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Two Sources of Bias in Estimating the Peak of the Laffer Curve

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Abstract: Important as it is for public policy, there is still no consensus about the size of the revenue-maximizing tax rate at the top of the Laffer curve. The purpose of this essay is not to supply a correct rate, but to identify difficulties in doing so. 1) Estimates of the revenue-maximizing tax rate are distorted by the discrepancy between the "elasticity of taxable income" at observed tax rates and as it would become at the revenue-maximizing tax rate, a discrepancy illustrated with reference to tax evasion as the source of the contraction in the tax base in response to increases in the tax rate. 2) When the response of tax revenue to tax rate is through the supply of labour, the Laffer curve may not be humped at all because the supply of labour may expand, rather than contract, in respond to an increase in the tax rate, causing tax revenue to rise more than proportionally to the tax rate all the way up to 100%.

JEL: Classification H21 and H26

Key Words: Duty to Vote, Tax Evasion, Labour-leisure Choice

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[1] I particularly appreciate the very helpful suggestions of my colleague John Hartwick

"In so far as men act rationally, they will at a higher (wage) rate divide their time between wage-earning and non-industrial uses in such a way as to earn *more money* but to work *fewer hours*."

Frank Kinght (1921, 117) quoted by Robbins (1930)

A major source of contention between left and right in contemporary politics is the location of the peak of the Laffer curve, the tax rate at which revenue is maximized so that no higher rate could ever be advantageous to anybody, rich or poor. People on the right see the revenue-maximizing tax rate as relatively low, placing an absolute limit on the size of government and, especially, on provision for the poor. People on the left see the revenue-maximizing tax rate as relatively high, leaving room for a wide range of social policies. Attempts to identify the revenue-maximizing tax rate are at once empirical and theoretical, inferring what the revenue-maximizing tax rate might be from evidence about the impact of observed changes in tax rates upon the size of the tax base and from inferences about how rational taxpayers might respond to changes in tax rates. This essay is on the theoretical side. No attempt is made to estimate the revenue-maximizing tax rate, but rather to identify biases in the way a revenue-maximizing tax rate is estimated.

A revenue-maximizing tax rate is identified as follows: Imagine taxation at a rate t imposed upon an observable base, Y, which may but need not be income, yielding tax revenue R where

$$R = tY \qquad (1)$$

and where Y is an increasing function of (1 - t), the taxpayer's return on the tax base once taxation has been imposed

$$\delta Y/\delta(1-t) > 0 \qquad (2)$$

One way or another, the taxpayer shrinks his observable tax base in response to an increase in the tax rate by working less, concealing income and so on. The effect upon tax revenue of a change in the tax rate is

$$\delta R/\delta t = Y - t\delta Y/\delta(1-t) = Y[1 - \varepsilon t/(1 - t)] \qquad (3)$$

where $\varepsilon = [\delta Y/\delta(1 - t)]/[Y/(1 - t)]$ (4)

which is commonly referred to as "the elasticity of taxable income". As long as $\delta Y/\delta(1\text{-}t)$ is positive, then ε must be positive as well. Tax revenue is maximized when

$$\delta R/\delta t = 0 \tag{5}$$

implying the revenue-maximizing tax rate, t*, to be

$$t^* = 1/(1 + \varepsilon) \tag{6}$$

which is necessarily less than 1 as long as $\varepsilon > 0$. Equation (6) allows the revenue-maximizing tax rate to be derived from the elasticity of taxable income estimated from historical evidence of tax rate and tax base. A selection of estimates of the revenue-maximizing tax rate is presented in table 1.

Table 1: Estimates of the Revenue-maximizing Tax Rate

[These estimates are not really comparable
because they are based upon different assumptions
and because earlier assumptions are modified later on.]

author	revenue-maximizing tax rate	
Stuart (1981), page 1020	70%	estimate for Sweden based up on elasticity of labour supply
Lindsey (1987), table 8, page 202	40%	introduces "new tax response" estimation
Feldstein (1995), table 2, page 565	25% to 50%	elasticity of taxable income estimated to be between 1 and 3
Diamond and Saez (2011), page 171	73%	estimate for top income bracket based upon the "new tax response"
Goolsbee (2000), page 375	between 70% and 100%	elasticity of taxable income estimated to be between 0 and .4

A serious problem in constructing these estimates is to distinguish between changes in the tax base in response to an increase (or decrease) in the tax rate and changes in the tax base reflecting other changes in the economy and destined to occur regardless. It is not sufficient just to observe the tax base before and after a change in the tax rate. To deal with this problem, estimates of the elasticity of taxable income are based upon the comparison of proportional changes in tax bases of two essentially similar groups, one confronted with a tax increase and the other not. Call the groups J and K. Suppose that, initially, the two groups are taxed at the same rate, t, but that, as part of a general change in the tax structure, the tax rate on people in group J is increased to $t + \Delta t$, while the rate on people in group K remains unchanged at t. [Alternatively, Δt may be the difference in the changes in their tax rates.] Both groups' tax bases change over time, but the two groups are presumed to be sufficiently alike that their tax bases would have changed proportionally if the tax increase on people in group J had not occurred. If the percentage changes in the observed tax bases of groups J and K are x and y, then the percentage change in the tax base of group J attributable to the increase in its tax rate is $x - y$.

Suppose that, initially, both groups are taxed at a rate of 40%, that the tax rate on group J is increased from 40% to 44% while the tax rate on group K remains unchanged at 40%, and that, over a period spanning the change in the tax rate, the tax base of group J is observed to rise by 9% while the tax base of group K is observed to rise by 12%. From this information, the elasticity of taxable income is estimated to be

$$\varepsilon = [\delta Y/Y]/[\delta(1-t)/(1-t)] = [.09 - .12]/[- .04/(1 - .40]$$

$$= [.03]/[.04/.6] = .45 \qquad (7)$$

and the estimated revenue-maximizing tax rate becomes

$$t^* = 1/[1 + .45] = 69\% \qquad (8)$$

Attempts to estimate the revenue-maximizing tax rate by some such procedure are referred to as the New Tax Responsiveness literature. There are many complications in this estimation procedure.[2] Typically, groups J and K are people in two adjacent tax brackets observed at times before and after the moment when tax rates on incomes in these brackets are changed.

Both components - [$\delta Y/Y$] and [$\delta(1-t)/(1-t)$] - of the definition of ε in equation (4) present complications. These must be approximated by [$\Delta Y/Y$] and [$\Delta(1-t)/(1-t)$] where ΔY and Δt are inferred from available data. In measuring $\Delta(1-t)/(1-t)$], it must be decided which taxes to take into account. In measuring [$\Delta Y/Y$], a time period must be chosen - from i years before the tax increase to i years after - over which the change in the tax base is observed. With group K as the control group, the estimate of ΔY for group J becomes

$$\Delta Y_J = Y_{J,\,i} - Y_{J,\,-i}[Y_{K,\,i}/Y_{K,\,-i}] \tag{9}$$

where $Y_{J,\,i}$ is the observed tax base of group J in year i,
$Y_{J,\,-i}$ is the observed tax base of group J in year -i,
$Y_{K,\,i}$ is the observed tax base of group K in year i
and $Y_{K,-i}$ is the observed tax base of group K in year -i.

The choice of i matters because the timing of the impact of tax rate on tax base can occur in several ways with very different impacts upon estimates of ΔY_J. Some possibilities are illustrated in table 2. The table is constructed on the assumption that, initially, groups J and K are taxed at the same rate and that their tax bases would have changed proportionally over time in response to economic conditions in society as a whole - specifically, that the tax base in group J would always be twice the tax base of group K - if their tax rates remained the same. Then, at time 0, which may be thought of as January 1 of the year 1, the tax rate is increased on group J, while the tax on group K remains as before.

Ignore for the moment the last two rows of the table which will be explained below. The first column of the table identifies years before (-i) or after (i) the tax increase on people in group J. The second column shows the postulated tax base of the control group K in each of these years. Numbers in this column are chosen arbitrarily. The remaining four columns show four alternative ways for the tax base of group J to respond to the tax increase: There may be no effect, in which case the

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[2]For a detailed description of how the elasticity of taxable income is estimated, see Saez, Slemrod and Giertz (2012).

tax base of group J remains at twice the tax base of group K. There may be a sudden and permanent decrease in tax base of group J starting immediately after the tax increase; specifically, beginning in year 1, the tax base of group J is assumed to 2 less than it would otherwise be. As shown in the second to last column, the tax base of group J may decrease gradually because it is costs less to make changes slowly than to make changes all at once. And as shown in the last column, there may be an increase in the observable tax base before the tax increase followed by a decrease afterwards, as taxpayers divert taxable income from a time when the tax rate is relatively high to a time when the tax rate is relatively low. This final possibility is illustrated - with no effect of the tax increase on the tax base except in the years immediately before and immediately after the tax increase.

Table 2: Estimating the Effect upon the Tax Base of Group J of a Tax Increase at Time 0 on Group H but not Group K

		alternative assumptions about the effect of the tax increase at time 0 upon the tax base of group J			
years (i) before (-) or after (+) the tax increase	tax base of group K in year i, $Y_{K,i}$	no impact of taxation	permanent and immediate contraction of tax base	gradual contraction of the tax base	switching some tax base from year 1 to year -1
-3	9	18	18	18	18
-2	8	16	16	16	16
-1	10	20	20	20	23
1	12	24	22	23	21
2	11	22	20	20	22
3	10	20	18	17	20
ΔY_J {-1 to +1}	----------	0	-2	-1	-6
ΔY_J {-3 to +3}	----------	0	-2	-3	0

In year -1, the observable tax base rises from 20 to 23. Then, in year 1, tax base falls from 24 to 21. Switching may be undertaken in several ways, among them by realizing capital gains or by cashing in stock options before rather than after a tax increase. Opportunities for switching are likely to be much greater for the wealthy than for the poor.

The last two rows in the table show how the estimate of ΔY_J changes in response to the length of the span over which it is observed. The second to last row is for the change over adjacent years, -1 and 1. The last row is for a change over six years from year -3 to year 3. Obviously, ΔY_J is 0 whenever the tax increase has no impact on the tax base. A immediate and permanent reduction in the tax base causes the estimate of the reduction (-2) to be the same regardless of the time period over which it is observed. A gradually increasing impact of the tax increase causes the estimated reduction in the tax base to be less when observed from year - 1 to year 1 than when observed from year -3 to year 3. Switching is just the opposite. Observed from year -1 to year 1, the tax base falls by 6, i.e. $\Delta Y_J = - 6$. Observed over a six year period from year -3 to year 3, there is no change in the tax rate at all. The difference between Goolsbee's high estimate of the revenue-maximizing tax rate in the last row of table 1 and the lower estimates in the rest of the table is due to this phenomenon. Additional complications in the estimation of ΔY_J arise from the possibility that, even without the tax increase, forces in the economy might have caused the tax bases of groups J and K to change at different rates.

From here on, a general observation about the New Tax Responsiveness literature leads to the two propositions which are the subject of this essay. The general observation is that the New Tax Responsiveness literature is a black box generating estimates without grounding in any specific mechanism - by which outcomes emerge from rational self-interested behaviour - connecting tax base to tax rate. The two propositions are that

a) when the mechanism connecting base to rate is tax evasion, equation (6) might be misleading because the elasticity of taxable income, ε, is not independent of the tax rate, and

b) when the mechanism connecting base to rate is the labour-leisure choice, an increase in the tax rate need not induce taxpayers to diminish their tax base.

That the New Tax Responsiveness Literature is a black box has one great advantage and several disadvantages. The great advantage is that in estimating the elasticity of taxable income one need not know which of the many forces that might affect the tax base are at work. The tax-induced contraction of the tax base may be by the labour-leisure choice, increased do-it-yourself activity, illegal tax evasion, legal tax avoidance, out-migration of highly-taxed people, or some combination of these. Only the final outcome matters.

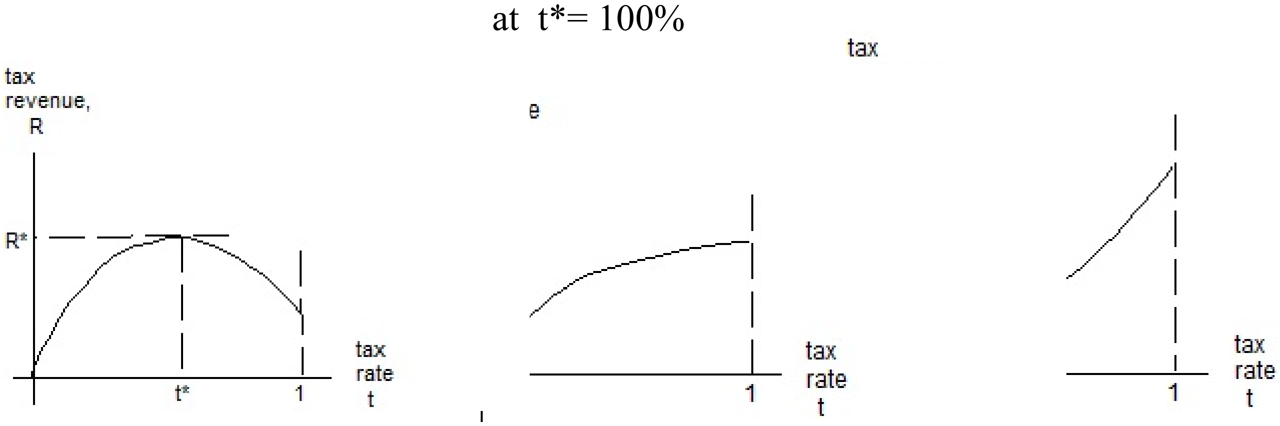
The flip-side of this virtue is that appropriate public policy may depend on why taxable income is elastic. Tougher enforcement of the tax code might be advantageous if and only if tax evasion is a major influence on the elasticity of taxable income. An understanding of the innards of the black box might be helpful in deciding which of the estimates of of the revenue-maximizing tax rate in Table 1 is more nearly correct. Also, as the sample of tax changes is small, it would be helpful to know whether the direction of the influence of tax rate on tax base is a necessary consequence of rational behaviour or just an observed fact at some given time and place. The two principal sections of this essay deal with the constancy or variability of ε when it is a reflection of tax evasion, and with the effect of tax rate on tax base when they are connected through the labour-leisure choice.

Figure 1: Three Possible Shapes of the Laffer Curve

a) A Humped Curve

b) Concavity with Revenue Maximized at $t^*= 100\%$

c) Convexity



Three possible shapes of the Laffer curve are compared in figure 1. The Laffer curve is generally believed to be humped, as in figure 1a, but there is still some question as to whether that is a logical requirement in the sense that people would have to be irrational otherwise. Are there plausible circumstances where the revenue-maximizing tax rate might rise to 100%? It will be shown that the revenue-maximizing tax rate can rise to 100% as shown in figure 1b and that even a convex Laffer curve as shown in figure 1c is not inconsistent with models of rational self-interested behaviour. Each of the three shapes is possible.

Tax Evasion and the Dependence of the Elasticity of Taxable Income on the Rate of Tax

Suppose for the moment that the elasticity of taxable income is a reflection of tax evasion exclusively where tax evasion is costly but undetectable. The phrase "underground economy" is being taken literally. In response to taxation, a part of one's income is hidden underground where the tax collector cannot find it, with no risk of detection or punishment once the required cost of concealment has been borne. The cost of concealment must be an increasing function of the proportion of income concealed so that, the higher the tax rate, the larger the proportion of one's income it becomes advantageous to conceal. This analysis of costly concealment applies equally-well to the switch from taxable work for pay to untaxable do-it-yourself activities or to legal but expensive accounting ploys to reduce one's tax bill. Special features of the labour-labour choice will be discussed in the next section.

The tax base is income, Y, as observed by the tax collector and as distinct from the original base, $\underline{Y}$, as it would be in the absence of taxation. When taxation at a rate t is imposed, it becomes advantageous for the taxpayer to conceal a proportion τ of the tax base where the cost of concealment, $C(\tau)$, depends upon the amount concealed. Let $c(\tau)$ be $C(\tau)/\underline{Y}$, the cost of concealment as a proportion of the original tax base. Assume that $\delta c/\delta\tau > 0$ and that $\delta^2 c/\delta\tau^2 > 0$, so that the cost of concealment is an increasing function of the fraction, τ, concealed and that the marginal cost increases as well. Add the simplifying assumption that

$$\delta c/\delta\tau = \beta\tau \qquad (10)$$

where β is a technical parameter signifying the efficiency of tax collection varying from 0 when no tax collection is possible to ∞ when tax avoidance is thwarted completely.[3] The higher β, the more costly does any given contraction of the observable tax base become and the smaller the proportion of pre-tax income concealed. An implication of equation (10) is that the proportion of one's original tax base concealed is independent of whether one's original income is large or small. All that matters is the efficiency of tax collection.

On these specifications, tax revenue, R, acquired from a person whose original income is $\underline{Y}$ becomes

$$R = \underline{Y}(1 - \tau)t \qquad (11)$$

and tax revenue as a proportion of the original tax base, r, becomes

$$r = R/\underline{Y} = (1 - \tau)t \qquad (12)$$

The full cost of taxation of the taxpayer who hides or removes a proportion τ of the original tax base becomes

$$\underline{Y}(1 - \tau)t + C(\tau) = \underline{Y}[(1 - \tau)t + c(\tau)] \qquad (13)$$

Seeking to minimize the full cost of taxation, the taxpayer confronted with a given tax rate t chooses τ such that

$$\delta[(1 - \tau)t + c(\tau)]/\delta\tau = 0 \qquad (14)$$

implying that $$t = \delta c(\tau)/\delta\tau = \beta\tau \qquad (15)$$

The taxpayer's choice of the proportion, τ, of the original tax base to evade is illustrated on figure 2 with the tax rate, t, on the vertical axis and the proportion of income concealed, τ on the horizontal axis.

The taxpayer conceals income up to the point where it is cheaper to pay the tax instead, as shown in equation (15) and illustrated in figure 2 by the crossing of the line showing the marginal cost of concealment with the horizontal line at a

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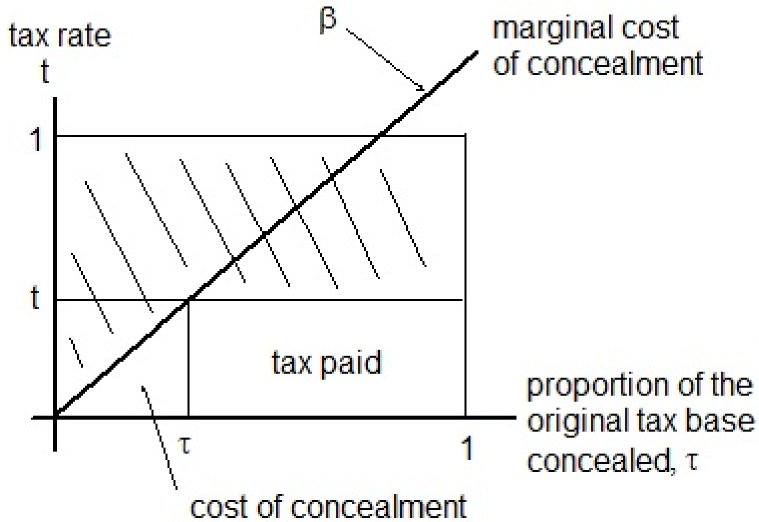
[3]No account is taken here of the cost to the government in detecting tax evasion or of the risk to the tax evader of punishment if detected. On the risk of detection, see Yitzaki (1987).

height t above the horizontal axis.

Figure 2: Taxpayer's Choice of the Proportion of the Original Tax Base to Conceal

(The shaded area is the taxpayer's share of the original tax base remaining after taxation and expenditure to avoid taxation.)



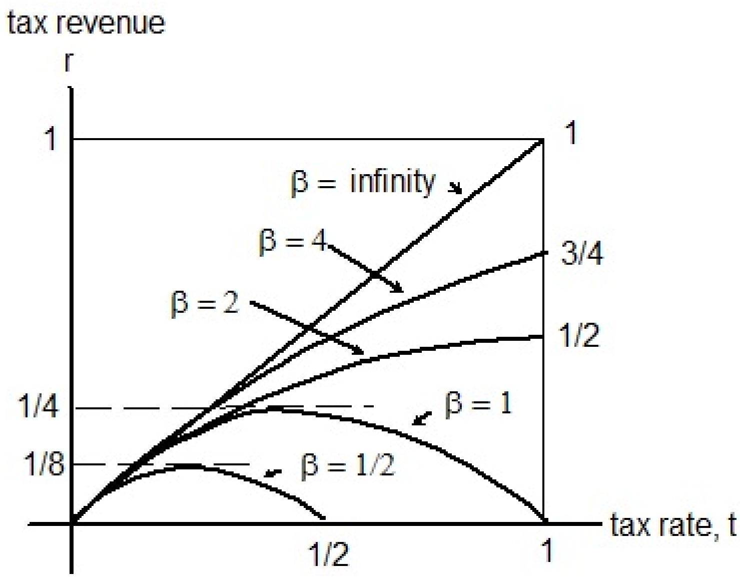
The, Laffer curve is now easily derived. From equations 11 and 15, it follows that, as a proportion of the original tax base, revenue, r, is

$$r = R/\underline{Y} = (1 - \tau)t = (1 - t/\beta)t = (t - t^2/\beta) \qquad (16)$$

which is maximized at a tax rate β/2 as long as the revenue-maximizing rate is less than 1, and at a tax rate of 1 (that is 100%) otherwise. Equation (16) shows revenue as a function of the tax rate t and the efficiency of tax collection β.

For several values of β, the corresponding Laffer curves are illustrated in figure 3 with r on the vertical axis and t on the horizontal axis. Regardless of β, all Laffer curves begin when t = 0 at a slope of 45 degrees. At β = ∞ where tax collection is so efficient that no contraction of the tax base is possible , the Laffer curve becomes a diagonal straight line with a maximum revenue of 1 when t = 1, meaning that the entire tax base may appropriated by the imposition of a tax rate of 100%. Otherwise, the slope diminishes with t to a greater or lesser extent depending on the value of β. As β falls from ∞ to 4 to 2 to 1 to ½, maximal revenue as a proportion of the original tax base falls from 1 to 3/4 to ½ to 1/4 to1/8. Maximal revenue is attained at a tax rate of 1 as long as $\beta \geq 2$. Only when $\beta \leq 2$ is the Laffer curve humped. If $\beta = ½$, revenue is maximized at t = 1/8.

Figure 3: Five Laffer Curves for Different Efficiencies of Tax Collection
(Figures not drawn exactly to scale)



When tax revenue is diminished by tax evasion, the Laffer curve is always concave, but not necessarily concave enough to generate a maximal revenue at a tax rate of less than 100%. The true magnitude of β is an empirical matter, but β is difficult to measure.

For any given β, it must be the case that $Y = \underline{Y}(1 - \tau) = \underline{Y}(1 - t/\beta)$ so that

$$\delta Y/\delta(1\text{-}t) = - \delta Y/\delta t = \underline{Y}/\beta \qquad (17)$$

and

$$\varepsilon = [(1\text{-}t)/Y]\delta Y/\delta(1\text{-}t)$$

$$= [(1\text{-}t)/\underline{Y}(1 - t/\beta)][\underline{Y}/\beta] = (1\text{-}t)/[(1 - t/\beta)\beta]$$

$$= (1\text{-}t)/(\beta - t) \qquad (18)$$

The first of the two main propositions in this essay follows immediately from equation (18). Since the elasticity of taxable income is dependent on the tax rate, a correctly observed elasticity at one tax rate need not be valid at another.

The elasticity of taxable income, ε, equals 1/β when t is set equal to 0, but either increases of decreases with t depending on whether β is less than or greater than 1. For β > 1, equation (18) shows that, the higher the tax rate, the lower the elasticity must be, so that equation (6) yields an underestimate of the revenue-maximizing tax rate when the elasticity is observed at a lower tax rate. When β is exactly equal to 1, the value of ε is 1 as well, regardless of the value of t, and t^{max} is 50% exactly as indicated in equation (6) above. Otherwise, t^{max} may or may not rise to 100% depending on whether or not β > 2.

Suppose, for example, that β = 2. At that value of β, the value of δr/δt is positive for all values of t from 0 to 1, meaning that r can only be maximized at a tax rate of 100%. Nevertheless, if ε were estimated in the usual way when the actual tax rate happened to be 25%, it would be inferred from equation (18) that

$$\varepsilon = (1-t)/(\beta - t) = (1- .25)/(2.1 - .25) = .405 \qquad (19)$$

and the revenue-maximizing tax rate would be estimated, in accordance with equation (6) to be about 71%.

The assumed invariance of ε in the "new tax responsiveness' literature may be a residue of the older "supply of labour model" where hours of work, H, is a diminishing function of the after-tax wage, w(1-t). The tax base becomes wH. The taxpayer's share of the base becomes (1-t). The elasticity of taxable income becomes the elasticity of supply of labour. The elasticity of taxable income is independent of t as long as the elasticity of supply of labour is constant. The analogy between tax evasion and the labour-leisure choice is close but not perfect. A tax rate of 100% destroys all incentive to work because the worker is left with nothing, but hard work may still be worthwhile when the tax base is contracted by tax evasion because a proportion of one's produce can be hidden from the tax collector.

With many routes from tax rate to tax base, ε may possibly be invariant as the new tax responsiveness literature would suggest, but that is by no means assured. Instead of assuming that Y is a function of (1-t) as in equation (2), suppose

$$Y = Y(t, R) \qquad (20)$$

where δY/δt < 0 and δY/δR > 0 because a part of any additional revenue would be devoted to thwarting tax evasion and to public services enhancing the productivity of labour and capital. The impact of a tax increase on tax revenue becomes

$$\delta R/\delta t = Y + t[\delta Y/\delta t + (\delta Y/\delta R)(\delta R/\delta t)] \qquad (21)$$

or, equivalently, $\varepsilon_{Rt} = [1 + \varepsilon_{Yt}]\backslash[1 - \varepsilon_{YR}]$ (22)

where $\varepsilon_{Rt} = (\delta R/\delta t)(t/R)$ which is the elasticity of revenue with respect to tax,

$\varepsilon_{Yt} = (\delta Y/\delta t)(t/Y)$ where ε in equation (5) above is equal to $[- (1\text{-}t)/t]\varepsilon_{Yt}$, and

$\varepsilon_{YR} = (\delta Y/\delta R)(R/Y)$ which is the elasticity of tax base with respect to tax revenue which could be negative but cannot exceed 1.

The revenue-maximizing tax rate is that for which $\varepsilon_{Yt} = - 1$. It would seem likely that the absolute value of ε_{Yt} increases steadily from something less than 1 at low values of t to something greater than 1 at high values of t, but there is no assurance that ε_{Yt} is proportional to t/(1-t) as is required to keep ε constant. The following section is about how ε_{Yt} might vary with t when the route from tax rate to tax base is the labour-leisure choice.

The Labour-leisure Choice

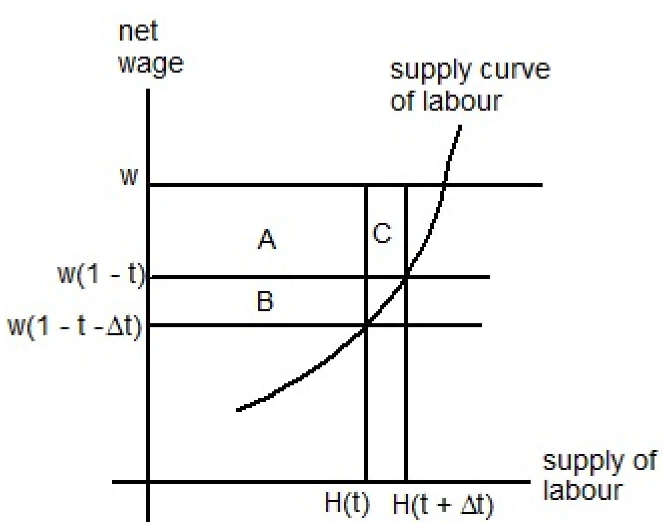
If the quotation from Frank Knight at the beginning of this essay is correct, the labour-leisure choice can never give rise to a humped Laffer curve because, when the rational response to a wage increase is to earn “more money”, tax revenue must necessarily increase too. Lionel Robbins, quoting Knight, disagreed, arguing instead that a rational person may or may not earn more money depending on the “elasticity of demand for income in terms of effort” (Robbins, 1930, 123). It is to be argued here that Robbins is correct. Even so, questions remain about when a tax increase leads to an increase in tax revenue, when a tax increase leads to a decrease in tax revenue, and the likelihood of each effect. The story will be told twice, once focussing on the elasticity of demand for goods and again focussing on the elasticity of substitution in use between goods and leisure, with emphasis on different aspects of the problem.

Replace the simple world where people are endowed with different incomes by a slightly more complex world where people are endowed with the same allocation of time and can transform some of their time into goods at prices, graduated in time rather than money, that differ from one person to the next. The fundamental choice in this world is between "goods" and "leisure" where the price of goods in terms of leisure is the inverse of one's post-tax wage and where "labour" is one's *expenditure* of time for the purchase of goods. Like any expenditure, expenditure of time may increase or decrease in response to the price of whatever the expenditure is for. Thus, when public revenue is raised by an income tax and when income is large or small depending on the taxpayer's supply of labour, tax revenue may increase more or less rapidly than the tax rate, and the Laffer curve may be concave or convex depending on whether the taxpayer's expenditure of time for the acquisition of goods is increased or decreased. Causation can go either way depending on the elasticity of demand for goods, in particular, as will be shown below, on whether the absolute value of the elasticity of demand for goods is greater or less than 1. The common presumption that the elasticity of supply of labour is upward-sloping is somewhere between misleading and wrong, wrong not in the sense that the supply curve of labour must be backward-bending, but in the sense that it may easily be so.

From the Slope of the Supply Curve of Labour to the Concavity of the Laffer Curve: Confining the influence of taxation to the labour-leisure choice, there is a revenue-maximizing tax rate of less than 100% if and only if the supply curve of labour is upward-sloping. Otherwise when the supply curve of labour is backward-bending, an increase in the tax rate leads to an increase in tax revenue all the way up to 100%.

Figure 4: Upward-sloping and Backward-bending Supply Curves of Labour

a) An Upward-sloping Supply Curve　　　　b) A Backward-bending Supply Curve



An upward-sloping supply curve of labour is shown on the left-hand side of figure 4 with net (after tax) wage on the vertical axis and hours of labour on the horizontal axis. When the tax rate is t, the net wage is w(1 - t), the supply of labour is H(t) and tax revenue, R(t), is twH(t) as represented by the area A + C. When the tax rate rises to t + Δt, the net wage falls to w(1 - t - Δt), supply of labour falls to H(t + Δt) and tax revenue, R(t + Δt), is (t + Δt) wH(t + Δt) as represented by the area A + B. The change in tax revenue, ΔR, brought about by the increase in the tax rate is [area B - area C] which, as is evident from the figure becomes progressively smaller and eventually turns negative as t increases.

Tax revenue is maximized when B = C where

$$B = w\Delta tH \tag{23}$$

and $$C = wt|\Delta H| \tag{24}$$

and where $|\Delta H|$ is the absolute value of ΔH. The elasticity of supply of labour is

$$[|\Delta H|/H] \div [\Delta t/(1 - t)] > 0 \tag{25}$$

which is essentially the elasticity of taxable income, ε, as defined in equation (4) with wH as the tax base. It follows that

$$|\Delta H| = \varepsilon H\Delta t/(1 - t) \tag{26}$$

so that B = C implies

$$w\Delta tH = wt\varepsilon H\Delta t/(1 - t) \tag{27}$$

or $$1 = \varepsilon t/(1 - t) \tag{28}$$

implying that $$t^* = 1/(1 + \varepsilon) \tag{29}$$

where t* is the revenue-maximizing tax rate, guaranteed to be less than 1 as long as ε > 0. Equation (6) above has been reproduced in a roundabout but instructive way.[4]

[4] With appropriate changes in the assumptions, equation (29) is easily converted into the Diamond-Saez formula for the revenue maximizing tax rate in the top bracket of a progressive

By contrast, tax revenue increases steadily with the tax rate - with no revenue-maximizing rate of less than 100% - when the elasticity of supply of labour is negative as illustrated by the backward-bending supply curve of labour on the right-hand side of figure 4. Now, ignoring the little rectangle between area B and area C, a tax increase from t to t + Δt leads to an increase in tax revenue from wtH(t) represented by the area A to w(t + Δt)H(t + Δt) represented by the sum of areas A, B and C. The change in revenue is

$$\Delta R = \text{area B} + \text{area C} = w\Delta tH + wt\Delta H$$

$$= w\Delta tH + wt|\varepsilon|H\Delta t/(1 - t) \qquad (30)$$

so that

$$\Delta R/\Delta t = wH[1 + |\varepsilon|t/(1 - t)] > 0 \qquad (31)$$

for all t less than or equal to 1. Tax revenue becomes an increasing function of t, and the Laffer curve is convex.

Thus, in so far as taxation influences the labour-leisure choice, the Laffer curve may peak at a tax rate of less than 100% if and only if there is an upward-sloping supply curve of labour. The question then becomes whether there are plausible circumstances where a backward-bending supply curve of labour may arise.

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income tax. See Peter Diamond and Emmanuel Saez, (2011), page 170. As it stands, equation (29) is for a proportional tax at a uniform rate. To focus on the top bracket in progressive taxation, let Y^T be the income at which the top rate first takes effect, let Y* be the average income in excess of Y^T for only those taxpayers whose tax base exceeds Y^T and on whom the top rate is imposed, and let t refer to the top marginal tax rate. The major analytical change in equation (29) is that the elasticity of base to rate is constructed for the *marginal* tax rate and the average tax base ($Y^T + Y^*$ rather than Y* alone). The elasticity in equation (29) is converted to

$$\varepsilon = [\Delta Y^*/(Y^T + Y^*)] \div [\Delta t/(1 - t)]$$

B and C in equations (23) and (24) become $B = Y^*\Delta t$ and $C = t\Delta Y^*$. Representing $Y^*/(Y^T + Y^*)$ by "a" and setting B = C yields the revenue-maximizing tax rate

$$t^* = 1/(1 + a\varepsilon)$$

which is the Diamond-Saez formula.

The Liklihood of a Backward-bending Supply Curve of Labour: Recognition of the dependence of the shape of the Laffer curve on the elasticity of demand for labour - whether the supply curve of labour is upward-sloping or backward-bending - leads to the question of whether the backward-bending supply curve of labour is a rare curiosum or an outcome that may be expected to occur from time to time. The supply curve of labour is commonly, but not universally, presumed to be upward-sloping. The quotation from Frank Knight at the beginning of this essay supports the opposite view that the supply curve of labour must be backward-bending. If either is possible, the problem becomes to identify the conditions in which each occurs.

As mentioned above, the supply of labour is an expenditure of time for the acquisition of goods and, like any expenditure, it may increase or decrease in response to an increase in price. To identify the requirements for a backward-bending supply curve of labour, an analogy may usefully be drawn between demand for "goods" in an environment where the taxpayer chooses between goods and leisure and demand for an ordinary good, like carrots.

Let p , q and y be the price of carrots, the quantity of carrots demanded and the income of the consumer. Expenditure, E, on carrots becomes

$$E = pq \qquad (32)$$

The derivative of expenditure with respect to price becomes

$$\delta E/\delta p = \delta(pq)/\delta p = q + p\delta q/\delta p \qquad (33)$$

or equivalently $$\varepsilon_{E,p} = 1 + \varepsilon_{q,p} \qquad (34)$$

where $\varepsilon_{E,p}$ and $\varepsilon_{q,p}$ are respectively the elasticities of expenditure on carrots and the quantity of carrors consumed with respect to the price of carrots, where $\varepsilon_{q,p}$ is negative unless carrots are Giffen goods, and where the absolute value of $\varepsilon_{q,p}$ may but need not exceed 1. The elasticity, $\varepsilon_{E,p}$, of expenditure with respect to price is negative if and only if the absolute value of $\varepsilon_{q,p}$ is greater than 1.

Substituting "goods", G, for carrots and designating the supply of labour as H when total time available is set equal to 1:

- q becomes G,

- p becomes 1/w(1 - t) which, since G = w(1 - t)H, is the price of goods with time as the numeraire,

- E [= pq] becomes G(1/w(1 - t)) which is equal to H,

and the elasticity formula becomes

$$\varepsilon_{H.1/w(1 - t)} = 1 + \varepsilon_{G,1/w(1 - t)} \tag{35}$$

where $\varepsilon_{H,1/w(1 - t)}$ and $\varepsilon_{G,1/w(1 - t)}$ are the "elasticity of hours of labour" and the "elasticity of demand for goods" with respect to the price of goods in terms of time rather than money.

But $\varepsilon_{H.1/w(1 - t)}$ is not quite the elasticity we are looking for. Whether the supply curve of labour is upward-sloping or backward-bending depends on the elasticity - $\varepsilon_{H,w(1-t)}$ defined as [δH/δw(1 - t)][w(1 - t)/H] - of hours of labour with respect to the wage rate, w(1 - t), rather than with respect to the price of goods, 1/w(1- t), in terms of labour. Fortunately, as a matter of simple arithmetic, the one is the negative of the other; $\varepsilon_{H,w(1-t)} = - \varepsilon_{H.1/w(1 - t)}$, so that the analogue of the elasticity formula for ordinary goods becomes

$$\varepsilon_{H,w(1-t)} = - \varepsilon_{G,1/w(1 - t)} - 1 \tag{36}$$

indicating that the supply curve of labour is upward sloping - i.e. $\varepsilon_{H,w(1-t)} > 0$ - if and . only if the demand curve for goods is elastic, meaning that the absolute value of the elasticity of demand for goods is greater than 1, i.e. $|\varepsilon_{G,1/w(1 - t)}| > 1$. Otherwise, if $- 1 < \varepsilon_{G,1/w(1 - t)} < 0$, the supply curve of labour must be backward-bending.

An increase in the tax rate, t, lowers the effective wage of labour, w(1- t), raising the price of goods in terms of time. The rise in price of goods leads to a fall in quantity of goods demanded, G, because the demand curve is downward sloping. The fall in quantity of goods demanded is sufficient to outweigh the rise in price, reducing expenditure of time, H, to acquire goods in accordance with an upward-sloping supply curve of labour if and only if the demand curve for goods is elastic. Otherwise, if the demand curve for goods is inelastic, the fall in the effective wage rate leads to an increase in expenditure of labour, so that the supply curve of labour is backward-bending. As there is no strong reason, a priori, for supposing the demand curve of goods to be elastic rather than inelastic, or vice versa, there can be no strong reason for supposing the supply curve of labour to be upward-sloping.

Note, finally, that the elasticity of revenue, R, with respect to the tax rate, t - the elasticity along the Laffer curve - is connected to the elasticity of demand for goods by the formula

$$\varepsilon_{R,t} = [1/(1\text{-}t)][1 + t\varepsilon_{G,1/w(1-t)}] \qquad (37)$$

so that[5] the Laffer curve is upward-sloping - i.e. $\varepsilon_{R,t} > 0$ - as long as

$$|\varepsilon_{G,1/w(1-t)}| < 1/t \qquad (38)$$

which is virtually certain to be true at low tax rates, but may remain true at t = 1.

Convexity is a stronger requirement. The Laffer curve would be convex - i.e. $\varepsilon_{R,t} > 1$ - whenever the demand curve for goods is inelastic, that is, whenever $|\varepsilon_{G,1/w(1-t)}| < 1$.

A distinction must be drawn here between the supply curve of labour as a whole, taking no account of the distribution of skill or variation in workers' preferences and the supply curve of labour to one particular firm. To the firm, the supply of labour may well be flat, or it may be upward-sloping because additional workers must be bid away from other firms. In the aggregate, the supply curve of labour may be backward-bending.

---

[5]For any x and y, $\varepsilon_{x,(1-y)} = -\varepsilon_{x,y}[(1-y)/y]$. Therefore, and since w is a constant,

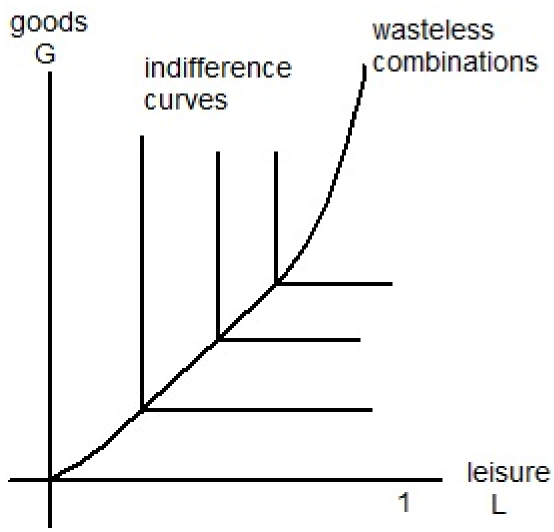
$$\varepsilon_{H,t} = -[t/(1\text{-}t)]\varepsilon_{H,w(1-t)}$$

Substituting for $\varepsilon_{H,w(1-t)}$ from equation (36), the elasticity of revenue, R, with respect to the tax rate, t, along the Laffer curve becomes

$$\varepsilon_{R,t} = 1 + \varepsilon_{H,t} = 1 - [t/(1\text{-}t)][-\varepsilon_{G,1/w(1-t)} - 1]$$
$$= [1/(1\text{-}t)][1 + t\varepsilon_{G,1/w(1-t)}]$$

Perfect Complementarity Between Goods and Leisure: The shape of the Laffer curve has been shown to depend on the elasticity of demand for goods, but this, in turn, must depend on the substitutability in use between goods and leisure. That being so, the analysis here may be extended by passing from demand curves to indifference curves, beginning with the extreme case of perfect complementarity where there is no substitutability at all.

Figure 5: Indifference Curves with No Substitution Between Goods and Leisure



Perfect complementarity between goods and leisure implies that indifference curves are L-shaped as illustrated in figure 5, with goods, G, on the vertical axis and leisure, L, on the horizontal axis. The path along the vertices of all indifference curves, called the "wasteless combinations" curve, is upward-sloping. The wasteless combinations curve shows all combinations of goods and leisure for which neither more goods nor more leisure would increase the person's utility unless combined with more of the other. The curve itself can be represented by a function L(G) which must be such that $\delta L/\delta G > 0$ and $\delta^2 L/\delta G^2 < 0$ to ensure that, no matter how large G, one never runs out of time altogether. Leisure, L, and labour, H, are measured not in hours, but as a proportion of total available time, so that

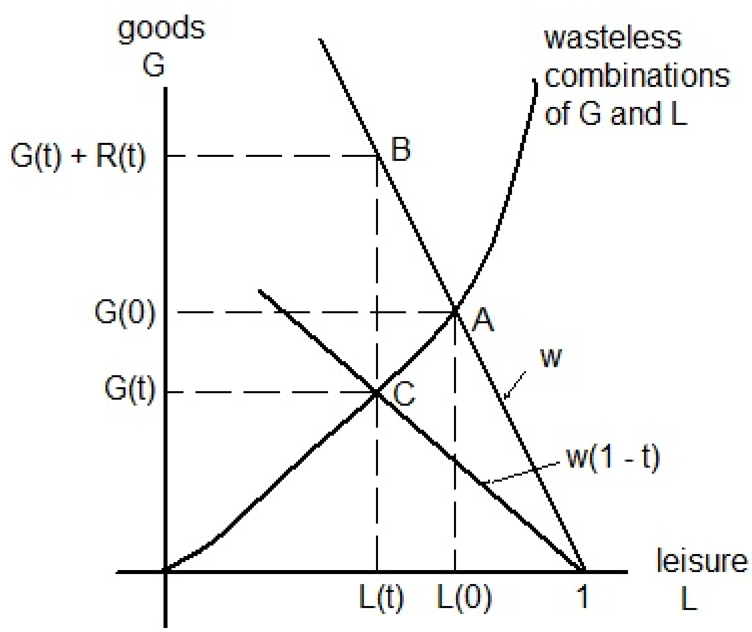
$$L(t) + H(t) = 1 \qquad (39)$$

For any combination of L and G, utility, u, can be represented as

$$u = \text{the smaller of } L \text{ or } L(G) \qquad (40)$$

The response to taxation is illustrated in figures 6 and 7. Figure 6 is an extension of figure 5 with the same "wasteless combinations" curve but with the addition of the person's budget constraints in the absence of taxation and when a tax rate of t is imposed. To avoid cluttering the diagram, the indifference curves are not shown. In the absence of taxation, the budget constraint of a person with a wage w is the diagonal line with slope w originating at the point 1 on the horizontal axis. The highest attainable indifference curve is at the crossing of the budget constraint and the wasteless combinations curve, yielding a combination, G(0) and L(0), of goods and leisure as indicated by the point A. When a tax t is imposed, the taxpayer's net wage falls from w to w(1 - t) causing a counter-clockwise swing in the budget constraint. Once again, the taxpayer, seeking to maximize utility, chooses a combination of goods and leisure at the crossing of the (new) budget constraint and the wasteless combination curve, yielding a combination, G(t) and L(t), of goods and leisure as shown by the point C.

Figure 6: A Person's Response to Taxation



Gross (pre-tax) income is wH(0), net (post-tax) income is wH(t)(1 - t) and tax paid (revenue) as a function of the tax rate, R(t), is wH(t)t. Revenue is shown in Figure 6 as the distance BC between gross and net income at L(t). Revenue, net income and gross income are indicated by R(t), G(t) and R(t) + G(t) on the vertical axis.

Two features of perfect complementarity should be emphasized. First, the supply curve of labour is necessarily backward-bending; the higher the tax rate, the lower the effective wage, the less leisure is demanded and, the more labour is supplied. Second, the Laffer curve is necessarily convex as shown in figure 1c. These are two sides of the same coin. Since

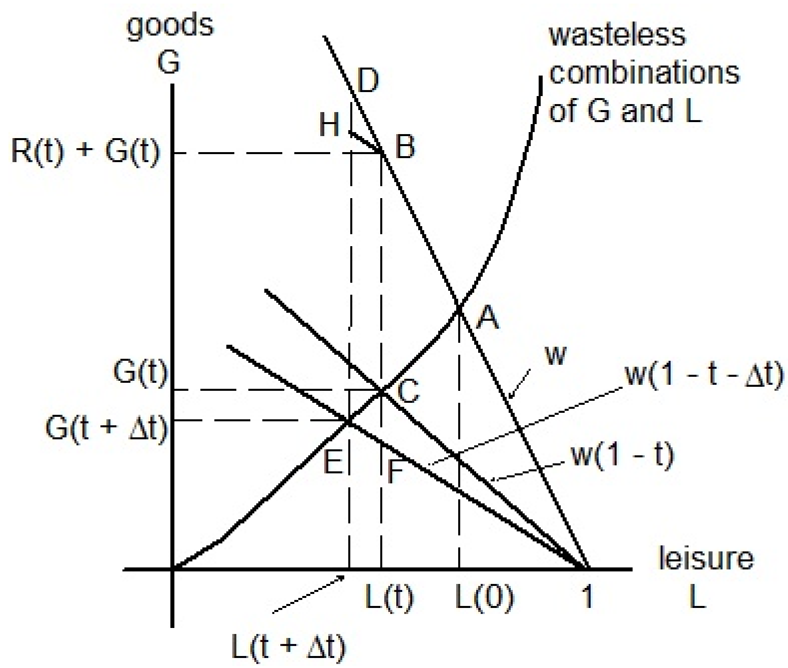
$$R(t) = wH(t)t \tag{41}$$

it must be the case that [6]

$$\varepsilon_{Rt} = 1 + \varepsilon_{Ht} \tag{42}$$

where ε_{Rt} is the elasticity of revenue with respect to the tax rate, and ε_{Ht} is the elasticity of the supply of labour with respect to the tax rate. If labour supply is unresponsive to the tax rate, then $\varepsilon_{Rt} = 1$ and the Laffer curve is an upward straight line with tax revenue increasing in proportion to tax rate all the way up to 100%. If $\varepsilon_{Ht} < 0$ the Laffer curve is concave. If $\varepsilon_{Ht} > 0$, the supply curve of labour must be backward-bending and the Laffer curve must be convex.

Figure 7: Increasing the Tax Rate from t to t + Δt



[6] Equation (41) implies that $\varepsilon_{Rt} = (\delta R/\delta t)(t/R) = wH(t)(t/R) + wt(\delta H/\delta t)(t/R) = 1 + \varepsilon_{Ht}$

That the Laffer curve is convex in this case is illustrated in figure 7. An increase in the tax rate from t to t + Δt is illustrated by a counter-clockwise swing in the budget constraint from slope w(1 - t) to w(1 - t - Δt). The wasteless combinations curve cuts the new budget constraint at the point E. If the supply of labour were invariant - if L remained at L(t) despite the increase in the tax rate from t to t + Δt - then revenue would increase in proportion to the tax rate. In that case,

$$R(t + \Delta t)/R(t) = BF/BC = w(1 - L(t))(t + \Delta t)/w(1 - L(t))(t) = (t + \Delta t)/t \quad (43)$$

generating an upward-sloping Laffer curve on the boundary between concave and convex. But since the supply of labour increases from (1 - L(t)) to (1 - L(t + Δt), revenue increases from BC to DE which is larger than BF, so that the proportional increase in revenue exceeds the proportional increase in the tax rate, making the Laffer curve convex and ensuring that the revenue-maximizing tax rate rises to 100%.[7]

Starvation, Redistribution and Progressivity: One may well protest that this cannot be so, for at a tax rate of 100%, all of one's income is appropriated by the tax collector with nothing left over for the taxpayer at all. There would be nothing to eat. There would be no point in working. The Laffer curve must surely be humped after all.

There are at least three responses to this objection. One response is to drop the assumption that the wage rate is invariant. Another is to relax the so far implicit assumption that all public revenue is devoted to the purchase of public goods with no impact on consumption. A third is to allow for progressive taxation. These responses will be considered in turn.

The simplest way of accounting for the impact of very high taxation upon the welfare of the taxpayer is by dropping the assumption that a person's wage rate is invariant. Instead of postulating a fixed wage w, it might be supposed that one's capacity to work is affected by one's standard of living. It might be supposed that w is invariant as long as $G > G^{min}$ and $L > L^{min}$ where G^{min} and L^{min} are minimal requirements for a person to work well, but that, otherwise,

---

[7] That DE is larger than BF is a matter of simple geometry. The line HB is drawn at slope w(1 - t - Δt) and with the point H on the line from D to E. Since HB is parallel to EF by construction, HE must equal BF so that DE > HE = BF.

$$w = w(G, L) \qquad (44)$$

where $\delta w/\delta G > 0$ and $\delta w/\delta L > 0$ whenever G and L are below minimal requirements

and where
$$w(0, 0) = 0 \qquad (45)$$

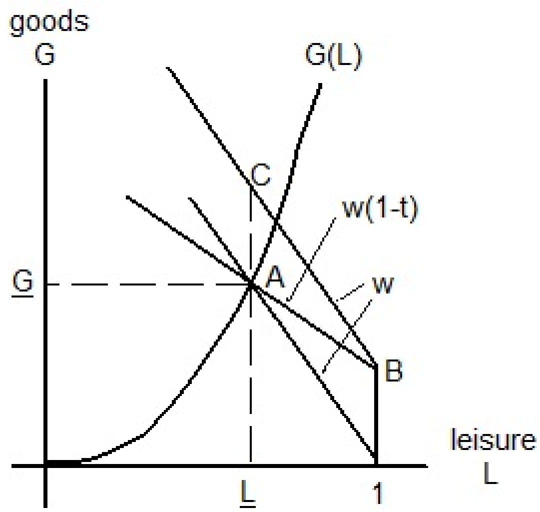
This assumption is sufficient to generate a humped Laffer curve, but it is not the mechanism one usually has in mind. Talk about the Laffer curve is usually with reference to manoeuvres by relatively prosperous people to evade tax. The taxpayer, while not actually starving, reacts to a tax increase by contracting the tax base enough to diminish total revenue.

Alternatively, minimal requirements can be restored through the redistribution of income. Public revenue is used in part for public purposes that do not augment goods or leisure directly and in part redistributed. Expenditure on the army is at one extreme; the negative income tax at the other. It has been assumed so far that public revenue is spent on public goods with no impact on the labour-leisure choice. New features of the Laffer curve emerge when it is assumed that tax revenue is redistributed.

Without abandoning the assumption that goods and leisure are perfect complements, suppose all public revenue is redistributed, increasing consumption of goods without affecting consumption of leisure except in so far as the taxpayer chooses to work more, or to work less, in response to a tax-financed transfer of goods. A distinction is required here between a person's tax paid, R, and subsidy received, S, where the two may be but are not necessarily the same.

A characteristic of such economies is that, regardless of the tax rate, the taxpayer's supply of labour is unaffected when the tax paid and the tax-financed subsidy are the same, as illustrated in figure 8. In the absence of taxation or subsidy, a person with a pre-tax wage w consumes $\underline{L}$ units of leisure and $\underline{G}$ units of goods as represented by the point A. Provision of a subsidy, S, raises one's initial endowment from 1 unit of labour and no goods to 1 unit of labour and S goods as represented by the point B which is a distance S above the horizontal axis. Now suppose that there is imposed a tax at a rate just high enough to pay for the subsidy. Tax revenue, R, must be such R = S. Being no better off and no worse off on account of the tax and subsidy together, the person's behaviour is unchanged. The upward shift in the person's budget constraint brought about the subsidy is exactly matched by the downward shift brought about by the imposition of the tax to finance it.

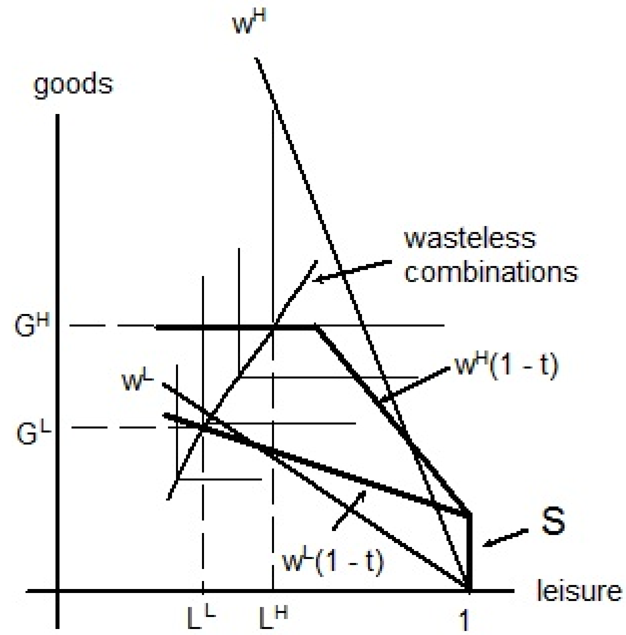
Figure 8: Taxation and Redistribution Leave Labour Supply Unchanged
(R = CA = the height of the point B above the horizontal axis)



For any subsidy, S, there is some tax rate - call it t(S) - that leaves leisure, goods, and the person's supply of labour, L, G and 1 - L, exactly as they would be in the absence of all tax and subsidy. The required tax rate is such that the slope of the post-subsidy budget constraint - the line starting at B and with slope w(1 - t) - passes through the point A. In a society of identical people, tax-financed redistribution neither increases nor decreases the welfare of the taxpayer for whom the subsidy is just equal to the tax paid. This proposition is true if and only if goods and leisure are prefect complements. Otherwise a tax-financed subsidy in a society of identical people makes everybody worse off, for everybody bears the cost of actions - working less or hiding income from the tax collector - to reduce one's tax bill.

In a society where people's wages differ and where total tax revenue is redistributed equally as in a negative income tax , the net beneficiaries of redistribution - people with less than average wage - are induced by the system to work less, and the net contributors are induced to work more. When all public revenue is redistributed equally and when goods and labour are prefect complements, an increase in the tax rate, by making the rich worse off, induces the rich to *increase* their supply of labour, raising additional tax revenue and making everybody else - and, in particular, the median voter - better off than they would be if everybody's labour supply were invariant. Where people's wages differ and where the income distribution is skewed in the usual way so that median wage is less than average wage, the median voter (the person with the median wage) favours a tax rate of 100%.

Figure 9: Progressive Taxation with a Top Marginal Tax Rate of 100%



Though a uniform tax rate of 100% is impossible without redistribution when a minimal consumption of goods and leisure is required, a top marginal rate of as much as 100% remains feasible as illustrated in figure 9. The figure compares two taxpayers with identical L-shaped indifference curves, different wages and confronted with a simple form of progressive taxation. One taxpayer has a high wage, w^H , and the other has a low wage, w^L. The tax schedule has three components: a uniform subsidy, S, a uniform tax rate t on all income less than some specified amount and a tax rate of 100% on all income above that amount. The figure is drawn on the assumption that only the person with the high wage ever earns enough for any of his income to be subject to the higher tax rate. The entire income of the person with the low wage is taxed at the low rate t. Thus, each person has two budget constrains, a pre-tax constraint represented by the upward-sloping straight lines through the point 1 on the horizontal axis, and a post-tax budget constraint represented by the kinked heavy lines. Each person`s chosen supply of labour is where the post-tax budget constraints cuts the wasteless combinations curve, placing him on the highest attainable indifference curve.

The figure could easily have been drawn with more than two tax brackets and with a top rate of less than 100%. The reason for the special assumption about the top bracket is to demonstrate that, though a tax rate of 100% is virtually impossible when all income is taxed at one flat rate, a top rate of 100% becomes possible when goods and leisure are perfect comlpements.[8] In fact, a top rate of more than 100% could be revenue-maximizing on the extreme assumptions that have been made so far. The rationale for these assumptions is not that they are likely to be valid in practice, but to add weight to the argument that the revenue-maximizing top rate may be higher than is often supposed and to serve as preface to the discussion of the substitutability between goods and leisure.

Substitutability in Use Between Goods and Leisure

The results of the preceding section - that a tax increase may increase the supply of labour and that the median voter may prefer a tax rate of 100% - are based upon the assumptions that, apart from redistribution, income is derived from labour alone and that labour and leisure are perfect complements - the former assumption will be discussed briefly at the end of this essay. The latter is the subject of this section.

Perfect complementarity is unnecessarily restrictive in that the results of the preceding section are unlikely to change very much if a smidgen of substitutability is introduced. The questions become how much substitutability is required to turn the Laffer curve from convex to concave and how much more is required to make it humped at some tax rate significantly less than 100%. Recall the result in equation (31) that the Laffer curve is concave if and only if $\delta H/\delta t < 0$, that is, if and only if a tax increase leads to a decrease in the supply of labour. That does not happen when goods and leisure are perfect complements, but it may happen if goods and leisure are substitutable in use.

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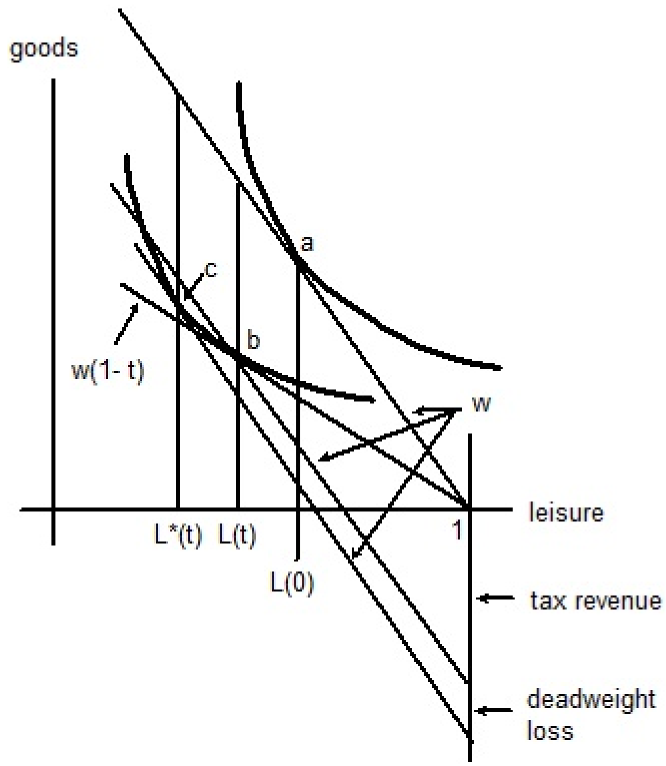
[8]This result is in sharp contrast to the proposition, derived on very different assumptions, that the appropriate top marginal tax rate is 0%. The proposition is derived on the assumption that the income for which the top rate applies can be set so high that it only applies to the additional income earned by the very richest person if and only if that income is not subject to tax. Diamond and Saez, op. cit, page 173 dismiss this result as irelevant in practice.

Figure 10: Deadweight Loss from the Substitutability between Goods and Leisure



Return to the earlier assumption that all tax revenue is devoted to something like national defence with no bearing on the rate of trade off in use between goods and leisure, and introduce a degree of substitutability between them. Figure 10 shows effects of taxation when indifference curves are no longer L-shaped but are, instead, curved as they are usually assumed to be. All income is taxed at a uniform rate, t. The two diagonal lines originating at the point 1 are a person's pre-tax and post-tax budget constraints with slopes w and w(1 - t). The two heavy curved lines are indifference curves; the higher indifference curve is tangent to the pre-tax budget constraint at the point a, and the lower indifference curve is tangent to the post-tax budget constraint at the point b. With no taxation, the demand for leisure is L(0). With taxation at a uniform rate t, the demand for leisure is L(t) which may be greater or less than L(0) depending on the curvature of the indifference curves.

Tax revenue is the difference between income earned and goods consumed. The full cost of taxation to the taxpayer is somewhat larger. It is the largest amount of money the taxpayer would be prepared to pay to have the income tax removed. It is the most the taxpayer could pay as a lump sum without placing himself on a lower indifference curve than that attained when the income tax at a rate t is imposed. Both tax revenue and the full cost of taxation are shown twice in figure 10. Tax revenue is shown as the distance, tw[1 - L(t)], below the pre-tax budget constraint of the point b (where the lower indifference curve is tangent to post-tax budget constraint) and again as the distance labelled "tax revenue" below the point 1 on the horizontal axis. The full cost of taxation is shown as the distance below the pre-tax budged constraint of the point c (where the slope of the lower indifference curve attainable after income taxation is tangent to the pre-tax budget constraint), and again as the sum of the distances "tax revenue" and "deadweight loss" below the point 1 on the horizontal axis. If taxation of income at a rate t were replaced with an equivalent lump sum tax, the taxpayer would reduce his demand for leisure from L(t) to L*(t) directly below the point c. As defined here, the deadweight loss is the area under the compensated demand curve for leisure constructed with reference to the lower indifference curve over the range from L*(t) to L(t).

Tax revenue and deadweight loss play essential roles in cost-benefit analysis. A project is deemed socially-desirable if and only if its ratio of benefit to cost (when all dollars of benefit are valued equally to whomsoever they may accrue) exceeds the "marginal cost of public funds", defined as the ratio of the increase in the "full cost of taxation" to the increase in "tax revenue" brought about by a small increase in the tax rate, where the full cost of taxation is defined as the sum of "tax revenue" and "deadweight loss" shown as distances below the point 1 on the horizontal axis of figure 10.

The impact, L(t) - L(0), of taxation at a rate t upon the demand for leisure (or, equivalently, the supply of labour) can be divided into an income effect, L*(t) - L(0), and a substitution effect, L(t) - L*(t). The income effect is the change in L as it would be if the income tax were replaced by an equivalent lump sum tax, equivalent in the sense that the taxpayer would be left equally well off. The income effect is usually, but not necessarily, negative because a person can be expected to consume less of all goods as income declines. The substitution effect is the change in L when the price of L decreases but utility remains the same. The substitution effect is positive except when indifference curve are L-shaped as shown in figure 5. The distinction between income effect and substitution effect is important in the context of the Laffer curve because, though only the substitution effect matters in the definition of the marginal cost of public funds, it is the total impact of taxation, L(t) - L(0), that matters in the determination of the revenue-maximizing tax rate. The substitution effect is irrelevant except as a component of the total effect of taxation on labour supply.

The question then arises of how large the substitutability between goods and leisure can be before the revenue-maximizing tax rate falls below 100%. Substitutability is introduced by giving the a utility function a constant elasticity of substitution in use between goods and leisure.

$$u = u(G, L) = \{aG^{\rho} + bL^{\rho}\}^{1/\rho} \qquad (46)$$

where a and b are constants, where ρ is a transformation of the elasticity in use between goods and leisure and where the attainable G depends on the wage rate, the tax rate and the supply of labour,

$$G= wH(1 - t) = w(1-L)(1 - t) \qquad (47)$$

so that

$$u = \{a[w(1-L)(1 - t)]^{\rho} + bL^{\rho}\}^{1/\rho} \qquad (48)$$

which depends on L and t rather than upon L and G. The elasticity of substitution in use between goods and leisure, σ, is a transformation of the parameter ρ [9].

---

[9]The elasticity of substitution, σ, is defined as

$$\sigma = [\% \text{ change in } G/L]/[\% \text{ change in } - \delta G/\delta L]$$

$$= \{d(G/L)/(G/L)\}/\{d(u_L/u_G)/(u_L/u_G)\}$$

where $u_G = \{aG^{\rho} + bL^{\rho}\}^{1/\rho-1}\{a\rho G^{\rho-1}\}$ and $u_L = \{aG^{\rho} + bL^{\rho}\}^{1/\rho-1}\{b\rho L^{\rho-1}\}$

so that $u_L/u_G = (b/a/)(G/L)^{1-\rho}$

and $d(u_L/u_G)/d(G/L) = (b/a)(1 - \rho)(G/L)^{-\rho}$

Rearranging the components of the definition of the elasticity, we see that

$$\sigma = \{1/d(u_L/u_G)/d(G/L)\}\{(u_L/u_G)/(G/L)\}$$

$$= \{1/[(b/a)(1 - \rho)(G/L)^{-\rho}]\}\{(b/a/)(G/L)^{1-\rho} /(G, L)\} = 1/(1 - \rho)$$

$$\sigma = 1/(1 - \rho) \tag{49}$$

lying somewhere between 0 (when indifference curves are L-shaped) and ∞ (when indifference curves are downward-sloping straight lines).

Choosing L to maximize u with given parameters, a, b, w, ρ and t, yields a first order condition

$$L/(1\text{-}L) = (a/b)^{1/(1-\rho)} w^{\rho/(1-\rho)} (1 - t)^{\rho/(1-\rho)} \tag{50}$$

A simplifying assumption is helpful at this point. The story remains essentially unchanged if it is assumed that a = b = w = 1, so that

$$L/(1\text{-}L) = (1 - t)^{\rho/(1-\rho)} \tag{51}$$

or equivalently $$H = 1/[1 + (1 - t)^{1-\sigma}] \tag{52}$$

showing the supply of labour as a function of the tax rate and the elasticity of substitution in use between goods and leisure.[10]

The taxpayer's Laffer curve - tax revenue as a function of the tax rate for this taxpayer alone - becomes

$$R = Ht = t/[1 + (1 - t)^{1-\sigma}] \tag{53}$$

Tax revenues computed from equation (53) are shown in Table 3 for the assumed parameters a = b = w =1, for a selection of tax rates, t, from 0 to 100%, and for a selection of elasticities of substitution, σ, from 0 to 5.

[10] From $\sigma = 1/(1-\rho)$, it follows that $\rho/(\rho - 1) = (\rho - 1)/(\rho - 1) + 1/(\rho - 1) = 1 - \sigma$. From $L/(1\text{-}L) = (1 - t)^{\rho/(1-\rho)} = (1 - t)^{1-\sigma}$, it follows that $L = (1 - t)^{1-\sigma}(1\text{-}L)$, so that $L[1 + (1 - t)^{1-\sigma}] = (1 - t)^{1-\sigma}$ and $H = 1 - L = 1 - \{(1 - t)^{1-\sigma}/[1 + (1 - t)^{1-\sigma}]\}$ or, equivalently, $H = 1/[1 + (1 - t)^{1-\sigma}]$

Table 3: Tax Revenue as a Function of the Elasticity of Substitution and the Tax Rate

(The revenue maximizing tax rate for each elasticity of substitution is indicated by *)

	σ = 0	σ = 0.5	σ = 1	σ = 1.5	σ = 2	σ = 3	σ = 5
t = 0	0	0	0	0	0	0	0
t = .1	.0526	.0513	.05	.0487	.0474	.0448	.0396
t = .2	.1111	.1056	.1	.0944	.0889	.0780	.05812 *
t = .3	.1765	.1633	.15	.1367	.1235	.0987	.05808
t = .4	.25	.2254	.2	.1746	.15	.1059 *	.0459
t = .5	.3333	.2929	.25	.2071	.1667	.1	.2071
t = .6	.4286	.3675	.3	.2325	.1714 *	.0828	.0150
t = .7	.5385	.4523	.35	.2477 *	.1615	.0578	.0056
t = .8	.6667	.5528	.4	.2472	.1333	.0308	.0013
t = .9	.8182	.6838	.45	.2162	.0818	.0089	.0001
t = 1	1 *	1 *	.5 *	0	0	0	0

Several features of the table should be noted.

- Since the taxpayer's endowment of time is set equal to 1 and since the wage is assumed to be 1 as well, the maximum conceivable tax revenue, when all available time is devoted to labour and when all income is taxed away, is 1. The maximal revenue is obtained at a tax rate of 100% whenever the elasticity of substitution is less than 1.

- From equation (53), it follows immediately that, regardless of the tax rate, exactly half of the taxpayer's available time is devoted to labour when the elasticity of substitution between goods and leisure is exactly 1, so that $R = Ht = t/2$. The Laffer curve is an upward-sloping straight line with a maximal revenue of ½.

- When the elasticity of substitution is greater than 1, tax revenue is always maximized at a tax rate of less than 100%, because, beyond that rate, the revenue loss from the diversion from labour to leisure caused by an increase in the tax rate exceeds the revenue gain from the higher tax on what is left of the original base. For each value of σ, the maximal revenue and the corresponding tax rate are indicated by *.

- The higher the elasticity of substitution, the lower the revenue-maximizing tax rate. At $\sigma = 1$, the revenue-maximizing tax rate is 100%. At $\sigma = 1.5$, the revenue-maximizing tax rate falls to 70%. At $\sigma = 2$, the revenue-maximizing tax rate falls to 70%. At $\sigma = 3$, the revenue-maximizing tax rate falls to 40%. At $\sigma = 5$, the revenue-maximizing tax rate falls to 20%.

- It is a peculiarity of the utility function with a constant elasticity of substitution that there is a drastic fall from maximal revenue to no revenue at all when the elasticity of substitution goes from 0 to infinity. This differs from the contraction of the tax base by tax evasion where, as shown in figure 3, the Laffer may be humped but not convex.

- There is an imposed symmetry in the utility function of equation (46); the rate of substitution between goods and leisure is assumed to depend on their ratio alone and is unaffected by a proportional increase or decrease in both together. That restriction does not apply to the L-shaped indifference curves in figure 5.

A revenue-maximizing tax rate of less than 100% must be such that

$$\delta R/\delta t = H + t\,\delta H/\delta t = 0 \qquad (54)$$

where, from equation (46), it follows that

$$\delta H/\delta t = (1 - \sigma)(1 - t)^{-\sigma}/[1 + (1 - t)^{1-\sigma}]^2 \qquad (55)$$

An immediate implication of equations (52) to (53) is that tax revenue is maximized at some rate less than 100% if and only if

$$[1 + (1 - t)^{1-\sigma}] = (1 - \sigma)(1 - t)^{-\sigma} \qquad (56)$$

which cannot be so unless $\sigma > 1$ as illustrated in the table.

Much depends on how public revenue is used. It has so far been assumed that revenue is separable in use from goods and leisure so that a person's welfare, can be described as

$$W = W(u(C, L), X) \qquad (57)$$

where u is utility as described in equation (46) above and X is whatever is financed by tax revenue. The effect of an increase in X is to augment the impact of any and every combination of G and L upon the citizen's well-being, with no change whatsoever in the impact of the tax rate upon labour supply.

Alternatively, some or all of tax revenue may be redistributed, augmenting G but having no effect upon L. An extreme form of this is the negative income tax where all tax revenue is redistributed equally among all citizens. Assume instead that an amount T is redistributed so that a person's value of G becomes

$$G = [1 - L]w(1 - t) + T \qquad (58)$$

so that utility of a person with pre-tax wage w becomes

$$u = \{a[(1 - L)w(1 - t) + T]^{\rho} + bL^{\rho}\}^{1/\rho} \qquad (59)$$

With t and T looked upon as invariant, the person chooses L to maximize u, setting $\delta u/\delta L = 0$ so that,

$$\{(1/\rho)u^{-1}\}\{\rho a[(1 - L)w(1 - t) + T]^{\rho-1}(-w(1 - t)) + \rho bL^{\rho-1}\} = 0 \qquad (60)$$

implying that

$$\rho a[(1 - L)w(1 - t) + T]^{\rho-1}(w)(1 - t) = \rho bL^{\rho-1}$$

or $$(24 - L)w(1 - t) + T = [(b/w(1 - t)a)^{1/(\rho-1)}]L$$

so that $$-w(1 - t)dL + dT = [(b/w(1 - t)a)^{1/(\rho-1)}]dL$$

and $$dL/dT = 1/\{w(1 - t) + [(b/w(1 - t)a)^{1/(\rho-1)}]\} > 0 \qquad (61)$$

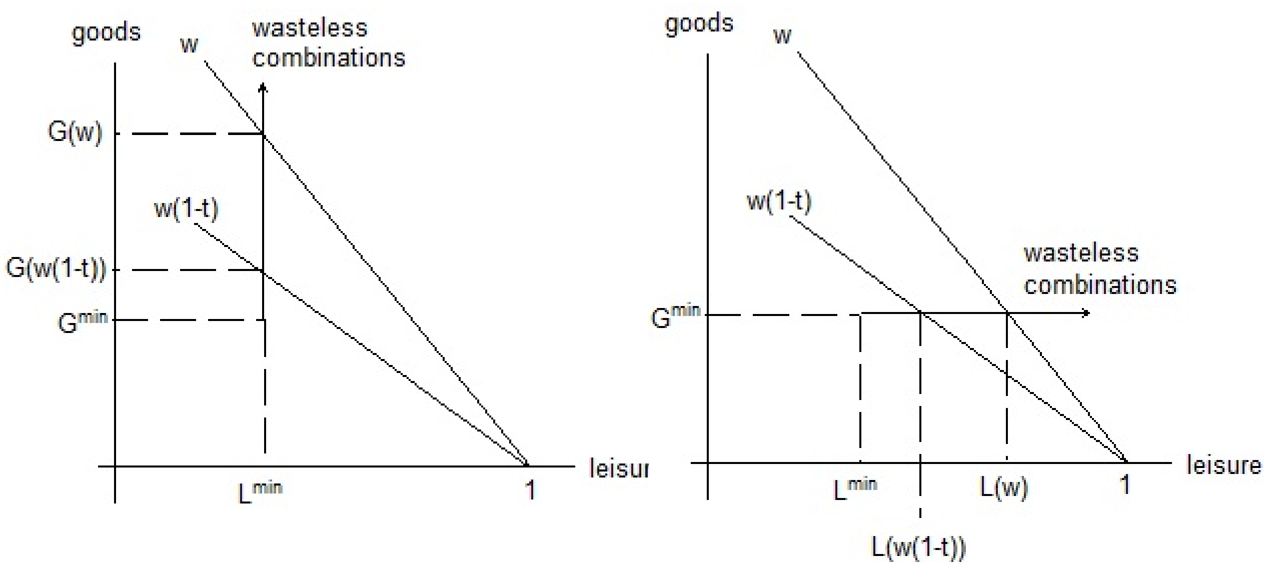
With the tax rate constant, an increase in the transfer T causes the taxpayer to consume more leisure and to decrease hours of work accordingly.

Target Goods and Target Leisure

The special case of perfect complementarity between goods and leisure - where $\sigma = 0$ - becomes more interesting and relevant to actual economies in the light of two additional restrictions that might be imposed. The taxpayer might have a target consumption of leisure, meaning that work is positively enjoyable up to but not beyond - say - 40 hours a week, that work beyond that limit is very burdensome and that he acquires as much goods as possible subject to that constraint. Alternatively, the taxpayer might have a target consumption of goods, meaning that, once the target is reached, all remaining time is devoted to leisure. Perhaps he is an ascetic who requires a minimum consumption to survive but who finds additional consumption distasteful. Perhaps his aim in life is to fly air planes where to do so requires a certain minimal income and where, once this minimal income is acquired, all available time is devoted to flight.

Figure 11: Comparing Targets

a) Target Leisure B) Target Goods



Target consumption of leisure and goods are illustrated on the right-hand and the left-hand sides of figure 11. The two sides of the figure are alike in that there is assumed to be minimal requirements, G^{min} and L^{min}, for goods and leisure, the same pre-tax budget constraint with slope w and the same post-tax budget constraint with slope w(1 - t). The two sides of the figure differ in that, on the right-hand side, L^{min} is the target consumption of leisure and, subject to this constraint, G is chosen by the taxpayer to be as large as possible, while, on the left-hand side, G^{min} is the target consumption of goods and L is chosen to be as large as possible.

With target leisure as shown on the left-hand side of the figure, the wasteless combinations curve is vertical, the supply of labour is invariant and the Laffer curve is an upward-sloping straight line with a revenue-maximizing tax rate of 100%. With target goods as shown on the right-hand side of the figure, the wasteless combinations curve is horizontal, and L is as large as one's budget allows. Taxation at a rate t, reduces leisure from L(w) to L(W(1 - t)) so that the supply curve of labour is backward-bending and the Laffer curve is convex.[11]

The simple story is that, as a tax increase lowers the real wage of labour, the taxpayer may work less, substituting leisure for goods because goods have become relatively expensive, or the taxpayer may work more to preserve his standard of living. A high elasticity of substitution between goods and leisure induces one course of action. A low elasticity of substitution - or in the extreme a target demand for goods - induces the other. The shape of the Laffer curve is determined accordingly.

Context

The Laffer curve has been discussed in this essay as though only two sources of deadweight loss in taxation need be considered, the cost of hiding taxable income from the tax collector and the labour-leisure choice. In fact, there is a much longer list of inter-connected influences.

- Implicit in the discussion of the cost of hiding income from the tax collector is the assumption that tax evasion is costly but undetectable. The cost of concealment

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[11]Scraps of evidence suggest that the supply curve of labour is backward-bending. The shape of the supply curve of labour should be reflected in the historical statistics of hours of work and the real wage. Between 1909 and 1999, real wage per hour (expressed in 1999 dollars) in the United States *rose* from \$3.80 in to \$13.90, but work per week *fell* from 53 hours to 42 hours, suggesting that the supply curve of labour really is backward-bending. See Fisk (2001)

is assumed to depend upon on the proportion of the true tax base concealed, but, once that cost is borne, there is assumed to be no chance of detection. Ignored in this portrayal of tax evasion are the risk to the tax evader and the corresponding cost to the government of detection and punishment. Cost to the government - of running the Internal Revenue Service, of prosecuting tax evaders and, occasionally, of imprisonment - should be accounted for in measuring the marginal cost of public funds and should be deducted from the measure of revenue in the construction of the Laffer curve. The revenue-maximizing tax rate may be reduced once such considerations are taken into account.

- The description of tax avoidance by reducing the supply of labour takes no account of income from land or capital. Almost by definition, the elasticity of base to rate - the value of ε in equation (6) - is zero for taxation of land. Acreage does not shrink in response to an increase in the tax rate. Capital is more complex because of the "double taxation of saving", once when income is earned and again on theyield from investment. Inherited wealth may be less sensitive to taxation than the creation of new wealth.

- Altogether ignored is the possibility of high taxation deterring enterprise and innovation. Better take a safe and steady job if a disproportionate share of the benefit of successful innovation will be taxed away. This is a questionable argument because it takes no account of how tax revenue is spent. Inherently risky innovation may be more encouraged by the infrastructure and social safety net that taxation supplies than discouraged by the taxation required to finance these programs. The would-be innovator who might earn $50 million from his innovation but who might equally-well go bankrupt may not be deterred on balance by a tax bill of $30 million in the event that the innovation is successful if the chance of success is increased by tax-financed roads, bridges and scientific research or when protected from utter destitution in the event of failure by a tax-financed old age pension and socialized medicine. The innovator may work harder to procure a target income when part of gross income is taxed away. Taxation may deter entrepreneurship, but it may equally-well have the opposite effect.[12]

- There may be migrational externalities. High and progressive taxation in any

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[12]Back in 1975, a would-be entrepreneur was considering whether to start a firm to be called Microsoft. He decided against it, and took an ordinary job instead, deterred by the marginal tax rate of 75% at that time.

jurisdiction may drive wealthy people to other jurisdictions where taxes are lower and less progressive. A distinction may be drawn in this context between the Laffer curve as it would be in the absence of migration and as it would be for the remaining residents of a jurisdiction if high taxation drives out many of the principal contributors to a system of redistribution. High taxation alone need have no such effect if combined with desirable and expensive public services, but rich people may be driven away when taxation is to finance a transfer from rich to poor. That is the standard argument for assigning redistributive powers to high levels of government, to provincial governments rather than to municipalities, to the federal government rather than to the provinces.

- High top marginal tax rates may be counter-productive because high income earners may avoid tax in ways that are not available to the rest of society, by complex financial manoeuvres that are excessively expensive unless one has a great deal of income to hide. More-taxed ordinary income may be converted to less-taxed capital gains. Money may be shielded from the tax collector in complex trust funds or in off-shore accounts. The relevant ε in equation (6) may be higher for the rich than for the poor. On the other hand, to oppose high tax rates on the grounds that wealthy people are especially adept at tax evasion is like the patricide who appeals for mercy on the grounds that he is an orphan. The argument is not altogether wrong but it adds weight to the case for closing loopholes as well as to the case for moderate top tax rates.

Taken as a whole, this essay is a collection of arguments about what the tax structure should be, arguments that do not add up to any definitive estimate but that may be of some use in debunking confident assertions about the location of the peak of the Laffer curve and in showing that it is at least possible for the Laffer curve not to be humped at all. Models showing a revenue-maximizing tax rate of 100% prove nothing, but they do cast doubt on claims that social policies are unworkable because the revenue required to finance them simply cannot be acquired. Simplistic as some of the arguments here may be, they are a defence against other, no less simplistic arguments, that looked upon in isolation may be misleading. It is difficult to say where the peak of the Laffer curve may be, but the commonly-told story about the labour-leisure choice is certainly inadequate, for there are circumstances where a tax increase increases the supply of labour.

Finally, a gap between the two main sections of this essay should be recognized. The first section, emphasizing to tax evasion, identifies an important bias in the procedure by which the revenue-maximizing tax rate is estimated. The second, pertaining to the labour-leisure choice, is about the mechanism by which the revenue-maximizing tax rate is determined. The theoretical possibility of very high revenue-maximizing rates would be of little importance if there were solid grounds for confidence in estimates yielding different results. Theoretical possibilities matter when the available estimating procedures are known to be untrustworthy or biassed.

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