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Optimal Unemployment Insurance and Redistribution

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Abstract

We characterize optimal income taxation and unemployment insurance in a search-matching framework where both voluntary and involuntary unemployment are endogenous and Nash bargaining determines wages. Individuals differ in utility when voluntarily unemployed (non-participants in the labour market) and decide whether to participate as a job seeker and if so, how much search effort to exert. Unemployment insurance trades off insurance versus moral hazard due to search. We show that it is optimal to have a positive linear wage tax without any redistributive concerns even if search is efficient so the Hosios condition is satisfied. We also allow for different productivity types so there is a redistributive role for the income tax and show that a proportional wage tax internalizes the macro effects arising from endogenous wages. Lump-sum income taxes and transfers can then redistribute between individuals of differing skills and employment states. Our analysis embeds optimal unemployment insurance into an extensive-margin optimal redistribution framework where transfers to the involuntary and voluntary unemployed can differ, and nests several standard models in the literature.

Key Words: Optimal Income Taxation, Unemployment Insurance

JEL: H21, H3, J6

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1 Introduction

Governments engage in both redistribution and social insurance. They redistribute among employed workers earning different incomes and those who choose not to work, and they provide unemployment insurance to those unable to find a job. Optimal redistribution and optimal unemployment insurance have typically been analyzed separately, the former in models where workers have heterogeneous productivities and the latter in single-productivity settings. The purpose of this paper is to embed optimal unemployment insurance analysis into an optimal income tax model of redistribution. This entails policy-makers making a distinction between transfers to the voluntary unemployed (non-participants) and the involuntary unemployed.

There is a large literature on optimal redistribution following the original approach of Mirrlees (1971), and it has been extended in several directions. See, for example, the summaries in Banks and Diamond (2010), Boadway (2012), Golosov and Tsyvinski (2015) and Tuomala (2016). Of relevance for us is the extension to allow for involuntary unemployment due to search frictions (Hungerbühler et al., 2006; Lehmann et al., 2011; Jacquet, et al., 2014; Kroft, et al. 2016). In these papers, following Diamond (1980) and Saez (2002), individuals can vary their labour supply only along the extensive margin: they decide between searching for work in a job market specific to their productivity or being voluntarily unemployed. Employment in each job market is determined by a static matching function (see Mortensen, 1977; Pissarides, 1990; and the survey in Mortensen and Pissarides, 1999), and wages are the outcome of bargaining between each hired worker and a firm. The government observes wages and chooses wage-specific taxes as well as a uniform transfer to all voluntary and involuntary unemployed. Workers are risk-neutral, which obviates the need for unemployment insurance. The analysis characterizes the pattern of optimal taxes and transfers, and compares them with those in the absence of involuntary unemployment. Redistributive taxes take into account their effect both on participation decisions and on wage-setting.

The literature on optimal unemployment insurance is also long-standing (Topel and Welch, 1980; Karni, 1999; Coles and Masters, 2006) and has recently been revisited by Chetty (2008). It focuses on the trade-off between insurance against involuntary unemployment and moral hazard due to search effort when there is no market for unemployment insurance and workers may not be able to self-insure because of liquidity constraints. This approach has been extended by Landais et al., 2016 to allow for endogenous search unemployment. Unemployment insurance continues to trade off insurance against search incentives, and they show how this trade-off varies over the macroeconomic business cycle (Landais et al. 2015). As well, the fact that unemployment insurance can affect the wage rate and that search can be inefficient leads to indirect — or macro — effects that must be taken into consideration. Since all workers are ex ante identical, there is no redistributive motive affecting the choice of unemployment insurance.

We analyze optimal redistribution and unemployment insurance jointly in a model that includes the main features of both the above approaches. We adopt an extensive-margin approach to the labour market augmented by search frictions. Individuals endowed with given productivity decide whether to search for a job at their skill level, and if so how intensively to search. There is a perfectly elastic supply of firms offering jobs at each skill level, and free entry subject to a zero-expected profit constraint determines the number of jobs offered. Successful matches are determined by a matching function, and wages are determined by Nash bargaining. Workers are risk-averse so value unemployment insurance. The government observes wages and therefore worker types, and imposes skill-specific taxes on the employed. The skill of those who do not find work—the involuntary unemployed—or who choose not to participate—the voluntary unemployed—is unobservable so transfers to them cannot depend on skill. However, job search activities are observable and consequently, the transfers to involuntary and voluntary unemployed may differ.

We highlight how incorporating risk aversion has implications for the assumed surplus-sharing rule determining equilibrium wages in the labour market. A key insight of our model is to show that it is optimal to use a proportional wage tax even though the government has access to a lump-sum tax. A skill-type specific proportional wage tax internalizes the various indirect or macro effects that arise through endogenous wages. Consequently, the lump-sum tax-transfer instruments can be used to redistribution without having to take into account of these indirect effects. We also show how incorporating a search effort decision gives rise to another incentive margin that the government must take into account when determining optimal participation taxes. Conversely, with separate transfers to the involuntary and voluntary unemployed the trade-off the government faces in trying to insure individuals against involuntarily unemployment while still providing them with incentives to engage in costly search effort is unaffected by the participation decision.

The remainder of the paper is organized as follows. In the next section, we outline the general model, including the government's problem. In Section 3, we characterize the optimal policies and discuss a simple dynamic extension in Section 4. Finally, we conclude in Section 5.

2 The Model

There is a discrete distribution of skill-types in the economy indexed by $i = 1, ...N$ where type $i+1$ has higher skill than type i. Following the pure extensive-margin labour supply model of Diamond (1980) and Saez (2002), there is job suitable for each skill-type. Denote y_i as the output of a type$-i$ skilled worker if employed, where $y_{i+1} > y_i$. Without loss of generality, the population of each type is the same and normalized to unity. Individuals of a given skill-type only differ in their utility if they choose not to participate, so all labour market participants (of a given skill-type) are identical. Among individuals of type i, n_i choose to participate and search for a job while $1 - n_i$ choose to be voluntarily unemployed. The n_i labour market participants or job-seekers decide how much search effort, denoted by s_i, to undertake.

Following Hungerbühler et al. (2006), there is a separate job matching market for each skill-type. In each market, there is free (costless) entry of firms. Each firm posts one vacancy and the total number of vacancies posted in the labour market for type$-i$ workers is denoted by o_i (for offers). An underlying assumption of these matching models is that there is a competitive lender who is risk neutral and fully insures firms when the lender gives credit to finance the costs of posting a vacancy. Assuming each firm who enters posts one vacancy is for simplicity and does not restrict the analysis given constant returns to scale in production. The n_i job seekers are matched to job vacancies by a constant returns to scale matching function, $m(s_i n_i, o_i)$, which depends on aggregate search effort in the labour market, $s_i n_i$, and total vacancies o_i. The matching function is assumed to be differentiable and increasing in both of its arguments and for simplicity is the same in all markets.

Define labour market tightness (from the firm's perspective) as the ratio of total vacancies to aggregate search effort, $\theta_i = o_i/(s_i n_i)$. Given constant returns to scale in the matching function, we can write total matches as $\ell_i = m(s_i n_i, o_i) = s_i n_i m(1, \theta_i) \equiv s_i n_i f(\theta_i)$ where $f'(\theta_i) > 0$. The probability that a job seeker finds a job is given by $p(s_i, \theta_i) \equiv \ell_i/n_i = s_i f(\theta_i)$.[1] The probability that a vacancy is filled is given by $q(\theta_i) \equiv m(s_i n_i, o_i)/o_i = m(1/\theta_i, 1) = f(\theta_i)/\theta_i$ where $q'(\theta_i) < 0$. Following Landais et al. (2016), we define $1 - \eta_i = \theta_i f'(\theta_i)/f(\theta_i) > 0$ and $\eta_i = -\theta_i q'(\theta_i)/q(\theta_i) > 0$, as the elasticities of $f(\theta_i)$ and $q(\theta_i)$, respectively. A skill-type i individual who is matched to a job produces y_i units of output and is paid wage w_i.

Following the pure extensive labour supply model (Diamond, 1980; Saez, 2002), the government observes individual wage rates or incomes w_i if employed. The government also observes whether someone looked for work, but not their skill type if they do not find a job. Nor can it observe the skill-types of the voluntary unemployed. Government policies include a type-specific linear wage tax rate τ_i, a transfer b_i for the employed, an involuntary unemployment income benefit b^I, and an income transfer b^V given to non-participants, where transfers can be positive or negative.

2.1 Individual Behaviour

Individuals make two decisions: whether to participate in the labour market and if they choose to participate how much search effort to undertake. We can think of these as the extensive and intensive search decisions. We characterize in sequence optimal search and participation decisions.

[1] Landais et al. (2016) assume a similar matching function in aggregate search effort, but the number of job seekers is fixed at unity and there is a single ability type. They define $f(\theta_i)$ as the rate a job seeker finds a job per unit of search effort, i.e., $f(\theta_i) = m(s_i n_i, o_i)/s_i n_i = m(1, \theta_i)$.

2.1.1 Search Effort Decision

A type$-i$ individual who chooses to participate in the labour market takes as given the tightness of the type$-i$ labour market θ_i, the market wage rate w_i and government policies, and chooses search effort s_i to maximize:

$$s_i f(\theta_i) v\big((1-\tau_i)w_i + b_i\big) + \big(1 - s_i f(\theta_i)\big) u(b^I) - \phi(s_i) \tag{1}$$

where $\phi(s_i)$ is the increasing and convex search cost function. The utility of consumption while in the employed state, $v(\cdot)$, can differ from the utility of consumption while in the involuntarily unemployed state, $u(\cdot)$, possibly reflecting some positive disutility of working. Any disutility of working is implicitly assumed to be the same for all workers in this formulation. All utility of consumption functions are assumed to be increasing and strictly concave, and the marginal utility of consumption tends to infinity as consumption tends to zero in all (un)employment states.

The first-order condition is

$$f(\theta_i)\big[v\big((1-\tau_i)w_i + b_i\big) - u(b^I)\big] - \phi'(s_i) = 0 \tag{2}$$

and the second-order condition, $-\phi''(s_i) < 0$, is satisfied. Provided the left-hand side of (2) evaluated at $s_i = 0$ is positive, then the unique interior optimum will have positive search effort. We assume that at the optimum employed individuals are strictly better off than involuntarily unemployed individuals. This implies that any individual will accept a job if they are matched to one. The solution to (2) gives optimal search effort $s_i = s(\tau_i, b_i, b^I; \theta_i, w_i)$. Comparative statics yields

$$\begin{aligned}
\frac{\partial s(\cdot)}{\partial \tau_i} &= -\frac{w_i v'((1-\tau_i)w_i + b_i) f(\theta_i)}{\phi''(s_i)} < 0; & \frac{\partial s(\cdot)}{\partial b^I} &= -\frac{u'(b^I) f(\theta_i)}{\phi''(s_i)} < 0; \\
\frac{\partial s(\cdot)}{\partial b_i} &= \frac{v'((1-\tau_i)w_i + b_i) f(\theta_i)}{\phi''(s_i)} > 0; & \frac{\partial s(\cdot)}{\partial \theta_i} &= \frac{f'(\theta_i)\Delta v_i}{\phi''(s_i)} > 0; \\
\frac{\partial s(\cdot)}{\partial w_i} &= \frac{(1-\tau_i) v'((1-\tau_i)w_i + b_i) f(\theta_i)}{\phi''(s_i)} > 0; & &
\end{aligned} \tag{3}$$

where

$$\Delta v_i \equiv v\big((1-\tau_i)w_i + b_i\big) - u(b^I) > 0 \tag{4}$$

is the utility gain from employment.

2.1.2 Labour Market Participation Decision

Individuals differ in their utility if they do not participate in the labour market, which could reflect differences in home productivities. Alternatively one could re-interpret individuals as differing in some fixed cost of participating or searching for work with the implication that those who are voluntarily unemployed would all be equally well-off as in Kroft et al. (2016). We do not, however, consider heterogeneity over the disutility of work to rule out the possibility of equally skilled workers earning different equilibrium wages. Let δ be the utility benefit of not participating in the labour market and $\mu(b^V)$ be the utility of consumption when voluntarily unemployed (or a non-participant), where as mentioned b^V is the income transfer to the voluntary unemployed. We assume that the cumulative distribution of δ is given by $G(\delta)$ with positive density $g(\delta)$, where δ is distributed on $[\delta_{min}, \delta_{max}]$. We assume the distribution $G(\delta)$ is the same for all skill-types, although that is not important for the qualitative results.

An individual choosing whether to participate in the labour market anticipates optimal search effort $s(\tau_i, b_i, b^I; \theta_i, w_i)$, and will participate if and only if

$$s(\cdot)f(\theta_i)v\big((1-\tau_i)w_i + b_i\big) + \big(1 - s(\cdot)f(\theta_i)\big)u(b^I) - \phi\big(s(\cdot)\big) \geq \mu(b^V) + \delta. \tag{5}$$

The participation constraint (5) is assumed to bind at some $\bar{\delta}_i = \bar{\delta}(\tau_i, b_i, b^I, b^V; \theta_i, w_i) \in (\delta_{min}, \delta_{max})$ so individuals with $\delta \leq \bar{\delta}_i$ participate and those with $\delta > \bar{\delta}_i$ choose not to participate where

$$\begin{aligned}
\frac{\partial \bar{\delta}(\cdot)}{\partial \tau_i} &= -w_i s_i f(\theta_i) v'(\cdot) < 0; & \frac{\partial \bar{\delta}(\cdot)}{\partial b_i} &= s_i f(\theta_i) v'(\cdot) > 0; \\
\frac{\partial \bar{\delta}(\cdot)}{\partial b^I} &= (1 - s_i f(\theta_i)) u'(\cdot) > 0; & \frac{\partial \bar{\delta}(\cdot)}{\partial b^V} &= -\mu'(\cdot) < 0; \\
\frac{\partial \bar{\delta}(\cdot)}{\partial \theta_i} &= s_i f'(\theta_i) \Delta v_i > 0; & \frac{\partial \bar{\delta}(\cdot)}{\partial w_i} &= (1 - \tau_i) s_i f(\theta_i) v'(\cdot) > 0.
\end{aligned} \tag{6}$$

The number of job seekers will be given by $n_i = n(\tau_i, b_i, b^I, b^V; \theta_i, w_i) \equiv G(\bar{\delta}(\cdot))$, so $1 - n(\tau_i, b_i, b^I, b^V; \theta_i, w_i)$ is the number of non-participants or voluntarily unemployed. For any given variable $x \in \{\tau_i, b_i, b^I, b^V, \theta_i, w_i\}$, we have

$$\frac{\partial n(\cdot)}{\partial x} = \frac{\partial \bar{\delta}(\cdot)}{\partial x} g(\bar{\delta}_i). \tag{7}$$

As expected, an increase in b^I or b_i, and a decrease in b^V or τ_i will increase labour market participation. Labour market participation will also be higher, the greater the wage rate and the more likely a job seeker will find a job (as determined by the tightness of the labour market, θ_i). We could assume, as in Saez (2002), that there is a separate group of persons who are unable to work, and if one could observe them, it might be desirable to offer them a different transfer. If they could not be observed, one would have to invoke some mechanism such as screening to identify them. For simplicity, we exclude them from our analysis.

2.2 Firms

Firms incur a cost of k per vacancy posted, which we assume is the same in all labour markets. Given the probability $q(\theta_i)$ that a vacancy is filled, profits per vacancy are $q(\theta_i)(y_i - w_i) - k$. Increases in vacancies/offers cause θ_i to rise, and that reduces q_i for a given w_i since $q'(\theta_i) < 0$. "Entry" of vacancies occurs until firms earn zero expected profits, $q(\theta_i)(y_i - w_i) = k$, so in equilibrium,

$$q(\theta_i) = \frac{k}{y_i - w_i} = \frac{f(\theta_i)}{\theta_i} \tag{8}$$

using $q(\theta_i) = f(\theta_i)/\theta_i$ from above. This implicitly yields

$$\theta\left(\frac{k}{y_i - w_i}\right) \equiv q^{-1}(\theta_i), \quad \text{with} \quad \theta'\left(\frac{k}{y_i - w_i}\right) < 0 \tag{9}$$

since $q'(\theta_i) < 0$. Therefore, we can write $\theta_i = \theta(w_i; y_i)$ which will be decreasing in w_i and increasing in y_i. More explicitly, differentiating (8) and using $1 - \eta_i = \theta_i f'(\theta_i)/f(\theta_i)$ from above, we can derive

$$\frac{\partial\theta}{\partial w_i} = -\frac{\partial\theta}{\partial y_i} = -\frac{\theta_i}{\eta_i(y_i - w_i)} < 0. \tag{10}$$

The probability $q(\theta_i)$ that a vacancy is filled can be written as $q\big(\theta(k/(y_i - w_i))\big)$, or simply as $q(w_i)$ with some abuse of notation, where $q'(w_i) > 0$. Intuitively, an increase in w_i reduces job offers and therefore market tightness θ_i, so the probability of filling a job q_i increases.

Recall that the probability of a job-seeker getting a job is $p(s_i, \theta_i) = s_i f(\theta_i)$. Therefore, using (9) and writing $p\big(s_i, \theta(w_i; y_i)\big)$ as $p(s_i, w_i; y_i)$ for simplicity, we have:

$$p(s_i, w_i; y_i) = s_i f\left(\theta\Big(\frac{k}{y_i - w_i}\Big)\right), \quad \text{with} \quad \frac{\partial p}{\partial s_i}, \frac{\partial p}{\partial y_i} > 0, \ \frac{\partial p}{\partial w_i} < 0 \tag{11}$$

since $f'(\theta_i) > 0$. This is what Kroft et al. (2016) refer to $p(s_i, w_i)$ as the labour demand relation (p.16).

2.3 Wage Determination Process

Following Lehmann and Van der Linden (2007), we assume the wage is determined by asymmetric Nash bargaining after a match is made. Nash bargaining has the important property that its solution is independent of affine transformations of the utility gain from employment. If, instead, the wage is determined by proportional bargaining where workers get some share β of the total surplus and the firm gets $1-\beta$, where the total surplus per match is given by $\Delta v_i + y_i - w_i$, the solution will not be independent of the cardinalization of worker utility, as shown by l'Haridon, Malherbet and Pérez-Duarte (2013). That is, a common affine transformation of utility functions $v(\cdot)$ and $u(\cdot)$ changes Δv_i and therefore the equilibrium wage rate.

Under asymmetric Nash bargaining, the wage w_i satisfies

$$w_i = \text{argmax } \big(\Delta v_i\big)^{\beta}\big(y_i - w_i\big)^{1-\beta} \tag{12}$$

where β is the worker's bargaining power, assumed again to be the same for all worker-types for simplicity and without loss of generality. As noted, the solution to this problem is not affected by affine transforms of the utility functions, e.g. $a + kv(\cdot), a + ku(\cdot)$, and we exploit this by assuming that utility functions are cardinal orderings. Note also that $v\big((1-\tau_i)w_i + b_i\big)$ is the same for all workers in a given search market so the same bargaining applies to all. As mentioned, the assumption that all workers have the same utility function is an important simplification. If workers differed in the disutility of work, then Δv_i would differ by the utility-type of worker and the wage would be utility-type-specific, which would complicate things. Most importantly though, this would result in workers of the same skill-type being paid different wages, something which we rule out with this assumption.

From the first-order condition for problem (12), we solve for the following wage:

$$w(\tau_i, b_i, b^I; y_i) = y_i - \frac{1-\beta}{\beta}\frac{\Delta v_i}{(1-\tau_i)v'((1-\tau_i)w_i + b_i)}. \tag{13}$$

Differentiating (13) we obtain:

$$\frac{\partial w(\cdot)}{\partial \tau_i} = \frac{\beta(y_i - w_i)v_i' - (1-\beta)v_i' w_i + \beta(1-\tau_i)(y_i - w_i)v_i'' w_i}{D_i} \gtreqless 0;$$

$$\frac{\partial w(\cdot)}{\partial b_i} = \frac{(1-\beta)v_i' - \beta(1-\tau_i)(y_i - w_i)v_i''}{D_i} < 0; \qquad \frac{\partial w(\cdot)}{\partial b^I} = \frac{-(1-\beta)u_i'}{D_i} > 0; \tag{14}$$

where

$$D_i \equiv -(1-\tau_i)v_i' + \beta(1-\tau_i)^2(y_i - w_i)v_i'' < 0$$

and

$$\frac{\partial w(\cdot)}{\partial y_i} = \frac{\beta}{1-(1-\beta)\Delta v_i v_i''/(v_i')^2} \in (0,1). \tag{15}$$

From (14), we have

$$\frac{\partial w(\cdot)}{\partial \tau_i} = \frac{\beta(y_i - w_i)v_i'}{D_i} - w_i \frac{\partial w(\cdot)}{\partial b_i} < -w_i \frac{\partial w(\cdot)}{\partial b_i}. \tag{16}$$

A change in τ_i has a different effect on the wage rate than a change in b_i, and this relationship will be useful in what follows. For $\tau_i > 0$ the surplus falls when w_i rises, and a higher τ_i should tend to a lower wage. That is, a rise in w_i does not just imply a transfer from firm to worker: part of the increase in w_i goes to the government due to higher taxes. That is not so with b_i. For any b_i, an increase in w_i is a pure transfer from firm to worker. In fact, the comparative statics show that an increase in b_i (which is like a reduction in tax payments) reduces w_i. However, the effect of τ_i on w_i is ambiguous. A sufficient condition for w_i to be increasing in τ_i is that $-(1-\tau_i)w_i v_i''/v_i' \geq 1$ where the left-hand side is the measure of relative risk aversion when $b_i = 0$.

By substituting (13) into (9) and (10), we can write $\theta\big(w(\tau_i, b_i, b^I; y_i); y_i\big)$ and

$$\frac{\partial \theta(\cdot)}{\partial w_i} = -\frac{\partial \theta(\cdot)}{\partial y_i} = -\frac{\theta_i}{\eta_i}\frac{\beta}{1-\beta}\frac{(1-\tau_i)v_i'}{\Delta v_i}. \tag{17}$$

Both of these will be useful in characterizing the optimal policies.

2.4 Labour Market Equilibrium

The individual's optimal search effort and the number of job seekers can be written solely as functions of the policy parameters, that is,

$$s_i(\tau_i, b_i, b^I) = s\big(\tau_i, b_i, b^I, \theta\big(w(\tau_i, b_i, b^I; y_i); y_i\big), w(\tau_i, b_i, b^I; y_i)\big) \tag{18}$$

and

$$n_i(\tau_i, b_i, b^I, b^V) = G\big(\bar{\delta}_i(\tau_i, b_i, b^I, b^V)\big) \tag{19}$$

where

$$\bar{\delta}_i(\tau_i, b_i, b^I, b^V) = \bar{\delta}\big(\tau_i, b_i, b^I, b^V, \theta(w(\tau_i, b_i, b^I; y_i); y_i), w(\tau_i, b_i, b^I; y_i)\big).$$

Tax/transfer policies $\{\tau_i, b_i, b^I\}$ have a direct effect on the individual's participation and search effort decisions as well as indirect or macro effects through their impact on the labour market equilibrium $\{\theta_i, w_i\}$. The overall impact of these policies on an individual's decisions depends on the relative magnitude of these various effects. The transfer to the voluntary unemployed b^V does not affect the surplus of a match and therefore, does not affect the equilibrium wage or tightness of the labour market. It only affects the participation decision (or, the extensive margin of the search decision) and not the decision about how much to search (the intensive search margin).

To make these various effects explicit, we have from (3), (6) and (7) that

$$\frac{\partial n(\cdot)/\partial w_i}{\partial n(\cdot)/\partial \theta_i} = \frac{\partial s(\cdot)/\partial w_i}{\partial s(\cdot)/\partial \theta_i} = \frac{(1-\tau_i) f(\theta_i) v_i'}{f'(\theta_i)\Delta v_i}$$

and consequently, using (10) we can write for any $x = \{\tau_i, b_i, b^I\}$

$$\frac{ds_i}{dx} = \frac{\partial s(\cdot)}{\partial x} + \frac{\partial s(\cdot)}{\partial w_i}\left[1 - \frac{\beta(1-\eta_i)}{\eta_i(1-\beta)}\right]\frac{\partial w(\cdot)}{\partial x}, \tag{20}$$

$$\frac{dn_i}{dx} = \frac{\partial n(\cdot)}{\partial x} + \frac{\partial n(\cdot)}{\partial w}\left[1 - \frac{\beta(1-\eta_i)}{\eta_i(1-\beta)}\right]\frac{\partial w(\cdot)}{\partial x}. \tag{21}$$

The first term on the right-hand side captures the direct effects of a change in the policy variables τ_i, b_i and b^I on individuals' search and participation behaviour and the second term captures the macro effects on search behaviour and participation decisions. The sign of the bracketed term will be $\gtreqless 0$ as $\eta_i \gtreqless \beta$.

Note that $\eta_i = \beta$ is the Hosios (1990) condition for job market i. With risk-neutral workers, if the Hosios condition is satisfied, it is well-known that search and participation are efficient in the sense that they maximize expected surplus. With risk aversion, things are more complicated since changes in w_i redistribute surplus between the worker and the firm. This affects social welfare unlike in the risk-neutral case where total surplus is all that counts. It is, however, still possible to show that search and participation maximize expected social surplus (normalized expected utility less total costs of job postings and search costs) when the Hosios condition is satisfied. Specifically, with free entry of firms who post costly vacancies and individuals who make participation and search effort decision, the resulting tightness of the labour market $\theta_i = o_i/(n_i s_i)$ in the private market equilibrium with Nash bargaining over wages maximizes expected social surplus when $\eta_i = \beta$ as shown in Appendix A.

2.5 The Government's Problem

Recall that the government observes wages and chooses type-specific proportional income taxes and transfers to the employed, τ_i and b_i for all $i = 1, ...N$, and transfers to the involuntary unemployed, b^I, and to the voluntary unemployed, b^V. We assume that the population is large enough such that the government faces no uncertainty. It knows the shares of the population made up of the employed and the voluntary and involuntary unemployed, and therefore it knows total tax revenues and transfers.

Policies are chosen to maximize the following utilitarian social welfare function:

$$\sum_{i=1}^{N}\Bigg(n_i(\tau_i, b_i, b^I, b^V)\Big[s_i(\tau_i, b_i, b^I)f\big(\theta_i(\tau_i, b_i, b^I)\big)v\big((1-\tau_i)w_i(\tau_i, b_i, b^I)+b_i\big)$$
$$+\big(1-s_i(\tau_i, b_i, b^I)f(\theta_i(\tau_i, b_i, b^I))\big)u(b^I)-\phi\big(s_i(\tau_i, b_i, b^I)\big)\Big] \tag{22}$$
$$+\int_{\bar{\delta}_i(\tau_i, b_i, b^I, b^V)}^{\bar{\delta}_{max}}\big(\mu(b^V)+\delta_i\big)g(\delta)d\delta\Bigg)$$

where the three terms represent the employed, the involuntary unemployed and the voluntary unemployed. The government's budget constraint is:

$$\sum_{i=1}^{N} n_i(\tau_i, b_i, b^I, b^V)\Big(s_i(\tau_i, b_i, b^I)f(\theta_i(\tau_i, b_i, b^I))b_i + (1 - s_i(\tau_i, b_i, b^I)f(\theta_i(\tau_i, b_i, b^I)))b^I\Big)$$
$$+\sum_{i=1}^{N}(1-n_i(\tau_i, b_i, b^I, b^V))b^V = \sum_{i=1}^{N} n_i(\tau_i, b_i, b^I, b^V)s_i(\tau_i, b_i, b^I)f(\theta_i(\tau_i, b_i, b^I))\tau_i w_i(\tau_i, b_i, b^I) \tag{23}$$

Using the Envelope Theorem from the individual's optimal search effort and participation decisions, the first-order conditions on τ_i and b_i for all $i = 1, ..., N$, b^I and b^V can be written as follows, where λ is the multiplier on the government's budget constraint:

$$-n_i s_i f(\theta_i) v_i'(\cdot) w_i + A_i \frac{\partial w_i}{\partial \tau_i} - \lambda \Big(- n_i s_i f(\theta_i) w_i - t_{pi} \frac{dn_i}{d\tau_i} - t_{ei} n_i f(\theta_i) \frac{ds_i}{d\tau_i} + \Big[- t_{ei} n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} - n_i s_i f(\theta_i) \tau_i \Big] \frac{\partial w_i}{\partial \tau_i} \Big) = 0 \qquad i = 1, ...N \tag{24}$$

$$n_i s_i f(\theta_i) v_i'(\cdot) + A_i \frac{\partial w_i}{\partial b_i} - \lambda \Big(n_i s_i f(\theta_i) - t_{pi} \frac{dn_i}{db_i} - t_{ei} n_i f(\theta_i) \frac{ds_i}{db_i} + \Big[- t_{ei} n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} - n_i s_i f(\theta_i) \tau \Big] \frac{\partial w_i}{\partial b_i} \Big) = 0 \qquad i = 1, ...N \tag{25}$$

$$\sum_{i=1}^{N} \Big(n_i \big(1 - s_i f(\theta_i)\big) u'(b^I) + A_i \frac{\partial w_i}{\partial b^I} \Big) - \lambda \sum_{i=1}^{N} \Big(n_i \big(1 - s_i f(\theta_i)\big) - t_{pi} \frac{dn_i}{db^I} - t_{ei} n_i f(\theta_i) \frac{ds_i}{db^I} + \Big[- t_{ei} n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} - n_i s_i f(\theta_i) \tau \Big] \frac{\partial w_i}{\partial b^I} \Big) = 0 \tag{26}$$

$$\sum_{i=1}^{N} \Big((1 - n_i) \mu'(\cdot) - \lambda \Big((1 - n_i) - t_{pi} \frac{\partial n_i}{\partial b^V} \Big) \Big) = 0 \tag{27}$$

where

$$\begin{aligned} A_i &\equiv n_i s_i f'(\theta_i) \frac{\partial \theta_i}{\partial w_i} \Delta v_i + n_i s_i f(\theta_i) v_i'(\cdot)(1 - \tau_i) \\ &= n_i s_i f(\theta_i) v_i'(\cdot)(1 - \tau_i) \left[1 - \frac{\beta(1 - \eta_i)}{\eta_i(1 - \beta)} \right] \gtreqless 0 \quad \text{for} \quad \eta_i \gtreqless \beta \text{ and } \forall i, \end{aligned} \tag{28}$$

with the second equality following from using (17). The interpretation of A_i is as follows. Holding participation and search effort constant, A_i shows the effect of a marginal increase in the equilibrium type$-i$ wage w_i on the value of the government objective. Social welfare (given n_i and s_i) may be increasing or decreasing in w_i depending on whether the elasticity of the probability of filling a vacancy, η_i, is greater or less than the worker's bargaining power, β.

The variable t_{pi} is the participation tax on ability-type i and is defined by

$$t_{pi} \equiv - \Big[s_i f(\theta_i)(b_i - \tau_i w_i) + (1 - s_i f(\theta_i)) b^I - b^V \Big] \tag{29}$$

and t_{ei} is the employment tax defined as

$$t_{ei} \equiv - \Big[b_i - b^I - \tau_i w_i \Big]. \tag{30}$$

The participation tax describes the net effect on government revenue as an individual moves from non-participation (voluntary unemployment) to participation in the labour market. The government no longer transfers b^V, it raises revenue $\tau_i w_i - b_i$ from an employed worker with probability $s_i f_i$, and it pays out an involuntary unemployment transfer b^I with probability $(1 - s_i f_i)$. The employment tax describes what happens to government revenue when a participating worker moves from being involuntary unemployed to employed. The government raises revenue $\tau_i w_i - b_i$ and no longer pays out b^I.

3 Optimal Policies

We begin by examining the role for a proportional wage tax alongside a lump-sum wage tax. We then characterize the optimal unemployment transfers and optimal participation taxes, highlighting how incorporating an intensive search effort decision within an extensive labour supply model affect standard results in the literature.

3.1 Role of Proportional Wage Taxes

We begin by rewriting the first-order conditions on τ_i, b_i, and b^I of the government's problem (24)–(26) using the expressions in (20), (21), the expressions for $\partial s(\cdot)/\partial w_i$ and $\partial n(\cdot)/\partial w_i$ from (3) and (7), respectively and where A_i, t_{pi} and t_{ei} are as defined in (28), (29) and (30):

$$-n_i s_i f(\theta_i) v_i'(\cdot) w_i + \Omega_i \frac{\partial w(\cdot)}{\partial \tau_i} + \lambda \left[n_i s_i f(\theta_i) w_i + t_{pi} \frac{\partial n(\cdot)}{\partial \tau_i} + t_{ei} n_i f(\theta_i) \frac{\partial s(\cdot)}{\partial \tau_i} \right] = 0 \qquad (31)$$

$$n_i s_i f(\theta_i) v_i'(\cdot) + \Omega_i \frac{\partial w(\cdot)}{\partial b_i} + \lambda \left[- n_i s_i f(\theta_i) + t_{pi} \frac{\partial n(\cdot)}{\partial b_i} + t_{ei} n_i f(\theta_i) \frac{\partial s(\cdot)}{\partial b_i} \right] = 0 \qquad (32)$$

$$\sum_{i=1}^{N} \Bigg[n_i (1 - s_i f(\theta_i)) u'(b^I) + \Omega_i \frac{\partial w(\cdot)}{\partial b^I} + \lambda \left(- n_i (1 - s_i f(\theta_i)) + t_{pi} \frac{\partial n(\cdot)}{\partial b^I} + t_{ei} n_i f(\theta_i) \frac{\partial s(\cdot)}{\partial b^I} \right) \Bigg] = 0 \qquad (33)$$

where

$$\Omega_i = A_i + \lambda \left(t_{pi} A_i \frac{g(\bar{\delta}_i)}{G(\bar{\delta}_i)} + t_{ei} A_i \frac{f(\theta_i)}{s_i \phi''(s_i)} + n_i s_i f(\theta_i) \left[- t_{ei} \frac{\beta(1-\eta_i)}{\eta_i(1-\beta)} \frac{(1-\tau_i) v_i'}{\Delta v_i} + \tau_i \right] \right). \qquad (34)$$

The expression Ω_i in (34) is the coefficient of the terms involving the direct effects of τ_i, b_i and b^I on the wage rate w_i in the first-order conditions for these variables, (31)–(33), the so-called macro effects of these policies (Landais et al. (2015)). Equivalently, (34) is derivative of the Lagrangian of the government problem with respect to the wage rate w_i. It captures all of the effects of a change in w_i on the Lagrangian including the direct effect on social welfare, as given by A_i, and the various revenue effects through a) changes in participation, n_i, as given by first term in the brackets, b) changes in search effort, s_i, which affects employment as given by the second term in the brackets, c) changes in the tightness of the labour market which also affects employment, as given by the first term in the square brackets, as well as d) the direct revenue effect of an increase in w_i as given by the last term in square brackets.

Using these first-order conditions, we can derive a series of results characterizing the optimal role for a proportional wage tax.

Result 1 *With exogenous wage rates, a type-specific proportional wage tax τ_i and a type-specific transfer to the employed b_i are perfect policy substitutes.*

Result 1 follows directly from the two first-order conditions (31) and (32). From (3) and (6), we obtain $\partial s(\cdot)/\partial \tau_i = -w_i \partial s(\cdot)/\partial b_i$ and $\partial n(\cdot)/\partial \tau_i = -w_i \partial n(\cdot)/\partial b_i$. Assuming that w_i is fixed, these imply that the first-order condition on b_i is equivalent to $(-w_i)$ times the first-order condition on τ_i. Therefore, any allocation that can be achieved by using τ_i could also be obtained by using b_i. The two policy instruments are perfect policy substitutes.

A corollary of Result 1 is that if, in addition to fixed wages, there were no search effort and participation decisions (s_i and n_i also fixed), the government could provide full unemployment insurance, that is, equalize the marginal utility of consumption for employed, involuntarily unemployed and voluntarily unemployed individuals, $v'((1-\tau_i)w_i + b_i) = u'(b^I) = \mu'(b^V)$ for all $i = 1, ..., N$. This first-best outcome follows directly from (32), (33) and (27) when w_i, s_i and n_i are all fixed. Given Result 1, this first-best outcome with full insurance against involuntary unemployment can be achieved by using only b_i and b^I as policy instruments: if there is under-insurance, so $w_i + b_i$ is too high relative to b^I for a given i, then b_i can be reduced until full insurance is achieved and w_i is not affected. This logic continues to hold when individuals decide both whether to participate in the labour market and how hard to look for work, so moral hazard prevents the government from providing full unemployment insurance.

In the standard extensive labour supply model, wages are observable and fixed (e.g., Saez (2002)). Extending this model to allow for involuntary unemployment with an intensive search decision, we have shown that the form of wage tax does not matter. We now show that this result no longer holds when wages are endogenous. When the wage rate is endogenous, reducing b_i increases w_i by (14), and changes in w_i will have effects on social welfare both directly and indirectly through changes in government revenue. Thus, moving toward full insurance using b_i cannot be guaranteed to be optimal, and this will be the case even without any search or participation decisions. Allowing for an additional policy instrument, τ_i, gives the government additional degrees of freedom. With an endogenous wage, there is now a separate role for the proportional income tax rate alongside b_i, b^I and b^V as stated in Result 2.

Result 2 *With endogenous wage rates, the optimal proportional wage tax τ_i ensures that optimal income transfers to the employed, b_i, and to the involuntary unemployed, b^I, are independent of changes in the equilibrium wage rate and the tightness of the labour market.*

To see this result, rewrite (31), the first-order condition on τ_i, using the two relationships from (3) and (6) noted above as well as the relationship in (16) (where all arguments of functions have been suppressed):

$$-n_i s_i f(\theta_i) v_i' w_i + \Omega_i \frac{\beta(y_i - w_i) v_i'}{D_i} - \Omega_i w_i \frac{\partial w}{\partial b_i} + \lambda w_i \left[n_i s_i f(\theta_i) - t_{pi} \frac{\partial n}{\partial b_i} - t_{ei} n_i f(\theta_i) \frac{\partial s}{\partial b_i} \right] = 0$$

Substituting the first-order condition for b_i, (32), into the above yields:

$$\Omega_i \frac{\beta(y_i - w_i) v_i'}{D_i} = 0 \qquad (35)$$

Eq. (35) represents the difference in the effect of τ_i on the Lagrangian expression relative to b_i. Given a positive surplus to any match and $D_i < 0$, it follows that $\Omega_i = 0$ for all

$i = 1, ..., N$. Substituting $\Omega_i = 0$ into (32) and (33) yields

$$n_i s_i f(\theta_i) v_i'(\cdot) + \lambda \Bigg[- n_i s_i f(\theta_i) + t_{pi} \frac{\partial n(\cdot)}{\partial b_i} + t_{ei} n_i f(\theta_i) \frac{\partial s(\cdot)}{\partial b_i} \Bigg] = 0 \tag{36}$$

$$\sum_{i=1}^{N} \Bigg[n_i(1 - s_i f(\theta_i)) u'(b^I) + \lambda \Bigg(- n_i(1 - s_i f(\theta_i)) + t_{pi} \frac{\partial n(\cdot)}{\partial b^I} + t_{ei} n_i f(\theta_i) \frac{\partial s(\cdot)}{\partial b^I} \Bigg) \Bigg] = 0 \tag{37}$$

These simplified first-order conditions on b_i and b^I are independent of changes induced in w_i for any i, implying that macro effects do not affect the conditions governing the choice of b_i or b^I as stated in Result 2. The choice of b^V, as given by (27), remains unaffected by the macro effects.

Result 2 arises from our assumed wage determination process since the proportional income tax rate and the income transfer to the employed differentially affect the bargained wage under Nash bargaining. We have, however, assumed Nash bargaining for a very specific and important reason. With a utilitarian social welfare function, the social ordering is unaffected by a common affine transformation of individual utilities. Therefore, to ensure that the optimal allocation is also unchanged with affine utility transformations, wage bargaining outcomes must also be unaffected. This will be satisfied by Nash bargaining as discussed above. This would not hold under proportional bargaining assumed by Landais et al. (2016) and Kroft et al. (2016). Under a proportional sharing rule, changes in τ_i and b_i have the same affect on the bargained wage. Consequently, they would be policy substitutes and the unemployment transfer would have to take macro effects into account. With risk aversion, however, the equilibrium wage rate under proportional bargaining would also be affected by affine transformations of utility. Therefore, the optimal allocation would change with any common affine transformation of individual utilities under proportional bargaining even though the social ordering has not. We have ruled out this possibility by assuming that wages are determined by Nash bargaining and in doing so have identified a separate role for a proportional wage tax alongside a lump-sum wage tax.

As long as the first-order condition on τ_i holds, any induced changes in w_i will have no net effect on social welfare. The values for b_i, b^I and b^V can be changed to obtain optimal unemployment insurance with moral hazard as well as optimal transfers to the voluntary unemployed. The induced effects of changes in these transfers on the wage rate — the macro effects — are no longer operational since these are neutralized by the choice of τ_i. A straightforward corollary of Result 2 then is that the government can ensure that with endogenous wages full unemployment insurance is obtained in the case of no moral hazard just as in the case of fixed wages.

As discussed above, when the Hosios condition is satisfied the resulting tightness of the labour market will be efficient under Nash bargaining. The bargained wage, however, may not be optimal given risk-averse individuals. In the analysis of Hungerbühler et al (2006) where workers are risk-neutral, wage bargaining yields the optimal wage when the Hosios condition is satisfied. There is no need for policies to correct the wage rate. That is no longer the case with risk-averse workers. Unlike the case with risk neutral workers where total surplus is all that matters, with risk averse workers changes in the wage rate redistributes surplus between the worker and the firm which in turn affects social welfare. Consequently, there can still be a role for a wage tax to correct the wage rate even when the Hosios condition is satisfied.

To show this, note that the Hosios condition implies that $\eta_i = \beta$, so $A_i = 0$ from (28), and the first-order condition on τ_i, (35), reduces to:

$$\lambda\left(n_i s_i f(\theta_i)\Big[-t_{ei}\frac{(1-\tau_i)v_i'}{\Delta v_i}+\tau_i\Big]\right)=0. \tag{38}$$

When the Hosios condition is satisfied, the equilibrium wage affects only revenues and it does so through two effects in (38) — a direct positive effect (given by τ_i times the number of employed) and a negative indirect effect through its effect on the tightness of the labour market (given by the employment tax times the change in the number of employed with a change in θ_i as a result of a change in w_i as shown in (17)). The optimal wage tax rate ensures these two effects exactly offset one another.

To see this more clearly, use (17) in (38) to obtain:

$$t_{ei}n_i s_i f'(\theta_i)\theta'(w_i)+n_i s_i f(\theta_i)\tau_i=0 \tag{39}$$

The first term is the employment tax t_{ei} times the change in employment when w_i increases (which is negative), that is, the indirect effect of a change in w_i on tax revenues via the induced change in employment. The second term is the direct effect of a change in w_i on income tax revenues from all $n_i s_i f(\theta_i)$ employed workers. Note that even if the Hosios condition is satisfied so search is optimal, an induced change in the wage rate will still affect maximized social welfare through its effect on government revenues. That is, the macro effect does not disappear when the Hosios condition is satisfied. There is now a revenue effect arising from endogenous wages that the proportional wage tax can be used to take into account. Whether the optimal proportional wage tax will be positive or negative will depend on the sign of the employment tax.

It is possible to sign the optimal proportional wage tax in the efficiency-only case when there is only a single skill-type so the government is not concerned with redistribution between skill-types. Suppressing i, we can obtain from the government's budget constraint an expression for the employment tax, $t_e = -(b - b^I - \tau w) = b^I/(sf) + (1-n)b^V/(nsf)$ which together with (38) can be used to solved for the optimal wage tax rate:

$$\frac{\tau}{1-\tau} = \left(\frac{b^I}{sf} + \frac{(1-n)b^V}{nsf}\right)\frac{v'}{\Delta v}.$$

Therefore, if $b^I, b^V > 0$, the optimal proportional wage tax is positive, $\tau > 0$. It is important to remember that the government also has access to a lump-sum wage tax which could be used to raise revenue to finance the positive transfers to the unemployed. The insight of our analysis is that the government will want to also impose a positive distortionary tax to ensure the wage rate itself is optimal.

This insight carries over to when the Hosios condition does not apply, but now the proportional wage tax is also being used to correct for search externalities and consequently, τ is not necessarily positive. To see this, rewrite (35) as follows using the definition of Ω given by (34), again suppressing the $i-$subscripts:

$$\frac{\tau}{1-\tau} = t_e \frac{v'}{\Delta v}\frac{\beta(1-\eta)}{\eta(1-\beta)} - \left(1 - \frac{\beta(1-\eta)}{\eta(1-\beta)}\right)\left(\frac{1}{\lambda} + t_p \frac{g(\bar{\delta})}{G(\bar{\delta})} + t_e \frac{f(\theta)}{\phi''(s)}\right).$$

The first-term on the right-hand side is positive for any values of η and β. The last term is negative if $\eta > \beta$, and could be large enough to cause $\tau < 0$ overall. The proportional wage tax is now used to correct for search externalities and they can be positive or negative overall.

Allowing for redistribution between skill-types does not change this fundamental role for the proportional wage tax. Whether the government will optimally set τ_i to be positive or negative when the Hosios condition is satisfied will depend on whether the employment tax on skill type$-i$ is positive or negative. Given that the government is transferring to the unemployed, $b^I, b^V > 0$, aggregate employment taxes must be positive, but it is possible employment taxes for some skill-types may be negative. In other words, some individuals may receive an employment subsidy.

3.2 Optimal Unemployment Insurance

By Result 2, the government can use the employment transfer b_i and the unemployment transfer b^I to redistribute between the employed and the involuntary unemployed without having to account for their macro effects on endogenously determined wages. Given that the government can observe who is voluntarily unemployed and transfers b^V to them, the insurance-incentive trade-off the government faces is the same as in a model with fixed wages and no participation decisions.

To see this, eliminate λ from the first-order conditions on b_i and b^I, (32) for all $i = 1, ...N$ and (33) respectively, and simplify to give:

$$\left(\frac{\sum_i n_i(1 - s_i f(\theta_i))}{\sum_i n_i}\right) \sum_i n_i s_i f(\theta_i) \left(\frac{v_i' - u'(b^I)}{v_i'}\right) = \sum_i t_{ei} n_i f(\theta_i) \frac{\partial s_i}{\partial b^I} \tag{40}$$

The left-hand side represents an insurance effect: the smaller the difference between u' and v_i', the more is a type$-i$ worker insured against changes in consumption. Insurance benefits all of the involuntary unemployed and the first-term on the left-hand side is the share of the population that is involuntarily unemployed. The right-hand side is a moral hazard effect: the more insurance is provided (the larger is b^I), the less intensively workers will search. The government trades off the effect of b^I on insurance versus its effect on search. This trade-off is the same as in Chetty (2008) except there are multiple skill-types and a participation decision.

This insurance-search incentive trade-off is independent of individuals' participation decisions, and we have Result 3 below. Increases in b_i and b^I both positively affect participation. Consequently, when solving for λ from the first-order conditions on b_i and b^I the resulting terms involving $\partial n/\partial b_i$ and $\partial n/\partial b^I$ cancel. This is not true with the terms involving search effort since b_i and b^I have opposing effects on search effort: an increase in b_i increases search effort and an increase in b^I reduces it. The government therefore faces a trade-off in its choice of b_i and b^I between insuring individuals against unemployment and inducing them to search for work. Further given the government can observe job search activities it can transfer a different amount to the voluntary unemployed.

Result 3 *The government's choice of unemployment insurance involves an incentive-insurance trade-off with endogenous search effort, and is independent of the participation decision.*

Finally, recall the important point that unlike Landais et al. (2016) macro effects of policy on the wage rates do not affect the conditions on transfers b_i and b^I, and therefore on optimal unemployment insurance. All effects arising from the social welfare and revenue effects of changes in the equilibrium wage rates are addressed by the choice of marginal tax rate τ_i. Of course, if the government could not distinguish between the voluntary and involuntary unemployed and was constrained to set $b^I = b^V$, then the latter part of Result 3 would not hold. But, even in this informational constrained case the government would continue to use the proportional wage tax to take into account of the various macro effects arising from having endogenous wages and the transfer to the employed and single transfer to the unemployed would again be independent of these macro effects. If, as in Chetty (2008), pre-tax wages are fixed and not affected by policies, then having a wage tax τ_i alongside an income transfer to the employed would be redundant.

3.3 Optimal Participation Taxes

We now turn to determining how allowing for involuntary unemployment and a search effort decision in an extensive labour supply model affects the optimal participation taxes.

Consider first the implications of involuntary unemployment without a search effort decision. To do so, assume search is fixed. By Result 2, the government uses the proportional wage tax to internalize all of the macro effects of having an endogenous wage and it follows by combining the first-order conditions on b_i for all i and b^I, (36) and (37) respectively, that the government would provide full unemployment insurance ($v_i'((1-\tau_i)w_i + b_i) = u'(b^I)$ for all $i = 1, ...N$). As individuals are risk averse, the government makes it more attractive to participate in the labour market by providing full unemployment insurance.

The optimal participation tax can then be derived directly from the first-order condition on b_i, (36), and is given by:

$$t_{pi} = n_i s_i f(\theta_i) \frac{1 - v_i'/\lambda}{\partial n/\partial b_i}. \tag{41}$$

This optimal participation tax rule is equivalent in interpretation to the one obtained by Saez (2002) who does not allow for involuntary unemployment and assumes fixed wages. The expression v_i'/λ in the numerator is the value in terms of government revenue of an increment in transfer to an employed individual and is the same as the social welfare weight, g_i as defined in Saez (2002). The expression in the denominator reflects the responsiveness of the tax base (participation) to the tax and will be positive give a higher employment income transfer induces participation.[2] If there are no income effects so the equilibrium number of participants is unaffected by individuals receiving an additional dollar of income regardless of being employed, involuntarily unemployed and voluntarily unemployed, then the sum of the welfare weights g_i over the discrete number of skill types would be unity. Further, as shown in Saez (2002), if the welfare weight is decreasing in skill-type and the government puts sufficient weight on the least-skilled individuals, i.e. $g_1 > 1$, then the optimal participation tax will be negative at the bottom of the skill distribution. Of course, by allowing for income effects this normalization of the welfare weights does not necessarily hold and v_i' could be greater or less than λ for any skill type$-i$.

Allowing for involuntary unemployment alone does not affect the structure of the optimal participation taxes obtained in extensive labour supply models with fixed wages given the government has access to a proportional wage tax to internalize the macro effects. Consider

[2] Expression (41) can be rewritten in a more familiar form as

$$\frac{T_{Pi}}{1 - T_{Pi}} = \left(1 - \frac{v_i'}{\lambda}\right) \frac{1}{\epsilon_{Pi}}$$

where $T_{Pi} = t_{pi}/s_i f(\theta_i) w_i$ is the share of expected wage individuals retain if they choose to participate, that is, $c_i - c_V = (1 - T_{Pi}) s_i f(\theta_i) w_i$, where $c_i = s_i f(\theta_i)((1 - \tau_i) w_i + b_i) + (1 - s_i f(\theta_i)) b^I$ is expected consumption from participating, $c_V = b^V$ is consumption from not participating and $\epsilon_{Pi} = \big(dn/d(c_i - c_V)\big)(c_i - c_V)/n_i$ is the participation elasticity. The above tax rule is obtained by considering the effect of a marginal change in b_i on the difference in consumption between participating and not, i.e., $d(c_i - c_V) = s_i f(\theta_i) db_i$, starting from the optimal tax/transfer system.

now the implications of allowing for a search effort decision. With endogenous search, the government can no longer equalize the marginal utility of consumption between the employed and the involuntary unemployed. Recall, an individual will only exert positive search effort if he is better of working than being involuntarily unemployed, that is, if $\Delta v_i = v((1-\tau_i)w_i + b_i) - u(b_i^I) > 0$. We have left unspecified the relationship between v and u, but under reasonable assumptions, e.g., $v(c) = u(c-\rho)$ or $v(c) = u(c) - \rho$ where $\rho \geq 0$ is some disutility from working, it follows directly from $v_i > u(b^I)$ that $v_i' < u'$ and individuals are not fully insured against involuntary unemployment.

The first-order condition (36) on b_i can be rewritten as

$$s_i f(\theta_i) \frac{v_i'}{\lambda} + t_{pi} \frac{\partial n(\cdot)}{\partial b_i} \frac{1}{n_i} + t_{ei} f(\theta_i) \frac{\partial s(\cdot)}{\partial b} = s_i f(\theta_i) \tag{42}$$

which equates the benefits and costs of an increase in b_i in terms of government revenue. The benefits are on the left-hand side. The first term is the so-called mechanical effect of the change, and includes the social value of the change in b_i to existing employed type$-i$ individuals, $s_i f(\theta_i) v_i'/\lambda$. The second term is the additional revenue raised by the increase in participation induced by the change in b_i, and the last term is the increase in revenue induced by an increase in employment resulting from more intensive search. The right-hand side is the revenue cost of the increased transfer to the $s_i f(\theta_i)$ employed type$-i$ workers. Whether the additional benefit through affecting the intensity of search is positive or negative depends on the sign of the employment tax, t_{ei}.

To see this more clearly, solving directly for t_{pi} yields:

$$t_{pi} = n_i s_i f(\theta_i) \frac{1 - v_i'/\lambda}{\partial n_i/\partial b_i} - t_{ei} n_i f(\theta_i) \frac{\partial s_i/\partial b_i}{\partial n_i/\partial b_i}. \tag{43}$$

The sign of the last term depends on the side of t_{ei}. As discussed above, it is possible in this case that the employment taxes for some skill-types are positive while for others they are negative. Consequently, whether endogenous search tends to increase or decrease the optimal participation tax depends on the sign of this employment tax and we have Result 4.

Result 4 *Endogenous search effort affects the structure of the optimal participation taxes.*

We now consider three special cases in which endogenous search puts downward pressure on the optimal participation tax: no redistribution between skill-types, allowing for a type-specific linear progressive income tax in which the lump-sum component can differ by employment status, and unobservable job search activities.

First, suppose there is no redistribution between skill-types by assuming a single skill-type. In this case, the optimal participation tax is given by (43) with the i subscript suppressed. Recall with a single-skill type, it follows from the government's budget constraint that the employment tax will be positive. In addition, a higher transfer to the employed induces both greater search effort and more participation. Therefore, the last term on the right-hand side of (43) will be negative. Moral hazard arising from the search effort decision puts *downward pressure* on the optimal participation tax relative to the case with fixed search effort in the case of a single skill-type.

Second, suppose the government was able to condition the involuntary unemployment transfer on skill-type by providing b_i^I. This would be the case of a type-specific linear income tax where the lump-sum component depends on the employment status of the individual. Any change in b_i^I would only affect the search effort decision of the type$-i$ individual. Moral hazard continues to prevent the government from providing full unemployment insurance, $u' > v_i'$, but the government will now face an insurance-search incentive trade-off for each skill-type which will be given by:

$$s_i\big(1 - s_i f(\theta_i)\big)\frac{v_i' - u'(b_i^I)}{v_i'} = t_{ei}\frac{\partial s_i}{\partial b_i^I}.$$

The above trade-off implies that all individuals face a positive employment tax $t_{ei} > 0$. Consequently, as in the single-type case moral hazard arising from the search effort decision puts *downward pressure* on the optimal participation tax relative to the case with fixed search effort provided the involuntary unemployed transfer can be conditioned on skill-type.

Third, suppose the government could not observe job search activities, so $b^I = b^V$. By definition, in this special case, $t_{pi} = s_i f(\theta_i) t_{ei}$ and (29) becomes:

$$t_{pi} = n_i s_i f(\theta_i)\frac{1 - v_i'/\lambda}{\frac{\partial n_i}{\partial b_i}\frac{1}{n_i} + \frac{\partial s_i}{\partial b_i}\frac{1}{s_i}}$$

In this case, endogenous search again puts *downward pressure* on the optimal participation tax.

The fact that the government cannot condition b^I on skill-type limits its ability to provide insurance as a change in b^I affects the search effort of all skill-types. As well, since the government observes job search activities it can provide a different transfer to the voluntary unemployed than to the involuntary unemployed and there is not a direct link between the sign of the participation tax and the sign of the employment tax. Taken together, these two facts imply that allowing for endogenous search can tend to either increase or decrease the optimal participation tax.

Finally, we can derive an expression relating the optimal income transfers to the employed, the involuntary unemployed and the voluntary unemployed. Dividing each of the first-order conditions for b_i for $i = 1, ..., N$, b^I and b^V — (36), (37), and (27) — by the relevant marginal utility, using the expressions for the changes in s_i and n_i in (3) and (7), and summing up yields:

$$\frac{N}{\lambda} = \sum_{i=1}^{N} n_i s_i f(\theta_i) \frac{1}{v'_i} + \sum_{i=1}^{N} n_i (1 - s_i f(\theta_i)) \frac{1}{u'} + \sum_{i=1}^{N} (1 - n_i) \frac{1}{\mu'} \tag{44}$$

where the right-hand side of the weighted average of the inverse of the marginal utilities of consumption. The inverse of the marginal utility of consumption is the marginal cost in terms of consumption of increasing utility by one util. Multiplying this inverse by the number of individuals of a given type (employed, involuntarily unemployed, voluntarily employed) yields the total amount of consumption needed to increase the utility of all of these individuals of a given skill-type by one util and summing up over the different skill-types gives the total amount of consumption needed to increase the utility of all individuals. The social benefit in terms of consumption of increasing everyone's utility by one util is given the total population divided by the marginal cost of public funds, λ.

4 Extension: Allowing for Self-Insurance

Our key insight that the government will want to use proportional wage taxes to mitigate the various macro effects arising from endogenous wages and then use lump-sum transfers to provide unemployment insurance and to redistribute between individuals carries over to a simple dynamic environment in which individuals can self-insure by saving. We describe this environment below.

Consider a single skill-type. Assume now that there are two periods and individuals are endowed with common level of initial assets so can save. The first period would be similar to the static model described above. The key difference is that at the end of the first period (after wages have been paid), individuals could decide how much of their income to consume and how much to save for the next period. We assume individuals are credit-constrained to have non-negative financial wealth. At the beginning of period 2, an exogenous proportion σ of employed workers are separated from employment and become involuntarily unemployed. We assume these individuals cannot search again. Those who did not find a job in the first period can again look for work. We assume that no one changes their participation decision at the end of the first period.

The firm's zero-profit condition and Nash bargaining need to take account of period-2 outcomes. We assume that the bargained wage in period 1 is constant for two periods since workers are risk-averse while firms are risk-neutral. That is, workers employed in period 1 who remain employed in period 2, receive w_1 in both periods. Thus, firms assume any risk. We denote by w_2 the equilibrium wage determined in period 2 and that is paid to households who were involuntarily unemployed in period 1 and employed in the second period. A consequent of this assumed structure is that the equilibrium tightness of the labour market for the long-term employed (determined in period 1) depends only on w_1 and the equilibrium tightness of the labour market in period 2 depends only on w_2. The equilibrium wage determined in period 1, however, will depend on policies set in both periods and with positive savings the equilibrium wage determined in period 2 will also depend on policies set in both periods.

The government implements a linear income tax at rate τ_j with a lump-sum transfer to the employed b_j where the subscript $j = 1, 2$ denotes the period in which the wage was determined. As in the previous cases considered, the government is assumed to be able to observe wage rates so the tax system can be conditioned on the observable wages. In period 2, there will be two different wages being paid: one to workers who have been employed for two periods (w_1) and one for workers who have only been employed one period (w_2). The government also provides an income transfer to the voluntary unemployed b^V. This transfer applies for both periods. The involuntary unemployed obtain b^I in the first period of their unemployment and b^{II} if they are unemployed for a second period. Unemployment insurance differs for long-term (periods 1 and 2) versus short-term (period 1 or 2) unemployed: b^{II} versus b^{I}.

The timing is as follows: At the beginning of period 1, the government sets policies $\{\tau_1, b_1, \tau_2, b_2, b^{I}, b^{II}, b^{V}\}$ to maximize a utilitarian social welfare function, all individuals make a participation decision and the participants make a search effort decision. Period 1 labour market equilibrium $\{\theta_1, w_1\}$ is then determined. Employed individuals are paid wages and pay taxes, and voluntary and involuntary unemployed receive income transfers. At the end of period 1, savings decision for individuals made by the voluntary unemployed, the involuntary unemployed and the employed. At the beginning of period 2, there is a search decision by individuals who were involuntarily unemployed in period 1 and exogenous separation of individuals who were employed in period 1. Period 2 labour market equilibrium $\{\theta_2, w_2\}$ is then determined and employed individuals receive wages and pay taxes, and voluntary and involuntary unemployed receive income transfers.

As in the static model, the proportional wage tax has a differential effect on the equilibrium wage relative to the lump-sum wage tax in any given period, but unlike the static case the wage taxes chosen in period 1 affect the second period equilibrium wage. The government still has sufficient instruments, however, to ensure that the equilibrium wages determined in each of the two periods are optimal, that is, maximize the government's Lagrangian. Consequently, Result 2 continues to hold and the first-order conditions on the transfers to the employed, the short-term involuntary unemployed, and the long-term involuntary unemployed are the same as when wages are fixed.

In this simple two-period environment, the ability to save affects the insurance-search incentive trade-off the government faces. Savings decision will be affected by the transfer the involuntary unemployed receive in period 1, the tax they pay if employed in period 2, and the transfer they receive if involuntarily unemployed in period 2. An individual's savings, in turn, will affect their search effort decision in period 2. Therefore, the effect of these three policies $\{b^{I}, b_2, b^{II}\}$ must be taken into account in choosing the optimal unemployment insurance and will affect the incentive-insurance trade-off the government faces in each period. But since job search activities are observable these trade-offs will not be affected by the participation decision.

5 Conclusion

We have explored the joint design of unemployment insurance and redistributive taxes and transfers in a setting with individuals of different skills who choose both whether to participate in job search and how intensively to search. The existing literature on optimal unemployment insurance sets aside redistributive considerations and focuses on the insurance-moral hazard trade-off in choosing an efficient unemployment insurance system. On the other hand, optimal redistribution models with involuntary unemployment take search effort as fixed and focus on the participation decision as in extensive-margin approaches to optimal income taxation. No distinction is made between transfers paid to the voluntary and involuntary unemployed. Our analysis combined these two approaches.

We considered how search decisions influence optimal participation taxes on the one hand, and how participation choices affect unemployment insurance. We find that second-best optimal policy requires using a proportional wage tax to address macro effects arising from changes in wage bargaining outcomes, leaving participation taxes and unemployment insurance to address redistributive and insurance objectives without concern for their impact on wage setting. While optimal participation taxes are moderated by search effort, unemployment insurance is not influenced by participation decisions. These insights carry over to a simple dynamic model in which individuals could save to self-insure.

We have adopted a number of simplifying assumptions to facilitate our analysis. We abstract from intensive-margin labour decisions by assuming that work effort is fixed in employment. We have assumed that the government can observe who is involuntarily unemployed so that workers cannot refuse employment or quit jobs to take advantage of unemployment insurance. If such behaviour was unobservable then some form of monitoring as shown in Boadway and Cuff (1999, 2014) would be needed. We have also assumed that workers direct their search only to labour markets catering to their skills. These assumptions are largely consistent with those that have been made in the related literature, and they enable us to obtain relatively clear and intuitive results. Relaxing them would take us too far afield for the scope of this paper.

References

[1] Banks, J. and P.A. Diamond (2010), 'The Base for Direct Taxation,' in J. Mirrlees, S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Gammie, P. Johnson, G. Myles, and J. Poterba (eds.), *Dimensions of Tax Design: The Mirrlees Review* (Oxford: Oxford University Press), 548–648.

[2] Boadway, R. (2012), *From Optimal Tax Theory to Tax Policy* (Cambridge, MA: MIT Press).

[3] Boadway, R. and K. Cuff (1999), 'Monitoring as an Instrument for Targeting Transfers,' *International Tax and Public Finance* 6, 317–337.

[4] Boadway, R. and K. Cuff (2014), 'Monitoring and Optimal Income Taxation with Involuntary Unemployment,' *Annals of Economics and Statistics* 113/114, 121–157.

[5] Chetty, R. (2008), 'Moral Hazard versus Liquidity and Optimal Unemployment Insurance,' *The Journal of Political Economy* 116(2), 173–234.

[6] Coles, M.G. and A. Masters (2006), 'Optimal Unemployment Insurance in a Matching Equilibrium,' *Journal of Labor Economics* 24, 109–38.

[7] Diamond, P. (1980), 'Income Taxation with Fixed Hours of Work,' *Journal of Public Economics* 13(1), 101-110.

[8] Golosov, M. and A. Tsyvinski (2015), 'Policy Implications of Dynamic Public Finance,' *Annual Review of Economics* 7, 147–171.

[9] Hosios, A.J. (1990) 'On the Efficiency of Matching and Related Models of Search and Unemployment,' *Review of Economic Studies* 57(2), 279–298.

[10] Hungerbühler, M., E. Lehmann E, A. Parmentier and B. Van der Linden B (2006), 'Optimal Redistributive Taxation in a Search Equilibrium Model,' *Review of Economic Studies* 73(3), 743-768,

[11] Jacquet, L., E. Lehmann, B. Van der Linden, (2014), 'Optimal Income Taxation with Kalai Wage Bargaining and Endogenous Participation,' *Social Choice and Welfare,* 42(2), 381–402.

[12] Karni, E. (1999), ‘Optimal Unemployment Insurance: A Survey,’ *Southern Economic Journal* 66(2), 442–465.

[13] Kroft, K., K. Kucko, E. Lehmann and J. Schmieder (2016), ‘Optimal Income Taxation with Unemployment and Wage Responses: A Sufficient Statistics Approach’, IZA Discussion Paper No. 9719.

[14] Landais, C., P. Michaillat, and E. Saez (2015), ‘A Macroeconomic Theory of Optimal Unemployment Insurance: Applications’, mimeo.

[15] Landais, C., P. Michaillat, and E. Saez (2016), ‘A Macroeconomic Theory of Optimal Unemployment Insurance: Theory’, *American Economic Journal: Economic Policy*, forthcoming.

[16] Lehmann E, A. Parmentier, B. Van der Linden (2011), ‘Optimal Income Taxation with Endogenous Participation and Search Unemployment,’ *Journal of Public Economics* 95(1112), 1523-1537.

[17] Lehmann, E. and B. Van der Linden (2007), ‘On the Optimality of Search Matching Equilibrium when Workers are Risk Averse’, *Journal of Public Economic Theory* 9(5), 867–884.

[18] l’Haridon, O., F. Malherbet and S. Pérez-Duarte (2013), ‘Does Bargaining Matter in the Small Firms Matching Model?’, *Labour Economics* 21, 42–58.

[19] Mirrlees, J.A. (1971), ‘An Exploration in the Theory of Optimum Income Taxation,’ *Review of Economic Studies* 38, 175–208.

[20] Mortensen, D.T. (1977), ‘Unemployment Insurance and Job Search Decisions,’ *Industrial and Labor Relations Review* 30, 505–17.

[21] Mortensen, D.T. and C.A. Pissarides (1999), ‘New Developments in Models of Search in the Labor Market,’ Volume 3 of Handbook of Labor Economics, Chapter 39, 2567-2627. Elsevier.

[22] Pissarides, C.A. (1990), *Equilibrium Unemployment Theory* (Oxford: Blackwell).

[23] Saez, E. (2002), ‘Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses,’ *Quarterly Journal of Economics* 117(3), 1039-1073.

[24] Topel, R. anf F. Welch (1980), 'Unemployment Insurance: Survey and Extensions,' *Economica* 47(187), 351–79.

[25] Tuomala, M. (2016), *Optimal Redistributive Taxation* (Oxford: Oxford University Press).

A Optimality of Search with Risk Aversion

Assume a single labour market. Let the number of job offers o, search effort s and participation, or the number of job-seekers n, be choice variables. The matching function is $m(sn, o)$ or $m(1, o/(sn)) \equiv f(o/(sn))$ where ns is aggregate search effort. Therefore, $nsf(o/(sn))$ is the expected number of matches. Then, $\theta = o/(ns)$ is labour market tightness, and depends on all three endogenous variables. Consider first the market solution and then the optimal one, and abstract from taxes and transfers, so $\tau = b = b^I = b^V = 0$. Worker utility is $v(w)$ if employed, u if involuntarily unemployed and μ if voluntarily unemployed. Where necessary, we normalize utility using consumption if employed as numeraire, so money-metric utility is $v(w)/v'(w)$, $u/v'(w)$ and $\mu/v'(w)$. We take u and μ as given in what follows, and we assume for simplicity that the elasticity of $f(o/(sn))$ is constant, so η is constant.

Labour market equilibrium

Given free entry of firms, the zero-profit condition applies:

$$nsf(o/(ns))(y - w) = ok \tag{A.1}$$

where y is output per worker and w is the wage rate. The latter is determined by the solution to the Nash bargaining problem between the firm and the worker when a match is made:

$$\max_{\{w\}} \ \big(v(w) - u\big)^{\beta}(y - w)^{1-\beta} = (\Delta v(w))^{\beta}(y - w)^{1-\beta}.$$

The first-order condition is:

$$\beta(\Delta v(w))^{\beta-1}(y - w)^{1-\beta}v'(w) - (1 - \beta)(\Delta v(w))^{\beta}(y - w)^{-\beta} = 0$$

which can be written:

$$y - w = (1 - \beta)\Big(y - w + \frac{\Delta v(w)}{v'(w)}\Big). \tag{A.2}$$

This determines w. Substituting it into the zero-profit condition (A.1) gives:

$$\frac{f(o/(ns))}{o/(ns)}(1 - \beta)\Big(y - w + \frac{\Delta v(w)}{v'(w)}\Big) = k.$$

Using $f'(o/(ns))(o/(ns))/f(o/(ns)) = 1 - \eta$, this can be written:

$$f'(o/(ns))\frac{1 - \beta}{1 - \eta}\Big(y - w + \frac{\Delta v(w)}{v'(w)}\Big) = k. \tag{A.3}$$

Equilibrium search effort satisfies (2), or in the absence of taxes and transfers:

$$f(o/(ns))\Delta v(w) = \phi'(s). \tag{A.4}$$

Eq. (A.2) from the Nash bargaining problem can be written

$$\frac{\Delta v(w)}{v'(w)} = \beta\Big(y - w + \frac{\Delta v(w)}{v'(w)}\Big). \tag{A.5}$$

Substituting this in (A.4) gives:

$$f(o/(ns))\beta\Big(y - w + \frac{\Delta v(w)}{v'(w)}\Big) = \frac{\phi'(s)}{v'(w)}. \tag{A.6}$$

The participation condition (4) in the absence of taxes and transfers becomes:

$$sf(o/(ns))v(w) + (1 - sf(o/(ns))u - \phi(s) \geqslant \mu + \bar{\delta}.$$

It is binding at $\bar{\delta}$ where:

$$sf(o/(ns))\Delta v(w) + u - \phi(s) - \mu - \bar{\delta} = 0.$$

Using (A.5), this can be written:

$$sf(o/(ns))\beta\Big(y - w + \frac{\Delta v(w)}{v'(w)}\Big) + \frac{u - \phi(s) - \mu - \bar{\delta}}{v'(w)} = 0. \tag{A.7}$$

Denote the market equilibrium outcomes as w^m, o^m, s^m, $\bar{\delta}^m$ and $n^m = G(\bar{\delta}^m)$. By (A.2), (A.3), (A.6) and (A.7) they satisfy

$$y - w^m = (1-\beta)\Big(y - w^m + \frac{\Delta v(w^m)}{v'(w^m)}\Big); \tag{A.2$'$}$$

$$f'(o^m/(n^m s^m))\frac{1-\beta}{1-\eta}\Big(y - w^m + \frac{\Delta v(w^m)}{v'(w^m)}\Big) = k; \tag{A.3$'$}$$

$$f(o^m/(n^m s^m))\beta\Big(y - w^m + \frac{\Delta v(w^m)}{v'(w^m)}\Big) = \frac{\phi'(s^m)}{v'(w^m)}; \tag{A.6$'$}$$

$$s^m f(o^m/(n^m s^m))\beta\Big(y - w^m + \frac{\Delta v(w^m)}{v'(w^m)}\Big) + \frac{u - \phi(s^m) - \mu - \bar{\delta}^m}{v'(w^m)} = 0. \tag{A.7$'$}$$

Planning equilibrium

We characterize the planning equilibrium for a given w, thereby focusing on the optimality of the search process. Since w is given, so is $v(w)$ and we can drop the argument w in what follows. The planner chooses o, s and $\bar{\delta}$ to maximize

$$nsf(o/(sn))\Big(y - w + \frac{v}{v'}\Big) + n\big(1 - sf(o/(sn))\big)\frac{u}{v'} + (1-n)\frac{\mu}{v'} + \int_{\bar{\delta}}^{1} \frac{\delta}{v'} dG(\delta) - ok - n\frac{\phi(s)}{v'}$$

where $n = G(\bar{\delta})$, $(\mu + \delta)/v'$ is the money metric utility for a non-participant with a given δ, and $\phi(s)/v'$ is money metric search costs.

The first-order conditions can be written as follows, using $\Delta v = v - u$ and the definition of η, and denoting optimal values by superscript s:

$$f'(o^s/(n^s s^s))\Big(y - w + \frac{\Delta v}{v'}\Big) = k; \tag{A.8}$$

$$f(o^s/(s^s n^s))\eta\Big(y - w + \frac{\Delta v}{v'}\Big) = \frac{\phi'(s^s)}{v'}; \tag{A.9}$$

$$s^s f(o^s/(s^s n^s))\eta\Big(y - w + \frac{\Delta v}{v'}\Big) + \frac{u - \mu - \bar{\delta}^s - \phi(s^s)}{v'} = 0. \tag{A.10}$$

We observe immediately that if $\beta = \eta$ (the Hosios condition) is satisfied, eqs. (A.3′), (A.6′) and (A.7′) are equivalent to (A.8), (A.9) and (A.10) respectively, and market outcomes are socially optimal. This assumes that the social optimum is evaluated at the wage determined in the private market equilibrium by Nash bargaining.

Suppose the Hosios condition is not satisfied. Then for a given wage rate the market outcome will not be socially optimal. To see this, let $\theta^m = o^m/(s^m n^m)$. From (A.3) and (A.8),

$$\frac{f'(\theta^m)}{f'(\theta^s)} = \frac{1-\eta}{1-\beta} \implies \eta \gtreqless \beta \text{ iff } f(\theta^m) \gtreqless f(\theta^s).$$

Then, from (A.6) and (A.9),

$$\frac{f(\theta^m)\beta}{f(\theta^s)\eta} = \frac{\phi'(s^m)}{\phi'(s^s)}.$$

Since $\eta \gtreqless \beta$ iff $f(\theta^m) \gtreqless f(\theta^s)$, we have $\beta f(\theta^m) \gtreqless \eta f(\theta^s)$ regardless of relative sign of η and β, so $s^m \gtreqless s^s$.

Finally, from (A.7) and (A.10), we have:

$$\frac{s^m f(\theta^m)\beta}{s^s f(\theta^s)\eta} = \frac{\bar{\delta}^m + \phi(s^m) + \mu - u}{\bar{\delta}^s + \phi(s^s) + \mu - u}$$

and $\bar{\delta}^m \gtreqless \bar{\delta}^s$.