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Mining Gold for the Currency during the Pax Romana

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Abstract

We set out a simple four sector macro model of the economy of the Roman Empire during a period of considerable economic prosperity. Our focus is on gold coins as currency and the seignorage which the government used to fund its activities. We solve numerically for a balanced growth representation of the economy of the empire, a solution that captures the intricacies of money creation, currency expansion and seignorage. We subscribe to the view that the exhaustion of low-cost gold and silver deposits contributed significantly to the ending of the economic prosperity enjoyed by Roman Italy and its provinces during the so-called Pax Romana (31 BC to 165 CE) and we attempt to capture significant shifts in variables during the decline.

- JEL Classification: E40; E10; N10
- Key words: Roman money supply; gold coinage; money during Pax Romana

1 Introduction

We are interested in a nation financing part or all of its public sector with seignorage. Seignorage is a surplus associated with the issuing of a new batch of currency: the purchasing value or capacity of the new currency less the cost of manufacturing the new currency.[1] We set out

*Peter Temin commented on an earlier draft based on a two region model and inspired me to reformulate matters as is done below.

[1]Buiter (2007) deals in part with seignorage in the sense we are using the term, but also with the accounts of the central bank fairly generally. The textbooks of Walsh

a four sector model (consumption goods, investment goods, government goods and a gold extraction activity) with an aggregate demand for money and consider injections of new money into the economy in each period. We link the size of these injections to the current size of the public sector and solve for a balanced growth path as a benchmark case.[2] Balanced growth is characterized by commodity prices unchanging period to period while quantities, including inputs, grow at the exogenous rate of population growth. The injections of particular interest to us are those of coins minted from precious metals (gold and silver) in the Roman Empire during its Pax Romana (31 BC to 165 BC[3]). This was something of a golden age for the economy of the Empire and we are interested in (a) how the economy and its money functioned over the period and (b) the possibility that exhaustion of low-cost silver and gold deposits contributed significantly to the "collapse" of the golden age at the end of the second century CE. We take up the case of an unchanging fraction of the production of current government services being paid for currently by citizens and the remaining fraction financed by the infusion of new money (the seignorage[4]), distributed to the owners of inputs to the government sector. This way of injecting new money into the economy plus the assumption of constant returns to scale in the production of goods in the four sectors, and our assumption of a textbook form

(2003; Chapter 4) and Romer (2006; Chapter 10) deal with seignorage. Seignorage is associated with the state issuing currency. When a counterfeiter counterfeit a $20 bill, she seeks the "seignorage", the difference in value between the cost of counterfeiting the bill and the value of goods which the bill allow her to purchase, namely twenty dollars.

[2]We do not attempt to calibrate our model to fit the very few bits of data that are available on the structure of the economy of Roman Italy. Scheidel (2012) draws together many fragments of data on revenues and expenditures of the central government in Rome during the Pax Romana.

[3]Some historians mark the end of the Pax Romana in 165 CE with the arrival of a new widespread disease. 180 CE is the date that Marcus Aurelius died, the last of the so-called five great emperors. One is then left without a precise dating of the end of the Pax Romana.

[4]When obtaining and minting new currency is costly, only part of the value of current new currency is seignorage. The rest is the cost of extraction and minting.

for the aggregate demand for money lets us obtain a growing economy that exhibits balanced growth. Though our attention is on an economic model of the Roman Empire during the Pax Romana, we have in mind that the depletion of high quality gold and silver deposits[5] contributed significantly to the collapse of the peace and prosperity associated with the Pax Romana. For simplicity, we abstract from a range of coin types of different metals that characterized currency to simply gold coin alone. Our extraction technology has constant returns to scale and seignorage takes the form of an exogenous mark-up of the current value of gold produced.[6] We have no explicit depletion of a gold stock in the ground and hence no depletion or Hotelling rents to take account of.

Our model with population growth exogenous has the government making two policy decisions: (1) the selection of a fraction of the current cost of government to be financed with current seignorage (faction γ remains funded by taxes or direct charges) and (2) the selection of the speed of extraction of gold. Essentially the government aims for price stability and a "large" funding of the public sector with seignorage. As the current flow of precious metals contracts, the government is faced with having to raise taxes or fees if it maintains a large public sector and/or a possible debasement of the coinage if it desires to maintain an unchanging nominal ΔM. This appears to be what occurred in the Third Century. There is much physical evidence that the coinage was debased regularly after about 165 CE and anecdotal evidence that rates of taxation became relatively high. The precise dating of episodes of coin quality debasement are known but the dates of tax rate increases and inflation are not known with precision. One must also distinguish

[5] We treat gold and silver deposits as located in the the city of Rome for the most part and thus abstract from actual transportation costs for gold and silver bullion to mints in the Empire. We also consider minting to have been done only in the city or Rome for the most part.

[6] Under constant returns to scale, K_R and N_R produce $(1+\phi)q_R$ of gold coins, with $p_R q_R = rK_R + wN_R$. $\phi p_R q_R$ is the surplus value generated by gold extraction. That is, $\phi p_R q_R$ is the current seignorage.

between debasement of the quality of individual coins and changes in the volume of currency of a particular metallic quality. Toward the end of the Third Century observers recorded the setting in of inflation. One route to inflation is of course rapid debasement of the currency. One exercise we conducted with our model was to explore what the economy, under balanced growth and no inflation, looked like with different values for γ. The idea is to see the implications of smaller injections of new currency. This contraction is offset by a rise in taxes and fees to maintain the government sector (the equilibrium size of the government sector remains endogenous). Under balanced growth different values of γ are associated with different values of ΔM and a different array of prices, prices that remain unchanging as the economy expands in accord with population growth, this latter is given exogenously. We also explore the implications of the government faced with different qualities of ore currently extracted (different values of ϕ). Equilibrium values turn of course on our choice of functional forms for the production functions for our the four commodities in our model and on parameter values used in our numerical solvings.

2 The Roman Economy during the Pax Romana

The early Roman Empire (the early Principate, spanning 31 BC to 180 CE, sometimes referred to as the era of Pax Romana) displayed some remarkable economic progress, particularly in Roman Italy. The era opens with the rule of Caesar Augustus (31 BC to 14 CE) and ends with that of Marcus Aurelius (161 to 180) and includes, toward its end, the consecutive administrations of "the five great emperors",[7] as so designated by historian, Edward Gibbon.[8] Temin (2006) emphasizes that both labor

[7]Nerva (96-98), Trajan (98-117), Hadrian (117-138), Antoninus Pius (138-161), Marcus Aurelius (161-180).

[8]Augustus was succeeded by so-called not great emperors (Tiberius (14-17), Caligula (37-41), Claudius (41-54), Nero (54-68), Galba (68-69), Otho (69), Vepasian (69-79), Titus (79-81), Domitian (81-96)). These in turn were followed by Edward

and capital markets functioned well[9] and banking[10] and long-distance trade flourished during the Pax Romana. Roberts (2011; Chapter 12) reports on the organization of firms and business more generally, among a variety of topics on the politics and economics of Rome.[11] The Empire stretched from modern Spain to modern Syria and from north Africa to northern England. Central were five large cities located around the edge of the Mediterranean Sea. There was considerable trade between widely separated locations[12] and regional specialization. Egypt became

Gibbon's five good or great emperors. Each of the five great emperors was succeeded by his adopted son, except for Marcus Aurelius who was succeeded by his natural son, Commodus, considered a poor quality leader.

[9]The slavery of the Roman Empire involved able individuals rising to senior positions in business, banking and agriculture. Most slaves would no doubt have toiled in laborious jobs. Slaves were denied many basic rights but the life of one on the bottom rung would not have been much different from that of an unskilled free person. Cases have been documented of a very poor free person selling himself into slavery in order to be better fed and housed. There were clear routes for an able slave to gain his freedom and many took advantage of this opportunity. See Temin (2004b). Temin argues that slavery in ancient Rome was an open system in which many slaves succeeded in getting their freedom and as well many slaves occupied positions of responsibility and lived comfortably. Slavery in the United States he argues was a closed system, with very few slaves being manumitted and incentives to work were of the "stick" variety rather than the "carrot" variety. "During the Second Century CE as peaceful conditions sharply reduced the supply of slaves, their average cost quadrupled." (Roberts (2011; p. 208). "At any one time, approximately 150,000 prisoners and slaves labored in Roman mines, 40,000 in the Spanish silver mines alone." (Roberts (2011; p. 218).

[10]Fractional reserves and money multiplication is not taken up by Temin (2004a). Presumably little money multiplication was taking place since the banks were not tightly linked in networks. The contribution of banks to the economy of ancient Rome was presumably in intermediating between lenders and borrower-investors.

[11]Publican societies, forms of joint stock companies, operated in many areas of the economy. The stock owners were usually the large land owners in the Empire. Small businesses were run by freedmen, often with a single slave and a "factory" joined to a residence.

[12]Archeologists point out that the numbers of wrecks of Roman ships in the Mediterranean exhibits a peak during the early Roman Empire. This supports the contention that the economy was doing relatively well at this time and that subsequent centuries were periods of less long-distance trade. (Temin (2004a; p. 729). Temin (2012) is a book-length argument for the view that the Roman Empire was a market-based entity. Roberts (2011; p. 229) cites Lionel Casson: "The Roman man in the street ate bread baked with grain grown in North Africa or Egypt, and fish that had been caught and dried near Gibralter. He cooked with oil from North Africa in pots and pans of copper mined in Spain, ate off dishware fired in French kilns, and drank wine from Spain or France."

the principal source of grain for the city of Rome. Spain became the principal source of gold and silver for the coinage. The standard of living (GDP per capita) in Roman Italy, the central region, attained during the Pax Romana has been estimated to have been at the level of the Netherlands (or Spain or Italy) in 1700. Per capita income in Roman Italy is thought to have been about twice that in the provinces of the Empire. The population of the city of Rome is estimated to have risen to one million over the first century CE. Nero (54 to 68 CE) supervised the re-building to the city's center to make it as splendid as the center of Alexandria after a huge fire consumed large tracts of "low income" housing in Rome, and a near-successor (Emperor Vepasian (69-79)) initiated the construction of the majestic Colosseum in the second half of the First Century. Around 100 CE, Trajan (98-119) supervised the reconstruction of Rome's port at Ostia on the coast of the Mediterranen Sea and had the massive Forum built in Rome itself. His new market in the city of Rome had three levels and comprised some one hundred and seventy-five shops. The Roman Empire is of course famous for its infrastructure: roads, ports, aqueducts, sewers, stadiums, temples and forums. Good infrastructure, particularly ports and roads, obviously helped to promote trade.[13] Showy stone temples and stadiums signalled a permenance and stability to the social system, which in turn probably encouraged orderly economic activity and social relations among individuals and groups more generally. Augustus encouraged Romans

With regard to high income folk, Roberts notes: "By the Antonine period, wealthy Romans dressed in wool from Melitus, linen from Egypt, cotton from Greece or India, and silk from China. Arabia and India supplied gens and pearls, Yeman and Ethiopia send perfumes. Colored marble quarried in Egypt or Anatolia faced Roman houses, and Romans at food seasoned with Indonesian pepper off Spanish silver dishes while quaffing African or Aegean wine in Syrian glasses, all while Greek statues gazed down on set tables of Moroccan lime wood."

[13]Temin (2004a) argues that land was bought and sold in markets. He emphasizes that agricultural societies that are based on mostly agricultural activity will have less market activity because isolated farms will be relatively autarkic. Information moved slowly in the absence of the post, phone and trains. Hence price adjustments would necessarily be sticky in economies before the nineteenth century.

to view him as connected to the prevailing gods and this practice may have contributed to social stability during his time as well as during succeeding administrations.

We focus attention here on the money supply during the Pax Romana. Trade flourished and there was a general absence of inflation. Our interpretation is that Augustus laid down a new system of governance and his successful successors embellished his system in various ways and his unsuccessful successors were unable to destroy the system by their various forms of mismanagement until the latter part of the Second Century. Since Augustus presided over the opening of a two hundred year period of Pax Romana, a distinguishing feature of the period was relative internal and external peace. Central to his system[14] was a balancing of the interests of power-seekers at the center. In addition he institutionalized payments for troops, at reasonable cost, both inside the Empire and at its frontiers. Another component of his system was a stable currency[15] which contributed stability to prices which in turn must have abetted local and long-distance trade.[16] The large dimensions of the Roman Empire as an economic entity were supported by the standardized currency, one that allowed for trade and regional specialization to flourish. The combination of a vast peaceful nation and a uniform system of currency and laws was not seen elsewhere in the world, with the

[14] "the forty-five years of his rule allowed him to painstakingly contruct a new social order" (Roberts (2011; p. 170).

[15] Shortly after the death of Augustus, there was a monetary crisis that failed to inflict lasting damage on the economy of Rome. "There even was a liquidity crisis in 33 CE in which interest rates rose, loans were called in, and land prices collapsed." (Temin (2001, p. 176). See also Roberts (2011; p.168)).

[16] "From the time Rome subjugated the Greek city-states of southern Italy to the beginning of the third century CE, its money supply was large and fairly stable. For instance, the total Roman coinage per capita in the early years of the Principate came to approximately 80 percent of the current US money supply. Rome's sources of silver allowed it to mint enough virtually pure silver denarii to reliably maintain this money supply. After Augustus conquered northern Spain, twenty thousand pounds a year of gold from its mines, joined later by those of Romania, furnished the eight-gram gold aureus as well. This was used for international trade, the payment of taxes, and other large payments." (Roberts (2011; p. 236-37)

possible exception of China,[17] until the early twentieth century. Banking in the private sector was much in evidence during the Pax Romana.[18] A system of laws and courts provided a social infrastructure which abetted business as well as civic order. There were no large-scale revolts by slaves, in contrast with say the period 100 to 31 BC. Augustus is known, among his other accomplishments, for conquering northern Spain and in so doing acquiring rich silver and gold deposits.[19] The bullion from

[17] "As part of the Unification of China, Qin Shi Huang (260 BC – 210 BC) abolished all other forms of local currency and introduced a national uniform copper coin based on the coins previously used by Qin. These coins were round with a square hole in the middle which was the common design for most Chinese copper coins until the 20th century. Due to the low value of an individual coin, the Chinese have traditionally strung a nominal thousand copper coins onto a piece of string. However government taxes were levied in both coins and in products such as rolls of silk. Salaries were also paid in both the Qin Dynasty and Han dynasties in "stones" of grain.

During the early Song dynasty (960–1279), China again reunited the currency system displacing coinages from ten or so independent states. Among pre-Song coins, the northern states tended to prefer copper coins. The southern states tended to use lead or iron coins with Sichuan using its own heavy iron coins which continued to circulate for a short period into the Song dynasty. By 1000, unification was complete and China experienced a period of rapid economic growth. This was reflected in the growth of coining. In 1073, the peak year for minting coins in the Northern Song, the government produced an estimated six million strings containing a thousand copper coins each. The Northern Song is thought to have minted over two hundred million strings of coins which were often exported to Inner Asia, Japan, and South-East Asia, where they often formed the dominant form of coinage. Song merchants rapidly adopted forms of paper currency starting with promissory notes in Sichuan called "flying money" (feiqian). These proved so useful the state took over production of this form of paper money with the first state-backed printing in 1024. By the 12th century, various forms of paper money had become the dominant forms of currency in China and were known by a variety of names such as jiaozi, qianyin, kuaizi, or guanzi." (http://en.wikipedia.org/wiki/Chinese_currency

#Imperial_China.... July 4, 2012.)

[18] "A good finanical system promotes growth, and indeed there appears to have been growth during the Roman Republic and early Roman Empire.... the existence [of growth] is consistent with the development of the Roman financial sophistication described here." (Temin 2004a; p. 729)

"The surprising result is that financial institutions in the early Roman Empire were better than those of eighteenth-century France, albeit not as developed as those of eighteeth-century England and Holland." (p. 729)

Temin never makes a clear claim that a source of great economic efficiency of the early Roman Empire was the presence of a smoothly functioning currency system throughout the Empire. He does note however: "Rural transactions in Rome were made with relatively uniform coins, as in eighteenth-century France, and possibly more easily than in eighteenth-century rural England." (p. 728).

[19] "After Augustus conquered northern Spain, twenty thousand pounds a year of

Spain flowed into the mints of the government of the Empire.

Our interest is in Augustus's monetary system, a system centered on silver and gold coins,[20] minted by the government in Rome as well as in satellite mints in other centers in the Empire. Of special interest is the system of government finance by seignorage. If a gold coin can be produced, including minting, at a cost of say \$Y in terms of capital and labor (ignoring Hotelling rent or user cost) and can then buy goods worth \$X in terms of capital and labor (X>Y), then the producer of the coin has some "free" purchasing power of \$X-\$Y. This surplus is the seignorage which can be used by the government to purchase goods and services. In our model, seignorage supports an explicit government sector (a sector employing labor and capital to produce government goods, flows per period). We have an explicit cost for gold ore extraction and a zero cost for refining and minting.[21] Households have explicit demands for money balances over each period.

Since Roman Italy has been judged to have enjoyed a per capita income about twice as high as that in "the provinces" a complete model of

gold from its mines, joined later by those of Romania, furnished the eight-gram gold aureus as well. This was used for international trade, the payment of taxes, and other large payments." (Roberts (2011; pp. 236-37) Between 200 BC and 200 CE an estimated 100 million denarii were brought to Rome per year from silver mines in Spain. (Rorberts (2011; Table 15, citing Badian (1972; p. 34). Scheidel (2012) reports that gold mines in Spain were yielding 6.5 tons per year (88 million sesteres) and in Bosnia 5.9 tons per year (80 million sesteres) An estimate for the first century BC has 35.4 tons of silver (44 million sesteres) being extracted in Spain. For the first century CE, Scheidel estimates some 200 million sesteres of value from gold and silver mines per year. He is unsure whether this represents net "profit" to the state or gross value of refined ores. "Mineral wealth alone might have covered a sizeable share of total expenditure, perhaps anywhere from a tenth to a quarter."

[20] There were coins of lesser metals in low denominations in addition to gold and silver coins.

[21] "Contrary to the widely held belief that ancient coinage was invented simply to promote trade, most scholars now believe this contention has not foundation. The more likely reason was that coins [rather than gold and/or silver bullion] were actually created for the purpose of facilitating the state's own expenditures for goods and services." Steve Coe on the Internet (http://ezinearticles.com/?Ancient-Silver-and-Gold-

Coinage&id=6225050), March 26, 2012.

the Roman economy would be a two region construct. We gloss over this issue of explicit interregional trade and simply treat the Roman economy as a single closed entity. In addition to the production of government goods and services, our Roman economy produces consumption goods and investment goods under constant returns to scale. Population (the labor force) in Roman Italy is assumed to be growing at a constant rate, presumably very small. We are interested in a period of sustained zero inflation or deflation. Our model ends up with a balanced growth trajectory, with prices unchanging and quantities, including the money stock, increasing at the constant rate of population increase. We assume no technical change or decay in durable capital. Of interest to us, and many others, is that currency problems in the Empire appear to co-incide with the un-ravelling of the prosperity of Pax Romana. Our view is that the currency problems (debasement of the coinage and later inflation) were likely triggered by the exhaustion of low-cost gold and silver deposits in the Empire.

The currency system that Augustus inherited and in a sense institutionalized was based largely on silver coins. Higher "denominations" of currency were gold coins and lower were bronze or copper coins.[22] Our analysis here was motivated directly by the question of the role of money during Pax Romana. However our model of Roman Italy is abstract and stripped-down and has not been calibrated to fit closely the various facts of the Pax Romana. This is in part because the facts, particularly the sizes of flows of gold and silver into Rome, are not well established. We

[22] "The monetary system of Rome was based on the silver denarius... The denarius was divided into four bronze sesterces, which were the common unit of commerce in the early Roman Empire. Sesterces were divided in turn into four copper asses, and the European, Latin set of coins was linked to a Middle Eastern, Greek set by a fixed exchange rate. The silver drachma was the equivalent of the sestertius, and it was divided into six and later seven bronze obols. For calibration, one modius (6.5 kilograms) of wheat cost four to six sesterces on the private market in Roman during the first century CE, and the daily wage was between three and four sesterces." (Temin (2006; p. 138).

remain convinced that the fairly well-documented economic growth of Roman Italy during the Pax Romana was directly tied to the orderly management of money by "the government" and we share the opinion of some historians, that the "collapse" of the Augustan system at the end of the Pax Romana was directly linked to the unravelling of the money control system that the center was able to maintain during the Pax Romana.

We present first some observations, culled from the internet in March and April, 2012.

"Ancient Rome was heavily dependant on trade, and golden coins were the main currency used....

The Romans maintained an Imperial Treasury consisting of gold coins, the treasury was supervised by the senate, and provided the Roman govenment with the necessary fianancial funds to maintain the Empire, pay salaries and import goods from other parts of the world.

Contrary to popular beliefs, the amount of gold produced and used in ancient Egypt was small, accounting for its limited Royal and religious use. The annual production of gold during pharaonic times is thought not to have exceeded one ton per year....

Spain alone, a major producing center in Roman times shipped 1400 tons of gold to Rome every year."[23] ("Gold" the Internet (www.aldokkan.com/art/gold.htm), March 27, 2012)

The scale of mining at Riotinto (in Spain) fundamentally altered the Roman economy. "Basically, it ensured Rome a constant supply of fresh metal for increased minting of silver and lower-denomination copper-based coins," says Jonathan Edmondson (York University). Rome used silver denarii to pay and feed its army, fund public building programs in its capital city, and subsidize the price of (and eventually allow free

[23] Healy (1978) deals exhaustively with Roman mining activity but leaves one without specifics about volumes of high-quality metals produced.

distribution of) grain to the city's residents. But following the invasion of Spain by the North African Mauri in the late second century, mining activity dropped off and the denarius plummeted from 97 percent silver to 40 percent, leading to outsized inflation as Roman minted ever-less-valuable coinage. "The Roman state experienced major problems, since taxes were paid in coin," Edmondson notes. "People started handing over these debased coins in payment of taxes, while hoarding the [older] higher-percentage silver coins." By the fourth century, he says, gold replaced silver as Rome's main currency. (http://factsanddetails.com/world.php?itemid=2050&catid=56&subcatid=369)

When the gold in Spain began to run out, the Romans noticed that there were also gold mines in Dacia (Romania today)... The Romans used the Dacian gold to pay their army... When the Dacian mines ran out of gold,[24] about 275 AD, the Romans abandoned Dacia and went home.[25] (http://www.historyforkids.org/learn/science/mining/gold.htm) ■

Duncan-Jones (1994) has analyzed hundreds of hoards of Roman coins, unearthed by archeologists over centuries in many different locations. In his Chapter 15, he documents the debasement of gold and silver coinage during the Third Century. "A pound of silver was producing twice as many denarii under Severus as it had under Augustus two centuries earlier. By the late Severan period the figure was more than twice what it had been under Antoninus Pius, only seventy years earlier." (p. 228) "The first decisive fall [in precious metal content] took place in AD 64, when fineness was reduced to about 93.5%." (p. 224)[26] "That

[24] Gold was mined by the Romans in Wales, also, presumably in relatively small quantities.

[25] Dacia was occupied from 106 AD to 275, from the time of Trajan's conquest to Aurelian's withdrawl. One might infer that Trajan was bent on exploiting the gold deposits in Dacia and by Aurelian's time, it had become too expensive for Rome to continue occupying the distant province just for its gold.

[26] "Caesar's aureus had 8.2 grams of gold content. Augustus's 7.8 and Nero's 7.25. Although the silver content of the denarius was halved by Trajan's reign (98 to 117 CE), it held it value because 25 denarii could always buy one arueus... There was

second-century armies received cash payment in precious-metal currency in not in serious doubt... No government has access to an unlimited supply of bullion, and in the case of Rome, reductions in the precious-metal content of the coinage suggests strain on the bullion supply." Duncan-Jones moves on directly to attempt to infer how the flow of coins was related to trade in the Empire and to payments to soldiers along the borders. "Since army pay was the biggest running expense of the Empire, the government's dependence on recycled cash to meet the expense was inevitable, and recycling must have taken place. (p. 176). Recycling involved "taxes paid in coin finding their way back from the provinces to Rome... and being sent out again to pay the troops in the provinces." This recycling "should have had a smoothing effect, which would tend to obliterate local characteristics in provincial coin-populations. Yet as we have seen, local characteristics remain clearly visible." (p. 176) "Trade did not "make the coin-population homogeneous throughout the empire. It is worth recalling Gaius' (110 to 179 CE) remark that in his world not only did the prices of grain, wine and oil, but also interest rates varied widely from place to place... that suggests an economy divided into small local cells, rather than something large and unified." (p. 176) We would expect that where transportation costs between places were relatively low, local prices would diverge little, as say between cities on the Mediterranean coast. Once goods had to be moved over land between places, prices of similar products would be expected to remain quite distinct. The fairly popular notion that considerable inter-regional trade took place during the Pax Romana should be taken as correct, until substantial contradictory evidence emerges. The work of Kessler and Temin (2007) on grain prices was pursued in order to shed light on market integration across regions in the Roman Empire during the Pax

little inflation and military pay was increased only slightly throughout this period... Interest rates remained at 4 to 6 percent in Rome and 8 to 12 precent in the provinces until the third century CE." (Roberts (2011, p. 237).

Romana. Temin is one of a number of observers who emphasize that long-distance trade was a distinguishing feature of the good economic performance of the Roman Empire during the Pax Romana. And these same observers note that the volume of this trade shrank considerably during the Third Century CE. Banking activity which has been found to be widespread during the Pax Romana also shrank hugely during the Third Century. There was some loss of coinage from the Empire to India and China. We do not build this into our formal analysis below.

Duncan-Jones's reflections about the recycling of payments to troops by the center are well worth pondering. During the Pax Romana large-scale warfare did not occur. Border security was maintained without large numbers of soldiers. When order is maintained over a period, people become accustomed to orderliness and order can be maintained with small-scale enforcers.[27] The cost of maintaining peace is often low for a period that follows a lengthy stretch of tranquility. Security during the Pax Romana appears to have been not hugely costly. Our view is that local tax revenues could have been adequate to maintain these border forces. Hodge (2002) contends that much infrastructure in the Roman Empire was organized by the military and they in turn could draw on slave workers for much of the physical laboring. Hence though large infrastructure projects in the Empire, such as sewer and aqueduct construction, may have appeared to be costly, they may in fact have required modest boosts in revenues to be successfully undertaken. The military were available to organize construction, given the absence of internal and external violence, and the military could control a large slave workforce on the various projects. The implicit wage of an unskilled slave would have been low. The center may well have supported certain projects in the provinces such as port construction but in our view, there was not a huge flow of revenue from taxation at the center which went out

[27]The British during their colonial period were able to maintain order over large areas and over millions of "subjects" with relatively few troops and police.

to the provinces. As the Third Century unfolded large raises were given to soldiers and large tax increases were imposed from Rome as well. Thus the flow of funds in the public sector during the Third Century was no doubt quite different from that during the Pax Romana. Our view is that indeed large-scale long-distance inter-regional trade took place during the Pax Romana and large flows of funds did not flow from the public sector in Rome to the provinces. We believe that relatively large-scale importing from the provinces was being done by Roman Italy during the Pax Romana and a principal source of funds for this importing was the direct exporting of gold coins. We have not however developed a two region model of the economy of the Roman Empire.

3 The Level of seignorage

One can think of the level of the mark-up as a gap in the cost of production of a coin between the government and an outsider. If outsiders have access to low-quality gold deposits that are associated with $X per unit for a minted coin (X is the cost of mining the gold, processing the ore and minting the coin) and the government monopolizes access to high-quality deposits with $Y per unit for minted coin, then the government is in a reasonable position to maintain the mark-up $[X-Y]. There is then a natural mark-up for gold coins.[28] Of course the cost of keeping outsiders away from high quality deposits is not trivial. The central government must back up control of all high-quality sources of supply to the system with force. Any outsider that is found with a high-quality deposit must have her deposit taken over by the government so that central control of the supply of new coins is maintained. We gloss over quantifying this control cost and including it in our analysis. In fact coins were minted

[28] Unit cost plus mark-up is referred to as the fiduciary value of a coin by Harris (2008). "When the earliest Greek and Roman coins were minted, they presumably had the same value in the marketplace as the equivalent quantity of metal. From very early on, however, states from time to time attempted to establish a conventional value for coins that, as metal, were worth less than their 'fiat' or 'fiduciary' value." (Harris (2008; p. 199).

in serveral places away from the city of Rome, though the latter was the center for minting.[29]

One can argue that Augustus inherited a good currency system and his support for the system led to it functioning smoothly over the long run of Pax Romana.[30] We share the view of some obervers: the depletion of the high quality gold and silver deposits toward the end of the Second Century CE led to a severe shrinkage in the size of the gap, X-Y, and the severe erosion of seignorage as a source of revenue for the center in the Roman Empire. An absense of government revenue from seignorage means that the government must turn to more taxation, the selling of additional government bonds or to clevery-managed inflation. Higher rates of taxation and a high level of inflation were apparent in the century following the Pax Romana.

One naturally reflects on the possibility of an adjustment in the value of a unit of currency in order to restore the previous level of seignorage. Debasement is the form of the typical adjustment and this shrinks the value of seignorage. In addition, any fiddling by the center with the metal content of the silver and gold coins would induce a strategic response by the public, typically the hoarding of older high-quality coins. Hoarding would contract the volume of money and would result in some seizing up of trade in goods and services. The details of such a process are beyond the scope of our analysis here. The spotty historical record indicates that the money system in the Roman Empire never functioned as smoothly after Pax Romana as it did during the Pax Romana.

[29] "Roman coins found at Oxyrhynchus in Egypt came from mints at Antioch, Alexandra, Nicomedia, Cyzicus, Constantinople, Rome, Aquilaea, Arles, Treves, Taraco, and even London." (Roberts (2011, p. 220).)

[30] Hollander (2008) confronts the rapid growth in the currency stock in the decades just before Augustus and the absence of inflation in that period. He introduces a demand for money that is independent of transactions needs by households.

4 Government Production Funded with current seignorage

It makes sense to distinguish phases or episodes in the history of the Roman Empire with respect to the Pax Romana. First would be the rule of Ceasar Augustus, the consolidation of a system of governance and an economic system. The economic system would have been one with a banking system functioning and a system or interregional trade and specialization, with markets for labor, capital and land functioning in accord with freely functioning supply and demand; and for our purposes, most importantly, a system of stable metallic currency. We conjure up an economy exhibiting gradual expansion, with some of the government sector financed by the injection of new currency (financed by seignorage). Second would be the phase of Pax Romana, two hundred years of relative peace along the borders and considerable within-country peace and security. This phase ended about 165 CE with an outbreak of a contagious, plague-like disease and after 180, mismanagement of the central government fianances and a gradual debasement of the currency. A fourth phase (about 201 CE to 280) followed, one of inflation, contraction of interregional trade and a large-scale contraction of the banking system.

To represent the phase of the two hundred year Pax Romana, we set out a four sector aggregate growth model, a model descended quite directly from Solow's 1956 neo-classical growth model, a model of balanced growth with an exogenously growing population (labor force) and a constant savings rate. Our extensions of Solow (1956) involve (a) an aggregate money supply, money demand "system", (b) a sector producing government goods distributed in part "for free" to citizens, and (c) a gold extraction sector. Given a balanced growth representation of two hundred years of the Pax Romana, we proceed to perturb the model in attempt to shed light on the "collapse" of the Pax Romana. Three

elements to consider are (a) debasement of the coinage, (b) inflation and (c) "high" taxes. We work almost entirely with diverse balanced growth paths. For such cases, the expectations of agents are fulfilled. Along transition paths, agents are generally up-dating their expectations and capturing such "rational" expectations can be complicated. We elect to keep matters relatively simple in our analysis at this time and to this end we work largely with balanced growth scenarios.

Given our four-sector balanced growth model, we numerically solve it with Matlab software. We try to gain insight into how the Roman economy was working. Of central interest is the financing of part of the government sector with seignorage. We do not have plausible parameters to use in the solving exercise. Our choice of parameters was constrained by what worked with the Matlab software (solving systems with non-linear equations with fifteen and sometimes sixteen unknowns).[31] We employ constant returns to scale production functions of the Cobb-Douglas form. Our money demand function is also independent of scale. The assumption of a constant savings rate makes this part of demand scale free, also. Novel is demand for government goods turning in part on the injection of new currency. The funding of the government sector also has the scale-free quality and thus balanced growth is both possible and is in fact solved for in our benchmark model. There is also money-system separability in our model: given balanced growth, we can perturb the money stock alone and observe the price level change while the "real quantities" remain unchanged.

The model is set out in detail in Appendix 1. We provide a sketch here. There are four production functions: for consumption goods, investment goods, government goods and ore with gold content:

[31] In addition we inferred that or choice of parameters was not unreasonable when the model generated meaningful outputs. We have not seen any detailed data that we felt that we needed to structure our model around.

$$q_C = f(K_C, N_C)$$
$$q_I = g(K_I, N_I)$$
$$q_G = h(K_G, N_G)$$
$$q_R = z(K_R, N_R)$$

with $K_C+K_I+K_G+K_R = K$, the capital stock and $N_C+N_I+N_G+N_R = N$, the current labor force (population). Inputs of K_R and N_R yield q_R of ore-for-gold. We say that q_R is tons of ore and translates costlessly into $(1+\phi)q_R$ gold coins, worth $p_R q_R(1+\phi)$. The low quality mines extract q_R for the same cost, $rK_R + wN_R$ but can only get q_R coins from their activity. Seignorage represents a cost advantage to officials of the government for getting gold coins. We assume that the owners of the low-quality mines remain inactive. They are in a zero profit position, indifferent to operating or not. The government monopolizes the high quality mines and reaps seignorage of $\phi p_R q_R$ in each period.

The constant savings rate, s, gives us demand equal to supply of investment goods in

$$s[p_C q_C + p_I q_I + p_G q_G + p_R q_R] = q_I p_I.$$

p_C, p_I, p_G and p_R are prices for consumption, investment, government goods and extraction activity respectively. $p_G q_G$ is the "dollars" paid directly by citizens for the production of the government product. We assume no decay of capital K.

Money supply, M , at a moment equals money demand in

$$M = \frac{1}{v} \times [p_C q_C + p_I q_I + p_G q_G + p_R q_R]$$

for $v = (\frac{1}{i})^{\psi}$, for i the interest rate, equal to r/p_I in balanced growth. $[p_C q_C + p_I q_I + p_G q_G + p_R q_R]$ captures demand for money balances for

transactions and $\frac{1}{v}$ captures demand for money balances that are sensitive to the interest rate.

The government sector is funded in part with current taxes or charges $p_G q_G$ in

$$p_G q_G = \gamma[rK_G + wN_G]$$

for γ the fraction of current cost of government production covered by direct charges ($0 < \gamma < 1$). The remaining cost of the government sector is financed by the current injection of new currency in

$$\phi p_R q_R = (1 - \gamma)[rK_G + wN_G],$$

where $\phi p_R q_R$ is current seignorage. Our government budget constraint is then $p_G q_G + \phi p_R q_R = rK_G + wN_G$. Our government does not issue bonds and thus there is no government debt.[32] The novelty of the model is (a) gold extraction q_R yields extra value ($\phi p_R q_R$) in terms of coins minted and (b) this extra value "tops up" revenue in a precise fashion from direct charges for current government production, q_G.

The model has the following national accounts representation (Table 1). Interior entries sum row-wise to the current value of product and column-wise to the current value of an input. Entries in the right column sum to the current value of national product while entries in the bottom row sum to the current value of aggreage primary input.

Table 1: Accounts Matrix for the Model

	capital values	labor values	money injection	NNP
cons. value	rK_C	wN_C	0	$=\mathrm{p}_C q_C$
invest. value	rK_I	wN_I	0	$=\mathrm{p}_I q_I$
govt product val.	rK_G	wN_G	$-(1-\gamma)[rK_G + wN_G]$	$=\mathrm{p}_G \mathrm{q}_G$
money inject. val.	0	0	$\phi p_R q_R$	$=\phi p_R q_R$
extract. cost	rK_R	wN_R		$=\mathrm{p}_R q_R$
NNI	=rK	=wN	=0	

[32] Temin remarks on the government in Rome not issuing bonds.

Christ (1968) and Blinder and Solow (1973) were early contributors to macroeconomics and the government budget constraint.

Observe that new money balances, $(1+\phi)p_Rq_R$ appear a part of current national product in the NNP column. In the money injection column, the value of seignorage "in production", namely ϕp_Rq_R equals the value of seignorage "in use", namely $(1-\gamma)[rK_G + wN_G]$. One can interpret the sum, $p_Gq_G + \phi p_Rq_R$ as the total current value of government product. Then p_Rq_R is net new money balances.

5 Numerical Solving

Given Cobb-Douglas production functions and the associated efficiency conditions, we can solve for $q_C, q_I, q_G, q_R, K_C, N_C, K_I, N_I, K_R, N_R, p_C, p_I, p_G, p_R, r$ and w given initial values for K, N and M. See Appendix 1. We employ Cobb-Douglas functions for each of our four production processes: $q_C = [K_C]^{0.1}[N_C]^{0.9}$; $q_I = [K_I]^{0.2}[N_I]^{0.8}$; $q_G = [K_G]^{0.6}[N_G]^{0.4}$ and $q_R = [K_R]^{0.7}[N_R]^{0.3}$. $\gamma = 0.4$, $s = 0.2$ and $\phi = 0.2$. Our base case endowments are $N = 11$, $K = 6$ and $M = 8$. We obtain outputs in Table 1.

Table 2: Base Case Solved Values

	q_C	q_I	q_G	q_R	K_C	N_C	K_I	N_I
ϕ =0.2,$\gamma = 0.4$	5.4059	1.9166	0.8002	2.5876	0.8402	6.6480	0.7009	2.4646

Table 2 continued

	K_R	N_R	p_C	p_I	p_G	p_R	r	w
ϕ =0.2,$\gamma = 0.4$	3.4680	1.3066	0.6681	0.7859	0.3548	0.8230	0.4298	0.4889

We verify in Appendix 1 that proportionate changes in all of K, N and M leave the prices in the system unchanged. This property is fundamental for obtaining a balanced growth trajectory.

6 Balanced Growth

One can extract a growth rate $n = q_I/K$ from numerically solving the model (obtaining value 1.9166/6) and re-formulate the model to one of balanced growth with n exogenous at value 1.9166/6. This we do in Appendix 2. We now have $q_I = n * K$ initially (the model becomes 15 equations in 15 unknowns). We verify that changes in M alone leave quantities unchanged while prices shift proportionately (money supply

separability). With this model[33] we change parameters to see how the balanced growth configuration looks under different assumptions.

Of interest is (a) less seignorage (smaller ϕ) available and (b) a smaller fraction of government production funded with charges or taxes (smaller γ). These two numerical investigations constitute our first-pass effort to capture the "collapse" of the Pax Romana. In Table 3, we report on initial outputs for balanced growth with change in parameter ϕ alone. The base case is the same as that in Table 2, except initial values of K, N and M are $(1+n)^2 6$, $(1+n)^2 11$ and $(1+n)^2 8$ respectively. (Observe that prices are the same as those in Table 2 for the case of $\phi = 0.2$.) ϕ is capturing "the amount of seignorage" being reaped from current mining activity. Hence less "free money" in the economy, smaller ϕ, should result in less q_G being produced. This we observe in Table 3: q_G declines with ϕ. And of importance q_G declines to zero when $\phi = 0$.

Table 3: ϕ Varies

ϕ	q_C	q_G	q_R	K_C	N_C	K_I	N_I	K_R
0.2	9.4111	1.3931	4.5048	1.4628	11.5736	1.2202	4.2907	6.0375
0.1	9.5214	0.7845	5.0712	1.4735	11.7148	1.2155	4.2948	6.7868
0.05	9.5877	0.4187	5.4116	1.4799	11.8000	1.2127	4.2973	7.2360
0.00	9.6638	0.0000	5.8011	1.4871	11.8976	1.2094	4.3002	7.7489

Table 3 continued.

ϕ	N_R	p_C	p_I	p_G	p_R	r	w
0.2	2.2747	0.6681	0.7859	0.3548	0.8230	0.4298	0.4889
0.1	2.5693	0.6456	0.7838	0.3437	0.7975	0.4171	0.4722
0.05	2.7474	0.6327	0.7825	0.3373	0.7829	0.4099	0.4627
0.00	2.9521	0.6184	0.7811	1.3253	0.7669	0.4019	0.4521

In addition we observe in Table 3, the prices of consumption, investment, government and mining goods decline with the decline in ϕ. Also the rental rate on capital and the wage also decline with ϕ. One can infer a general price level decline with the decline and ϕ and a substitution into private consumption goods from government goods. A general

[33] We verify that proportionate changes in M and K leave prices unchanged and a change in M alone leaves quantities unchanged. See Appendix 2.

summary is that the shrinkage in seignorage induces a lowering of the price level and a considerable shrinkage in the amount of government product.

In our next experiment, we leave ϕ unchanging and we alter the value of γ, the fraction of current government product paid for directly by citizens as taxes or charges. $(1-\gamma)$ is the fraction which is paid for with seignorage. There is roughly speaking the same amount of "free money" being injected into the economy but it is being injected in a different fashion as the fraction of $p_G q_G$ being paid for with direct charges for use declines. When government covers more of the cost of $p_G q_G$ with seignorage, the price p_G declines significantly along with quantity q_G. p_G declines to zero when $\gamma = 0$. See Table 4.

Table 4: γ Varied

γ	q_C	q_G	q_R	K_C	N_C	K_I	N_I	K_R
0.9	1.2403	7.1668	3.5272	0.0851	1.6704	0.5899	5.1455	3.5996
0.7	8.0790	2.6423	4.1918	1.0583	10.1261	1.0481	4.4569	5.3066
0.3	9.5957	1.2045	4.5572	1.5297	11.7675	1.2479	4.2666	6.1594
0.2	9.7334	1.0611	4.5979	1.5817	11.9110	1.2694	4.2485	6.2542
0.1	9.8402	0.9483	4.6305	1.6232	12.0216	1.2864	4.2344	6.3301
0.05	9.8850	0.9005	4.6444	1.6410	12.0679	1.2937	4.2284	6.3626
0.00	9.9254	0.8573	4.6571	1.6571	12.1094	1.3003	4.2230	6.3922

Table 4 continued

γ	N_R	p_C	p_I	p_G	p_R	r	w
0.9	3.3638	0.4948	0.8238	0.9311	1.0511	0.7210	0.3306
0.7	2.4178	0.6406	0.7952	0.6548	0.8844	0.4890	0.4600
0.3	2.2563	0.6717	0.7845	0.2638	0.8135	0.4213	0.4929
0.2	2.2542	0.6743	0.7834	0.1747	0.8063	0.4149	0.4959
0.1	2.2325	0.6763	0.7826	0.0869	0.8006	0.4100	0.4982
0.05	2.2282	0.6771	0.7822	0.0433	0.7982	0.4079	0.4992
0.00	2.2243	0.6778	0.7819	0.0000	0.7961	0.4060	0.5000

We observe in Table 4 that the decline in γ corresponds to q_G shrinking and q_C rising. Gold production is rising as γ declines. One might argue that the system is seeking out more seignorage as the share of direct payment for q_G declines. Citizens are substituting private con-

sumption for government goods consumption as their direct share in payment for goverment goods declines. Prices p_C and w rise while other prices decline.

In Table 5, we change parameters ϕ and γ simultaneously from business-as-usual (base case in Table 2). That is, we have balanced growth values as a base case and we recalculate with a simultaneous change in ϕ and γ in an attempt to capture (a) higher tax charging for maintaining the public sector (γ larger) and (b) lower quality ore being available for the coinage (ϕ smaller). See Table 5.

Table 5: Simultaneous Shift for ϕ and γ

ϕ; γ	q_C	q_G	q_R	K_C	N_C	K_I	N_I	K_R
0.2;0.4	5.4059	0.8002	2.5876	0.8402	6.6480	0.7009	2.4646	3.4680
0.1;0.6	5.2588	0.6650	2.8502	0.7749	6.5056	0.6684	2.4940	3.7526

Table 5 continued

ϕ; γ	N_R	p_C	p_I	p_G	p_R	r	w
0.2;0.4	1.3066	0.6681	0.7859	0.3548	0.8230	0.4298	0.4889
0.1;0.6	1.5002	0.6386	0.7865	0.5240	0.8151	0.4334	0.4646

The change from the base case in Table 5 corresponds very roughly to the transition "out of" the Pax Romana. In the "shift" to the post Pax Romana period, we have aggregate private consumption declining, government product declining and the mining sector output increasing (see Table 5). The price of a unit of government output rises significantly. The wage declines and the rental on capital rises. $p_G q_G$ is tax revenue and it rises from 0.2839 to 0.3485 in the transition to the post Pax Romana period. Society is getting less government output and paying more in taxes in the shift forward in time. Some historians argue for a significant population decline in the Empire as it declined into the fifth and sixth centuries. We have not built long run population decline into our analysis.

7 Inflation and Debasement

Conventional wisdom emphasizes that the Roman Empire in its decline exhibited significantly higher tax rates. The need for more revenue to pay for more soldiers on the frontiers is the picture that is painted. The problem with debasing a currency is the uneven-ness of the impact that is experienced by users. Some coins become worth more than others and transactors can dispute the true value of a payment. Transactions become difficult to effect when each side has a different valuation of the means of payment. There is the well-known tendency for high-quality coins to be hoarded and lower-quality coins to be "passed off" relatively rapidly for purchasing (Gresham's Law: "bad currency drives out good currency"). A contraction in the stock of active currency can choke off the over-all volume of transacting. Debasement followed the end of the Pax Romana and posed a persistent problem during the Third Century. Inflation can result from minting large volumes of low-quality coins. This seems to have occured as the government faced a need for more revenue and turned to reaping "income" from seignorage. The new rounds of minting would have occured with debased coins. The quality of coins remained very high during the Pax Romana. Well into the Third Century, observers commented on the inflation that had set in. We capture this in Table 6, along with debasement of the currency and tax rate increases. We start with the base case and increase K and N at rate n and M somewhat more rapidly than n, at $1.2\times n$, n=1.9166/6. At the same time, we simultaneously contract ϕ (less seignorage) and increase γ (larger fraction of q_G paid for by direct charges). See Table 6.

Table 6: K and N grow at rate n; M multiplies at $(1+n)^{1.2\times t}$

	q_C	q_G	q_R	K_C	N_C	K_I	N_I	K_R
ϕ =0.2,$\gamma = 0.4$	5.4059	0.8002	2.5876	0.8402	6.6480	0.7009	2.4646	3.4680
ϕ =0.15,$\gamma = 0.5$	7.0155	0.9973	3.5714	1.0584	8.6562	0.9006	3.2736	4.7391
ϕ =0.12,$\gamma = 0.6$	9.0676	1.3514	4.8229	1.3263	11.2266	1.1561	4.3490	6.3342
ϕ =0.10,$\gamma = 0.7$	11.5890	2.0059	6.4138	1.6273	14.4138	1.4709	5.7906	8.3097
$\phi = 0.08, \gamma = 0.8$	14.4905	3.2004	8.4692	1.9128	18.1466	1.8371	7.7458	10.7491
$\phi = 0.06, \gamma = 0.9$	16.4248	6.2720	10.8929	1.8826	20.8943	2.1380	10.5460	13.1895

Table 6 continued

$M = M_0 * (1 + n)^t$	N_R	p_C	p_I	p_G	p_R	r	w
ϕ =0.2,$\gamma = 0.4$	1.3066	0.6681	0.7859	0.3548	0.8230	0.4298	0.4889
ϕ =0.15,$\gamma = 0.5$	1.8457	0.6906	0.8313	0.4661	0.8678	0.4578	0.5037
ϕ =0.12,$\gamma = 0.6$	2.5531	0.7174	0.8798	0.5911	0.9202	0.4904	0.5215
ϕ =0.10,$\gamma = 0.7$	3.5050	0.7454	0.9319	0.7331	0.9826	0.5309	0.5394
$\phi = 0.08, \gamma = 0.8$	4.8559	0.7695	0.9885	0.8951	1.0570	0.5829	0.5530
$\phi = 0.06, \gamma = 0.9$	6.9707	0.7738	1.0535	1.0952	1.1677	0.6751	0.5474

In Table 6, we observe that q_G and p_G rise significantly (high tax payments as inflation and debasement take place). q_R and q_C each rise at about the same rate. p_R and p_C also rise at about the same rate. There is then a general rise in the price level (inflation) and a shift to higher direct outlays by citizens for government product. The case ($\phi = 0.06, \gamma = 0.9$) is roughly speaking turning the model into a case of no seignorage and a currency that is costly to produce. This is not really a textbook case since seignorage will be approximately zero.

8 Dynamics

The analysis to this point has dealt with situations in which the model is in balance or at a stationary configuration. We turn briefly to the issue of the stability of the model around a balanced growth path. In Appendix 3, we restructure the model to be expressed in per capita terms and we "extract" a value for the population growth rate n that yields balanced growth, given the current initial values for K/N and M/N. We then perturb the model by having it start with the "wrong" initial value for K/N and investigate whether the model converges to the balanced growth path in a sequence of moves. In this experiment, we

have the monetary authority maintain the money supply so that M/N is unchanging. Our example exhibited convergence for K/N.

9 End of the Pax Romana

An epidemic (a plague of some sort, brought in from Persia by soldiers returning from the frontier) took hold of the Empire in 165 CE and disrupted society.[34] Roberts (2011) considers the arrival of this plague to have triggered the end of the Pax Romana. 180 CE marked the death of Emperor Marcus Aurelius. His successor, his natural son, Emperor Commodus launched an expenditure binge that emptied the treasury at the center. He was in due course assassinated. During the Third Century CE, there is evidence of the banking system and long-distance trade in the Empire shrinking severely. The Colosseum in Rome remained active for contests into the sixth century but, the economy of Roman Italy essentially contracted after about 180 CE until the Renaissance.[35] A civil war of sorts occurred near the end of the Pax Romana, followed by the Severan Dynasty (193-235). Stability returned during the Fourth Century but, relative to the situation during the Pax Romana, a less productive political and economic order prevailed. Political power in Roman Italy was slowly ceded after the Third Century to leaders in Constantinople. As early as 285 Diocletian established a co-emperor for

[34] "Egyptian wages doubled after the major Antonine plague of 165-175 CE." Temin (2004b; p. 519).

[35] "Around the start of the third century CE, the early Roman Empire came to an end under the pressure of a number of problems: several emperors who were exceptionally autocratic and excessive and a series of revolts by the army which in turn led to Rome being ruled by a series of short-term emperors. The disruption manifested itself in many ways, including increased inflation in the third century CE that is visible to us through debased coinage and occasional price quotations. Inflation was less than 1 percent in the first and second centuries CE, but prices doubled after the Antonine plague of the late second century and doubled again soon thereafter. The denarius began to be progressively debased at this time. (Temin (2006, p. 149)). Roberts emphasizes the disruption of social stability occasioned by the plague. We, with others, argue for a significant disruption in the stability of government caused by the exhaustion of low-cost silver and gold mines in the late second century CE.

the Eastern part of the empire. These co-leaders held different amounts of power until the government in Rome simply disintegrated. The Romans retreated from governing in England in 410 CE. The Visigoths briefly held the city of Rome in 410 but the loss of Africa by Rome triggered the final fall. (Roberts (2011; p. 252).

The remarkable aspect of the Roman Empire is that it was able to function as a fairly unified entity over so much territory for such a long spell. The eastern part never gave up using the Greek language and the subjects could not be said to have been assimilated by Roman Italy. One might argue, with hindsight, that the government in Rome simply bit off more than it could chew and would inevitably be faced with a splitting off of the eastern provinces from those in the west. Roberts (p. 245) suggests that "Business survived [the collapse of the Roman Empire] in Africa and the eastern half of the Roman Empire; Byzantium and the Arab forces that triumphed in the seventh century inherited an urban system of money, markets and entrepreneurial business that it would return to Europe during the early Renaissance."

There is considerable documentation of currency problems in the Empire during the Third Century CE.[36] Historians have not however traced a path from the later dates of the well-functioning of the money system during the Pax Romana to the relative instability of the money system during the Third Century. What is clear is that the economy of

[36]Peter Temin emphasizes that the period, post Pax Romana, became one of inflation, a period of a seeming unchanging volume of currency chasing a shrinking volume of transactions. Contemporary observers noted this inflation in their writing. In addition, there is abundant evidence in coin finds of a debasement of the currency in the period post Pax Romana (third century CE), debasement via the mixing of baser metals with gold and silver in coins. The inflation could then certainly not be the result of a currency volume of unchanging quality getting smaller, but it remains compatible with a smaller volume of high-quality gold and silver bullion being available for the currency. Aggressive debasement must have led to a rapid increase in the volume of currency in circulation and to the related inflation. In the short run, debasement provides a burst of extra seignorage to the government while in the longer run it generally results in some pathology with the payments system, inflation being one plausible scenario.

the Empire was prosperous during the period of a well-functioning money system and was becoming fragmented during the Third Century. A popular interpretation is that stable government at the center fell apart; many emperors held office for short periods and many failed to impose order on the Empire at a reasonable cost. Tax rates became high and prices were not stable. Another interpretation is that the high-quality gold and silver mines in Spain and Dacia were depleted and those in power at the center were unable to develop a new well-functioning money system, similar to that which was in effect during the Pax Romana. A well-functioning money system, sound fiscal management and law and order were in some sense sufficient for markets to open up and trade to flourish. There is reasonable evidence of vast quantities of gold and silver being mined in Spain, Dacia and elsewhere during the Pax Romana. This is indirect evidence for the view that depletion of high-quality deposits around 180 CE became an issue for those in power. We do not have good evidence for a serious scarcity of gold and silver bullion around 180 CE and a crisis of management of the money system after that date. There is no evidence to contradict this view, either.

It makes sense to distinguish leadership problems from management problems. Frequent turnover of leaders often accompanied by coups and assassinations are leadership issues. It is possible for managers at the center to keep the nation fairly peaceful and prosperous even with leadership problems. One usually links the two, however. A power vaccuum (leadership crisis) aften co-incides with a breakdown in management. The reverse can occur. A management crisis can trigger and leadership crisis. Historians emphasize the leadership problems that developed in the center after the Pax Romana. We are putting forth the idea that management problems may well have triggered leadership crises. Our argument is that without a steady flow of gold and silver bullion to the center, managers or senior civil servants could not do their jobs well.

They turned to new taxes and tax rates in order to get revenue and the high rates met with various forms of resistence. Some observers argue that owners of large estates loosened their ties with the larger economy, becoming somewhat autakic, and avoiding the payment of taxes to the center. In addition the managers turned to inflationary finance in an attempt to maintain fiscal order. The orderly currency system of the Pax Romana became a fairly disorderly system as debasement set in. Leadership problems co-incided with the management problems and the economy of Roman Italy as well as that of the Empire as a whole never returned to its earlier properous long-run trajectory.

10 Seignorage and the Economy

"Printing money" for a modern economy is centrally about "setting" the price level or altering the pace of aggregate economic activity and peripherally about getting "free" government output. "Historic" Spain however is a case study of a nation with a large share of government activity financed with imports of gold and silver bullion that was almost free. After 1530, Spain had large quantities of gold and silver flowing into the treasury from mines in Central and South America as well as from direct plunder. This has been viewed as the case of a government floating on a sea of "free" revenue. One argument is that much of the "free" gold and silver went toward "excessive" importation of goods to Spain and in so doing "choked off" the development of local productive activities. Local producers could not compete with low cost imports.[37] Local producers got shouldered out of supplying local markets and as well were generally unable to sell to foreign markets. A central mechanism here becomes the relatively high cost of local products to buyers abroad because the exchange rate has made local products expensive in

[37] This is an early form, or a variant, of what today is referred to as Dutch Diesease or the resource curse. Abundant exports such as oil in a nation today fund a large volume of imports and local producers are often unable to get rooted and to compete successfully with exporters located abroad.

international markets (so-called Dutch Disease).

Conventional wisdom, in addition, accepts that the large volumes of gold and silver flowing to the center in Spain in the sixteenth and seventeenth centuries contributed directly to a significant inflation in Europe. Under the reign of Philip II, Spain saw a fivefold increase in prices.[38] The low-cost flows of gold and silver moving into Spain during the sixteenth and seventeenth centuries are thus considered to have been deleterious for the economic development of Spain.[39] We would however be remiss to fail to note that Spain managed to build up a huge colonial empire and to conquer large areas of Europe during its lengthy period of massive "importing" of gold and silver bullion. Spain built an empire of sorts largely on "free" gold and silver. Nevertheless it is not unreasonable to say that Spain fell victim to a version of "resource curse" whereas the Roman Empire, during the Pax Romana, managed to avoid such a trap. Some might argue that "the Augustan money system" stunted the economic development of Roman Italy during the Pax Romana because "excessive" imports were made available to Roman Italy via payment with the low-cost gold and silver. This negative judgement might include a stifling of technical progress because firms could not get rooted locally and expand and innovate. Roman Italy during the Pax Romana was essentially an importing nation in this view. This view is worth reflecting on. Roman Italy has, however, been accepted as the high income region of the early Empire. The local economy may not have been import-oriented but was apparently doing well. In contrast with Sixteenth century Spain, there was no significant inflation in the Empire during the Pax Romana. The Emperor may have been simply

[38] (http://en.wikipedia.org/wiki/Philip_II_of_Spain#Economy)

[39] "Even though Philip II was bankrupt by 1596 (for the fourth time, after France had declared war on Spain), more silver and gold were shipped safely to Spain in the last decade of his life than ever before. This allowed Spain to continue its military efforts, but led to an increased dependency on the precious metals and jewels." (http://en.wikipedia.org/wiki/Philip_II_of_Spain#Economy)

lucky in his management of the currency or he may have been intelligent and prudent. A by-product of having abundant gold and silver for funding imports, was the expansion in the stock of currency each year. The rate of expansion in the currency appears to have been right since inflation and deflation were avoided for about two hundred years.

It is not difficult to see how early observers would define a nation as an economic successs if the nation was coping militarily with its neighbors, possessing large cities with bustling activity, and engaging in the importing of goods in large volumes. A nation with large inflows of low-cost gold and silver could with a little luck pass this test of economic success. Adam Smith's *Wealth of Nations* emphasizes the folly of achieving economic success via gold and silver accumulation, the folly of mercantilism. Smith emphasized that successful economic development was about high standards of productivity of local workers and high standards of living for the citizens of a nation, not about the accumulation of large reserves of gold via unbalanced trade. For Smith balanced international trade involved both efficient local production and access to foreign and often exotic goods. Access to imports was to be achieved via successful low-cost local production of goods that foreigners were keen on importing, not by the corralling of a stream of low-cost gold and silver that was for funding the importing. Successful economies would be active exporters of locally-produced goods, not of low-cost gold and silver coins and bullion.

Our view of the economy of Roman Italy during the Pax Romana is not that it was free of mercantilism. Seignorage was at the center of the economic system. However as gold and silver coins flowed out to fund imports to Roman Italy, the outer provinces were benefitting twice-fold. They were getting infusions of new currency to lubricate trade in their own economies and they were being subjected to significant demand for their products from the center. Though Roman Italy may have been

operating as a mercantilist nation, the drawbacks of such a system were offset to a considerable extent by benefits accruing to the economies of the provinces. The economy of the empire was a special center-periphery model with mercantilist characteristics. The managers in Rome appear to have found the Goldilocks point – the expansion in the coin stock in circulation was not too hot and not too cold – in contrast with what managers achieved in Spain. There was a balance of sorts in injecting new coinage into the Empire at large. In addition, the demand for imports by Roman Italy meant revenues and encouragement for producers in the provinces. An interesting balance in economic development between the center and the periphery took place. Good management by government led to relative tranquility within the Roman Empire. Borders were kept secure. Internal security allowed for the development of a business culture. A unified and stable currency led to large-scale inter-regional trade.

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APPENDIX 1: THE MODEL AND A NUMERICAL SOLVING

We begin with a simple closed economy without money. We then amend this basic model to incorporate money, which determines a price level and our key seignorage channel: signorage finances the public sector. That is, we drop two equations from our basic model and add two replacements and in so doing we end up with our simple monetary growth model with seignorage financing the public sector. First the basic model without money.

We have four constant returns to scale activities in our closed economy producing consumption goods, investment goods, government goods and gold coins. Hence

1) consumption goods

$$\begin{aligned} q_C &= f(K_C, N_C) \\ \frac{f_{K_C}}{f_{N_C}} &= \frac{r}{w} \\ p_C q_C &= rK_C + wN_C \end{aligned}$$

One can think of this as a system of 3 equations in q_C, K_C and N_C. p_C is taken as a parameter for the moment. The subscript C indicates the consumption goods sector and K_C and N_C are capital and labor employed in the consumption goods sector. $f(.)$ is the production function and r and w are the rental rate for capital and the wage respectively.

2) investment goods

$$\begin{aligned} q_I &= g(K_I, N_I) \\ \frac{g_{K_I}}{g_{N_I}} &= \frac{r}{w} \\ p_I q_I &= rK_I + wN_I \end{aligned}$$

3 equations in q_I, K_I and N_I with p_I treated as a parameter for the moment.

3) government good

$$\begin{aligned} q_G &= h(K - K_C - K_I, N - N_C - N_I) \\ \frac{h_{K-K_C-K_I}}{h_{N-N_C-N_I}} &= \frac{r}{w} \\ p_G q_G &= \gamma\{r[K - K_C - K_I] + w[N - N_C - N_I]\} \end{aligned}$$

3 equations in q_G, r, and w with p_G treated as a parameter for the moment. (γ is the fraction of the current cost of government output that is paid for by direct charges or taxes. $(1-\gamma)$ is the fraction covered by current seignorage.)

4) extraction activity

$$\begin{aligned} q_R &= z(K_R, N_R) \\ \frac{z_{K_R}}{z_{N_R}} &= \frac{r}{w} \\ p_R q_R &= rK_R + wN_R, \end{aligned}$$

3 equations in q_R, K_R, and N_R with p_R treated as a parameter for the moment.

We proceed to treat the four output prices above as endogenous. We have four additional equations.

The constant savings rate, s, gives us demand equal to supply of investment goods in

$$s[p_C q_C + p_I q_I + p_G q_G + p_R q_R] = q_I p_I.$$

Money supply, M , at a moment equals money demand in

$$M = \frac{1}{v} \times [p_C q_C + p_I q_I + p_G q_G + p_R q_R]$$

for $v = (\frac{1}{i})^{\psi}$, for i the interest rate, equal to r/p_I in balanced growth. $[p_C q_C + p_I q_I + p_G q_G + p_R q_R]$ captures demand for money balances for

transactions and $\frac{1}{v}$ captures demand for money balances that are sensitive to the interest rate.

The fraction $(1-\gamma)$ of the cost of the government sector is financed by the current injection of new currency in

$$\phi p_R q_R = (1-\gamma)[rK_G + wN_G],$$

where $\phi p_R q_R$ is current seignorage. (We have in mind that outsiders can produce gold at cost $p_R q_R$ but there is no surplus that can be used for purchases.)

Current money supply increase is

$$\Delta M = (1+\phi)[rK_R + wN_R].$$

Our government budget constraint is then $p_G q_G + \phi p_R q_R = rK_G + wN_G$.

BASIC RUN GIVES US n

....

basic set up run for n (16 equations in 16 unknowns) corrected Wed July 10, 2013....

function f=bgmbsx(x)

%

n=1.8704/6;ac=.1;ai=0.2;ag=0.6;ar=0.7;N=11*(1+n)^1;K=6*(1+n)^1;M=8*(1+n)^1;

% fraction 0.4 of pg*qg is directly paid in current period. 0.6 by seign.

gm=0.4;s=0.2;fi=0.2;

% define terms needed in solving...

qc=x(1);

qi=x(2);

qg=x(3);

qr=x(4);

Kc=x(5);

Nc=x(6);

```
Ki=x(7);
Ni=x(8);
Kr=x(9);
Nr=x(10);
Kg=K-Kc-Ki-Kr;
Ng=N-Nc-Ni-Nr;
pc=x(11);
pi=x(12);
pg=x(13);
pr=x(14);
r=x(15);
w=x(16);
%
f(1)=qc-(Kc^(ac)*Nc^(1-ac));
f(2)=qi-(Ki^(ai)*Ni^(1-ai));
f(3)=qg-(Kg^ag*Ng^(1-ag));
f(4)=qr-(Kr^ar*Nr^(1-ar));
%
f(5)=ac*Nc*w-(1-ac)*Kc*r;
f(6)=ai*Ni*w-(1-ai)*Ki*r;
f(7)=ag*Ng*w-(1-ag)*Kg*r;
f(8)=ar*Nr*w-(1-ar)*Kr*r;
%
f(9)=pc*qc-(r*Kc+w*Nc);
f(10)=pi*qi-(r*Ki+w*Ni);
% revenue is "limited" cost of govt product
f(11)=pg*qg-gm*(r*Kg+w*Ng);
f(12)=pr*qr-(r*Kr+w*Nr);
f(13)=M*((r/pi)^0.1)-(pc*qc+pi*qi+(pg*qg)+pr*qr);
% delta M equals "residual" cost of govt product.
```

f(14)=(qi/K)*M-((1+fi)*(r*Kr+w*Nr));

f(15)=fi*(r*Kr+w*Nr)-(1-gm)*(r*Kg+w*Ng);

f(16)=s*(pc*qc+pi*qi+(pg*qg)+pr*qr)-pi*(qi);

% (1+n)^0... wed. July 10, 2013

% 5.4060 qi 1.9166 qg 0.8002 qr= 2.5876 0.8403 6.6481 0.7009 2.4646

% 3.4680 1.3066 pc 0.6681 pg= 0.7859 pi= 0.3548 pr= 0.8230 r=0.4298 w=0.4889

% (1+n)^1... n=qi/K=1.8704/6.... prices preserved...

% 7.0912 2.5140 1.0496 3.3942 1.1022 8.7206 0.9194 3.2329

% 4.5491 1.7139 pc= 0.6681 0.7859 0.3548 0.8230 r= 0.4298 w= 0.4889

% ////////////////

APPENDIX 2: POPULATION GROWTH RATE GIVEN

Our basic model above can be altered so that population growth n is exogenous. In this case, we have $q_I = nK$ and the model above becomes one of 15 equations in 15 unknowns. With this model we investigate how balanced growth looks different when (a) the amount of seignorage varies (vary ϕ) and (b) the amount of the public sector financed with seignorage varies (vary $(1-\gamma)$).

wed experiments with system ...γ varies

.....

function f=bgmrx(x)

%

n=1.9166/6;ac=.1;ai=0.2;ag=0.6;ar=0.7;N=11*(1+n)^2;K=6*(1+n)^2;M=8*(1+n)^2;

% fraction 0.4 of pg*qg is directly paid in current period. 0.6 by seign.

gm=0.0;s=0.2;fi=0.2;

% define terms needed in solving...

qi=n*K;

qc=x(1);

```
% qi=x(2);
qg=x(2);
qr=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
Kr=x(8);
Nr=x(9);
Kg=K-Kc-Ki-Kr;
Ng=N-Nc-Ni-Nr;
pc=x(10);
pi=x(11);
pg=x(12);
pr=x(13);
r=x(14);
w=x(15);
%
f(1)=qc-(Kc^(ac)*Nc^(1-ac));
f(2)=qi-(Ki^(ai)*Ni^(1-ai));
f(3)=qg-(Kg^ag*Ng^(1-ag));
f(4)=qr-(Kr^ar*Nr^(1-ar));
%
f(5)=ac*Nc*w-(1-ac)*Kc*r;
f(6)=ai*Ni*w-(1-ai)*Ki*r;
f(7)=ag*Ng*w-(1-ag)*Kg*r;
f(8)=ar*Nr*w-(1-ar)*Kr*r;
%
f(9)=pc*qc-(r*Kc+w*Nc);
% f(10)=pi*qi-(r*Ki+w*Ni);
```

```
% revenue is "limited" cost of govt product
f(10)=pg*qg-gm*(r*Kg+w*Ng);
f(11)=pr*qr-(r*Kr+w*Nr);
f(12)=M*((r/pi)^0.1)-(pc*qc+pi*qi+(pg*qg)+pr*qr);
% delta M equals "residual" cost of govt product.
f(13)=(qi/K)*M-((1+fi)*(r*Kr+w*Nr));
f(14)=fi*(r*Kr+w*Nr)-(1-gm)*(r*Kg+w*Ng);
f(15)=s*(pc*qc+pi*qi+(pg*qg)+pr*qr)-pi*(qi);
% n exog, (1+n)^0... fi=0.2... (checks with BGMBSX)
% 5.4059 0.8002 2.5876 0.8402 6.6480 0.7009 2.4646 3.4680 Nr=1.3066
% pc=0.6681 pi= 0.7859 pg=0.3548 pr= 0.8230 r= 0.4298 w= 0.4889
%
% (TEST OF M-change for prices change alone... M^(1+n) alone CHECKS OUT...)
% NEUTRALITY OF M STOCK.....quantities unchanged.
% 5.4059 0.8002 2.5876 0.8402 6.6480 0.7009 2.4646 3.4680 Nr=1.3066
% 0.8815 1.0370 0.4682 1.0859 0.5671 0.6451
% (1+n)^1 in K,N,M... fi=0.2 ... BG prices non-varying...
% 7.1327 1.0558 3.4142 1.1086 8.7716 0.9248 3.2519 4.5758 Nr=1.7240
% 0.6681 0.7859 0.3548 0.8230 0.4298 0.4889
% (1+n)^2 ...fi=0.2.. prices unchanging
% 9.4111 1.3931 4.5048 1.4628 11.5736 1.2202 4.2907 6.0375 Nr=2.2747
% 0.6681 0.7859 0.3548 0.8230 0.4298 0.4889
% REDON WED AM
% gm=0.9
% 1.2403 7.1668 3.5272 0.0851 1.6704 0.5899 5.1455 3.5996 3.3638 Nr=0.4948
% 0.8238 0.9311 1.0511 0.7210 0.3306
% gm=1.0 did not compute...
% gm=0.7
```

% 8.0790 2.6423 4.1918 1.0583 10.1261 1.0481 4.4569 5.3066 Nr=2.4178

% 0.6406 0.7952 0.6548 0.8844 0.4890 0.4600

% gm=1.0 did not compute...

% xxxxx gm down to 0.30 from 0.4,... fi at 0.2 unchanging

% 9.5957 1.2045 4.5572 1.5297 11.7675 1.2479 4.2666 6.1594 Nr=2.2563

% 0.6717 0.7845 0.2638 0.8135 0.4213 0.4929

% gm down to 0.20

% 9.7334 1.0611 4.5979 1.5817 11.9110 1.2694 4.2485 6.2542 Nr=2.2542

% 0.6743 0.7834 0.1747 0.8063 0.4149 0.4959

% gm down to 0.10

% 9.8402 0.9483 4.6305 1.6232 12.0216 1.2864 4.2344 6.3301 Nr=
2.2325

% 0.6763 0.7826 0.0869 0.8006 0.4100 0.4982

% gm down to 0.05

% 9.8850 0.9005 4.6444 1.6410 12.0679 1.2937 4.2284 6.3626 Nr=
2.2282

% 0.6771 0.7822 0.0433 0.7982 0.4079 0.4992

% gm down to 0.0 (note zero price for qr !!)

% 9.9254 0.8573 4.6571 1.6571 12.1094 1.3003 4.2230 6.3922 Nr=
2.2243

% 0.6778 0.7819 0.0000 0.7961 0.4060 0.5000

////////////

Wed experiments with ϕ varying

function f=bgmrx(x)

%

n=1.9166/6;ac=.1;ai=0.2;ag=0.6;ar=0.7;N=11*(1+n)^2;K=6*(1+n)^2;M=8*(1+n)^2;

% fraction 0.4 of pg*qg is directly paid in current period. 0.6 by
seign.

gm=0.4;s=0.2;fi=0.0;

% define terms needed in solving...

```
qi=n*K;
qc=x(1);
% qi=x(2);
qg=x(2);
qr=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
Kr=x(8);
Nr=x(9);
Kg=K-Kc-Ki-Kr;
Ng=N-Nc-Ni-Nr;
pc=x(10);
pi=x(11);
pg=x(12);
pr=x(13);
r=x(14);
w=x(15);
%
f(1)=qc-(Kc^(ac)*Nc^(1-ac));
f(2)=qi-(Ki^(ai)*Ni^(1-ai));
f(3)=qg-(Kg^ag*Ng^(1-ag));
f(4)=qr-(Kr^ar*Nr^(1-ar));
%
f(5)=ac*Nc*w-(1-ac)*Kc*r;
f(6)=ai*Ni*w-(1-ai)*Ki*r;
f(7)=ag*Ng*w-(1-ag)*Kg*r;
f(8)=ar*Nr*w-(1-ar)*Kr*r;
%
```

```
f(9)=pc*qc-(r*Kc+w*Nc);
% f(10)=pi*qi-(r*Ki+w*Ni);
% revenue is "limited" cost of govt product
f(10)=pg*qg-gm*(r*Kg+w*Ng);
f(11)=pr*qr-(r*Kr+w*Nr);
f(12)=M*((r/pi)^0.1)-(pc*qc+pi*qi+(pg*qg)+pr*qr);
% delta M equals "residual" cost of govt product.
f(13)=(qi/K)*M-((1+fi)*(r*Kr+w*Nr));
f(14)=fi*(r*Kr+w*Nr)-(1-gm)*(r*Kg+w*Ng);
f(15)=s*(pc*qc+pi*qi+(pg*qg)+pr*qr)-pi*(qi);
% n exog, (1+n)^0... fi=0.2... (checks with BGMBSX)
% 5.4059 0.8002 2.5876 0.8402 6.6480 0.7009 2.4646 3.4680 Nr=1.3066
% pc=0.6681 pi= 0.7859 pg=0.3548 pr= 0.8230 r= 0.4298 w= 0.4889
%
% (TEST OF M-change for prices change alone... M^(1+n) alone CHECKS OUT...
% NEUTRALITY OF M STOCK.....quantities unchanged.
% 5.4059 0.8002 2.5876 0.8402 6.6480 0.7009 2.4646 3.4680 Nr=1.3066
% 0.8815 1.0370 0.4682 1.0859 0.5671 0.6451
% (1+n)^1 in K,N,M... fi=0.2 ... BG prices non-varying...
% 7.1327 1.0558 3.4142 1.1086 8.7716 0.9248 3.2519 4.5758 Nr=1.7240
% 0.6681 0.7859 0.3548 0.8230 0.4298 0.4889
% (1+n)^2 ...fi=0.2.. prices unchanging
% 9.4111 1.3931 4.5048 1.4628 11.5736 1.2202 4.2907 6.0375 Nr=2.2747
% 0.6681 0.7859 0.3548 0.8230 0.4298 0.4889
% fi= 0.1
% 9.5214 0.7845 5.0712 1.4735 11.7148 1.2155 4.2948 6.7868 Nr= 2.5693
% 0.6456 0.7838 0.3437 0.7975 0.4171 0.4722
% fi=0.05
```

% 9.5877 0.4187 5.4116 1.4799 11.8000 1.2127 4.2973 7.2360 Nr= 2.7474

% 0.6327 0.7825 0.3373 0.7829 0.4099 0.4627

% fi=0.0.. ZERO FOR qg with ZERO SEIGNORAGE available...

% 9.6638 -0.0000 5.8011 1.4871 11.8976 1.2094 4.3002 7.7489 Nr= 2.9521

% 0.6184 0.7811 1.3253 0.7669 0.4019 0.4521

.....

APPENDIX 3: CONVERGENCE

We can formulate our model in per capita terms, population growth n exogenous, and verify that in balanced growth, $k_{t+1} = k_t$ for $k_t = K_t/N_t$. In this per capita formulation we observe that $I_t/N_t = dk_t/dt + nk_t$. Here is the per capita formulation with results reported for a sequence of runs to test for convergence of k_t to its balanced growth value, 0.5455. We started with $k(0)$ at 7/11 instead of the balanced growth value of 6/11=0.5455. After 15 updates in a sequence, k_t was at 0.5465 and convergence was apparent.

.....

function f=bgmx(x)

% K0=(6/N) Base case and no (1+n)^8 expansion...

n=1.9166/6;ac=.1;ai=0.2;ag=0.6;ar=0.7;N=11*(1+n)^0;K0=0.5469*(1+n)^0;M=(8/N)*(1+n)

% fraction 0.4 of pg*qg is directly paid in current period. 0.6 by seign.

gm=0.4;s=0.2;fi=0.2;

% define terms needed in solving...

% qi=n*K;

qc=x(1);

% qi=x(2);

qg=x(2);

qr=x(3);

```
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
Kr=x(8);
Nr=x(9);
Kg=K0-Kc-Ki-Kr;
Ng=1-Nc-Ni-Nr;
pc=x(10);
pi=x(11);
pg=x(12);
pr=x(13);
r=x(14);
w=x(15);
qi=x(16);
%
f(1)=qc-(Kc^(ac)*Nc^(1-ac));
f(2)=(qi-K0+n*K0)-(Ki^(ai)*Ni^(1-ai));
f(3)=qg-(Kg^ag*Ng^(1-ag));
f(4)=qr-(Kr^ar*Nr^(1-ar));
%
f(5)=ac*Nc*w-(1-ac)*Kc*r;
f(6)=ai*Ni*w-(1-ai)*Ki*r;
f(7)=ag*Ng*w-(1-ag)*Kg*r;
f(8)=ar*Nr*w-(1-ar)*Kr*r;
%
f(9)=pc*qc-(r*Kc+w*Nc);
% f(10)=pi*qi-(r*Ki+w*Ni);
% revenue is "limited" cost of govt product
f(10)=pg*qg-gm*(r*Kg+w*Ng);
```

```
f(11)=pr*qr-(r*Kr+w*Nr);
f(12)=M*((r/pi)^0.1)-(pc*qc+pi*(qi-K0+n*K0)+(pg*qg)+pr*qr);
% delta M equals "residual" cost of govt product.
f(13)=n*M-((1+fi)*(r*Kr+w*Nr));
f(14)=fi*(r*Kr+w*Nr)-(1-gm)*(r*Kg+w*Ng);
f(15)=s*(pc*qc+pi*(qi-K0+n*K0)+(pg*qg)+pr*qr)-pi*(qi-K0+n*K0);
f(16)=pi*(qi-K0+n*K0)-(r*Ki+w*Ni);
% n exog, (1+n)^0... fi=0.2... (checks with BGMBSX)
% 5.4059 0.8002 2.5876 0.8402 6.6480 0.7009 2.4646 3.4680 Nr=1.3066
% pc=0.6681 pi= 0.7859 pg=0.3548 pr= 0.8230 r= 0.4298 w= 0.4889
% Thurs test for k(t+1).... success... BASE CASE
% qc=0.4914 qg= 0.0727 qr= 0.2352 0.0764 0.6044 0.0637 0.2241
0.3153
% 0.1188 pc= 0.6681 pi= 0.7859 pg= 0.3548 pr=0.8230 r= 0.4298
w=0.4889 k(t+1)= 0.5455
% START WITH WRONG k(0)=7/11
% 0.4960 0.0804 0.2639 0.0878 0.6013 0.0738 0.2247 0.3693
% 0.1205 0.6493 0.7529 0.3209 0.7336 0.3670 0.4820 0.6129
% 6129
% 0.4949 0.0785 0.2566 0.0848 0.6021 0.0712 0.2245 0.3553
% 0.1201 0.6538 0.7608 0.3288 0.7544 0.3814 0.4837 0.5955
% continue with k(0)=0.5955 and search for convergence...
% 0.4941 0.0770 0.2512 0.0827 0.6026 0.0693 0.2244 0.3450
% 0.1197 0.6573 0.7669 0.3351 0.7708 0.3928 0.485 0.5827
% .5827
% 0.4934 0.0759 0.2471 0.0811 0.6031 0.0678 0.2243 0.3374
% 0.1195 0.6600 0.7716 0.3399 0.7834 0.4017 0.4860 0.5732
% 0.5732
% 0.4929 0.0751 0.2441 0.0799 0.6034 0.0668 0.2242 0.3317
% 0.1193 0.6620 0.7751 0.3435 0.7931 0.4085 0.4867 0.5661
```

% 0.5661

% 0.4926 0.0745 0.2418 0.0790 0.6036 0.0660 0.2242 0.3275

% 0.1192 0.6635 0.7778 0.3463 0.8005 0.4138 0.4873 0.5608

% 0.5608

% 0.4923 0.0741 0.2402 0.0783 0.6038 0.0654 0.2242 0.3244

% 0.1191 0.6647 0.7799 0.3485 0.8061 0.4178 0.4877 0.5569

% 0.5569

% 0.4921 0.0737 0.2389 0.0778 0.6040 0.0650 0.2241 0.3221

% 0.1190 0.6655 0.7814 0.3501 0.8103 0.4208 0.4880 0.5540

% 0.5540

% 0.4919 0.0735 0.2380 0.0775 0.6041 0.0647 0.2241 0.3203

% 0.1190 0.6662 0.7825 0.3513 0.8135 0.4230 0.4882 0.5518

% current k(0)= 0.5518 ....

% 0.4918 0.0733 0.2373 0.0772 0.6041 0.0644 0.2241 0.3190

% 0.1189 0.6667 0.7834 0.3522 0.8159 0.4248 0.4884

% 0.5502... 0.5490...0.5481...0.5474...0.5469... 0.5465

% 0.4915 0.0729 0.2357 0.0766 0.6043 0.0639 0.2241 0.3161

% 0.1188 0.6678 0.7854 0.3542 0.8214 0.4287 0.4888 0.5465

.....

The value M/N is held at the balanced growth value in this exercise. We could redo our exercise with K/N held at the balanced growth value and test for convergence in the value of M/N. We did not pursue this.