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Firm Reputation and Employee Startups

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Abstract

This paper studies a repeated-game model in which firms can build a reputation for rewarding innovative employees. In any Pareto efficient equilibrium, low-value innovations get developed in established firms, while high-value innovations get developed in startups. The threshold level can be discontinuous, so otherwise similar firms may exhibit very different levels of innovation. The paper also shows that the optimal incentive contract for innovative employees has an option-like form, and that a firm may want to worsen the distribution of possible innovations. The model's predictions are consistent with a broad set of observed regularities regarding the creation of employee startups.

Keywords: Startups, innovation, reputation, venture capital markets

JEL classifications: L14, L26, O31, O34, M13

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Ninety three percent of the executives at large U.S., U.K., and French companies surveyed recently by the consulting company Accenture believe their firm's long-term success depends on its ability to innovate (Koetzier and Alon, 2013). A major challenge to a company's ability to innovate is the threat of departure of innovative employees, who always have the option to pursue their ideas on their own. Indeed, a significant fraction of startups are founded by former employees of established companies (Cooper, 1985; Bhidé, 1994; Gompers et al, 2005). To fully understand what allows companies to remain innovative, we therefore need to understand why employees leave established firms to join startups. Such employee departures do not appear to be random; rather, empirical researchers have documented a number of regularities about the characteristics of companies that spawn startups, about the employees who join startups, and about the economic environments that are conducive to creation of startups:

- Public companies located in Silicon Valley and Massachusetts spawn substantially more startups than public companies located elsewhere (Gompers et al, 2005).

- Employees in large and mature companies are less likely to become entrepreneurs than employees in smaller and younger firms (Gompers et al, 2005; Eriksson and Kuhn, 2006; Elfenbein et al, 2010; and others).

- The smaller entrepreneurial spawning in large and mature firms is at least partly offset by the employees pursuing more venturing opportunities inside the established firm (Kacperczyk, 2012).

- Venture capital (VC) markets seem to have an ambiguous effect on firm creation: Samila and Sorenson (2011) and Popov and Roosenboom (2013) document that an improvement in VC markets stimulates new firm creation, whereas Zucker et al (1998) find that the number of VC firms in a region has a negative effect on the number of startups.

- However, when a greater supply of VC funds does induce creation of startups, it can generate many more new firms than it funds. Samila and Sorenson (2011) find that investing in an additional firm induces the entry of two to twelve startups.

- Better performing employees are more likely to start new ventures (Eriksson and Kuhn, 2006; Braguinsky et al, 2012; Groysberg et al, 2009; Campbell et al, 2012).

- A firm's spawning rate increases much less than proportionally with its number of patents (Gompers et al, 2005).

- The spin-off activity is lower in economic upturns than in economic downturns (Eriksson and Kuhn, 2006).

- Startups established when the GDP growth is low have a higher risk of exiting (Eriksson and Kuhn, 2006).

This paper proposes a model that is consistent with all of the above regularities. The model builds on Arrow's (1962) insight, later developed by Anton and Yao (1994, 1995, 2002) and others, that ideas are hard to sell, especially those that are not readily protected through patents. The problem is that to convince a buyer that her idea is worth buying, the seller first needs to divulge to the buyer at least a part of the idea. But once informed, the buyer may no longer need the inventor to implement the innovation and may therefore decide to expropriate her. A well known example is the case of Robert Kearns, the inventor of the intermittent windshield wiper system. Kearns offered his idea for licensing to all three big US automakers, who all turned him down but soon started to install such systems in their cars. Another example is the quick release socket wrench invented by Peter Roberts, then a Sears employee. In 1964, Roberts offered his invention to Sears as an "employee suggestion." Sears paid him $10,000 and proceeded to make more than $40 million from the invention within ten years. Roberts's subsequent lawsuit against Sears took twenty years to settle.

Anticipating this kind of property rights problem, innovators may prefer to leave their initial employer to pursue their idea in a startup. The central point of the present model is that firms can try and resolve this issue by building a reputation for not expropriating innovators. Such a reputation is valuable because, having more resources, better expertise, established routines, and so on, existing companies can typically implement new projects more efficiently than new enterprises (Stinchcombe, 1965; Nelson and Winter, 1982; Teece, 1986). However, unless the firm is sufficiently patient, reputation for rewarding *all* ideas is impossible to maintain because the temptation to expropriate the most valuable ideas is too strong. Thus, the paper shows that, in general, a Pareto-efficient reputation equilibrium is characterized by a threshold value such that employees with ideas whose expected values exceed this threshold will leave their current employer and develop their ideas in startups, even though it would be more efficient for these ideas to be implemented by the established firm. Employees whose ideas have values below the threshold level will share them with the firm and these innovations will be developed in-house (e.g., through corporate venturing or spin-offs).

Building on this basic result, the paper studies how the firm's reputational concerns affect both the firm's and the employees' incentives to invest in generating innovations and how the cutoff value for creating a startup depends on firm and worker characteristics, on the stage of the business cycle, and on the availability of outside financing for new startups. These analyses yield a rich set of implications that are broadly consistent with the empirical observations on innovation and startups highlighted above.

Apart from predictions that conform with existing evidence, the model yields several novel theoretical results that could be of help in designing future empirical studies:

First, employee startups whose parent firms are growing relatively fast should perform better than those whose parent firms exhibit below average growth rates.

Second, in a comparison across otherwise similar industries, the average quality of employee startups should be positively correlated with the average quality of the innovations that established firms develop in-house.

Third, the contract that optimally motivates risk-averse employees to engage in innovative activities has a simple and plausible form: Employees with no innovative ideas receive only a base salary, those with low-value ideas receive a bonus independent of the value of the innovation, and, lastly, those employees who come up with sufficiently valuable ideas receive bonuses proportional to the value of the innovation.

Fourth, firms may have an incentive to deliberately *worsen* the distribution of potential innovations that they face.

Finally, compared to the first-best efficient outcome, firms may overinvest in increasing the frequency with which innovations arrive.

Related theoretical literature

Several earlier papers have explored models of employee startups. In Pakes and Nitzan (1983), a scientist hired by an entrepreneur to develop the entrepreneur's idea sometimes leaves (when it is efficient to do so) and sets up a rival firm. Anton and Yao (1995) show that in a setting with adverse selection, a wealth constrained employee with an innovative idea may find it optimal to develop her idea in a startup even if this is not an efficient outcome.[1] Both of these papers assume that profits from inventions are contractible. In contrast, the present paper focuses on situations in which the marginal profit from a new idea is not contractible, for example because it is hard to disentangle it from the profits generated by the established firm's other operations.

More recent literature on the topic includes Hvide (2009), Klepper and Thompson (2010), Hellmann (2007), Hellmann and Perotti (2011), and Chatterjee and Rossi-Hansberg (2012).[2] In both Hvide (2009) and Klepper and Thompson (2010), employees leave established firms for startups because established firms are unable to perfectly assess the value of the employees' innovations. In Hellmann (2007), an established firm refuses to implement its employees' ideas in order to motivate them to focus on their employment tasks. Nevertheless, some employees do come up with profitable innovations and subsequently leave the firm to implement them in startups.[3]

[1] Anton and Yao (1994) and (2002) sidestep the possibility an inventor can implement her idea in a startup and instead study mechanisms that may allow her to sell the idea even in the absence of well-defined property rights.

[2] See also Cassiman and Ueda (2006), Silveira and Wright (2010), and Spulber (2012).

[3] Additional papers that model a firm's incentive policies to encourage innovation include Aghion

Like the present paper, Hellmann and Perotti (2011) model firms with reputation for rewarding employees for generating ideas. However, they assume parameter specifications under which all ideas are disclosed to the firm and focus their analysis on how firms and markets complement each other in generating and completing ideas that are initially "half baked" and need to be circulated before they yield a valuable innovation.

In Chatterjee and Rossi-Hansberg (2012), a worker who takes her idea to a startup gives up the option of coming up with an even better idea in the future. The idea therefore has to be sufficiently good to be worth implementing in a startup. Thus, their paper shares with the present model the result that high quality ideas are implemented in startups whereas lower quality ideas are developed in established firms.

The theory developed here differs from the previous literature on startups in its focus on how entrepreneurial spawning depends on the firm's reputation for rewarding innovative ideas when formal contingent contracts are not feasible. This approach stands in contrast to the frequent assumption in this literature that firms can commit to their policies, including policies for rewarding innovative ideas, even if these turn out to be ex post inefficient (e.g., Hellmann, 2007; Bernardo et al, 2009). Another distinguishing feature of the present theory is its fit with a range of empirical regularities on startups. Some of these regularities can also be explained by the various models proposed in the previous literature, but none of those papers offers a single unified framework that generates all of the highlighted regularities.

In its focus on reputation, the paper is also related to the vast literature on repeated games, in particular to models of collusion among firms over the business cycle (e.g., Bagwell and Staiger, 1997) and to models of relational employment contracts (e.g., Baker et al, 1994; Levin, 2003). One difference compared to the latter strand is that, unlike in most of the relational contracting models, the firm and the worker can see the realization of the value of the relationship before they decide whether to enter the relationship in a given period, and they can condition the play on this value. Another

and Tirole (1994), de Bettignies (2008), Bernardo et al (2009), Hellmann and Thiele (2011), and Manso (2011). These papers do not study startups.

novel feature of the model is that the firm and the workers can invest in changing the random process that determines the value of the relationship.

The rest of the paper is organized as follows. Section 1 outlines the details of the model. Section 2 analyzes a benchmark version of the model in which workers are risk neutral and neither the firm nor the workers can influence the arrival of new ideas. Section 3 considers the effects of business cycles. Section 4 allows the firm and the workers to affect the arrival of new ideas through investments. It also characterizes the optimal incentive contracts for both risk-neutral and risk-averse workers, and examines the effects of external financial markets. Section 5 links the model's theoretical results to the available empirical evidence and discusses a few additional predictions that could be used to test the validity of the proposed theory. Section 6 concludes.

1 Model

Basic setup. Time is discrete and indexed by $t = 1, 2, ...$ There is an infinitely lived "established" firm which in every period employs N agents (employees), who each work for one period and then retire. The firm and the employees are risk-neutral throughout the majority of the paper, although the analysis of optimal incentive contracts in Section 4 will allow for risk-averse employees. The firm discounts future income using a discount factor $\delta < 1$. The employees are liquidity constrained, which means that monetary transfers from workers to the firm are not feasible and that if an employee decides to implement his idea in a startup, he needs to seek outside financing. This financing stage is modeled in a reduced-form way through a parameter k (introduced below), which captures the ease with which financing can be secured by new entrepreneurs.

Innovation process. At the beginning of each period, with probability $q(N)$ the Nature chooses a single employee in the firm (each with conditional probability $1/N$), who then gets a chance to discover an idea for a new project (an innovation). The probability function $q(.)$ increases in N, which reflects that, all else equal, a larger group of people generates ideas more frequently than a smaller group. The employee selected by the Nature discovers an innovation with probability p; with unconditional probability $1 - pq(N)$ no innovation is discovered in the given period.

If in period t an employee comes up with an innovation, its value to the firm, v_t, is a random variable drawn from the interval $[0, \bar{v}_t]$ according to a cumulative distribution function $F_t(.)$, which has a continuously differentiable density $f_t(.)$.

Growth rate of the economy. The distribution F_t and its support evolve over time as the economy grows. In particular, the value of an innovation discovered in period t is $v_t = g_t v_{t-1}$, where g_t is the growth rate of the economy in period t. The initial values v_0 are drawn from the interval $[0, \bar{v}_0]$ according to the cdf $F_0(v_0)$, with a density $f_0(v_0)$ which is finite for all v_0.

The purpose of allowing the economy to grow is to study the effects of business cycles, which will be introduced in Section 3. For now, it will be assumed that the economy maintains a constant rate of growth g, i.e., $g_t = g$ for each t, with $g\delta < 1$ to ensure finite payoffs.

Implementation of ideas. An employee can report his innovation to the firm, which involves disclosing enough details about it so that the firm can implement the innovation without the employee's help. Alternatively, the employee can quit without reporting his idea and exploit it in a startup firm. The profitability of the innovation is v_t if it is implemented by the established firm, and kv_t if developed by the agent.

Although one might imagine circumstances in which an innovation is more efficiently developed by a startup than an established firm, for the sake of focus I will follow Anton and Yao (1995) and others in assuming $k < 1$, which means that the most efficient arrangement is for all ideas to be developed by the established firm. This captures the notion that established firms have in place the infrastructure, know-how, capital, and marketing channels that are needed to implement new projects effectively and at an efficient scale, whereas startups first have to raise costly funding, establish operations, and recruit suitable employees.[4] The parameter k thus reflects the ease with which a new firm can be created in this economy, the ease with which entrepreneurs obtain loans from the banking sector, the availability of venture capital financing, and so on.

If the innovator quits, the firm can replace him with a new worker for the rest of the period, but no new idea arrives within the firm until the next period.

Contracting and reputation. Even though the value of an employee's idea, v_t, is observed by both the worker and the firm (if the worker reports it), it is not possible for a third party to verify it. This implies that formal contracts contingent on v_t are not feasible. Consequently, if an employee divulges his idea to the firm, the firm can implement the innovation and expropriate the employee. To overcome this problem, the firm can develop a reputation for compensating employees for their innovations. A general relational contract will consist of a sharing rule $s_t(v_t)$ for innovators and a (contractible) salary w_t for those workers who do not come up with ideas.

Although v_t is not verifiable, the firm's future employees can see whether the firm has paid $s_t(v_t)$ for a current employee's idea. One possible interpretation of this assumption is that employees communicate and, *ex post,* a current employee has nothing to gain by falsely accusing the firm of expropriating her idea if this was not the case.[5]

Investments in innovation. At the beginning of period $t = 1$, the firm can undertake a one-time investment that augments either the distribution F_t or the frequency $q(N)$ with which ideas arrive. Similarly, the workers can change the process through which new ideas are generated, but only in the period in which they are employed in the firm. The details of the investment technology will be described later, after the analysis of a case in which the distributions and the frequency of ideas are exogenously given.

[4] Allowing for some ideas to have $k > 1$ would be relatively easy. The main cost would be in terms of additional notation and somewhat more complicated exposition. The paper's results would be unaffected, as the ideas with $k > 1$ would in equilibrium be simply developed in startups and it would be efficient to do so.

[5] More generally, one could assume that future employees only observe a noisy signal about whether the firm has cheated.

2 The baseline case with no investments

I will start by analyzing a baseline case in which p, $q(N)$, and $F_t(.)$ are all fixed and neither the firm nor the employees can affect them through investments. The solution concept is subgame-perfect Nash equilibrium (SPNE). As is known from Abreu (1988), to verify whether an outcome of a repeated game can be sustained as a subgame-perfect Nash equilibrium it is enough to assume that any deviation is met with a maximum possible punishment. In the present setting, this corresponds to assuming that if in period t the firm pays an innovator less than $s_t(v_t)$, then starting in period $t+1$ all innovators stop offering their ideas to the firm and instead develop them in startups.

The following proposition contains the first main result of the paper; it characterizes each innovator's optimal choice between reporting his idea to the firm and developing it in a startup. All proofs are in the Appendix.

Proposition 1. *Any Pareto-efficient SPNE has the following form: In every period t, there is a threshold level $v_t^* \in [0, \bar{v}_t]$, such that all the ideas with $v_t \leq v_t^*$ (up to sets of measure zero) get developed in the established firm, while the ideas with $v_t > v_t^*$ get developed in startups.*

Although other subgame-perfect Nash equilibria may exist, Proposition 1 shows that any Pareto efficient SPNE has the simple form of a cutoff equilibrium, in which workers sell low-value ideas ($v_t \leq v_t^*$) to the firm and these innovations get developed in-house, whereas employees with high-value ideas ($v_t > v_t^*$) quit without disclosing their ideas to the firm and these innovations get developed in startups. This result rests on the fact that the firm's benefit from continuing reputation is proportional to the expected value of all the future innovations sold to the firm (i.e., those with $v_t \leq v_t^*$), but its temptation to renege is proportional to the value of the current period's innovation, say v_t'. Thus, if the firm is willing to pay a worker the promised amount $s(v_t') \geq kv_t'$ for an innovation of value v_t', it should be willing to pay him at least his outside option kv_t for any innovation $v_t < v_t'$, because the temptation to renege is smaller for v_t than for v'_t. Moreover, viewed from periods prior to t, adding v_t to the set of innovations that are developed in-house increases the future reputational rents, which helps to deter the firm from expropriating innovations in those earlier periods.

The prediction that established companies pursue low value innovations while high value innovations are developed in startups is stark, but the broad pattern is plausible. For example, Baumol (2005) has argued that anecdotal evidence suggests exactly this pattern. In Baumol's words, a "disproportionate share of breakthrough inventions is contributed by independent inventors, entrepreneurs, and small or startup firms, while the large firms specialize in incremental improvements."

I now turn to characterizing the thresholds v_t^*. I will start by focusing on equilibria in which the fixed salary is $w_t = 0$ for each t and in which the innovators' payoffs are equal to their outside opportunities, i.e., $s_t(v_t) = kv_t$. This is motivated by the standard observation that the firm is induced to cooperate by the threat of losing future rents if cheating, which means that an equilibrium in which the firm receives all the surplus from continuing cooperation is the easiest one to sustain.[6]

Under this assumption about $s_t(v_t)$, the game is stationary in the sense that once the values of the ideas are normalized to $\frac{v_t}{g^t}$, the period-t subgames look identical for all t. Thus, there is a cutoff level $v_0^* \in [0, \bar{v}_0]$ such that in a stationary Pareto-efficient SPNE, each period-t cutoff level satisfies $v_t^*/g^t = v_0^*$. The expected value to the firm of an idea that arrives in period t is then given by $g^t \int_0^{v_0^*} v f_0(v) dv$.

Suppose that in all of the periods $\tau \leq t-1$ the firm and the workers had played cooperative strategies according to which each innovator with $v_\tau \leq g^\tau v_0^*$ brings his idea to the firm and the firm pays him $s_\tau(v_\tau) = kv_\tau$. Then the firm does not expropriate an idea v_t that arrives in period t if

$$\frac{v_t}{g^t} \leq A \int_0^{v_0^*} v f_0(v) dv, \qquad \text{where } A \equiv \frac{\delta g(1-k)}{(1-\delta g)\, k} pq(N).$$

[6]This argument applies when the employees cannot invest in improving the probability distribution of ideas. Once investing is allowed, a sharing rule that gives the employees a part of the surplus can make reputation easier to sustain, as will be shown in Section 5.3.

Thus, if

$$A\int_0^{\bar{v}_0} v f_0(v)dv \geq \bar{v}_0, \tag{1}$$

then $v_0^* = \bar{v}_0$, i.e., workers never leave the firm to create a startup and all innovations get developed in-house. If (1) does not hold, then $v_0^* < \bar{v}_0$ and the cutoff level v_0^* is determined by the following equilibrium condition:

$$A\int_0^{v_0^*} v f_0(v)dv = v_0^*. \tag{2}$$

Note that for given parameter values and a given F_0 there may exist multiple solutions to (2). However, given the paper's focus on Pareto-efficient equilibria, whenever I refer to "equilibrium" I have in mind an SPNE with the largest cutoff value v_0^*.

Proposition 2. (i) *If condition (1) holds, then in (the Pareto efficient) equilibrium all innovations get developed in-house, i.e.,* $v_0^* = \bar{v}_0$ *(and* $v_t^* = \bar{v}_t$*). If (1) does not hold, then the threshold* v_0^* *is from* $[0, \bar{v}_0)$ *and given by (2).*

(ii) *The largest* v_0^* *weakly increases in* δ, g, p, *and* N, *and decreases in* k.

(iii) *The threshold value* v_0^* *can be discontinuous in each of* δ, g, p, N, *and* k.

Part (i) of Proposition 2 has already been discussed. The comparative statics in part (ii) confirm the familiar intuition that the set of equilibria is bigger when the surplus from future innovations is larger, because the threat of punishment is then more disciplining. As a result, the set of innovations the firm is willing to reward increases. In the present setting the expected future surplus increases in δ, p, g, and N and falls in k. Consequently, a firm that is, say, more patient finds it easier to maintain a reputation for compensating innovators, so an increase in δ increases the set of innovations that can be undertaken in-house.

The last part of the proposition is more subtle. It shows that a small change in any of δ, g, p, N, or k can trigger a disproportionately large change in the firm's behavior. Thus, two firms that look almost identical in all respects but differ slightly in size, in how they value future payoffs, or in how often their employees come up with new ideas, can differ dramatically in their levels of innovation. Similarly, two countries or regions that have comparable growth rates or that offer entrepreneurs almost identical access to external financing (as captured by k) can differ substantially in the proportion of innovations that are developed in established firms versus those developed in startups.

Such discontinuities are due to an unraveling process that can arise when the surplus is relatively small: Suppose the firm's temptation to renege when $\frac{v_t}{g^t} = \bar{v}_0$ is larger than its future reputational rents even if future play involves all innovations being developed in-house. Removing $v_t = g^t \bar{v}_0$ from the set of the ideas reported to the firm removes this temptation, but it also decreases the expected future surplus from cooperation, so that now even the ideas with slightly smaller normalized values than $\bar{v}_0$ present too strong a temptation to renege. Removing these smaller value ideas from the set of reported ideas further decreases the future surplus, and so on. Starting from an equilibrium in which $v_0^* > 0$, a small decrease in δ, g, p, N, or an increase in k, can trigger such unraveling and cause a precipitous drop in v_0^*. For example, the proof of Proposition 2 shows that when F_0 is uniform, a small increase in, say, the efficiency of capital markets can cause a switch from an SPNE in which all innovations are developed in-house to an SPNE in which no innovator offers his idea to the firm.

Note that for *any* distribution F_0 we have that $v_0^* = 0$ if $\delta \to 0$ and $v_0^* = \bar{v}_0$ if $\delta \to 1/g$. The next proposition gives a necessary and sufficient condition for the existence of an interior v_0^*.

Proposition 3. (i) *For any distribution F_0, there exist δ_1 and δ_2, $0 < \delta_1 \leq \delta_2 < 1/g$, such that $v_0^* = 0$ iff $\delta \leq \delta_1$ and $v_0^* = \bar{v}_0$ iff $\delta \geq \delta_2$.*

(ii) *Suppose there is a $y \in (0, \bar{v}_0)$ such that*

$$\frac{(1-k)g\delta_2}{k(1-g\delta_2)} pq(N) \int_0^y v f_0(v) dv > y. \tag{3}$$

Then $\delta_1 < \delta_2$, so that for all $\delta \in (\delta_1, \delta_2)$ the highest threshold is in the interior, *i.e.,* $v_0^* \in (0, \bar{v}_0)$.

(iii) *If the reverse of (3) holds for all* $y \in (0, \bar{v}_0)$, *then* $\delta_1 = \delta_2$, *that is, an interior threshold does not exist for any* δ.

The reason an interior cutoff level equilibrium may be feasible even if $v_0^* = \bar{v}_0$ is not is that if the distribution has a relatively thin right tail, high value innovations are relatively scarce. Therefore, the expected value of continuing cooperation is relatively small, so the firm would renege if presented with a high value idea. Removing such high value innovations from consideration does not decrease the expected value of the future surplus by much because of their small likelihood, but it removes the cases of biggest temptation. Thus, the proportion of ideas that get developed in startups rather than in the established firm depends on the distribution F_0. Heuristically, if F_0 is "top heavy," i.e., the average innovation value $E(v_0)$ is close to $\bar{v}_0$, then the value of reputation is high and v_0^* can be large, possibly equal to $\bar{v}_0$. If, on the other hand, high value ideas are relatively rare so that $E(v_0)$ is small, then v_0^* is also small, possibly zero.

3 Startup creation over the business cycle

As mentioned in the Introduction, evidence suggests that the rate of startup creation depends on the state of the business cycle. This section therefore explores how business cycles affect startup creation in the present model. To that end, assume that the growth rates g_t follow a business cycle, which will be modeled using the approach taken in Bagwell and Staiger (1997). Specifically, in any given period the economy can be in one of two possible states: when $g_t = r > 0$, the economy is in a recession, when $g_t = b$, where $b > r$, the economy is in a boom. Thus, the value of any given invention is higher in a boom period than in a recession period. To ensure bounded payoffs, let $b\delta < 1$.

The transition between the two states is described by a Markov process such that in period t the economy will be in the boom state ($g_t = b$) with probability λ if in period $t-1$ it was in a recession state ($g_{t-1} = r$) and with probability $1-\rho$ if in period

$t-1$ the economy was in a boom state ($g_{t-1} = b$). Thus, λ and ρ give the respective probabilities of switching from a current recession to a boom and vice versa. To capture the significant positive autocorrelation in output growth observed in U.S. data, it will be assumed that $1-\rho > \lambda$; that is, there is a positive serial correlation among states, so that a boom state is more likely to follow after a boom state than after a recession state. For concreteness, the initial state is assumed to be a boom state, i.e., $g_1 = b$.

To derive the firm's non-reneging conditions for the two states, let ω_t^b denote the firm's period-t expected payoff along the equilibrium path if t is a boom period, i.e.,

$$\omega_t^b \equiv q(N)p(1-k)b\Pi_{\tau=1}^{t-1} g_\tau \int_0^{v_0^{*b}} v f_0(v)dv, \tag{4}$$

where v_0^{*b} denotes the normalized threshold level in a boom period. The firm's expected payoff and threshold level in a recession period, ω_t^r and v_0^{*r}, are defined similarly.

Suppose that the firm has cooperated in all periods prior to t and that in period t the economy is in a boom state. At the beginning of period t, the firm's continuation value from staying on the equilibrium path is

$$V_t^b = \omega_t^b + \delta\left[\rho V_{t+1}^{b,r} + (1-\rho)V_{t+1}^{b,b}\right], \tag{5}$$

where $V_{t+1}^{b,b}$ ($V_{t+1}^{b,r}$) denotes the firm's $t+1$ continuation values if in period $t+1$ the economy is in a boom (recession). Similarly, if in period t the economy is in a recession, the firm's continuation value is given by

$$V_t^r = \omega_t^r + \delta\left[\lambda V_{t+1}^{r,b} + (1-\lambda)V_{t+1}^{r,r}\right], \tag{6}$$

where $V_{t+1}^{r,b}$ and $V_{t+1}^{r,r}$ are defined analogously to $V_{t+1}^{b,b}$ and $V_{t+1}^{b,r}$. The firm's incentive compatibility constraints in period t are then as follows:

$$\text{Boom:} \qquad kv_t \leq \delta\left[\rho V_{t+1}^{b,r} + (1-\rho)V_{t+1}^{b,b}\right]$$

$$\text{Recession:} \qquad kv_t \leq \delta\left[\lambda V_{t+1}^{r,b} + (1-\lambda)V_{t+1}^{r,r}\right]$$

Now, note that in a stationary equilibrium it must be $V_{t+1}^{b,b} = bV_t^b$ and $V_{t+1}^{r,r} = rV_t^r$. Furthermore, the only difference between $V_{t+1}^{r,b}$ and $V_{t+1}^{b,b}$ is that $V_{t+1}^{r,b}$ follows after a period of growth rate r, whereas $V_{t+1}^{b,b}$ follows after a period of growth rate b. Thus, the two continuation values are related through $V_{t+1}^{r,b} = \frac{r}{b}V_{t+1}^{b,b} = rV_t^b$. Similarly, the relationship between $V_{t+1}^{r,r}$ and $V_{t+1}^{b,r}$ satisfies $V_{t+1}^{b,r} = \frac{b}{r}V_{t+1}^{r,r} = bV_t^r$. Equations (5) and (6) can therefore be restated as

$$V_t^{\sigma} = \omega_t^b + \delta\left[\rho b V_t^r + (1-\rho) b V_t^b\right] \quad (7)$$

and

$$V_t^r = \omega_t^r + \delta\left[\lambda r V_t^b + (1-\lambda) r V_t^r\right]. \quad (8)$$

Similarly, using $v_t = g_t v_{t-1}$, the firm's incentive compatibility conditions reduce to:

$$\text{Boom:} \qquad k v_{t-1} \leq \delta\left[\rho V_t^r + (1-\rho)V_t^b\right] \quad (9)$$

$$\text{Recession:} \qquad k v_{t-1} \leq \delta\left[\lambda V_t^b + (1-\lambda)V_t^r\right] \quad (10)$$

An analysis of these conditions yields the following result.

Proposition 4. *In the Markov growth economy, any Pareto-efficient SPNE has two threshold levels, v_0^{*b} and v_0^{*r}, such that $0 \leq v_0^{*r} \leq v_0^{*b} \leq \bar{v}_0$ and such that if in period t the economy is in state $\sigma \in \{r, b\}$, all the ideas with $\frac{v_t}{\Pi_{\tau=1}^t g_\tau} \leq v_0^{*\sigma}$ get developed in the established firm, while the ideas with $\frac{v_t}{\Pi_{\tau=1}^t g_\tau} > v_0^{*\sigma}$ get developed in startups. Moreover, if $v_0^{*r} \in (0, \bar{v}_0)$, then $v_0^{*r} < v_0^{*b}$.*

Proposition 4 tells us that it is easier for the firm to maintain a reputation for rewarding innovative employees when the economy is in a boom state than when it is in a recession state. This is because the positive serial correlation of growth rates makes the future look more optimistic in a boom state than in a recession state, which means that the benefits from maintaining a good reputation are higher in a boom state. Consequently, the threshold level for innovations to be developed in startups is higher during a high-growth period than it is during a low-growth period, $v_0^{*b} \geq v_0^{*r}$, and strictly so when the recession threshold is interior.

4 Investment in Innovation

If the firm and the workers can invest in augmenting the process through which ideas are generated, should we expect them to invest efficiently? And how do such investments alter the relationship between startup creation and the availability of outside financing? To explore these and related questions, I now enrich the model to allow the players to influence the idea generating process. I will consider two cases. In the first, the firm and the workers will be able to increase the frequency $q(N)$, respectively p, with which ideas arrive; in the second, they will be able to augment the distributions $F_t(.)$.

To ease the exposition and economize on notation, I will from now on assume that the rate of economic growth is constant and equal to $g_t = 1$ in all periods. This means that $v_t = v_0$ for each t, so that the time subscripts and the subscript 0 are no longer needed. In what follows, I will therefore instead write v, v^*, $\bar{v}$, F, and so on.

The analysis will start with a worker's choice of p and its interaction with k, where the latter will be interpreted as the availability of VC financing. Subsequently, I will derive the optimal incentive contract offered by the firm. The firm's investment in $q(N)$ will be analyzed next, followed by an analysis of the players' incentives to affect the distribution F.

4.1 Workers' choice of p and the effects of financial markets

Suppose first that the workers can invest in increasing the arrival rate of innovations by enhancing their human capital and by increasing their effort aimed at generating new ideas. Formally, let $c(p^i)$ be worker i's cost of choosing the probability p^i with which he discovers an innovation if chosen by the Nature, and assume that $c(.)$ is increasing and convex, with $c(0) = c'(0) = 0$. Also, to provide a benchmark case and to develop intuition for the main effects of investments, maintain for now the initial assumption that the firm keeps the entire surplus $(1-k)v$ from an idea developed in-house.

Because an individual worker cannot affect v^* and receives only a fraction k of his idea's value, it immediately follows that each worker under-invests in p^i. A less straightforward point is that the workers' investments alter in a non-trivial way how their outside option, measured by k, affects the threshold v^*. However, before exploring these comparative statics, a few remarks about the interpretation of k are in order. While k may capture a number of factors that allow an innovator to appropriate part of the value of his innovation, an interpretation I will focus on in this section is that k measures the efficiency of financial markets, especially the markets for venture capital. These markets are particularly relevant in the present context because VC firms not only finance new firms, but also provide expertise which can help an entrepreneur to develop his idea to its full potential.[7] Furthermore, most of the empirical studies that document the effects of outside financing on startup creation focus on VC financing.

Of course, just as in the case of disclosing his idea to his employer, a worker may worry about opportunistic behavior by the VC firm from which he seeks financing. Nevertheless, there are reasons to expect that expropriation by a VC firm is not too serious a threat, at least as a first approximation. Atanasov et al (2012), for instance, document that reputation and the threat of litigation limit the opportunistic behavior of VCs. Arguably, these mechanisms are more effective in the case of VCs than in the case of corporate employers. For example, it would be hard for an employee to sue her employer for stealing her innovation, as companies are often deemed to have property rights on ideas developed by their employees. Moreover, it might be less credible for a VC firm than for an employer to claim that they have developed the idea independently and it would be more difficult for the VC firm to actually implement the innovation without the employee's help. As pointed out by Atanasov et al, "if opportunistic behavior [by VC firms] were too widespread, venture capital could not flourish as it has, nor could formal contracts be written, in equilibrium, in the strongly pro-VC manner documented by Kaplan and Stromberg (2003)." Thus, for the sake of retaining focus on public corporations, I will proceed under the assumption that the

[7] For an analysis of an entrepreneur's choice between bank credit and venture capital as alternative sources for financing her startup, see Ueda (2004).

threat of being expropriated by a VC firm is not of first-order concern to entrepreneurs.[8]

Coming back to the effects of k on v^*, the comparative statics results of Proposition 2 indicate that v^* decreases in k. Under the interpretation that k captures the availability of VC financing, this suggests that an improvement in VC financing results in more ideas being developed in startups and fewer in established firms. This result is not unexpected, but it has the somewhat surprising implication that an increase in the efficiency of VC markets can decrease welfare. This is because in the present model established firms exploit innovations more effectively than startups; consequently, a decrease in the proportion of ideas that get developed in-house entails a deadweight loss, which can offset the direct efficiency gain due to the startups' higher profits.

This possibility is easily illustrated: Suppose $v^* = \bar{v}$ and $\delta = \delta_2$, where δ_2 is as in part (i) of Proposition 3. Suppose also that the reverse of condition (3) holds for all $y \in (0, \bar{v})$, as in part (iii) of Proposition 3. In this (knife-edge) case, condition (2) holds for $v^* = \bar{v}$ at the current level of k (denote it $\hat{k}$), but cannot hold for any $v^* > 0$ if $k > \hat{k}$. Hence, a small increase in k would trigger a total collapse of innovation development within established firms. This would be accompanied by a large increase in the number of startups, so it might look like the improvement in VC financing encouraged innovation, but the net effect would be a dramatic decline in efficiency.

A more interesting observation, however, is that although the conclusion that increased availability of VC financing encourages creation of startups may seem intuitive and straightforward, it is in fact not valid in general when generating new ideas requires worker investments. Focusing again on the equilibrium with the largest v^* (when multiple equilibria exist), this point is demonstrated by Proposition 5 below.

Proposition 5. *Suppose* $\frac{c''(p^*)}{c'(p^*)}p^* < 1 - k$*, which holds if the cost function* $c(.)$ *is not too convex. Then*

(i) v^* *increases in* k*;*

[8] Alternatively, we can think of k as incorporating this threat (in an admittedly stylized manner).

(ii) *there is a class of distribution functions F such that $p^*(\bar{v}-v^*)$ decreases in k. That is, an improvement in VC markets* decreases *the expected number of startups.*

Proposition 5 shows that the comparative statics conclusions of Proposition 2 can be reversed if we take into account that the rate at which new ideas arrive depends on the time and effort the firm's employees put into the innovation process. Thus, paradoxically, an improvement in VC financing could lead to a *decrease* in the number of startups, as shown in part (ii) of the proposition. The logic behind this result is as follows: A higher k means that innovators get to benefit more from any given idea, which encourages them to invest in generating ideas. All else equal, the resulting increase in the arrival rate of ideas would directly increase the number of new startups. However, given that ideas now arrive more often, the firm finds it easier to build a reputation for compensating innovators, so that a smaller proportion of innovators are forced to pursue their ideas in startups. When the workers' cost function is not too convex as measured by the relative Arrow-Pratt coefficient $\frac{c''(p^*)}{c'(p^*)}p^*$, this second effect dominates, so that an increase in k leads to a decrease in the number of startups.

4.2 Optimal incentive contracts

Another implication of Proposition 5 that is of interest is that if workers can invest in improving the innovation process, then contrary to the case of an exogenously given p cooperation can be easier to sustain if the workers receive a part of the surplus from the ideas developed by the firm. The implication of this observation is that the firm may therefore find it optimal to share the surplus with the workers, in order to encourage them to invest. This subsection explores such surplus sharing and derives the optimal relational contract (i.e., a sharing contract supported by the firm's reputational concerns), both for the case of risk-neutral and for the case of risk-averse workers. It also shows that a (weaker) version of Proposition 5 continues to hold when one takes into account that a change in k may affect the optimal contract.

4.2.1 Optimal relational contracts for risk-neutral workers

Recall that $s(v)$ denotes the sharing scheme for innovators and w the salary for those workers who do not come up with ideas. When the workers are risk-neutral, as assumed so far, deriving an optimal scheme $(w, s(v))$ is straightforward: There are multiple optimal relational contracts, each characterized by $w = 0$ and by the same threshold v^*, but the exact sharing rule $s(v)$ is indeterminate. The latter point can be readily seen by inspecting the workers' and the firm's expected payoffs. Let $\Omega \equiv \int_0^{v^*} [s(v) - kv] f(v)dv$ denote the expected surplus the relational contract gives to an innovator. The players' payoffs are then respectively given as follows:

$$\text{Firm:} \qquad \frac{q(N)}{1-\delta} p \left[(1-k) \int_0^{v^*} v f(v) dv - \Omega \right]$$

$$\text{Worker:} \qquad \frac{q(N)p}{N} \left[\Omega + k \int_0^{\bar{v}} v f(v) dv \right] - c(p)$$

These expressions make it clear that if an $s(v)$ is optimal, then any other contract $s'(v)$ such that $\int_0^{v^*} [s'(v) - kv] f(v)dv = \Omega$ and $s'(v) \in [kv, kv^*]$ for $v \leq v^*$ is also optimal.[9]

Incentive contracts complicate the effect of k on v^*, because k now affects the workers' investments not only directly, but also indirectly, through the firm's adjustments in the sharing scheme $s(v)$. Nevertheless, Proposition 6 below shows that the qualitative insights of Proposition 5 carry over to the present setting.

Proposition 6. *Suppose workers can invest in increasing the frequency of new ideas. Suppose also that* $c(p) = \frac{p^\eta}{\eta}$*, with* $\eta \in (1, 2-k)$*. Then if the workers are risk-neutral, there is a class of distribution functions* F *such that* v^* *is interior and*

(i) $\Omega^* = 0$*; (ii)* v^* *increases in* k*; and (iii)* $p^*(\bar{v} - v^*)$ *decreases in* k*.*

In addition to showing that the insights of Proposition 5 continue to hold under certain conditions when optimal relational contracts are taken into account, Propo-

[9] The requirement $s(v) \geq kv$ for $v \leq v^*$ ensures that the worker does not leave the firm to develop his idea in a startup. The condition $s(v) \leq kv^*$ for $v \leq v^*$ is implied by the firm's non-reneging constraint, as will be shown in the proof of Proposition 6.

sition 6 contains the new result that the firm may find it optimal not to strengthen the workers' incentives beyond those provided by their outside opportunity kv. The somewhat surprising aspect of this result is that it holds even when an increase in k would raise the threshold level v^*, as in part (ii) of the proposition. Given that for a fixed k an increase in v^* must mean higher profits for the firm (as can be seen from condition (2)), why wouldn't the firm strengthen the workers' incentives?

The answer is that a relational contract cannot perfectly replicate the incentive effects of an increase in k because k affects the workers' payoffs from *all* innovations, including those with $v > v^*$ implemented in startups. In contrast, a relational contract can only increase the workers' payoffs from the innovations implemented in-house. But when v^* is relatively small to start with, matching the incentive effects of an exogenous increase in k would mean sharing with the workers a lot of surplus from a small set of low-value innovations. Such a contract may not be sustainable as an SPNE – and if it is, it could decrease the firm's profit even if an increase in k would raise it.

4.2.2 Optimal contracts for risk-averse workers

Given that the optimal contract for risk-neutral workers is indeterminate (as is often the case in principal-agent models), it is of interest to ask what the optimal contract looks like when workers are risk-averse. To address this question, let $u(.)$ be a representative worker's strictly concave and differentiable utility function. Defining $M \equiv \frac{\delta q(N)}{1-\delta}$ and assuming that the threshold v^* is interior, the firm's optimization problem is as follows:

$$\max_{s(v),v^*,p} \frac{M}{\delta} p \int_0^{v^*} [v - s(v)] \, f(v) dv$$

subject to

$$
\begin{aligned}
Mp \int_0^{v^*} [v - s(v)]\, f(v)dv &= s(v^*) && (11) \\
\int_0^{v^*} u(s(v)) f(v)dv + \int_{v^*}^{\bar{v}} u(kv) f(v)dv &= c'(p) && (12) \\
s(v) &\geq kv \quad \text{for all } v \leq v^* && (13) \\
w &\geq 0 && (14)
\end{aligned}
$$

Here, (11) is a generalization of the firm's non-reneging condition (2), condition (12) is the workers' incentive compatibility constraint, (13) is their individual rationality constraint, and (14) is the limited liability constraint.

Proposition 7. *The optimal sharing scheme* $(w, s(v))$ *for risk-averse workers is given by* $w = 0$ *and a* $v^+ \leq v^*$ *such that*

$$
\begin{aligned}
s(v) &= kv^+ \quad \textit{for} \quad v \leq v^+; \\
s(v) &= kv \quad \textit{for} \quad v \in [v^+, v^*]; \textit{ and} \\
s(v) &< kv \quad \textit{for} \quad v > v^*.
\end{aligned}
$$

Proposition 7 shows that the optimal sharing rule for risk-averse workers rewards disproportionately low value ideas (those with $v < v^+$), for which the workers are paid more than what they would get if they took the idea to a startup. On the other hand, innovators with higher value ideas (between v^+ and v^*) are paid their opportunity cost kv, so for this region the contract coincides with what was assumed in the baseline model. One interpretation of this sharing rule is that it entails a fixed reward for innovating, kv^+, which does not depend on the value of the worker's innovation, plus a bonus component $k(v - v^+)$ that is only paid for relatively high value innovations.

Note that this contract resembles an option contract such that the worker receives a fixed salary of kv^+, plus options with a strike price v^+ on k shares of the profit from the worker's innovation if implemented by the firm. However, one cannot interpret the relational contract literally as an option contract because the maintained assumption throughout the paper is that the profits from the innovation are not contractible.

4.2.3 Choice of q by the firm

Suppose now the workers cannot influence p (so that we can revert to assuming that $s(v) = kv$), but the firm can increase the frequency of new inventions through q, whose dependence on N will be suppressed here to economize on notation. For example, the firm could invest in research laboratories and equipment, establish a worker training program, and so on, all of which would increase q.

Formally, q is chosen by the firm at the beginning of the first period at cost $\phi(q)$, which is twice continuously differentiable, increasing, and strictly convex, with $\phi(0) = \phi'(0) = 0$ for all N. The firm then chooses q to maximize

$$\frac{1-k}{1-\delta} pq \int_0^{v^*} vf(v)dv - \phi(q),$$

subject to v^* being determined by (1) and (2).

Does the firm invest efficiently? To answer this question, denote the firm's optimal choice by q^* and compare it with the first-best level of investment (which entails all innovations being developed in-house). Specifically, the first-best level of investment, q^{FB}, maximizes the total surplus, given by

$$\frac{1}{1-\delta} pq \int_0^{\bar{v}} vf(v)dv - \phi(q).$$

Thus, when $v^* = \bar{v}$ is sustainable as an outcome of an SPNE for some $q < q^{FB}$, the firm under-invests compared to the first-best level, i.e., $q^* < q^{FB}$. The argument is familiar: because $k > 0$, the firm only captures a part of the total surplus generated by its investment and therefore invests less than would be efficient. However, this conclusion may not hold when v^* is in the interior. This is demonstrated by the next proposition.

Proposition 8. *When firms invest in affecting the frequency of new inventions, both under-investment ($q^* < q^{FB}$) and over-investment ($q^* > q^{FB}$) are possible, depending on the parameter values and on F.*

Proposition 8 tells us that the firm might have an incentive to invest *more* than would be optimal in the first-best scenario. The reason is that when $v^* < \bar{v}$, an increase in the rate at which new ideas arrive makes it easier for the firm to maintain a reputation for rewarding innovation. This in turn increases the cutoff level v^* and with it the firm's expected profit. The social planner, on the other hand, is not concerned with the effect of q on v^* because in the first-best outcome *all* ideas are developed in the established firm, that is, $v^* = \bar{v}$. This extra benefit may add enough to the firm's investment incentive to push its optimal investment over the first-best level q^{FB}.[10]

4.3 Investments in augmenting the distribution F

An alternative channel through which the firm and the workers might affect the idea generating process is by altering the distribution of innovations F. One natural way to measure changes in F is in terms of first-order stochastic dominance. At a first blush, one might think that both the workers and the firm would always be in favor of improving the distribution in the sense of first-order stochastic dominance. Such is indeed the case if $v^* = \bar{v}$ is feasible under the improved distribution. In fact, if the firm could costlessly choose *any* distribution, it would trivially choose the degenerate distribution under which $v = \bar{v}$ with probability one.

In reality, however, a firm's ability to choose a distribution of ideas is likely to be limited. In such a case, the model implies that the firm may in fact prefer a distribution that is first-order stochastically dominated by its current distribution F. The reason is that the firm's expected profit increases with v^* and a worsening in F in terms of FOSD can increase v^*. Proposition 9 below states this result more formally.

Proposition 9. *Let v_F^* and v_H^* be the respective equilibrium thresholds under distribution functions F and H. For any distribution F under which $v_F^* \in (0, \bar{v})$ there exists a class $\mathcal{H}$ of distribution functions that are first-order stochastically*

[10] An alternative benchmark would be the second-best outcome, in which the total surplus is maximized subject to the constraint that the innovations with $v > v^*$ get developed in startups. It is relatively straightforward to show that the firm always under-invests compared to this alternative benchmark.

dominated by F, such that $v_H^* > v_F^*$ for each $H \in \mathcal{H}$.

Proposition 9 suggests that a firm may have an incentive to invest in *worsening* the distribution from which innovations are drawn. In particular, the firm may choose to invest in replacing the default distribution F with a first-order stochastically dominated distribution H, even if this is costly. This result offers a possible explanation for a surprising finding in the Accenture study mentioned in the Introduction, according to which a majority of the surveyed companies (64 percent) appear to deliberately focus on pursuing limited incremental line extensions rather than developing transformative ideas that would introduce totally new products or services.

The intuition behind the result is as follows: If $v^* < \bar{v}$, then an improvement in F that puts more weight on $v > v^*$ and less weight on $v < v^*$ may negatively affect the firm's ability to maintain a reputation for rewarding innovation. This is because (holding v^* fixed) such a change decreases the expected surplus from continuing cooperation, which only includes those ideas whose values are below v^*. Proposition 9 tells us that this may induce a decrease in v^*, which results in a decrease in the firm's expected profits because the firm only earns profit on the innovations that are developed in-house. In such a case, the firm may want to do the opposite, i.e., to change F in a way that puts more weight on low value innovations ($v < v^*$).

Now consider the workers' incentives to augment F. Assume first an innovator's payoff from idea v is kv whether the idea is developed in-house or in a startup. Then the worker clearly benefits if F improves in the sense of first-order stochastic dominance. Thus, in this case the firm and the workers may have diametrically opposite preferences regarding the process through which ideas are generated – the workers prefer any process that puts more weight on high value ideas, whereas the firm may have a preference for a process that favors low value ideas. The firm may therefore adopt a technology and implement R&D processes that favor relatively low value innovations, as appears to be the case in the firms surveyed for the Accenture study mentioned above, but the workers may adjust their efforts so as to focus primarily on high value ideas. It is worth noting that in a situation like this, the firm may again find it optimal to share with the workers the surplus from the ideas developed in-house, to refocus their attention towards relatively low value ideas.

5 Empirical Implications

The theoretical results developed above are consistent with the empirical regularities that were highlighted in the Introduction. This section discusses the match between the model and the documented regularities in greater detail and also offers a few additional implications that follow from the model but have not yet been tested.

5.1 Availability of venture capital financing

The empirical evidence on how the rate of startup creation depends on the supply of venture capital is mixed. Recent studies by Samila and Sorenson (2011) using US data and Popov and Roosenboom (2013) using international data show that an improvement in VC markets stimulates new firm creation. On the other hand, using data from the biotech industry, an influential study by Zucker et al (1998) finds that the number of VC firms in a region has a significant negative impact on the number of startups.

In the model of this paper, the availability of VC financing is captured in a reduced form through the parameter k that measures the value of the innovation when developed in a startup relative to its value when developed in an established firm. Thus, an increase in the supply of VC funds would in the model correspond to an increase in k. The comparative statics with respect to k show that the model can give rise to both of the above relationships, depending upon the underlying parameter values.

Specifically, part (ii) of Proposition 2 shows that in the absence of investments an increase in k stimulates creation of new startups. The logic is that an easier access to VC financing improves the innovators' outside option and, by the same token, decreases the established firm's expected future surplus from cooperation. This makes it less valuable for the firm to maintain a reputation for rewarding innovators, which induces more innovators to leave for startups (formally, the cutoff level v^* decreases).

Interestingly, Samila and Sorenson (2011) find that an increase in the supply of VC funds generates more new firms than it funds: their estimates indicate that "investing in an additional firm would stimulate the entry of two to twelve establishments." This fits well with part (iii) of Proposition 2 which shows that the creation of new firms can be discontinuous in k; that is, a small improvement in VC financing can generate a large increase in the number of startups.

Zucker et al (1998) document a negative relationship between the number of venture capital firms and the number of startups. They find this relationship surprising, but propositions 5 and 6 show that such a negative relationship is a distinct theoretical possibility. An increase in k motivates the workers to invest in generating new ideas, which in turn increases the firm's gains from maintaining a reputation for rewarding innovators. When the workers' cost functions are not too convex, the workers respond to an increase in k by increasing their investments considerably. As a result, the cutoff level v^* increases and the number of new startups drops.

The above discussion suggests that it might be possible to empirically distinguish the environments in which the relationship between VC financing and startup creation is positive from those in which the relationship is negative: A positive relationship should prevail when worker investments are relatively unimportant for the frequency with which new ideas arrive, while a negative relationship may arise when worker investments are crucial and easy to spur.

5.2 Firm location

Besides the availability of VC financing, parameter k can represent other factors that make it easier for an innovator to implement her idea in a startup, including the location of the innovator's current employer. This interpretation relates the model to the evidence in Gompers et al (2005), who find that the companies located in Silicon Valley and in Massachusetts tend to spawn substantially more startups (38% and 24%

more respectively) than companies located elsewhere.

The explanation offered by Gompers et al (which in turn goes back to Saxenian, 1994) is consistent with the present model: Employees of entrepreneurial firms located in Silicon Valley and Massachusetts are likely to have a better access to a network of customers and suppliers – including suppliers of capital – than do employees of other firms. Moreover, they tend to interact more with experienced entrepreneurs, from whom they learn how to start a new firm. In the present model, these effects would be captured by a higher k for firms based in Silicon Valley and Massachusetts, which would imply a lower v^* for these firms and therefore a greater number of startups. Thus, the interpretation offered by this paper is that established firms located in these two regions find it harder to maintain a reputation for rewarding employee innovation, which means that more employee innovators leave to pursue their ideas in startups.

Further, note that k could also reflect the strength of trade secret laws and the enforcement of non-compete clauses, where a smaller k means stronger laws and/or stronger enforcement. Given that non-compete laws are weaker in California than in Massachusetts, the model would predict more startups being spawned by companies located in California than by those located in Massachusetts. This reinforces the conclusion that the model is consistent with the above evidence.

Here again the model suggests that there could be more to these relationships than meets the eye: If better outside opportunities (due to location advantages) spur employees to generate substantially more ideas, the result could be a *decrease* in the number of startups, as discussed in the previous subsection.

5.3 Firm size

Robust empirical evidence documents that employees of large companies are less likely to start new firms than employees in smaller firms (Gompers et al, 2005; Dobrev and Barnett, 2005; Eriksson and Kuhn, 2006; Sørensen, 2007; Elfenbein et al, 2010; Avnimelech and Feldman, 2010). On the other hand, employees of large firms are more likely to pursue venturing opportunities inside the established firm (Kacperczyk, 2012).

These relationships arise in the model in a straightforward way. All else equal, firms with more employees (captured by a larger N) are more often confronted with a situation where an employee has an innovative idea. Larger companies therefore find it easier to build a reputation for rewarding innovative employees, which means that the cutoff value v^* for ideas that get developed in startups is higher in these companies, as shown in part (ii) of Proposition 2. The flip side of this result is that employees of large companies pursue more ideas in-house. Note, however, that despite a larger v^*, a large firm may well spawn a greater *total* number of startups than a small one, as is indeed the case for the firms studied by Gompers et al (2005).

5.4 The business cycle and firm growth

Eriksson and Kuhn (2006) study startup creation using a large matched employer-employee data set covering the entire Danish private sector in years 1981 to 2000. Their analysis reveals that the rate of employee startup creation depends upon the growth rate of the whole economy, as well as upon the growth rate of the parent firm. Specifically, they show that (i) the rate of startup creation is counter-cyclical, (ii) startups established when the economy is growing relatively slowly are more likely to exit, even after controlling for GDP growth as an additional explanatory variable, and (iii) the rate at which employees leave to start new firms is larger in periods in which the parent firm's sales per employee grow relatively slowly.

All three of these regularities fit well with the analysis of Section 3, which allowed for business cycles. As shown in Proposition 4, the model implies that the cutoff value for new startups is larger during economic upturns than during downturns, because the expected future growth rates (and therefore also the expected future surplus) are higher when the economy currently experiences an upturn. Consequently, the model predicts that the rate of new startup creation is counter-cyclical, in line with Eriksson and Kuhn's finding (i). Moreover, given that the threshold value for startup creation is higher during economic upturns, the startups created during upturns are founded around more valuable ideas and are therefore of better average quality, as in the above empirical finding (ii). Finally, given that growth rates of (large) individual firms appear to exhibit positive autocorrelation (Coad and Hölzl, 2009), matching the model with Eriksson and Kuhn's finding (iii) only requires that the coefficients g_t are reinterpreted as growth rates of the firm's sales rather than growth rates of the whole economy. Proposition 4 then implies that the cutoff value for new startups is lower in periods in which the parent firm experiences low rates of sales growth, which means that the rate of startup creation is higher during these periods.

5.5 Employee ability

Another well documented empirical regularity is that more productive employees are more likely to start new companies. For example, Campbell et al (2012) examine data on U.S. firms in the legal services sector and find that better performing employees, as measured by their earnings, are more likely to create new ventures. Braguinsky et al (2012) conclude the same using data on US scientists and engineers. Similarly, in their study of research analysts in investment banks over 1988-1996, Groysberg et al (2009) document that star analysts are more likely than non-star analysts to become entrepreneurs. Finally, the Eriksson and Kuhn's (2006) study mentioned earlier shows that individuals starting spin-offs are on average better educated and more likely to be from the upper end of the skills distribution.

To see that the model is consistent with these regularities, we need to enrich it to allow for heterogenous employees. Specifically, assume that employees differ in their abilities and, without loss of generality, let them be ordered so that the higher is an employee's index i, the higher is her ability. The fact that employee i is more able than employee $i-1$ will be captured through the assumption that employee i's distribution of ideas is better in the sense of first-order stochastic dominance: $F_0^i \succ_{FOSD} F_0^{i-1}$, $i = 2, 3, ..., N$. For simplicity, all employees draw their ideas from the same support.

The main modification in the analysis of the baseline model of Section 2 that is required to account for such skill heterogeneity is that the density function $f_0(.)$ in the equilibrium conditions (1) and (2) needs to be replaced by the average density $\bar{f}_0(.)$, defined by $\bar{f}_0(.) \equiv \frac{1}{N}\sum_{i=1}^{N} f_0^i(.)$, where f_0^i is the density function of the cdf F_0^i. With this modification in place, all of the arguments behind the results of Section 2 go through in a relatively straightforward way, so that all of the results carry over. In particular, propositions 1 and 2 continue to hold as stated, i.e., the equilibrium is again characterized by a threshold v_0^* such that all innovations with normalized values higher than v_0^* are developed in startups. Given that more able employees are by assumption more likely to come up with ideas whose values exceed the threshold, they are also more likely to leave the firm to start their own company. If higher skills also translate into the employee's better performance while employed in the established firm, which is a reasonable assumption, then in line with the above empirical studies the model predicts that more productive employees are more likely to create startups.

5.6 Patenting and startups

Gompers et al (2005) document that a firm's rate of generating new startups increases much less than proportionally with its number of patents. This finding, too, can be understood in light of the current model. Suppose that two otherwise identical firms differ in the probability p with which their employees generate ideas: in a firm H the employees have a high probability, p_H, of generating ideas, whereas in a firm L the probability is low, $p_L < p_H$. Then Proposition 2 implies that the H-firm would exhibit a higher (interior) cutoff level for internal development of ideas, $v_H^* > v_L^*$. As long as both firms convert into patents roughly the same share of the ideas that are not used to spawn startups, the H-firm would therefore produce more patents than the L-firm, $p_H F(v_H^*) > p_L F(v_L^*)$, but it would also be true that $\frac{p_H\left[1-F(v_H^*)\right]}{p_L\left[1-F(v_L^*)\right]} < \frac{p_H F(v_H^*)}{p_L F(v_L^*)}$, that is, a doubling in the rate of a firm's patents generates less than twice as many startups. In other words, the more innovative firm would have more patents, but this would not be reflected in a proportional increase in the number of startups.

5.7 Additional predictions

In addition to shedding light on a range of documented empirical regularities, as discussed above, the model yields predictions that have not yet been tested, but could be of help in guiding future empirical studies on employee startups.

First, similar to the effects of business cycles discussed in subsection 5.4, the model predicts that the quality of employee startups should be positively related to the economic performance of the parent firm. In particular, the quality of the startups spawned by firms experiencing relatively high rates of sales growth should be higher than the quality of the startups spawned by firms with below average growth rates. This again follows from Proposition 4, which shows that the cutoff value for startup creation is positively related to the rate of growth, so that the ideas implemented in startups are of higher average value when sales growth is high.

Second, the cutoff nature of the equilibria suggests a positive relationship between the quality of startups spawned by a firm and the quality of the innovations that the firm develops internally. Thus, if two industries, A and B, are otherwise similar but employee startups are of higher average quality in industry A than in B, then the quality of the innovations developed in established firms should also be higher in industry A than in industry B.

Third, Proposition 7 can be used to derive predictions about the structure of the optimal bonuses for innovators. In particular, it implies that relatively low-value ideas are all rewarded with the same-size bonus, whereas the bonus for relatively high value innovations is proportional to the value of the innovation. Moreover, the bonus for low value innovations represents a larger share of the innovation's value than the bonuses for large value innovations. Both of these predictions appear to be testable.

Finally, empirical tests of the model could in principle exploit differences in distributions of ideas across industries. While it is not easy to perform comparative statics with respect to the distribution F without assuming a specific distribution, some suggestive observations are possible. For example, if, say, the internet commerce industry has a relatively long-tailed distribution of innovations whereas the biotech industry has a relatively higher frequency of high value ideas, then all else equal the internet commerce industry is likely to exhibit a (relatively) higher rate of startup creation. On the other hand, the biotech startups should on average perform better.

6 Conclusion

This paper builds a theory in which startup creation is the result of a reputational failure. New ideas are more efficiently developed in established firms than in startups, but sale of ideas suffers from the property rights problem identified by Arrow (1962). Inventors therefore offer their ideas to an established firm only if the firm has a reputation for rewarding ideas of similar value. Since it is more tempting for a firm to steal a high-value idea than a low-value idea, innovators tend to pursue high-value ideas in startups and offer to established firms only relatively low-value ideas. This simple framework has rich empirical implications for how the rate of startup creation depends on firm and employee characteristics, on the stage of the business cycle, and on the availability of financing for entrepreneurs, which mesh well with the regularities documented in the empirical literature on startups. The theory also offers as yet untested predictions that could be of use to future empirical studies on the topic, as well as theoretical insights into the structure of optimal relational contracts for innovative employees and into the firms' and employees' incentives to invest in generating new ideas.

The paper leaves unexplored several potentially interesting extensions of the basic framework. First, some firms implement substantial innovations through "corporate venturing," where the innovation is developed and commercialized through a separate, although not entirely independent, entity, a "spin-off." Such a spin-off venture makes the profits generated by the innovation easier to disentangle from the parental firm's profits, which might make it feasible to write formal contracts contingent on profits. It would be of interest to enrich the present model by allowing a fraction of ideas to be contractible and to explore how the spin-off option affects the firm's ability to maintain a reputation for rewarding innovators.

Another potentially important consideration suppressed in the analysis of this paper is the possibility of competition, both among firms for innovative workers and between the established firm and a startup in case the firm expropriates an innovator who then leaves and sets up a competing venture. From Anton and Yao (1994) we know that the latter option increases the worker's bargaining power, but it would be of interest to explore how this affects the rate of startup creation when reputational concerns matter.

Finally, one might want to know how the analysis would be affected if the workers were long-lived. This would introduce a host of technical complications, but also new interesting considerations. Some of these issues have already been explored elsewhere in the literature – in particular, Chatterjee and Rossi-Hansberg (2012) study whether it is optimal for a long-lived innovator to leave for a startup now or to wait for a better idea that might come in the future. Nevertheless, interactions between such dynamic considerations and reputation building might prove to be a source of additional insights.

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Appendix: Proofs

Proof of Proposition 1: Let $I_t \subseteq [0, \bar{v}_t]$ be the set of potential period-t ideas that in equilibrium get developed inside the firm and let $\mathcal{D}_x$ denote the family of all subsets of the interval $[0, x]$ that have positive measure. To prove the claim, I will show that if $v_t \in I_t$ then in any Pareto efficient SPNE it must be that $D_t \subseteq I_t$ for all $D_t \in \mathcal{D}_{v_t}$.

Thus, suppose, as a way of contradiction, that there is a Pareto efficient SPNE in which for some $\tilde{v}_t \in I_t$ there exists a set $D_t \in \mathcal{D}_{\tilde{v}_t}$ such that $D_t \subsetneq I_t$. Since $\tilde{v}_t \in I_t$, the firm must be willing to pay $s_t(\tilde{v}_t)$ for the innovation $\tilde{v}_t$ rather than stealing it and then being punished forever by all future innovators. That is, it has to be that

$$q(N) \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} p_\tau \Pr\{v_\tau \in I_\tau\} E(v_\tau - s_\tau(v_\tau) | v_\tau \in I_\tau) \geq s_t(\tilde{v}_t). \tag{A1}$$

Now, define $\hat{s}_t(v_t) \equiv kv_t$ for all $v_t \in D_t$. Given that $\tilde{v}_t > v_t$ for all $v_t \in D_t$ (by the choice of D_t), and given that $s_t(\tilde{v}_t) \geq k\tilde{v}_t$ (because otherwise the worker would not be willing to offer the innovation $\tilde{v}_t$ to the firm), we have $s_t(\tilde{v}_t) \geq k\tilde{v}_t > kv_t = \hat{s}_t(v_t)$. Condition (A1) therefore implies

$$q(N) \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} p_\tau \Pr\{v_\tau \in I_\tau\} E(v_\tau - s_\tau(v_\tau) | v_\tau \in I_\tau) \geq \hat{s}_t(v_t),$$

which shows that it is incentive compatible for the firm to compensate the workers also for the period-t ideas $v_t \in D_t$. Since the strategies and payoffs after period t are not affected by the inclusion in period t of the ideas from D_t, it must be that there exists a period-t continuation SPNE in which all the ideas from $I_t \cup D_t$ get developed in-house. Moreover, again using $s_t(\tilde{v}_t) \geq k\tilde{v}_t > kv_t = \hat{s}_t(v_t)$, we have

$$\begin{aligned} \Pr\{v_t &\in (I_t \cup D_t)\} E(v_t - s_t(v_t) | v_t \in (I_t \cup D_t)) = \int_{v_t \in (I_t \cup D_t)} (v_t - s_t(v_t)) f_t(v_t) dv_t \\ &> \int_{v_t \in I_t} (v_t - s_t(v_t)) f_t(v_t) dv_t = \Pr\{v_t \in I_t\} E(v_t | v_t \in I_t). \end{aligned}$$

This shows that the new period-t continuation equilibrium yields a higher payoff to the firm than the original one. Consequently, all of the payments $s_{t-1}(v_{t-1})$ in the original sets I_{t-1} remain incentive compatible under this new equilibrium, and so do, by backward induction, all of the payoffs $s_\tau(v_\tau)$ in the original sets I_τ for $\tau = 1, 2, .., t-2$.

Since the new SPNE equilibrium constructed above yields the same surplus for the workers as the original equilibrium and a strictly higher expected profit for the firm, the original equilibrium could not have been Pareto efficient. Q.E.D.

Proof of Proposition 2: (i) These two claims follow from the arguments in the text.

(ii) Total differentiation of condition (2) with respect to A yields

$$\frac{dv_0^*}{dA}\left[Av_0^* f_0(v_0^*) - 1\right] = -\int_0^{v_0^*} v f_0(v) dv. \tag{A2}$$

Although v_0^* can be discontinuous (and hence not differentiable) in A, $\frac{dv_0^*}{dA}$ exists at any v_0^* such that $Av_0^* f_0(v_0^*) - 1 \neq 0$. Furthermore, at the largest such interior v_0^*, it must be $Av_0^* f_0(v_0^*) - 1 < 0$. Otherwise, it would be that

$$A \int_0^x v f_0(v) dv > x \tag{A3}$$

for x slightly larger than v_0^*, which would imply that either a $v_0^{*\prime} > v_0^*$ exists that satisfies (2) or $v_0^* = \bar{v}_0$ is feasible. In either case, the assumption that v_0^* is the largest feasible cutoff level and that $v_0^* < \bar{v}_0$ would be contradicted. Thus, for v_0^* such that $Av_0^* f_0(v_0^*) - 1 < 0$ it must be that $\frac{dv_0^*}{dA}$ exists; moreover, inspection of (A2) reveals that $\int_0^{v_0^*} v f_0(v) dv > 0$ implies $\frac{dv_0^*}{dA} > 0$. The claim in part (ii) then follows from the fact that A strictly increases in δ, g, N, and p, and strictly decreases in k.

Now suppose v_0^* is such that $Av_0^* f_0(v_0^*) - 1 = 0$. Then the function $\Phi(x) \equiv A \int_0^x v f_0(v) dv - x$ must have a local maximum or an inflexion point at $x = v_0^*$ because if v_0^* represented a local minimum, then again (A3) would hold for x slightly larger than v_0^*, which would contradict the assumption that v_0^* is the largest feasible cutoff level and that $v_0^* < \bar{v}_0$. For the same reason, if v_0^* is an inflection point of $\Phi(x)$, then $\Phi(x)$ must be decreasing in the neighborhood of v_0^*. In either case, an increase in

A must result in an increase in v_0^*.

(iii) The possibility of discontinuity of v_0^* in A (and hence in δ, g, p, N, and k) will be demonstrated through an example. Let F be uniform, so that (2) becomes $\frac{A}{\bar{v}_0}\int_0^{v_0^*} v dv = v_0^*$, or, after integrating and rearranging,

$$\frac{A}{2\bar{v}_0}v_0^{*2} - v_0^* = 0. \tag{A4}$$

This yields two solutions, $v_0^* = 0$ and $v_0^* = \frac{2\bar{v}_0}{A}$. However, differentiating the LHS of (A4) with respect to v_0^* and evaluating at $v_0^* = \frac{2\bar{v}_0}{A}$ yields $\frac{dLHS(A4)}{dv_0^*}|_{v_0^*=\frac{2\bar{v}_0}{A}} = 1 > 0$, which by the argument above implies that if $\frac{2\bar{v}_0}{A} \leq \bar{v}_0$, then $v_0^* = \bar{v}_0$ can be sustained as an SPNE. On the other hand, if $\frac{2\bar{v}_0}{A} > \bar{v}_0$, then $v_0^* = 0$ is the unique SPNE. Hence, v_0^* exhibits a discontinuity at $A = 2$: For $A < 2$, the cutoff value is $v_0^* = 0$, but for $A \geq 2$ we have $v_0^* = \bar{v}$. Q.E.D.

Proof of Proposition 3: (i) Similarly to the proof of Proposition 2, let $\Phi(x,\delta) \equiv A\int_0^x vf_0(v)dv - x$, where $A \equiv \frac{\delta g(1-k)}{(1-\delta g)k}pq(N)$ and note that $\Phi_\delta(x,\delta) > 0$.[11]

Now, given that $f_0(x) < \infty$ for all $x \in [0,\bar{v}_0]$, we have that $\sup_{x\in[0,\bar{v}_0]} xf_0(x) < \infty$. There must therefore exist a $\hat{\delta} > 0$ such that $\Phi_x(x,\delta) = Axf_0(x) - 1 < 0$ for all $x \in [0,\bar{v}_0]$ and all $\delta \leq \hat{\delta}$. Combined with $\Phi(0,\delta) = 0$, this implies $\Phi(x,\delta) < 0$ for all $x \in [0,\bar{v}_0]$ and all $\delta \leq \hat{\delta}$. Hence, there exists a δ_1 such that $v_0^* = 0$ if $\delta \leq \delta_1$. To see that the "only if" part of the claim holds, note that $\Phi_\delta(x,\delta) > 0$ for all $x > 0$ implies that if $v_0^* > 0$ for some δ, then it must be $v_0^* > 0$ for all $\delta' > \delta$.

Next, observe that the corner solution $v_0^* = \bar{v}_0$ exists iff $\Phi(\bar{v}_0,\delta) \geq 0$ and that $\Phi(\bar{v}_0,\delta) = -\bar{v}_0$ for $\delta = 0$ and $\lim_{\delta\to 1/g}\Phi(\bar{v}_0,\delta) = \infty$. Hence, by continuity of $\Phi(\bar{v}_0,\delta)$ in δ, there exists a unique $\delta_2 \in (0,1/g)$ such that $\Phi(\bar{v}_0,\delta) \geq 0$ iff $\delta \geq \delta_2$.

(ii) Let $\delta = \delta_2$ and suppose (3) holds for some $y \in (0,\bar{v}_0)$, i.e., $\Phi(y,\delta_2) > 0$. Then by continuity, there must exist a $\delta^+ < \delta_2$ such that $\Phi(y,\delta) > 0$ for all $\delta \in (\delta^+,\delta_2)$. Furthermore, given that $v_0^* = \bar{v}_0$ is not feasible for $\delta < \delta_2$, it must be $\Phi(\bar{v}_0,\delta) < 0$ for all $\delta < \delta_2$. Thus, by continuity of $\Phi(\bar{v}_0,\delta)$, for any $\delta \in (\delta^+,\delta_2)$ there must exist a

[11] A subscript denotes the partial derivative with respect to the given variable.

$v_0^* \in [y, \bar{v}_0)$ such that $\Phi(v_0^*, \delta) = 0$, i.e., (2) holds and an interior cutoff value exists. It therefore has to be that $\delta_1 < \delta_2$ and $v_0^* \in (0, \bar{v}_0)$ for all $\delta \in (\delta_1, \delta_2)$, as claimed in part (ii) of the proposition.

(iii) Conversely, suppose (3) does not hold for any $y \in (0, \bar{v})$, i.e., we have $\Phi(x, \delta_2) \leq 0$ for all $x \in [0, \bar{v}]$. Then $\Phi_\delta(x, \delta) > 0$ for $x > 0$ implies $\Phi(x, \delta) < 0$ for all $\delta < \delta_2$ and all $x \in (0, \bar{v}_0]$, which means that if $\delta < \delta_2$, (2) cannot hold for any $v_0^* > 0$. This in turn implies $\delta_1 = \delta_2$. Q.E.D.

Proof of Proposition 4: Assume, contrary to the first claim in the proposition, that $v_0^{*r} > v_0^{*b}$. Conditions (9) and (10) imply that this requires RHS(10) > RHS(9), which holds iff $(1 - \rho - \lambda)(V_t^b - V_t^r) < 0$. Since $1 - \rho - \lambda > 0$, this in turn holds iff $V_t^b < V_t^r$. Solving (7) and (8) yields

$$\begin{aligned} V_t^b &= X\left[\omega_t^b\left[1 - \delta\left(1 - \lambda\right)r\right] + \omega_t^r \delta\rho b\right] \text{ and} \\ V_t^r &= X\left[\omega_t^r\left[1 - \delta\left(1 - \rho\right)b\right] + \omega_t^b \delta\lambda r\right], \end{aligned}$$

where $X \equiv [1 - \delta(1 - \lambda)r][1 - \delta(1 - \rho)b] - \delta^2 rb\rho\lambda$. It is straightforward to verify that $X > 0$ for all $\delta < 1/b$ (see Bagwell and Staiger, 1997). Thus, we have that $V_t^b < V_t^r$ iff $\omega_t^b(1 - \delta r) < \omega_t^r(1 - \delta b)$, which requires $\omega_t^b < \omega_t^r$, because $1 - \delta r > 1 - \delta b$.

Now, holding v_0^{*r} fixed, increase v_0^{*b} to a level $\hat{v}_0^{*b}$ such that the new continuation values $\hat{V}_t^b$ and $\hat{V}_t^r$ satisfy $\hat{V}_t^b = \hat{V}_t^r$. The expression for ω_t^b in (4) and a similar expression for ω_t^r imply that if $v_0^{*b} = v_0^{*r}$ then $\omega_t^b = \frac{b}{r}\omega_t^r > \omega_t^r$, which yields $V_t^b > V_t^r$. Thus, such a $\hat{v}_0^{*b}$ exists and satisfies $\hat{v}_0^{*b} < v_0^{*r}$. This increase in v_0^{*b} increases both V_t^r and V_t^b, so that $\frac{v_t}{\Pi_{\tau=1}^t g_\tau} = v_0^{*r}$ remains incentive compatible in a recession state (i.e., (10) continues to hold). Moreover, given that RHS(9) = RHS(10) when $\hat{V}_t^b = \hat{V}_t^r$, (14) must hold as well for this innovation, i.e., all the innovations v_t such that $\frac{v_t}{\Pi_{\tau=1}^t g_\tau} = v_0^{*r}$ must be incentive compatible also in a boom state. Thus, there must exist an SPNE such that $v_0^{**b} > v_0^{*b}$ and $v_0^{**r} \geq v_0^{*r}$, which Pareto-dominates the initial equilibrium. Thus, in a Pareto-efficient equilibrium it cannot be that $v_0^{*r} > v_0^{*b}$.

To prove the second claim, assume, again as a way of contradiction, that $v_0^{*r} = v_0^{*b} \in (0, \bar{v}_0)$. As shown above, this yields $\omega_t^b = \frac{b}{r}\omega_t^r > \omega_t^r$, which in turn implies $V_t^b > V_t^r$. But then RHS(9) $>$ RHS(10), so that innovations such that $\frac{v_t}{\Pi_{\tau=1}^t g_\tau} \in (v_0^{*b}, v_0^{*b} + \varepsilon)$ satisfy (9) for small $\varepsilon > 0$, which means that an SPNE exists with cutoff levels $v_0^{**b} > v_0^{*b}$ and $v_0^{**r} \geq v_0^{*r}$. This new SPNE Pareto-dominates the initial equilibrium, which implies that if $v_0^{*r} \in (0, \bar{v}_0)$ then a Pareto-efficient equilibrium must have $v_0^{*b} > v_0^{*r}$. Q.E.D.

Proof of Proposition 5: (i) Let $\hat{v} \equiv \frac{q(N)}{N} \int_0^{\bar{v}} v f(v) dv$ and define $\gamma(.) \equiv c'^{-1}(.)$, so that $p^* = \gamma(k\hat{v})$. Then the firm's non-reneging constraint (2) can be written as

$$\frac{\delta q(N)(1-k)}{(1-\delta)\, k} \gamma(k\hat{v}) \int_0^{v^*} v f(v) dv = v^*. \tag{A5}$$

Thus, v^* increases in k iff the LHS of (A5) increases in k (holding v^* constant), which in turn holds iff $\frac{(1-k)}{k}\gamma(k\hat{v})$ increases in k. Differentiating, we get

$$\begin{aligned} \frac{d}{dk}\left[\frac{(1-k)}{k}\gamma(k\hat{v})\right] &= \hat{v}\gamma'(k\hat{v})\frac{(1-k)}{k} - \frac{\gamma(k\hat{v})}{k^2} > 0 \\ &\Longleftrightarrow \\ \gamma'(k\hat{v})(1-k) &> \frac{\gamma(k\hat{v})}{k\hat{v}}. \end{aligned} \tag{A6}$$

Now, we have $\gamma(k\hat{v}) = \gamma(c'(p^*)) = p^*$, from which $\gamma'(c'(p^*))c''(p^*) = 1$, so that $\gamma'(k\hat{v}) = \gamma'(c'(p^*)) = \frac{1}{c''(p^*)}$. Plugging this into (A6) along with $k\hat{v} = c'(p^*)$ and rearranging, condition (A6) becomes

$$1 - k > \frac{c''(p^*)p^*}{c'(p^*)}.$$

(ii) The claim holds if $\frac{d[p^*(\bar{v}-v^*)]}{dk} < 0$ is possible. We have

$$\begin{aligned} \frac{d\left[p^*(\bar{v}-v^*)\right]}{dk} &= \frac{dp^*}{dk}(\bar{v}-v^*) - p^*\frac{dv^*}{dk} \\ &= \hat{v}\gamma'(k\hat{v})(\bar{v}-v^*) - \gamma(k\hat{v})\frac{dv^*}{dk}, \end{aligned}$$

which is negative if

$$\gamma'(k\hat{v})(\bar{v}-v^*) < \frac{\gamma(k\hat{v})}{k\hat{v}}\frac{dv^*}{dk}. \tag{A7}$$

Now, (A6) and (A7) can hold simultaneously only if

$$\frac{dv^*}{dk} > \frac{\bar{v}-v^*}{1-k}. \tag{A8}$$

To see that this condition can be satisfied, let $M \equiv \frac{\delta q(N)}{1-\delta}$ and differentiate condition (A5) with respect to k to get

$$\frac{dv^*}{dk}\left[M\left(1-k\right)v^* m(k\hat{v})f(v^*)-k\right] = M\left[\gamma(k\hat{v})-\hat{v}\gamma'(k\hat{v})(1-k)\right]\int_0^{v^*} vf(v)dv. \tag{A9}$$

The bracketed term on the LHS of (A9) must be negative at the largest v^* (as the LHS of (A5) must cross the RHS from above). Moreover, when condition (A6) holds, the bracketed term on the RHS of (A9) is also negative.

Now, consider any distribution $F(.)$ such that for some $\tilde{M} > 0$ the cutoff level v^* is given by the tangent point between $\frac{\tilde{M}(1-k)}{k}\gamma(k\hat{v})\int_0^x vf(v)dv$ and x. Denote this cutoff level as $\tilde{v}^*$ and note that it must be both $\tilde{M}\frac{(1-k)}{k}\gamma(k\hat{v})\int_0^{\tilde{v}^*} vf(v)dv = \tilde{v}^*$ and $1-\tilde{M}\frac{(1-k)}{k}\gamma(k\hat{v})\tilde{v}^* f(\tilde{v}^*) = 0$. Further, for any $M > \tilde{M}$ we have $\frac{M(1-k)}{k}\gamma(k\hat{v})\int_0^{\tilde{v}^*} vf(v)dv > \tilde{v}^*$, so that $v^* > \tilde{v}^*$ and v^* is differentiable in k for M close to $\tilde{M}$, with $\frac{dv^*}{dk} > 0$ and $\lim_{M\downarrow\tilde{M}} \frac{dv^*}{dk} = \infty$. Combined with $\gamma'(k\hat{v}) = \frac{1}{c''(p^*)} < \infty$, this implies that both (A7) and (A8) must hold for all M greater than, but sufficiently close, to $\tilde{M}$. Q.E.D.

Proof of Proposition 6: *Step 1.* Note first that $s(v) \leq kv^*$ must hold for all $v \leq v^*$, because otherwise v^* would not be a Pareto efficient SPNE. To see this, suppose $s(v') > kv^*$ for some $v' \leq v^*$. The firm's non-reneging constraint yields

$$Mp\int_0^{v^*}\left[v-s(v)\right]f(v)dv \geq s(v') > kv^*,$$

where $M \equiv \frac{\delta q(N)}{1-\delta}$. This implies that there exists a $\tilde{v} > v^*$ such that

$$Mp \int_0^{v^*} [v - s(v)] \, f(v) dv \geq kv \quad \text{for all } v \leq \tilde{v}.$$

Let $\tilde{v}^*$ be the largest such $\tilde{v}$ within $(v^*, \bar{v}]$. By construction, there must exist an SPNE characterized by the cutoff $\tilde{v}^*$ and by an alternative contract $\tilde{s}(v)$ that coincides with $s(v)$ for $v \leq v^*$ and is given by $\tilde{s}(v) = kv$ for $v \in (v^*, \tilde{v}]$. Clearly, this alternative SPNE Pareto dominates the original equilibrium as it expands the set of innovations that are developed in-house while giving the workers exactly the same incentives and the same expected utility as the original contract $s(v)$.

Step 2. If claim (i) in the proposition holds (i.e., if $\Omega^* = 0$), then the problem is identical to the one that yielded Proposition 5. Claims (ii) and (iii) then follow immediately from Proposition 5. Therefore, all that needs to be proven is that when $c(p) = \frac{p^\eta}{\eta}$, with $\eta \in (1, 2-k)$, there exist distribution functions such that v^* is interior and $\Omega^* = 0$, i.e., $s^*(v) = kv$ for all $v \leq v^*$.

Using $\Omega \equiv \int_0^{v^*} [s(v) - kv] \, f(v) dv$ instead of $s(.)$ as the firm's choice variable, the worker's incentive compatibility constraint for the choice of p can be written as

$$\frac{q(N)}{N} \left[\Omega + k \int_0^{\bar{v}} v f(v) dv \right] = c'(p), \tag{A10}$$

which shows that p is independent of v^* and is an increasing and differentiable function of Ω: $p = p(\Omega)$.

Assuming an interior v^* (which will be verified in Step 3) and defining

$$\pi(\Omega, v^*) \equiv \frac{M}{\delta} p(\Omega) \left[(1-k) \int_0^{v^*} v f(v) dv - \Omega \right],$$

the firm's optimization problem can then be stated as

$$\max_{\Omega \geq 0, v^*} \pi(\Omega, v^*)$$

$$\text{subject to} \qquad \delta \pi(\Omega, v^*) = kv^*, \tag{A11}$$

where $s(v^*) = kv^*$ on the RHS of (A11) follows from the workers' participation constraint $s(v) \geq kv$ combined with $s(v) \leq kv^*$ (proven in Step 1).

Applying the Implicit Function Theorem to constraint (A11) yields

$$\frac{dv^*}{d\Omega} = -\frac{\delta\pi_\Omega}{\delta\pi_{v^*} - k}. \tag{A12}$$

The firm's first order condition with respect to Ω is then $\pi_\Omega + \pi_{v^*}\frac{dv^*}{d\Omega} \leq 0$, or, using (A12),

$$\pi_\Omega \left[1 - \frac{\delta\pi_{v^*}}{\delta\pi_{v^*} - k}\right] \leq 0. \tag{A13}$$

Now, the same argument as in the proof of part (ii) in Proposition 2 shows that at the largest interior v^* it must be $\delta\pi_{v^*} - k < 0$. Moreover, $\pi_{v^*} = \frac{M}{\delta}p(\Omega)\,(1-k)\,v^* f(v^*) > 0$. The bracketed term in (A13) is therefore strictly positive, so that (A13) reduces to $\pi_\Omega \leq 0$. This implies that $\Omega^* = 0$ if $\pi_\Omega < 0$.

We have

$$\begin{aligned}
\pi_\Omega &= Mp'(\Omega)\left[(1-k)\int_0^{v^*} vf(v)dv - \Omega\right] - Mp(\Omega) \\
&= p'(\Omega)\frac{kv^*}{p(\Omega)} - Mp(\Omega).
\end{aligned}$$

Using $p'(\Omega) = \frac{1}{c''(p)}$ obtained from condition (A10), we thus get that $\pi_\Omega < 0$ iff

$$pc''(p) > (1-k)\int_0^{v^*} vf(v)dv - \Omega, \tag{A14}$$

or, using constraint (A11),

$$Mp^2c''(p) > kv^*. \tag{A15}$$

Given that $c(p) = \frac{p^\eta}{\eta}$, with $\eta \in (1, 2-k)$, and recalling that $\hat{v} \equiv \frac{q(N)}{N}\int_0^{\bar{v}} vf(v)dv$, we have $c'(p) = p^{\eta-1}$, $c''(p) = (\eta-1)p^{\eta-2}$, $p = \left(k\hat{v} + \frac{q}{N}\Omega\right)^{\frac{1}{\eta-1}}$, and $p^2c''(p) = (\eta-1)p^\eta$. Condition (A15) can therefore be written as

$$M(\eta-1)\left(k\hat{v} + \frac{q}{N}\Omega\right)^{\frac{\eta}{\eta-1}} > kv^*. \tag{A16}$$

Now, suppose (A16) holds at $\Omega = 0$. Then it also has to hold for all $\Omega > 0$ because $\eta > 1$ implies that the LHS of (A16) increases in Ω, whereas (A12) shows that at any point at which (A16) holds the RHS of (A16) decreases in Ω. Hence, if (A16) holds at some Ω, it also has to hold for all $\Omega' > \Omega$. Therefore, to show that $\Omega^* = 0$, it is sufficient to show that (A16) holds at $\Omega = 0$.

Thus, set $\Omega = 0$ in (A14) and use $pc''(p) = (\eta - 1)k\hat{v}$ to get $\pi_\Omega < 0$ iff

$$(\eta - 1)\hat{v} > \frac{1-k}{k} \int_0^{v^*} vf(v)dv. \tag{A17}$$

Now, clearly $\int_0^{v^*} vf(v)dv < v^*$, which implies that (A17) holds if

$$\frac{v^*}{\hat{v}} < \frac{1-k}{k(\eta - 1)}. \tag{A18}$$

Step 3. To see that there exist distribution functions such that (A18) holds, let $f(v) = \frac{1}{2}\varepsilon\,(3 + 2\varepsilon)\,v^{\varepsilon-1} - \varepsilon\,(1 + 2\varepsilon)\,v^{2\varepsilon-1}$, with $\varepsilon \in (0, \frac{1}{2})$ and support $(0, 1]$. Then $\hat{v} = \frac{q}{N}\int_0^1 vf(v)dv = \frac{\varepsilon q}{2N(1+\varepsilon)}$ and

$$\int_0^{v^*} vf(v)dv = \varepsilon\frac{(3 + 2\varepsilon)}{2\,(1 + \varepsilon)}v^{*(1+\varepsilon)} - \varepsilon v^{*(1+2\varepsilon)}$$

Defining $A \equiv M\,(1 - k)\,p = \frac{\delta q(N)}{1-\delta}\,(1 - k)\,p$, the non-reneging condition (A11) is $A \int_0^{v^*} vf(v)dv = v^*$, which for $v^* > 0$ holds iff

$$(3 + 2\varepsilon)\,v^{*\varepsilon} - 2\,(1 + \varepsilon)\,v^{*2\varepsilon} - \frac{2\,(1 + \varepsilon)}{A\varepsilon} = 0. \tag{A19}$$

Now, for $x > 0$, the expression $T(x) \equiv (3 + 2\varepsilon)\,x^\varepsilon - 2\,(1 + \varepsilon)\,x^{2\varepsilon}$ is single-peaked and maximized at $x = x^+ \equiv \left(\frac{3+2\varepsilon}{4(1+\varepsilon)}\right)^{\frac{1}{\varepsilon}} < 1$, with $T(x^+) = \frac{(3+2\varepsilon)^2}{8(1+\varepsilon)} > 1$ and $T(1) = 1$. Hence, for any A such that $\frac{2(1+\varepsilon)}{A\varepsilon} \in \left(1, \frac{(3+2\varepsilon)^2}{8(1+\varepsilon)}\right)$, condition (A19) yields an interior threshold v^*, which is between x^+ and 1 and converges to x^+ as $A \to A^+ \equiv \frac{16(1+\varepsilon)^2}{\varepsilon(3+2\varepsilon)^2}$. Noting that p is independent of δ and therefore any $A \in R^+$ can be chosen through an appropriate choice of δ, x^+ is the infimum of the set of v^* that can be supported through a choice of δ. Let δ^+ be the δ defined by $A = A^+$. Given that $\hat{v}$ is also independent of δ, we then have

$$\begin{aligned}\lim_{\delta \to \delta^+} \frac{v^*}{\hat{v}} &= \frac{x^+}{\hat{v}} = \frac{2(1+\varepsilon)}{\varepsilon}\left(\frac{3+2\varepsilon}{4(1+\varepsilon)}\right)^{\frac{1}{\varepsilon}} \\ &= \frac{(3+2\varepsilon)^2}{8\varepsilon(1+\varepsilon)}\left(\frac{3+2\varepsilon}{4(1+\varepsilon)}\right)^{\frac{1-2\varepsilon}{\varepsilon}}\end{aligned}$$

Using the L'Hôpital's rule, we get

$$\begin{aligned}&\lim_{\varepsilon \to 0} \frac{(3+2\varepsilon)^2}{8\varepsilon(1+\varepsilon)}\left(\frac{3+2\varepsilon}{4(1+\varepsilon)}\right)^{\frac{1-2\varepsilon}{\varepsilon}} \\ &= \lim_{\varepsilon \to 0} \frac{(3+2\varepsilon)^2}{8\varepsilon(1+\varepsilon)} \lim_{\varepsilon \to 0}\left(\frac{3+2\varepsilon}{4(1+\varepsilon)}\right)^{\frac{1-2\varepsilon}{\varepsilon}} \\ &= \lim_{\varepsilon \to 0} \frac{(3+2\varepsilon)}{2(1+2\varepsilon)} \lim_{\varepsilon \to 0}\left(\frac{3+2\varepsilon}{4(1+\varepsilon)}\right)^{\frac{1-2\varepsilon}{\varepsilon}} = 0,\end{aligned}$$

which shows that condition (A18) holds for all ε sufficiently close to zero. Therefore, if ε is small it must be $\Omega^* = 0$ and the conclusions of Proposition 5 apply. Q.E.D

Proof of Proposition 7: *Step 1.* Observe that as long as $s(v) \leq v^*$ for $v \leq v^*$ and $s(v) < kv$ for $v > v^*$, the firm's non-reneging constraint (11) is not affected by the exact functional form of $s(v)$. Thus, holding v^* fixed, we can ignore constraint (11) when finding the optimal $s(v)$.

Using $m(v) \equiv s(v) - kv$, the firm's optimization problem can be written as

$$\max_{m(v),p} \frac{q(N)}{1-\delta} p \int_0^{v^*} [(1-k)v - m(v)] f(v) dv$$

subject to $m(v) \geq 0$ for all $v \leq v^*$ and

$$\frac{q(N)}{N}\left[\int_0^{v^*} u(m(v)+kv) f(v) dv + \int_{v^*}^{\bar{v}} u(kv) f(v) dv\right] = c'(p). \tag{A20}$$

The constraint $s(v) \leq v^*$ for $v \leq v^*$ will be ignored for now, but will be shown to hold for the contract that solves the above problem.

Letting μ be the multiplier associated with constraint (A20), pointwise optimization with respect to m yields the first-order condition

$$-\frac{q(N)}{1-\delta}pf(v)+\mu\frac{q(N)}{N}u'(m+kv)f(v)\leq 0,\quad \text{with equality if } m>0.$$

Thus, if $m>0$, then $s(v)$ is given by

$$u'(s)=\frac{pN}{\mu\left(1-\delta\right)},$$

so that $s(v)$ is constant for all v for which $s(v)>kv$. Hence, there must exist a $v^{+}\leq v^{*}$ such that $s(v)=kv^{+}$ for all v for which $s(v)>kv$. Furthermore, if $s(v)=kv$, then the first-order condition requires

$$u'(kv)\leq\frac{pN}{\mu\left(1-\delta\right)}.$$

Step 2. Now suppose there exist v_1 and v_2, $v_1<v_2\leq v^{*}$, such that $s(v_1)=kv_1$ and $s(v_2)>kv_2$. Then $s(v_2)>s(v_1)$, which, due to the strict concavity of u, implies

$$\frac{pN}{\mu\left(1-\delta\right)}=u'(s(v_2))<u'(s(v_1))\leq\frac{pN}{\mu\left(1-\delta\right)},$$

a contradiction. Thus, combining all of the above results, it must be that $s(v)=kv^{+}$ for $v\leq v^{+}$ and $s(v)=kv$ for $v\in[v^{+},v^{*}]$, where v^{+} is such that (A20) holds, i.e.,

$$\frac{q(N)}{N}\left[\int_0^{v^{+}}u(kv^{+})f(v)dv+\int_{v^{+}}^{\bar{v}}u(kv)f(v)dv\right]=c'(p).$$

Finally, the optimal p is given by the first-order condition

$$\frac{q(N)}{1-\delta}\int_0^{v^{*}}\left[\left(1-k\right)v-m(v)\right]f(v)dv=\mu c''(p).$$

Q.E.D.

Proof of Proposition 8: The possibility of $q^{*}<q^{FB}$ has already been demonstrated

in the text. To see that the second claim holds, note that $q^* > q^{FB}$ if

$$\frac{1-k}{1-\delta}p\left[\int_0^{v^*} vf(v)dv + q\frac{dv^*}{dq}v^*f(v^*)\right] > \frac{p}{1-\delta}\int_0^{\bar{v}} vf(v)dv$$

or, after rearranging,

$$(1-k)\,qv^*f(v^*)\frac{dv^*}{dq} > \int_0^{\bar{v}} vf(v)dv - (1-k)\int_0^{v^*} vf(v)dv \tag{A21}$$

Now, define $B \equiv \frac{\delta(1-k)}{(1-\delta)k}p$ so that (2) can be written as $Bq\int_0^{v^*} vf(v)dv = v^*$. The Implicit Function Theorem then yields

$$\frac{dv^*}{dq} = \frac{B}{[1 - Bqv^*f(v^*)]}\int_0^{v^*} vf(v)dv.$$

As shown in the proof of part (ii) in Proposition 2, at the largest interior v^* it must be $1 - Bqv^*f(v^*) > 0$. Now, consider any F such that for some $\tilde{B} > 0$ the cutoff level v^* is given by the tangent point between $\tilde{B}q\int_0^x vf(v)dv$ and x. Denote this cutoff level $\tilde{v}^*$ and note that it must be both $\tilde{B}q\int_0^{\tilde{v}^*} vf(v)dv = \tilde{v}^*$ and $1 - \tilde{B}q\tilde{v}^*f(\tilde{v}^*) = 0$. Further, for any $B > \tilde{B}$ we have $Bq\int_0^{\tilde{v}^*} vf(v)dv > \tilde{v}^*$, so that $v^* > \tilde{v}^*$ and v^* is differentiable in B for B sufficiently close to $\tilde{B}$, with $\frac{dv^*}{dq} > 0$ and $\lim_{B\downarrow\tilde{B}} \frac{dv^*}{dq} = \infty$. This implies that (A21) holds for all B greater than, but sufficiently close, to $\tilde{B}$. Q.E.D.

Proof of Proposition 9: From (2), for any two distributions F and H we have $v_H^* > v_F^*$ if $\int_0^{v_F^*} vh(v)dv > \int_0^{v_F^*} vf(v)dv$. Integrating by parts yields

$$\int_0^{v_F^*} vf(v)dv = v_F^*F(v_F^*) - \int_0^{v_F^*} F(v)dv$$

and similarly for H. Thus,

$$\int_0^{v_F^*} v[h(v) - f(v)]dv = v_F^*\left[H(v_F^*) - F(v_F^*)\right] - \int_0^{v_F^*} \left[H(v) - F(v)\right]dv. \tag{A22}$$

Now consider a cdf H such that (i) $H(v) \geq F(v)$ for all $v \in [0, \bar{v}]$, (ii) $H(v_F^*) > F(v_F^*)$, and (iii) $[H(v) - F(v)] < H(v_F^*) - F(v_F^*)$ for all $v \in [0, v^*)$. Then (i) and (ii) imply that H is strictly first-order stochastically dominated by F, while (ii) and (iii) imply that (A22) is strictly positive, so that $v_H^* > v_F^*$. Q.E.D.