This condition is exactly the same as the optimal club condition derived in equation (8'). The optimum club condition says that the marginal benefit from a change in tutoring time, T , is exactly equal to the marginal congestion cost from a change in the number of students, n . We can also solve for the teachers choice

variables in the same way as in the homogeneous section and will have the same properties.

In this case, we worked out a case of discriminatory club service when the teacher charged different prices for the same service. This is a standard case of price discrimination where the teacher charges different prices for the same service. This is possible because the teacher can observe the income of the students. This is consistent with the observation that teachers do allow different types of students to attend the same tutorial class, but charge different prices.

ii. Full Discrimination

In the case of full discrimination, the teacher discriminates between different types of students by charging different prices and providing different service packages. The teacher is assumed to offer a club package, $\{T^i, p^i, n^i\}$, where $i = 1$ and 2 . In the partial discrimination case, the teacher was providing the same service to different types of students in the same club. In the present case, he provides tutoring in different types of clubs. Thus, the teacher has to devote separate time to each type from the total time available for tutoring. We assume that the teacher gives z^1 hours of time to type 1 students and z^2 hours of time to type 2 students such that the following time constraint is satisfied:

$$1 - S - z^1 - z^2 = 0 \quad (23)$$

where $z^i = \frac{N^i}{n^i} T^i$.

We use a different approach to solve this problem.²⁸ First, we solve the teacher's maximization problem by providing tutoring to type i students given the time available to that type, z^i , as defined above. This gives us a teacher's indirect

²⁸ If we solve this problem in the same way as in the partial discrimination case, then we are left with five equations (two optimal club rules, one time constraint and two utility constraints) and six unknowns, $\{T^i, p^i, n^i\}$. The problem will be under determined.

utility function, V^i , as a function of z^i for each type of student, i . Second, we find out the maximum utility to the teacher which satisfies the time constraint, (23).

Step I

In this step, the teacher is assumed to maximize his income from each type of student. We can formulate the Lagrangean in the following way:

$$\begin{aligned} \max_{\{T^i, p^i, n^i\}} \mathbf{L}(T^i, p^i, n^i) = & V^i(N^i \cdot p^i) \\ & + \lambda_i [u^i(Y^i - p^i, \gamma\{S \cdot e + g(T^i, n^i)\}) - \bar{u}^i(Y^i, \gamma \cdot S \cdot e)] \\ & + \chi_i [z^i - \frac{N^i}{n^i} T^i]. \end{aligned} \quad (24)$$

We can solve this problem and derive the optimal rules as:

$$V^{i'} n^i \frac{u_2^i}{u_1^i} \gamma g_1 = \chi_i \quad (OTR)$$

$$V^{i'} n^i \frac{u_2^i}{u_1^i} \gamma g_2 = -\chi_i \frac{T^i}{n^i}. \quad (OCR)$$

In the case of full discrimination, we could also derive the optimal rules for both types, since the teacher provides different services to different students. These rules have similar interpretations. Using both rules, we derive the optimal club rule as:

$$g_1 \cdot T^i + g_2 \cdot n^i = 0. \quad (25)$$

Equation (25) is the same club rule we derived in the previous sections, and it holds for each group separately. This says that the marginal benefit to the teacher by increasing tutoring time by one unit must be equal to the marginal loss to the teacher due to the congestion from admitting one more student to the club. We solve for T^i , n^i from equations (25) and the time constraint for each group, $z^i = \frac{N^i}{n^i} T^i$ as:

$$T^{i*} = T^i(z^i, N^i) \quad (26.a)$$

$$n^{i*} = n^i(z^i, N^i). \quad (26.b)$$

Using the utility constraint of the student, i , we solve for p^i as:

$$p^{i*} = p^i(z^i, N^i, Y^i, \gamma, e). \quad (26.c)$$

In this case, we showed that the optimal tutoring time and the optimal number of students in a club depends on the time allocated to each type, z^i , and the number of students in each type, N^i . Optimal price, p^i , depends on the students' income and ability, time allocated to each type, z^i , and the effort of the teacher at school. Their relationships have been derived in the previous sections and those relationships also hold in this case. It remains to determine the value of z^i ; the time allocated to each type.

Step II

In Step I, we maximized the utility of the teacher with the revenue from each type of student given the value of z^i . We have a contour of V as a function of z . In this step, we are seeking a point on that contour that satisfies the time constraint, (23). This will help us to derive a condition of the inter-allocation of time between each type. We formulate this problem in the following way:

$$\max_{\{z^1, z^2\}} R(z^1, z^2) = V(N^1 p^1(\cdot) + N^2 p^2(\cdot)). \quad (27)$$

Subject to the constraint

$$1 - S - z^1 - z^2 = 0 \quad (23)$$

$$z^1 > 0, \text{ and } z^2 > 0$$

where $R(\cdot)$ can be interpreted as the revenue function of the teacher. The teacher must maximize his revenue from tutoring to maximize his utility, V . θ is the Lagrangean multiplier for the time constraint (23), and is the shadow value of the

teacher's time available for tutoring. The teacher allocates positive time to each type.

We derive the optimal condition of the allocation of time between the two groups of students as:

$$\frac{\partial V}{\partial z^1} = \frac{\partial V}{\partial z^2}. \quad (28)$$

Equation (28) implies that the teacher will allocate time between the two groups in such a way that the marginal utility (revenue) derived from each group must be same. This condition maximizes his utility (revenue) received from both groups, and at the same time satisfies the time constraint. Using equation (28), we derive the following timing rule between the two groups of students.

$$\frac{z^1}{z^2} = \frac{N^1 p^1 \epsilon_{p^1 z^1}}{N^2 p^2 \epsilon_{p^2 z^2}}. \quad (29)$$

Where $\epsilon_{p^i z^i}$ refers to the elasticity of p^i with respect to z^i .

Equation (29) implies that the ratio of time allocated to both groups must be equal to their ratio of change in revenues. By assuming $N^1 = N^2$, we can replicate the results of the partial discrimination case from equation (29) and show that $p^2 > p^1$ when $z^1 = z^2$. The rich students pay a higher price than the poor students when the services available to the students are the same i.e. $z^1 = z^2$. This uses the fact that the elasticity of the rich students are lower than the elasticity of the poor students. Thus the partial discrimination case is a special case of the full discrimination case. In the full discrimination case, although there is implicit assumption of zero opportunity cost of time, the teacher now has two alternative uses of time. He devotes time to both types separately. The opportunity cost of time to one type is the revenue foregone from the other type. Since he is maximizing his revenue by allocating time to both types, the time devoted to tutoring in a club must, therefore, depend on the characteristics of the students. This result is different from the one we derived in the partial discrimination case.

4.2 Ability Heterogeneity

4.2.1 The Students

We assume in this case that the students are differentiated only by their ability. The abilities of the students are represented by γ^ι where $\iota = 1$ and 2 . 1 stands for low ability and 2 stands for high ability. There are N students of which N_1 are low ability and N_2 are high ability. The teacher offers all of the students a similar package, $\{T, p, n\}$. The purpose of this exercise is again to show whether there are any conditions under which a uniform offer by the teacher to all of the students, when they are heterogeneous in respect to ability, is optimal.

We can write the student's membership constraint as:

$$u^\iota(Y - p, \gamma^\iota\{S \cdot e + g(T, n)\}) \geq \bar{u}^\iota(Y, \gamma^\iota \cdot S \cdot e). \quad (30)$$

Following the similar arguments developed in proposition 3, we state in the following proposition how the effect of a teacher's offer on the utility of a student changes as his ability changes.

Proposition:4 *If for a given value of $T > 0$, $p > 0$, $n > 0$, and $\gamma^\zeta > \gamma^\iota$, and with the standard properties of the student's utility function*

$$u^\iota(Y - p, \gamma^\iota\{S \cdot e + g(T, n)\}) = \bar{u}^\iota(Y, \gamma^\iota \cdot S \cdot e),$$

then

$$u^\zeta(Y - p, \gamma^\zeta\{S \cdot e + g(T, n)\}) < \bar{u}^\zeta(Y, \gamma^\zeta \cdot S \cdot e).$$

This can be proven in the same way we proved proposition 3. Proposition 4 proves that if a student with a given ability derives the same utility from a club as his reservation utility, then his utility from the membership of the same club will be strictly less than his reservation utility as his ability increases.

4.3.2 The Teacher

The teacher's problem is no different from the income heterogeneity section. To avoid repetition, we will not go into further details. Using the results of Proposition 4, we can show that the teacher will charge a lower price to the high ability students and a higher price to the low ability students in order to make the tutoring attractive to the high ability students. This is mainly due to the fact that the high ability students derive more utility from education at school than do the low ability students. This means that the marginal utility from education of the high ability students is lower than the low ability students. In order to attract the high ability students to the club, the teacher has to lower the price, otherwise, the high ability students will not find the package attractive.

The implication of this result is that the discriminatory policy of the teacher maximizes revenue from private tutoring by charging a higher price to the low elastic (low ability) students and a lower price to the high elastic (high ability) students. It, therefore, encourages the high ability students and discourages the low ability students to take up more education through club services. This can also be justified from an efficiency point of view, since this policy helps the high ability students to accumulate more human capital in the economy.

4.3 General Discussion of Heterogeneous Cases

The economic justification of discrimination not only arises from the point of view of the teacher as he is able to maximize his revenue, but it also arises on the grounds of equity. In the absence of discrimination, the poor students may not be able to buy private tutoring if the uniform price is higher than his reservation price. The price discrimination makes it possible for the poor students to demand more private tutoring and acquire higher education. So, we can justify discriminatory pricing on the grounds of equity as the rich subsidizes the education of the poor by paying a higher price to the teacher. One of the main policy issues, in regard to price discrimination, is its effect on income distribution. As we saw, price discrimination

redistributes income away from the low-elasticity groups toward students in the high-elasticity groups and the teacher. This could be one of the reasons why we are observing a lower drop out rate over the years, as the practice of providing private tutoring is becoming more and more popular.

We also consider the implications of ability heterogeneity for an efficiency. Since the teacher charges a higher price to the low ability students and a lower price to the high ability students, it encourages a high ability student to acquire more education from tutoring while discouraging a low ability type. This can be justified as an improvement in efficiency. In this model, if we allow the students to have both the characteristics simultaneously, then the teacher will unambiguously charge a lower price to the high ability and poor students and a higher price to the low ability and rich students.

The analysis of the equity and efficiency issues raised above, from an economic point of view, remains incomplete without formalizing a planner's problem. To analyze these issues, a welfare maximization approach seems more appropriate than a cost minimization approach adopted in the "homogeneous student" section. Such welfare maximization, however, is easier said than done. We know that the equilibrium described here cannot be efficient. There are several reasons for this. First, the teacher (tutor) is a monopolist. Second, it could be more efficient for the planner to change the rules so that no shirking occurs. Third, we know from Faulhaber (1975) that when there are joint products with inputs and outputs of the goods distinguishable, then efficiency requires that price equals marginal cost for each output separately. Cross subsidization is inefficient.

So far as equity is concerned, it is not clear whose welfare the planner should maximize. If the planner maximizes the welfare of the students, then the question that arises here is whether the students should maximize their utility or their education. Secondly, since there are more than one group of students involved, the determination of social welfare weights attached to different groups is another important issue. It is not clear whether a rich student should acquire more education

than a poor student, or whether a high ability student should be given a chance to acquire more education than a low ability student, etc. It should be clear from the above that the question of welfare optimum is a highly complex one. We have opted for a simple framework of cost minimization which circumvents these problems. Needless to say, addressing these issues will be an important constituent of future research in this area.

To conclude this section, we need to clarify our assumption about the monitoring technology²⁹ when there are heterogeneous students. We know that the planner is interested in the education of the students. Now the question arises, whose education does the government monitors observe. This is because the students are heterogeneous. The monitors may have problems in identifying and distinguishing between the types. If they can perfectly observe the characteristics of the students like the teachers, then there is no problem. Unfortunately, this is not so simple. For the purpose of this paper, we therefore assume that the monitors observe only the average level of education. There is a problem in this approach as there will always be some students whose education is lower than the average level. So even if the government fulfills its target of achieving the average level of education, some students will always be below that target. This explains why there are some students who acquire less than the minimum education despite the imposition of this constraint.

We can similarly analyze the behavior of the teacher and the planner's problems previously explained in the homogeneous section and thus will not be presented to avoid repetition.

²⁹ We write the monitoring technology in the heterogeneous model as: $q(\tilde{E}, m)$ where $\tilde{E} = \frac{\sum_i (S \cdot e + g_i)}{n}$ is the average level of education in the economy when there are N students per teacher.

5 Conclusion

The United Nations adopted its *Universal Declaration of Human Rights* in December 10, 1948. Article 26 of the Declaration states that “*Everyone has the right to education. Education shall be free, at least in the elementary and fundamental stages. Elementary education shall be compulsory. Technical and professional education shall be made generally available and higher education shall be equally accessible to on the basis of merit.*” Even after four decades, the objective of the United Nations to achieve universalization of education is far from being achieved in most of the developing countries.

This paper develops a new model of private tutoring. We explained that monitoring is expensive in the absence of a good monitoring technology. The government finds it cheaper to allow the teachers to offer tutoring rather than to monitor the teachers’ performance at school. The teachers make up their salary by providing tutoring services to the students. Thus we showed that the teachers, by charging a price for their services outside the school, divert the benefit of free public education towards themselves. This model can be considered when the teachers are heterogeneous. This assumption will differentiate the schools’ quality. It will be interesting to consider when the students choose their own school.

We also extend the model where the students are heterogeneous in respect to income and ability. The provision of a uniform club service and price in the presence of heterogeneity is not optimal from the teacher’s point of view. If the teacher charges a uniform price, then poor students may not demand tutoring and their education may be less than the required minimum. If the teacher discriminates between the types of students, then he charges a higher price to the rich and low ability students than to the poor and high ability students. This way, the rich and the low type students subsidize the tutoring education of the poor and the high type students, which can be advocated on both equity and efficiency grounds.

In developing countries, we observe that the government gives scholarships

to the poor but meritorious students. This is known as a merit-cum-means principle which helps the government to decide which students are eligible for scholarships and provides an indication of the goals of the social planner. We also show, through our heterogeneous model, that the teacher charges a lower price to poor students and high ability students. It meets the merit-cum-means principle criteria. This way, the poor and high ability students acquire a higher level of education from private tutoring. So we can justify the role of private tutoring where the planner can, at a lower social cost, delegate some of its responsibilities to the teachers.

This model predicts that the difference in education levels among students who go to the public school does not come from differences in their income when the teacher provides the same club service to everybody at different prices. The difference in educational levels among the students arise due to the differences in the ability of students. This result is entirely different from the results derived in Biswal (1993), where tutoring has been modeled as a private good and the students acquire a higher level of education when they are differentiated by income. In this model, the teachers will always supply the same amount of tutoring under any regime but the cost of tutoring to the students will be different. Thus the government can influence the cost of tutoring through the adoption of an appropriate policy. We also showed that the cost of tutoring falls with increasing monitoring but at the cost of increasing the salaries of the teachers.

We interpret the cost of tutoring as an indirect tax on the students to subsidize the public education. The government, observing this transaction, optimally decides the amount of monitoring and the salary of the teachers so that the students get their minimum level of education and the teachers get at least their reservation utility at a minimum cost to the society. The low level of monitoring technology makes monitoring expensive. The planner finds it cheaper to do less monitoring and to allow the teacher to offer private tutoring. The presence of private tutoring in developing countries explains the low official salary of the teachers. As the monitoring becomes less expensive with the increase in the technology of monitoring, the

need and the demand for tutoring falls, teachers get a higher salary, and the economy increasingly resembles that of a developed economy. Thus, the existence of the institution of private tutoring can indicate the stage of the economic development reached by a country.

References

- Barham, V., R. Boadway, M. Marchand, and P. Pestieau, 1991, Education and the poverty trap, Discussion Paper No. 830, Queen's University, Canada.
- Becker, G.S., 1975, Human Capital (The University of Chicago Press, Chicago, IL).
- Berglas, E., 1976, On the theory of clubs, *American Economic Review* 66, 116-121.
- Berglas, E., 1981, The market provision of club goods once again, *Journal of Public Economics* 15, 389-393.
- Besley, T. and S. Coate, 1991, Public provision of private goods and the redistribution of Income, *American Economic Review*, September, 979-984.
- Biswal, B.P., 1993, Providing Education in Developing countries: A Theoretical Analysis, Unpublished Mimeo, Queen's University, Canada.
- Boadway, R., 1980, A note on the market provision of club goods, *Journal of Public Economics* 13, 131-137.
- Bowles, S. and H. Gintis, 1976, *Schooling in Capitalist America* (Basic Books, New York.)
- Buchanan, J., 1965, An economic theory of clubs, *Economica* 32, 1-14.
- Butts, R., 1978, *Public education in the United States* (Holt, Rinehart and Winston, New York).
- Card, D. and A. Krueger, 1992, Does school quality matter? Returns to education and characteristics of public schools in the United States, *Journal of Political Economy* 100, 1-40.
- Cornes, R. and T. Sandler, 1986, *The Theory of Externalities, Public Goods, and Club Goods* (Cambridge University Press, Cambridge).
- Creedy, J. and P. Francois, 1990, Financing higher education and majority voting, *Journal of Public Economics* 43, 181-200.
- Faulhaber, A., 1975, Cross Subsidization: Pricing in Public Enterprises, *American Economic Review* 65, 966-977.
- Flatters, F. and B. Macleod, 1991, Administrative Corruption and Taxation, Mimeo, Queen's University, Kingston, Canada.
- Hanushek, E., 1986, The Economics of Schooling: Production and Efficiency in Public Schools, *Journal of Economic Literature* 24, 1141-1177.
- Holmstrom, B., 1979, Moral Hazard and Observability, *Bell Journal of Economics* 10, 74-91.
- Holmstrom, B., 1982, Moral Hazard in Teams, *Bell Journal of Economics* 13, 324-340.
- ILO, 1991, *Teachers in developing countries: A survey of employment conditions* (International Labour Office, Geneva).
- Ireland, N.J., 1990, The mix of social and private provision of goods and services, *Journal of Public Economics* 43, 201-219.
- Johnson, G.E., 1984, Subsidies for higher education, *Journal of Labor Economics* 2(3), 303-318.

- Kennedy, P.W., 1988, The market provision of club goods, Discussion Paper No. 732, Queen's University, Canada.
- Krueger, A.O., 1974, The political economy of the rent seeking society, *American Economic Review*, 291-303.
- Lott, J.R. Jr., 1990, An explanation of the public provision of schooling: The importance of indoctrination, *Journal of Law and Economics* 33(1).
- Lucas, R.E. Jr., 1988, On the mechanics of economic development, *Journal of Monetary Economics* 22, 3-42.
- Mirrlees, J.A., 1975, The Theory of Moral Hazard and Unobservable Behavior, Part I, Mimeo.
- Oakland, W., 1972, Congestion, Public Goods and Welfare, *Journal of Public Economics* 1, 339-357.
- Posner, R.A., 1975, The social cost of monopoly and regulation, *Journal of Political Economy* 83, 807-827.
- Rasmusen, E., 1990, *Games and Information: An Introduction to Game Theory* (Cambridge University Press, Cambridge).
- Romer, P., 1986, Increasing returns and long-run growth, *Journal of Political Economy* 94, 1002-1037.
- Stiglitz, J.E., 1974, The Demand for Education in Public and Private School Systems, *Journal of Public Economics* 3, 349-385.
- Stiglitz, J.E., 1982, Self-selection and pareto efficient taxation, *Journal of Public Economics* 17, 213-240.
- Tullock, G., 1967, The welfare cost of tariffs, monopolies, and theft, *Western Economic Journal* 5, 224-232.
- UNDP, 1991, *Human Development Report*, (Oxford University Press, Oxford).
- UNESCO, 1992, *Statistical Yearbook* (United Nations Educational, Scientific and Cultural Organisation, Geneva).
- UNESCO, 1991, *World Education Report* (United Nations Educational, Scientific and Cultural Organisation, Geneva.)
- Usher, D., 1977, The Welfare Economics of the Socialization of Commodities, *Journal of Public Economics* 8, 151-168.
- World Bank Staff, 1986, *Financing education in developing countries: An exploration of policy options* (Education and Training Department, World Bank, Washington D.C.)
- World Bank Staff, 1987, *World Development Report* (Oxford University Press, New York).

Appendix 1

Labor-Leisure Choice in a Club Model

We can rewrite the teacher's problem in the following way:

$$\max_{\{T,p,n\}} V(\bar{w} + Np, 1 - S - \frac{N}{n}T). \quad (A.1)$$

Subject to the constraints:

$$u(Y - p, \gamma\{S \cdot e + g(T, n)\}) - \bar{u}(Y, \gamma \cdot S \cdot e) = 0 \quad (A.2)$$

$$T > 0, p > 0, \text{ and } n > 0.$$

All notations carry their usual meaning. Equation (A.1) is the utility function of the teacher. The first argument of the teacher's utility function represents his total income from salary and private tutoring. The second argument of the teacher represents his leisure. His utility is increasing and concave in both the arguments. Equation (A.2) is the utility constraint of the students. This represents the students get at least as much utility from a club as they get without it. We do not need a separate time constraint as it is already reflected in the second argument of the teacher's utility function.

The first order conditions are:

$$-V_2 \cdot \frac{N}{n} + \lambda \cdot u_2 \cdot \gamma \cdot g_1 = 0 \quad (: T)$$

$$V_1 \cdot N - \lambda \cdot u_1 = 0 \quad (: p)$$

$$V_2 \cdot \frac{N}{(n)^2} \cdot T + \lambda \cdot u_2 \cdot \gamma \cdot g_2 = 0. \quad (: n)$$

λ is the Lagrangean multiplier to the constraint (A.2) and it can be interpreted as the shadow value of the student's utility.

We can rearrange the first order equations to derive the following optimal tutoring rule and congestion rule of a club:

$$V_1 \cdot n \cdot \frac{u_2}{u_1} \cdot \gamma \cdot g_1 = V_2 \quad (OTR)$$

$$-V_1 \cdot n \cdot \frac{u_2}{u_1} \cdot \gamma \cdot g_2 = V_2 \frac{T}{n}. \quad (OCR)$$

OTR implies that the teacher gives time to each club in such a way that the marginal benefit to him from providing tutoring to all students in a club is equal to the cost of its time, leisure. Similarly, OCR implies that the teacher determines the optimal number of students in a club in such a way that the marginal cost to him in terms of loss of income due to congestion from admitting one more student to the club equals the marginal utility gain from the fall in the average time devoted to each student.

By rearranging both of the optimal rules, we can derive the same optimal club rule:

$$g_1 \cdot T + g_2 \cdot n = 0.$$

This shows that the optimal club rule does not change even if we introduce the labor-leisure choice of the teacher into the model. But we can check from the optimal rules that the club fee will increase since the teacher values leisure.

Appendix 2

Proof of Proposition 1: By assumption when there is no monitoring, the chance of getting fired is equal to zero ,i.e. $q(E, 0) = 0$. Thus the teacher chooses effort to maximize the monopoly gains from tutorials. Recognizing that $e \geq 0$, the first order condition (12) can be rederived as:

$$V' \cdot N \cdot p_e < 0. \quad (i)$$

This implies that the teacher supplies zero effort in the school in case of no monitoring. So we get: $e(\bar{w}, 0) = 0$. Q.E.D.

Appendix 3

Proof of Proposition 2: By totally differentiating the teacher's indirect utility function with respect to v , \bar{w} and m , and by setting $dv = 0$, we get the condition that:

$$\frac{d\bar{w}}{dm}|_v > 0. \quad Q.E.D.$$

Appendix 4

Proof of Proposition 3: We have:

$$u^i(Y_i - p, \gamma\{S \cdot e + g(T, n)\}) = \bar{u}^i(Y_i, \gamma \cdot S \cdot e).$$

This implies that

$$u_1^i p = u_2^i \gamma g.$$

From the assumption of concavity, we can show that

$$u_1^j p < u_2^j \gamma g.$$

Thus,

$$u^j(Y^j - p, \gamma\{S \cdot e + g(T, n)\}) > \bar{u}^j(Y^j, \gamma \cdot S \cdot e). \quad Q.E.D.$$

Table 1 Comparison of teacher's incomes with those of other occupations¹ requiring comparable training (1982-85)

Regions/cities	Income ratio teacher/ electrical engineer		Income ratio teacher/ bank cashier(male) ²		Income ratio teacher/ executive secretary (female) ²	
	1982	1985	1982	1985	1982	1985
<i>Africa</i>						
Cairo	0.28	0.20	0.47	0.34	0.38	0.40
Lagos	-	0.24	-	0.61	-	0.43
<i>Latin America</i>						
Bogota	0.41	0.22	0.90	0.36	0.98	0.30
Buenos Aires	0.17	0.37	0.56	0.47	0.51	0.67
Caracas	0.64	0.26	1.20	0.80	0.60	0.40
Mexico	0.26	0.32	0.62	0.66	0.80	0.62
Panama	0.22	0.21	0.42	0.42	0.45	0.47
Rio de Janeiro	0.25	0.20	0.79	0.54	0.31	0.36
Sao Paulo	0.15	0.20	0.67	1.26	0.25	0.42
<i>Asia</i>						
Abu Dhabi	0.50	-	1.27	-	0.78	-
Bahrain	0.66	0.65	0.66	1.08	1.08	0.84
Bangkok	0.29	0.26	0.85	0.58	0.58	0.70
Bombay	0.44	0.32	0.52	0.53	0.60	0.53
Djakarta ³	0.31	0.20	0.63	0.40	0.25	0.23
Manila	0.28	0.34	0.60	0.38	0.68	0.75

- = Not available

¹ Gross annual income.

² Ratios calculated on earnings converted into US dollars.

³ Net income before tax, and without employer's contribution.

Source: Teachers In Developing Countries, ILO 1991.

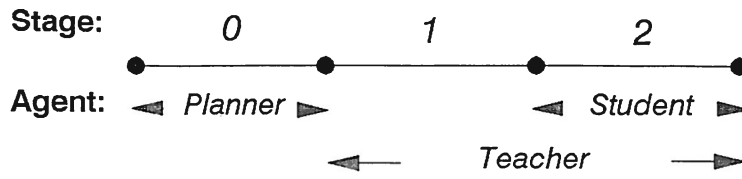


Figure 1

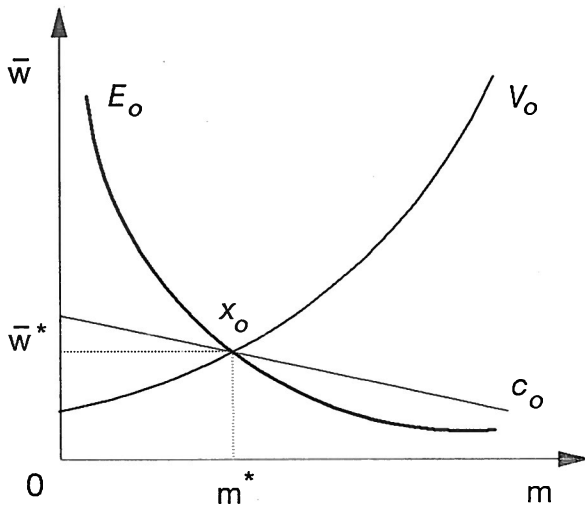


Figure 2

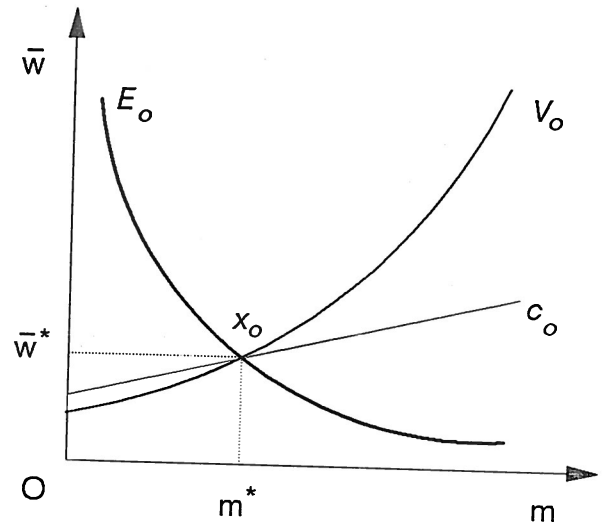


Figure 3

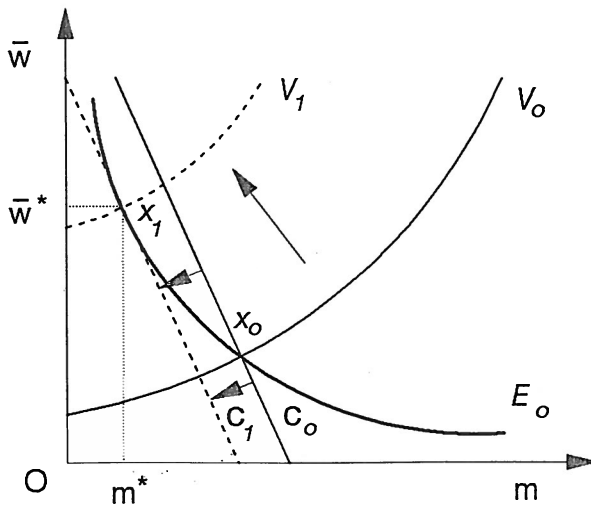


Figure 4

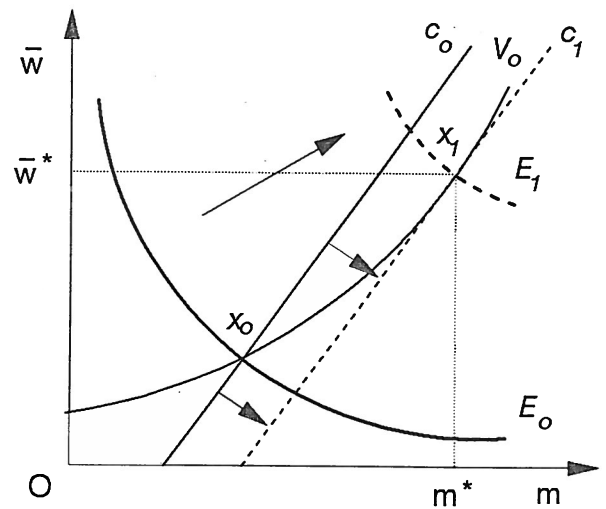


Figure 5