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Default, Mortgage Standards and Housing Liquidity

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Abstract

The effects of households' indebtedness on their house-selling decisions are studied in a dynamic equilibrium model with search in the housing market and defaultable long-term mortgages. In equilibrium, both sellers' asking prices and time-to-sell increase with the relative size of their outstanding mortgages. In turn, the *liquidity* of the housing market associated with time-to-sell determines the mortgage standards of competitive lenders, measured by the maximum loan-to-value (LTV) ratio offered at origination. Calibrated to the U.S. economy, the model generates, as observed, positive correlations over time between house prices and LTV's at origination and across sellers among asking prices, time-to-sell, and LTV's outstanding.

JEL Classification: E30, G21, R10, R31

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1 Introduction

In this paper we study the influence of households' mortgage debt on their house-selling decisions and the effects of the resulting housing market *liquidity* (by which we mean the speed with which houses can be sold) on mortgage lending standards. To these ends, we develop a dynamic model with a frictional housing market and defaultable long-term mortgage debt. The model gives rise in equilibrium to distributions of both house prices and debt, as well as to endogenous default probabilities that differ across households. Aggregate shocks drive mortgage standards through their effect on liquidity, and this together with the mortgage terms offered to buyers affect the selling decisions of households.

Our theory demonstrates that (i) house-selling decisions depend critically on sellers' levels of home equity, and (ii) liquidity in the housing market is an important factor in determining the extent to which households can borrow. Calibrated to U.S. data, the economy captures, qualitatively, both the observed relationship between households' mortgage loan-to-value (LTV) ratios and asking prices, and the negative correlation between house prices and mortgage lending standards (measured by down-payments) over time.

Our paper is motivated first by the observation that U.S. house prices are positively correlated with mortgage LTV's at origination (or, equivalently, correlated negatively with down-payment ratios) for first-time home buyers (see Figure 1)[1]. This phenomenon is particularly noticeable during the period leading up to the U.S. sub-prime mortgage crisis and subsequent house price collapse. Over this time period, U.S. housing markets were "booming" in the sense that prices were rising, sales volume increasing and time-to-sell declining (Ngai and Sheedy, 2015). At the same time, mortgage lending standards were relaxed, specifically and significantly in the sense of lowered down-payment requirements.[2] It has been argued that requirements were lowered to a greater extent than can be explained

[1] Actual down-payments do not necessarily reflect mortgage standards. First-time buyers, however, are the most likely to be affected by down-payment constraints.

[2] Lending standards may include mortgage approval rates, down-payment ratios, document requirements, interest rates, *etc.*. We focus on down-payment ratios, or LTV at origination. Gerardi et al. (2008) show that many subprime loans in this period were characterized by such high LTV's. Duca et al. (2011) construct a series for loan-to-value ratios (LTVs) faced by first-time home buyers and show that the overall LTV ratio increased from 2000 to 2005. Barlevy and Fisher (2011) use data compiled for over 200 U.S. cities between 2000 and 2008 to find that interest-only (IO) mortgages were used sparingly in cities in which an elastic housing supply kept housing prices in check, but were common in cities with an inelastic supply in which housing prices rose sharply and then crashed. Dell'Ariccia et al. (2012) document and show that lending standards (denying rates) declined more in areas that experienced larger credit booms (more applicants) and greater price appreciation. Mian and Sufi (2011) find that regions with high latent demand from 2001 to 2005 experienced large relative decreases in denial rates, increases in mortgages originated, and increases in housing price appreciation, despite the fact that the same regions experienced significantly negative relative income and employment growth over this time period.

by explicit changes in regulatory constraints (see *e.g.* Belsky and Richardson, 2010). Here, we explore the incentive of profit maximizing lenders to relax lending standards in response to a "hot" housing market, by which we mean specifically one in which prices are high and time-to-sell is low by historical standards.

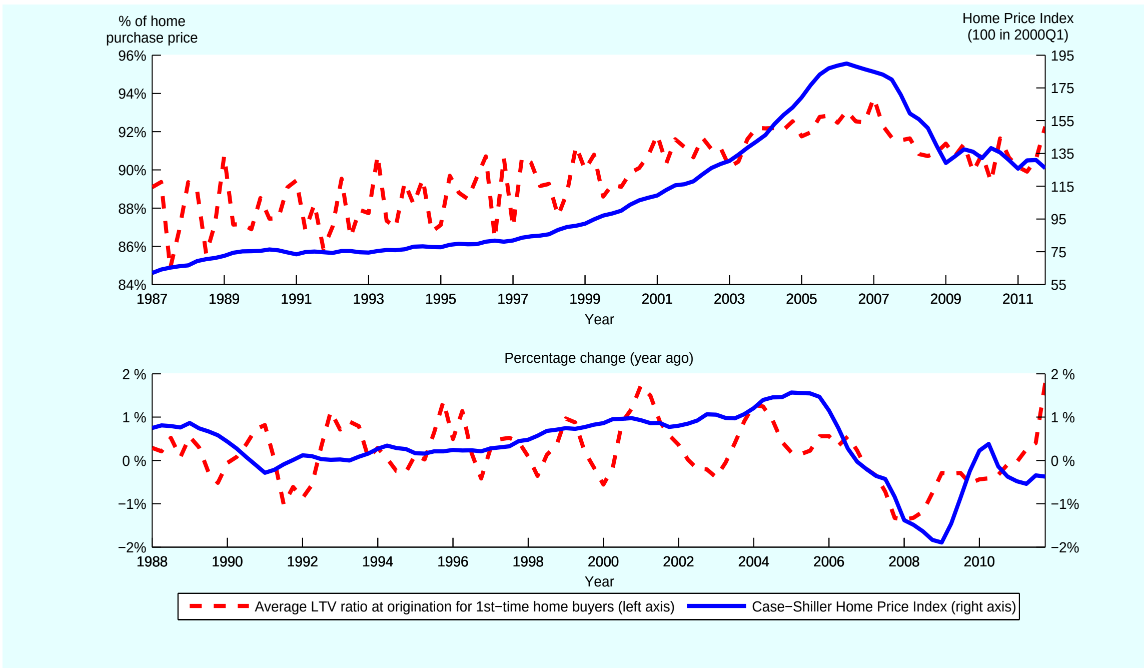

Figure 1: Values and percentage changes (from one year earlier) in average first-time home buyers' down-payment ratios and S&P/Case-Shiller U.S. National Home Price Index. Source: American Housing Survey (AHS) 2007, 2009, 2011 national data.

An important consideration for lenders determining mortgage standards is default risk. A homeowner in financial distress can in principle avoid foreclosure by selling. Thus, the default risk of an indebted homeowner is to some extent tied to her strategy for selling her house. With this in mind, note that as observed by Genesove and Mayer (1997, 2001) and Anenberg (2011), house sellers' leverage affects both their asking prices and time-to-sell. Specifically, sellers with high LTV's post higher asking prices, wait longer to sell, and sell ultimately at higher prices. This observation suggests that mortgage debt affects not only prices, but also households' incentive to sell.

In our theory, we consider a growing population of *ex ante* identical households, each of which lives either in a single city (on which we focus) or in a largely unmodeled rest-of-the-world. All residents of the city require housing and may either live as a renter or own one of a large number of identical houses, which are produced and sold initially by a competitive development industry. Households enter the city when the value of doing so exceeds their exogenous outside option. Once there, households remain in the city either as renters or homeowners until they leave as a result of exogenous shocks.

Houses are sold following the protocol of directed search as described by Moen (1997). Sellers offer houses for sale in a variety of sub-markets, within each of which prospective buyers and sellers are randomly matched. Each sub-market is characterized by a unique combination of a posted price and matching probabilities for buyers and sellers. These probabilities determine the expected time-to-buy and time-to-sell for buyers and sellers, respectively. Search is directed in the sense that buyers and sellers choose sub-markets optimally given the trade-off between the posted price and the matching probabilities.

House purchases are financed by mortgages offered by competitive mortgagees (*e.g.* banks) which control the terms offered. Specifically, mortgagees decide the *size* of the mortgage to offer, and this determines the LTV at origination or, equivalently, the down-payment ratio. Mortgages are of a fixed length and at an exogenous interest rate, which we model as determined by aggregate conditions rather than those within the city. Households who do not own pay rent each period equal to a fixed and exogenous fraction of income.

Mortgage holders are subject to random financial distress shocks which force them to either sell their homes through the search process or default and face foreclosure. Households in distress are not committed to sell, and based on their specific situations decide whether and how to do so. Thus, they effectively choose optimally their likelihood of default on mortgage debt. If a household defaults, its house is seized by the mortgage company, a foreclosure flag is placed on the its record, and it is prohibited from participating in the housing market until the flag is lifted, which also occurs randomly.

A homeowner in financial distress can reduce her likelihood of foreclosure by posting a low price and raising her matching probability. In our calibrated model, however, in equilibrium all distressed households choose prices associated with substantial probabilities of default. This probability generally rises with the household's outstanding mortgage debt. Moreover, households can choose to default outright, and some do. This occurs in equilibrium if house price movements render home equity sufficiently negative.

Because sellers are heterogeneous, there arises in equilibrium a distribution of house prices which evolves over time owing to aggregate shocks.[3] Home *buyers* remain identical, however, as we assume that goods are non-storable and rule out household saving. Free-entry of homogeneous buyers into the housing market gives rise to the aforementioned

[3] Houses are sold by construction firms, mortgagees, and home-owners differentiated by both their reasons for selling and levels of mortgage debt.

trade-off between house prices and matching probabilities. Heterogeneous sellers separate themselves optimally into various sub-markets based on their individual states. As a result, the individual decision problem is independent of the distribution of sellers and the model is *block recursive* as in Shi (2009) and Menzio and Shi (2010).

Regardless of the aggregate state, above a certain LTV the prices posted in equilibrium by selling homeowners are steeply increasing in their outstanding mortgage debt, a result consistent with the empirical findings of Genesove and Mayer (1997). More highly levered sellers thus are more likely to default than less levered ones. As a result, negative shocks to city-wide income cause particularly severe waves of default and foreclosure if they occur when the economy has a high proportion of highly levered homeowners.

Housing market liquidity affects mortgage lending standards both through the expected default rate and lenders' expected losses upon default. The more liquid the market, the higher the probability with which indebted households sell and thus the lower the rate of default and foreclosure. At the same time, mortgagees sell foreclosed houses more quickly, lowering their expected carrying cost and thus the cost of default. Finally, houses typically sell at higher prices in a hotter market, regardless of who sells, further lowering the cost of default. Overall, mortgagees are willing to offer larger mortgages (allowing a higher LTV at origination), when the housing market is more liquid. This tends to occur when both income and house prices are high.

The paper contributes to the growing literature on search frictions in the housing market (see, *e.g.* He et al. (2015), Head and Lloyd-Ellis (2012), and Wheaton (1990)). Specifically, we extend the theory of Head et al. (2014) (HLS14) which studies the dynamics of house prices and construction in an environment with homogeneous buyers and sellers and complete financial markets. Here, we introduce a role for mortgage debt and a form of limited commitment which allows households to default under certain circumstances. These features generate heterogeneity among households, *ex post*. Also, while HLS14 focuses mainly on random search, directed search is integral to our analysis.[4]

For the most part, the literature neither models in detail long-term mortgage contracts nor examines the relationship between lending and liquidity. That said, Hedlund (2015) and Hedlund (2016) also consider models featuring directed search, long-term mortgages, and limited commitment. The models studied in both of these papers, however, feature a different market arrangement in which buyers and sellers interact only indirectly via intermediaries who buy houses from heterogenous sellers and then sell them, along with newly constructed ones, to heterogeneous buyers. Like ours, this setup renders the model

[4] HLS14 does consider a case with competitive search, but only as a robustness check.

block recursive and tractable. Here, as we focus on the differential decisions of *sellers* exclusively, we find it useful to abstract from heterogeneity on the part of buyers not only for tractability, but also to simplify the calibration, presentation and interpretation of our findings.[5]

Our theory differs from those studied Hedlund (2015) and Hedlund (2016) also in that we consider finite mortgages at fixed interest rates rather than infinite-horizon mortgage contracts with either fixed (as in Hedlund, 2015) or flexible (as in Hedlund, 2016) rates. While finite-horizon contracts add complexity, they enable us to study differences in optimal house-trading decisions across households at different *stages* of mortgage repayment. Similarly, shocks affect households who have purchased at different times in the past differentially in our environment. We think this interesting as in the U.S. conventional mortgages typically have a 30-year term and about 70% are at fixed interest rates.[6] Finally, while we focus on housing and mortgage markets at the city level, Hedlund studies them at the national level.

In a New Keynesian model along the lines of Iacoviello (2005) with credit-constrained consumers and housing market frictions, Ungerer (2015) shows that expansionary monetary policy leads to higher leverage among homeowners. In his model, a decrease in mortgage interest rates increases housing demand, bringing more buyers into the market and allowing lenders to liquidate foreclosed houses more quickly. This reduces the expected carrying cost of a foreclosed house and induces lenders to increase mortgage LTV's at origination.

Our model differs from that studied by Ungerer (2015) in several respects: First, in Ungerer (2015) there is no default in equilibrium and thus no liquidation of houses. Financial frictions take the form of a Kiyotaki-Moore collateral constraint with lenders offering debt to the extent that default is prevented in equilibrium. In contrast, in our theory lending standards reflect default probabilities and the foreclosure inventory is a component of housing supply. As a result, market liquidity affects both the expected carrying cost of a foreclosed house *and* the expected default rate. Both factors contribute to the positive correlation between house prices and LTV's at origination. Second, we model debt as long-term rather than one-period. As noted above, we are able to trace out the default decision of indebted households at every stage of the mortgage repayment

[5] Directed search and sorting with two-sided heterogeneity is a challenging problem to tackle. There are a handful of papers that characterize the steady state of such an economy under certain conditions (see Shi (2001), Shimer and Smith (2000), Smith (2006), and Eeckhout and Kircher (2010)). Shi (2005) further shows that dynamics of sorting with two-sided heterogeneity can be tractable in some settings. Nevertheless, our model would be intractable for the purposes of the exercise we construct if we allowed for two-sided heterogeneity with endogenous saving alongside mortgage choices.

[6] This percentage, however, has declined in recent years.

process. Finally, in Ungerer (2015) houses are divisible, the housing stock is fixed and there is no construction sector.

By focusing on the selling decisions of households, our paper is also related to those of Ngai and Tenreyro (2014) and Ngai and Sheedy (2015). These papers consider, respectively, the effect of seasonal fluctuations in demand and aggregate conditions on the decisions of homeowners to sell. Similarly, our focus is related to that of Guren and McQuade (2015) who consider the importance of foreclosure (which is exogenous in their case) and liquidation for the dynamics of house prices, with specific reference to the experience of the U.S. housing crisis. None of these papers consider the two principal issues studied here; the differential selling decisions of heterogeneous homeowners distinguished by their levels of mortgage debt and the effect of housing market liquidity on mortgagees' willingness to extend credit. As such, we see these papers as complements to our own.

The remainder of this paper is organized as follows. Section 2 describes our baseline search economy. Section 3 formalizes the directed search equilibrium. Section 4 presents an alternative environment with a frictionless housing market, rather than one featuring directed search. Section 5 describes our baseline calibration and Section 6 characterizes the balanced growth path of our search economy parameterized in this way. Section 7 considers dynamics in response to aggregate shocks. Section 8 concludes.

2 The Environment

Time is infinite and discrete, with time periods indexed by t. The economy consists of the *city*, and the "rest of the world". Together they are populated by a measure $Q(t)$ of *ex ante* identical households, which grows exogenously at net rate μ. Each household lives indefinitely and supplies one unit of labor inelastically every period. In period t, this unit of labor earns income $y(t)$, in units of a single date t consumption good. The level of income follows a stationary stochastic process in logarithms.

Households in the city require housing, and may either rent or own *one* of a large number of symmetric housing units. Households' preferences are represented by

$$\mathcal{U} = \mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t \left[u(c(t)) + z(t)\right]\right], \tag{1}$$

where $c(t)$ denotes consumption and $z(t)$ housing in period t, respectively. We assume that $z(t) = z_H$ if the household owns the house in which they live and $z(t) = 0$ otherwise. The function $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable, with the boundary properties: $\lim_{c\to\infty} u'(c) = 0$ and $\lim_{c\to 0} u'(c)$ sufficiently large. All households have the common discount factor, $\beta \in (0, 1)$. Both consumption goods and housing services are non-storable and there is no technology for households to save across periods.[7]

The process by which households enter the city follows HLS14. At the beginning of each period, measure $\mu Q(t)$ of new households arrive in the economy. Each of these households has a best alternative value to entering the city, denoted ε. These values are independently and identically distributed across new households via the distribution function $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$. Households that enter the city are separated randomly and permanently into two groups; those that value home-ownership and those that do not. The former we refer to as *buyers* and the latter as *perpetual renters.* Each period there exists a critical alternative value, $\varepsilon_c(t)$, below which a new household strictly prefers to enter the city:

$$\varepsilon_c(t) = \psi V_b(t) + (1-\psi) V_p(t), \tag{2}$$

where $V_b(t)$ and $V_p(t)$ are lifetime (from time t) values of being a buyer and a perpetual renter, respectively, and $1-\psi$ is the probability of the entrant becoming a perpetual renter.

New houses are built by a construction industry comprised of a large number of identical, competitive developers. Each new house requires one unit of land, which can be purchased in a competitive market at price $q(t) = \mathcal{Q}(N(t))$. The developer also incurs construction cost $k(t) = \mathcal{K}(N(t))$, where $N(t)$ denotes the total quantity of new houses built in period t. Houses require one period to build; those constructed in period t become available for sale at the beginning of period $t+1$.

Home-ownership results immediately in both a utility benefit to the owner-occupier and a per-period maintenance cost. Houses depreciate over time, regardless of whether or not they are occupied. Depreciation is, however, offset by the owner at maintenance cost d each period.

Households in the city that do not own houses rent. We abstract from most aspects of the rental market, and assume that rent is equal to a fixed fraction of the city-level income; *i.e.* $R(t) = \varsigma y(t)$. The supply of rental accommodation is totally elastic, and is not considered part of the city's housing stock.

At the end of each period, all households in the city, regardless of their ownership status, may experience a shock which induces them to leave the city permanently. For perpetual

[7] These assumptions render all buyers in the housing market identical.

renters and buyers (regardless of whether or not they currently own a house) these shocks occur with probabilities $\pi_p \in (0,1)$ and $\pi_h \in (0,1)$, respectively. All households which exit the city receive continuation utility, $\overline{V}$. Exiting homeowners also have vacant houses that they may want to sell, depending in part on their outstanding mortgage debt, if any. These households also have the option of defaulting.

In the city, the housing market is characterized by directed search. We imagine there being a large variety of potential *sub-markets* indexed by a price, p, and a pair of matching probabilities; one each for both buyers and sellers. Within each sub-market, matching takes place via a matching function, $\mathcal{M}(B, S)$, which is increasing in both arguments and has constant returns to scale. Given this form we can index sub-markets by (θ, p), where θ denotes market tightness (*i.e.,* the ratio of the measures of buyers, B, and sellers, S, present in the sub-market) and p the posted transaction price.

Both buyers and sellers take (θ, p) for all sub-markets as given and decide which to enter. The matching probabilities for buyers $\gamma(\theta)$ and sellers $\rho(\theta)$ are given by

$$\gamma(\theta) = \frac{\mathcal{M}(B,S)}{B} = \mathcal{M}\left(1, \frac{1}{\theta}\right) \tag{3}$$

$$\rho(\theta) = \frac{\mathcal{M}(B,S)}{S} = \mathcal{M}(\theta, 1) = \theta\gamma(\theta). \tag{4}$$

Each buyer and seller can enter only a single sub-market in a given period, and there is no cost of entry. Free-entry generates endogenously a trade-off between the house price and the matching probability across active sub-markets. Intuitively, higher-price sub-markets have lower levels of tightness as buyers (who are all identical) are willing to pay a higher price only if they are compensated with higher probability of matching with a seller.

The stock of searching buyers includes both newly entered households and those which have been searching unsuccessfully for some time. As noted above, these households are identical. Sellers, however, are of a number of different types. First, developers sell newly built homes. Second, homeowners who receive exit shocks as described above may decide to sell. Note that these buyers are heterogeneous to the extent that they have different outstanding mortgages. Homeowners may also sell as a result of a financial distress shock (described below), and again they are differentiated by their outstanding mortgage. Finally, mortgagees sell foreclosed houses (see below).

In our calibration, prices typically exceed per period income and as there is no saving, households must borrow to finance house purchases.[8] Mortgages are provided by a large number of perfectly competitive mortgagees owned by risk-neutral investors who consume all profits and losses *ex post*.[9] To finance their loans, mortgagees trade one-period risk-free bonds at an exogenous interest rate, i, in an external bond market. They also incur a proportional service cost, ϕ, per period associated with the administration of mortgages.

The debt contract is a fixed-rate mortgage with finite maturity T. Let m^t and r^t represent the size and interest rate on a mortgage loan issued in period t, respectively. Contract, (m^t, r^t, T), specifies a constant payment per period:

$$x(m^t, r^t) = \frac{r^t}{1 - (1 + r^t)^{-T}}\ m^t. \tag{5}$$

As the homeowner makes payments, the principle balance on a period t mortgage, m^t_n, evolves via

$$m^t_{n+1} = \left(1 + r^t\right) m^t_n - x\left(m^t, r^t\right) \tag{6}$$

where $n \in \{0, T-1\}$ and $m^t_0 = m^t$. Since T is fixed exogenously and r^t is constant over the life of the contract, $x(\cdot)$ is unrelated to t after origination, and m^t_n, for $n \in \{0, T-1\}$, represents the mortgage balance at the beginning of the period in which the $n+1$st payment will take place.[10]

A borrower can terminate his/her mortgage contract at any time by paying off the remaining balance. Mortgages, however, are issued only on new home purchases.[11] A mortgage termination is a default if the borrower does not repay all of the outstanding balance. Default leads to *foreclosure*, whereby the mortgagee takes control of the house, remitting to the borrower any surplus value of the house in excess of the outstanding loan balance. Mortgages are non-recourse, in that lenders do not have direct access to homeowners' current and/or future income in the event of a default.

Homeowners with outstanding mortgage debt receive, with probability π_d each period, a financial distress shock. We interpret these shocks as representing circumstances such as

[8] In the absence of saving, households would prefer to borrow to smooth consumption even if the house price were less than period income.

[9] Alternatively, these firms could be owned by households to whom they would transfer their *ex post* profits and losses lump-sum. This formulation would, however, complicate the computation without changing our results significantly.

[10] That is, at the beginning of period t, m^t_n represents the remaining balance on a mortgage issued in period $t - n - 1$, for $n = 0, \ldots, T - 1$.

[11] In the absence of saving, a limitation on refinancing is required to generate a distribution of outstanding mortgage debts on the balanced growth path. In the absence of such a restriction homeowners would have incentive to refinance every period in order to smooth consumption.

accidents or unexpected illnesses that render the household unable to continue mortgage payments. Recipients of such shocks are referred to as *distressed owners*. They must terminate their current mortgage contract at the end of the current period and either pay their outstanding debt or default.

In the event of default, a borrower's mortgage balance is set to zero and a foreclosure flag is placed on her credit record. The mortgage company repossesses the house, puts it in real-estate-owned (REO) inventory, and decides whether and how to sell it starting the following period. As noted above, the defaulting homeowner receives the difference between the value of a house in REO inventory and the outstanding mortgage balance, if positive. Upon a successful sale, the mortgage company loses a fraction $\chi \in (0,1)$ of the revenue to cover an exogenous cost, which we think of as representing, for example, legal fees. As a penalty for defaulting, buyers with foreclosure flags lose access to the mortgage market and are thus effective excluded from home ownership. Beginning with the following period, the foreclosure flag remains on a buyer's record with probability $\pi_f \in (0,1)$.[12]

In equilibrium, the mortgage rate is given by $r^t = i+\phi+\varrho$, where i and ϕ are exogenously given as described above. The component ϱ represents a risk premium, which compensates for the risk of default.[13]

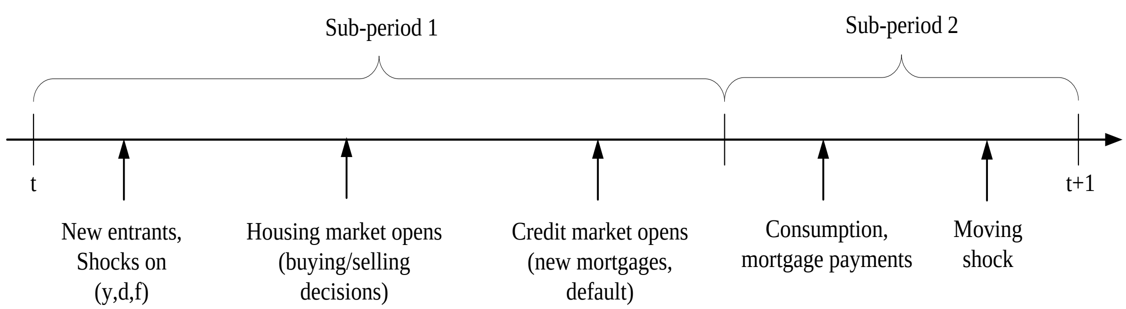

Figure 2: Time Line

Each period consists of two sub-periods. At the beginning of sub-period 1, new households with $\varepsilon \leq \varepsilon_c(t)$ enter the city, income and , financial distress shocks are realized, and the foreclosure flags are randomly lifted. Immediately thereafter, the housing mar-

[12] According to the policies of Fannie Mae and Freddie Mac, foreclosure filings stay on a borrower's credit record for a finite number of years.

[13] Without ϱ, a mortgage contract earns strictly negative expected profit due to the positive probability of default.

ket opens: Buyers and sellers choose sub-markets, (p, θ), in which to search and list their houses for sale, respectively. After matching takes place, new owners take out mortgages to finance their purchases and current mortgage holders decide whether or not to default.

In sub-period 2, households receive income, make payments (for maintenance, new house purchases, mortgages and/or rents), and consume the remainder. At the end of the period, moving shocks are revealed for all households and those who receive them leave the city immediately. Figure 2 provides an illustration of the timing of decisions.

3 Equilibrium

We begin by describing in detail the behavior of agents and then define an equilibrium for the environment described above, which we refer to as our *baseline search* economy.

3.1 Households

Consider households' value functions sequentially for the two sub-periods of a typical time period t. For each period, throughout $V(t)$'s will be used to denote agent and house values at the beginning of the period; while $W(t)$'s will be used to denote agents' values at the beginning of the second sub-period. Sub-scripts will be used to distinguish agent and house states. In general, household values at the beginning of the second sub-period also depend on intra-period asset holdings, a. For agents who are home-owners, their values, $V_o(m_n, t)$ and $W_o(m_n, t)$, depend also on their outstanding mortgage balance. Finally, as house purchases take place in the first sub-period, new homeowners values depend also on the price at which they purchased, $W_o(p, m_0, t)$. From this point we will suppress the dependence of values on time where possible. In so doing, primes (*e.g.,* "V_o'") will be used to denote future values.

3.1.1 The first sub-period

House trading and mortgage default decisions are both made in the first sub-period. Let V_b denote the value function for a buyer. These households are either new entrants or those not owning a house and without a foreclosure flag, *i.e.* for whom $f = 0$:

$$V_b = \max_{(p,\theta)} \left[\gamma(\theta) W_o(p, m_0) + (1 - \gamma(\theta)) W_b(0) \right]. \tag{7}$$

In sub-period 1, a buyer will search for a house to buy, choosing optimally to enter sub-market (p, θ). The buyer is matched with a seller with probability $\gamma(\theta)$, in which case she proceeds to sub-period 2 as a new owner with value $W_o(p, m_0)$. The price paid and initial loan balance m_0 (offered by the mortgage company and specified below) determine the homeowner's down-payment.[14] With probability $1 - \gamma(\theta)$, the buyer fails to get a match, remains a buyer, and proceeds to sub-period 2 with value $W_b(0)$. As noted above, the argument of $W_b(\cdot)$ indicates the intra-period asset balance, a. This will be non-zero *only* in the event that the buyer has sold a house in the sub-period that has just ended. A buyer who enters the current period without a house, necessarily enters the second sub-period with $a = 0$.

With buyers free to choose among them, all active sub-markets must offer the same value, V_b. Using (7) yields

$$\theta = \gamma^{-1}\left(\frac{V_b - W_b(0)}{W_b(p, m_0) - W_b(0)}\right) \equiv \theta\left(p\right). \tag{8}$$

Thus, free-entry of buyers determines the relationship between the transaction price and market tightness across sub-markets.

Let $V_o\left(m_n\right)$ denote the value for a resident owner with mortgage m^{t-n-1} at origination, who has made $n - 1$ payments and is *not* in financial distress:

$$V_o\left(m_n\right) = \max_{p^s}\left\{\begin{array}{l} \rho(\theta(p_s))W_b(\max[0, p_s - m_n]) + \\ \left[1 - \rho(\theta(p_s))\right] \max\limits_{D_n \in \{0,1\}} \left\{\begin{array}{l} (1 - D_n)W_o(m_n) \\ +D_n W_f(\max[0, \beta E\left[V'_{REO}\right] - m_n]) \end{array}\right\} \end{array}\right\}. \tag{9}$$

This homeowner decides whether and in which sub-market to sell her house. This can be represented by choice of asking price alone, given (8). If she enters sub-market p_s, then with probability, $\rho(\theta(p_s))$, her house is successfully sold. In this case, she repays as much outstanding debt as possible, and keeps the remaining profit, if any, to that $a = \max[0, p_s - m_n]$. She then proceeds to the second sub-period as a buyer without the foreclosure flag and value $W_b(a)$. Note that the constraint $p_s \geq m_n$ is not imposed on on sellers. That is, the choice of selling price is not constrained to meet or exceed the outstanding debt. *Short sales* are an option in the sense that mortgage lenders allow indebted homeowners to clear their debt (without the consequence of foreclosure) with an amount lower than the outstanding balance, as long as the homeowner makes the effort to

[14] We measure loan-to-value (LTV) ratios by the mortgage balance relative to the value of a house in REO inventory. Thus, while home buyers make different down-payments depending on the sub-market in which they purchase, at each point in time the LTV at origination is the same for all mortgages.

list the house and successfully sells at the listed price.[15]

If the homeowner chooses not to sell her house, or has failed to sell it, she then decides whether or not to default on her current mortgage contract. Here, for $n = 0, \ldots, T-1$, $D_n = 1$ if a homeowner who has made $n-1$ payments on a mortgage (issued in period $t-n-1$) chooses to default rather than making the nth payment; and $D_n = 0$ otherwise. The value of a homeowner who has not defaulted at the beginning of the second sub-period is $W_o(m_n)$. A homeowner who has defaulted has value $W_f(a)$, where $a = \max[0, \beta E\left[V'_{REO}\right] - m_n]$, at the beginning of the second sub-period. Such a homeowner effectively "sells" her house to the mortgagee for the expected discounted value of a vacant house in REO inventory at the beginning of the next period, $\beta E\left[V'_{REO}\right]$. If this value is less than the homeowner's outstanding mortgage debt, m_n, her assets are set to zero. The expectation here is taken with respect to aggregate shocks which affect the value of vacant houses. If the value of the vacant house exceeds the debt, the defaulting homeowner keeps the residual value. In either case, she acquires a foreclosure flag.

Next, consider a resident homeowner who receives a financial distress shock at the beginning of period t.[16] This homeowner must terminate her mortgage contract within the same period. If her house is sold, the homeowner receives the residual value net of debt and then becomes a buyer without a foreclosure flag, $W_b(\max[0, p_{sd} - m_n])$. If the house is not sold, the owner defaults, the foreclosure flag is placed on her credit record. In this case, the homeowner receives the residual value of the house net of the debt and enters the

[15] For quantitative exercises, we have computed the seller decision problems *both* with *and* without the constraint $p^s \geq m_n$. The constraint does not make a difference quantitatively. In particular, we find this constraint non-binding for all types of indebted sellers in both the steady state and the dynamics given the parameters and income shocks we adopt in the paper. Theoretically, it is straightforward to see why an indebted seller may find it optimal to list at a price $p^s > m_n$, even when short sale is permitted. In the seller's problem (9), for any choice of $p^s \leq m_n$ the problem amounts to choosing p^s such that $\rho(\theta(p^s))$ is maximized. Since $\rho(\theta(p^s))$ is decreasing in p^s, this is equivalent to minimizing the choice of price. Say, p^s is chosen such that the resulting selling probability is one, ignoring whether such price exists or not. For any $p^s \leq m_n$, the maximized value of the right-hand side of (9) can be no greater than that of the value given $\rho = 1$. Given that W_b increases with p^s, the seller will optimally choose to list the house at a price higher than the outstanding debt as long as there exists some $\tilde{p}^s > m_n$ such that the maximized value is greater than the seller's.

We allow for short sales of this type to illustrate that the positive relationship between LTV and asking prices does not depend on short sales being ruled out.

[16] In the event of financial distress, it is always in an owner's best interest to attempt to sell if they have positive equity. If the housing market is liquid, distressed owners with positive equity would never default because they could sell immediately pay their mortgage debt. According to the RealtyTrac report, however, less than 50% of homeowners who go into foreclosure have negative equity. In our model, time-consuming search and matching account for this feature of the housing market.

next sub-period with value $W_f(\max[0, \beta E[V'_{REO}] - m_n])$.[17] Thus the value of a distressed resident owner with debt m_n is given by:

$$V_f(m_n) = \max_{p^{sd}} \left\{ \begin{array}{l} \rho(\theta(p_{sd}) W_b(\max[0, p_{sd} - m_n]) \\ + [1 - \rho(\theta(p_{sd}))] W_f(\max[0, \beta E[V'_{REO}] - m_n]) \end{array} \right\}. \tag{10}$$

A resident homeowner without a mortgage decides whether and how to sell her house. If the house is successfully sold, the owner moves on as a buyer with value $W_b(p_{nd})$. Otherwise, she moves onto the next sub-period as an owner without debt W_{nd}. Such a homeowner has value

$$V_{nd} = \max_{p_{nd}} \{\rho(\theta(p_{nd}))W_b(p_{nd}) + [1 - \rho(\theta(p_{nd}))] W_{nd}\}. \tag{11}$$

Next, consider homeowners who have left the city. Such households become irrelevant once they are no longer homeowners. As long as they are, however, they still make sales and default decisions. Such a homeowner has value:

$$V_L(m_n) = \max_{p_L} \left\{ \begin{array}{l} \rho(\theta(p_L)) \left\{u\left(\max_{p_L}[0, p_L - m_n] + y_L - R_L\right) + \beta\overline{V}\right\} + \\ [1 - \rho[\theta(p_L)]] \times \\ \max\limits_{D_n^L} \left\{ \begin{array}{l} (1 - D_{Ln}) \left\{ \begin{array}{l} u(y_L - R_L - x_n - d) \\ +\beta E[V'_L(m_{n+1})] \end{array} \right\} \\ +D_{Ln} \left\{ u \left(\begin{array}{l} \max[0, \beta[EV'_{REO}] - m_n] \\ +y_L - R_L \end{array} \right) + \beta\overline{V} \right\} \end{array} \right\} \end{array} \right\}. \tag{12}$$

Here, y_L, R_L, and d are income, rent and maintenance costs paid by the exiting household while living outside the city.[18] Also, $x_n = x(m^t, r^t)$ denotes the household's nth payment on her mortgage issued $n+1$ periods prior. Once the homeowner has either sold her house or defaulted, she receives exogenous continuation value, $\overline{V}$.

The value of an owner who has left the city *without* debt prior to moving is given by:

$$V_{Lw} = \max_{p_{Lw}} \left\{ \begin{array}{l} \rho(\theta(p_{Lw})) \left\{u(p_{Lw} + y_L - R_L) + \beta\overline{V}\right\} \\ + \ [1 - \rho[\theta(p_{Lw})]] \left\{u(y_L - R_L - d) + \beta E[V'_{Lw}]\right\} \end{array} \right\}. \tag{13}$$

Such a household's only decision is with regard to whether and at what price to sell.

[17] Note that distressed resident owners can use proceeds from sales, but not labor income, to pay off outstanding mortgage debt. Relaxing this constraint would complicate the model without affecting the results significantly.

[18] These quantities are necesssary as long as the household remains a homeowner, because they impinge on its default and pricing decisions.

3.1.2 Vacant Houses

At the beginning of the current period, the values of vacant houses in developers' inventories, V_c, and of foreclosed houses, V_{REO} are given, respectively, by

$$V_c = \max_{p_c} \{\rho(\theta(p_c)) p_c + [1 - \rho(\theta(p_c))] [-d + \beta E [V_c']]\} \tag{14}$$

$$V_{REO} = \max_{p_{REO}} \{\rho(\theta(p_{REO})) (1 - \chi) p_{REO} + [1 - \rho(\theta(p_{REO}))][-d + \beta E [V_{REO}']]\} . \tag{15}$$

Note that in (15), it can be seen that the mortgage company loses fraction χ of the proceeds of its sales as a cost of foreclosure.

3.1.3 Households in sub-period 2

As there is no saving, households' behavior in sub-period 2 effectively is trivial: They consume their income net of rent and mortgage payments, as well as whatever assets with which they enter the sub-period. Here, we establish the value functions for the various household states at the beginning of this sub-period, which were used in the expressions above.

A perpetual renter remains a renter (never seeking to purchase a house) the entire time she stays in the city. Such a household's value is given by:

$$V_p = W_p = u(y - R) + \pi_p \beta \overline{V} + (1 - \pi_p) \beta E [W_p'] . \tag{16}$$

With probability π_p, the perpetual renter receives a moving shock, leaves the city immediately and receives the continuation value $\beta \overline{V}$. Otherwise, she moves onto the next period again as a renter. She consumes her income net of rent.

A buyer with the foreclosure flag on her credit record has access neither to credit nor the housing market. She will remain renting until she moves out of the city or her foreclosure flag is lifted. As above, $W_f(a)$ is the value of such a buyer with asset a at the beginning of sub-period 2. Thus,

$$W_f(a) = u(y + a - R) + \pi_h \beta \overline{V} + (1 - \pi_h) \beta \{\pi_f \ E [W_f'(0)] + (1 - \pi_f) E [V_b']\} \tag{17}$$

Conditional on staying in the city, with probability π_f the foreclosure flag remains and the household moves onto the following period with expected value $W_f'(0)$ (such households are inactive in the first sub-period of next period, as the foreclosure flag prevents them from purchasing a house). With probability $1-\pi_f$, the foreclosure flag is lifted and this household will enter the next period as a buyer with value V_b'.

A buyer without a foreclosure flag at the beginning of sub-period 2 is either a resident owner who just successfully sold her house or a buyer who has failed to purchase a house in sub-period 1. Such a buyer may have a positive intra-period asset balance, a, coming from net sales proceeds in the previous sub-period. She will move on with value V_b' and participate in the housing market in the next period if not hit by the moving shock at the end of the current period. The value of such a buyer is given by

$$W_b(a) = u(y + a - R) + \pi_h\beta\overline{V} + (1-\pi_h)\beta E\left[V_b'\right]. \tag{18}$$

A resident homeowner with a mortgage has the principle balance m_n. The owner's periodic income is used to cover repayment, maintenance cost and consumption. Let $W_o(m_n)$ denote the value of such an owner. It follows that for $n \in [0, T-2]$,

$$\begin{aligned} W_o(m_n) \quad &= \quad u(y - x_n - d) + z_H + \pi_h\beta E\left[V_L'(m_{n+1})\right] + \\ &\quad\quad (1-\pi_h)\left\{\pi_d\beta E\left[V_f'(m_{n+1})\right] + (1-\pi_d)\beta E\left[V_o'(m_{n+1})\right]\right\}. \end{aligned} \tag{19}$$

If the owner receives a moving shock, she exits the city immediately and continues with value $V_L'(m_{n+1})$. Note that her mortgage debt does not vanish because she has relocated. Conditional on not relocating, in the next period the owner receives a financial distress shock with probability π_d. In this case, she continues as a distressed resident owner with debt $V_f'(m_{n+1})$. Otherwise, she enters the next period as a non-distressed owner with value $V_o'(m_{n+1})$.

For $n = T-1$, a resident homeowner with a mortgage has value

$$W_o(m_{T-1}) = u(y - x_{T-1} - d) + z_H + \pi_h\beta V_{Lw}' + (1-\pi_h)\beta E\left[V_{nd}'\right]. \tag{20}$$

In this case, the current mortgage payment is the homeowner's last. Thus, she will continue on with value V_{Lw}' if hit by the moving shock (in which case she leaves the city owning a house but having no debt) and with value V_{nd}' if she remains in the city.

A new owner who has purchased a house in the preceding sub-period pays the difference between the purchase price and total debt m_0; that is, she makes a down-payment. Her periodic mortgage payments begin the following period. Let $W_o(p, m_0)$ denote the value of a new homeowner:

$$\begin{aligned} W_o(p, m_0) &= u(y-(p-m_0)-d) + z_H + \pi_h \beta V_L'(m_0) + \\ &\quad (1-\pi_h)\left\{\pi_d \beta E\left[V_f'(m_0)\right] + (1-\pi_d)\beta E\left[V_o'(m_0)\right]\right\}. \end{aligned} \tag{21}$$

Finally, homeowners without mortgage debt do not suffer financial distress shocks. They remain in the city until they leave randomly. The value of such a homeowner is given by

$$W_{nd} = u(y-d) + z_H + \pi_h \beta V_{Lw}' + (1-\pi_h)\beta E\left[V_{nd}'\right]. \tag{22}$$

3.2 Developers

As noted above, the construction industry is comprised of a large number of competitive firms. Free entry ensures that in equilibrium the cost of building a house equals the expected value of a vacant house for sale in period $t+1$:

$$\mathcal{Q}(N) + \mathcal{K}(N) = \beta E\left[V_c'\right]. \tag{23}$$

3.3 Mortgagees

Because a mortgagee has access to funds at a fixed cost, it issues mortgages until it earns zero profit on each contract. In particular, the expected return net of expected foreclosure costs on mortgages will equal the opportunity cost of funds, that is, the interest rate i of the external bonds plus the servicing cost ϕ. Houses are identical, households cannot save over time, and regular repayments of all new mortgages start from the period following that in which the house is purchased and the mortgage originated. As such, all new borrowers are identical to the mortgage company at the point of loan origination. Therefore, mortgagees loan the same amount, m^t, to all new borrowers in period t, regardless of the price they pay for their house.

Let $P(m_n)$, for $n \in \{0, \cdots, T-1\}$, be the present value, at the beginning of the current sub-period 2, of a mortgage issued in $n+1$ periods before and held by a resident homeowner. That is, the value of a mortgage of original size m^{t-n-1} after $n-1$ payments have been made. Correspondingly, let $P_L(m_n)$, be the present value of such a mortgage held by an owner that has relocated.[19] Then, for $n \in \{0, 1, \cdots, T-1\}$,

$$
\begin{array}{l}
P(m_n) = x_n \mathbb{I}_{\{n \neq 0\}} + \dfrac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \times \\
E \left[\begin{array}{ll}
\pi_h & \left[\begin{array}{l}
\rho(\theta(p'_L)) \min \left[p'_L, m'_{n+1} \right] + \\
\left[1 - \rho(\theta(p'_L))\right] \left\{ \begin{array}{l}
D'_{Ln+1} \min \left[\beta V''_{REO}, m'_{n+1} \right] \\
+ \ (1 - D'_{Ln+1}) P'_L(m'_{n+1})]
\end{array} \right\}
\end{array} \right] + \\
(1-\pi_h) & \left[\begin{array}{ll}
\pi_d \left\{ \begin{array}{l}
\rho(\theta(p'_{sd})) \min \left[p'_{sd}, m'_{n+1} \right] \\
+ \ \left[1 - \rho(\theta(p'_{sd}))\right] \min \left[\beta V''_{REO}, m'_{n+1} \right]
\end{array} \right\} + & \\
(1-\pi_d) \left\{ \begin{array}{l}
\rho(\theta(p'_s)) \min \left[p'_s, m'_{n+1} \right] \\
+ \left[1 - \rho(\theta(p'_s))\right] \\
\times \left\{ \begin{array}{l}
D'_{n+1} \min \left[\beta V''_{REO}, m'_{n+1} \right] \\
+ \ (1 - D'_{n+1}) P'(m'_{n+1})
\end{array} \right\}
\end{array} \right\} &
\end{array} \right]
\end{array} \right]
\end{array}
\tag{24}
$$

and for all $n \in \{1, \cdots, T-1\}$,

$$
P_L(m_n) = x_n + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} E \left[\begin{array}{l}
\rho(\theta(p'_L)) \min \left[p'_L, m_{n+1} \right] + \\
\left[1 - \rho(\theta(p'_L))\right] \left\{ \begin{array}{l}
D'_{Ln+1} \min \left[\beta V''_{REO}, m_{n+1} \right] \\
+(1 - D'_{Ln+1}) P'_L(m_{n+1})
\end{array} \right\}
\end{array} \right] \tag{25}
$$

where $p'_s,\ p'_{sd},\ p'_L,\ D'_{n+1},\ D'_{Ln+1}$ are household policies (sales prices and default decisions) in period $t+1$ contingent on having mortgage balance m_{n+1}. Also, $I_{\{n \neq 0\}}$ and $I_{\{n \neq T-1\}}$ are index functions, with

$$
\mathbb{I}_{\{n \neq 0\}} = \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{otherwise} \end{cases} \tag{26}
$$

$$
\mathbb{I}_{\{n \neq T-1\}} = \begin{cases} 0, & \text{if } n = T-1 \\ 1, & \text{otherwise.} \end{cases} \tag{27}
$$

Here, 26 indicates a mortgage on which the borrower is making regular repayments beginning with the period after origination, while (27) indicates a mortgage that matures after the current repayment is made. The present value, $P(m_n)$, equals the current period payment x_n on a mortgage originated $n+1$ periods ago, plus the discounted expected value of the mortgage in the next period. The latter is affected by the probabilities of the borrower receiving a moving and/or financial distress shock, and the decisions she will

[19] For such owners, we have $n \geq 1$ as one repayment has already been made by the beginning of the first sub-period 2 following the household's relocation.

make regarding pricing and/or default in the event that such shocks are realized. Note again that these decisions do *not* depend on the price at which she originally purchased.

To compute the present value of a mortgage contract at origination, we proceed recursively. First, we compute $P(m_{T-1})$, and then use backward induction to obtain $P(m_n)$ for $n \in \{0, \dots, T-2\}$. The value $P_L(m_n)$ is determined in a similar way except that once relocated borrowers experience neither moving nor distress shocks.

If a borrower sells her house in period $t+1$, the amount that the mortgage company will receive is the minimum of the sale proceeds and the outstanding debt m_n. The equilibrium mortgage loan size, m^t, is then determined using the mortgage lender's zero-profit condition:

$$P(m^t) - m^t = 0. \tag{28}$$

3.4 Laws of motion

Now consider the evolution of the distributions of households and houses across states. We express all quantities in *per capita* terms (*i.e.* divided by the total economy population, Q_t) At the beginning of period t, the households are divided into renters; perpetual renters, $F(t)$; buyers without a foreclosure flag, $B(t)$; "buyers" with a foreclosure flag, $B_f(t)$, and homeowners. This latter group are either residents in the city or have relocated and still own a house.

Within each type, households are further differentiated by their mortgage balance, if any. For $n = 0, \dots, T-1$, let $H_n(t)$ denote the period t measure of city resident homeowners who were issued a mortgage in period $t-n-1$ and thus have made $n-1$ payments prior to period t. Similarly, $H_{Ln}(t)$ denotes the measure of homeowners who have relocated by period t holding such a mortgage. Let $H_\emptyset(t)$ and $H_{L\emptyset}(t)$, respectively denote resident and re-located homeowners who no longer have an outstanding mortgage. Note that there is no need to keep track of relocated households once they cease to be homeowners (because they have already either sold or defaulted) .

Houses for sale are either held by relocated homeowners, developers, mortgagees, or distressed homeowners. Denote the inventories of construction firms and mortgage lenders by $H_c(t)$ and $H_{REO}(t)$ respectively. Suppressing the time indicator, the total stock of houses for sale in the current period is then given by:

$$H_s = \underbrace{H_{L\emptyset} + \sum_{n=1}^{T-1} H_{Ln} + H_c + H_{REO}}_{\text{vacancies}} + \underbrace{\pi_d \sum_{n=0}^{T-1} H_n}_{\text{distressed sellers' homes}} . \tag{29}$$

Similarly, the total current period measure of buyers searching to trade in the housing market, B_{sum}, can be written:

$$\begin{aligned} B_{sum} &= \sum_{n=0}^{T-1} \left[(1-\pi_d)\theta\left(p_s\right) H_n + \pi_d \theta\left(p_{sd}\right) H_n + \theta\left(p_L\right) H_{Ln} \right] \\ &\quad + \theta(p_{nd}) H_{\emptyset} + \theta(p_{Lnd}) H_{L\emptyset} + \theta(p_c) H_c + \theta(p_{REO}) H_{REO}. \end{aligned} \tag{30}$$

In particular, the measure of buyers in an active sub-market equals the measure of sellers in that sub-market multiplied by the the corresponding market tightness. For example, the measure of buyers searching for foreclosed houses sold by a mortgagee equals the measure of REO houses, H_{REO}, multiplied by the tightness of the mortgagee's optimally chosen sub-market, $\theta(p_{REO})$.

We now write out the laws of motion for the stocks households and houses in the various states. To begin with, the *per capita* measure of permanent renters in the next period consists of those remaining from the current period and those who will have newly entered:

$$(1+\mu)F' = (1-\pi_p)F + (1-\psi)G(\varepsilon_c')\mu. \tag{31}$$

Similarly, the measure of buyers with foreclosure flags next period includes those remaining from the current period who have neither moved nor had their flag removed. To this is added the measure of resident homeowners who default this period. These homeowners may either have received a financial distress shock and failed to sell or have defaulted strategically. Thus, we have

$$(1+\mu)B_f' = (1-\pi_h) \left\{ \begin{array}{l} \pi_f B_f + \\ \pi_d \sum_{n=0}^{T-1} (1-\rho\left(\theta\left(p_{sd}\right)\right)) H_n \} + \\ (1-\pi_d) \sum_{n=0}^{T-1} (1-\rho\left(\theta\left(p_s\right)\right)) D_n H_n \end{array} \right\} \tag{32}$$

where as above, p_{sd}, p_s, and D represent optimal pricing and default decisions. Note that in general these depend on homeowners' outstanding mortgages.

The measure of buyers *without* foreclosure flags at the beginning of next period consists of newly-entering buyers, previously flagged buyers whose flag has been removed, and non-relocating buyers from the current period who fail to buy a house. Note that the measure of buyers who successfully match in the current period equals the sum of the measures of the prospective sellers of various types multiplied by their corresponding matching probabilities. Thus, we have

$$(1+\mu)B' = \psi G(\varepsilon_c')\mu + (1-\pi_f)B_f + (1-\pi_h)\left\{ \begin{array}{l} B - \rho\left(\theta(p_{Lw})\right) H_{L\emptyset} - \\ \rho\left(\theta(p_c)\right) H_c - \rho\left(\theta(p_{REO})\right) H_{REO} - \\ \sum_{n=1}^{T-1} \rho\left(\theta(p_L)\right) H_{Ln} \end{array} \right\}. \tag{33}$$

The measure of indebted owners who have made n periodic payments by the beginning of period $t+1$ on a mortgage of size m^{t-n-1} at origination evolves (for $n > 0$) via:

$$(1+\mu)H_n' = (1-\pi_h)(1-\pi_d)(1-\rho\left(\theta(p_s)\right)(1-D_n)H_{n-1}. \tag{34}$$

That is, the indebted owners with an ongoing mortgage going into the next period are the indebted owners from the current period who do not move, experience financial distress, successfully sell their house, or default strategically.

For $n = 0$, H_0' is the measure of resident homeowners who successfully purchase a house, remain in the city and do not experience financial distress. This measure can be recovered from the number of sales in the current period. Thus we have

$$(1+\mu)H_0' = (1-\pi_h)\left\{ \begin{array}{l} (1-\pi_d)\sum_{n=0}^{T-1} \rho\left(\theta\left(p_s\right)\right) H_n \\ +\pi_d \sum_{n=0}^{T-1} \rho\left(\theta\left(p_{sd}\right)\right) H_n \\ +\sum_{n=1}^{T-1} \rho\left(\theta\left(p_L\right)\right) H_{Ln} \\ +\rho\left(\theta(p_{nd})\right) H_\emptyset + \rho\left(\theta(p_{Lw})\right) H_{L\emptyset} \\ +\rho\left(\theta(p_c)\right) H_c + \rho\left(\theta(p_{REO})\right) H_{TE"O} \end{array} \right\}. \tag{35}$$

Finally, the measure of resident owners without a mortgage evolves via

$$(1+\mu)H_\emptyset' = (1-\pi_h)\left\{ \begin{array}{l} (1-\pi_d)(1-\rho\left(\theta\left(p_d\right)\right))(1-D_{T-1})H_{T-1} \\ +(1-\rho\left(\theta(p_{nd})\right))H_\emptyset \end{array} \right\}. \tag{36}$$

This group is comprised of its previous members who have neither move nor sell plus resident homeowners who make their last mortgage payment in the current period.

Proceeding similarly for relocated homeowners, H_{Ln}' is the measure who made their $(n+1)$st payment in the period t. Again, the loan size at origination is m^{t-n-1} and $H_{L\emptyset}$

is the current period measure of relocated owners without debt:

$$
\begin{aligned}
(1+\mu)H'_{Ln} &= (1-\rho\left(\theta(p_L)\right))(1-D_{Ln-1})H_{Ln-1} \\
&\quad +\pi_h(1-\pi_d)(1-\rho\left(\theta(p_s)\right))(1-D_{n-1})H_{n-1};
\end{aligned} \tag{37}
$$

$$
(1+\mu)H'_{L0} = \pi_h \left\{ \begin{array}{l} (1-\pi_d)\sum_{n=0}^{T-1}\rho(\theta\left(p_s\right))H_n \\ +\pi_d\sum_{n=0}^{T-1}\rho(\theta\left(p_{sd}\right))H_n \\ +\sum_{n=0}^{T-1}\rho(\theta\left(p_L\right))H_{Ln} \\ +\rho(\theta(p_{nd})H_{\emptyset}+\rho(\theta(p_{Lw})H_{L\emptyset} \\ +\rho(\theta(p_c))H_c+\rho(\theta(p_{REO}))H_{REO} \end{array} \right\} \tag{38}
$$

$$
\begin{aligned}
(1+\mu)H'_{L\emptyset} &= \pi_h \left\{ \begin{array}{l} (1-\rho(\theta(p_{nd}))H_{\emptyset} \\ +(1-\pi_d)(1-\rho(\theta(p_s)))(1-D_{T-1})H_{T-1} \end{array} \right\} \\
&\quad +(1-\rho(\theta(p_L)))(1-D_{LT-1})H_{LT-1} \\
&\quad +(1-\rho(\theta(p_{Lnd}))H_{L\emptyset}.
\end{aligned} \tag{39}
$$

As depreciation is offset by maintenance, the *per capita* city housing stock evolves via

$$
(1+\mu)H' = H+N, \tag{40}
$$

where N is the measure houses built in the current period and available for sale in the next.

The *per capita* stock of houses in developers' inventory at the beginning of the next period includes those houses that go unsold in the current period plus those that are newly built:

$$
(1+\mu)H'_c = (1-\rho(\theta(p_c))\,H_c+N. \tag{41}
$$

Finally, the stock of houses in the REO inventory at the beginning of the next period,

H'_{REO}, includes those that go unsold in the current period plus the new foreclosures:

$$\begin{aligned}(1+\mu)H'_{REO} &= (1-\rho(\theta(p_{REO})))\, H_{REO} \\ &+\pi_d \sum_{n=0}^{T-1}(1-\rho(\theta(p_{sd}))H_n \\ &+(1-\pi_d)\sum_{n=0}^{T-1}(1-\rho(\theta(p_s))D_n H_n \\ &+\sum_{n=1}^{T-1}(1-\rho(\theta(p_L)D_{Ln}H_{Ln}. \end{aligned} \tag{42}$$

3.5 A Directed Search Equilibrium

Definition. Given a mortgage interest rate, r; rent level, R; terminal continuation value, $\overline{V}$; and a stochastic process for city-level income, y, a *directed search equilibrium* is, for all periods, a collection of

1. Household value functions,

$$V_b,\ W_b;\ V_o,\ W_o;\ V_f,\ W_f,\ V_{nd},\ W_{nd};\ V_L,\ V_{Lw},\ V_p,\ W_p, \tag{43}$$

   with associated policy functions (choices of sub-market to enter and whether to default):

$$p_s,\ p_{sd},\ p_{nd},\ p_L,\ p_{Lw},\ D_n,\ D_{Ln}, \qquad n=0,\dots,T-1; \tag{44}$$

2. house values:

$$V_c,\ V_{REO} \tag{45}$$

   with associated policies for developers and mortgagees:

$$p_c,\ p_{REO}; \tag{46}$$

3. an entry cut-off and mortgage contract:

$$\varepsilon_c, m_0; \tag{47}$$

4. and *per capita* measures of households and houses

$$\underbrace{F,\ B,\ B_f,\ H_n,\ H_{Ln},\ H_{\emptyset},\ H_{L\emptyset}}_{\text{households}},\ n=0,\dots T-1; \qquad \underbrace{H,\ N,\ H_c,\ H_{REO}}_{\text{houses}}. \tag{48}$$

Such that:

1. New households enter the city optimally so that (2) holds;
2. All agents optimize such that the value and policy functions listed in (43) - (46) satisfy (7), (9) - (22);
3. Free entry of developers: N satisfies (23);
4. Free entry of mortgagees: $m_0 = m^t$ satisfies (28);
5. The stocks of households and inventories of houses evolve according to (31) - (40);
6. "Market clearing": $B = B_{sum}$.

Requirements 1-5 in the above definition are standard and have been described in detail above. Requirement 6 states that in equilibrium the measure of buyers without foreclosure flags must be consistent with the total measure of buyers actively participating in housing search. Alternatively, it means that all eligible buyers enter some sub-market.

As has been mentioned, sellers are heterogeneous, and each period their distribution is characterized by $(H_n, H_{Ln}, H_{\emptyset}, H_{L\emptyset}, H_c, H_{REO})$. The decision problems faced by households, developers and mortgagees are, however, not affected by this distribution. In fact, as can be seen from (7), and (9) - (28), all of the value and policy functions listed in (43) - (46), together with the mortgage contract m_0, are independent of the stocks listed in (48). This is true despite the fact that the stocks themselves depend on individual decisions, and that the distribution of sellers does affect aggregate statistics.

Thus, the model is block recursive in the sense of Shi (2009). As discussed there, block recursivity arises because heterogeneous sellers select themselves optimally into separate sub-markets through the directed search mechanism. In doing so, they take the trade-off between the price and the matching probability as given. Given a particular target transaction price, the only factor that matters for a seller's trading decision is the probability with which she will be matched with a buyer; the distribution of sellers over other price targets is irrelevant. *Vice versa*, for a given matching probability a seller cares only about the price at which she can sell.

Block recursivity greatly aids tractability by eliminating the role of the distribution of sellers in individual decisions. It is especially useful here as it enables us to examine the dynamics of the model in response to aggregate shocks.

4 An Economy without Search

To illustrate the role of search frictions in the economy, we consider also an environment in which the housing market is perfectly competitive. Here, houses are perfectly liquid in that buyers (without a foreclosure flag) and sellers are able to trade immediately and neither developers nor mortgagees hold houses in inventory.

In this setting, financial distress is extreme — at the beginning of period t, with probability π_d an indebted resident owner may experience a default shock which forces her to default immediately. A borrower not hit by such a shock may choose to default only in the case in which their housing equity becomes negative.[20]

4.1 Value functions

Household decisions in sub-period 2 are identical to those in the search economy. Sub-period 1 household values here are distinguished by the superscript w. A buyer without the foreclosure flag purchases a house at competitive price p_t and immediately becomes a homeowner with value $V_o^w(m_0)$. An indebted resident owner who does not receive a default shock decides whether and how to sell and whether or not to default. As before, let $D^w \in \{0,1\}$ be the default indicator. If the owner sells, she repays as much of her outstanding debt as possible, keeps any remaining profit, and becomes a buyer without the foreclosure flag. If she decides not to sell, then she decides whether to default:

$$V_o^w(m_n) = \max \left\{ \begin{array}{c} \underbrace{W_b^w(\max[0, p - m_n)]}_{\text{sell}}, \\ \max_{D^w \in \{0,1\}} \left\{ \begin{array}{c} (1 - D^w) W_b(m_n) + \\ \underbrace{D^w W_f^w(\max[0, \beta E[V_{REO}'^w] - m_n])}_{\text{don't sell}} \end{array} \right\} \end{array} \right\}, \tag{49}$$

[20]Note, however, that as default is costly, not all owners with negative equity will default.

where $V'^w_{REO} = (1-\chi)V'^w_c$ is the value of a vacant house in the next period, net of the foreclosure cost, χ.

An indebted owner who experiences a distress shock immediately defaults. Such an owner has the value:

$$V^w_f(m_n) = W^w_f(\max[0, \beta E[V'^w_{REO}] - m_n). \tag{50}$$

A resident owner without debt decides whether or not to sell and has value:

$$V^w_{nd} = \max\left\{W^w_b(\max[0, p - m_n]), W^w_o(m_n)\right\}. \tag{51}$$

Relocated owners with and without mortgage debt make similar selling and default decisions and have values $V^w_L(m_n)$ and V^w_{Lnd}, respectively:

$$V^w_L(m_n) = \max\left\{\begin{array}{l} \underbrace{u(\max[0, p - m_n] + y^L - R^L) + \beta\overline{V}}_{\text{sell}} + \\ \max_{D^w_t \in \{0,1\}} \left\{\underbrace{\begin{array}{c} ((1-D^w_L)(u(y^L - R^L_t - x_n - d) + \beta E[V'^w_L(m_{n+1})]) + \\ +D^w_L(u(\max[0, \beta E[V'^w_{REO}] - m_n] + y^L - R^L) + \beta\overline{V} \end{array}}_{\text{don't sell}}\right\} \end{array}\right\} \tag{52}$$

$$V^w_{Lnd} = \max\left\{\underbrace{u(p + y^L - R^L) + \beta\overline{V}}_{\text{sell}}, \underbrace{u(y^L - R^L - d) + \beta E[V'^w_{Lnd}]}_{\text{don't sell}}\right\}. \tag{53}$$

The values of vacant houses to both construction firms and mortgage companies are given respectively by:

$$V^w_c = p \tag{54}$$

$$V^w_{REO} = (1-\chi)p. \tag{55}$$

The present values of mortgage contract, $(m^t; r^t)$, at the beginning of sub-period 2 after $n-1$ payments have been made, for relocated and resident homeowners are given, respectively, by:

$$P_L(m_n) = x_n + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \times$$
$$E\left\{ \max \left\{ \begin{array}{l} \min\left[p', m_{n+1}\right], \\ \max_{D'^w_{Ln+1}} \left\{ \begin{array}{l} \left[D'^w_{n+1} \min\left[\beta V''^w_{REO}, m_{n+1}\right]\right. \\ +(1-D'^w_{n+1}) P'_L(m_{n+1}) \end{array} \right\} \end{array} \right\} \right\} \tag{56}$$

for $n \in \{1, T-1\}$, and

$$P^\iota_t(m_n) =$$
$$x_n \mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \tag{57}$$
$$\times E \left\{ \begin{array}{l} \pi_h \max \left\{ \begin{array}{l} \min[p, m_{n+1}], \\ \max_{D^w_{Ln+1}} \left\{ \begin{array}{l} D'^w_{Ln} \min\left[\beta V''^w_{REO}, m_{n+1}\right] \\ +(1-D'^w_{Ln}) P'_L(m_{n+1}) \end{array} \right\} \end{array} \right\} \\ +(1-\pi_h) \left\{ \begin{array}{l} \pi_d \min\left[\beta V''^w_{REO}, m_{n+1}\right] + \\ (1-\pi_d) \left\{ \max \left\{ \begin{array}{l} \min\left[p', m_{n+1}\right], \\ \max_{D^w_{n+1}} \left\{ \begin{array}{l} D'^w_{n+1} \min[\beta V''_{REO}, m_{n+1,} \\ (1-D'^w_{n+1}) P^\iota_{t+1}(m_{n+1}) \end{array} \right\} \end{array} \right\} \right\} \end{array} \right\} \end{array} \right\}$$

for all $n \in \{0, \cdots, T-1\}$, where, D'^w_{n+1} and D'^w_{Ln+1} are household default choices in the next period, conditional on the aggregate shocks and mortgage balance, m_{n+1}.

4.2 Equilibrium

The definition of equilibrium is similar to that for the search economy except that the housing market now clears each period in the Walrasian sense. All households (other than permanent renters) who begin the period without a house are buyers. If the measure of buyers exceeds the sum of the measures of new and foreclosed houses, then the price of housing adjusts until the appropriate measure of current homeowners chooses to sell. A shortage of buyers (and thus $p = 0$) is avoided by the continual entry of buyers without homes driven by population growth. Finally, the *per capita* laws of motion for households are listed in Appendix A.

5 Calibration

We now choose parameters for both the baseline search and non-search economies to match selected characteristics of the U.S. economy, under the assumption that the economy is on a balanced growth path. In this steady-state, the housing stock grows at the rate of population growth and all other components of the equilibrium, including the distribution of agents across states and real house values, remain constant.

To begin with, we specify the following functional forms:

$$\begin{aligned} u(c) &= \ln(c) \\ \mathcal{M}(B,S) &= \varpi B^{\eta} S^{1-\eta} \\ k &= \frac{1}{\kappa} N^{\frac{1}{\varsigma}} \\ q &= \bar{q} N^{\frac{1}{\xi}} \end{aligned} \tag{58}$$

where η is the elasticity of the measure of matches with respect to the measure of buyers, ξ represents the elasticity of new land supply with respect to land prices, and the matching function is relevant only for the baseline search economy.

Table 1 lists parameter values for the baseline search economy. Parameters above the separating line are set to match the corresponding targets directly, while those below are determined jointly to match the targets in the right-most column. A time period is defined as one year.[21] The discount factor β is set to reflect an annual interest rate of 4%. Income in the steady state is normalized to one. Thus, all present values and prices are measured relative to steady-state *per capita* income. The terminal continuation value, $\overline{V}$, is equal to the steady-state value of being a perpetual renter, $\overline{V}_p$. To determine the mortgage rate r, the annual yield on international bonds i is set at 4%. The values of ϕ and ϱ are determined jointly in calibration.

The following parameters and targets are chosen following HLS14: The rate μ is chosen to match the annual population growth during the 1990s. The value of π_p is set to match the annual fraction of renters that move between counties and π_h to match the annual fraction of home owners who move between counties according to the Census Bureau.

[21] Setting a time period as one year is due to specifics of our model. In this setting, households cannot save (so that all buyers are homogeneous) and thus house buyers can only finance the down payment with their periodic labor income. Empirically, the average housing price is about 12.8 times of quarterly income. If one period takes a quarter, then a buyer's periodic income is too small to afford a typical down payment of 15% - 20%. In fact, borrowers could only afford a down payment less than 7.9% of the average housing price if the time period was set as a quarter.

Table 1: Calibration Parameter Values

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_p	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land-price-to-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
ς	0.16	Rent-price ratio	5%
		Parameters determined jointly	
χ	0.440	Loss severity rate	27%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.060	Annual foreclosure rate	1.6%
z_H	0.3280	Average loan-to-income ratio at origination	2.72
ϖ	0.56	Average fraction of delinquent loans repossessed	33.5%
κ	0.137	Average housing price relative to annual income	3.2
η	0.1880	Relative volatility of sales growth	1.32
α_p	6.200	Relative volatility of population growth	0.17

The supply elasticity parameter is set to $\xi = 1.75$ following Saiz (2010). There, the supply elasticity for 95 U.S. cities is estimated for the period between 1970 and 2000. The estimates vary from 0.60 to 5.45 with a population-weighted average of 1.75 (2.5 unweighted). The steady-state unit price of land $\bar{q}$ is set such that the relative share of land in the price of housing is 30% (see Davis and Palumbo (2008) and Saiz (2010)). The elasticity of new construction with respect to the price of housing, ζ, is set equal to the median elasticity for the 45 cities studied by Green et al. (2005), $\zeta = 5$.

The maintenance cost d is chosen to be 2.5% of the steady-state housing price according to Harding et al. (2007). Moreover, the average house price is 3.2 times annual income. The value of ψ is calibrated so that the ownership rate in the city $\overline{H}/(\overline{H}+\overline{B}+\overline{F}) = 66.7\%$, where

$$\overline{H} = \sum_{n=0}^{T-1} \overline{H}_n + \overline{H}_\emptyset$$

denotes the total measure of homeowners in the steady state. The Census Bureau reports the ownership rate among households whose head is between age 35 and 44 is roughly 66.7%.

We set the rent-to-income ratio to $\varsigma = 0.16$ as follows. Based on empirical findings of the Lincoln Institute of Land Policy, the average rent-to-price ratio is around 5% prior to the most recent housing boom leading up to the 2008 financial crisis. Then we compute ς as the product of rent-to-price ratio and the average house price.

The remainder of the parameters listed in Table 1 are determined to match jointly a number of targets based on the model. First, we set the average length of time following a foreclosure until a borrower is again allowed to access the mortgage market to five years. This time frame is consistent with the policies of Fannie Mae and Freddie Mac, which guarantee most U.S. mortgages. Thus we set the probability that a foreclosure flag remains on a borrower's credit record to $\pi_f = 0.8$.

According to the Federal Housing Finance Board, the average contract rate on conventional, fixed-rate mortgages between 1995 and 2004 was 7.2%. We target an average down-payment ratio of 20% and an annual default rate of 1.6%, which is close to the average annual foreclosure rate among all mortgages during the 1990s according to the National Delinquency Survey by the Mortgage Bankers Association.

Foreclosed houses tend to cause losses to lenders due to associated transaction and time cost. The loss severity rate is defined as the present value of all losses on a given loan as a fraction of the balance on the default date. Estimates of the loss severity rates range widely; from as low as 2% during the period 1995-1999 (Pennington-Cross, 2003) to more than 75% during the Great Recession (Andersson and Mayock, 2014). In this paper, we choose parameters so that in the event of a default,

$$\frac{\min\{\beta \overline{V}_{REO}, m_n\}}{m_n} = 0.73 \tag{59}$$

implying an average loss severity rate of 27%.

Phillips and Vanderhoff (2004) find that 30% of defaulted conventional fixed-rate loans and 50% of defaulted conventional adjustable-rate loans transition to REO and Ambrose and Capone (1996, 1998) report that 32% to 38% of defaulted FHA loans transition to foreclosure. Based on these numbers, we choose parameters such that in the event of financial distress, the average probability of a successful sale is 66.5%. That is, 33.5% of the homeowners who experience financial distress ultimately end up in foreclosure in the steady-state.

Evidence available from the American Housing Survey (AHS) suggests that prior to 2003 the ratio of the original loan size to yearly income averages 2.72. Accordingly, we choose parameters such that the steady-state loan-to-income ratio at origination is given by $\overline{m}_0/\overline{y} = 2.72$.

Finally, the economy's dynamics depend crucially on two elasticities: that of entry; $\alpha_p = \varepsilon_c g(\varepsilon_c)/G(\varepsilon_c)$ (where g is the density of G) and that of the matching function with respect to the number of buyers, denoted by η. These two parameters are calibrated jointly using estimates of the relative standard deviations of population growth and housing sales growth in response to income shocks as in HLS14.

For the non-search economy, all parameters remain at their values in Table 1 except for π_d, z_H and ψ, which are adjusted so that the steady-state statistics match the relevant targets again. In the steady-state of the non-search economy, the construction cost parameter is adjusted so that $P^* = 3.2$ given the rate of population growth. Appendix B provides lists of parameter values re-calibrated to the non-search economy.

6 The Balanced Growth Path

We now characterize the steady-state of the baseline search economy. Along this balanced growth path, *per capita* income remains constant over time. The steady-state satisfies the definition of equilibrium established in Section 3.5, plus the requirement that all functions and values listed in (43) - (48) are time invariant.

In the steady-state, all owners have strictly positive home equity[22]. Resident owners who receive neither moving nor financial distress shocks do not attempt to sell their houses regardless of their outstanding mortgage balances. All relocated owners continue to make repayments until a successful sale occurs or their mortgage is completely paid off. Finally, all distressed owners attempt to sell their houses. That is, there are no strategic defaults, in the sense that foreclosure occurs only as the result of financial distress shocks followed by unsuccessful attempts to sell.

Figure 3 presents the distributions of resident homeowners (upper panel) and sellers (lower panel) by mortgage status. In the steady-state, the distribution of homeowners is

[22] Here home equity represents the difference between the average housing price and outstanding mortgage debt

driven solely by exogenous shocks. The measures of owners decrease with the number of payments for $n = 1, \cdots, 29$, owing to the effects of both moving and financial distress shocks which affect homeowners at constant rates over time. The large bin at $n = 30$ represents the stock of homeowners who have re-payed their entire mortgage before experiencing either shock. While these homeowners no longer face a risk of financial distress, they remain subject to moving shocks and exit the city eventually with probability one. Similarly, the measure of distressed sellers decreases with the number of payments fulfilled, for $n = 1, \cdots, 29$, although there are no such sellers with $n = 30$ by construction.

The distribution of relocated sellers, in contrast, is driven by households' choice of selling probability. These households are not required to sell, and they are no longer hit by relocation shocks. The fact that some enter sub-markets with high prices and low sales probabilities accounts for the hump-shape of the distribution. The spike at $n = 30$ arises from the fact that resident homeowners who have paid off their mortgages are still subject to moving shocks, at which point they become relocated sellers without a mortgage.

Figure 4 illustrates the distribution of housing prices in the steady state.

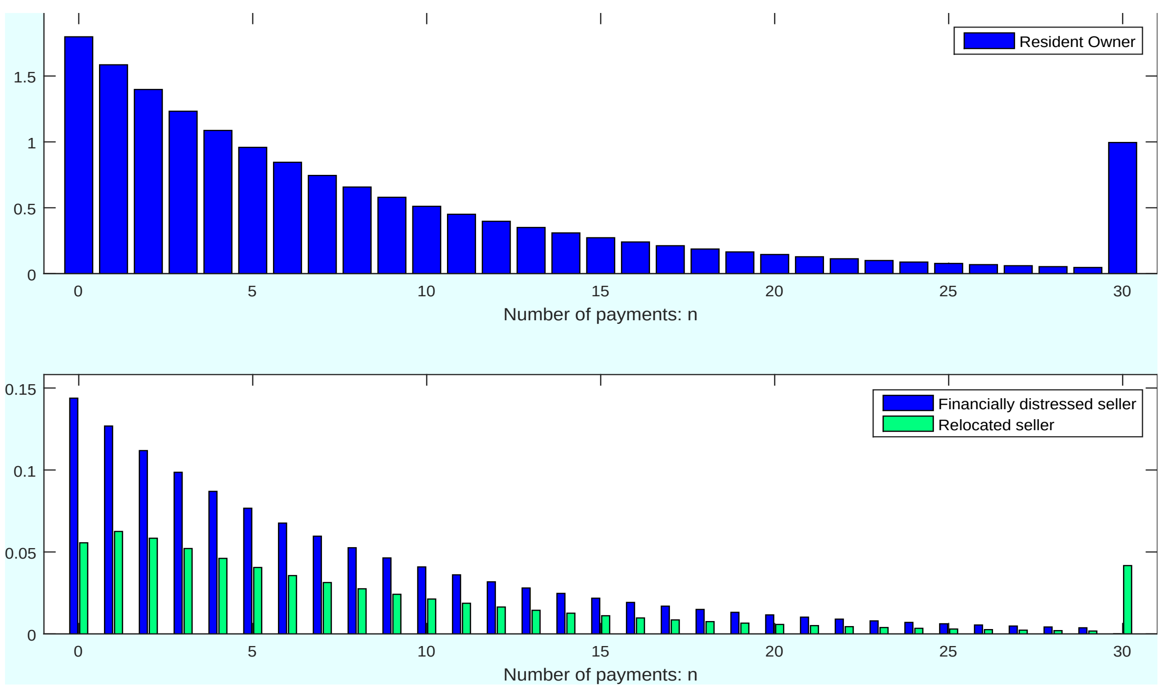

Figure 3: Steady-state distributions of mortgage status respectively among resident owners (upper panel) and household sellers (lower panel)

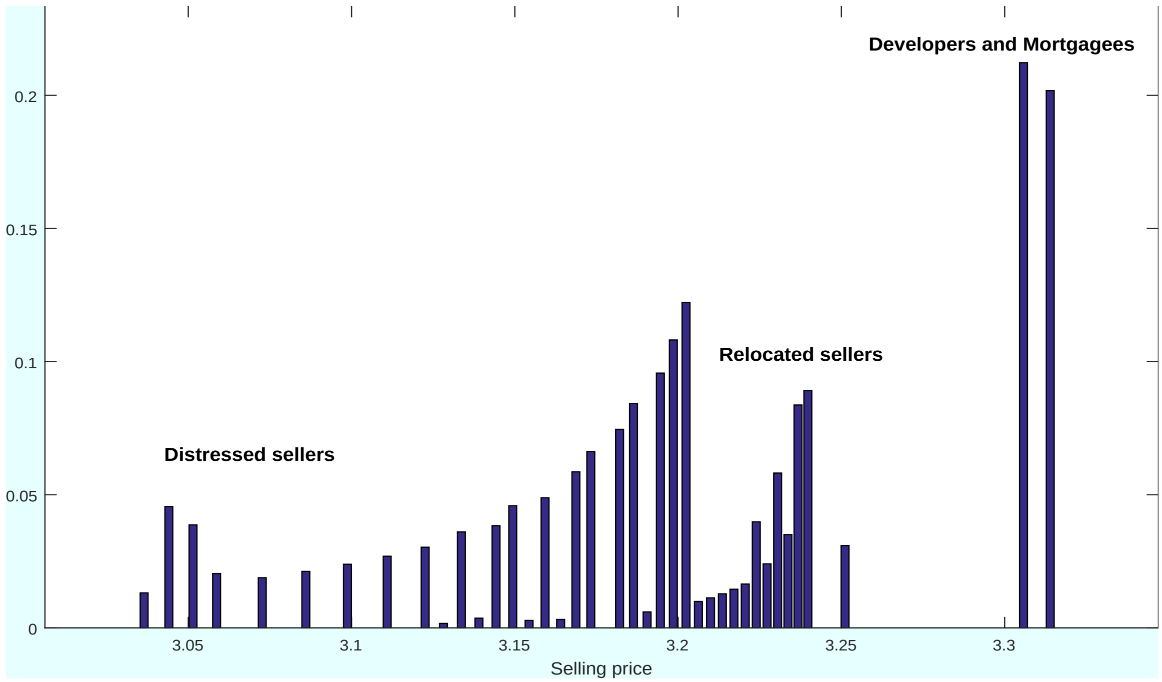

Figure 4: Steady-state distribution of housing prices

6.1 Leverage and seller behavior

Figure 5 illustrates the relationship between a seller's optimal choice of sub-market (which determines both her asking price and sales probability) and her debt position. Overall, a distressed seller is more eager to sell than a relocated seller and therefore, conditional on debt position (represented here by the LTV ratio), posts a lower price and sells with a higher probability. The cost of failing to sell is higher for distressed sellers for two reasons. First, a distressed seller has no choice but default if she fails to sell her house within the period, while a relocated seller does not, and moreover retains the choice of whether to default in the next period. Second, a relocated seller receives continuation value $\overline{V}$, which is independent of her credit record, while a distressed seller who has defaulted on her mortgage and remains in the city is excluded from the housing market until her foreclosure flag is lifted (five periods on average).

All developers and all mortgagees are identical, and so each of these sets of sellers enters a single sub-market. In the calibration, both set higher prices than individual homeowners

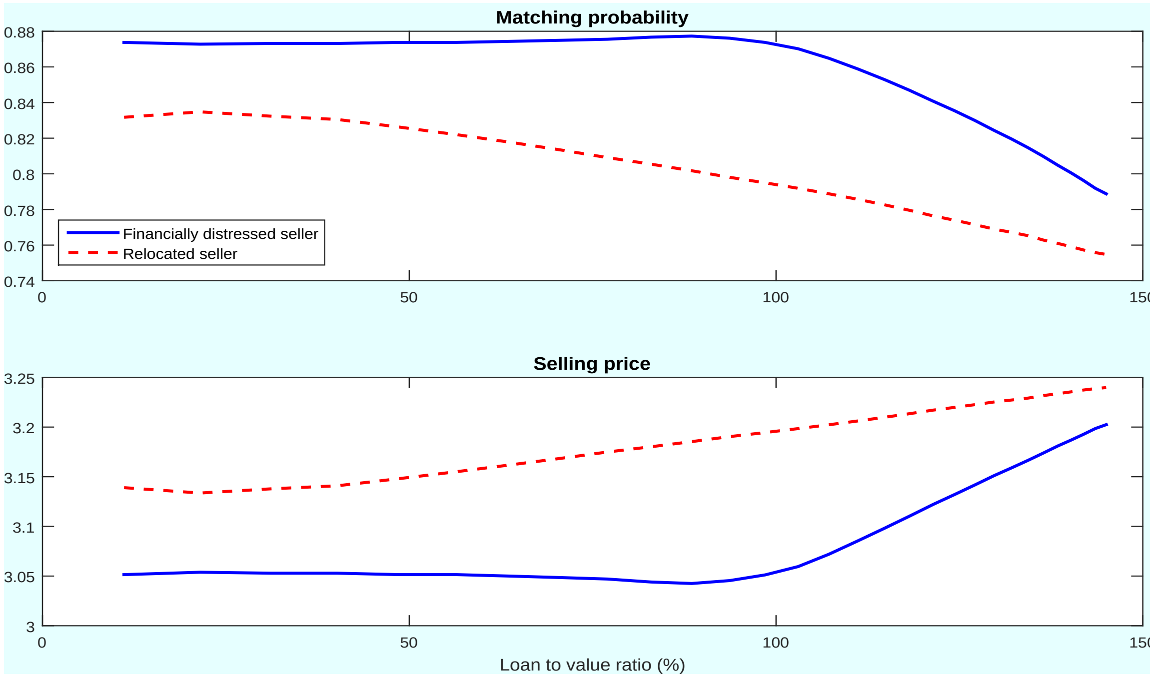

Figure 5: Leverage and seller behavior. The top panel shows the choices of selling probability by distressed and relocated sellers. Correspondingly, the bottom panel demonstrates the choices of selling price by the two types of sellers.

(mortgagees set the highest prices) and sell at commensurately lower rates.[23]

Note also the relationship between the posted asking price and LTV. For both types of seller, the posted price is initially (very) weakly decreasing in LTV. At some point, and this is more dramatic for distressed sellers, the relationship becomes strongly increasing. For distressed sellers in particular, the relationship resembles closely that reported by Genesove and Mayer (1997) (see Figure 2, p. 267). In their empirical study of condominium sales in Boston during the 1990's and is also consistent with the findings of Anenberg (2011). Figure 6 combines their results with ours in common units.[24]

To understand the leverage-price relationship illustrated in Figures 5 and 6, consider

[23] The high price asked by mortgagees for houses in REO inventory and the resulting long average time on the market are counterfactual. This characteristic of the equilibrium arises from simplifications in the modeling of mortgagees; specifically their risk neutrality and the fact that the foreclosure cost is independent of the time-to-sell. It could be addressed in a number of ways, all of which would add complexity to the model and yet have little effect on our main results. Both the selling behaviour of households and the extent to which mortgagees are willing to lend depend mainly on the overall default rate and foreclosure costs. These are both calibration targets and the overall results will not change much as long as their values remain the same.

[24] In our economy, the posted (asking) price is proportional to the mark-up, as all vacant (non-foreclosure) houses have a common value.

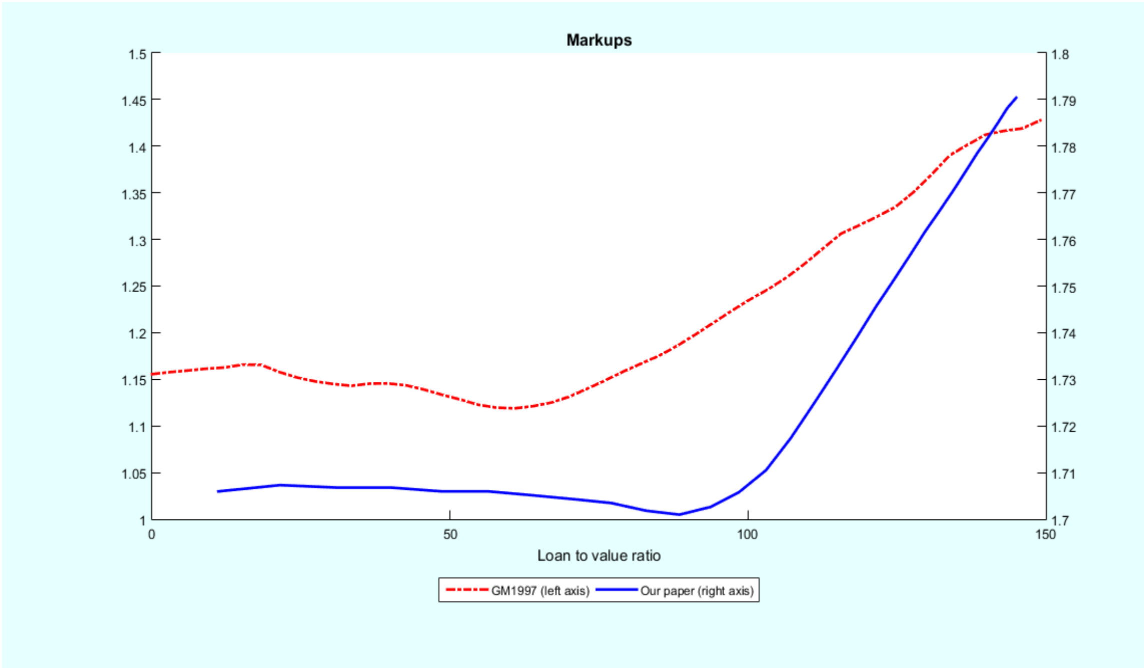

Figure 6: The red dot-dash curve (left axis) depicts the ratio of asking price to assessed value as measured by Genesove and Mayer (1997) plotted against sellers' LTV. The blue curve (right) depicts the same relationship for the ratio of the posted price to the value of a house in REO inventory for our baseline search economy.

the case of a distressed seller. Using (10), in the steady state, for any given $p > m_n$ (see footnote 15), the gain from trade for a such a seller as a function of her outstanding debt, m_n, is given by:

$$\Psi(m_n) = W_b(p - m_n) - W_f(\max[0, \ \beta V_{REO} - m_n]) \tag{60}$$

$$= \begin{cases} W_b(p - m_n) - W_f(\beta V_{REO} - m_n), & \text{if } m_n < \beta V_{REO} \\ W_b(p - m_n) - W_f(0), & \text{if } m_n \geq \beta V_{REO} \end{cases} \tag{61}$$

$$= \pi_f(1 - \pi_h)\beta [W_f(0) - V_b] + \begin{cases} u(y - R + p - m_n) - u(y - R + \beta V_{REO} - m_n), & \text{if } m_n < \beta V_{REO} \\ u(y - R + p - m_n) - u(y - R), & \text{if } m_n \geq \beta V_{REO} \end{cases} \tag{62}$$

Differentiating (62) with respect to the level of debt, m_n, we have

$$\Psi'(m_n) = \begin{cases} u'(y - R + \beta V_{REO} - m_n) - u'(y - R + p - m_n), & \text{if } m_n < \beta V_{REO} \\ -u'(y - R + p - m_n), & \text{if } m_n \geq \beta V_{REO} \end{cases} \tag{63}$$

where here $\Psi'(\cdot)$, $u'(\cdot)$ denote differentiation. Given that $u' > 0$ and $u'' < 0$, we have:

Proposition 1 *Conditional on $p > m_n$, we have:*

(i) *If $m_n \geq \beta V_{REO}$, then $\Psi'(m_n) < 0$;*

(ii) *If $m_n < \beta V_{REO}$, $\Psi'(m_n) > 0$ for any given $p > \beta V_{REO}$ and $\Psi'(m_n) < 0$ for any given $p < \beta V_{REO}$.*

In the steady-state a distressed seller chooses a sub-market to maximize her expected gain from trade. Given the matching function and free-entry of buyers, the optimal sub-market decision in (10) is equivalent to

$$\max_{p,\theta} \ \rho(\theta)\Psi(m_n; p) \tag{64}$$

where

$$\theta(p) = \gamma^{-1}\left(\frac{V_b - W_b(0)}{V_o(p, m_0) - W_b(0)}\right) \tag{65}$$

follows directly from (8) evaluated at the steady-state. It is straightforward to show that if $p > m_n$, (i) $\rho(\theta(p))$ is strictly decreasing in p given the properties of the matching function listed in (3) and (4); and (ii) the gain from trade $\Psi(m_n; p)$ increases with price p for any debt level m_n, since $u' > 0$. Thus, a higher selling price raises the gain from trade, but reduces selling probability. The optimal sub-market choice reflects this trade-off.

The shape of the relationship depicted in Figures 5 and 6 can be understood using Proposition 1. When a seller is sufficiently indebted ($m_n \geq \beta V_{REO}$), given $p > m_n$, the gain from trade $\Psi(m_n; p)$ is strictly decreasing in debt level, m_n. As they receive residual profit only if they sell at a sufficiently high price, heavily indebted sellers worry less about the likelihood that they successfully sell than about the gain they receive if they do. Their cost of foreclosure is *fixed*; the marginal cost of defaulting on a larger debt is borne entirely by the lender.

A less indebted seller (*i.e.* one with $m_n < \beta V_{REO}$) has greater incentive to sell, as failure to do so results in the loss of residual profit as well as the cost of the foreclosure tag. Moreover, for $p > \beta V_{REO}$, the gain from trade Ψ is strictly increasing in debt m_n. As such, a more indebted seller (but with $m_n < \beta V_{REO}$) will chose a lower price/higher sales probability. Overall, for $m_n < \beta V_{REO}$, the effect of debt on the gain from trade (*i.e.*, $\Psi'(m_n)$) is likely to be small in that m_n affects symmetrically the returns both to selling and failing to do so (see (63)). Thus the relationship is essentially flat for lower LTV's but rapidly increasing for higher LTV's.

It is worthwhile clarifying that the condition $p > \beta V_{REO}$ is not particularly restrictive. For example, in our baseline calibration, the steady-state value of $\beta V_{REO} = 1.79$, while the minimum selling price chosen by a seller is 3.04. In general, βV_{REO} tends to be much lower than the choice of selling price by any seller due to the foreclosure and carrying costs associated with houses in REO inventory.

Note that while the proof of Proposition 1 relies on the assumption that consumption goods are non-storable, the actual result does not. In particular, the derived properties of $\Psi'(d)$ require only that $W_b'(p - m_n) > 0$ and $W_f'(\beta V_{REO} - m_n) - W_b'(p - m_n) > 0$ for $m_n < \beta V_{REO}$. The former condition requires that the value of a buyer without a foreclosure flag is a strictly increasing function of her asset holdings. The latter requires that the slope of the value of a buyer with a flag at asset position $\beta V_{REO} - m_n$ exceed that of the value of an unflagged buyer at $p - m_n$ for lower levels of debt. Observing that $p > \beta V_{REO}$ in general, this requirement is not overly restrictive for value functions such as $W_f(\cdot)$ and $W_b(\cdot)$, which are strictly concave.

6.2 Matching and lending standards

We now conduct comparative statics exercises to illustrate the roles of two specific parameters of the matching function, the coefficient ϖ and the elasticity η. In our calibration, these determine the fundamental trading conditions of the housing market. Here we examine they affect the liquidity of housing and mortgage lending. Overall, these exercises demonstrate that mortgage lending standards are lower the more liquid is the housing market.

The top left and right panels of Figure 7 illustrate the effect of changes to ϖ on the average mortgage loan as a fraction of the purchase price and the probability of a mortgage ending in foreclosure, respectively.[25] Consider a mortgage issued in the current period. The

[25] In the steady-state all buyers receive the same loan an thus have the same LTV origination measured relative to V_{REO}. Because, they pay different prices, however, they finance different fractions of their purchase price. The left-hand panels of Figure 7 depict the effects of matching parameters on average, maximum and minimum shares of the house price financed at origination, respectively. The probability that a mortgage ends in foreclosure, however, is unrelated to the original purchase price.

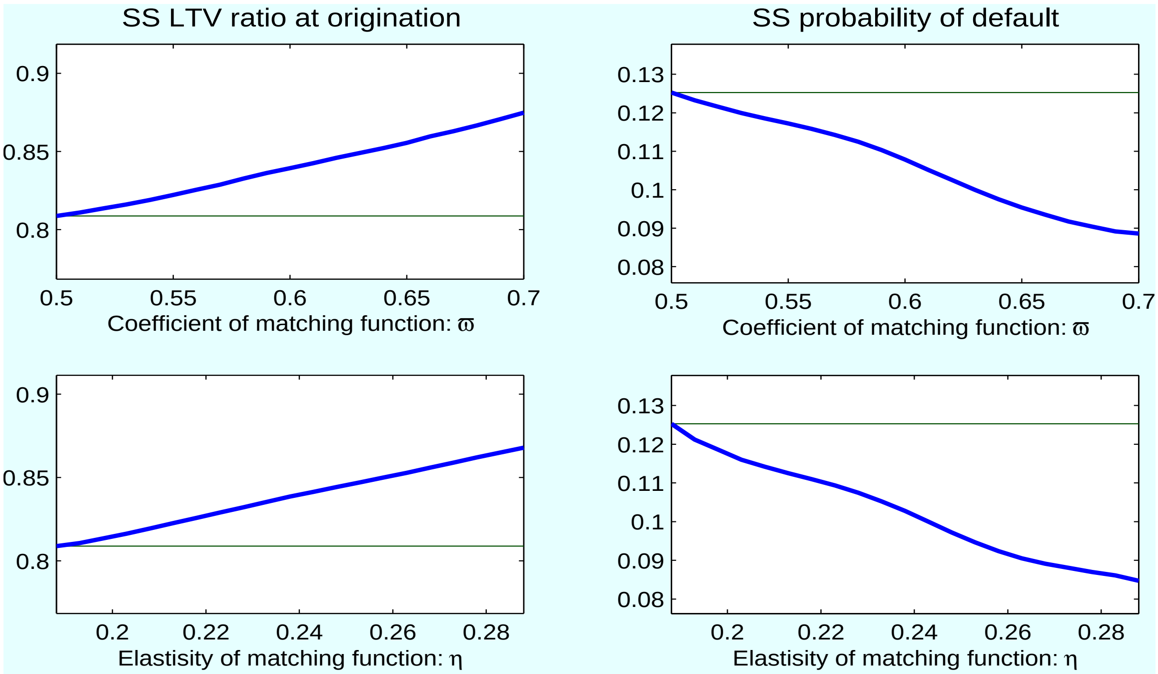

Figure 7: Effects of matching function characteristics on average loan sizes as fractions of the purchase price (left panels) and default rates (right panels) in the steady state.

probability of such a mortgage ending in foreclosure, Π_d, is given by

$$\begin{aligned}\Pi_d &= \sum_{n=1}^{T}(1-\pi_h)^n(1-\pi_d)^{n-1}\pi_d[1-\rho(\theta(p_{sd}))]m_{n-1} \\ &\quad + \sum_{n=1}^{T}(1-\pi_h)^{n-1}\pi_h[1-\rho(\theta(p_{Ld}))]m_{n-1} \qquad (66)\end{aligned}$$

where $\rho(\theta(p_{sd}))$ and $\rho(\theta(p_{Ld}))$ are the trading probabilities in the optimal sub-markets chosen at period $t+i$ for resident and relocated borrowers, respectively, who default after having made $n-1$ payments. The first term is the summation of probabilities of default over the entire duration of mortgage conditional on staying in the city. Similarly, the second term is the summation of probabilities of default conditional on having relocated elsewhere.

As is shown in Figure 7, loan sizes at origination are increasing and default probabilities decreasing with the value of ϖ. *Ceteris paribus*, the higher the value of ϖ, the more likely a seller is to match, or equivalently, the more liquid the housing market. Thus, the expected default rate is lower because more homeowners experiencing distress successfully sell their houses. At the same time, houses in REO inventory also sell more quickly. Overall, with both the likelihood and cost of default and foreclosure reduced, mortgage firms are willing to make larger loans to home buyers.

The lower two panels in Figure 7 demonstrate similar results for varying the value of η, the elasticity of matches with respect to the measure of buyers. As η increases, the surplus resulting from housing transactions that accrues to buyers rises, implying a higher value of being a buyer. This increases the value of living in the city, lowering the entry cutoff, ε_c. Conversely, the return to construction is lower as firms receive less of the surplus associated with new houses. Overall, the housing market is tighter in the steady-state, and *all* houses sell with relatively higher probability. Again, this lowers both the expected rate and cost of default, leading mortgagees to issue larger loans.

7 Equilibrium Dynamics

We now consider the dynamics resulting from aggregate shocks in equilibrium focusing on the effects of endogenous variation in liquidity over time. In this analysis the baseline search economy is compared to the *non-search* (NS) economy described in Section 4.

To begin with, we posit an first-order autoregressive process for the log of income, $\ln y_t$:

$$\ln y(t) = \lambda \ln y(t-1) + \epsilon(t), \qquad \epsilon \sim N(0, \sigma_\epsilon). \tag{67}$$

We set $\lambda = 0.96$ and $\sigma_\epsilon = 0.02$. Here we report results for positive shocks, that is random increases in city income. Responses to negative shocks are reported for both economies in Appendix C.

7.1 Population growth, house prices and construction

Figure 8 illustrates the responses of city population growth, the average house price, and construction to a shock to local income which evolves via (67). For each of the three endogenous variables, the responses to the shock in both the baseline and NS economy are similar to those reported by HLS14. This is not surprising as our baseline economy has been constructed in part to preserve the dynamics of basic housing market variables generated in that paper. For this reason we discuss them only briefly.[26]

A positive shock to local income induces immediate entry of households to the city,

[26] For a detailed discussion, see Section V.A. of HLS14.

raising the population growth rate. The response of population growth is, however, much larger in the search economy.[27] The responses of house prices and construction rates differ both qualitatively and quantitatively across the two economies. The search model generates serial correlation in both price growth and construction, whereas the non-search economy does not generate such dynamics. Rather, without search the house price jumps immediately (by a relatively large amount) and then returns monotonically to its steady-state. This initial jump in house prices, followed by a long and slow decline, actso to limit the entry of households to the city, accounting for the smaller population response overall.

Figure 8: Impulse responses to a positive income shock: population, prices and construction

7.2 Market tightness and matching probabilities.

In the baseline, serial correlation in both house price growth and the construction rate is driven by the change in housing market liquidity due to search and matching. To illustrate this, Figure 9 depicts responses of overall market tightness, and respective average matching

[27] In their experiments HLS14 adjusted the distribution of alternative values so that the variance of population growth in the search and non-search economies was equal. Here we do not do this so as to highlight the difference in the responses of house prices in the two models.

probabilities for buyers and sellers.[28] Changes in housing market liquidity are associated

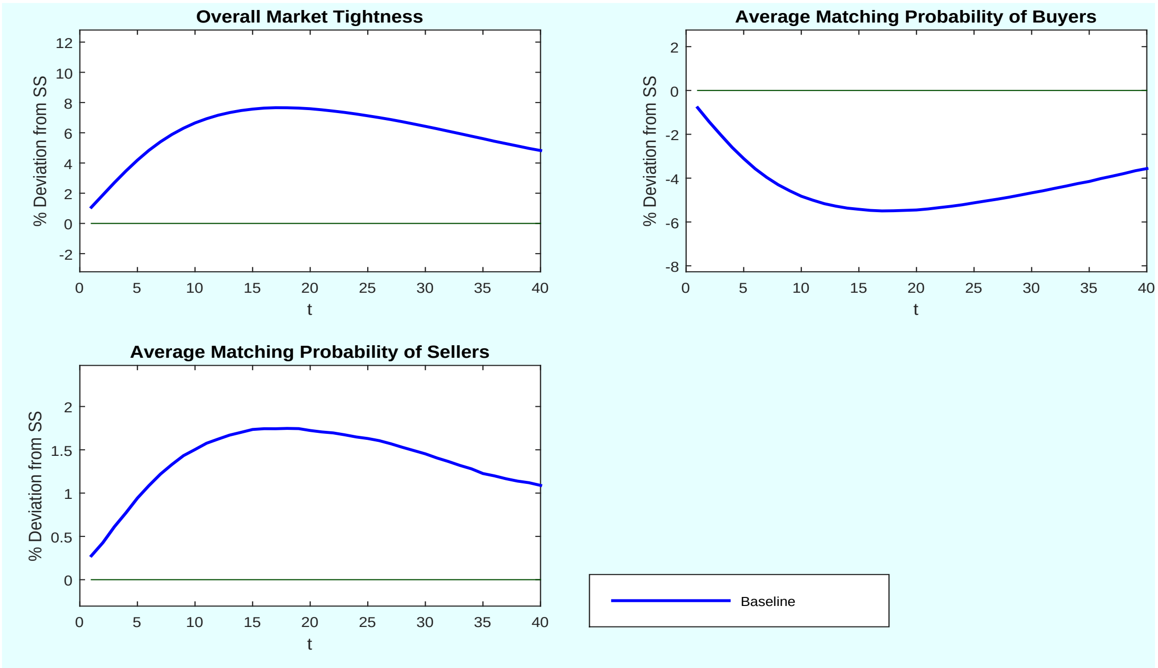

Figure 9: Impulse responses to a positive income shock: matching

with the dynamics of both tightness and the matching probabilities. Following a positive city income shock, increased entry directly raises the measure of prospective buyers. As construction takes time, overall market tightness (*i.e.*, the ratio of the *total* measures of buyers to sellers across sub-markets) increases immediately. Moreover, tightness continues to rise for a prolonged period for several reasons. First, the persistence of the income shock leads to further entry of buyers. Second, unmatched buyers remain in the market. Third, the entry of sellers via construction, is mitigated by both the expected future decline in income and, as seen below, reduced foreclosures. Overall, entry of buyers dominates and tightness both rises and remains above the steady-state persistently.

Higher market tightness implies higher (lower) matching probabilities for sellers (buyers) at any given trading price. The top-right and bottom-left panels in Figure 9 demonstrate these relationships very clearly. As houses become increasingly more liquid in the sense that it takes less and less time to sell them, their values and thus their sales prices continue to rise. This leads to serial correlation both in house price growth and construction, as the latter is driven by the value of new houses. As income returns to its steady-state

[28] These phenomena do not occur in the NS economy. This accounts for the lack of momentum in the impulse responses.

level, entry of households to the city slows. As fewer households enter, searching buyers match, and new houses come on the market, tightness falls. Eventually, house prices and construction return to their steady-state levels.

7.3 The default rate, mortgage size, and the LTV at origination

In the baseline, the average selling probability for sellers increases on impact and continues to rise for several periods before gradually reverting to its long-run level. As noted above this benefits distressed sellers by lowering the probability with which they face foreclosure. As such, the default rate moves opposite the selling rate, as shown in the first panel of Figure 10. In contrast, as the default rate is exogenous in the NS economy it does not vary over time.

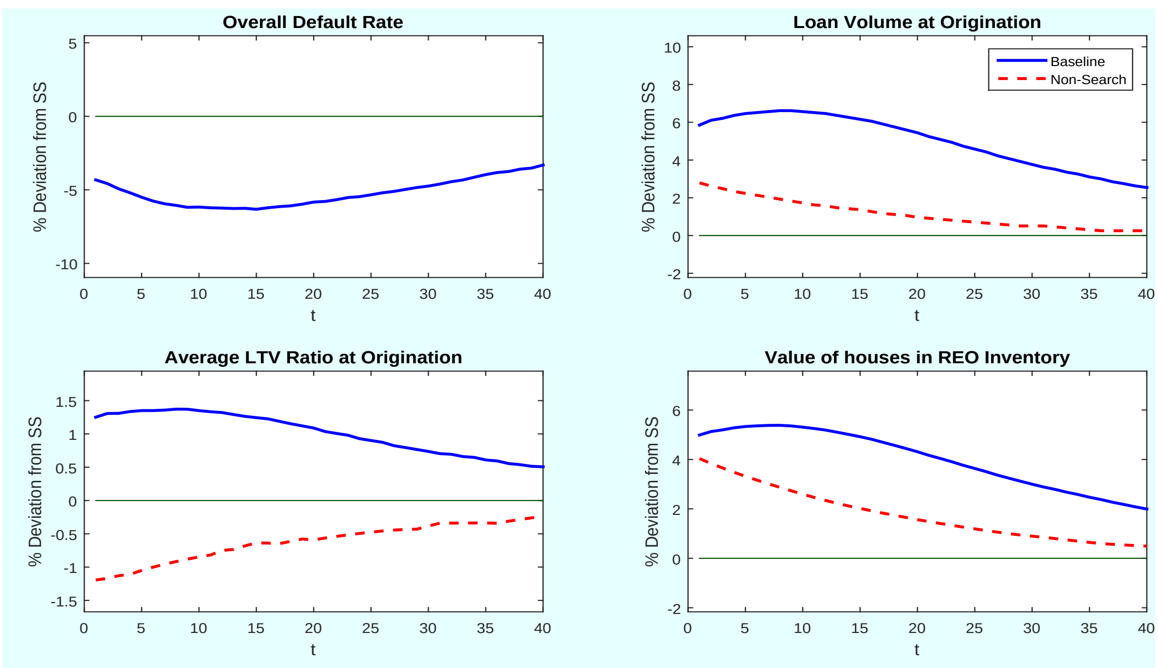

Figure 10: Impulse responses to a positive income shock: mortgage

The responses of loan size at origination ($i.e.$, m_0) differ significantly across the two economies (see Figure 10). From the bottom-left panel of Figure 8, it is clear that loan size largely follows the path of house prices. For example, in the baseline, loan size increases on impact and exhibits momentum following the house price. In the NS economy, loan size also tracks the house price, which displays different dynamics, as noted above.

The equilibrium loan size, m_0, is determined by (28) and thus depends on the expected rate and cost of default in addition to the house price. The close tracking of equilibrium loan size to the house price illustrates that ultimately the value of houses *must* be reflected in mortgage size, in both economies. As discussed above, it is movements in housing demand relative to construction (supply) that drive home values, including the component associated with default risk.

The responses of LTV at origination differ significantly depending on whether there is search. In the baseline, the initial LTV rises immediately following the shock. Several forces contribute to this result: First, the expected default rate on new mortgages declines and remains low for an extended time as houses become increasingly liquid. Similarly, lenders' exposure to risk associated with mortgages issued in earlier periods declines as well. Since the mortgage market is competitive and the interest rate fixed, in equilibrium lower risk translates into loans being larger relative to the purchase price. We refer to this expansion of lending as the *market tightness* effect.

Second, borrowers holding mortgages at the time of the shock experience a relatively large increase in home equity (and a corresponding *reduction* in LTV) as a result of the increase in house values. As illustrated earlier, a decline in LTV is associated with lower asking prices and higher sales probabilities, especially for sellers in financial distress. This *home equity* effect also lowers the default rate and hence the riskiness of lenders' portfolios of outstanding mortgages. Again, competition results in this being passed through to buyers in the form of a higher LTV at origination.

Third, the proceeds of foreclosure sales rise and remain high for several periods reflecting the increases in both house values and the selling rate. This increases the value of houses in REO inventory ($V_{REO}(t)$) and reduces the cost of default, again raising both the return to lending and the LTV at origination. The response of $V_{REO}(t)$ is shown in the bottom-right panel of Figure 10.[29]

Expected reductions of both the default rate and loss upon default induce mortgagees to make larger loans at origination. Because mortgages are long-term, this effect is magnified by the fact that the LTV's on all pre-existing mortgages are instantaneously and persistently *reduced*. Eventually, as tightness and the selling rate return to their steady-states, the LTV at origination does as well, following the path of the average house price.

In contrast, in the NS economy the LTV at origination *falls* significantly in response to the shock, gradually returning to the steady-state monotonically thereafter. In this

[29] As the response of $V_{REO}(t)$ is almost identical to that of housing prices, it is driven mostly by changes in house prices, rather than by the higher selling rate. This is not surprising given that the carrying (*i.e.* maintenance) cost is less than 1.5% of the average house price in the calibration.

economy the default rate is fixed exogenously and the mechanism discussed above for the baseline is not operative. As house prices rise in response to the shock and are expected to fall monotonically back to their steady-state levels in the future, mortgagee' expected loss upon default is *higher*, rather than lower. As the default rate cannot fall to compensate, the mortgagee must require a higher down-payment to break even given the increase in default risk. As an overall result, the baseline and NS economies exhibit very different co-movements between average house prices and LTV's at origination. In Figure 11 it can be seen that the baseline generates a clearly positive co-movement between the two variables while the NS economy predicts a strong negative relationship.

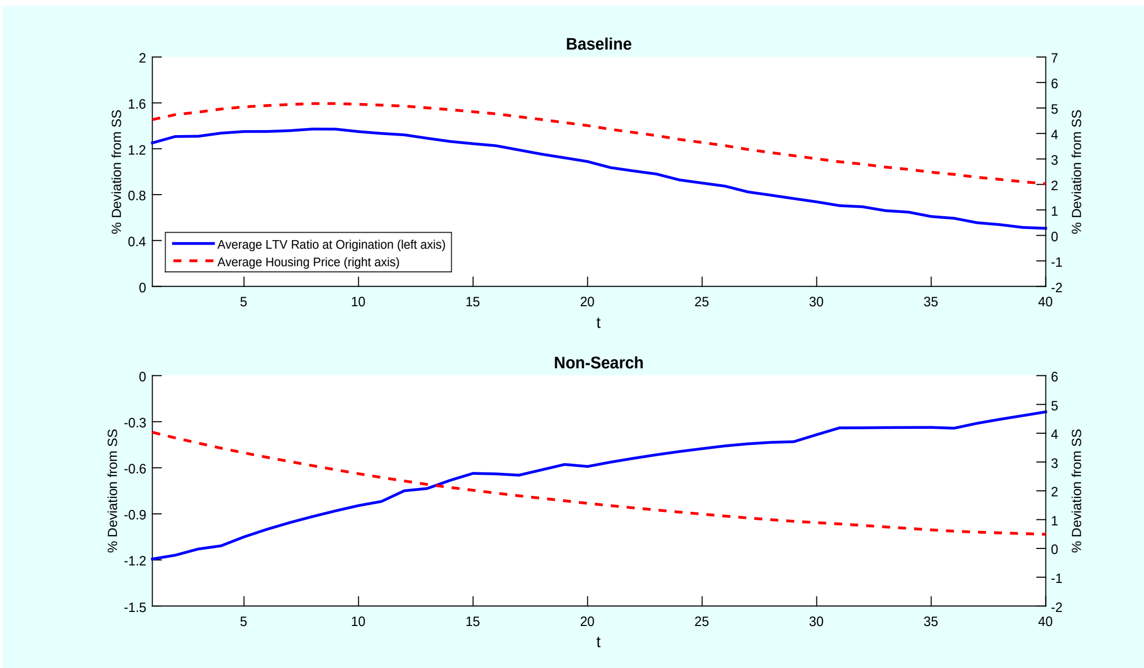

Figure 11: Co-movements between average down-payment ratio and average housing price in baseline and non-search economies.

7.4 The decisions of indebted sellers

Figure 12 depicts the responses of sellers' *probability of sale* (associated with optimal pricing decisions) at four different stages of the mortgage-repayment process.[30] The first three

[30]For example, the first panel of the figure depicts the probability of sale for an optimally pricing seller who has not yet made her first payment ($n = 0$), t periods following the shock. That is, it depicts the sales probabilities for a cross-section of sellers at the same stage of repayment but with loans originated at different times.

panels depict choices of distressed sellers and the lower-right for newly relocated sellers. The figure depicts responses only for the baseline as the NS economy as no counterparts for these measures. All four panels demonstrate a pattern consistent with an average sales probability as shown in the bottom panel of Figure 9.[31] That is, all panels display patterns reflecting the time-paths of tightness and the average sales rate.

The main departure from this pattern involves distressed sellers who have just purchased and taken out a mortgage in the period *before* the shock occurs ($n = 0$). The shock increases the value of the house and thus substantially reduces these households' LTV's. When such a household receives a financial distress shock, it faces the prospect of losing this capital gain if it fails to sell and must default. The household thus has strong incentive to sell, and so posts a low price resulting in a relatively high sales probability.[32]

Buyers who purchase *following* the shock experience no such unanticipated capital gain, as both house prices, current and future, and future matching rates are taken into account when a new mortgage is issued. This explains the large *drop* in the selling-probability for distressed sellers with $n = 0$ in *subsequent* periods. The choice of relocated sellers with $n = 0$ also displays a similar initial responses, albeit of smaller magnitude. These sellers neither face imminent foreclosure nor experience such a large capital gain because they are on average less levered than new homeowners.

Consider next the case of sellers one period from paying off their mortgage ($n = 29$) in the period before the shock. These sellers experience a *lower* probability of a sale (and a higher default probability), as they *raise* their asking prices. Recall that in the event of a default, the mortgagee keeps the outstanding mortgage balance and returns any residual value beyond that of a house in REO inventory. For sellers with $n = 29$, the outstanding balance is low precisely because the mortgage has been nearly paid in full. Thus, the cost of default is relatively low for these homeowners.

The responses of sellers in the middle of their mortgage repayment term ($n = 15$ in the figure) lie between those of sellers at the beginning and end of their terms. The effects discussed above combine for these sellers and largely cancel, leaving the response to reflect largely the movements of the average sales probability.

Responses of these variables to a negative income shock are contained in Appendix C. For the baseline, the dynamics are nearly symmetric to the responses to a positive income shock. One significant exception is that immediately following a negative income

[31] Note that Figure **??** displays a panel whereas Figure 9 depicts a time-series. Each point in the lower-right panel of Figure 9 represents a weighted average of the corresponding points in Figure **??** together with those for all sellers with n's not shown, construction firms, and mortgagees holding REO inventories.

[32] The increase in the sales probability is further augmented by the entry of additional buyers.

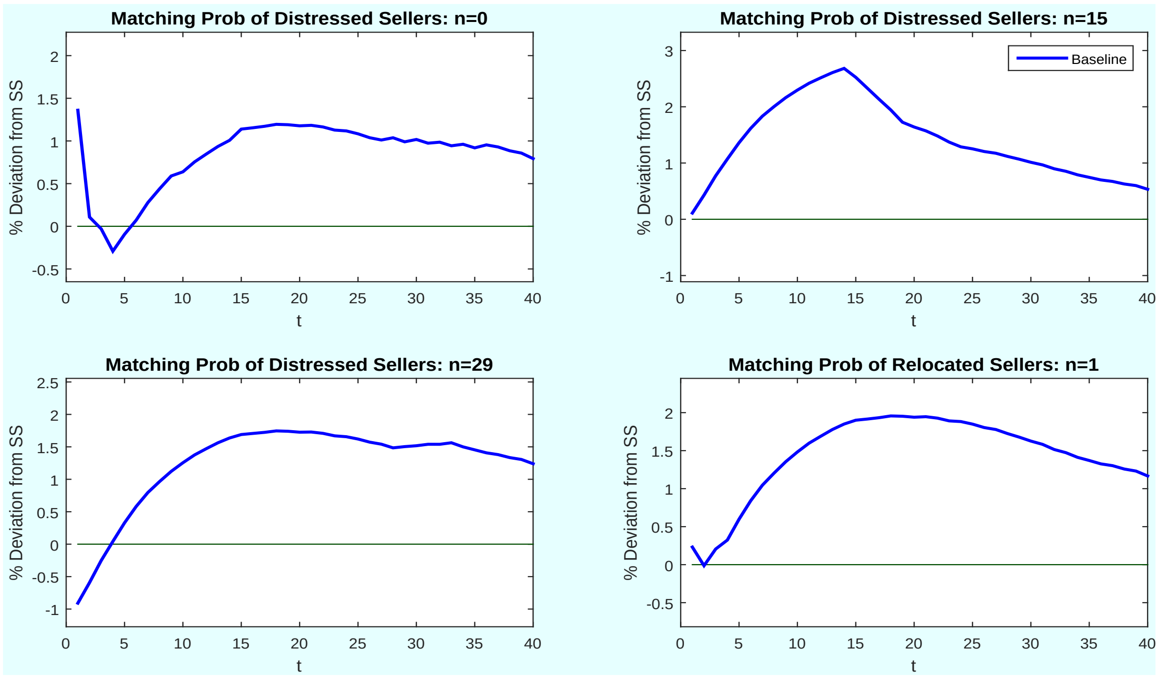

Figure 12: Impulse responses to a positive income shock: house-selling choices (probability)

shock, non-distressed owners may experience such a large increase in LTV that home equity becomes sufficiently negative that they choose to default on their mortgages outright. In this case the set sellers includes some *non-distressed* homeowners who attempt to sell before defaulting at the end of sub-period 1.

Table 2 contains the default rates implied by optimal pricing following a positive income shock. Table 4 in Appendix C contains the corresponding probabilities for the case of a negative shock. Overall, sellers with relatively high leverage are much more likely to default on mortgages than those with less. Outside of the steady-state. the distribution of indebted sellers matters for the response of the economy to shocks. All else equal, a negative shock occurring when the economy has a high proportion of high-leverage home owners will cause much more severe defaults at the aggregate level than will one occurring when leverage is lower overall.

8 Conclusion

A dynamic equilibrium model of housing transactions in which purchases are financed by long-term defaultable mortgages is used to study (i) the effect of sellers' degree of leverage

Table 2: Default probabilities: Borrowers having made n payments before a positive shock

	n=0	1	5	10	15	29
t=1	0.2005	0.1944	0.1822	0.158	0.1342	0.1342
2	0.2166	0.1919	0.1766	0.1554	0.1315	0.1315
3	0.2141	0.2110	0.1740	0.1526	0.1286	0.1286
4	0.2143	0.2081	0.1709	0.1495	0.1254	0.1254
5	0.2119	0.2056	0.1683	0.1469	0.1226	0.1226
6	0.2108	0.2045	0.1671	0.1456	0.1213	0.1213
7	0.2088	0.2025	0.1836	0.1433	0.1190	0.1190
8	0.2075	0.2044	0.1824	0.1420	0.1176	0.1176
9	0.2062	0.2030	0.1810	0.1405	0.1160	0.1160
10	0.2055	0.2024	0.1803	0.1398	0.1153	0.1153

on their pricing behavior and likelihood of default; and (ii) the effects of housing market liquidity on size of mortgages offered by competitive mortgagees. In the model, house prices, liquidity, mortgage standards, and default probabilities all respond endogenously to income shocks.

Sellers' asking prices are shown to be decreasing in and relatively insensitive to increased leverage when LTV's are low, but become steeply increasing in leverage at higher debt ratios. This result matches the shape of the leverage-price relationship estimated by Genesove and Mayer (1997), Anenberg (2011) and others. Moreover, seller behavior also differs with leverage along the dynamic path. Also, housing market liquidity influences lending behavior significantly. In particular, the theory generates consistent positive co-movement between house prices and LTV's at origination. This observation accords qualitatively with observations regarding lending standards both during the period leading up to the recent house price collapse in the U.S., and during the current and on-going period of house price growth in Canada. An alternative model without search (*i.e.*, a frictionless housing market) fails to capture either of these phenomena.

References

Ambrose, B. and Capone, C. (1996). Do lenders discriminate in processing defaults. *Cityscape*, 2(1):89–98.

Ambrose, B. and Capone, C. (1998). Modeling the conditional probability of foreclosure in the context of single-family mortgage default resolutions. *Real Estate Economics*, 26(3):391–429.

Andersson, F. and Mayock, T. (2014). Loss severities on residential real estate debt during the great recession. *Journal of Banking & Finance*, 46:266–84.

Anenberg, E. (2011). Loss aversion, equity constraints and seller behavior in the real estate market. *Regional Science and Urban Economics*, 41(1):67–76.

Barlevy, G. and Fisher, J. (2011). Mortgage choices and housing speculation. manuscript, Federal Reserve Bank of Chicago.

Belsky, E. and Richardson, N. (2010). Understanding the boom and bust in nonprime mortgage lending. Working paper, Cambridge, MA: Harvard University, Joint Center for Housing Studies.

Davis, M. A. and Palumbo, M. (2008). The price of residential land in large u.s. cities. *Journal of Urban Economics*, 63(1):352–84.

Dell'Ariccia, G., Igan, D., and Laeven, L. (2012). Credit booms and lending standards: Evidence from the subprime mortgage market. *Journal of Money, Credit and Banking*, 44(2-3):367–384.

Duca, J., Muellbauer, J., and Murphy, A. . (2011). House prices and credit constraints: Making sense of the u.s. experience. *Economic Journal*, 121(1):533–51.

Eeckhout, J. and Kircher, P. (2010). Sorting and decentralized price competition. *Econometrica*, 78(2):539–74.

Genesove, D. and Mayer, C. (1997). Equity and time to sell in the real estate market. *The American Economic Review*, 87(3):255–269.

Genesove, D. and Mayer, C. (2001). Loss aversion and seller behavior: Evidence from the housing market. *The Quarterly Journal of Economics*, 116(4):1233–60.

Gerardi, K., Lehnert, A., Sherland, S., and Willen, P. (2008). Making sense of the subprime crisis. *Brookings Papers on Economic Activity*, 2008(2):69–159.

Green, R., Malpezzi, S., and Mayo, S. (2005). Methropolitan-specific estimates of the price elasticity of supply of housing, and their sources. *American Economic Review*, 95(2):334–39.

Guren, A. and McQuade, T. (2015). How do foreclosures exacerbate housing downturns? manuscript, Boston University and Stanford University.

Harding, J. P., Rosenthal, S. S., and Sirmansa, C. (2007). Depreciation of housing capital, maintenance, and house price inflationation: Estimates from a repeat sales model. *Journal of Urban Economics*, 61(2):193–217.

He, C., Wright, R., and Zhu, Y. (2015). Housing and liquidity. *Review of Economic Dynamics*, 18(3):435–55.

Head, A. and Lloyd-Ellis, H. (2012). Housing liquidity, mobility, and the labour market. *The Review of Economic Studies*, 79(4):1559–89.

Head, A., Lloyd-Ellis, H., and Sun, H. (2014). Search, liquidity and the dynamics of house prices and construction. *American Economic Review*, 104(4):1172–1210.

Hedlund, A. (2015). Illiquidity and its discontents: Trading delays and foreclosures in the housing market. Working paper, University of Missouri.

Hedlund, A. (2016). The cyclical dynamics of illiquid housing, debt, and foreclosures. *Quantitative Economics*, 7(1):289–328.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 95(3):739–64.

Menzio, G. and Shi, S. (2010). Block recursive equilibria for stochastic models of search on the job. *Journal of Economic Theory*, 145(10):1453–94.

Mian, A. and Sufi, A. (2011). House prices, home equity-based borrowing, and the u.s. household leverage crisis. *American Economic Review*, 101(5):2132–56.

Moen, E. (1997). Competitive search equilibrium. *Journal of Political Economy*, 105(2):385–411.

Ngai, R. and Sheedy, K. (2015). Moving house. CEPR Discussion Paper No. DP10346.

Ngai, R. and Tenreyro, S. (2014). Hot and cold seasons in the housing market. *American Economic Review*, 104(12):3991–4026.

Pennington-Cross, A. (2003). Subprime and prime mortgages: Loss distributions. Office of Federal Housing Enterprise Oversight Working Paper (03-1).

Phillips, R. and Vanderhoff, J. (2004). The conditional probability of foreclosure: An empirical analysis of conventional mortgage loan defaults. *Real Estate Economics*, 32(4):571–88.

Saiz, A. (2010). The geographic determinants of housing supply. *The Quarterly Journal of Economics*, 125(3):1253–96.

Shi, S. (2001). Frictional assignment, part i, efficiency. *Journal of Economic Theory*, 98(2):232–60.

Shi, S. (2005). Frictional assignment, part ii: Infinite horizon and inequality. *Review of Economic Dynamics*, 8(1):106–37.

Shi, S. (2009). Directed search for equilibrium wage-tenure contracts. *Econometrica*, 77(2):561–84.

Shimer, R. and Smith, L. (2000). Assortative matching and search. *Econometrica*, 68(2):343–69.

Smith, L. (2006). The marriage model with search frictions. *Journal of Political Economy*, 114(6):1124–44.

Ungerer, C. (2015). Monetary policy, hot housing markets and leverage. FEDS Working Paper No.2015-048.

Wheaton, W. (1990). Vacancy, search, and prices in a housing market matching model. *The Journal of Political Economy*, 98(6):1270–92.

A Laws of motion for the non-search economy

For the non-search economy, we have the following laws of motion:

$$(1+\mu)F' = (1-\pi_p)F + (1-\psi)G(\varepsilon_c')\mu. \tag{68}$$

$$(1+\mu)B_f' = (1-\pi_h)\left\{\begin{array}{l} \pi_f B_f + \pi_d \sum_{n=0}^{T-1} H_n \\ +(1-\pi_d)\sum_{n=0}^{T-1}(1-I_n)D_n H_n \end{array}\right\} \tag{69}$$

where $I_n = 1$ if the owner chooses to sell, and 0 otherwise.

$$(1+\mu)B' = \psi G(\varepsilon_c')\mu + (1-\pi_f)B_f + (1-\pi_h)\sum_{n=0}^{T-1} I_n H_n. \tag{70}$$

$$(1+\mu)H_n' = (1-\pi_h)(1-\pi_d)(1-I_{n-1})(1-D_{n-1})H_{n-1}; \tag{71}$$

$$(1+\mu)H_0' = (1-\pi_h)(1-\pi_d)B; \tag{72}$$

$$(1+\mu)H_\emptyset' = (1-\pi_h)\left\{\begin{array}{c} (1-\pi_d)(1-I_{T-1})(1-D_{T-1})H_{T-1} \\ +(1-I_T)H_\emptyset \end{array}\right\}. \tag{73}$$

$$\begin{aligned}(1+\mu)H_{Ln}' &= (1-I_{Ln-1})(1-D_{Ln-1})H_{Ln-1} \\ &\quad + \pi_h(1-\pi_d)(1-I_{n-1})(1-D_{n-1})H_{n-1};\end{aligned} \tag{74}$$

$$(1+\mu)H_{L0}' = \pi_h(1-\pi_d)B; \tag{75}$$

$$\begin{aligned}(1+\mu)H_{L\emptyset}' &= \pi_h\left\{\begin{array}{c} (1-\pi_d)(1-I_{T-1})(1-D_{T-1})H_{T-1} \\ +(1-I_T)H_\emptyset \end{array}\right\} \\ &\quad + (1-I_{LT-1})(1-D_{LT-1})H_{LT-1} + (1-I_{LT})H_{L\emptyset}.\end{aligned} \tag{76}$$

$$(1+\mu)H_c' = N. \tag{77}$$

$$\begin{aligned}(1+\mu)H_{LREO}' &= \pi_d\sum_{n=0}^{T-1} H_n + \sum_{n=1}^{T-1}(1-I_{Ln})D_{Ln}H_{Ln} \\ &\quad + (1-\pi_d)\sum_{n=0}^{T-1}(1-I_n)D_n H_n.\end{aligned} \tag{78}$$

B Calibration parameters for the non-search economy

Table 3: Calibration Parameter Values: Non-Search Economy

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_p	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed-rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land price-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
ς	0.16	Rent-price ratio	5%
		Parameters determined jointly	
χ	0.460	Loss severity rate	27%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.016	Annual foreclosure rate	1.6%
z_H	0.3280	Average loan-to-income ratio at origination	2.72
κ	0.137	Average price of a house	3.2
α_p	6.200	Relative volatility of population growth	0.17

C Responses to a negative local income shock

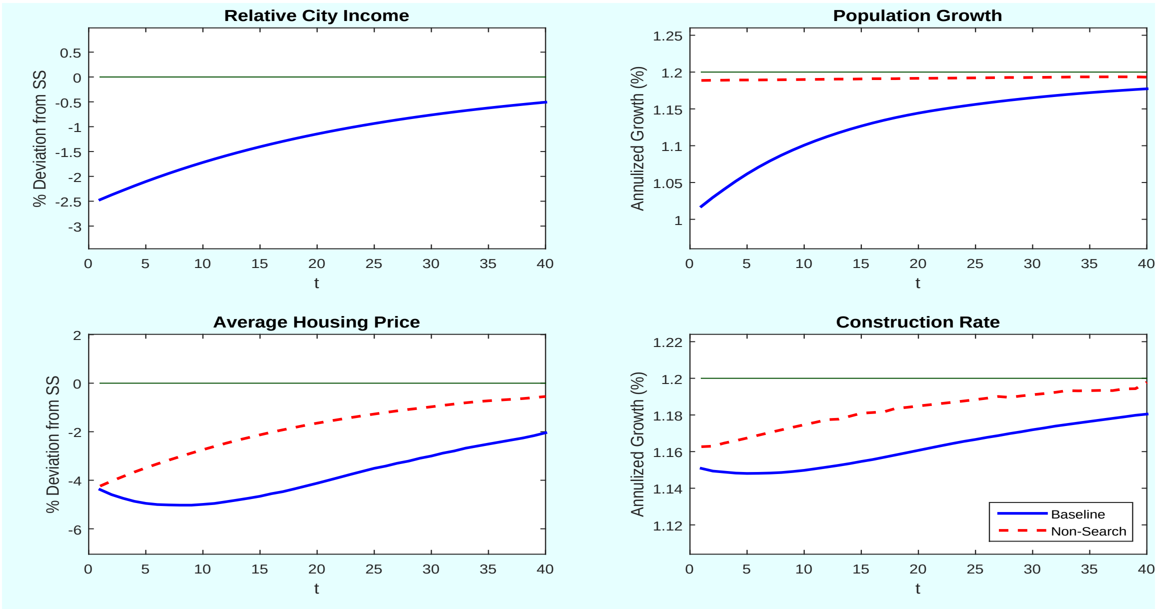

Figure 13: Impulse responses to a negative income shock: population, prices and construction

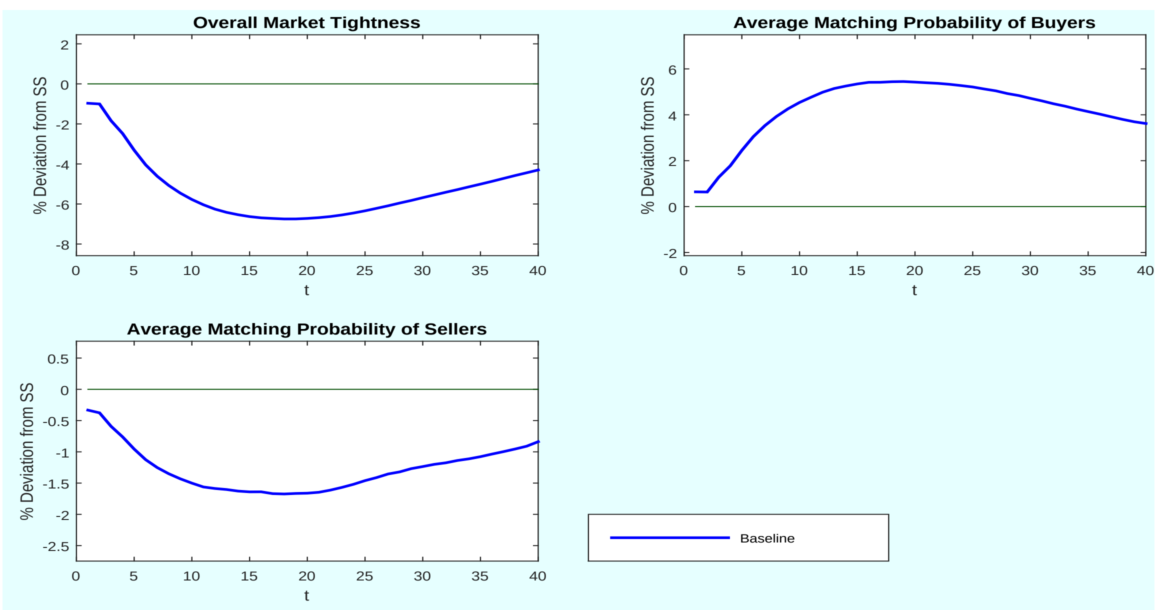

Figure 14: Impulse responses to a negative income shock: matching

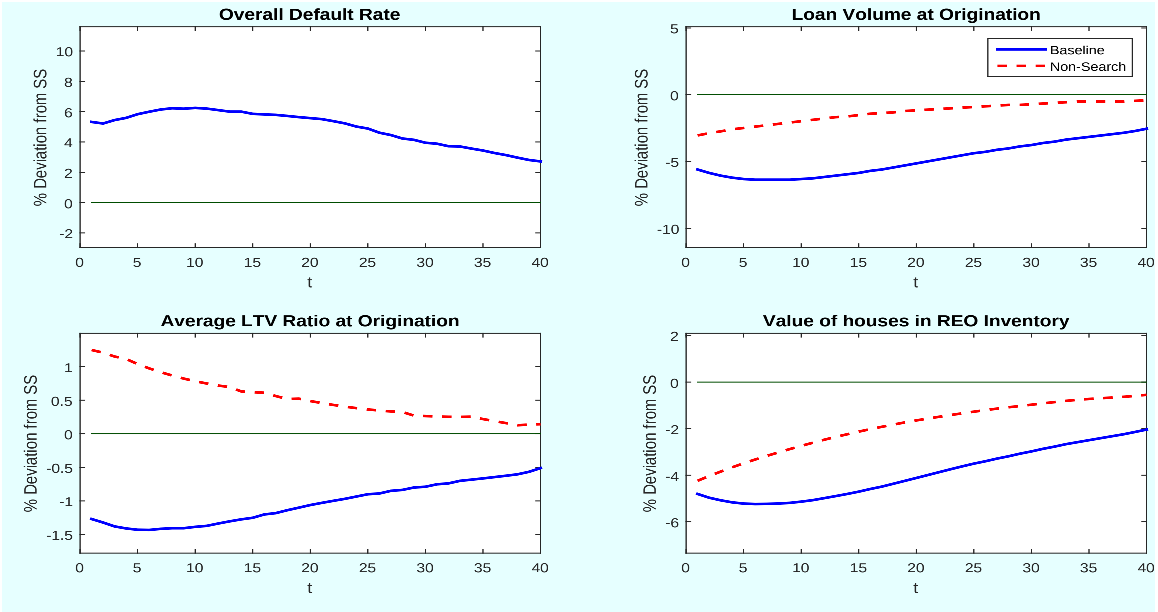

Figure 15: Impulse responses to a negative income shock: mortgage

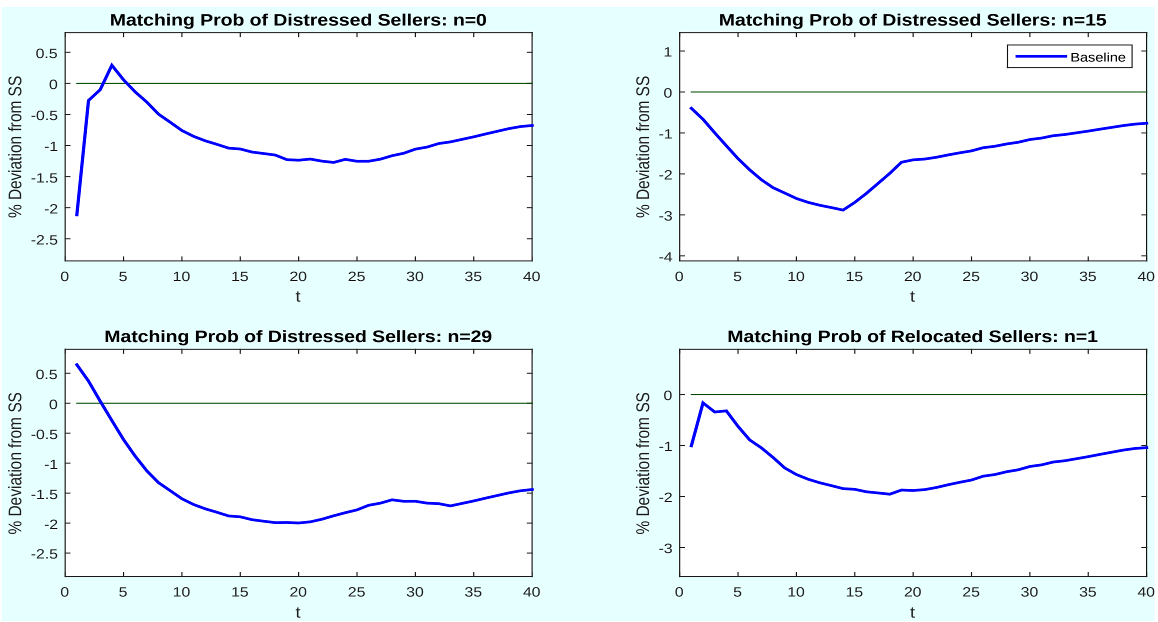

Figure 16: Impulse responses to a negtive income shock: house-selling choices

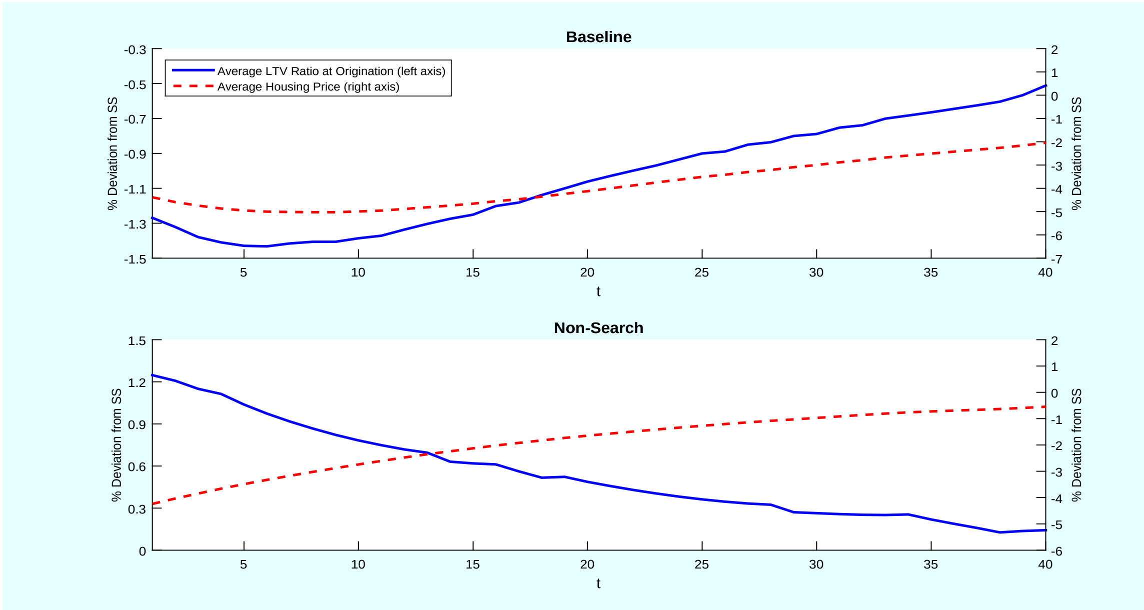

Figure 17: Co-movements between average down-payment ratio and average housing price in baseline and non-search economies.

Table 4: Default probabilities: Borrowers having made n payments before a negative shock

	n=0	1	5	10	15	29
t=1	0.2279	0.2217	0.2030	0.1721	0.1386	0.1206
2	0.2047	0.2233	0.2047	0.1738	0.1404	0.1225
3	0.2076	0.2045	0.2076	0.1769	0.1437	0.1259
4	0.2101	0.2070	0.2101	0.1796	0.1465	0.1288
5	0.2098	0.2098	0.2129	0.1825	0.1496	0.1319
6	0.2124	0.2093	0.2154	0.1852	0.1523	0.1347
7	0.2139	0.2109	0.1957	0.1867	0.1540	0.1364
8	0.2153	0.2123	0.1972	0.1882	0.1555	0.1380
9	0.2166	0.2136	0.1986	0.1896	0.1570	0.1395
10	0.2175	0.2145	0.1994	0.1905	0.1579	0.1405