Queen's Economics Department Working Paper No. 1356

A Utilitarian Measure of Economic Growth

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2-2016

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Dan Usher, Latest version March 1, 2016

Abstract: A utilitarian measure of economic growth combines changes in the distribution of income with changes in real income per person to show how much better off people are becoming over time. It is the rate of growth of the dollar value of average utility of income. As such , it is seen differently by people with different utility of income functions. A growth rate in U.S. household income of 0.63% per year as ordinarily measured disappears altogether - is transformed into a decline of 0.086% per year - when the utility of income function is sufficiently concave. Strengths, weaknesses and implicit assumptions of the utilitarian measure are discussed.

Key Words: national income, utilitarian, certainty-equivalence.

JEL Codes: E31, E32, O40

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We rely on statistics of economic growth to tell us how much better off we are becoming in the course of time, but a given rate of economic growth as ordinarily defined seems more desirable when the benefits of growth are widely shared than when they are concentrated upon the privileged few. Starting from full equality, a 2% rate of growth is seen as more socially desirable when everybody's income rises by 2% then when the entire increase in income accrues to the top one percent, while the income of the remaining ninety-nine percent does not rise at all. Recognition of this aspect of most people's sense of social welfare suggests that the ordinary measure of economic growth might be supplemented - supplemented, not replaced - by a measure that takes changes in the distribution of income into account.

The purpose of this note is to weigh average income and the distribution of income on a common scale that is somehow representative of people's preferences for the comparison of societies or of the same society at different periods of time. Relevance of the analysis to societies with different populations will be discussed at the end of this note.

In the formation of law and public policy, we commonly try to maximize some notion of of total utility rather than total income, on the understanding that an additional dollar of income provides more utility to a person whose initial income is low than to a person whose initial income is high. Correspondingly, we would like a measure of progress in the economy as the growth of the dollar value of average utility rather than of average income. This note is an attempt to define a utilitarian measure precisely, to present a simple computation of how growth rates are affected by the switch from the ordinary measure to the utilitarian measure, and to discuss some of the statistical and analytical difficulties involved.

Consider an example. Table 1 shows quintiles of mean household income in the United States in constant (2014) dollars for the years 1973 and 2014, together with the percentage increases over this period. On average, as shown in the last column, real household income rose by 29.4% over the entire period, but, as shown in the bottom row, percentage increases were lower for the poor than for the rich, varying steadily from a decline of 3.4% for the poorest fifth to a rise of 51.1% for the richest fifth. If the overall increase in average household income had been spread out evenly among quintiles, mean household income in the poorest quintile would have risen from $12,097 to $15,653 instead of falling to $11,676. Of growth in total income, 76.3% accrued to the richest fifth of the population. Looking at such data, we are ordinarily inclined to say that average income has increased but the distribution has deteriorated, without

Table 1: United States Mean Household Income Received by Each Fifth of the Population (in 2014 dollars) US Census, Table H3

	lowest fifth	second fifth	third fifth	fourth fifth	highest fifth	average
1973	$12,097	$30,416	$49,889	$71,768	$128,413	$58,517
2014	$11,676	$31,087	$54,041	$87,834	$194,053	$75,738
growth	- 3.4%	2.1%	8.3%	22.2%	51.1%	29.4%

trying to place these changes on a common scale, but, just as consumption of apples and oranges is weighted on a common scale in the measurement of ordinary income, it may be useful to ask how changes in average income and the distribution of income might be compared.

Consider a simpler example. Imagine two societies, A and B, which may be the same society in two consecutive years, each with two people, 1 and 2, whose incomes are y_1^A and y_2^A in society A and y_1^B and y_2^B in society B. Suppose y_1^A and y_2^A are both equal to 10, $y_1^B = 6$ and $y_2^B = 16$, so that average incomes are 10 and 11, but there is a standard deviation of 5 in society B. Which is the better society? In which society can people be said to be better off? Is the standard deviation of 5 a fair price to pay for the increase in average income of 1? In evaluating these societies, one might simply recognize that size and distribution of income are both important, or one might attempt to weigh size and distribution on a common scale.

The way to compare desirability of societies with different patterns of income is to imagine yourself having to select one of these societies on the understanding that, whichever society you select, you will have equal chances of acquiring the incomes of each and every person there. If you knew which place you would occupy in each society, you would, of course, choose the society that provides you with the highest income, but behind what Rawls called "the veil of ignorance" you would take the entire distribution of income into account.[1] A useful intermediate step in this choice is to equate the pattern of income in each society with a certainty-equivalent income, **I**, the income which, if you had it for sure, would leave you as well off as you would be with equal chances of each and every income in that society. The best society, as you see it, would then be the society with the highest certainty-equivalent income. In the example in the preceding paragraph, the choice between society A and society B depends whether the higher average income in society B compensates for its higher standard deviation as indicated in your measure of certainty equivalence. If you place a low weight on standard deviation, the certainty-equivalent income would be higher in society B. If you place a high weight on standard deviation, the certainty-equivalent would be higher in society A. If A and B refer to the same society t years apart, the utilitarian measure of the rate of economic growth becomes the growth rate of **I** over time.

The veil of ignorance test identifies a person's ranking of societies with different distributions of income, but it does not go far enough, for we want to measure certainty-equivalent incomes in order to infer a person's preference among societies from information about patterns of income and about the person's preferences defined independently of the choice at hand. To combine size and distribution of income in a person's certainty equivalent income, assume that i) the person has a concave utility of income function, u(y) where $u'(y) > 0$ and $u''(y)$

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[1]Harsanyi called this *Impersonality*. "Individual's preferences satisfy this requirement of impersonality if they indicate what social situation he would choose if he did not know what his personal position would be in the new situation chosen (and in any of its alternatives) but rather had an equal *chance* of obtaining any of the social positions." Harsanyi, 1955, page 316)

< 0, ii) behind the veil of ignorance, the person chooses the society with the higher average utility rather than just the higher average income and iii) the general form of the person's utility of income function is

$$u(y) = (y)^{\alpha} \qquad (1)$$

where α, the elasticity of utility with respect to income, may differ from one person to another. The assumed concavity of the utility of income function ensures that $\alpha < 1$, but all values between 1 and $-\infty$ are possible, each corresponding to a different intensity of one's willingness to put up with uncertainty of income.

If α were the same for everybody, everybody' utilitarian measures of economic growth would be the same, but, when α differs from one person to the next, so too does the utilitarian measure. It turns out that the lower one's value of α, the more weight one places on equality as opposed to average size of income in one's assessment of the utilitarian measure of the rate of economic growth.

As seen by a person with an elasticity α of utility to income, the certainty-equivalent income, **I**, in a two person society with incomes y_1 and y_2 is determined implicitly in the equation

$$(\mathbf{I})^{\alpha} = (½)[(y_1)^{\alpha} + (y_2)^{\alpha}] \qquad (2)$$

In the numerical example where $y_1^A = y_2^A = 10$, $y_1^B = 6$ and $y_2^B = 16$, the certainty equivalent income, $\mathbf{I^A}$, in society A is 10, and the certainty-equivalent income, $\mathbf{I^B}$, in society B depends on α as shown in the third column of table 2. When A and B are consecutive years, the utilitarian rate of economic growth, g, as shown in the fourth column is also dependent on α.

Table 2: Certainty-equivalent Incomes and the Utilitarian Measure of Economic Growth as Seen by a Person with an Elasticity, α, of Utility with Respect to Income

α	$\mathbf{I^A}$	$\mathbf{I^B}$	g
1	10	11	10%
1/2	10	10,4	4%
1/10	10	9.92	-0.08%
0	10	9.80	- 2%
-2	10	7.95	-20.5%
-10	10	6.43	-35.7%
$-\infty$	10	6	-40%

The value of **I** must be 10 in the first year, A, regardless of the value of α, but the value of **I** in the second year, B, is a decreasing function of α as shown in Table 2. When the years are consecutive, the utilitarian measure of economic growth for any given α, is $100\{(\mathbf{I}^B - \mathbf{I}^A)/\mathbf{I}^A\}\%$. The least possible concavity of the utility of income function is where α =1 at which **I** is just the average income, so that the utilitarian measure of economic growth and the growth rate of average income are the same. The greatest possible concavity is where α = - ∞, at which the certainty-equivalent income is the lowest income, the smaller of y_1 and y_2 in accordance with Rawls' maximin principle.[2] Other values of **I** in the year B vary steadily in between. Equation (2) morphs into a Cobb-Douglas function when α = 0.[3] Depending on the value of α, the utilitarian measure of economic growth in this two-person, two period economy may be anywhere from 10% to - 40%.

The example is easily extended to societies with larger populations and with populations that vary over time. Suppose now that A and B are the same society t years apart and that population has changed between these years from n to m. The incomes of the n people in society A and the m people in society B are $(y_1^A, y_2^A, \ldots, y_n^A)$ and $(y_1^B, y_2^B, \ldots, y_n^B)$. Mean incomes are Y^A and Y^B where

$$Y^A = (1/n)(y_1^A + y_2^A + \ldots + y_n^A) \qquad (3)$$

and

$$Y^B = (1/m)(y_1^B + y_2^B + \ldots + y_n^B) \qquad (4)$$

As ordinarily measured, the rate of economic growth, of average rather than total income, between year A and year B is

$$g(Y) = (1/t)\ln(Y^B/Y^A) \qquad (5)$$

By contrast, certainty-equivalent incomes - the dollar value of expected social welfare - in years A and B as seen by a person with a utility of income function u(y) are defined implicitly by the equations

$$u(\mathbf{I}^A) = (1/n)[u(y_1^A) + u(y_2^A) + \ldots + u(y_n^A)] \qquad (6)$$

and

$$u(\mathbf{I}^B) = (1/m)[u(y_1^B) + u(y_2^B) + \ldots + u(y_m^B)] \qquad (7)$$

so that a person's valuations of $\mathbf{I}^A$ and $\mathbf{I}^B$ are dependent on α and are the same as Y^A and Y^B when α = 1. For any given α, the *utilitarian measure of economic growth*, g(**I**), becomes

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[2]From equation (2) and as long as $y_2 > y_1$, it follows that $\mathbf{I} = y_1[(½)^{(1/\alpha)}][1 + (y_2/y_1)^\alpha]^{(1/\alpha)} = y_1$ when α = - ∞ because both of the expressions in square brackets are equal to 1 in this case..

[3] This proved, using l'Hopital's rule, in Chiang (1974) pp 419-421.

$$g(\mathbf{I}) = (1/t)\ln(\mathbf{I}^A/\mathbf{I}^B) \qquad (8)$$

How might α be identified? There are two logically-similar tests by which "your" value of α might be discovered. The first is a variant of the example in table 2. Imagine yourself with equal chances of acquiring each of two possible incomes, a high income, y_2, and a low income, y_1, every year for the rest of your working life. Your value of α can be inferred from your answer to the question: How much income each year, if you had it for certain, would leave you as well of as you would be with the risky prospect? Your answer, **I** , to that question is your certainty-equivalent income. Your value of α can be then inferred from equation (2) above.

Suppose, for example, that y_2 = \$200,000 and y_1 = \$50,000. If your certainty-equivalent income were as high as \$125,000, you would be risk neutral and your value of α would be 1. If you would accept \$112,500, your value of α would be ½. If, being very risk averse, you would accept as little as \$68,599, your value of α would be -2, Whatever your answer to the question, your assessment of the utilitarian measure of economic growth in the two-period example in table 2 would be the value of g for your imputed value of α. If most people's certainty equivalent income were about \$95,000, then the common value of α would be approximately equal to 0.[4]

Alternatively, your value of α can be inferred from your answer to a different question: Starting from given high and low incomes, y_2 and y_1, you might ask yourself: How much would I be willing to see the high income reduced in return for an increase in the low income of \$1? What is the decrease in the high income which together with an increase of \$1 in the low income leaves me as well off as I was were before? The answer to this question is implicit in equation (2) with **I** held constant. Specifically, your answer to this question must be $-\delta y_2/\delta y_1$ where

$$\delta y_2/\delta y_1 = -(y_2/y_1)^{1-\alpha} \qquad (9)$$

If you are not a all risk averse, you would trade changes in y_2 and y_1 dollar for dollar; $\delta y_2/\delta y_1$ would equal -1 and your inferred value of α would be 1. If you are prepared to give up \$2 in the event that you are prosperous - with an income of \$200,000 - in return for an extra dollar in the event that you are not - with an income of \$50,000 - then your inferred value of α must be ½. If you are prepared to give up as much as \$5, your inferred value of α must be - 0.16.

The expression $\delta y_2/\delta y_1$ can also be interpreted as your valuation, as a basis for public policy, of the acceptable cost to the rich per dollar of benefit to the poor, for example, in the redistribution of income. The higher your absolute value of $\delta y_2/\delta y_1$, the lower your value of α, and the greater your concern for the poor must be. Imagine a society with a very high marginal cost of public funds and with two distinct social classes: the rich with incomes of \$200,000 per head and the poor with an incomes of \$50,000 per head. A person with α = 0 would tax the rich

[4]Utilitarian in this context harks back to the Neumann-Morgenstern concept of utility as that for which one seeks to maximize the expected value in risky situations. See von Neumann and Morgenstern, third edition, Princeton University Press, 1953, chapter I section 3.

up to $4 for every $1 acquired by the poor. Note however that the four-to-one trade-off only applies at the margin; the trade-off is automatically adjusted as incomes are equalized. This test can be extended from the two-class society in this example to a continuous distribution of income with a unique value of $\delta y_2 / \delta y_1$ for every pair of incomes.

A difficulty with both of these tests is the inference from behaviour under uncertainty to concern for other people. The assumption behind this inference is that a person's perception of utility in equations (6) and (7) is the same when incomes accrue to different people as when they accrue with equal probabilities to one and the same person. That may but need not always be so. Someone who is quite willing to bear risk himself may nonetheless favour public policy to narrow the distribution of income, even at the sacrifice of many dollars from the rich per dollar acquired by the poor. The original objective of utilitarianism was to design public policy for the maximization of some notion of aggregate happiness, where income brings happiness but each extra dollar brings progressively less and less. Of the two tests, the second would seem to be closer to that ideal.

Even concavity has its exceptions. A person may be risk loving rather than risk averse. There may be a minimal income below which one cannot survive, so that a person who is risk averse as long as there is no danger of income falling below subsistence, might be prepared to gamble when the consequence of not gambling is that one's income is lethally small. Suppose the income below which one cannot survive is $10,000. A person who is ordinarily risk averse and whose income if he does not gamble is $8,000 would accept fifty-fifty chances of a gain of $3,000 and a loss of $6,000 because a 50% chance of survival is better than none. At the other end of the scale, a person whose one great passion is space exploration, who requires an income of $300,000 to engage in space exploration, but whose income is only $100,000 might be prepared to pay $50,000 for a one-in-five chance of winning $200,000 if that were the only bet available. If he wins, he engages in space exploration. If he loses, he spends his days in the public library reading about space exploration.

Table 3: Dollar Values of Average Utility per Household,
Trade-offs between Incomes in Highest and Lowest Quintiles
and Utilitarian Measures of Economic Growth

postulated α	1	½	0	- ½	- 1	- 5	- ∞
$\mathbf{I}^{1973}$	$58,517	$51,437	$44,225	$37,481	$31,785	$16,654	$12,097
$\mathbf{I}^{2014}$	$75,738	$62,873	$50,680	$40,412	$32,710	$16,084	$11,676
$-\delta y_T/\delta y_B$ 1973	1	3.26	10.62	34.59	112.68	1,430,829.50	∞
$-\delta y_T/\delta y_B$ 2014	1	4.08	16.62	67.75	276.22	21,074,518.00	∞
growth rate	0.63%	0.49%	0.33%	0.18%	0.07%	- 0.085%	- 0.086%

Consider once again the changes in quintiles of household income in the United States

between 1973 and 2014 as shown in table 1 above. For these data, the dollar equivalents of average utility, the trade-offs in social welfare between dollars to the rich and dollars to the poor and the implied utilitarian measures of rates of economic growth are shown for selected values of α in Table 3. For each value of α, the dollar values of average utility in 1973 and 2014 are shown in the second and third rows. The next two rows show the social welfare preserving trade-offs in 1973 and 2014, the amount, $-\delta y_T / \delta y_B$, by which mean income in the top fifth of the distribution can be reduced in compensation for an increase of \$1 in the mean income in the bottom fifth without altering social welfare in the nation as a whole. These trade-offs are derived from variants of equations (6) and (7) with n = m = 5 and with $\mathbf{I}^A$, $\mathbf{I}^B$ and all but the top and bottom incomes held constant. The last row shows the utilitarian rates of economic growth computed for each value of α from equation (8).

The story in the table is that, though the ordinary measure of the growth rate of real income per year is 0.63%, the utilitarian growth rate diminishes steadily as the utility of income function becomes increasingly concave until, eventually, the rate turns negative because all weight is placed upon the lowest quintile of the distribution.

If most people's values of α were about 0 - implying a willingness to see incomes in the highest quintile reduced by about \$16 in return for a \$1 increase in incomes in the lowest quintile - the utilitarian rate of economic growth would be about half the rate as ordinarily computed, 0.33% per year rather than 0.63%.

There are statistical and theoretical problems with these measures. The statistical problem is the discrepancy between the numbers in table 1 and the ideal numbers for illustrating the principles in this note. Convenient as it is for illustrating alternative measures of social welfare, the distribution of income in table 1 overlooks several important considerations. Ideally, the measures should take account of the entire distribution of income rather than the quintile means. Utilities might be compared over people's entire lives rather than at a moment of time. Changes in the age distribution of the population may be important because people who consume the same amount each year of their lives would have higher observed incomes in their working years than in retirement. Allowance should be made for changes over time in tax rates and provision of public services. Ideally, income as the basis for the utilitarian measure should include transfers in kind such as food stamps, public provision of medical care and public education together with some accounting for shares of corporate profits and capital gains and for changes over time in family size.For any given distribution of income as it is measured, a society with substantial public services is likely to have more real equality than a society where each person is entirely on his own. No such corrections are included here. The numbers in the table remain useful, though they must be taken with a grain of salt.

The theoretical problems are more interesting. What is being called the utilitarian measure of economic growth is utilitarian in one sense but not another. Utilitarianism as normally understood is a *prescription* for personal ethics or public policy to seek the greatest good for the greatest number. Here it is merely a *description* of how size and distribution of income may be combined in a measure of what somebody sees as best for society as a whole, without specifying

how people ought to behave. The utility of income function is central to both the measure and the ethical doctrine, but several similarities and differences should be noted: i) The utilitarian doctrine is no less dependent than the utilitarian measure upon differences among people in the concavity of the utility of income function. ii) The utilitarian measure is an average, but the utilitarian doctrine is sometimes seen as combining total population together with the happiness of the typical person in the measure of the "greatest good for the greatest number" which one should seek to maximize.[5] iii) Unlike the utilitarian measure which may, without contradiction, be observed for any community, large or small, the utilitarian prescription must apply to some specific community, one's city, one's country or the entire world including even people yet to be borne. Consider these in turn.

Classical utilitarianism is sometimes looked upon as promoting the maximization of total happiness as the objective of public policy, on the working assumption that people's happiness at any given time and place is an observable fact of nature rather than something seen differently from one person to the next. It is as though the value of α in people's utility functions, as shown in equation (1), were the same for everybody and could somehow be observed, or as though there were a true value of α that people would be mistaken not to respect. The utilitarian measure of economic growth would then be a single society-wide fact. Of the different utilitarian measures of economic growth for the different values of α in tables 2 and 3, one measure would be right, and the others, based upon wrong measures of α, would be wrong. Without that assumption, utilitarianism as a moral doctrine is no less subjective and no less different in its implications from one person to another than the utilitarian measure of economic growth. In particular, people with values of α would differ in their perceptions of the morally-obligated redistribution of income. Like the utilitarian measure of economic growth, it is entirely dependent on the postulated concavity of the utility of income function, mandating redistribution from nothing to full equality of income depending on the concavity of one's utility of income function. [6]

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[5]Jeremy Bentham speaks of "the sum of his total pleasures", "the sum of his total pains" and "the happiness of the community" as though happiness and pains could be measured and added up for any given collection of people. (Bentham, 1789)

[6]Imagine a society with just two people (or two groups with equal numbers of people in each), a rich person with pre-tax, pre-transfer income y_H and a poor person with pre-tax, pre-transfer income y_L. There may be a transfer of income, T, from the rich person to the poor person, but taxation is costly in the sense that, of T dispatched from the rich person, only T(1-k) arrives to the poor. Think of k as deadweight loss in taxation, but suppose k to be the same regardless of T.

The post-tax, post-transfer incomes of rich and poor become $[y_H - T]$ and $[y_L + (1-k)T]$ where T must lie between 0 and $[y_H + y_L]/(2 - k)$ as long as taxation is genuinely redistributive, greater than 0 to ensure that income is transferred from rich to poor (not the other way round) but less than $[y_H + y_L]/(2 - k)$ beyond which rich and poor would change places, the post-tax, post-transfer income of the person who was originally poor exceeding that of the person who was

If there is one true measure of social welfare - based upon the comparability and measurability of people's happiness - I have no idea how that measure might be discovered. Otherwise, the best one can hope for is that people's values of α may not be too far apart. Of course, the general form of the utility function in equation (1) is arbitrary. For example, a person may place a high value on income transferred to people whose initial incomes are less than - say - $20,000, but be unconcerned about the distribution of income among people whose incomes are well above that limit.

The utilitarian measure of economic growth differs from utilitarianism as an ethical doctrine in the former's total disregard of population. A utilitarian measure of economic growth can be for any population, large or small. A doubling of the population with no change in the distribution of income leaves the utilitarian measure unchanged, but, at least on some versions of utilitarianism, it doubles total social welfare and is morally desirable on that account. Let **I** be the income equivalent of average utility in some society. Utilitarianism may be seen as an injunction to maximize n**I** where n is total population, to maximize **I** alone, or perhaps to maximize f(n)**I** where $f' > 0$ and $f'' < 0$ (with perhaps the additional constraint that f' falls to 0 when n exceeds some sufficiently large number). Interpreting **I** as average happiness, the first criterion is classical utilitarianism, to supply the greatest good for the greatest number, the second is the maximization of average happiness, and the third is what Ng (1986, page 379) calls "number damped total

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originally rich.

Consider a person whose utility of income function is as shown in equation (1) for some given value of α. For such as person, the utilitarian mandate to seek the greatest good for the greatest number requires a tax T to maximize social welfare W where

$$W = [y_H - T]^{\alpha} + [y_L + T(1 - k)]^{\alpha}$$

Setting $\delta W/\delta T = 0$, the welfare-maximizing value of T is identified by the first order condition

$$[y_H - T] / [y_L + T(1 - k)] = [1/(1 - k)]^{1/(1 - \alpha)}$$

where $1/(1 - k) > 1$ and $1/(1 - \alpha) > 0$ as long as the person's utility of income function is concave. When the utility of income function is as concave as it can ever be (i.e. when $\alpha = -\infty$), then $[1/(1 - k)]^{1/(1 - \alpha)} = 1$ and post-tax, post-transfer incomes are equalized by setting $T = [y_H + y_L]/(2 - k)$. When all concavity vanishes (i.e. when $\alpha = 1$), the first order condition requires the ratio of post-tax, post transfer incomes, $[y_H - T] / [y_L + T(1 - k)]$, to be made as large as possible. Since T cannot be negative, T must be set equal to 0. In between, an increase in the concavity of the utility of income function reduces α, reducing $1/(1-\alpha)$, reducing $[y_H - T] / [y_L + T(1 - k)]$ and increasing the morally-required degree of redistribution of income as reflected in the tax rate T.

utility".[7]

The relative merits of the three variants of utilitarianism as an ethical injunction depend very much on why population has increased. Average utilitarianism, the maximization of **I** alone, would seem to be the more appropriate indicator of social welfare when population has increased by immigration, where the increase in population in one's country is matched by a decrease elsewhere. We would not want to say that social welfare as the sum total of happiness to be maximized by private behaviour or in public choice has increased for no other reason than that people have chosen to live in one country rather than another. Population growth may influence **I** as more people come to share a given national endowment of resources or because of economies of scale, but that is automatically captured in the definition of **I** and no adjustment for the international distribution of population would seem to be required.

It is less clear how to account for population growth that is not just migration from one country to another, but is the birth of new people in one's country, so that an extra million people in my country is at the same time an extra million people in the world. Here the case for classical utilitarianism or the maximization of number-damped utility seems stronger, the argument being that *all* lives are valuable, lives of people who are, or might be, born tomorrow as well as the lives of people here today. Number-damped utility has the advantage of valuing additional people without at the same time giving rise to Parfitt's (1984) famous "repugnant conclusion" of classical utilitarianism that social welfare is lower in any society, no matter how prosperous, than in some other society, no matter how poor, as long as the latter has a sufficiently large population and as long as people's utilities in that society are all positive no matter how small. Ultimately, the choice among the three variants of utilitarianism as prescriptions for individual behaviour or public choice would seem to depend on what one feels compelled to maximize in one's own behaviour or through the intermediary of the state.

An analogy may be drawn between actual families and the family of man. Parents love their children and know that they would love additional children too, but choose to have fewer children than they might. People are concerned about future generations and would sacrifice a great deal to ensure the continuance of mankind forever, but, beyond some limit, may not much care whether the population a hundred years hence is large or small. Especially in view of the limits on land and natural resources, we are far less concerned, if we are concerned at all, about a permanent decrease in world population from 7.4 billion to 6.4 billion than about a decrease some time in the future from 1 billion to 0.

There is also some question of whose utility is taken into account: one's own exclusively, one's family, one's social class, one's nation or the entire present and future population of the world. It is not unreasonable to suppose that one's sense of obligation to other people varies according to proximity. Clearly, one puts more weight on one's own income than upon the

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[7]For an exposition and critique of these criteria, see Blackorby, Bossert and Davidson (2005).

incomes of other people. Common interest among people in the same nation takes precedence over concern for other people throughout the world, if only because of the requirement for fellow citizens to make public decisions collectively. One is prepared to sacrifice more for the poor in one's own country than for equally poor people throughout the world, though one is concerned with world poverty to some extent.

A standard objection to strict utilitarianism - that one is morally required to seek the greatest good for the greatest number - is that it demands too much. The objection is that people's concern for one another is genuine but limited. We feel an obligation to help people who are less well off than ourselves, directly or through the intermediary of the government, but most people's generosity stops well short of giving one's entire fortune to the poor as one who seeks the greatest good for the greatest number utilitarian ideal would be obliged to do. We admire good King Wenceslas for his generosity, but we would think him a fool if he gave so generously to all yonder peasants that his standard of living became no higher than theirs."Ultimate ends are a matter of pure choice"[8]. It would seem to be a fact of life that, though people are concerned about other people today and about other people yet to be born, one's degree of concern is less than the utilitarian ideal would seem to require. We are prepared to sacrifice for other people but not to that extent.

Welfare of other people, poor people today or people yet to be born, is to a large extent a public good. Just as roads and the police force convey benefits to everybody regardless of how these public services are financed so too does everybody, or almost everybody alive today want civilization to be perpetuated and people to be prosperous tomorrow. Just as the cost of roads and the police force is too great to warrant anybody financing these services all by himself, so too is the cost of perpetuating and assisting future generations. Private charity may help, but improvement in the prospects of of the poor and of future generations must be financed collectively if at all. Alleviation of poverty today and the preservation of mankind tomorrow become a public good that nobody all by himself can do much about but that can be significantly affected by public policy financed by nation-wide, or perhaps world-wide, taxation. The repugnant conclusion vanishes because, beyond some limit, numbers of future people are of little concern to people alive today, though a deep concern for the future is imbedded in each person's utility function. Such considerations are abstracted away in the utilitarian measure of economic growth by the assumption in equation (2) above that utility is a function of income alone.

Promotion of the greatest good for the greatest number makes sense as an ideal for the government of a country when "number" refers to the people of that country alone but where "good" is as they see it and where their good includes the alleviation of poverty and concern for future generations. A difficulty with this formulation is that a government in office is expected to

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[8]The quotation is from Harrod (1936, 145). Harrod adds that "I hold that when people use the terms good, right, virtue, etc., promotion of the common interest is what they mean. I have no reason for promoting the common interest. It is beyond the power of reason to prescribe ends." (page 144).

keep election promises and to serve the constituency that elected it, promises and services which may conform imperfectly to the utilitarian ideal. Campaign promises may benefit segments of the electorate at the expense of the nation of the whole. Nevertheless, a "government house" (Goodin, 1995) utilitarianism where utility is what citizens desire regardless of why they desire it remains as an ideal. A democratically-elected government may also be utilitarian by delegation, placed in office in the expectation that it will maximize happiness of the electorate as a whole. A prime example of this is the food and drug administration which, if it performs its assigned task, maximizes the nation's welfare within the domain of its authority. The government in office may legally overturn decisions of the food and drug administration, but except in rare cases, would be very foolish to do so. This narrowly-defined utilitarianism converts government into a vicarious altruist, serving its citizens exclusively, but in circumstances where citizens demand concern for the poor, at home and abroad, and for future generations.

Government house utilitarianism can be modelled as the maximization by the government of the total welfare, W, of its citizens where

$$W = \sum_{i=1}^{n} v_i \qquad (10)$$

where n is population, and where v_i is the welfare of citizen i including his satisfaction from his own income and his satisfaction from the well-being of other people elsewhere or yet to be born. Specifically,

$$v_i = u(y_i) + \sum_{j=1}^{g} w_{ij} f^i(n_j) I_j \qquad (11)$$

where $u(y_i)$ is person i's utility from his own income and the second expression on the right hand side of the equation is person i's satisfaction from the dollar values of utility in each of the g groups about which he is concerned. The entire population of the world or people yet to be born may be included among the g groups. Groups of people are signified by j. The dollar value of utility per person in group j is I_j. Person i's degree of concern for people in group j is w_{ij}. Person i's weighting of the number of people in group j is $f^i(n_j)$, a concave function to represent a supposed number-damped utility. The government would maximize W with due regard to whatever the constraints may be in transferring income from rich to poor and to the cost of caring for future generations.

In this formulation, government maximizes citizens' expected utility but has no moral preferences of its own. Citizens have moral preferences that a utilitarian government must respect.

The measure of economic growth in this note is utilitarian in the sense that size and distribution of income are combined into a natural indicator of how well off people are becoming over time. Classical utilitarianism is a doctrine that privte actions or public policy should be directed to maximizing the sum of happiness in the nation or the world. A less demanding version of utilitarianism would have government maximizing the welfare of its citizens as they see it, where citizens' welfare, broadly defined, depends to some extent on the welfare of others. Both the utilitarian measure and the utilitarian prescription differ from one person to the next according

to one's value of α, unless people's values of α are the same or some objective value of α can be identified.

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