Queen’s Economics Department Working Paper No. 827

Testing for Structural Breaks

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7-1991

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Discussion Paper #827

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July 1991

Abstract

The purpose of this paper is to investigate the tests of Hansen (1991) to detect structural breaks in cointegrated relations using Monte Carlo methods . The evaluation takes place within the linear quadratic model. The evidence for a single regressor suggests that the tests have proper size and that the power is good provided the cost of adjustment is low. In addition to the tests of Hansen, we consider the sensitivity of the augmented Dickey-Fuller (ADF) test for cointegration in the presence of a structural break. Our Monte Carlo experiments show that the ADF test suffers a substantial loss of power (a failure to reject the null of no cointegration). As a practical example we consider the stability of the long-run coefficients in annual U.S. money demand.

KEY WORDS: linear quadratic; cointegration; structural breaks

JEL Classification Numbers: C12, C15, C22, C52

The authors thank Don Andrews, Anindya Banerjee, Bruce Hansen, Jeremy Rudin, and Gregor Smith for helpful suggestions and Robert Lucas for kindly supplying the data. The first author thanks the Department of Economics at the University of British Columbia and acknowledges the financial support of the Social Sciences and Humanities Research Council of Canada.

Testing for Structural Breaks in Cointegrated Relations

1. Introduction

Recently a series of researchers have developed tests for a single structural break with unknown break point in dynamic models. These include Andrews (1990), Banerjee, Lumsdaine, and Stock (1990), Bates (1990), Chu (1989), Hansen (1990 and 1991), Krämer, Ploberger and Alt (1988), Perron (1990a, b), Perron and Vogelsang (1991), and Zivot and Andrews (1990)[1]. Except for the Hansen (1991) paper, these tests are designed to test for a structural change in regression coefficients with stationary series or to test for a unit root (possibly with a break point) against a stationary alternative with a single (unknown) break point. The tests for unit roots are well suited to analyze break points in such variables as real GNP, real exchange rates and other integrated processes (see Banerjee, Lumsdaine, and Stock, 1990; Christiano, 1988; Banerjee, Dolado and Galbraith, 1990, Perron and Vogelsang, 1991 and Zivot and Andrews, 1990). The Hansen (1991) paper is the only one to date to consider structural breaks in *cointegrating* relations with unknown break points. This is clearly an important advance and permits a much wider range of economic application.

The purpose of this paper is to investigate using Monte Carlo methods the tests of Hansen (1991) to detect structural breaks in cointegrated

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[1] This list reflects only the more recent econometric contributions excluding such important papers as Brown, Durbin and Evans (1975), Chow (1960) and Quandt (1960). The interested reader is directed to the surveys by Krishnaiah and Miao (1988) and Zacks (1983) and the discussion in Andrews (1990) for a more complete list of references.

relations. The evaluation takes place within the class of linear quadratic models that has been extensively employed in modelling labor demand, the permanent income hypothesis, money demand, and investment (see Sargent, 1987). These models give rise to linear decision rules (in the variables) and hence have well understood properties for the integrated variables.

For a single regressor the evidence from the Monte Carlo work suggests that Hansen's tests have proper size (when there are no structural breaks in the cointegrating relation) and that the power is good provided the cost of adjustment is low. Power falls dramatically as the stable root becomes large (high cost of adjustment); a fact that is somewhat disturbing given the slow adjustment speeds obtained in application. As a practical example we return to a problem that has received a great deal of attention in the applied literature over the years, namely the stability of the long-run coefficients in annual U.S. money demand (for recent discussions see Lucas, 1988 and Stock and Watson, 1991).

In addition to the tests of Hansen we consider the sensitivity of the augmented Dickey-Fuller (ADF) test for cointegration in the presence of a structural break. Perron (1989) demonstrates that when there is a trend break in a trend stationary regression, standard tests of the null hypothesis of a unit root are biased towards the null. We examine whether breaks in the cointegrating vector have similar effects on the ADF tests. Our Monte Carlo work shows that the ADF test exhibits a considerable fall in power (a failure to reject the null of no cointegration) when there is a single structural break in the cointegrating relation[2]. This may in part explain the low

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[2]This should not be interpreted as a flaw in the ADF test since one would expect a fall in power in the presence of a structural break. In fact Hansen (1991) has suggested using his tests for structural breaks as a test of the

rejection rates of no cointegration observed in the applied literature.

The organization of this paper is as follows. Section 2 reviews the linear quadratic model, discusses tests for cointegration in the context of this model and describes Hansen's tests for structural change. Section 3 describes the Monte Carlo design and presents the results. Section 4 estimates and tests the stability of the U.S. money demand using annual data. Section 5 closes with some final thoughts and suggests some extensions.

2. Linear Quadratic Models, Cointegration and Tests For Structural Breaks

(i) Linear Quadratic Model

The linear quadratic model is a popular and tractable dynamic model in which agents minimize a dynamic quadratic objective function. Agents are assumed to track the long-run target variable y_s^* as given by a static equilibrium theory and choose the actual y_s to minimize the present discounted value of deviating from equilibrium $(y_s - y_s^*)$ and the costs of adjustment $(y_s - y_{s-1})$. The problem is to minimize the infinite horizon objective function over the uncertain stream $\{y_s\}$:

$$\min_{\{y_s\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} [\delta(y_s - y_s^*)^2 + (y_s - y_{s-1})^2], \tag{2.1}$$

for $s \geq t$, where the expectation is taken with respect to information available to the agent at time t $(\mathbb{F}_t)$, $\beta \in (0,1)$ is the discount factor and $\delta > 0$ is a weighting factor (see Kennan, 1979).

The static equilibrium relationship is $y_t^* = \theta x_t + e_t$, where e_t is a mean

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null of cointegration against the alternative of no cointegration. The idea is that a lack of cointegration (a spurious regression) leads to coefficient estimates that appear to be non-constant.

zero, independently and identically distributed error with variance σ_e^2, and x_t is a (kx1) vector of forcing variables. We assume that e_t is in $\mathbb{F}_t$ but unknown to the investigating econometrician whose information set is $\mathbb{G}_t \subset \mathbb{F}_t$; hence there is no stochastic singularity.

The forward solution to (2.1) is:

$$y_t = \lambda\, y_{t-1} + (1 - \lambda)\,(1 - \beta\lambda)\, E_t \sum_{s=t}^{\infty} (\beta\lambda)^{s-t}\, y_s^* \,, \tag{2.2}$$

where $\lambda < 1$ is the stable root of the Euler equation obtained from the first-order conditions.

This model has been used to explain, for example, the demand for labor by firms (Sargent, 1978, and Hansen and Sargent, 1980), the demand for labor and capital by firms (Meese, 1980), the demand and supply of labor (Kennan, 1979, 1988), natural resource extraction (Hansen, Epple, and Roberds, 1985), the demand for money (Cuthbertson and Taylor, 1987, Domowitz and Hakkio, 1990, and Gregory, Smith, and Wirjanto, 1990), the supply of money (Mercenier and Sekkat, 1988), optimal inventory holdings (West, 1986b) and the permanent income hypothesis (Nason, 1991). Hansen and Sargent (1990) have also analyzed and developed software for computable general equilibrium linear quadratic models.

The Wiener-Kolmogorov prediction formula can be used to replace the expectations in (2.2) given the law of motion for the forcing variables (see Sargent, 1987). In this paper we shall be concerned with the case where x_t is a kx1 vector of integrated processes of order 1 denoted I(1):

$$(I - L)\, R(L)x_t = \varepsilon_t, \tag{2.3}$$

where $\{\varepsilon_t\}$ is independently and identically distributed with a mean of 0 and variance of Σ and the roots of $R(L) = I - R_1L - \ldots - R_pL^p$ lie outside the unit circle. To simplify the solution of the model, consider the example of a scalar x_t $(k = 1)$. Given the stochastic process for x_t in (2.3), equation (2.2) can be solved. For instance, if $\Delta x_t = \varepsilon_t$ $(k = 1)$, the error correction model (ECM) can be obtained as:

$$\Delta y_t = (\lambda - 1)(y_{t-1} - \theta x_{t-1}) + (1 - \lambda)\, \theta \Delta x_t + (1 - \beta\lambda)(1 - \lambda) e_t. \tag{2.4}$$

Alternatively with $\Delta x_t = \rho \Delta x_{t-1} + \varepsilon_t$ and $|\rho| < 1$, then:

$$\Delta y_t = (\lambda-1)(y_{t-1}-\theta x_{t-1}) + (1-\lambda)\, \theta \Delta x_t \,/(1-\rho\lambda\beta) + (1 - \beta\lambda)(1 - \lambda) e_t. \tag{2.5}$$

In general, the solution will depend upon the serial correlation properties of Δx_t. However regardless of the exact nature of (2.3), the following relation always holds:

$$y_t = \theta x_t + \eta_t, \qquad t = 1,\ldots,T \tag{2.6}$$

where η_t is a stationary error. Hence y_t and x_t are cointegrated and θ is the cointegrating vector.[3]

The general form for η_t $(k = 1)$ is:

$$\eta_t = [\Psi(L)\ \lambda/(1-\lambda L)]\ \varepsilon_t + [\delta\ \lambda/(1-\lambda L)]\ e_t, \tag{2.7}$$

where $\Psi(L)$ depend upon the nature of x_t in (2.3). For instance $\Delta x_t = \varepsilon_t$, $\Psi(L) = -\theta$ and for $\Delta x_t = \rho\Delta x_{t-1} + \varepsilon_t$, $\Psi(L) = -\theta(1-\rho\beta)/[(1-\rho\beta\lambda)(1-\rho L)]$.

The stable root $\lambda < 1$ satisfies:

$$\lambda^2\beta+1 = \lambda+\lambda\beta+\lambda\delta,$$

where $\lambda \to 1$ as $\delta \to 0$. That is, as the cost of adjustment gets large (a small δ) the stable root approaches 1 and η_t in (2.6) is nearly integrated.

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[3] Single equation estimation of the linear quadratic model with integrated processes have been considered by Dolado, Galbraith and Banerjee (1991) and Gregory, Pagan and Smith (1990).

Applied work has yielded point estimates for the root that have typically been 0.9 or greater (see for example, Nason, 1991; Meese, 1980; Mendis and Muellbauer, 1982; Nickell, 1984, 1986 and Sargent, 1978).

(ii) Testing For Cointegration in the Linear Quadratic Model

The most widely used cointegration test is the augmented Dickey-Fuller (ADF) t-ratio test (see Said and Dickey, 1984), recommended by Engle and Granger (1987). The test is based on the residuals from a cointegrating regression and is constructed to test the null hypothesis of no cointegration. Hence the null of a unit root in the residuals is tested against the alternative that the root is less than unity. One first estimates equation (2.6) by ordinary least squares (OLS) and tests the null hypothesis of no cointegration using a scalar unit root test $t(\hat{\alpha})$ on the residuals:

$$\Delta\hat{\eta}_t = \alpha\ \hat{\eta}_{t-1} + \sum_{i=1}^{m} \hat{\phi}_i\ \Delta\hat{\eta}_{t-i} + \hat{\nu}_t, \tag{2.8}$$

where the lag length m is chosen sufficiently large in order for $\hat{\nu}_t$ to be serially uncorrelated. The distribution of $t(\hat{\alpha})$ depends upon the number of regressors in (2.6); critical values are found in MacKinnon (1990) and Phillips and Ouliaris (1990).

Gregory (1991) has examined the finite sample properties of a number of tests for cointegration under a variety of parameter settings for the linear quadratic model. The results indicate sharp differences in the various tests to detect cointegrating relations especially when the cost of adjustment term becomes large ($\delta \rightarrow 0$) and the number of regressors is large. The Monte Carlo evidence suggests that the ADF test as well as Phillip's (1987) Z_α test have proper size compared to their asymptotic values and possess the best power.

(iii) Testing For Structural Breaks in the Cointegrating Vector

We will give a brief description of the tests proposed by Hansen (1991); see that paper for further details. Since the critical values for Hansen's test include a constant in the cointegrating regression, we rewrite equations (2.6) and (2.3) as:

$$y_t = \gamma \tilde{x}_t + \eta_t \tag{2.9}$$

and

$$x_t = x_{t-1} + \xi_t, \tag{2.10}$$

where $\tilde{x}_t = (1, x_t^T)^T$ is a (k+1) x 1 vector, $\gamma = (\mu, \theta)$ (with $\mu = 0$) and $\xi_t = R(L)^{-1}\varepsilon_t$. Define the vector $u_t = (\eta_t, \xi_t^T)^T$ and the following matrices (the long-run variance matrices):

$$\Omega = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{T} E\left[u_j \, u_t^T\right] \tag{2.11}$$

$$\Lambda = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{t} E\left[u_j \, u_t^T\right] ,$$

partitioned in conformity with u:

$$\Omega = \begin{bmatrix} \Omega_{\eta\eta} & \Omega_{\eta\xi} \\ \Omega_{\xi\eta} & \Omega_{\xi\xi} \end{bmatrix} \quad \text{and} \quad \Lambda = \begin{bmatrix} \Lambda_{\eta\eta} & \Lambda_{\eta\xi} \\ \Lambda_{\xi\eta} & \Lambda_{\xi\xi} \end{bmatrix}. \tag{2.12}$$

Also define $\Omega_{\eta.\xi} = \Omega_{\eta\eta} - \Omega_{\eta\xi}\Omega_{\xi\xi}^{-1}\Omega_{\xi\eta}$ and $\Lambda_{\eta\xi}^{+} = \Lambda_{\xi\eta} - \Lambda_{\xi\xi}\Omega_{\xi\xi}^{-1}\Omega_{\xi\eta}$.

The test procedure requires an estimator of θ which has a mixture normal asymptotic distribution. As in Hansen (1991) the fully modified (FM) estimator of Phillips and Hansen (1990) is used. This estimator which has been studied in Monte Carlo experiments by Gregory, Pagan and Smith (1990), Phillips and Loretan (1991) and Stock and Watson (1991) appears to have good finite sample properties in terms of bias and coverage probabilities.

To begin, we estimate (2.9) by OLS and obtain the residuals $\hat{\eta}_t = y_t - \hat{\gamma}\tilde{x}_t$ and define $\hat{u}_t = (\hat{\eta}_t, \Delta x_t^T)^T$. With the $\hat{u}_t$, form estimates of Ω and Λ, denoted by $\hat{\Omega}$ and $\hat{\Lambda}$. The estimators of these long-run matrices used in this paper are due to Andrews (1991) and Andrews and Monahan (1990). They are obtained from a prewhitened quadratic spectral kernel with a vector autoregression of order one for the prewhitening. The automatic bandwidth estimator is also a vector autoregression of order one; see the Appendix for details.

Partition $\hat{\Omega}$ and $\hat{\Lambda}$ as Ω and Λ with: $\hat{\Omega}_{\eta.\xi} = \hat{\Omega}_{\eta\eta} - \hat{\Omega}_{\eta\xi}\hat{\Omega}_{\xi\xi}^{-1}\hat{\Omega}_{\xi\eta}$ and $\hat{\Lambda}_{\eta\xi}^{+} = \hat{\Lambda}_{\xi\eta} - \hat{\Lambda}_{\xi\xi}\hat{\Omega}_{\xi\xi}^{-1}\hat{\Omega}_{\xi\eta}$. Define the transformed dependent variable:

$$y_t^+ = y_t - \hat{\Omega}_{\eta\xi}\hat{\Omega}_{\xi\xi}^{-1}\,\Delta x_t.$$

The FM estimator of γ is:

$$\hat{\gamma}^+ = \left(\sum_1^T \left(y_t^+\tilde{x}_t^T - (0\ \ \hat{\Lambda}_{\xi\eta}^{+T})\right)\right)\left(\sum_1^T \tilde{x}_t\tilde{x}_t^T\right)^{-1}, \tag{2.13}$$

with the associated residual vector:

$$\hat{\eta}_t^+ = y_t^+ - \hat{\gamma}^+\,\tilde{x}_t\,.$$

Form the "score" s_t:

$$\hat{s}_t = \left(\tilde{x}_t\hat{\eta}_t^+ - \begin{bmatrix} 0 \\ \hat{\Lambda}_{\xi\eta}^{+} \end{bmatrix}\right).$$

Two properties of the OLS regression function are:

$$\frac{1}{T}\sum_{t=1}^T \tilde{x}_t\hat{\eta}_t^+ = \begin{bmatrix} 0 \\ \hat{\Lambda}_{\xi\eta}^{+} \end{bmatrix} \qquad \text{and} \qquad \sum_{t=1}^T \hat{s}_t = 0.$$

To allow for possible parameter instability, we modify (2.9):

$$y_t = \gamma_t\,\tilde{x}_t + \eta_t\,. \tag{2.14}$$

In this paper we investigate the four tests for parameter instability in cointegrating relations suggested by Hansen (1991). For each test the null hypothesis is that γ_t is constant. The tests differ in the treatment of the (implicit) alternative. For the first two tests the alternative hypothesis is:

$$\gamma_t = \begin{cases} \gamma_1, & t \le [T\tau] \\ \gamma_2, & t > [T\tau] \end{cases} ; \qquad \tau \in (0,1),$$

where $[\cdot]$ denotes the integer part. Thus we are testing for a single break point at time $[T\tau]$. The null hypothesis is:

$$H_o: \gamma_1 = \gamma_2 .$$

The first test assumes τ is known under the alternative:

$$H_1: \gamma_1 \neq \gamma_2; \qquad \tau \text{ known.}$$

A test of H_o against H_1 is given by the statistic:

$$F = F(\tau) = S_T(\tau)^T V_T(\tau)^{-1} S_T(\tau)\ \hat{\Omega}^{-1}_{\eta.\xi} , \tag{2.15}$$

where:

$$S_T(\tau) = S_{[T\tau]} = \sum_{t=1}^{[T\tau]} \hat{s}_t ,$$

and

$$V_T(\tau) = M_T(\tau) - M_T(\tau) M_T(1)^{-1} M_T(\tau),$$

$$M_T(\tau) = \sum_{t=1}^{[T\tau]} \tilde{x}_t \tilde{x}_t^T .$$

The F test (2.15) for known (fixed) τ is asymptotically distributed under H_o as χ^2 with k+1 degrees of freedom.

The second test treats the timing of the break point as unknown:

$$H_2: \gamma_1 \neq \gamma_2; \ \tau \in \mathcal{T} ,$$

where $\mathcal{T}$ is some compact subset of (0,1). The test is:

$$F_{sup} = \sup_{\tau \in \mathcal{T}} F(\tau). \tag{2.16}$$

The third and fourth test treat γ_t as a martingale process:

$$\gamma_t = \gamma_{t-1} + \omega_t, \quad E\left[\omega_t | \mathcal{F}_{t-1}\right] = 0, \quad E\left[\omega_t \omega_t^T\right] = \kappa^2 G_t,$$

where $\mathcal{F}_t$ is some increasing sequence of σ-fields to which γ_t is adapted and G_t is some known covariance array which measures the parameter stability in the t'th period. A convenient choice for G_t is some constant so that there is a constant hazard of parameter instability over the sample (see Hansen, 1990).

The null hypothesis may be written as the restriction that the variance of the martingale differences is zero:

$$H_o: \kappa^2 = 0.$$

One possible alternative is:

$$H_3: \quad \kappa^2 > 0, \qquad G_{[T\tau]} = \left(\hat{\Omega}_{\eta.\xi} \otimes V_T(\tau)\right)^{-1}.$$

The test statistic is:

$$F_{mean} = \frac{1}{T} \sum_{t=1}^{T} F(\tau). \tag{2.17}$$

Another alternative is:

$$H_4: \quad \kappa^2 > 0 \qquad G_{[T\tau]} = \left(\hat{\Omega}_{\eta.\xi} \otimes M_T(1)\right)^{-1},$$

with the test statistic:

$$L_c = \frac{1}{T} \, tr\left\{M_T(1)^{-1} \sum_{i=1}^{T} S_i \; \hat{\Omega}_{\eta.\xi}^{-1} \; S_i^T\right\}. \tag{2.18}$$

The limiting distributions and critical values for (2.16)-(2.18) can be found in Hansen (1991).

The F test for fixed (known) τ is simple but has the disadvantage that power may be low when τ is chosen in an arbitrary way. This test is asymptotically equivalent to the usual Wald statistic for testing $\gamma_1 = \gamma_2$ on two subsamples with the full sample used to estimate the variance. Stock and Watson (1991) use a similar test to evaluate the stability of U.S. money demand for a fixed break point. As has been demonstrated in a related context by Zivot and Andrews (1990), if a break point is arrived at by a "data search" over the sample, say by choosing the highest F (F_{sup}) as in (2.16), then the usual χ^2 critical values are smaller than the appropriate critical values for F_{sup} so that inference is biased against the null hypothesis.

A test similar in spirit to the F_{sup} test has been used in a number of studies for breaks in I(1) variables (see Chu, 1989; Hansen 1990; and Perron and Vogelsang, 1991 and Zivot and Andrews, 1990). In the present context of testing for breaks in cointegrated relations, the principal advantage of the F_{sup} over the other tests is that it provides an estimate of τ, which may be useful for model respecification in the event that the null hypothesis of no structural break is rejected. A drawback is in implementation a region for $\mathcal{T}$ must be specified (otherwise at the endpoints 0 and 1, the test statistic diverges almost surely). Following Andrews (1990) we choose $\mathcal{T} = [.15,.85]$. One would expect power loss from structural breaks that occur outside this region.

The F_{mean} test and the L_c test differ in their treatment of the variance in the martingale process. The L_c test may be viewed as a Lagrange multiplier-like test (see Andrews, 1990 and Hansen, 1990) and requires no trimming. Although the F_{mean} test requires no trimming, Hansen (1991) advises that some trimming be done. In the Monte Carlo analysis we follow this advice and choose the same rule as used in the supF test: $\mathcal{T} = [.15,.85]$[4].

This completes the discussion of the various tests. In the next section

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[4]In private correspondence Bruce Hansen has suggested that this kind of trimming will have a small effect on the the asymptotic size of the F_{mean}. He has also found in Monte Carlo experiments that the size biases in finite samples from using critical values in Hansen (1991) based on no trimming are small.

we develop a simple Monte Carlo experiment to evaluate these stability tests as well as the sensitivity of the ADF tests to structural breaks. The data generating process is the linear quadratic model.

3. Monte Carlo Design and Results

There is some conceptual difficulty in allowing for structural change in any optimizing model. For instance, what beliefs do we endow agents with in forming expectations of a possible structural break in the future and how does such information influence their current behavior? Since the break point $\tau \in (0,1)$ is itself nonrandom in the testing procedures, it seems sensible to treat the occurrence (if $\tau \neq 1$) of a structural break as unanticipated and not reoccurring. Thus for example, if we analyze a change in the cointegrating vector $(k = 1)$ from say θ_1 to θ_2 at time τT, the cointegration relation (2.6) would be specified as[5]:

$$y_t = \theta_t x_t + \eta_t, \quad \begin{cases} \theta_t = \theta_1, & t \leq [\tau T] \\ \theta_t = \theta_2, & t > [\tau T] \end{cases}, \quad t = 1,\ldots T . \tag{3.1}$$

The Monte Carlo experimental design is similar to Gregory (1991), Gregory, Pagan and Smith (1990) and West (1986a). The computer package used in the analysis is GAUSS386 and the programs are available from the authors upon request. For each experiment we do 1000 replications with observation set $T = 100$, 200 and 500 and record the rejection frequencies of the tests (2.15)-(2.18) using the five percent asymptotic critical values[6].

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[5] For all the Monte Carlo experiments the regressions include a constant which is set to zero in the data generating process.

[6] We have also calculated the rejection frequencies based on the one and ten percent significance levels. These are qualitatively similar to the five percent values and are available upon request.

F_{mid} (2.15) is the F-test at the midpoint and the other tests are labelled as in Section 2. We set $\tau = 1$, .25, .5 and .75. When $\tau = 1$ there is no structural break in the cointegrating relation (for these experiments $\theta = 1$) and rejection frequencies will provide information regarding the size of the tests in finite samples. For $\tau \neq 1$ there is a structural break and we set $\theta_1 = 1$ for $t \leq \tau T$ and $\theta_2 = .5$ for $t > \tau T$. We have done some experimentation with other values for θ and as would be expected the larger the change the better the tests are at detecting the structural break.

In Table 1 we vary the stable root $\lambda = .1$, .7 and .9 (labelled experiments 1, 2 and 3). The other parameter settings are: $k = 1$, $\beta = .97$, e_t and ε_t are normally and independently distributed with mean zero, $COV[e_t, \varepsilon_s^T] = 0$ for all t and s, $VAR[(1-\lambda)(1-\beta\lambda) \; e_t] = 1$ and $VAR[\varepsilon_t] = 1$. Thus we start with the situation in which Δx_t is exogenous and is serially uncorrelated. The variance for e_t has been scaled up in order to avoid singularities which would be caused by λ's approaching unity in the cointegrating relation (see equation (2.7)).

In Table 1 for $\tau = 1$ over the various values for λ, we see that the test sizes for F_{mid} and L_c are close to their asymptotic values for the sample sizes considered; there is a slight tendency for overrejection for F_{mean} and underrejection for F_{sup} (for the latter this is particularly true when $\lambda = .9$). Since there are so few instances of substantial overrejection for the size calculations, there appeared to be little need to size-correct the power calculations. Moreover using asymptotic critical values better mimics the actual situation applied researchers face.

For cases with $\tau \neq 1$ we see that power falls substantially as λ gets large. For instance, with $\lambda = .1$, $T = 200$, and $\tau = .5$ all of the tests reject stability more than 80 percent of the time; with $\lambda = .9$ and the same setting the rejection frequency falls to at most 7 percent. As might be expected when $\tau = .5$, the F_{mid} has the best power compared to the others; however when $\tau \neq .5$ we see poorer test performance with F_{mid} relative to the others. The F_{mean} appears to have the best power of the other three (with the poorest results from F_{sup}).

Unfortunately there is a considerable fall in the ability of all of the tests to detect structural breaks in modest size data sets (T = 200) as the cost of adjustment rises (a high λ)[7]. In fact it is apparent from these results that if $\lambda = .9$, $\tau = .25$ and unknown, it would be very unlikely to find the break even with as many observations as 500 since the rejection frequency is only .14 for the best test (F_{mean}). On the other hand, if $\tau = .25$ were known for the same experiment $F_{.25}$ has a 27 percent rejection frequency[8]. This result holds in all experiments: when we calculate the appropriate F test at the true break point F_{τ} always has the highest power. Thus in situations where the researcher has strong *a priori* reason to expect a structural break, our Monte Carlo evidence suggests it may be advisable to use it. Nevertheless if the chosen τ is a long way away from the true one, the loss of power is sizeable. Further from Table 1, for the larger λ it also seems that the structural breaks are easier to detect when they occur at the end of the sample rather than at the beginning.

In Table 4 we provide the average estimated value of τ from the F_{sup} over the 1000 replications together with its standard deviation (the numbers for the experiments refer to experiments with the same number in Table 1).

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7 The same test feature emerged in Gregory (1991) where there was a dramatic fall in power of the tests for cointegration when the stable root is near unity.

8 These results are not shown in the table but are available from the authors upon request.

For $\tau = 1$, the average estimated τ is around .5 with a standard deviation of around .23. When $\tau \neq 1$, the average estimated τ from the F_{sup} test is extremely close to its true value for $\lambda = .1$ at $T = 100$ and given the standard deviations appears to be tightly estimated. Not surprisingly, given the power results, as λ increases the average estimate of τ is close to .5 with a standard error of a similar magnitude to the value obtained when $\tau = 1$. In Figure 1 we estimate the density of $\hat{\tau}_i$ $i = 1,...1000$ for experiment 2 with $\lambda = .5$ and $\tau = .5$ using a normal kernel (see Silverman, 1986) for $T = 100$, 200 and 500. As is clear from the figure with a sample size of 100 the estimates are widely distributed over the interval [.15, .85]. At $T = 200$ the estimate is more tightly distributed around .5 but still has fairly large tails. By $T = 500$ the estimator is reasonably precise with negligible tails.

In Table 3 we present the results from testing for cointegration using the ADF tests in the presence of structural change. The tests are performed with a lag length of one and six in the test regression (see equation (2.8)). For $\lambda = .1$ and .7 with $\tau = 1$ (no structural break) we see that by $T = 200$ the ADF tests with one and six lags reject 100 percent of the time (the exception is for $\lambda = .7$ and six lags where it is .94). For $\lambda = .9$ we see a substantial fall in the power of the ADF tests (for instance $\tau = 1$, $T = 200$ the rejection frequency of the ADF with one lag is 50 percent). The effect of a structural break in the cointegrating relation is to lower the rejection frequency for all the ADF tests in all experiments, particularly when the break is not at the beginning of the sample. For example, the ADF rejection frequency falls to about 72 percent (from 100 percent) for the lag length of one and to 17 and 38 percent (from 100 and 94 percent) for a lag length of six at $\tau = .5$ with $\lambda = .1$ and $\lambda = .7$ respectively. The numbers with six lags are probably more relevant given that in practice the lag length is likely chosen on the basis of some statistical test for serial correlation and that the breaks are likely to manifest themselves as correlated errors. Clearly structural breaks result in dramatic reductions in rejection frequencies of the ADF tests.

In Table 2 we consider several other experiments involving structural breaks. Except for the changes indicated, the parameter settings are identical to those in Table 1 experiment 2 ($\lambda = .7$). In experiment 4 and 5 there is positive ($\rho = .8$) and negative ($\rho = -.5$) correlated Δx_t respectively. In experiment 6 we allow for an endogenous regressor with $\Delta x_t = \varepsilon_t$ being correlated with e_t ($\sigma_{e\varepsilon} = .8$). We know from several papers (see for example Phillips and Hansen, 1990 and Stock and Watson, 1991) that this correlation creates additional nuisance parameters for estimation. In Table 2 we see the rejection frequencies are similar to those in experiment 2, except experiment 4 where there is positive serial correlation. With large positive serial correlation in Δx_t, the rejection frequency for all the tests drops sharply (compare $\tau = .5$ and $T = 500$ for experiment 2 against experiment 4). For experiment 4 it is interesting to note that power is larger for $\tau = .75$ than for $\tau = .5$. In Tables 3 and 4 the ADF and estimated τ results are given for experiments 3-6. Again special attention should be given to experiment 4 where for the ADF test with six lags the rejection frequency for $T = 500$ is only 3 percent (compared to 100 percent when there is no break).

Lastly in Table 5 we investigate the sensitivity of the F tests, ADF and estimated τ to structural breaks in the short-run parameters like the cost of adjustment term in (2.1). For the first part of the sample $\lambda = .7$ and for the second part $\lambda = .5$ or $.9$, with $\theta = 1$ over the whole sample. The various F tests have some power to detect this kind of structural change. However, for the $\lambda = .5$ and .9 at $T = 100$, the results are close to what would be expected from size alone. By $T = 500$ most tests are rejecting around the 10 percent mark for $\tau = .5$. For this kind of structural change, the ADF tests are hardly changed with results very close to those obtained in experiment 2 with $\tau = 1$. The estimated τ from F_{sup} (λ changing to .5 in the second half of the sample) is poorly estimated with a average value around .5 regardless of the true value of τ. In contrast with $\lambda = .9$ in the second part of the sample, the estimated value of τ rises on average with higher sample sizes especially at $\tau = .5$

4. Stability Of U.S. Money Demand

As an application of the tests for structural stability, we examine the stability of the coefficients in the long-run money-demand equation. The two most recent discussions of this issue are in Lucas (1988) and Stock and Watson (1991). The long-run cointegrating relation that is at the center of the debate is:

$$\ln(m_t) - \ln(p_t) = \mu + \theta_1 \ln(y_t) + \theta_2 r_t + \eta_t, \tag{4.1}$$

where m is M1, p is the implicit price deflator, y is real net national product and r is the 6 month commercial paper rate. This specification is identical to Lucas (1988) and Stock and Watson (1991). The data (the same as used in Lucas, 1988) are annual from 1901-1985 giving 86 observations.

In Table 6 part A we estimate the coefficients over the entire sample using the Phillips and Hansen (1990) fully modified (FM) procedure discussed in Section 2. The estimates are quite similar to the OLS estimates found in Lucas (1988) and virtually identical to Stock and Watson's FM estimates (they use a different estimator for the long-run covariance matrices). Also in this table are the tests for structural breaks in the cointegrating relation. All of them indicate a rejection of the null hypothesis of structural stability at the five percent level of significance[9]. The F_{sup} test estimates the break around 1966. The ADF test with one lag rejects the null of no cointegration at the 5 percent level.

The results in Table 6 conflict with the conclusions reached by Stock and Watson (1991) who have argued that the U.S money demand is stable over the same period. They perform a Wald test of structural change with a fixed (known) break point at 1946 and find no significant structural break. Our results and those of Stock and Watson can be reconciled using Figure 2 where we graph the $F(\tau t)$. In the figure we have also placed the appropriate critical values (at the 5 percent level) for the F test with known (fixed) break point and those from F_{sup}. It is evident that there are two spikes in the F test over the sample (1928-1931 and post 1956) that are higher than the critical value appropriate for the F_{sup}. On the other hand, with a fixed alternative there is a local minimum right around the period Stock and Watson calculate their statistic (1946) and this value is close to the critical value for fixed τ.

To help us better interpret our applied results, we conduct a simple Monte Carlo experiment which is calibrated to the U.S. money demand data. The set up for this is given in Table 6 part B. The estimated λ from the data is .72 with a standard error of (.02) (see Gregory, Pagan and Smith, 1990) for details on how to estimate the short-run parameters of the linear quadratic model). For the experiment we set $\lambda = .7$, $\beta = .97$ and the variances and covariances for the disturbances e_t, ε_{1t} and ε_{2t} are estimated

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9 A puzzling feature of the results is that if we recast r_t in natural logarithms and rerun (4.1), we find that all of the F tests retain the null of no structural change at the five percent level of significance.

from $\Delta[\ln(m_t)-\ln(p_t)]$, $\Delta\ln(y_t)$, and $\Delta(r_t)$ respectively. For the case where there is no structural break $\tau = 1$, we use the estimates of θ obtained in part A of Table 6. When $\tau \neq 1$ we use estimates of the split sample (1903-1945, 1946-1987) given in Table 3 labeled PHFM in Stock and Watson (1990). All other features of the exercise are identical to the Monte Carlo experiment 2 in Section 3. Note that this Monte Carlo experiment differs from the earlier ones because there are two regressors in the cointegrating relation.

For the small sample size $T = 100$ and $\tau = 1$ all of the tests reject the null of no break too frequently compared to the appropriate asymptotic critical value. This overrejection disappears by $T = 200$. From this overrejection, it is clear that for all the experiments with $\tau \neq 1$ and $T = 100$, the rejection frequencies (roughly 10 percent) merely reflect the size bias. In fact at $T = 200$ when there is a structural break the rejection frequencies are very close to the corresponding size results. It is not until we have sample sizes of $T = 500$ that we obtain much higher rejection frequencies or better estimates of the break point from the F_{sup}. This last observation is especially true for breaks later in the sample ($\tau = .75$). In all cases we find the rejection frequency for the ADF test is virtually the same regardless of whether there is a break or not.

The Monte Carlo results make it clear that we cannot have much confidence in a conclusion that there are breaks in the U.S money demand. At the same time we believe that conclusions of a stable money demand recently expressed in the literature are premature and need a careful reassessment. Certainly additional work with larger data sets is likely to shed more light on this issue.

5. Conclusion

Econometricians have long understood that a structural break in a regression estimated over the entire sample invalidates conventional significance testing and can produce misleading estimates. Until very recently, we have been unable to test for a structural break with unknown break point in cointegrating relations. Hansen (1991) has developed a series of tests appropriate to this problem using the fully modified estimators of Phillips and Hansen (1990).

The purpose of this paper has been to evaluate these tests in the environment of the quadratic-adjustment model. In the case of a single regressor we have found that the tests have size close to their asymptotic values for reasonably small sample sizes. Power is good provided the stable root is small. However when the cost of adjustment is high producing a large stable root, the data set needs to be large before acceptable rejection frequencies occur. The Monte Carlo evidence for two regressors in the applied example of U.S. money demand suggests that there is a need for further research to document the size as well as the power for larger dimensional problems.

The Monte Carlo evidence indicates that F_{mean} has the best finite sample properties among tests with unknown break point. If the break point is known, then there can be a substantial increase in power from using it in the standard way against χ^2 critical values. Nonetheless this same test will have very poor power properties when the chosen break point is far away from the true point. Given the experience in the Monte Carlo experiment together with the applied U.S. money example, we advise against using arbitrary break points. We also found that the tests have some power in detecting structural change in short-run parameters like the cost of adjustment.

One possible avenue to improve power is to correct for (some of) the serial correlation (see Bewley, 1979) due to the cost of adjustment. This may be done by estimating the λ and constructing a new dependent variable, $y_t+[\hat{\lambda}/(1-\hat{\lambda})]\Delta y_t$ for the cointegrating relation. Gregory, Pagan and Smith (1990) have found that this transformation, which is appropriate in any linear quadratic model regardless of the underlying forcing process for Δx_t, resulted in better coverage probabilities in Monte Carlo experiments for the FM estimators. This extension is currently being pursued.

There are many ways to estimate and conduct inference on cointegrating vectors besides the FM procedure of Phillips and Hansen (1990). These include the maximum likelihood estimator of Johansen (1988) and (1990), the full information estimator of Phillips (1991), and the forward and backward ordinary least squares estimator of Saikkonen (1991) and Stock and Watson (1991). For these procedures the setup is similar to Hansen (1991), suggesting structural change tests might also be developed for these estimators as well.

Another concern of this paper has been to examine tests for cointegration like the augmented Dickey-Fuller in the presence of a structural break. Our Monte Carlo work indicates that the rejection frequency for the ADF test falls considerably when there is a break in the cointegrating vector (but is not greatly affected by changes in short-run parameters like the cost of adjustment). A common practice in applied work is to test for cointegration and proceed to estimate a cointegrating relation only if the null of no cointegration is rejected. Gregory (1991) has shown that these tests suffer a large power loss when the stable root is near unity and the number of regressors is large. When there is a structural break in the cointegrating vector, we have shown a similar reduction in power occurs in these tests. With this in mind, it would be useful to develop a test of the null of no cointegration against the alternative of two cointegrating regimes, say by using the maximal ADF statistic.

Appendix: Estimating Long Run Covariance Matrices

The approach used throughout the Monte Carlo work for estimating long run covariance matrices is due to Andrews (1991) with some important modifications in Andrews and Monahan (1990). Park and Ogaki (1991) have investigated this vector autoregressive prefiltering procedure in the context of estimating cointegrated models. They found using Monte Carlo methods that this estimator of the long run covariance matrix has small bias and mean square error.

Let V_t be a nx1 vector whose long-run covariance matrix is given by Ω. Prewhiten V_t by a finite vector autoregression. Obtain the residuals from this and use an automatic bandwidth for a kernel estimator of the heteroskedastic-autocorrelation consistent (HAC) variance covariance matrix. Recolor to obtain the estimate of the long-run covariance matrix. The procedure is as follows:

1. Prewhitening

$$V_t = \sum_{r=1}^{b} \hat{A}_r \, V_{t-r} + V_t^* \qquad t = b+1 \ldots T \ , \tag{A.1}$$

where $\hat{A}_r$ are (nxn) parameter estimates and V_t^* are the corresponding residuals (in the Monte Carlo work b = 1).

2. HAC estimation of V_t^*

$$\hat{\Omega}^*(S_T) = \sum_{j=-T+1}^{T-1} k\left(j/S_T\right) \, \hat{\Gamma}^*(j), \qquad \hat{\Gamma}^*(j) = \begin{cases} T^{-1}\sum_{t=j+1}^{T} V_t^* \, V_{t-j}^{*T} & \text{for } j \geq 0 \\ T^{-1}\sum_{t=-j+1}^{T} V_{t+j}^* {}^* V_t^T & \text{for } j < 0 \end{cases} \tag{A.2}$$

where S_T is the data dependent (automatic) bandwidth and $k(\cdot)$ is the real-valued quadratic spectral kernel:

$$k(x) = 25/(12\pi^2x^2) \left\{\frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5)\right\}. \tag{A.3}$$

For the quadratic spectral kernel $S_T = 1.3221\ (\hat{\alpha}^*\ T)^{1/5}$ where $\hat{\alpha}^*$ is obtained by regressing V_t^* on V_{t-1}^* with associated coefficient matrix A nxn and innovation covariance matrix Σ and then calculating:

$$\hat{\alpha}^* = \left\{\frac{2\ \text{vec}\ \hat{g}^T\ W_T\ \text{vec}\ \hat{g}}{\text{tr}\ W_T\ (I + K_{nn})\ \hat{f}\otimes\hat{f}}\right\}, \tag{A.4}$$

where

$$\hat{f} = 1/2\pi\ (I-\hat{A})^{-1}\hat{\Sigma}\ (I-\hat{A}^T)^{-1} \tag{A.5}$$
$$\hat{g} = 1/2\pi\ (I-\hat{A})^{-3}\left[\hat{A}\hat{\Sigma}+\hat{A}^2\hat{\Sigma}\hat{A}^T+\hat{A}^2\hat{\Sigma}-6\hat{A}\hat{\Sigma}\hat{A}^T+\hat{\Sigma}(\hat{A}^T)^2+\hat{A}\hat{\Sigma}(\hat{A}^T)^2+\hat{\Sigma}\hat{A}^T\right](I-\hat{A}^T)^{-3}.$$

W_T is a n^2xn^2 diagonal weight matrix with 2's for diagonal elements that correspond to diagonal elements of Ω and 1's for diagonal elements that correspond to non-diagonal elements of Ω, vec is the vectorization operator, $\otimes$ is the Kronecker cross-product and K_{nn} is an n^2xn^2 commutation matrix that transforms vec(A) into vec(A^T).

3. Recolor

$$\hat{\Omega} = \hat{D}\ \hat{\Omega}^*(S_T)\ \hat{D}^T \quad \text{and} \quad \hat{D} = \left[I_n - \sum_{r=1}^{b} \hat{A}_r\right]^{-1}. \tag{A.6}$$

To calculate $\hat{\Lambda}$ for the Stock-Watson test (3.6) we do not prewhiten but follow Andrews (1991) directly. That is:

$$\hat{\Lambda} = = \sum_{j=0}^{T} k(j/S_T)\ \hat{\Gamma}(j) \qquad \hat{\Gamma}(j) = T^{-1}\sum_{t=j+1}^{T} \hat{\nu}_t\ \hat{\nu}_{t-j}^T. \tag{A.7}$$

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Table 1 Structural Breaks and Cost of Adjustment

$$y_t = \theta_t x_t + \eta_t, \quad \begin{cases} \theta_t = 1\ , & t \le [\tau T] \\ \theta_t = .5, & t > [\tau T] \end{cases}, \quad t = 1,\dots T$$

$$x_t = x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim NID(0,1) \qquad (k = 1)$$

	1. $\lambda = .1$				2. $\lambda = .7$				3. $\lambda = .9$			
	F_{mid}	F_{sup}	F_{mean}	L_c	F_{mid}	F_{sup}	F_{mean}	L_c	F_{mid}	F_{sup}	F_{mean}	L_c
τ=1												
T=100	.06	.03	.07	.04	.03	.01	.05	.04	.03	.03	.04	.01
T=200	.06	.04	.08	.06	.03	.02	.05	.04	.04	.02	.06	.04
T=500	.05	.06	.06	.04	.06	.03	.07	.05	.05	.02	.06	.03
τ=.25												
T=100	.25	.60	.52	.38	.04	.01	.06	.06	.02	.02	.03	.01
T=200	.49	.97	.91	.69	.09	.04	.21	.17	.03	.01	.04	.02
T=500	.75	1.0	1.0	.92	.33	.82	.69	.49	.07	.04	.14	.11
τ=.5												
T=100	.93	.62	.73	.56	.13	.02	.09	.06	.03	.03	.05	.02
T=200	.98	.90	.91	.81	.58	.08	.36	.27	.06	.03	.07	.04
T=500	.99	.96	.97	.92	.88	.66	.76	.63	.44	.13	.34	.24
τ=.75												
T=100	.37	.53	.64	.57	.06	.04	.11	.08	.04	.04	.06	.03
T=200	.59	.79	.83	.78	.19	.11	.33	.27	.06	.07	.10	.05
T=500	.74	.89	.91	.88	.43	.61	.71	.61	.22	.22	.40	.29

Notes:
Rejection frequencies at the five percent level of significance using asymptotic critical values in 1000 replications. F_{mid} is the F-test at the midpoint equation (2.15), F_{sup} is the largest F over the interval τ = (.15,.85) equation (2.16), F_{mean} is the average over the same interval equation (2.17) and L_c is the Lagrange multiplier like test equation (2.18).

Table 2 Structural Breaks, Serial Correlation and Endogenous Regressors

$$y_t = \theta_t x_t + \eta_t, \quad \begin{cases} \theta_t = 1, & t \le [\tau T] \\ \theta_t = .5, & t > [\tau T] \end{cases}, \quad t = 1,\ldots T$$

$$x_t = x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim NID(0,1) \qquad (k = 1)$$

	4. $\Delta x_t = .8\Delta x_{t-1} + \varepsilon_t$				5. $\Delta x_t = -.5\Delta x_{t-1} + \varepsilon_t$				6. $\sigma_{e\varepsilon} = .8$			
	F_{mid}	F_{sup}	F_{mean}	L_c	F_{mid}	F_{sup}	F_{mean}	L_c	F_{mid}	F_{sup}	F_{mean}	L_c
τ=1												
T=100	.03	.05	.07	.02	.03	.03	.05	.03	.03	.00	.02	.02
T=200	.04	.02	.04	.03	.04	.02	.06	.04	.03	.00	.04	.03
T=500	.06	.03	.07	.04	.05	.03	.06	.04	.04	.01	.05	.04
τ=.25												
T=100	.03	.06	.08	.03	.03	.01	.05	.03	.06	.03	.12	.07
T=200	.03	.05	.10	.05	.08	.03	.13	.11	.13	.15	.30	.18
T=500	.05	.09	.13	.07	.31	.73	.65	.45	.35	.85	.72	.43
τ=.5												
T=100	.13	.14	.16	.03	.08	.02	.07	.05	.16	.03	.12	.07
T=200	.14	.13	.18	.05	.50	.07	.27	.23	.57	.08	.34	.22
T=500	.14	.11	.15	.06	.97	.81	.87	.73	.78	.49	.58	.42
τ=.75												
T=100	.23	.31	.34	.10	.05	.05	.11	.06	.07	.05	.11	.07
T=200	.22	.29	.31	.13	.16	.10	.30	.26	.11	.06	.23	.18
T=500	.18	.28	.29	.13	.51	.73	.81	.74	.26	.40	.47	.37

Notes:
Rejection frequencies at the five percent level of significance using asymptotic critical values in 1000 replications. F_{mid} is the F-test at the midpoint equation (2.15), F_{sup} is the largest F over the interval τ = (.15,.85) equation (2.16), F_{mean} is the average over the same interval equation (2.17) and L_c is the Lagrange multiplier like test equation (2.18).

Table 3 Structural Breaks and Testing For Cointegration

ADF	EXPERIMENTS 1.	2.	3.	4.	5.	6.
τ = 1						
T = 100	1.0 (.79)	.85 (.35)	.13 (.08)	.34 (.20)	.81 (.36)	.89 (.44)
T = 200	1.0 (1.0)	1.0 (.94)	.50 (.30)	.95 (.72)	1.0 (.93)	1.0 (.95)
T = 500	1.0 (1.0)	1.0 (1.0)	1.0 (1.0)	1.0 (1.0)	1.0 (1.0)	1.0 (1.0)
τ = .25						
T = 100	.92 (.34)	.74 (.33)	.17 (.12)	.19 (.19)	.79 (.36)	.52 (.18)
T = 200	.95 (.48)	.95 (.67)	.49 (.34)	.24 (.23)	.99 (.82)	.78 (.44)
T = 500	.97 (.55)	.98 (.82)	.96 (.86)	.19 (.25)	1.0 (.95)	.87 (.61)
τ = .5						
T = 100	.65 (.11)	.48 (.18)	.13 (.09)	.03 (.05)	.60 (.23)	.28 (.09)
T = 200	.72 (.17)	.72 (.38)	.35 (.23)	.03 (.05)	.85 (.52)	.44 (.16)
T = 500	.74 (.19)	.78 (.49)	.71 (.59)	.02 (.06)	.92 (.71)	.55 (.25)
τ = .75						
T = 100	.53 (.09)	.45 (.14)	.12 (.09)	.02 (.03)	.52 (.18)	.22 (.08)
T = 200	.60 (.13)	.61 (.31)	.30 (.19)	.03 (.04)	.76 (.43)	.33 (.14)
T = 500	.61 (.77)	.65 (.42)	.61 (.48)	.04 (.05)	.82 (.58)	.39 (.18)

Notes:

The augmented Dickey-Fuller test (ADF) uses one lag (see equation(2.8)). Beside these in parentheses are the the ADF test with six lags.

Table 4 Structural Breaks and Estimating Break Points

$\hat{\tau}$	EXPERIMENTS 1.	2.	3.	4.	5.	6.
$\tau = 1$						
T = 100	.51 (.22)	.52 (.22)	.54 (.22)	.53 (.21)	.52 (.22)	.52 (.22)
T = 200	.51 (.23)	.51 (.22)	.51 (.22)	.52 (.23)	.51 (.23)	.50 (.22)
T = 500	.51 (.23)	.51 (.23)	.52 (.22)	.51 (.23)	.51 (.23)	.51 (.23)
$\tau = .25$						
T = 100	.28 (.12)	.46 (.21)	.53 (.22)	.37 (.21)	.46 (.22)	.40 (.19)
T = 200	.24 (.02)	.35 (.18)	.48 (.22)	.33 (.19)	.39 (.20)	.29 (.12)
T = 500	.25 (.01)	.26 (.06)	.39 (.20)	.32 (.19)	.27 (.11)	.26 (.05)
$\tau = .5$						
T = 100	.49 (.07)	.49 (.17)	.52 (.22)	.50 (.21)	.50 (.19)	.51 (.15)
T = 200	.49 (.04)	.47 (.13)	.50 (.20)	.48 (.21)	.49 (.14)	.50 (.12)
T = 500	.49 (.02)	.47 (.09)	.47 (.16)	.48 (.20)	.49 (.08)	.51 (.09)
$\tau = .75$						
T = 100	.71 (.09)	.62 (.20)	.54 (.22)	.56 (.24)	.58 (.21)	.64 (.19)
T = 200	.73 (.06)	.65 (.17)	.58 (.22)	.56 (.23)	.64 (.18)	.67 (.17)
T = 500	.74 (.03)	.69 (.13)	.63 (.19)	.57 (.23)	.71 (.11)	.70 (.15)

Notes:
$\hat{\tau}$ is the estimated break point in the supF test (equation (2.16)) and its standard error (in parentheses).

Table 5 Testing for Structural Breaks when Cost of Adjustment Changes

$$y_t = \theta\, x_t + \eta_t, \quad \begin{cases} \theta = 1; & \lambda_t = .7, \; t \le [\tau T] \\ & \lambda_t = \lambda_2, \; t > [\tau T] \end{cases} , \quad t = 1,\ldots T$$

$$x_t = x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \text{NID}(0,1) \qquad (k = 1)$$

	$\lambda_2 = .5$						$\lambda_2 = .9$					
	F_{mid}	F_{sup}	F_{mean}	L_c	ADF	$\hat{\tau}$	F_{mid}	F_{sup}	F_{mean}	L_c	ADF	$\hat{\tau}$
τ=.25												
T=100	.05	.01	.07	.05	.99	.42 (.21)	.03	.02	.03	.02	.17	.54 (.21)
T=200	.07	.06	.11	.07	1.0	.35 (.20)	.04	.02	.04	.03	.63	.55 (.20)
T=500	.08	.16	.15	.10	1.0	.33 (.20)	.05	.03	.06	.04	1.0	.56 (.20)
τ=.5												
T=100	.06	.01	.05	.04	.97	.45 (.19)	.04	.02	.04	.03	.32	.59 (.28)
T=200	.11	.03	.09	.06	1.0	.40 (.18)	.08	.02	.07	.04	.78	.61 (.16)
T=500	.11	.07	.10	.08	1.0	.39 (.18)	.14	.10	.13	.08	1.0	.64 (.15)
τ=.75												
T=100	.03	.02	.04	.03	.95	.53 (.21)	.03	.02	.06	.04	.48	.64 (.19)
T=200	.05	.01	.06	.05	1.0	.48 (.21)	.06	.07	.12	.08	.94	.71 (.15)
T=500	.06	.03	.05	.05	1.0	.49 (.22)	.10	.30	.25	.13	1.0	.75 (.10)

Notes:
Rejection frequencies at the five percent level of significance using asymptotic critical values in 1000 replications. F_{mid} is the F-test at the midpoint equation (2.15), F_{sup} is the largest F over the interval τ = (.15,.85) equation (2.16), F_{mean} is the average over the same interval equation (2.17) and L_c is the Lagrange multiplier like test equation (2.18). ADF is the augmented Dickey-Fuller test with one lag and $\hat{\tau}$ is the average estimated break point in 1000 replications and in parentheses are the standard deviation.

Table 6 Testing for Structural Breaks in U.S Money Demand

A. Annual Data 1901-1985*

$$\ln(m_t)-\ln(p_t) = \mu + \theta_1 \ln(y_t) + \theta_2 r_t + \eta_t$$

$\hat{\mu}$	$\hat{\theta}_1$	$\hat{\theta}_2$	F_{mid}	F_{sup}	F_{mean}	L_c	ADF	$\hat{\tau}$
1.75	.996	-.101	8.61	17.98	8.83	.817	-4.5.	.80
(.13)	(.03)	(.005)	(7.81)	(14.6)	(5.71)	(.701)	(-3.77)	

*Below the coefficient estimates are the standard errors and below the test statistics are the five percent asymptotic critical values.

B. Monte Carlo Simulations and Structural Breaks (λ = .7)

$$y_t = \theta_{1t}x_{1t} + \theta_{2t}x_{2t} + \eta_t, \quad \begin{cases} \theta_{1t} = \theta_{11}, \ \theta_{2t} = \theta_{21}, & t \le [\tau T] \\ \theta_{1t} = \theta_{12}, \ \theta_{2t} = \theta_{22} & t > [\tau T] \end{cases}, \quad t = 1,\ldots T$$

$$\Delta x_{1t} = .38\, \Delta x_{1t-1} + \varepsilon_{1t}$$
$$\Delta x_{2t} = .06\, \Delta x_{2t-1} + \varepsilon_{2t}$$
$$\{e_t, \varepsilon_{1t}, \varepsilon_{2t}\}^T \sim N(0,\Omega)$$

$$\Omega = \begin{bmatrix} .0075 & .0007 & -.0286 \\ & .0040 & .0165 \\ & & 1.580 \end{bmatrix}$$

$\tau = 1$: $\theta_{11} = .995$, $\theta_{21} = -.101$

$\tau = .25$: $\theta_{11} = .911$, $\theta_{21} = -.102$; $\theta_{11} = .205$, $\theta_{21} = -.018$

	τ = 1						τ = .25					
	F_{mid}	F_{sup}	F_{mean}	L_c	ADF	$\hat{\tau}$	F_{mid}	F_{sup}	F_{mean}	L_c	ADF	$\hat{\tau}$
T=100	.08	.11	.13	.10	.95	.51 (.22)	.07	.11	.13	.08	.94	.51 (.22)
T=200	.04	.01	.04	.05	1.0	.51 (.21)	.04	.01	.05	.05	1.0	.50 (.22)
T=500	.04	.01	.05	.04	1.0	.53 (.22)	.07	.06	.11	.10	1.0	.46 (.22)
	τ = .5						τ = .7					
T=100	.08	.11	.13	.08	.94	.51 (.22)	.08	.11	.13	.08	.93	.52 (.22)
T=200	.07	.01	.07	.06	1.0	.51 (.21)	.06	.02	.07	.08	1.0	.55 (.21)
T=500	.39	.18	.31	.26	1.0	.50 (.18)	.24	.27	.38	.35	1.0	.62 (.20)

Notes:
For part A of the table m is M1, p is the implicit price deflator, y is real net national product and r is 6 month commercial paper rate (see text for further details). For part b of the table, see table 1 for an explanation of the symbols used.

Fig.1: SupF Structural Break Test

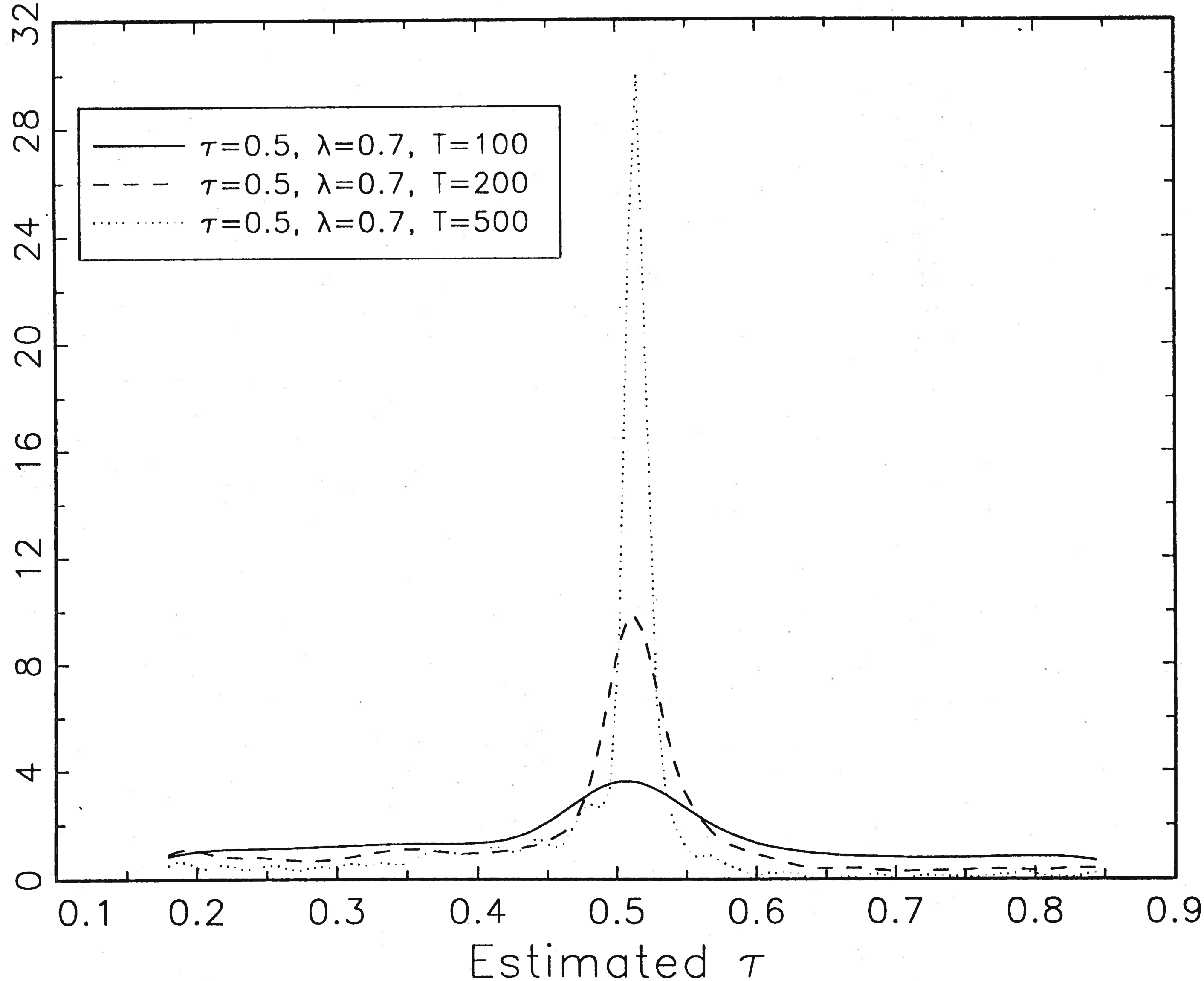

Fig.2: U.S. Money Demand, 1903–85