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Cash-Flow Business Taxation Revisited: Bankruptcy, Risk Aversion and Asymmetric Information

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Abstract

It is well-known that cash-flow business taxes with full loss-offset, and their present-value equivalents, are neutral with respect to firms' investment decisions when firms are risk-neutral and there are no distortions. We study the effects of cash-flow business taxation when there is bankruptcy risk, when firms are risk-averse, and when financial intermediaries face asymmetric information problems in financing heterogeneous firms. Cash-flow taxes remain neutral under bankruptcy risk alone, but can distort the entry and investment decisions of firms under both risk-aversion and asymmetric information. We characterize the nature of such distortions and show that cash-flow taxes can increase social welfare in this context. An ACE tax is equivalent to a cash-flow tax but is easier to implement under asymmetric information.

Key Words: cash-flow tax, risk-averse firms, asymmetric information
JEL: H21, H25

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1 Introduction

A classic result in the design of business taxes due to Brown (1948) concerns the neutrality of cash-flow taxation. Investment decisions undertaken in a world of full certainty will be unaffected by a tax imposed on firms' cash flows, assuming there is full loss-offsetting (and the tax rate is constant, as shown by Sandmo, 1979). In effect, a cash-flow tax will divert a share of the pure profits or rents of the firm to the government. This is of obvious potential policy interest since it represents a non-distorting source of tax revenue.

Not surprisingly, the so-called Brown tax inspired a sizable literature on neutral business tax design, much of which generalized Brown's neutrality result to taxes that are equivalent to cash-flow taxes in present value terms. Boadway and Bruce (1984) show that the cash-flow tax is a special case of a more general class of neutral business taxes that have the property that the present value of deductions for future capital costs (interest plus depreciation) arising from any investment just equals initial investment expenditures. (Current costs are assumed to be fully deductible on a cash basis when incurred, though they can be capitalized as well with no difficulty.) They characterized a general neutral business tax satisfying this property as follows. Investment expenditures are added to a capital account each year, and each year the capital account is depreciated at a rate specified for tax purposes. Capital costs deducted from the tax base in each tax year consist of the cost of capital and the depreciation rate applied to the book value of the capital account. We refer to this as a Capital Account Allowance (CAA) tax. The CAA tax is neutral regardless of the depreciation rate used, as long as full loss-offsetting applies. Moreover, the depreciation rate used for tax purposes can be arbitrary and can vary from year to year. Its pattern can be chosen so that negative tax liabilities are mitigated. Indeed, the firm itself can choose the depreciation rate to use, possibly contingent on minimizing tax losses in any given year. In effect, the CAA tax system allows the firm to carry forward any unused deductions for investment at the interest rate.

More generally, neutrality can be achieved by a business tax in which the present value of future tax bases just equals the present value of cash flows. An example of a cash-flow equivalent tax system of this sort is the Resource Rent Tax (RRT) proposed by Garnaut and Clunies-Ross (1975) for the taxation of non-renewable natural resources. In their version, firms starting out are allowed to accumulate negative cash flows in an account that rises each year with the cost of capital. Once the account becomes positive, cash flows are taxed as they occur. Like the Brown tax or the CAA tax, the RRT is neutral with respect to decisions by the firm, including extraction in the case of resource firms. Negative cash flows are carried forward at the cost of capital thereby achieving the equivalent of cash-flow taxation.

These basic results continue to apply if returns to investment are uncertain, provided firms are risk-neutral. Fane (1987) shows that neutrality holds under uncertainty as long as tax credits and liabilities are carried forward at the risk-free nominal interest rate, and that tax credits and liabilities are necessarily redeemed eventually. Bond and Devereux (1995) show that the CAA tax remains neutral in the presence of uncertainty and the possibility of bankruptcy provided that a risk-free interest rate applied to the value of the capital account is used for the cost of capital deduction, that any unused negative tax credits are refunded in the event of bankruptcy, and that the valuation of risky assets satisfy the value additivity principle.[1] The use of a risk-free discount rate reflects the assumption that there is no risk to the firm associated with postponing capital deductions into the future (i.e., no political risk). Boadway and Keen (2015) show that the same neutrality result applies to the RRT in the presence of uncertainty.

Bond and Devereux (2003) extend these results to the Allowance for Corporate Equity (ACE) tax, which is a version of the CAA tax that allows actual interest deductions alongside a cost of capital deduction for equity-financed investment. (They assume that there are no rents earned by bond-holders.) They also show that neutrality can be achieved using the more general case of cash-flow taxation proposed by Meade (1978), referred to as (R+F)-base cash-flow taxation, in which both real and financial cash flows are included in the base. Notably, Bond and Devereux assume full information in the sense that banks can observe incomes of firms that claim to be bankrupt, and they do not allow firms to choose the amount of investment; that is, there is no intensive margin. Recently, these results have been extended to consider the effect of cash-flow taxation on the entry decision of firms/entrepreneurs. Kanniainen and Panteghini (2012) show, using an option-value model for determining entry (and exit) of entrepreneurs, that cash-flow taxation distorts the entry decision unless the cash-flow tax rate is same as the wage tax rate potential entrepreneurs face in alternative employment.

These neutrality results have inspired some well-known policy proposals, some of which have been implemented. A cash-flow business tax was recommended by the US Treasury (1977), Meade (1978) in the UK, and the President's Panel (2005) in the USA. The latter two both recommended additional cash-flow taxation to apply to financial institutions. At the same time, refundability of outstanding tax losses on firms that wind up was not proposed. The Australian Treasury (2010) (the Henry Report) recommended an RRT for

[1] The value additivity principle implies that the present value of the sum of stochastic future payoffs is equal to the sum of the present values of these payoffs, and is consistent with a no-arbitrage principle in the valuation of assets.

the mining industries in Australia. Several bodies have recommended an ACE corporate tax system, including the Institute for Fiscal Studies (1991), the Mirrlees Review (2011) and Institut d'Economia de Barcelona (2013), and some instances of cash-flow-equivalent taxes have been implemented. ACE taxes have been deployed in a few countries, including Brazil, Italy, Croatia and Belgium. Reviews of their use may be found in Klemm (2007), de Mooij (2011), Panteghini, Parisi, and Pighetti (2012), and Princen (2012). Cash-flow-type taxes with full loss-offset are used in the Norwegian offshore petroleum industry (reviewed in Lund, 2014), and the RRT was applied temporarily in the Australian mining industry.

The neutrality of cash-flow taxation no longer applies when the simple assumptions are relaxed. Suppose firms' owners are risk-averse so that part of the return to investment is compensation for risk. A cash-flow tax applies to both rents and returns to risk-taking, and these two streams cannot be distinguished. As Domar and Musgrave (1944) famously show, risk-averse savers faced with a proportional tax on capital income with full loss-offset would be expected to increase the proportion of their portfolio held as risky assets, although the results become murkier when the proceeds from the tax are returned to savers by the government, as thoroughly discussed in Atkinson and Stiglitz (1980) and Buchholz and Konrad (2014). These results readily extend to an entrepreneurial firm, as shown in Mintz (1981). This non-neutrality is not necessarily a bad thing if the government is better able to diversify risk that private savers, but for our purposes the cash-flow tax is no longer a neutral tax on rents.

Additional problems arise if there are credit-market imperfections due to asymmetric information. This has been approached as a problem of adverse selection, first studied by Stiglitz and Weiss (1981) and subsequently by de Meza and Webb (1987). The latter authors show that if banks cannot observe the probability of a firm going bankrupt, there will be excessive credit extended to entrepreneurs. These results have been generalized by Boadway and Keen (2006) and Boadway and Sato (2011) to allow for ex post monitoring by banks to verify that firms claiming bankruptcy are truly insolvent. Moral hazard in credit markets can also give rise to market failure, as in Townsend (1979), Diamond (1984), Williamson (1986, 1987) and Bernanke, Gertler and Gilchrist (1996), where the costs of observing returns realized ex post by the lender generate an agency cost that tends to distort the market equilibrium. Parts of the literature on credit market failures resulting from asymmetric information are surveyed in Boadway and Tremblay (2005).

In this paper, we revisit the use of cash-flow taxation as a rent-collecting device when firms might be risk-averse and asymmetries of information may exist in capital markets. We do so in a simple partial equilibrium model of risk-averse entrepreneurs who vary in their productivity, so that returns to inframarginal entrepreneurs generate rents. We study the effects of cash-flow taxation on both the entry decision of potential entrepreneurs (the extensive margin) and on the decision as to how much to borrow and invest (the intensive margin). The object is both to uncover distortions a cash-flow tax may impose on these two margins when there is asymmetric information and risk aversion. The effect of the tax on intensive-margin decisions turns out to be especially important. We begin with a base case where entrepreneurs with projects of differing productivity are risk-neutral, and where banks can observe entrepreneurs' types. Investment outcomes are uncertain and entrepreneurs face the possibility of bankruptcy, which banks can verify only by engaging in costly monitoring. We then extend the analysis to allow for risk-averse entrepreneurs. Conducting the analysis using entrepreneurial firms is for simplicity. The analysis could be readily extended to public corporations.

Some of our main results are as follows. With risk-neutral entrepreneurs and the possibility of bankruptcy, the cash-flow tax is neutral in the absence of asymmetric information as in Bond and Devereux (1995, 2003), but may distort investment decisions if banks must incur monitoring costs when firms go bankrupt. Surprisingly, if banks can deduct monitoring costs from taxable income on bankrupt projects, the cash-flow tax increases investment while leaving bankruptcy risk and firms' expected profits unchanged. Expected rents, government expected revenues and social surplus all increase with the tax rate. With risk-averse entrepreneurs, the cash-flow tax results in some risk-sharing between firms and the government. If there are no monitoring costs, or if such costs are tax deductible for banks, the cash-flow tax induces higher investment but is neutral with respect to bankruptcy risk and expected utility as in Domar and Musgrave (1944). The cash-flow tax will also tend to increase social welfare, provided the government is not too risk averse. We show that an ACE tax achieves the same outcome as a cash-flow tax with monitoring costs deductible, but does so in an administratively simpler way.

We begin by outlining the main elements of the model under risk-neutrality. We then study the effects of cash-flow taxation in the basic model, and consider some extensions including the ACE tax. Finally, we assume firms are risk-averse and analyze the effects of cash-flow taxation.

2 The Basic Model with Risk-Neutral Entrepreneurs

There is a population of potential entrepreneurs with an identical endowment of wealth but different productivities as entrepreneurs. We assume that there is a single period. That means that we can suppress the entrepreneurs' consumption-savings decision, which simplifies our analysis considerably by allowing us to focus on production decisions. At the beginning of the period, potential entrepreneurs decide whether to enter a risky industry and invest their wealth there. If they do not enter, they invest their wealth in a risk-free asset and consume the proceeds at the end of the period. Also for simplicity, we suppress their labor income: all income comes from profits they earn if they enter the risky sector, or their initial wealth if they do not. Adding labor income (as in Kanniainen and Panteghini (2012)) would make no substantial difference for our result on business taxation as we discuss later.

Entrepreneurs who enter the risky industry choose how much to borrow to leverage their own equity investment, which determines their capital stock. After investment has been undertaken, risk is resolved. Those with good outcomes earn profits for the entrepreneur. Those with bad outcomes go bankrupt. Their production goes to their creditors, which are risk-neutral competitive banks. There are thus two decisions made by potential entrepreneurs. First, they decide whether to enter, which we can think of as an extensive-margin decision, and second, they decide how much to borrow to expand their capital, which is an intensive-margin decision. To begin with, we assume that entrepreneurs are risk-neutral. The details of events are as follows.

A continuum of potential entrepreneurs are all endowed with initial wealth E. For simplicity, we assume that the production function is linear in capital K. The average product of capital, denoted R, is constant, but differs across entrepreneurs, and is distributed over $[0, \overline{R}]$ by the distribution function $H(R)$. The value of output is subject to risk, and the stochastic value of a type$-R$ entrepreneur's output is $\tilde{\varepsilon}RK$, where $\tilde{\varepsilon}$ is distributed uniformly over $[0, \varepsilon_{\max}]$, with density $g = 1/\varepsilon_{\max}$. The expected value of ε is

$$\overline{\varepsilon} \equiv \mathbb{E}[\tilde{\varepsilon}] = \frac{\varepsilon_{\max}}{2} = \frac{1}{2g} \tag{1}$$

We assume that the distribution of $\tilde{\varepsilon}$ is the same for all entrepreneurs, so they differ only by their productivity R. Capital is financed by the entrepreneur's own equity and debt, and depreciates at the proportional rate δ per period. Entrepreneurs who do not enter invest all their wealth in a risk-free asset with rate of return ρ, so consume $(1 + \rho)E$. Since all potential entrepreneurs have the same alternative income, those with the highest productivity as entrepreneurs will enter the entrepreneurial sector. Let $\widehat{R}$ denote the average product of the marginal entrepreneur.

Entrepreneurs who enter invest all their wealth in the risky firm. Then, E will be the common value of own equity of all entrepreneurs. The type$-R$ entrepreneur who has entered borrows an amount $B(R)$ so his aggregate capital stock is $K(R) = E + B(R)$. Let $B(R) \equiv \phi(R)K(R)$, where $\phi(R)$ is the leverage rate. Then $K(R)$ can be written:

$$K(R) = \frac{E}{1 - \phi(R)} \tag{2}$$

We assume that there is a maximum value of the capital stock, such that $K(R) \leqslant \overline{K}$, and moreover that $E < \overline{K}$ so the entrepreneur's wealth is less than the maximum size of the capital stock. By (2), this implies that $0 \leqslant \phi(R) \leqslant 1 - E/\overline{K} < 1$. Since we assume that all the entrepreneur's wealth is invested, the minimum level of capital for entrepreneurs who enter is E. Allowing entrepreneurs to invest only part of their wealth would complicate the analysis slightly without adding any insight.[2] The entrepreneur's capital stock is therefore in the range $K(R) \in [E, \overline{K}]$. The assumption of a maximal capital stock reflects the notion that after some point additional capital is non-productive. It is like a strong concavity assumption on the production function, which precludes extreme outcomes that would otherwise occur with linear production.

By the assumptions we make below, equilibrium analysis applies separately to entrepreneurs of each type. Accordingly, consider a representative type$-R$ entrepreneur and drop the indentifier R from all functions for simplicity. The entrepreneur's ex post after-tax profits (or return to own equity) is given by:

$$\widetilde{\Pi} = \tilde{\varepsilon} RK + (1-\delta)K - (1+r)B - \widetilde{T} \tag{3}$$

where $\widetilde{T}$ is the tax paid and r is the interest rate, so $(1+r)B$ is the repayment of interest and principal on the borrowing B. Note that the interest rate r is specific to type R since as we assume later banks observe entrepreneurs' types. The term $(1-\delta)K$ is the value of capital remaining after production, given the depreciation rate δ.

The tax liability under cash-flow taxation, evaluated after $\tilde{\varepsilon}$ is revealed, is given by:

$$\widetilde{T} = \tau\Big(\tilde{\varepsilon} RK - (1+\rho)K + (1-\delta)K\Big) = \tau\big(\tilde{\varepsilon} RK - \rho K - \delta K\big) \tag{4}$$

where τ is the tax rate and, as noted above, ρ is the risk-free interest rate. We assume this equation applies as long as the firm is not bankrupt. As discussed below, if the firm goes bankrupt, we assume the bank pays taxes on the bankrupt cash flows and gets to deduct costs of monitoring the firm for bankruptcy. The cash-flow tax base in the middle expression consists of three terms. The first is the revenue of the firm, $\tilde{\varepsilon} RK$. The second, $(1+\rho)K$, is the deduction for investment. Since this occurs at the beginning of the period,

[2] If entrepreneurs were allowed to invest part of their wealth in the safe asset, doing so would increase leverage for any given level of investment. As will be discussed below, that would tend to increase bankruptcy risk and the interest rate faced by the entrepreneur. If entrepreneurs are facing unlimited liability in the case of bankruptcy, there would be no incentive to invest less than total wealth in the risky project since the interest rate on borrowing will tend to be higher than the rate of return on the safe asset. If there is limited liability in the case of bankruptcy, entrepreneurs may choose to hold wealth in the safe asset although that would result in higher interest costs on borrowing.

we assume that the tax savings from deducting investment are either refunded immediately or are carried over to the end of the period with interest at rate ρ. Eq. (4) applies whether $\widetilde{T}$ is positive or negative, so implicitly assumes that the tax system allows full loss-offsetting. Finally, the cash-flow tax is levied on selling or winding-up the business assets, $(1-\delta)K$, at the end of the period. Using (3) and (4), ex post after-tax profits may be written:

$$\widetilde{\Pi} = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) - (1+r)B + \tau(1+\rho)K \tag{5}$$

Entrepreneurs are confronted with bankruptcy when ε is too low to meet debt repayment obligations, that is, when $\widetilde{\Pi} < 0$. This occurs for entrepreneurs with $\varepsilon < \hat{\varepsilon}$, where $\hat{\varepsilon}$, which is specific to type R, satisfies:

$$0 = (1-\tau)\Big(\hat{\varepsilon}RK + (1-\delta)K\Big) - (1+r)B + \tau(1+\rho)K \tag{6}$$

In what follows, we refer to $\hat{\varepsilon}$ as *bankruptcy risk*. The higher the value of $\hat{\varepsilon}$, the greater the chances of the entrepreneur going bankrupt. In the event of bankruptcy, the loan is not repaid, and the remaining after-tax profits $(1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) + \tau(1+\rho)K$ go to the creditor. Combining (5) and (6), we obtain:

$$\widetilde{\Pi} = (1-\tau)(\tilde{\varepsilon} - \hat{\varepsilon})RK \quad \text{for} \quad \tilde{\varepsilon} \geqslant \hat{\varepsilon} \tag{7}$$

Eq. (6) can be written, using $\phi = B/K$, as

$$(1-\tau)\big(\hat{\varepsilon}R + (1-\delta)\big) - (1+r)\phi + \tau(1+\rho) = 0$$

For given R, the relationship between bankruptcy risk $\hat{\varepsilon}$ and leverage ϕ depends on how the lending rate r facing entrepreneurs is obtained. This is determined by a competitive banking sector. Assume that banks are risk-neutral and can observe R for each entrepreneur, but cannot observe $\tilde{\varepsilon}$. Thus, there is no adverse selection since banks know entrepreneurs' types, but there is moral hazard: part of the risk of bankruptcy is borne by banks rather than entrepreneurs. Imperfection of the financial market due to asymmetric information is captured by an ex post verification or monitoring cost in the event a firm declares bankruptcy. Following the financial accelerator model of Bernanke *et al* (1999), we assume that the verification cost is proportional to ex post output so takes the form $c\tilde{\varepsilon}RK$, for $\tilde{\varepsilon} \leqslant \hat{\varepsilon}$. This might reflect the fact that the verification cost includes the costs of seizing the firm's output in a default.[3] The expected total verification cost for a given type of entrepreneur is:

$$\int_0^{\hat{\varepsilon}} c\tilde{\varepsilon}RKgd\tilde{\varepsilon} = cRKg\frac{\hat{\varepsilon}^2}{2} \tag{8}$$

so the expected verification or monitoring cost increases with bankruptcy risk, $\hat{\varepsilon}$.

In the event of bankruptcy, the firm no longer pays $(1+r)B$, and its profits go to the bank. Using (5) these profits become:

$$\widetilde{\Pi} = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) + \tau(1+\rho)K \tag{9}$$

Competition among banks ensures that expected profits earned from lending to the representative entrepreneur of each type are zero. We assume that banks will not go bankrupt, so they pay the risk-free interest rate ρ on their deposits. Assuming also that banks incur no costs of operation and that they can deduct bankruptcy costs from taxable income, zero-expected bank profits imply the following, using the fact that debt is only repaid if $\tilde{\varepsilon} > \hat{\varepsilon}$,

$$(1+\rho)B = (1+r)B\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} gd\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \widetilde{\Pi}gd\tilde{\varepsilon} - (1-\tau)cRKg\hat{\varepsilon}^2/2$$

Substituting (9) for $\widetilde{\Pi}$ and integrating, we obtain:

$$(1+\rho)B = (1+r)B\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} gd\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \Big((1-\tau)\big(\tilde{\varepsilon}RK+(1-\delta)K\big)+\tau(1+\rho)K\Big)gd\tilde{\varepsilon} - (1-\tau)cRKg\frac{\hat{\varepsilon}^2}{2}$$

$$= (1+r)B(1-g\hat{\varepsilon}) + (1-\tau)RKg\frac{\hat{\varepsilon}^2}{2} + \big((1-\tau)(1-\delta)+\tau(1+\rho)\big)Kg\hat{\varepsilon} - (1-\tau)cRKg\frac{\hat{\varepsilon}^2}{2} \tag{10}$$

Eq. (10) reflects the fact that for $\varepsilon \leqslant \hat{\varepsilon}$, the banks retain after-tax profits, but face the verification costs $c\tilde{\varepsilon}RK$. This zero-profit condition applies for each type of entrepreneur, so in general each faces a different interest rate.

Note that the banks pay no cash-flow tax on their own profits, although they incur whatever taxes are owing (positive or negative) on the profits they receive on bankrupt projects. While we assume initially that the bank deducts bankruptcy costs, we consider later the possibility that these costs are non-deductible. The absence of cash-flow taxation on banks is innocuous given our assumption that banks earn no pure profits or rents and are risk-neutral. Expected cash flows, and therefore expected tax liabilities, would be zero. In

[3] Bernanke and Gertler (1989) introduced a fixed verification cost in a business cycle model where there is asymmetric information between lenders and borrowers about the realized return on risky projects, while Townsend (1979) explored the design of debt contracts with verification costs that could either be fixed or functions of realized project output. See also Bernanke *et al* (1996) for an analysis of the implications of agency costs in lending contracts arising from asymmetric information about project outcome.

a world where banks earn rents, the cash-flow tax could be extended to them, as proposed initially by the Meade Report (1978) and discussed in the Mirrlees Review (2011). If banks were subject to a cash-flow tax, the assumption that they can deduct monitoring costs would be unquestionable. It is useful to begin the analysis with the case where monitoring costs are deductible. This case serves as a benchmark for the purpose of comparison with the social optimum and with the outcome under the ACE tax.

Using the zero-profit condition (6), interest and principal repayments $(1+r)B$ can be eliminated from (10) to give the following lemma.[4]

Lemma 1 *The leverage rate* $\phi \equiv B/K$*, for* $0 < \phi < 1 - E/\overline{K}$*, is given by:*

$$\phi(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1+\rho}\left(\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R + (1-\delta)\right) + \tau \tag{11}$$

We assume that entrepreneurs recognize how leverage affects the interest rate in (10), and therefore the probability of bankruptcy. This is taken into account by (11). While in practice, entrepreneurs choose leverage ϕ, it is convenient for us to assume in our analysis that they choose bankruptcy risk $\hat{\varepsilon}$, which is related to leverage via (11). Routine differentiation of (11) gives properties of $\phi(\hat{\varepsilon}, R, \tau, c)$ that are useful in what follows:

$$\phi_{\hat{\varepsilon}} = \frac{1-\tau}{1+\rho}(1 - g\hat{\varepsilon} - cg\hat{\varepsilon})R; \quad \phi_c = -\frac{1-\tau}{1+\rho}\frac{Rg\hat{\varepsilon}^2}{2}; \quad \phi_R = \frac{1-\tau}{1+\rho}\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon};$$

$$\phi_\tau = \frac{1-\phi}{1-\tau}; \quad \phi_{\hat{\varepsilon}\hat{\varepsilon}} = -\frac{1-\tau}{1+\rho}(1-c)Rg\hat{\varepsilon}; \quad \phi_{\hat{\varepsilon}\tau} = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{1+\rho}R \tag{12}$$

Prior to ε being revealed, the representative entrepreneur's expected profits are $\overline{\Pi} \equiv \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon}$. (Recall that for $\tilde{\varepsilon} < \hat{\varepsilon}$, profits are claimed by the bank.) Given the expression for $\widetilde{\Pi}$ in (5), this becomes:

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) - (1+r)B + \tau(1+\rho)K\Big) g d\tilde{\varepsilon} \tag{13}$$

[4] Proof: Using (6), the righthand side of (10) can be written:

$$\big((1-\tau)(\hat{\varepsilon}R + (1-\delta)) + \tau(1+\rho)\big)K(1-g\hat{\varepsilon}) + (1-\tau)RK\frac{g\hat{\varepsilon}^2}{2} + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)Kg\hat{\varepsilon} - (1-\tau)\frac{cRKg\hat{\varepsilon}^2}{2}$$

$$= (1-\tau)\left(\hat{\varepsilon}(1-g\hat{\varepsilon}) + \frac{g\hat{\varepsilon}^2}{2}\right)RK + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)K - (1-\tau)\frac{cRKg\hat{\varepsilon}^2}{2}$$

$$= (1-\tau)\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon}RK + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)K \qquad \blacksquare$$

Using (10), (1) and (2) along with $B = K - E$, this may be written:

$$\overline{\Pi} = \left(\frac{1-\tau}{1-\phi(\hat{\varepsilon}, R, \tau, c)} \Big(\overline{\varepsilon} R - \delta - \rho - cgR\frac{\hat{\varepsilon}^2}{2} \Big) + 1 + \rho \right) E \equiv \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \qquad (14)$$

where $\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \overline{\Pi}/E$ is expected profit per unit of own equity. Using (11), $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$ satisfies the following lemma.[5]

Lemma 2

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1-\phi(\cdot)} \frac{Rg}{2} (\varepsilon_{\max} - \hat{\varepsilon})^2 \qquad (15)$$

Using (15) along with (7) and (2), expected profits, $\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon}$, can be rewritten as:

$$\overline{\Pi} = (1-\tau)RK \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (\tilde{\varepsilon} - \hat{\varepsilon}) g d\tilde{\varepsilon} = \frac{(1-\tau)}{1-\phi(\cdot)} \frac{Rg}{2} (\varepsilon_{\max} - \hat{\varepsilon})^2 E = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \qquad (16)$$

Since we assume risk-neutrality in this base case, the expected utility of entrepreneurs is given by $\overline{\Pi}$ in (14) or (16).

For future use, differentiate $\overline{\pi}(\cdot)$ in (14) with respect to $\hat{\varepsilon}$ to obtain:

$$\overline{\pi}_{\hat{\varepsilon}} = \frac{1-\tau}{1-\phi} \left(\Delta(\hat{\varepsilon}, R, \tau, c) \Big(\overline{\varepsilon} R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2} \Big) - c\hat{\varepsilon} gR \right) \qquad (17)$$

where

$$\Delta(\hat{\varepsilon}, R, \tau, c) \equiv \frac{\phi_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\hat{\varepsilon}, R, \tau, c)} \qquad (18)$$

Differentiating (18) by τ and using the properties of $\phi(\hat{\varepsilon}, R, \tau, c)$ in (12), we obtain:[6]

$$\Delta_\tau = 0 \qquad (19)$$

[5] Proof: Rewrite $\overline{\pi}$ in (14) as

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1}{1-\phi} \left((1-\tau) \Big(R\overline{\varepsilon} - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2} \Big) + (1+\rho)(1-\phi) \right)$$

From (11), we obtain

$$(1+\rho)(1-\phi) = -(1-\tau) \Big(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2} \Big) \hat{\varepsilon} R + (1-t)(\rho + \delta)$$

Substituting this in the expression for $\overline{\pi}$ gives, using $\overline{\varepsilon} = \varepsilon_{\max}/2$ and $\varepsilon_{\max} = 1/g$:

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1-\phi} R \Big(\overline{\varepsilon} - \hat{\varepsilon} \Big(1 - \frac{g}{2} \hat{\varepsilon} \Big) \Big) = \frac{1-\tau}{1-\phi} \frac{Rg}{2} (\hat{\varepsilon} - \varepsilon_{\max})^2 \qquad \blacksquare$$

[6] Proof: Differentiating $\Delta(\hat{\varepsilon}, \tau, c)$, we have $\Delta_\tau = \phi_{\hat{\varepsilon}\tau}/(1-\phi) + \phi_\tau \phi_{\hat{\varepsilon}}/(1-\phi)^2$. Using (12) for ϕ_τ, $\phi_{\hat{\varepsilon}}$ and $\phi_{\hat{\varepsilon}\tau}$,

$$\Delta_\tau = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{(1+\rho)(1-\phi)} R + \frac{1-\phi}{1-\tau} \frac{1-\tau}{1+\rho} (1 - g\hat{\varepsilon} - cg\hat{\varepsilon}) R \frac{1}{(1-\phi)^2} = 0 \qquad \blacksquare$$

3 Behavior of Entrepreneurs

Recall that entrepreneurs make two choices in sequence. First, they decide whether to undertake risky investments, given their productivity R. This is the extensive-margin decision. Then, if they enter, they decide how much to borrow to acquire more capital over and above their own equity, E. This is their intensive-margin decision. Once their shock $\tilde{\varepsilon}$ is revealed, their after-tax profits and therefore ex post utility are determined. We consider the intensive and extensive decisions in reverse order for a representative entrepreneur of a given type, and continue to suppress the type identifier R for simplicity.

3.1 Choice of leverage: intensive margin

As mentioned, given (11) the choice of leverage ϕ, which determines $K = E/(1-\phi(\cdot))$, is essentially the same as the choice of $\hat{\varepsilon}$. This follows because, even though ϕ is not necessarily monotonic in $\hat{\varepsilon}$, $\phi_{\hat{\varepsilon}\hat{\varepsilon}} < 0$ by (12). Differentiating (16) with respect to $\hat{\varepsilon}$ and using (17), we obtain:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \frac{1-\tau}{1-\phi}\left(\Delta(\hat{\varepsilon}, R, \tau, c)\left(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\right) - c\hat{\varepsilon}gR\right)E \tag{20}$$

where $\Delta(\hat{\varepsilon}, R, \tau, c)$ is defined in (18). Alternatively, differentiating $\overline{\pi}$ in (15),

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \left(\Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}}\right)\frac{(1-\tau)Rg}{1-\phi}(\varepsilon_{\max} - \hat{\varepsilon})^2 E \tag{21}$$

Let $\hat{\varepsilon}^*$ be the optimal choice of $\hat{\varepsilon}$. If it is in the interior, $d\overline{\Pi}/d\hat{\varepsilon} = 0$, so

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}^*} > 0 \tag{22}$$

From (22) we obtain the following lemma.[7]

Lemma 3 *Assume $\hat{\varepsilon}^*$ is in the interior. Then,*

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0$$

Thus, the cash-flow tax does not affect bankruptcy risk for firms with $\hat{\varepsilon}^*$ in the interior, although it does affect leverage ϕ by (12) and therefore K.

[7]Proof: Differentiate the first-order condition (22) to obtain:

$$\Delta_\tau d\tau + \left(\Delta_{\hat{\varepsilon}} - \frac{2}{(\varepsilon_{\max} - \hat{\varepsilon}^*)^2}\right)d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition, and $\Delta_\tau = 0$ by (19). ■

Alternatively, $\hat{\varepsilon}^*$ may take on corner solutions at the top or bottom. From (11), $\hat{\varepsilon}^*$ takes on a minimum value of $\hat{\varepsilon}^* = 0$ when $\phi \leqslant \phi(0, R, \tau, c) = (1-\tau)(1-\delta)/(1+\rho) + \tau > 1$. The maximum value of $\hat{\varepsilon}^*$ satisfies $\phi(\hat{\varepsilon}, R, \tau, c) = 1 - E/\overline{K}$, which is assumed to be smaller than $\varepsilon_{\max}$ for any entrepreneur type.

To see how the level of R affects leverage and bankruptcy risk, assume again that $\hat{\varepsilon}^*$ is in the interior so $d\overline{\Pi}/d\hat{\varepsilon} = 0$ in (21). If the second-order conditions are satisfied, we have:

$$\frac{d^2\overline{\Pi}}{d\hat{\varepsilon}^2} \;\propto\; \Delta_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c) - \frac{2}{(\varepsilon_{\max} - \hat{\varepsilon})^2} \;<\; 0$$

Rewrite elements of (12) as

$$\phi_{\hat{\varepsilon}} = \frac{1-\tau}{1+\rho}\Big(1 - g\hat{\varepsilon} - cg\hat{\varepsilon}\Big)R, \quad \phi_R = \frac{1-\tau}{1+\rho}\Big(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}$$

This implies that if $\phi_{\hat{\varepsilon}} > 0$, then $\phi_R > 0$ and $\phi_{\hat{\varepsilon}R} > 0$. The following lemma then applies.[8]

Lemma 4 *Assuming that $\phi_{\hat{\varepsilon}} > 0$ at $\hat{\varepsilon}^*$ and the second-order conditions are satisfied,*

$$\frac{d\hat{\varepsilon}^*}{dR} \;>\; 0 \tag{23}$$

Thus, the probability of bankruptcy increases with the productivity R of the entrepreneur.

3.2 Decision to undertake risky investment: extensive margin

Ex ante, entrepreneurs decide whether to undertake the risky investment or to opt for the risk-free option. In the risk-free option, they invest their wealth E at a risk-free return ρ, leading to consumption of $(1+\rho)E$. They enter if their expected income as an entrepreneur, given by $\overline{\Pi}$ in (14) or (16), is at least as great as their certain income if they invest their wealth in a safe asset and obtain consumption of $(1+\rho)E$, that is,

$$\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E \quad \text{or} \quad \overline{\pi}(\hat{\varepsilon}, R, \tau, c) \geqslant 1+\rho \tag{24}$$

From (16), $\overline{\Pi}$ is increasing in R. Given that $\hat{\varepsilon}$ is being optimized, the cutoff value of R, denoted $\widehat{R}$, will be uniquely determined by $\overline{\pi}(\hat{\varepsilon}, \widehat{R}, \tau, c) = 1+\rho$. Using the expression for $\overline{\pi}$ in (14), the following lemma is apparent.

[8] Proof: Differentiate $d\overline{\Pi}/d\hat{\varepsilon} = 0$ in (21), and use $\Delta_R = \phi_{\hat{\varepsilon}R}/(1-\phi) + \phi_{\hat{\varepsilon}}\phi_R/(1-\phi)^2 > 0$ and the second order-conditions on $\hat{\varepsilon}$. ■

Lemma 5 *The cutoff value of R is determined by:*

$$\overline{\varepsilon}\widehat{R} - \delta - \rho - \frac{c\widehat{R}g\hat{\varepsilon}^2}{2} = 0 \tag{25}$$

Entrepreneurs with $R > \widehat{R}$ enter the risky sector and earn a rent. Those with $R < \widehat{R}$ invest their wealth in a risk-free asset, so earn no rent. The issue is to what extent does the cash-flow tax system serve as a tax on rents. To study this, consider first the social optimum as a benchmark.

4 The Constrained Social Optimum

For purposes of comparison with the outcome achieved under cash-flow taxation, it is useful to assume that the government confronts the same asymmetric information constraint that the banks do. That is, they must monitor ex post incomes of entrepreneurs who declare bankruptcy and incur the same cost as the banks. The constrained social optimum maximizes the surplus net of monitoring costs of entrepreneurs who invest in the risky sector since no surplus is generated either by the banks, which earn zero expected profits, or by potential entrepreneurs who invest in the safe outcome and earn $(1+\rho)E$. Expected social surplus, denoted $\overline{S}$, can be defined as the expected value of production by entrepreneurs less the opportunity cost of financing the entrepreneurs' capital less expected monitoring costs. Financing costs include the cost of both debt and equity finance, $(1+\rho)B + (1+\rho)E = (1+\rho)K$.

For the representative entrepreneur with given R, $\overline{S}$ can be written:

$$\begin{aligned}\overline{S} &= \int_0^{\varepsilon_{\max}} \big(\tilde{\varepsilon}RK + (1-\delta)K\big)gd\tilde{\varepsilon} - (1+\rho)K - cRKg\frac{\hat{\varepsilon}^2}{2} \\ &= \overline{\varepsilon}RK + (1-\delta)K - (1+\rho)K - cRKg\frac{\hat{\varepsilon}^2}{2} \end{aligned} \tag{26}$$

where $\hat{\varepsilon}$ is determined by (6) with $\tau = 0$, or equivalently from (11),

$$\phi(\hat{\varepsilon}, R, 0, c) = \frac{1}{1+\rho}\left(\left(1 - \frac{g\hat{\varepsilon}}{2} - \frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R + (1-\delta)\right) \tag{27}$$

Note that $\overline{S}$ includes the surplus earned by the investments of entrepreneurs who go bankrupt less ex post verification costs since this accrues to the banks. This expression for $\overline{S}$ applies whether taxes are in place or not.

We can show that if there are no taxes, entrepreneurial expected profit maximization leads to the constrained social optimum. That is, there is no market failure. To see this, note that with no taxes, expected profits in (13) become:

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\Big) g d\tilde{\varepsilon} \tag{28}$$

and the banks' zero profits expression (10) reduces to:

$$(1+\rho)B = (1+r)B \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} g d\tilde{\varepsilon} + \int_{0}^{\hat{\varepsilon}} \big(\tilde{\varepsilon}RK + (1-\delta)K\big) g d\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} \tag{29}$$

Adding (28) and (29), we obtain:

$$\overline{\Pi} - (1+\rho)E = \int_{0}^{\varepsilon_{\max}} \big(\tilde{\varepsilon}RK + (1-\delta)K - (1+\rho)(B+E)\big) g d\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} = \overline{S} \tag{30}$$

Therefore, if the entrepreneur maximizes expected profits $\overline{\Pi}$, expected social surplus $\overline{S}$ will be maximized as well since $(1+\rho)E$ is constant. We can think of $\overline{\Pi} - (1+\rho)E$ as the expected rent of entrepreneurs.

To verify this in another way, rewrite $\overline{S}$ as follows using $K = E + B = E/(1-\phi)$:

$$\overline{S} = \Big(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\Big)\frac{E}{1-\phi(\cdot)}$$

where $\phi(\cdot)$ is given by (27). Differentiating with respect to $\hat{\varepsilon}$ yields:

$$\frac{d\overline{S}}{d\hat{\varepsilon}} = \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\Big(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\Big)E - \frac{cRg\hat{\varepsilon}}{1-\phi}E = \frac{d\overline{\Pi}}{d\hat{\varepsilon}} \tag{31}$$

where the second equality follows from $d\overline{\Pi}/d\hat{\varepsilon} = \overline{\pi}_{\hat{\varepsilon}}E$, with $\overline{\pi}_{\hat{\varepsilon}}$ given by (17) when $\tau = 0$. This result obviously applies for all entrepreneurial types. Therefore, the choice of leverage by entrepreneurs is efficient in the no-tax case.

Consider now the extensive-margin decision. Entry by a type$-R$ entrepreneur will be efficient if $\overline{S} > 0$. Let R^S be the marginal entrepreneur-type in the social optimum. For this entrepreneur, $\overline{S} = 0$, or using (26),

$$\overline{\varepsilon}R^S - \delta - \rho - cR^S g\frac{\hat{\varepsilon}^2}{2} = 0 \tag{32}$$

This corresponds with (25) when $\tau = 0$, so $\widehat{R} = R^S$ and entry is efficient as well. In this case, for the marginal entrepreneur, expected rent is zero, and equals expected social surplus.

When $\tau > 0$, following the same procedure of combining (13) with (10), we obtain:

$$\overline{\Pi} - (1+\rho)E = \int_{0}^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) - (1+\rho)(B+E)\Big) g d\tilde{\varepsilon} - (1-\tau)cRKg\frac{\hat{\varepsilon}^2}{2}$$

$$= (1 - \tau)\overline{S} \tag{33}$$

Now, maximizing entrepreneurs' net surplus, $\overline{\Pi} - (1+\rho)E$, will generally not be equivalent to maximizing expected social surplus $\overline{S}$. We turn to this in the next section.

Finally, it is useful to characterize the unconstrained social optimum, that is, the full-information outcome where the banks can observe the output of the bankrupt firms without monitoring. When $c = 0$, (31) and (32) become

$$\frac{d\overline{S}}{d\hat{\varepsilon}} = \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\big(\overline{\varepsilon}R - \delta - \rho\big)E \quad \text{and} \quad \overline{\varepsilon}R^U - \delta - \rho = 0$$

where R^U is the marginal entrepreneur in the unconstrained social optimum. Note first that $R^U < R^S$ so there are too few entrepreneurs in the constrained social optimum. Second, for the marginal entrepreneur R^U, $d\overline{S}/d\hat{\varepsilon} = 0$, so he is indifferent to the levels of leverage and K. For $R > R^U$, $d\overline{S}/d\hat{\varepsilon} > 0$ and independent of $\hat{\varepsilon}$. Inframarginal entrepreneurs will therefore maximize leverage such that $K = \overline{K}$. The implication is that there will be too little investment in the constrained social optimum with asymmetric information.

5 Cash-Flow Taxation with Risk-Neutral Entrepreneurs

The base-case model discussed in the previous section includes both bankruptcy, when entrepreneurs are unable to repay their loans fully, and asymmetric information, in the sense that banks can only verify bankruptcy with costly ex post monitoring. In this section, we consider the effect of cash-flow taxation in the base-case economy, in particular whether such a tax is neutral.

The leverage decision for an inframarginal type$-R$ entrepreneur is governed by (20), where $d\overline{\Pi}/d\hat{\varepsilon} = 0$ if $\hat{\varepsilon}^*$ is in the interior. In this case, differentiating $\phi(\cdot)$ gives:

$$\frac{d\phi}{d\tau} = \phi_{\hat{\varepsilon}}\frac{d\hat{\varepsilon}^*}{d\tau} + \phi_\tau \tag{34}$$

By Lemma 3, for $\hat{\varepsilon}^*$ in the interior, $d\hat{\varepsilon}^*/d\tau = 0$. Therefore, (34) reduces to

$$\frac{d\phi}{d\tau} = \phi_\tau = \frac{1-\phi}{1-\tau} > 0 \tag{35}$$

While the tax does not affect bankruptcy risk, it does increase leverage and investment so is not neutral.

The tax is also neutral with respect to the extensive margin. The marginal entrepreneur $\widehat{R}$ satisfies(25). Since $\hat{\varepsilon}$ independent of τ, so is $\widehat{R}$. Thus, entry decisions are unaffected by the tax.

Next, consider the effect of the tax on expected profits of the firm. Lemma 2 applies, so by (15), $\overline{\pi}$ is given by:

$$\overline{\pi} = \frac{1-\tau}{1-\phi}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 \equiv D\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2$$

Using (35), $D \equiv (1-\tau)/(1-\phi)$ is independent of τ,[9] so expected profits are as well. Thus, while the tax increases leverage and therefore K, it leaves expected after-tax profits unchanged.

Expected government revenue from the cash-flow tax, using (4) and the tax deductibility of monitoring costs, is

$$\overline{T} = \tau \int_{\widehat{R}}^{\overline{R}} \left(\overline{\varepsilon}R - \rho - \delta - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{1-\phi(\cdot)}dH(R) \equiv \tau\overline{Y} \tag{36}$$

where $\overline{Y}$ is the aggregate expected tax base and $H(R)$ has been defined as the distribution of entrepreneur types. Given from above that neither $\widehat{R}$ nor $\hat{\varepsilon}$ are affected by the tax, differentiating $\overline{T}$ yields:

$$\frac{d\overline{T}}{d\tau} = \overline{Y} + \tau \int_{\widehat{R}}^{\overline{R}} \left(\overline{\varepsilon}R - \rho - \delta - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{(1-\phi)^2}\frac{d\phi}{d\tau}dH(R) > 0 \tag{37}$$

where the inequality follows from (35). The first term is the mechanical effect of an increase in the tax rate on revenues which is positive. The second term is also positive given that leverage increases with the tax rate as shown above. Since leverage increases with τ, more rents are created by the additional investment and this induces an increase in $\overline{Y}$.

Finally, consider the effect of the tax on constrained expected social surplus $\overline{S}$. Using $\overline{\Pi} - (1+\rho)E = (1-\tau)\overline{S}$ from (33), we have:

$$\overline{S} = \frac{\overline{\Pi} - (1+\rho)E}{1-\tau}$$

Since a tax increase leaves $\overline{\Pi} = \overline{\pi}E$ unchanged, it will increase $\overline{S}$. In effect, the tax induces the firms to increase leverage while keeping $\hat{\varepsilon}$ constant. To see how the tax affects K, start by substituting $D = (1-\tau)/(1-\phi)$ into (2) which gives $K = ED/(1-\tau)$. Let K_0 be the level of capital chosen by the entrepreneur in the absence of taxation, so $K_0 = ED$. Therefore, $K = K_0/1-\tau$, and

$$\Delta K = K - K_0 = \frac{K_0}{1-\tau} - K_0 = \frac{\tau K_0}{1-\tau} > 0$$

[9] Proof:

$$D_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\phi_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\frac{1-\phi}{1-\tau} = 0 \qquad \blacksquare$$

Thus, introducing the tax increases K and as can be seen from (26) $\overline{S}$ increases.[10] Equivalently, the increase in K holding $\hat{\varepsilon}$ constant generates more pre-tax profits, or rents. The government taxes away those profits, leaving after-tax expected profits unchanged and improving constrained expected social surplus. Thus, while the no-tax outcome replicates the constrained social optimum, implementing a cash-flow tax improves social outcomes without changing firms' expected profits. It does so by breaking the connection between leverage and bankruptcy risk. This reflects the fact that levels of K in the absence of the tax are less than in the unconstrained social optimum as discussed above.

The optimal tax rate for a given entrepreneur type would be that which just induced the entrepreneur to choose maximum leverage. The government cannot observe R so cannot implement optimal type-specific tax rates.

The main results of the analysis in the base-case model are summarized as follows.

Proposition 1 *With risk-neutral entrepreneurs, equilibrium has the following properties:*

i. Entrepreneurs with average product R above some threshold level $\widehat{R}$ enter the risky industry and earn a rent. For those with K in the interior, leverage ϕ and bankruptcy risk $\hat{\varepsilon}^*$ are increasing with R.

ii. In the absence of taxation, the equilibrium leverage and entry decisions are socially efficient.

iii. Assuming that the banks can deduct monitoring costs from the income tax on bankrupt projects, leverage increases with the tax rate, while bankruptcy risk and expected profits remain unchanged, and expected tax revenues increases. Expected rents and expected social surplus both increase.

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[10]The marginal effect of the tax on capital can be obtain as follows: Differentiating (2), we obtain:

$$\frac{dK}{d\phi} = \frac{E}{(1-\phi)^2} = \frac{K}{1-\phi}$$

Differentiating $D = (1-\tau)/(1-\phi)$, which is constant, gives

$$\frac{d\phi}{d\tau} = \frac{1-\phi}{1-\tau}$$

Combining these equations, we obtain,

$$\frac{dK}{d\tau} = \frac{dK}{d\phi}\frac{d\phi}{d\tau} = \frac{K}{1-\phi}\frac{1-\phi}{1-\tau} = \frac{K}{1-\tau}$$

6 ACE Tax System

The cash-flow system may be difficult to implement since it requires deductions for ex post monitoring costs c incurred by banks. Consider the following tax system that achieves the same outcome in present value terms but does not require deductibility of c. Taxes are paid by the firm as long as it is profitable, but if it goes bankrupt the bank claims a tax credit for unclaimed capital deductions. Given that bankrupt firms do not repay their loans and that their operations are transfered to creditors, tax credits for capital are also transfered to banks. The form of the tax on profitable firms and the tax credit the banks obtain for bankrupt firms are as follows.

For non-bankrupt firms, the end-of-period tax liability is:[11]

$$\widetilde{T} = \tau\big(\tilde{\varepsilon}RK - K - \rho E - rB + (1-\delta)K\big)$$

where the deduction for investment K occurs at the end of the period, while the firm gets to deduct costs of financing by equity and debt at the rates ρ and r respectively. The last term is the taxation of the sale of depreciated final assets. This expression simplifies to:

$$\widetilde{T} = \tau(\tilde{\varepsilon}RK - \rho E - rB - \delta K) \quad \text{for} \quad \varepsilon \geqslant \hat{\varepsilon} \tag{38}$$

In the case of bankruptcy, the firm does not repay its debt, and there is an unclaimed tax credit on its equity-financed investment of $\tau(1+\rho)E$. This is grossed-up by the tax rate and the amount refunded to the bank is[12]

$$T = -\frac{\tau}{1-\tau}(1+\rho)E \quad \text{for} \quad \varepsilon < \hat{\varepsilon} \tag{39}$$

The bankrupt firm gets nothing for its investment since that is turned over to the bank.

The ex-post profit of the firm when $\varepsilon \geqslant \hat{\varepsilon}$ becomes $\widetilde{\Pi} = \tilde{\varepsilon}RK + (1-\delta)K - (1+r)B - \widetilde{T}$, which using (38) can be written as

$$\widetilde{\Pi} = (1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\big) + \tau(1+\rho)E \tag{40}$$

Bankruptcy again occurs when $\widetilde{\Pi} < 0$, where bankruptcy risk $\hat{\varepsilon}$ at $\widetilde{\Pi} = 0$ satisfies:

$$0 = \hat{\varepsilon}RK + (1-\delta)K - (1+r)B + \frac{\tau(1+\rho)}{1-\tau}E \tag{41}$$

[11] This is equivalent to the CAA system when the tax depreciation rate is 100 percent at the end of the period. Firms put investment K into a capital account at the beginning of the period, and deduct $K + \rho E + rB$ from taxes at the end of the period.

[12] The reason for the difference between the firm's unclaimed tax credit and the amount refunded to the bank is explained in Bond and Devereux (2003) as follows. Providing a refund to the bank reduces the interest rate. The benefit to the firm will be equal to the reduction in the interest rate multiplied by $(1-\tau)$. Since the refund is provided to the bank rather than the firm, it must be multiplied by $1/(1-\tau)$.

The bank zero-expected profit condition, given that it obtains the after-tax profits of the bankrupt firms including the tax credit on unused equity deductions, is

$$(1+\rho)B = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1+r)Bgd\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \Big(\tilde{\varepsilon}RK + (1-\delta)K + \frac{\tau(1+\rho)}{1-\tau}E\Big)gd\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} \quad (42)$$

Using (41) to eliminate $(1+r)B$ from (42) we obtain after routine simplification the same expression for leverage (11) as in the cash-flow tax case and its properties in (12).

Using (40), expected profits are

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi}gd\tilde{\varepsilon} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\big) + \tau(1+\rho)E\Big)gd\tilde{\varepsilon} \quad (43)$$

Substituting (42) into (43) to eliminate $(1+r)B$, we obtain:

$$\overline{\Pi} = (1-\tau)\Big(\overline{\varepsilon}R + (1-\delta) - cgR\frac{\hat{\varepsilon}^2}{2}\Big)K + (1+\rho)E - (1-\tau)(1+\rho)(B+E)$$

Using $B+E=K=1/(1-\phi)$, this becomes

$$\overline{\Pi} = \frac{1-\tau}{1-\phi}\Big(\overline{\varepsilon}R - \delta - \rho - cgR\frac{\hat{\varepsilon}^2}{2}\Big)E + (1+\rho)E \quad (44)$$

This is the same as (14) in the cash-flow tax case. Lemma 3 still applies, so $\hat{\varepsilon}$ is independent of τ. With $\hat{\varepsilon}$ independent of τ, Lemma 5 implies that $\widehat{R}$ is also independent of τ. Then by (35), $d\phi/d\tau = (1-\phi)/(1-\tau) = 1/D$, so D is independent of τ as before. Also, $\overline{\pi}$ is independent of τ by Lemma 2.

Finally, using (38) and (39), expected tax revenues from a type$-R$ entrepreneur can be written

$$\overline{T}_R = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \tau(\tilde{\varepsilon}RK - \rho E - rB - \delta K)gd\tilde{\varepsilon} - \int_0^{\hat{\varepsilon}} \frac{\tau}{1-\tau}(1+\rho)Egd\tilde{\varepsilon}$$

Using the bank zero-profit condition (42) to eliminate rB, this expression reduces to

$$\overline{T}_R = \int_0^{\varepsilon_{\max}} \tau(\tilde{\varepsilon}R - \rho - \delta)Kgd\tilde{\varepsilon} - \tau cRg\frac{\hat{\varepsilon}^2}{2}$$

which is equivalent to expected taxes for a type$-R$ entrepreneur reflected in $\overline{T}$ for the cash-flow tax case (36). Thus, the ACE tax has the same effect on entrepreneurial behaviour and on efficiency as the cash-flow tax.

This equivalence holds despite the fact that bankruptcy costs are not deductible under the ACE but they are under the cash-flow tax. The intuition is as follows. The equilibrium interest rate reflects expected bankruptcy costs, and because actual interest payments are deductible from the firm's profits under the ACE, the tax system effectively provides a deduction indirectly for expected bankruptcy costs. In other words, with the cash-flow tax actual bankruptcy costs are deductible, whereas under the ACE expected bankruptcy costs are implicitly deductible. From an ex ante perspective, both are equivalent for the firm's extensive and intensive margin decisions.

The equivalence between the ACE and cash-flow taxation depends on the tax credit for bankrupt firms accruing to the banks. If we assume instead, following Bond and Devereux (2003), that the tax credit $\tau(1+\rho)E$ goes to shareholders in the event of bankruptcy, the equivalence breaks down. Ex post profits of the firm are still (40) if $\varepsilon \geq \hat{\varepsilon}$, but in the event of bankruptcy profits are $-\tau(1+\rho)E$ so expected profits in (43) become

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\Big) g d\tilde{\varepsilon} + \tau(1+\rho)E$$

Condition (41) determining $\hat{\varepsilon}$ still applies, but the bank's zero expected profit condition (42) becomes

$$(1+\rho)B = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (1+r)B g d\tilde{\varepsilon} + \int_{0}^{\hat{\varepsilon}} \Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) g d\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2}$$

Using this zero-profit condition to eliminate $(1+r)B$ from the expression for $\overline{\Pi}$ leads to (44) as above. From this we can infer that the extensive-margin decision is unaffected by the tax. However, if the bank's zero-profit condition is combined with (41), the expression for ϕ that results differs from (11), and as a result changes in τ will affect $\hat{\varepsilon}$ unlike in the cash-flow tax case. Thus, an ACE tax in which the tax credit on bankruptcy goes to the firm will not be equivalent to a cash-flow tax.[13]

We summarize these results on the ACE as follows.

Proposition 2 *Under the ACE system, the tax is equivalent to the cash-flow tax with bankruptcy costs deductible as long as unclaimed tax credits on bankrupt firms accrue to the banks. Therefore, the ACE tax system increases leverage without affecting bankruptcy risk and expected profits and leads to higher expected tax revenues, rents and social surplus. This equivalence does not apply if the tax credits go to bankrupt firms instead of the banks.*

7 Extensions of the Basic Model with Risk-Neutrality

We now consider four extensions of the basic model under the cash-flow tax. First, we analyze the unconstrained case considered earlier where banks are fully informed in the sense that they know whether firms are bankrupt. This is the case where monitoring is

[13]Bond and Devereux (2003) find that wind-up and bankruptcy decisions are not affected by the ACE when the tax credit goes to the firm. However, they do not have an intensive-margin decision in their model.

unnecessary, so $c = 0$. It corresponds with Bond and Devereux (1995) where entrepreneurs are risk-neutral and there is no asymmetric information, although here we allow for limited liability in the event of bankruptcy. Second, we assume that monitoring costs c incurred by the banks are not deductible from the tax base of bankrupt firms. Third, we consider the case where potential entrepreneurs can earn some income in the risk-free sector if they choose not to enter the risky sector. Finally, we assume that there is a minimum capital requirement larger than the entrepreneur's initial wealth so investing in the risky sector necessarily involves borrowing.

7.1 Costless monitoring (c = 0)

Consider first the extensive-margin decision, where (25) applies. When $c = 0$, $\widehat{R}$ satisfies

$$\bar{\varepsilon}\widehat{R} - \delta = \rho \tag{45}$$

which is independent of τ. Equivalently, $\overline{\pi} \geqslant 1+\rho$ by (24), so for the marginal entrepreneur, $\overline{\pi} = 1 + \rho$. Eq. (45) then follows from (14). Thus, the cash-flow tax does not distort the entry decision in this case. The value of $\widehat{R}$ that satisfies (45) is the value R^S that satisfies (32) in the constrained social optimum.

Next, consider the effect of the cash-flow tax on leverage. With $c = 0$, (20) becomes:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}} E = \frac{1-\tau}{1-\phi}\Delta(\hat{\varepsilon}, R, \tau, c)(\bar{\varepsilon}R - \delta - \rho)E$$

For the marginal entrepreneur, (45) implies that $\overline{\pi}_{\hat{\varepsilon}} = 0$, so $d\overline{\Pi}/d\hat{\varepsilon}|_{R=\widehat{R}} = 0$. Therefore, leverage ϕ and thus K are indeterminate for the marginal entrepreneur and independent of τ. For inframarginal entrepreneurs, $\bar{\varepsilon}R - \delta - \rho > 0$ since $R > \widehat{R}$, so $\overline{\pi}_{\hat{\varepsilon}}$ has the same sign as $\Delta(\cdot) = \phi_{\hat{\varepsilon}}/(1-\phi)$. This is positive given that $\phi_{\hat{\varepsilon}} > 0$ when $c = 0$ by (12). Therefore, $\hat{\varepsilon}$ takes its maximum value with $\phi(\hat{\varepsilon}^*, R, \tau, c) = 1 - E/\overline{K}$. Since $\phi_{\hat{\varepsilon}} > 0$ and $\phi_\tau > 0$ by (12), we have that $d\hat{\varepsilon}^*/d\tau < 0$ to keep ϕ constant. While $\hat{\varepsilon}^*$ changes, τ does not distort ϕ or the capital stock $K = \overline{K} = E/(1-\phi)$.

Note that differentiating (14) with $c = 0$ and given that ϕ is fixed at its maximum value, we obtain

$$\frac{d\overline{\Pi}}{d\tau} = -\frac{\bar{\varepsilon}R - \delta - \rho}{1-\phi}E \ < \ 0 \text{ for } R > \widehat{R}$$

That is, the cash-flow tax decreases after-tax expected profits or rents for all inframarginal firms without changing their behavior. Moreover, the reduction is increasing in productivity R. Marginal firms are not affected since their expected profits are zero. Moreover, since leverage ϕ is not affected by changes in τ, the effect of a tax increase on expected tax revenues in (37) reduces simply to:

$$\frac{d\overline{T}}{d\tau} = \overline{Y}$$

Thus, the tax does not affect the size of the tax base since it does not affect leverage or investment. It simply diverts an increased share of pre-tax profits to the government from the firm. This also implies that the tax leaves pre-tax profits unchanged. This can be seen by simply noting that pre-tax profits, which by (14) are equal to $(\overline{\varepsilon}R-\delta-\rho)E/(1-\phi)+(1+\rho)E$ when $c=0$, are unaffected by the tax given that ϕ remains fixed.

We can verify that the choice of capital in this special case is the same as in the unconstrained social optimum. From (26), when $c=0$ expected social surplus for a type$-R$ firm becomes $\overline{S} = (\overline{\varepsilon}R^U - \rho - \delta)K$, recalling that R^U is the marginal entrepreneur in the unconstrained social optimum. For the marginal entrepreneur, expected social surplus is zero and independent of K by (45), so the latter is indeterminate and remains so under cash-flow taxation. For inframarginal entrepreneurs, $\overline{S}$ is increasing in K, so they all borrow and invest such that $K=\overline{K}$, just as we have found under cash-flow taxation.

In summary, when $c=0$, the no-tax outcome is the same as the unconstrained social optimum and τ affects neither ϕ nor $\widehat{R}$. This corresponds with the standard case of cash-flow tax neutrality when entrepreneurs are risk-neutral and capital markets are perfect. The tax applies to rents, and there are no rents for the marginal entrepreneur. Note that the CAA and RRT tax systems will also be neutral provided the rate at which unused capital deductions or cash flows are carried forward is the risk-free interest rate ρ.

7.2 Monitoring costs non-deductible

Consider now the case where banks cannot deduct bankruptcy costs from taxable income. In this case, condition (10) for banks' zero expected profits becomes:

$$(1+\rho)B = (1+r)B(1-g\hat{\varepsilon})+(1-\tau)RKg\frac{\hat{\varepsilon}^2}{2}+\big((1-\tau)(1-\delta)+\tau(1+\rho)\big)Kg\hat{\varepsilon}-cRKg\frac{\hat{\varepsilon}^2}{2} \quad (46)$$

and the leverage rate, derived in Lemma 1 for the case where monitoring costs are deductible, is now given by the following:

$$\phi(\hat{\varepsilon},R,\tau,c) = \frac{1-\tau}{1+\rho}\Big(1-\frac{g\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}R + \frac{1-\tau}{1+\rho}(1-\delta)+\tau-\frac{cRg\hat{\varepsilon}^2}{2(1+\rho)} \quad (47)$$

Differentiation of (47) gives:

$$\phi_{\hat{\varepsilon}} = (1-\tau)(1-g\hat{\varepsilon})\frac{R}{1+\rho}-\frac{cRg}{1+\rho}\hat{\varepsilon}; \quad \phi_c = -\frac{Rg}{2(1+\rho)}\hat{\varepsilon}^2; \quad \phi_R = \frac{1-\tau}{1+\rho}\Big(1-\frac{g\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}-\frac{cg}{2(1+\rho)}\hat{\varepsilon}^2;$$

$$\phi_\tau = -\Big(1-\frac{g\hat{\varepsilon}}{2}\Big)\frac{\hat{\varepsilon}R}{1+\rho}-\frac{1-\delta}{1+\rho}+1 = \frac{1-\phi}{1-\tau}-\frac{cRg\hat{\varepsilon}^2}{2(1-\tau)(1+\rho)} \quad (48)$$

$$\phi_{\hat{\varepsilon}\hat{\varepsilon}} = -(1-\tau+c)\frac{Rg}{1+\rho} \ < \ 0; \quad \phi_{\hat{\varepsilon}\tau} = -(1-g\hat{\varepsilon})\frac{R}{1+\rho} \ < \ 0$$

Finally, (20) which characterizes the intensive-margin decision is now given by

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \left(\frac{\Delta(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\cdot)}\Big((1-\tau)(\overline{\varepsilon}R-\delta-\rho) - \frac{cRg\hat{\varepsilon}^2}{2}\Big) - \frac{c\hat{\varepsilon}Rg}{1-\phi(\cdot)}\right)E \tag{49}$$

while the extensive-margin choices of entrepreneurs are still guided by (24). However, (25) for the cutoff value of R becomes

$$(1-\tau)(\overline{\varepsilon}\widehat{R}-\delta-\rho) - \frac{c\widehat{R}g\hat{\varepsilon}^2}{2} = 0 \tag{50}$$

Consider first the marginal entrepreneur $\widehat{R}$ for whom (50) applies. Substituting this into (49), we obtain:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = -\frac{c\widehat{R}g\hat{\varepsilon}}{1-\phi}E < 0$$

This implies that $\hat{\varepsilon} = 0$ for $R = \widehat{R}$, so $\widehat{R}$ satisfies (45) and is independent of τ. Thus, the marginal entrepreneur takes no bankruptcy risk, which implies the extensive-margin decision is not affected by a cash-flow tax.

Eq. (49) characterizes the leverage decision for an inframarginal entrepreneur, where $d\overline{\Pi}/d\hat{\varepsilon} = 0$ if $\hat{\varepsilon}^*$ is in the interior. Differentiating $\phi(\cdot)$ gives (34) for $d\phi/d\tau$, as in the base case. To evaluate the sign of $d\phi/d\tau$ in this case where bankruptcy costs are not deductible, it is useful to derive the following lemma.[14]

Lemma 6 *Assume $\hat{\varepsilon}^*$ is in the interior. Then,*

$$\frac{d\hat{\varepsilon}^*}{d\tau} < 0 \quad \text{as} \quad c > 0$$

[14]Proof: Differentiate the first-order condition (22) to obtain:

$$\Delta_\tau d\tau + \Big(\Delta_{\hat{\varepsilon}} - \frac{2}{(\varepsilon_{\max}-\hat{\varepsilon}^*)^2}\Big)d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition. Since $\Delta > 0$ by (22), $\phi_{\hat{\varepsilon}} > 0$ by the definition of Δ in (18). Differentiating (18) by τ, we have $\Delta_\tau = \phi_{\hat{\varepsilon}\tau}/(1-\phi) + \phi_\tau\phi_{\hat{\varepsilon}}/(1-\phi)^2$. Using (48) for ϕ_τ and $\phi_{\hat{\varepsilon}\tau}$,

$$\Delta_\tau = -\frac{1}{1-\phi}\frac{R}{1+\rho}(1-g\hat{\varepsilon}) + \frac{\phi_{\hat{\varepsilon}}}{1-\phi}\frac{1}{1-\tau} - \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\frac{1}{1-\tau}\frac{cRg\hat{\varepsilon}^2}{2(1+\rho)}$$

Using (48) for $\phi_{\hat{\varepsilon}}$, we obtain:

$$\Delta_\tau = -\frac{c\hat{\varepsilon}Rg}{(1+\rho)(1-\tau)(1-\phi(\cdot))}\Big(1 + \frac{\phi_{\hat{\varepsilon}}(\cdot)}{1-\phi(\cdot)}\frac{\hat{\varepsilon}}{2}\Big) \tag{51}$$

Lemma 6 follows since $\Delta_\tau < 0$ as $c > 0$ by (51) when $\phi_{\hat{\varepsilon}} > 0$.

By Lemma 6, $d\hat{\varepsilon}^*/d\tau < 0$ when $c > 0$ and $\hat{\varepsilon}^* > 0$, and by (18) and (22), $\phi_{\hat{\varepsilon}} > 0$, while $\phi_\tau \gtreqless 0$ by (48). Therefore, $d\phi/d\tau$ is ambiguous in sign, but in general cash-flow taxation is distortionary.

Next, consider the effect of the tax on expected after-tax profits of the firm. Lemma 2 applies, so $\overline{\pi}$ is given by (15). Differentiating this by τ, we obtain

$$\frac{d\overline{\pi}}{d\tau} = -\frac{1}{1-\phi}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 - \frac{1-\tau}{1-\phi}Rg(\varepsilon_{\max} - \hat{\varepsilon})\frac{d\hat{\varepsilon}^*}{d\tau} + \frac{1-\tau}{(1-\phi)^2}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2\frac{d\phi}{d\tau} \quad (52)$$

The first term in (52) is negative representing the (mechanical) effect of a change in τ on after-tax profits. The second term is positive, and results from the fact that the tax reduces bankruptcy risk for any given level of leverage, as shown in Lemma 6. Specifically, because of loss offsetting, increasing the tax rate reduces bankruptcy risk holding leverage constant, and this tends to increase expected profits. The last term is ambiguous in sign by (34). If the firm increases leverage, K will increase, resulting in more pre-tax rents for the firm, and vice versa. Eq. (52) reflects the non-neutrality of the cash-flow tax in this case.

This non-neutrality has implications for how expected tax revenue responds to the tax rate. Expected tax revenues are still given by (36), and $d\overline{T}/d\tau$ by (37), although the sign of $d\overline{T}/d\tau$ is now ambiguous. As before, the first term in (37) is the mechanical increase in tax revenues from the pre-existing base. The sign of the second term is ambiguous depending on whether leverage and therefore K increase or decrease with the tax.

7.3 Earned income in the risk-free sector

Suppose next that entrepreneurs who choose not to enter the risky sector obtain a fixed income denoted y in the risk-free sector, and get to invest their wealth E at the risk-free interest rate as well. In this case, an entrepreneur will enter as long as $\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E+y$, by analogy to (24). Consider the special case where $c = 0$. Eq. (25) determining $\widehat{R}$ becomes

$$(1-\tau)(\overline{\varepsilon}\widehat{R} - \delta - \rho) = \frac{y}{E}$$

Thus, entry would appear to be deterred, though the intensive-margin decision would not be affected.

Neutrality would be achieved if income in the risk-free sector, y, were taxed at the same tax rate applied to entrepreneurs' cash-flows, τ. Kanniainen and Panteghini (2012) also find that cash-flow taxation does not affect the choice of entrepreneurs to undertake a risky investment as long as the same tax rate applies to earnings in the safe occupation.

One could complicate matters further by assuming that part of the income earned in the risky sector reflects labor income of the entrepreneur. If that income were y and the entrepreneur was allowed to deduct it from cash flows, neutrality would prevail as long as the income y earned in the risk-free sector was taxed at the same rate as income y attributed to the entrepreneur in the risky sector.

7.4 Minimum capital requirements

We assumed so far that the entrepreneurs invested all their asset wealth E in the risky firm, and could borrow on top of that. Marginal entrepreneurs would either choose $E = K$ in the case where $c > 0$, or were indifferent about the amount of borrowing with $c = 0$. This accounted for the fact that marginal entrepreneurs might as well choose $\hat{\varepsilon} = 0$ in which case $r = \rho$, resulting in no distortion of the entry decision.

Suppose however that there is a minimum level of $K > E$ that is needed for a firm to operate. This implies that all firms must borrow at least some fixed amount, and generally there will be some bankruptcy risk, $\hat{\varepsilon} > 0$ (although that is not necessarily the case). In the special case where $c = 0$, the cash-flow tax will still be neutral. The marginal entrepreneur will still be indifferent to leverage so does not mind taking on the minimum required B, and the inframarginal firms will all go to the maximum K as before.

In the case with $c > 0$, in the absence of the minimum K, the marginal firm chooses $\hat{\varepsilon} = 0$, so there is no risk of bankruptcy and $r = \rho$. Inframarginal firms have an incentive to borrow as in the case , but they do not go to the maximum K because of the additional monitoring cost c faced by the banks, which serves to increase r. When the minimum capital constraint is imposed, marginal firms with productivity $\widehat{R}$ now have to go to the minimum, and if that causes $\hat{\varepsilon}$ to become positive for them, their entry and leverage decisions will both be distorted by the cash-flow tax.

The main findings of this section are as follows.

Proposition 3 *Under the cash-flow tax and risk-neutrality,*

i. With costless monitoring ($c = 0$)*, the cash-flow tax does not distort either leverage or entry decisions. Leverage and capital are indeterminate for the marginal entrepreneur. For inframarginal entrepreneurs, leverage and capital take their maximum values and bankruptcy risk decreases with the level of the cash-flow tax. A cash-flow tax diverts expected rents from the firms and banks to the government.*

ii. With costly monitoring ($c > 0$)*, the cash-flow tax with monitoring costs non-deductible does not distort entry, but distorts leverage and reduces bankruptcy risk for inframarginal entrepreneurs. The marginal entrepreneur incurs no debt and faces no bankruptcy risk. For inframarginal entrepreneurs, the effect of the cash-flow tax on leverage is ambiguous, but results in lower bankruptcy risk than in the social optimum.*

iii. If potential entrepreneurs who do not enter the risky sector earn an alternative income in the risk-free sector, in addition to the risk-free return on their wealth, if $c = 0$ the cash-flow tax will be neutral with respect to both entry and leverage as long as income earned in the risk-free sector is taxed at the same rate as cash flows in the risky sector. Otherwise, entry decisions will be distorted.

iv. If there is a minimum capital requirement greater than E, the cash-flow tax will remain neutral if monitoring is costless. With costly monitoring, if the minimum capital requirement leads the marginal entrepreneur to face strictly positive bankruptcy risk, entry and leverage will be distorted.

8 Risk-Averse Entrepreneurs

Recall that all consumption takes place at the end of the period. Entrepreneurs who enter the risky industry invest all their wealth in their firm at the beginning of the period and consume the after-tax profits $\widetilde{\Pi}$ at the end, where the base-case cash-flow tax outlined above applies. As in the base case with risk neutrality, entrepreneurs get no income in the event of bankruptcy since that goes to the bank. Let their end-of-period expected utility V be:

$$V = \frac{1}{1-\gamma}\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi}^{1-\gamma} g d\tilde{\varepsilon} \tag{53}$$

where $0 < \gamma < 1$, so the entrepreneur's utility function exhibits constant relative risk aversion. Using (7) and (2), expected utility becomes:

$$V = \frac{\big((1-\tau)RK\big)^{1-\gamma}}{1-\gamma}\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (\tilde{\varepsilon}-\hat{\varepsilon})^{1-\gamma} g d\tilde{\varepsilon} = \frac{E^{1-\gamma}}{1-\gamma}\Big(\frac{R(1-\tau)}{1-\phi(\cdot)}\Big)^{1-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{2-\gamma}\frac{g}{2-\gamma} \tag{54}$$

Alternatively, using (15) for $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$, expected utility in (54) may be written:[15]

$$V = \frac{E^{1-\gamma}}{1-\gamma}\overline{\pi}(\hat{\varepsilon}, R, \tau, c)^{1-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{\gamma}\frac{2^{1-\gamma}g^{\gamma}}{2-\gamma} \tag{55}$$

These alternative representations of expected utility will be useful in what follows.

As before, the entrepreneur decides whether to enter, and if so, how much to borrow and therefore how much risk to take on. Consider these decisions in reverse order.

[15] Proof: Rewrite (54) as:

$$V = \frac{E^{1-\gamma}}{1-\gamma}\Big(\frac{1-\tau}{1-\phi}R\frac{g}{2}(\varepsilon_{\max}-\hat{\varepsilon})^2\Big)^{1-\gamma}\cdot\Big(\frac{g}{2}\Big)^{\gamma-1}(\varepsilon_{\max}-\hat{\varepsilon})^{2(\gamma-1)}\frac{g}{2-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{2-\gamma}$$

Using (15), this becomes (55). ■

8.1 Intensive-margin decision

The choice of leverage ϕ is again equivalent to the choice of bankruptcy risk $\hat{\varepsilon}$ through (11). Differentiating (54) with respect to $\hat{\varepsilon}$ and using the definition of $\Delta(\cdot)$ in (18), we obtain after straightforward simplification:

$$\frac{dV}{d\hat{\varepsilon}} \;\propto\; \Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2-\gamma}{1-\gamma}\frac{1}{\varepsilon_{\max} - \hat{\varepsilon}} \tag{56}$$

Let $\hat{\varepsilon}^*$ be the optimal choice of $\hat{\varepsilon}$. If $\hat{\varepsilon}^*$ is in the interior, $dV/d\hat{\varepsilon} = 0$, and (56) gives:

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2-\gamma}{1-\gamma}\frac{1}{\varepsilon_{\max} - \hat{\varepsilon}^*} > 0 \tag{57}$$

From (57) we obtain the following analogue to Lemma 3.[16]

Lemma 7 *If $\hat{\varepsilon}^*$ is in the interior, then*

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0 \qquad \text{and} \qquad \frac{d\hat{\varepsilon}^*}{d\gamma} < 0$$

Note that Lemma 4 applies here as well, so bankruptcy risk and therefore leverage are increasing in entrepreneurial productivity: $d\hat{\varepsilon}^*/dR > 0$.

8.2 Extensive-margin decision

The entry decision involves comparing expected utility as an entrepreneur with that obtained from the safe alternative, where end-of-period consumption is $(1+\rho)E$. Entrepreneurs will enter if $V \geqslant \big((1+\rho)E\big)^{1-\gamma}/(1-\gamma)$. From (54), V is increasing in R, so the cutoff value $\widehat{R}$ will be uniquely determined.

To characterize the extensive-margin decision, we show first that in equilibrium $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur, $\widehat{R}$. Let R_0 be the value of R such that $\hat{\varepsilon}^*$ just becomes zero. Then, $\widehat{R}$ satisfies the following:

Lemma 8 $\widehat{R} \lessgtr R_0$ *as* $\gamma \gtrless 0$

[16] Proof: Differentiate the first-order condition (57) to obtain:

$$\Delta_\tau d\tau - \Big(\frac{1}{\varepsilon_{\max} - \hat{\varepsilon}^*}\frac{1}{(1-\gamma)^2}\Big)d\gamma + \Big(\Delta_{\hat{\varepsilon}} - \frac{2-\gamma}{1-\gamma}\frac{1}{(\varepsilon_{\max} - \hat{\varepsilon}^*)^2}\Big)d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition. Lemma 7 follows since $\Delta_\tau = 0$ by (19).

The proof is provided in the Appendix. The implication of Lemma 8 is that $\hat{\varepsilon}^* = 0$ at $R = \widehat{R}$ when $\gamma \geqslant 0$, and this applies regardless of the values of c or τ. Note that this confirms the result that we found earlier in the risk-neutral model with $\gamma = 0$ where the marginal entrepreneur choose $\hat{\varepsilon}^* = 0$ as well. In the risk-neutral case, $\widehat{R} = R_0$, so $\hat{\varepsilon}^* > 0$ for all inframarginal entrepreneurs.

Lemma 8 implies that with $\gamma > 0$, $\hat{\varepsilon}^* = 0$ for all entrepreneurs with $R \in [\widehat{R}, R_0]$. In that range, there is no risk of bankruptcy, so $r = \rho$ by the banks' zero-profit conditions (10) and ex post after-tax profits in (5) can be written, using $B = K - E = E/(1-\phi) - E$:

$$\widetilde{\Pi} = \Big(\frac{1-\tau}{1-\phi}\big(\tilde{\varepsilon}R - \delta - \rho\big) + 1 + \rho\Big)E = \big(D(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)E$$

where, recall, $D \equiv (1-\tau)/(1-\phi)$. Therefore, expected utility can be written:

$$V = \int_0^{\varepsilon_{\max}} \frac{\widetilde{\Pi}^{1-\gamma}}{1-\gamma} g d\tilde{\varepsilon} = \frac{E^{1-\gamma}}{1-\gamma}\int_0^{\varepsilon_{\max}} \big(D(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} \tag{58}$$

The value of $\widehat{R}$ is determined where $V = (1+\rho)^{1-\gamma}E^{1-\gamma}/(1-\gamma)$ or using (58),

$$\int_0^{\varepsilon_{\max}} \big(D(\tilde{\varepsilon}\widehat{R} + 1 - \delta - \rho) + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} = (1+\rho)^{1-\gamma} \tag{59}$$

Entrepreneurs will enter the risky sector only if $R \geqslant \widehat{R}$. Those with $R < \widehat{R}$ will invest their wealth in a safe asset.

The expression for V in (58), which applies for all $R \in [\widehat{R}, R_0]$, has a further implication for the intensive margin. In this range, an entrepreneur of given R will choose leverage, or equivalently $D \equiv (1-\tau)/(1-\phi)$, to maximize V. The first-order condition $\partial V/\partial D = 0$ reduces to:

$$\int_0^{\varepsilon_{\max}} \big(D^*(\tilde{\varepsilon}R + 1 - \delta - \rho) + \rho\big)^{-\gamma} g d\tilde{\varepsilon} = 0 \tag{60}$$

where D^* is the optimal choice of D. This value of D^*, and therefore ϕ will be increasing in R. As $R \to R_0$, $\phi \to (1-\tau)(1-\delta)/(1+\rho) + \tau$, which is the value of ϕ that just satisfies (11) when $\hat{\varepsilon} = 0$.

8.3 The effects of cash-flow taxation

We next turn to the effects of the base-case cash-flow tax on the entrepreneurs' intensive- and extensive-margin decisions when entrepreneurs are risk-averse. Later we consider the ACE case where firms can deduct interest, and where tax credits on unused capital costs go to the banks. We begin with the extensive and intensive margin decisions, and then consider the effects on expected profit and on government revenues.

8.3.1 Extensive margin decision

Lemma 8 applies here since $\gamma > 0$, so $\widehat{R} < R_0$ implying that $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur. For a given tax rate τ, the marginal entrepreneur $\widehat{R}$ chooses leverage such that the first-order condition (60) is satisfied at $R = \widehat{R}$. This leads to a unique value of the optimal $D^* = (1-\tau)/(1-\phi)$. As τ increases, ϕ must increase by the same amount to keep D^* constant. Recall that $\widehat{R}$ satisfies (59). Since the choice of D^* by the marginal entrepreneur is independent of the tax rate, the value of $\widehat{R}$, and therefore the extensive margin decision, that satisfies (59) is independent of τ.

8.3.2 Intensive margin decision

Next, consider the intensive-margin decision, beginning with entrepreneurs in the range $R \in [\widehat{R}, R_0]$. As we have seen, for these entrepreneurs, $\hat{\varepsilon}^* = 0$ and the first-order condition (60) again applies. This yield a unique value of D^* for each R that is independent of the tax rate. Since $D^* = (1-\tau)/(1-\phi)$, leverage will rise by the same amount as an increase in τ. And, since D^* is invariant with the tax rate, so will V be as seen by (58). Thus, for entrepreneurs in this range, a result analogous to Domar-Musgrave (1944) applies. Entrepreneurs offset an increase in τ by increasing private risk-taking, while achieving the same level of expected utility V. The increase in leverage ϕ entails more investment and therefore more rent generation, which accrues to the government in increased tax revenues as discussed below.

For entrepreneurs of type $R > R_0$, $\hat{\varepsilon}^* > 0$. Their intensive-margin decision is similar to the base case with risk-neutrality above. Eq. (34) again applies, and since $d\hat{\varepsilon}^*/d\tau = 0$ by Lemma 7 (34) simplifies to $d\phi/d\tau = \phi_\tau$. By (12), $\phi_\tau = (1-\tau)/(1-\phi)$, so τ encourages leverage. However, the increase in leverage does not translate into an increase in bankruptcy risk $\hat{\varepsilon}$, since $d\hat{\varepsilon}^*/d\tau = 0$. As well, as we have shown in footnote 9, $D = (1-\tau)/(1-\phi)$ is constant when $\phi_\tau = (1-\tau)/(1-\phi)$. Therefore, with D and $\hat{\varepsilon}$ constant, so is expected utility in (54) as long as leverage ϕ and K are in the interior. This again is analogous to the Domar and Musgrave (1944) result: a tax on capital income with full loss-offset encourages risk-taking by risk-averse individuals because the government is sharing the risk with the entrepreneur on actuarially fair terms. In the case of a cash-flow tax, the government is sharing the risk of the entrepreneur, and as a consequence private risk $\hat{\varepsilon}$ does not change as a result of the tax.

8.3.3 Expected profits and tax revenues

In addition, an increase in τ has no effect on expected profits. To see this, consider the expression for the expected rate of return on equity, $\overline{\pi}$, in (15). Since both $\hat{\varepsilon}^*$ and $D^* = (1-\tau)/(1-\phi)$ are independent of τ, $\overline{\pi}$ will also be invariant with τ. Therefore, entrepreneurs are able to offset the effect of the tax on their expected after-tax profits by increasing their leverage. The increase in leverage will correspond to an increase in borrowing and investment, which in turn will increase expected before-tax profits and therefore expected tax revenue to the government since after-tax profits are unchanged. To see that expected tax revenues will rise, note that (36) still applies, and the change in expected tax revenues will again be given by (37), which is positive since $d\phi/d\tau > 0$. Expected tax revenues rise due both to a mechanical effect and to an increase in the tax base because of increased leverage and investment.

In summary, if $c > 0$ and $\gamma > 0$, leverage is encouraged by the cash-flow tax, but entrepreneurs' expected profits are not affected. As well, entry is not affected. Expected tax revenues of the government increase, reflecting the fact that the increase in leverage increases before-tax profits or rents while leaving after-tax profits unchanged. Social welfare will increase if the increase in expected tax revenues is valuable to the government, and that depends on how the government evaluates the increase in risk that might accompany the tax revenues.

Consider two extreme cases. In the first, assume that ε is an idiosyncratic shock and that the government can fully pool risk. Then, expected tax revenues in (36) are non-stochastic and the increase in tax revenues (37) are unambiguously beneficial. The increase in the tax rate therefore improves social welfare.

In the other extreme, suppose ε is a common shock to all firms, so government revenue is stochastic. Total tax revenue can then be written $T(\varepsilon)$, and expected tax revenue is $\overline{T} = \mathbb{E}_\varepsilon[T(\varepsilon)]$, and continues to be given by (36). The variance of government revenue can be written:

$$\mathrm{Var}[T] = \tau^2 \Big(\int_{\widehat{R}}^{\overline{R}} \frac{R}{1-\phi} dH(R)E \Big)^2 \sigma_\varepsilon^2$$

Suppose the social value of government revenue is given by the concave function $G(T)$. Then, a second-order Taylor expansion gives:

$$\begin{aligned} \mathbb{E}_\varepsilon[G(T(\varepsilon))] &\approx G(\overline{T}) + G'(\overline{T})(T(\varepsilon) - \overline{T}) + \frac{1}{2} G''(\overline{T})(T(\varepsilon) - \overline{T})^2 \\ &= G(\overline{T}) + \frac{1}{2} G''(\overline{T}) \mathrm{Var}[T] \end{aligned}$$

If $G''(\overline{T}) < 0$, this expression is generally ambiguous in sign. The more concave is $G(T)$, the less willing is the government to take on risk, and the more likely will an increase in the tax rate reduce social welfare even though it increases expected tax revenues. If an increase in the tax rate increases social welfare, the optimal cash-flow tax rate on entrepreneur R will be the one that just causes the entrepreneur to go to the maximal leverage. Of course, since the government cannot identify entrepreneurs' types, it cannot apply the optimal type-specific tax rate.

This compares with the standard analysis of capital income taxation and risk-taking discussed by Atkinson and Stiglitz (1980) and Buchholz and Konrad (2014). If the government is better able to pool risk than individuals, the no-tax equilibrium will entail too little risk-taking. A tax on capital income that results in more risk-taking—that is, more leverage—will improve social outcomes without affecting the expected utility of individuals. In the extreme case where the government can pool risk perfectly, the increase in pre-tax expected income does not affect the total risk faced by the economy, since the risk of individuals has not changed. However, if the government is unable to pool risk any better then the private sector, private sector outcomes will be optimal. The increase in expected pre-tax income resulting from capital income taxation will be accompanied by an increase in risk that individuals were not willing to bear on their own, so excessive risk-taking will be induced from a social point of view.

8.4 The ACE tax system

Consider again the ACE tax introduced above. Firms pay the tax in (38) if they are profitable, and banks receive the tax credit (39) on behalf of bankrupt firms. It is straightforward to show that with risk aversion this tax system yield the same results as the base-case cash-flow tax. Eq. (7) still applies, so expected utility is given by (54). The first-order condition on $\hat{\varepsilon}$, (57) holds, and therefore so does Lemma 7. As we have seen above, the expressions for leverage and its properties, (11) and (12), still applies, and therefore so does the proof of Lemma 8 in the Appendix. Therefore, (59) determining $\widehat{R}$ continues to apply.

These equivalencies imply that the effects of ACE tax changes replicate those of the cash-flow tax. The entry decision is independent of τ, while an increase in τ increases leverage without affecting bankruptcy risk. Both expected after-tax profits and expected utility are unchanged, while the government obtains more revenue, some of which comes from the increase in expected rents resulting from the increase in leverage and investment.

Our findings with risk-averse entrepreneurs are summarized as follows.

Proposition 4 *With risk-averse entrepreneurs, equilibrium has the following properties:*

i. There is a range of entrepreneurs with $R \in [\widehat{R}, R_0]$ for whom there is no risk of bankruptcy, so $\hat{\varepsilon}^* = 0$ and $r = \rho$. For these entrepreneurs, the cash-flow tax increases *leverage, but leaves expected utility unchanged.*

ii. For all entrepreneurs for which $\hat{\varepsilon}^*$ is in the interior, leverage and expected tax revenues increase with the cash-flow tax rate, while bankruptcy risk, expected profits and expected utility are unchanged, so constrained social surplus increases.

iii. The cash-flow tax is neutral with respect to entry decisions.

iv. The ACE tax with a tax credit on bankrupt firms accruing to the banks has the same effects as a cash-flow tax.

Without asymmetric information ($c = 0$) and with risk-averse entrepreneurs, it can be shown that the cash-flow tax increases borrowing and leverage while it is neutral with respect to bankruptcy risk. These effects arise because the cash-flow tax involves some risk-sharing between the government and the entrepreneur. In fact, the effect of the tax on expected profits is exactly offset by increased leverage, so that the tax leaves after-tax profits unchanged.

9 Concluding Remarks

In this paper, we analyzed the impact of cash-flow business taxation on firms' choice of leverage and on decisions to enter a risky industry in a setting where entrepreneurs may be risk-averse and face bankruptcy risk, and where there is asymmetric information between entrepreneurs and financial intermediaries. We focused in particular on whether the neutrality of cash-flow taxation found by Bond and Devereux (1995, 2003) in the absence of asymmetric information and risk-aversion continues to hold under these features. The main results of the analysis are as follows.

With risk-neutral entrepreneurs, cash-flow taxation taxes rents only. Without asymmetric information in the credit market (i.e. with costless monitoring), the cash-flow tax affects neither entry nor leverage decisions, so the standard neutrality results apply. CAA and RRT systems are also neutral in this case provided that the risk-free interest rate is used to carry forward untaxed cash flows. When banks must undertake costly monitoring of firms that declare bankruptcy, the tax does not affect entry decisions, given that the marginal entrepreneur earns no rent, but it distorts leverage. If banks can obtain a tax deduction for the monitoring costs they incur on the bankrupt firms they seize, the tax will increase leverage but will leave bankruptcy risk and the firms' after-tax profits unchanged. Surprisingly, the tax will increase social welfare in this case. By inducing firms to increase leverage, and therefore capital, the tax leads to higher pre-tax profits, or rents, without affecting bankruptcy risk. These additional pre-tax profits are effectively taxed away by the government leading to a higher constrained social surplus.

An ACE tax in which firms can deduct actual interest payments can be designed that is equivalent to the cash-flow tax. With such a system no tax is applied to bankrupt firms, but the banks obtain a tax credit for unused deductions by the firm. The ACE tax has the advantage over cash-flow taxation that monitoring costs incurred by the banks need not be deducted from the tax in the event of bankruptcy: interest deductibility indirectly accomplishes that. If the tax credit owing on bankrupt firms is given to the firms instead of the banks, the equivalence of the ACE with cash-flow taxation no longer applies.

With risk-averse entrepreneurs, cash-flow taxation taxes both rents and return to risk. Without monitoring costs, or if monitoring costs are deductible, the cash-flow tax increases leverage, but is neutral with respect to bankruptcy risk, expected profits and expected utility. The cash-flow tax involves some risk-sharing between the government and firms, and increases social welfare in this case for firms will levels of capital in the interior. The tax increases rents and tax revenues without affecting expected utility.

We have assumed that asymmetric information involved moral hazard rather than adverse selection, so banks can observe firm types but not their profits. If banks cannot observe the productivity of entrepreneurs ex ante, there would be an adverse selection problem as in Stiglitz and Weiss (1981) and de Meza and Webb (1987), among others. In this case, if banks are unable to offer separating contracts, all entrepreneurs will face the same interest rate. In contrast to the case considered here, the market equilibrium without taxation will be inefficient along the extensive margin and cash-flow taxation will generally not be neutral. In particular, there will be excessive entry by the least-productive entrepreneurs to take advantage of the favorable interest rate. A cash-flow tax discourages entry, thereby improving efficiency. It would be interesting to extend the analysis to the case where banks are able to offer contracts in which the interest rate varies with the size of loan, so firms can be separated by type. There will be informational rents that might influence the effect of cash-flow taxes.

Appendix

Proof of Lemma 8: Note first that the following lemma indicates when $\hat{\varepsilon}^* = 0$.

Lemma 9

$$\hat{\varepsilon}^* = 0 \qquad \mathit{if} \qquad \frac{\overline{\varepsilon} R}{\delta + \rho} < \frac{1}{2}\frac{2-\gamma}{1-\gamma} \tag{61}$$

Proof: $\hat{\varepsilon}^* = 0$ if $dV/d\hat{\varepsilon}|_{\hat{\varepsilon}=0} < 0$, or $\Delta(0, R, \tau, c) < (2-\gamma)/((1-\gamma)2\overline{\varepsilon})$ by (56). By (18),

$$\Delta(0, R, \tau, c) = \frac{\phi_{\hat{\varepsilon}}(0, R, \tau, c)}{1 - \phi(0, R, \tau, c)} = \frac{(1-\tau)R/(1+\rho)}{1 - (1-\tau)(1-\delta)/(1+\rho) - \tau} = \frac{R}{\rho + \delta}$$

using (11) and (12). ■

Next, we can derive the conditions under which $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur $\widehat{R}$. From Lemma 9, R_0 satisfies:

$$\frac{\overline{\varepsilon} R_0}{\delta + \rho} = \frac{1}{2} \frac{2-\gamma}{1-\gamma} \tag{62}$$

The expected utility of this entrepreneur becomes from (55) and using $\varepsilon_{\max} = 1/g$:

$$V_0 = \frac{E^{1-\gamma}}{1-\gamma} \overline{\pi}(0, R_0, \tau, c)^{1-\gamma} \frac{2^{1-\gamma}}{2-\gamma}$$

Using (15) for $\overline{\pi}$, (11) for ϕ, and (62), this may be written after some manipulation:

$$V_0 = \frac{(1+\rho)^{1-\gamma} E^{1-\gamma}}{1-\gamma} \frac{1}{(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma}} \tag{63}$$

Recall that for the marginal entrepreneur, $\widehat{R}$,

$$V(\widehat{R}) = \frac{(1+\rho)^{1-\gamma} E^{1-\gamma}}{1-\gamma}$$

We have that $\widehat{R} \leqslant R_0$ if $V(\widehat{R}) \leqslant V_0$, or from (63) if $(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma} \leqslant 1$, or equivalently $\gamma \geqslant 0$, where the equality applies for $\gamma = 0$. ■

References

Atkinson, Anthony B. and Joseph E. Stiglitz (1980) *Lectures on Public Economics* (New York: McGraw-Hill) 97–127.

Australian Treasury (2010), *Australia's Future Tax System* (Canberra: Commonwealth of Australia) (the Henry Report).

Bernanke, Ben and Mark Gertler (1989), 'Agency Costs, Net Worth, and Business Fluctuations,' *American Economic Review* 79, 14–31.

Bernanke, Ben, Mark Gertler and Simon Gilchrist (1996), 'The Financial Accelerator and the Flight to Quality,' *The Review of Economics and Statistics* 78, 1–15.

Bernanke, Ben, Mark Gertler and Simon Gilchrist (1999), 'The Financial Accelerator in a Quantitative Business Cycle Framework,' in John B. Taylor and Michael Woodford (eds), *Handbook of Macroeconomics* (Elsevier), 1341–1393.

Boadway, Robin and Michael Keen (2015), 'Rent Taxes and Royalties in Designing Fiscal Regimes for Nonrenewable Resources,' in Robert Halvorsen and David F. Layton (eds), *Handbook on the Economics of Natural Resources* (Cheltenham, UK: Edward Elgar), 97–139.

Boadway, Robin and Neil Bruce (1984), 'A General Proposition on the Design of a Neutral Business Tax,' *Journal of Public Economics* 24, 231–9.

Boadway, Robin and Michael Keen (2006), 'Financing and Taxing New Firms under Asymmetric Information,' *FinanzArchiv* 62, 471–502.

Boadway, Robin and Motohiro Sato (2011), 'Entrepreneurship and Asymmetric Information in Input Markets,' *International Tax and Public Finance* 18, 166–92.

Boadway, Robin and Jean-François Tremblay (2005), 'Public Economics and Start-up Entrepreneurs,' in Vesa Kanniainen and Christian Keuschnigg (eds), *Venture Capital, Entrepreneurship and Public Policy* (MIT Press), 181–219.

Bond, Stephen R. and Michael P. Devereux (1995), 'On the Design of a Neutral Business Tax under Uncertainty,' *Journal of Public Economics* 58, 57–71.

Bond, Stephen R., and Michael P. Devereux (2003), 'Generalised R-based and S-based Taxes under Uncertainty,' *Journal of Public Economics* 87, 1291–1311.

Brown, E. Cary (1948), 'Business-Income Taxation and Investment Incentives,' in *Income, Employment and Public Policy: Essay in Honor of Alvin H. Hansen* (New York: Norton), 300–316.

Buchholz, Wolfgang, and Kai A. Konrad (2014), 'Taxes on Risky Returns – An Update,' Max Planck Institute for Tax Law and Public Finance, Working Paper 2014-10, Munich.

Diamond, Peter (1984), 'Financial Intermediation and Delegated Monitoring,' *Review of Economic Studies* 51, 393–414.

Domar, Evsey D. and Richard Musgrave (1944), 'Proportional Income Taxation and Risk-taking,' *Quarterly Journal of Economics* 58, 388–422.

Fane, George (1987), 'Neutral Taxation Under Uncertainty,' *Journal of Public Economics* 33, 95–105.

Garnaut, Ross and Anthony Clunies-Ross (1975), 'Uncertainty, Risk Aversion and the Taxing of Natural Resource Projects,' *Economic Journal* 85, 272–87.

Institut dEconomia de Barcelona (2013), *Tax Reform*, IEB Report 2/2013.

Institute for Fiscal Studies (1991), *Equity for Companies: A Corporation Tax for the 1990s*, Commentary 26 (London).

Kanniainen, Vesa and Paolo Panteghini (2012), 'Tax Neutrality: Illusion or Reality? The Case of Entrepreneurship,' Helsinki Center of Economic Research Discussion Paper No. 349.

Klemm, Alexander (2007), 'Allowances for Corporate Equity in Practice,' *CESifo Economic Studies* 53, 229–62.

Lund, Diderik (2014), 'State Participation and Taxation in Norwegian Petroleum: Lessons for Others?,' *Energy Strategy Reviews* 3, 49–54.

Meade, James E. (1978), *The Structure and Reform of Direct Taxation*, Report of a Committee Chaired by Professor James Meade (London: George Allen and Unwin).

De Meza, David, and David C. Webb (1987), 'Too Much Investment: A Problem of Asymmetric Information,' *Quarterly Journal of Economics* 102, 281–92.

Mintz, Jack (1981), 'Some Additional Results on Investment, Risk-Taking, and Full Loss Offset Corporate Taxation with Interest Deductibility,' *Quarterly Journal of Economics* 96, 631–42.

Mirrlees, Sir James, Stuart Adam, Timothy Besley, Richard Blundell, Stephen Bond, Robert Chote, Malcolm Gammie, Paul Johnson, Gareth Myles and James Poterba (2011), *Tax by Design: The Mirrlees Review* (London: Institute for Fiscal Studies).

de Mooij, Ruud (2011), 'Tax Biases to Debt Finance: Assessing the Problem, Finding Solutions,' IMF Staff Discussion Note, SDN/11/11, Washington.

Panteghini, Paolo, Maria Laura Parisi, and Francesca Pighetti (2012), 'Italys ACE Tax and its Effect on Firm's Leverage,' CESifo Working Paper No. 3869, Munich.

President's Advisory Panel on Federal Tax Reform (2005), *Simple, Fair and Pro-Growth: Proposals to Fix Americas Tax System* (President's Advisory Panel: Washington).

Princen, Savina (2012), 'Taxes Do Affect Corporate Financing Decisions: The Case of Belgium ACE,' CESifo Working Paper No. 3713, Munich.

Sandmo, Agnar (1979), 'A Note on the Neutrality of the Cash Flow Corporation Tax,' *Economics Letters* 4, 173–76.

Stiglitz, J.E. and A. Weiss (1981), 'Credit Rationing in Markets with Imperfect Information,' *American Economic Review* 71, 393–410.

Townsend, R. (1979), 'Optimal Contracts and Competitive Markets with Costly State Verification,' *Journal of Economic Theory* 21, 265–293.

United States Treasury (1977), *Blueprints for Basic Tax Reform* (Washington: Treasury of the United States).

Williamson, S. (1986), 'Costly Monitoring, Financial Intermediation and Equilibrium Credit Rationing,' *Journal of Monetary Economics* 18, 159–79.

Williamson, S. (1987), 'Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing,' *Quarterly Journal of Economics* 101, 135–45.