Queen's Economics Department Working Paper No. 1207

Long memory in stock market volatility and the volatility-in-mean effect: the FIEGARCH-M model

Bent Jesper Christensen
University of Aarhus and CREATES

Jie Zhu
University of Aarhus and CREATES

Morten Ørregaard Nielsen
Queen's University and CREATES

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

6-2009

Long Memory in Stock Market Volatility and the Volatility-in-Mean Effect: The FIEGARCH-M Model*

Bent Jesper Christensen†
University of Aarhus and CREATES

Morten Ørregaard Nielsen
Queen's University and CREATES

Jie Zhu
University of Aarhus and CREATES

June 27, 2009

Abstract

We extend the fractionally integrated exponential GARCH (FIEGARCH) model for daily stock return data with long memory in return volatility of Bollerslev and Mikkelsen (1996) by introducing a possible volatility-in-mean effect. To avoid that the long memory property of volatility carries over to returns, we consider a filtered FIEGARCH-in-mean (FIEGARCH-M) effect in the return equation. The filtering of the volatility-in-mean component thus allows the co-existence of long memory in volatility and short memory in returns. We present an application to the daily CRSP value-weighted cum-dividend stock index return series from 1926 through 2006 which documents the empirical relevance of our model. The volatility-in-mean effect is significant, and the FIEGARCH-M model outperforms the original FIEGARCH model and alternative GARCH-type specifications according to standard criteria.

JEL Classification: C22.

*We are grateful to David Bates, Tim Bollerslev, Christian Dahl, Bruce Hansen, George Tauchen, and participants at the 2007 NBER-NSF Time Series Conference in Iowa City and the CREATES conference in Aarhus, 2007, for useful comments, and to the Social Sciences and Humanities Research Council of Canada (SSHRC grant no. 410-2009-0183), the Danish Social Science Research Council (FSE grant no. 275-05-0220), the Center for Analytical Finance (CAF), and the Center for Research in Econometric Analysis of Time Series (CREATES, funded by the Danish National Research Foundation) for financial support. Some of this research was carried out when Christensen and Zhu were visiting the Economics Departments at Harvard University and Cornell University, respectively, and the hospitality of both departments is gratefully acknowledged.

†Correspondence: Bent Jesper Christensen, School of Economics and Management, University of Aarhus, 322 University Park, DK-8000 Aarhus C, Denmark; phone: +45 89 421 547; email address: bjchristensen@creates.au.dk

Keywords: FIEGARCH, financial leverage, GARCH, long memory, risk-return tradeoff, stock returns, volatility feedback.

1 Introduction

Many of the salient features of daily stock returns are well described by the FIEGARCH (fractionally integrated exponential generalized autoregressive conditional heteroskedasticity) model introduced by Bollerslev & Mikkelsen (1996). Thus, in addition to time-varying volatility and volatility clustering (the ARCH and GARCH effects, as in Engle (1982) and Bollerslev (1986)), and the resulting unconditional excess kurtosis or heavier than normal tails, the model accounts for long memory in volatility (fractional integration, as in the FIGARCH model of Baillie, Bollerslev & Mikkelsen (1996)), as well as asymmetric volatility reaction to positive and negative return innovations (the exponential feature, as in Nelson's (1991) EGARCH model).

In this paper, we introduce a filtered in-mean generalization of the FIEGARCH model, which we label FIEGARCH-M. The generalization allows a volatility feedback or risk-return relation effect of changing conditional volatility on conditional expected stock returns, and generates unconditional skewness. Following recent literature (Ang, Hodrick, Xing & Zhang (2006) and Christensen & Nielsen (2007)), it is changes in volatility that enter the return equation. The filtering of volatility when entering it in the return specification implies that the long memory property of volatility (the fractionally integrated feature) does not spill over into returns, which would be empirically unrealistic.

That volatility exhibits long memory is well established in the recent empirical literature. This finding is consistent across a number of studies[1], and financial theory may accommodate long memory in volatility as well, see Comte & Renault (1998). Many of the studies use GARCH-type frameworks, but none of them consider in-mean specifications, i.e., parametric relations across conditional means and variances[2]. The FIEGARCH-M model of the present paper fills this gap.

Three related effects may introduce a relation between volatility and mean returns, namely, (i) a risk-return tradeoff capturing the risk premium required by investors as compensation for taking on additional risk, (ii) a financial leverage effect, and (iii) a volatility feedback effect. We briefly discuss each of these in turn.

Early theoretical and empirical contributions on the risk-return relation were due to Merton (1973, 1980). In equilibrium, investors taking on additional risk should be compensated

[1] See, e.g., Robinson (1991), Crato & de Lima (1994), Baillie et al. (1996), Ding & Granger (1996), Breidt, Crato & de Lima (1998), Robinson (2001), and Andersen, Bollerslev, Diebold & Labys (2003).

[2] To the best of our knowledge, the only study of the relation between volatility with long memory and conditional mean returns is Christensen & Nielsen (2007), which is outside the GARCH-class, using instead a stochastic volatility model and basing inference on realized (from high-frequency returns) volatility or implied (from option prices) volatility.

through higher expected return, which implies a positive coefficient in the risk-return relation. The GARCH-M (GARCH-in-mean) model proposed by Engle, Lilien & Robins (1987) allows for the direct effect of volatility changes on asset prices through required returns in a short memory GARCH-type model, by introducing the conditional volatility function into the conditional mean return equation. Empirical studies of the risk-return tradeoff using GARCH-type models for stock returns obtain mixed results regarding both the sign and the significance of the in-mean effect, see e.g. Bollerslev, Engle & Wooldridge (1988), Chou (1988), Glosten, Jagannathan & Runkle (1993), Nelson (1991), Campbell & Hentschel (1992), and Chou, Engle & Kane (1992). Recent work in asset pricing examines cross-sectional risk premia induced by covariance between innovations in volatility and stock returns. This literature finds negative premia, e.g. Ang et al. (2006). The idea is that since innovations in volatility are higher during recessions, stocks which co-vary with volatility are stocks that pay off in bad states, and these should require a smaller risk premium. For a survey of related studies, see Lettau & Ludvigson (2004).

While time-varying volatility in itself generates excess kurtosis in unconditional distributions, which is common to most financial return series, the phenomenon that negative return innovations induce higher volatility than positive innovations of the same magnitude, observed particularly in stock return distributions, may be accommodated using the EGARCH model of Nelson (1991). The asymmetric volatility reaction pattern may stem from a financial leverage effect, see e.g. Black (1976), Engle & Ng (1993), and Yu (2005). The standard argument from Black (1976) is that bad news decrease the stock price, hence increasing the debt-to-equity ratio (i.e. financial leverage), and equity carries all asset risk, making the stock relatively riskier after the price drop and increasing future expected volatility.

An alternative source of a negative volatility-return relation is the volatility feedback mechanism of Campbell & Hentschel (1992), that is, if volatility is increased, then so is the risk premium, in case of a positive tradeoff between risk and conditional expected return. Hence, the discount rate is also increased, which in turn for an unchanged dividend yield lowers the stock price. Presumably, the volatility feedback effect should be strongest at the market level, whereas the leverage effect should apply to individual stocks.

Our FIEGARCH-M model includes both the exponential (asymmetry) and in-mean features, thus allowing tests of whether both are empirically relevant. Although the causality is reversed, the leverage and volatility feedback effects may be seen as supplementing each other as explanations of the negative return-volatility relation documented in empirical stock market research. In the empirical model, the negative relation may show up both through the exponential and the in-mean feature. Of these, only the latter generates unconditional skewness (see He, Silvennoinen &

Terasvirta (2008)). It is worth noting that the volatility feedback mechanism induces a negative volatility-return relation even in the presence of a positive equity premium or risk-return tradeoff, and for a given data frequency the negative feedback effect may dominate the positive tradeoff effect in the estimation of the in-mean volatility-return relation. At the relatively high, say daily, frequencies where GARCH-style models are most useful, the initial price reaction through the change in discount rate (the feedback mechanism) is relatively more important than the change in mean return (asset pricing or tradeoff) effect of a volatility change, and so the estimated in-mean effect may to a larger extent reflect feedback. Our model allows estimating both the exponential and volatility-in-mean effects simultaneously, and the estimated in-mean volatility-return relation will point to a feedback or tradeoff effect operating alongside the leverage effect.

We apply our FIEGARCH-M model to the CRSP value-weighted cum-dividend stock index return series using daily data from 1926.1.2 through 2006.12.29. We estimate the model by quasi-maximum likelihood (QML). The validity of the robust (sandwich-formula) standard errors is confirmed using the wild bootstrap algorithm. We compare the model to a number of alternative GARCH-type specifications, including IGARCH, Spline-GARCH, FIGARCH, Adaptive FIGARCH, EGARCH, FIEGARCH, and associated models with in-mean effects, such as GARCH-M. The comparison confirms that FIEGARCH is preferred over other models without in-mean features. Furthermore, in-mean features in fact further improve the fit. The best model according to standard information criteria rewarding both goodness-of-fit and parsimony as well as to out-of-sample forecasting performance is the new FIEGARCH-M specification. In particular, the volatility-in-mean effect is statistically significant, even when controlling for autocorrelation in daily returns. Thus, the results demonstrate that the volatility-in-mean effect indeed is an empirically important extension of the original FIEGARCH model.

In the next section, we present our FIEGARCH-M model, which incorporates all the above mentioned features. Section 3 presents the application to the daily CRSP data, and Section 4 concludes.

2 The FIEGARCH-M Model

We extend the FIEGARCH model by introducing volatility into the return equation, i.e., the in-mean feature, along the lines of the GARCH-M literature, thus yielding a new FIEGARCH-M model. Since long memory in volatility introduced into the return equation in a linear fashion generates long memory in returns, which may not be empirically warranted, we follow Ang et al. (2006) and Christensen & Nielsen (2007) and consider the possibility that it is changes in volatility rather than volatility levels that enter the in-mean specification and induce a volatility-return relation.

Let the daily continuously compounded returns on the stock or stock market index be given by

$$r_t = \ln(P_t) - \ln(P_{t-1}), \tag{1}$$

where t is the daily time index and P_t the stock price or index level at time t. In the FIEGARCH-M model, we use the conditional mean specification

$$r_t = \mu + \lambda h_t + \varepsilon_t, \tag{2}$$

where volatility changes enter in the form of h_t, defined in (7) below as the filtered (fractionally differenced) conditional variance. Thus, the specification allows for a volatility-return relation through the parameter λ. Letting $\mathcal{F}_{t-1}$ denote the information in returns through $t-1$, i.e., the σ-field generated by $\{r_{t-1}, r_{t-2}, ...\}$, it is noted that h_t is $\mathcal{F}_{t-1}$-measurable, so the return innovations are $\varepsilon_t = r_t - E(r_t|\mathcal{F}_{t-1})$ with $E(\cdot|\mathcal{F}_{t-1})$ denoting conditional expectation given $\mathcal{F}_{t-1}$. It follows that ε_t in (2) is a martingale difference sequence (with respect to $\mathcal{F}_t$).

The key is the modeling of the conditional return variance

$$\sigma_t^2 = Var(r_t|\mathcal{F}_{t-1}) = E(\varepsilon_t^2|\mathcal{F}_{t-1}). \tag{3}$$

As in the FIEGARCH model, the specification is

$$\phi(L)(1-L)^d(\ln\sigma_t^2 - \omega) = \psi(L)g(z_{t-1}), \tag{4}$$

where ω is the mean of the logarithmic conditional variance, $\phi(L)$ and $\psi(L)$ are polynomials in the lag operator, $\phi(L) = (1-\phi_1 L) \times \ldots \times \left(1-\phi_p L\right)$ and $\psi(L) = (1+\psi_1 L) \times \ldots \times \left(1+\psi_q L\right)$, and $(1-L)^d$ is the fractional difference operator defined by its binomial expansion

$$(1-L)^d = \sum_{i=0}^{\infty} \frac{\Gamma(i-d)}{\Gamma(-d)\Gamma(i+1)} L^i, \tag{5}$$

where d is the order of fractional integration in log-variance and $\Gamma(\alpha) = \int_0^\infty x^\alpha e^{-x} dx$ is the Gamma function. The fractional difference with $0 < d < 1$ allows for stronger volatility persistence than that of the GARCH-type generated by the lag-polynomials $\phi(L)$ and $\psi(L)$. The exponential or asymmetry feature is ensured by modeling $\ln\sigma_t^2$ in (4), as opposed to σ_t^2, and by the definition of the news impact function $g(\cdot)$ governing the manner in which past returns impact current volatility,

$$g(z_t) = \theta z_t + \gamma(|z_t| - E|z_t|), \tag{6}$$

where $z_t = \varepsilon_t/\sigma_t$ is the normalized innovation. This follows Nelson's (1991) EGARCH specification. Here, γ is the rate at which the magnitude of the normalized innovations in deviations from mean, i.e., $|z_t| - E|z_t|$, enter into current volatility[3], and θ generates an asymmetry in news impact on volatility. Thus, if $\theta < 0$ then negative innovations induce higher volatility than positive innovations of the same magnitude. However, this asymmetric reaction to innovations of different sign does not induce unconditional skewness in returns, which is instead produced by the in-mean feature (see He et al. (2008)) and hence also accommodated by the FIEGARCH-M specification.

Bollerslev & Mikkelsen (1996) in fact use the model with $p = q = 1$. Defining $h_t = (1 - L)^d(\ln \sigma_t^2 - \omega)$ as the fractionally differenced log-variance in deviation from the long run level, it is convenient to rewrite the resulting FIEGARCH(1,d,1) model as

$$h_t = (1 - L)^d(\ln \sigma_t^2 - \omega) = \phi_1 h_{t-1} + g(z_{t-1}) + \psi_1 g(z_{t-2}). \tag{7}$$

Thus, the relevant measure of volatility changes h_t follows a special ARMA(1,1) process. The presence of h_{t-1} on the right hand side of (7) is a GARCH-effect, i.e., volatility (here, its fractional difference) depends on its own lag, whereas the ARCH-effect stems from past returns feeding into current volatility, namely, via the news impact $g(z_{t-1})$ (and its lagged value) in (7).

In addition to the volatility-return relation where fractionally differenced volatilities h_t enter mean returns as in (2), Ang et al. (2006) and Christensen & Nielsen (2007) also consider a specification where volatility innovations enter instead. In the present GARCH-framework, the innovation to volatility is best understood as the news impact $g(z_{t-1})$, yielding the alternative return equation

$$r_t = \mu + \lambda g(z_{t-1}) + \varepsilon_t. \tag{8}$$

Thus, $g(z_{t-1})$ is the most recent innovation to σ_t^2, and it is $\mathcal{F}_{t-1}$-measurable, so in (8) the return innovations are again the martingale differences $\varepsilon_t = r_t - E(r_t|\mathcal{F}_{t-1})$, as in (2).

3 Application to the CRSP Value-Weighted Index, 1926-2006

Our application uses daily cum-dividend returns on the CRSP value-weighted index from January 2, 1926, the starting date of the CRSP series, to December 29, 2006, for a total of $T = 21,519$ return observations. The CRSP series is more than twice as long as the S&P 500 series that was considered by Bollerslev & Mikkelsen (1996) in the original FIEGARCH study. That series covered the period January 2, 1953, to December 31, 1990, for a total of $T = 9,559$ observations. The

[3]Note that if z_t is Gaussian, then $E|z_t| = \sqrt{2/\pi}$.

CRSP and S&P series are very similar over the common subperiod, with a correlation coefficient of 0.9880.

Following Nelson (1991) and Bollerslev & Mikkelsen (1996), we include a variable N_t equal to the number of nontrading days between $t-1$ and t to account for the fact that volatility tends to be higher following weekend and holiday nontrading periods, but with each nontrading day contributing less to volatility than a trading day. Thus, our volatility equation with $p = q = 1$ becomes

$$h_t = (1-L)^d(\ln \sigma_t^2 - \ln(1+\delta N_t) - \omega) = \phi_1 h_{t-1} + g(z_{t-1}) + \psi_1 g(z_{t-2}). \tag{9}$$

Here, the parameter δ measures the contribution of each nontrading day to variance, as a fraction of the contribution from a trading day. To calculate the fractional differences h_t, we truncate the infinite sum in (5) at $i = \min\{t-1, 1000\}$, following Baillie et al. (1996) and Bollerslev & Mikkelsen (1996).

Using (9) for volatility and either (2) or (8) to define the return innovations ε_t, the model is estimated by quasi maximum likelihood (QML). Thus, the sample log-likelihood for return data $r_t, t = 1, ..., T$, is

$$\ln L(\eta) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln \sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\right), \tag{10}$$

where $\eta = (\mu, \lambda, \omega, \delta, \theta, \gamma, \psi_1, ..., \psi_q, \phi_1, ..., \phi_p, d)$ is the unknown parameter vector to be estimated, of dimension $p+q+7$. Estimation is carried out by numerical maximization of $\ln L(\eta)$. To initialize the recursions on (9) and (2) respectively (8) we use the unconditional sample average and variance of r_t for the presample $(t = 0, -1, \ldots)$ values of r_t and σ_t^2, and we use $\varepsilon_t = 0$ for $t = 0, -1, \ldots$. The distributional assumption behind the likelihood function is that the return innovations ε_t are conditionally normal. For robustness against departures from Gaussianity, we calculate robust standard errors based on the sandwich-formula $H^{-1}VH^{-1}$, where H is the Hessian of $\ln L(\eta)$ and V the sum of the outer products of the individual quasi score contributions. Below, we verify the validity of the QML robust standard errors using the wild bootstrap (Wu (1986)).

Table 1 about here

Estimation results for a number of alternative GARCH-type specifications are shown in Table 1, using a simple constant mean return equation $r_t = \mu + \varepsilon_t$. In addition to the FIEGARCH model (9), we consider the special case of the EGARCH model with $d = 0$, often used to model stock returns, as well as a standard GARCH model and its fractional extension, given by

$$\sigma_t^2 = \omega(1+\psi_1)^{-1} + [1 - (1+\psi_1 L)^{-1}(1-\phi_1 L)(1-\phi_2 L)(1-L)^{-d}](\varepsilon_t^2 - \delta N_t) + \delta N_t. \tag{11}$$

The IGARCH model has $d = 1$, and the standard GARCH model has $d = 0$. In the alternative parametrization of the standard GARCH model given by

$$\sigma_t^2 - \delta N_t = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\left(\sigma_t^2 - \omega - \delta N_t\right),$$

where $\alpha(L) = \sum_{i=1}^{p} \alpha_i L^i$ and $\beta(L) = \sum_{i=1}^{q} \beta_i L^i$ are the ARCH and GARCH polynomials, we have the equivalences $\phi(L) = 1 - \alpha(L) - \beta(L)$ and $\psi(L) = 1 - \beta(L)$.

Recent literature has suggested a possible need for time-variation in unconditional variances, in addition to that in conditional variances. This may be relevant in our case, considering the length of our sample period (more than 80 years). The Adaptive FIGARCH (A-FIGARCH) model of Baillie & Morana (2007) replaces the term $\omega(1+\psi_1)^{-1}$ in (11) with the trigonometric series

$$\omega_t = \omega_0 + \sum_{j=1}^{k}[\gamma_j \sin(2\pi jt/T) + \delta_j \cos(2\pi jt/T)]. \tag{12}$$

In our estimation, γ_j, $j \geq 0$ and δ_j, $j \geq 3$ were insignificant, as in Baillie & Morana (2007), so these parameters are not estimated in the specifications reported in our tables. A similar effect is modeled by Engle & Rangel (2008) in their Spline-GARCH model where $\sigma_t^2 = g_t \tau_t$ with

$$\begin{aligned} g_t &= \omega(1+\psi_1)^{-1} + [1 - (1+\psi_1 L)^{-1}(1-\phi_1 L)(1-\phi_2 L)](\frac{\varepsilon_t^2}{\tau_t} - \delta N_t) + \delta N_t, \qquad (13) \\ \tau_t &= c\exp[w_0 t + \sum_{i=1}^{k} w_i \max\{(t - t_{i-1})^2, 0\}], \end{aligned}$$

and $t_i = iT/k$. In our specification we use $k = 7$ knots (estimated knot coefficients not reported in the tables) as in Engle & Rangel (2008).

The results in Table 1 confirm the empirical relevance of each of the elements of the FIEGARCH model. Thus, volatilities exhibit long memory, with the fractional differencing parameter d positive and strongly significant (robust standard errors in parentheses). The special parameters (θ, γ) of the news impact function present in the EGARCH and FIEGARCH models are strongly significant, including in particular the asymmetry parameter θ, which takes a negative value, corresponding to a leverage effect. The nontrading-day count N_t gets a coefficient δ estimated to about 0.2 in the EGARCH and FIEGARCH models, showing that weekend and holiday contributions to variance per day are about 20% of those for trading days. The results for the FIEGARCH model in the last column of Table 1 may be compared to those in Bollerslev & Mikkelsen (1996). In particular, the point estimate of d, at 0.54, is slightly smaller for our longer data series than their estimate of 0.63. The robust t-statistic takes the value 19.31 in our data, compared to 10.05 for the shorter sample.

The Ljung-Box portmanteau statistics for serial correlation in the standardized return innovations $\hat{z}_t = \hat{\varepsilon}_t/\hat{\sigma}_t$, reported as Q_{10} and Q_{100} for 10 and 100 lags, respectively, take the values 272.61 and 379.16. In GARCH-type models, p-values from standard χ^2-distributions are not reliable, but the statistics are still useful for model comparison. So are the similar Ljung-Box statistics for absolute standardized return innovations $|\hat{z}_t|$, indicated with a superscript A in the table, since absolute returns are serially correlated in GARCH models even when raw returns are not.

The table also shows the maximized log likelihood, the Akaike and Schwartz (Bayesian) information criteria, reported as AIC and SIC, and Engle & Ng (1993) sign bias and size bias misspecification tests, for which one and two asterisks denote rejection at the 5% and 1% level, respectively. Of all the models, FIEGARCH clearly has the best AIC and SIC values, as well as the best Engle & Ng (1993) tests.

Finally, the last two rows of the table show mean absolute forecast errors (MAFE) and squared correlations (R^2) for one-day-ahead out-of-sample forecasts of σ_t^2 for the last 200 days of our sample period. For the construction of each forecast, the model is re-estimated using data through $t-1$. To measure true volatility we use realized volatility based on 5-minute returns throughout trading day t. Among all the models, FIEGARCH has both the best (lowest) MAFE and the best (highest) R^2.

Table 2 about here

Estimation results for the models with in-mean effects are shown in Table 2. The return equation is

$$r_t = \mu + \lambda\sigma_t^2 + \varepsilon_t \tag{14}$$

in the GARCH-M, Spline-GARCH-M, FIGARCH-M, and A-FIGARCH-M specifications (first four columns in the table) where σ_t^2 is given by (11)-(13). The EGARCH-M specification in the fifth column of the table uses the return equation (2). This is also used in the first of the two FIEGARCH-M specifications in the table, denoted FIEGARCH-M$_h$, i.e., the FIEGARCH-M generalization with volatility changes h_t in-mean. The last column is the specification FIEGARCH-M$_g$ with news impacts $g(z_{t-1})$ entering the return equation as in (8).

The reported estimates in Table 2 show that the in-mean parameter λ governing the volatility-return relation is negative throughout, and strongly significant except in the Spline-GARCH-M and EGARCH-M cases. The robust t-statistic for λ is -6.80 in the FIEGARCH-M$_h$ model and -8.01 in the FIEGARCH-M$_g$ model. These two models are considerably better than the other models with in-mean effects in the table in terms of the AIC and SIC information criteria, Engle & Ng (1993) tests, portmanteau statistics Q_{10} and Q_{100}, and out-of-sample forecasting performance. On the same criteria, they also clearly outperform the original FIEGARCH model without in-mean effect from the last column of Table 1. Indeed, the Ljung-Box statistics show a dramatic drop in value when including the in-mean effect in the FIEGARCH-M models, compared to the pure FIEGARCH case, showing that changes in volatility account for a considerable portion of changes in returns. Neither of the two FIEGARCH-M specifications is rejected by the Engle & Ng (1993) tests. The FIEGARCH-M$_g$ model with news impact $g(z_{t-1})$ entering the mean equation does somewhat better than the FIEGARCH-M$_h$ model in terms of the AIC and SIC criteria and so is perhaps the preferred specification, based on these results. From the R^2 statistic (last row), volatility forecasts from the model explain 31% of the variation in future realized volatility, which is the highest in the table.

The dramatic drop in the Ljung-Box statistics in the FIEGARCH-M models compared to the pure FIEGARCH model suggests that the volatility-return relation might account for serial dependence in observed daily returns. Bollerslev & Mikkelsen (1996) alternatively control for return dependence using AR(m) specifications, i.e., the return equation is

$$r_t = \mu_0 + \mu_1 r_{t-1} + ... + \mu_m r_{t-m} + \varepsilon_t. \tag{15}$$

We therefore turn to the encompassing specifications including AR(m) as well as current and lagged volatility-in-mean effects.

Table 3 about here

Results including lagged returns and in-mean effects are shown in Table 3, which is laid out as Table 2. The return equation in the GARCH-M, Spline-GARCH-M, FIGARCH-M, and A-FIGARCH-M cases is now

$$r_t = \mu_0 + \mu_1 r_{t-1} + ... + \mu_m r_{t-m} + \lambda_1 \sigma_t^2 + ... + \lambda_m \sigma_{t-m+1}^2 + \varepsilon_t. \tag{16}$$

The EGARCH-M and FIEGARCH-M$_h$ models use the return equation

$$r_t = \mu_0 + \mu_1 r_{t-1} + ... + \mu_m r_{t-m} + \lambda_1 h_t + ... + \lambda_m h_{t-m+1} + \varepsilon_t \tag{17}$$

and the FIEGARCH-M$_g$ model uses the return equation

$$r_t = \mu_0 + \mu_1 r_{t-1} + ... + \mu_m r_{t-m} + \lambda_1 g(z_{t-1}) + ... + \lambda_m g(z_{t-m}) + \varepsilon_t. \tag{18}$$

In the estimation, the parameter vectors $(\mu_0, \ldots, \mu_m)$ and $(\lambda_1, \ldots, \lambda_m)$ replace μ and λ in the definition of η in the log-likelihood function (10), so there are now $p+q+2m+6$ parameters in the most general specifications (except for the Spline-GARCH-M model, which of course has more). The table shows results for $m = 3$, following Bollerslev & Mikkelsen (1996). In the FIEGARCH-M models (the last two columns of the table), several of the parameters in both the autoregressive and the volatility-in-mean terms are significant at conventional levels. This suggests that the in-mean terms indeed pick up a volatility-return relation, rather than only serial dependence in returns, which is now controlled for.

The two FIEGARCH-M specifications are now about equally good in terms of information criteria, Ljung-Box statistics, sign/size bias tests, and out-of-sample forecasting. These two models are not rejected by the Q_{10} and Q_{100} tests, or the size bias and joint Engle & Ng (1993) tests. The FIEGARCH-M models are clearly better than the other models in the table according to the AIC and SIC information criteria and out-of-sample forecasting, showing the importance of both the fractional and exponential features. These FIEGARCH-M specifications with both autoregressive and volatility-in-mean effects also clearly outperform the specifications without autoregression in the previous tables, both in terms of the information criteria, and, particularly, in terms of the portmanteau statistics Q_{10} and Q_{100}.

Table 4 about here

Throughout, we have relied on the standard "sandwich-formula" robust QML standard errors. To check the validity of the approach in our application, we also compute standard errors by the wild bootstrap algorithm (999 replications) and compare. The results are shown in Table 4. We focus on the FIEGARCH-M models from the last two columns of the previous table, and the point estimates and robust standard errors from there are repeated in Table 4 for convenience. The table in addition reports wild bootstrap standard errors in the second set of parentheses. From the table, robust and wild bootstrap standard errors are quite similar, particularly for the autoregressive and volatility-in-mean terms of the return equation. In the remaining cases, i.e., in the variance equation, the robust standard errors almost always exceed the wild bootstrap standard errors, suggesting that the QML approach is valid and indeed perhaps conservative. The biggest difference is for the parameter δ in the FIEGARCH-M$_g$ model, where the robust standard error is approximately ten times the wild bootstrap standard error.

Table 5 about here

The results in Tables 3 and 4 suggest that only the first lagged return is significant in the autoregressive specification in the FIEGARCH-M models, once the volatility-in-mean effects are allowed for. Table 5 shows results for the final FIEGARCH-M models, in both cases maintaining only the first lag in the return equation. In the FIEGARCH-M_g model we also drop the insignificant third lag of the in-mean effect. Curiously, λ_1 is estimated to be negative and λ_2 positive. It is conceivable that both a volatility feedback effect and a risk-return relation are present at several lags, but that which dominates varies with lag length. A possible nonlinearity in either relation would make it more difficult to separate the two effects.[4] If anything, the volatility feedback effect should induce an immediate price drop as the discount rate increases in response to an increase in volatility, whereas the risk-return relation increases expected returns, which would show up in realized returns with a lag, see Christensen & Nielsen (2007). Thus, it makes sense that coefficients are initially negative, then positive, under this interpretation.

From Tables 3 and 5, the likelihood ratio (LR) tests for the two final FIEGARCH-M models against the corresponding full models take the values 3.82 and 6.44, for p-values of 14.8% and 9.2% in their asymptotic χ^2_2 and χ^2_3 distributions. The last column of the table shows results for the pure FIEGARCH model with $m = 3$ autoregressive terms and no in-mean effects selected in Bollerslev & Mikkelsen (1996). Compared to Table 3, the pure FIEGARCH model comes about by dropping the three in-mean terms in either of the FIEGARCH-M models, and the associated LR-statistics take the values 15.56 and 15.60, each with a p-value of 0.1% in the asymptotic χ^2_3-distribution. At conventional levels, the reduction to FIEGARCH is rejected, whereas reduction to either of the FIEGARCH-M models with only one lagged return in the mean equation is not.

The LR-statistics for joint significance of the volatility-in-mean λ parameters in the models in Table 5 take the values 32.94 and 30.36, respectively, for p-values $< 0.1\%$ in the asymptotic χ^2_3 and χ^2_2 distributions. The Ljung-Box and sign/size bias tests are similar for all three models in Table 5, except that the sign bias test rejects the pure FIEGARCH model. The AIC and SIC information criteria in Table 5 are better (lower) for the FIEGARCH-M models than for the original FIEGARCH model. Thus, starting from the encompassing FIEGARCH-M_h model in Table 3, dropping two lagged returns yields better information criteria than dropping three in-mean terms. Similarly, in the FIEGARCH-M_g model in Table 3, dropping two lagged returns

[4] A recent strand of literature argues that the risk-return relation may be nonlinear. For example, Linton & Perron (2003) suggest a semiparametric EGARCH-M model, while Conrad & Mammen (2008) propose a specification test for the functional form of the risk premium. Another potential explanation for the apparent negative risk-return relation is an omitted variable bias (relevant pricing factors are omitted), as suggested by Scruggs (1998). Further investigation of either of these possibilities is beyond the scope of this paper.

and the last in-mean term yields better information criteria than dropping three in-mean terms. Finally, comparing the two FIEGARCH-M specifications in Table 5, the AIC criterion selects the model with volatility changes h_t in-mean, whereas the SIC criterion, which rewards parsimony more highly, points to the specification with news impacts $g(z_{t-1})$ in-mean as the final model. All in all, the evidence points to an important role for the in-mean effect, capturing a volatility-return relation that remains significant even when controlling for lagged returns in the return equation and the standard financial leverage effect ($\theta < 0$) in the volatility equation.

4 Concluding Remarks

We have introduced an in-mean version of the FIEGARCH model in which the long memory property of volatility does not carry over to returns. This is accomplished through a filtering (fractional differencing) of the in-mean volatility measure. Our empirical application of the resulting FIEGARCH-M model to the daily CRSP value-weighted cum-dividend stock index returns confirms the long memory property of volatility and establishes the empirical relevance of including the filtered in-mean term.

Consistently across specifications, we find a negative coefficient on the most recent filtered volatility-in-mean term. As we have discussed, a negative volatility-return relation could correspond to a leverage effect, a volatility feedback effect, or both. According to asset pricing theory, increased volatility should require investor compensation in the form of higher conditional expected returns, although this has proved hard to establish empirically, and would likely only apply to holding periods considerably longer than a single day. The volatility feedback effect considered here is actually consistent with a positive tradeoff between risk and conditional expected return, since it simply captures the initial drop in price following an increase in volatility, and hence in the discount rate. The evidence suggests that at the daily frequency, any positive effect of the risk-return tradeoff on the most recent volatility-in-mean term in the return equation is dominated empirically by a negative financial leverage or volatility feedback effect. When including more lagged in-mean-terms, the second gets a positive coefficient, possibly picking up a positive risk-return tradeoff effect at this lag. Our results are consistent with the notion that when volatility is increased, the immediate consequence is an increase in discount rate and hence a drop in stock price, producing a negative contemporaneous volatility-in-mean or feedback effect, whereas the subsequent impact through increased conditional expected return generates a positive risk compensation or tradeoff in-mean effect at a one period lag. In the final models, these in-mean effects are jointly significant, even when controlling for autocorrelation in returns as well as a classical financial leverage effect (the asymmetric or exponential feature) in the volatility equation.

Although the financial leverage and volatility feedback effects are mutually consistent, we conjecture that our results on the negative sign of the first in-mean term more likely reflect the volatility feedback effect, since this should be strongest at the market level which we consider, whereas financial leverage should show up most strongly for individual stocks. Recent developments in asset pricing, e.g., Ang et al. (2006), also point to negative premia in the return equation in cross-sectional regressions where innovations to volatility rather than volatility levels enter the return equation, as in our FIEGARCH-M model with news impact in-mean. Thus, we contribute with aggregate time series evidence complementing the cross-sectional findings on the sign of the volatility-return relation.

References

Andersen, T. G., Bollerslev, T., Diebold, F. X. & Labys, P. (2003), 'Modelling and forecasting realized volatility', *Econometrica* **71**, 579–625.

Ang, A., Hodrick, R. J., Xing, Y. & Zhang, X. (2006), 'The cross-section of volatility and expected returns', *Journal of Finance* **61**, 259–299.

Baillie, R. T., Bollerslev, T. & Mikkelsen, H. O. (1996), 'Fractionally integrated generalized autoregressive conditional heteroskedasticity', *Journal of Econometrics* **74**, 3–30.

Baillie, R. T. & Morana, C. (2007), 'Modeling long memory and structural breaks in conditional variances: an adaptive FIGARCH approach', *Working paper No. 593, Queen Mary University of London*.

Black, F. (1976), 'Studies of stock market volatility changes', *Proceedings of the American Statistical Association, Business and Economic Statistics Section* pp. 177–181.

Bollerslev, T. (1986), 'Generalized autoregressive conditional heteroskedasticity', *Journal of Econometrics* **31**, 307–327.

Bollerslev, T., Engle, R. F. & Wooldridge, J. M. (1988), 'A capital asset pricing model with time-varying covariances', *Journal of Political Economy* **96**, 116–131.

Bollerslev, T. & Mikkelsen, H. O. (1996), 'Modeling and pricing long memory in stock market volatility', *Journal of Econometrics* **73**, 151–184.

Breidt, F. J., Crato, N. & de Lima, P. (1998), 'The detection and estimation of long-memory in stochastic volatility', *Journal of Econometrics* **83**, 325–348.

Campbell, J. Y. & Hentschel, L. (1992), 'No news is good news: An asymmetric model of changing volatility in stock returns', *Journal of Financial Economics* **31**, 281–318.

Chou, R., Engle, R. F. & Kane, A. (1992), 'Measuring risk aversion from excess returns on a stock index', *Journal of Econometrics* **52**, 201–224.

Chou, R. Y. (1988), 'Volatility persistence and stock valuations: Some empirical evidence using GARCH', *Journal of Applied Econometrics* **3**, 279–294.

Christensen, B. J. & Nielsen, M. Ø. (2007), 'The effect of long memory in volatility on stock market fluctuations', *Review of Economics and Statistics* **89**, 684–700.

Comte, F. & Renault, E. (1998), 'Long-memory in continuous-time stochastic volatility models', *Mathematical Finance* **8**, 291–323.

Conrad, C. & Mammen, E. (2008), 'Nonparametric regression on latent covariates with an application to semiparametric GARCH-in-mean models', *Working Paper, University of Heidelberg.*

Crato, N. & de Lima, P. (1994), 'Long-range dependence in the conditional variance of stock returns', *Economics Letters* **45**, 281–285.

Ding, Z. & Granger, C. W. J. (1996), 'Modelling volatility persistence of speculative returns: A new approach', *Journal of Econometrics* **73**, 185–215.

Engle, R. F. (1982), 'Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation', *Econometrica* **50**, 987–1006.

Engle, R. F., Lilien, D. M. & Robins, R. P. (1987), 'Estimating time-varying risk premia in the term structure: The ARCH-M model', *Econometrica* **55**, 391–407.

Engle, R. F. & Ng, V. (1993), 'Measuring and testing the impact of news in volatility', *Journal of Finance* **43**, 1749–1778.

Engle, R. F. & Rangel, J. G. (2008), 'The Spline-GARCH model for low-frequency volatility and its global macroeconomic causes', *Review of Financial Studies* **21**, 1187–1222.

Glosten, L. R., Jagannathan, R. & Runkle, D. E. (1993), 'On the relation between the expected value and the volatility of the nominal excess return on stocks', *Journal of Finance* **48**, 1779–1801.

He, C., Silvennoinen, A. & Terasvirta, T. (2008), 'Parameterizing unconditional skewness in models for financial time series', *Journal of Financial Econometrics* **6**, 208–230.

Lettau, M. & Ludvigson, S. C. (2004), Measuring and modeling variation in the risk-return trade-off, *in* Y. Ait-Sahalia & L. P. Hansen, eds, 'Handbook of Financial Econometrics (Forthcoming)', North-Holland, Amsterdam.

Linton, O. & Perron, B. (2003), 'The shape of the risk premium: evidence from a semiparametric generalized autoregressive conditional heteroscedasticity model', *Journal of Business and Economic Statistics* **21**, 354–367.

Merton, R. C. (1973), 'An intertemporal capital asset pricing model', *Econometrica* **41**, 867–887.

Merton, R. C. (1980), 'On estimating the expected return on the market', *Journal of Financial Economics* **8**, 323–361.

Nelson, D. B. (1991), 'Conditional heteroskedasticity in asset returns: A new approach', *Econometrica* **59**, 347–370.

Robinson, P. M. (1991), 'Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regressions', *Journal of Econometrics* **47**, 67–84.

Robinson, P. M. (2001), 'The memory of stochastic volatility models', *Journal of Econometrics* **101**, 195–218.

Scruggs, J. T. (1998), 'Resolving the puzzling intertemporal relation between the market risk premium and conditional variance: a two-factor approach', *Journal of Finance* **53**, 575–603.

Wu, C. F. J. (1986), 'Jackknife, bootstrap and other resampling methods in regression analysis (with discussion)', *Annals of Statistics* **14**, 1261–1350.

Yu, J. (2005), 'On leverage in a stochastic volatility model', *Journal of Econometrics* **127**, 165–178.

Table 1: GARCH models for CRSP value-weighted cum-dividend returns, 1926.1.2–2006.12.29

Parameter	GARCH	IGARCH	Spline-GARCH	FIGARCH	A-FIGARCH	EGARCH	FIEGARCH
μ	7.147×10^{-4}	7.117×10^{-4}	7.285×10^{-4}	7.362×10^{-4}	7.453×10^{-4}	5.530×10^{-4}	5.781×10^{-4}
	(5.193×10^{-5})	(5.621×10^{-5})	(5.087×10^{-5})	(5.132×10^{-5})	(5.063×10^{-4})	(5.488×10^{-5})	(5.211×10^{-4})
ω	6.702×10^{-7}	5.362×10^{-7}	8.020×10^{-7}	2.272×10^{-6}	7.249×10^{-7}	-9.029	-8.941
	(1.128×10^{-7})	(9.173×10^{-8})	(2.253×10^{-7})	(4.276×10^{-8})	(8.747×10^{-8})	(0.1250)	(0.1500)
δ	7.493×10^{-6}	7.029×10^{-6}	5.108×10^{-6}	7.312×10^{-6}	6.639×10^{-6}	0.2328	0.2297
	(1.434×10^{-6})	(1.327×10^{-6})	(1.652×10^{-6})	(1.403×10^{-6})	(1.370×10^{-6})	(0.03698)	(0.03771)
θ	–	–	–	–	–	-0.1093	-0.1151
						(0.01147)	(0.01333)
γ	–	–	–	–	–	0.2321	0.2236
						(0.01769)	(0.01697)
ϕ_1	0.9953	1.000	0.9825	0.2807	0.2752	0.9916	0.7910
	(1.917×10^{-3})		(3.118×10^{-3})	(0.03072)	(0.03235)	(1.302×10^{-3})	(0.07042)
ϕ_2	0.03868	0.04224	0.03462	–	–	–	–
	(0.01627)	(0.01668)	(0.01565)				
ψ_1	-0.9238	-0.9247	-0.9057	-0.5989	-0.5776	-0.4648	-0.5990
	(7.068×10^{-3})	(6.989×10^{-3})	(8.379×10^{-3})	(0.04113)	(0.04737)	(0.05297)	(0.1107)
d	–	–	–	0.4274	0.4179	–	0.5368
				(0.03561)	(0.04060)		(0.02782)
δ_1	–	–	–	–	0.02008	–	–
					(8.339×10^{-3})		
δ_2	–	–	–	–	-0.03152	–	–
					(9.261×10^{-3})		
$\ln L(\eta)$	71,870.97	71,864.58	71,960.76	71,939.22	71,966.12	72,149.68	72,222.39
AIC	−143,729.94	−143,719.17	−143,891.51	−143,866.46	−143,912.23	−144,285.35	−144,428.79
SIC	−143,682.08	−143,679.28	−143,771.86	−143,818.60	−143,832.47	−144,229.52	−144,364.97
Q_{10}	267.76	273.30	285.19	288.56	298.65	255.20	272.61
Q_{100}	364.02	369.32	381.68	390.29	400.43	354.35	379.16
Q^A_{10}	21.91	21.03	18.25	11.39	11.80	42.03	38.68
Q^A_{100}	105.61	106.97	144.29	132.06	146.46	128.93	197.59
Sign Bias	65.31**	64.90**	73.90**	69.32**	71.07**	8.23**	5.35**
Negative Size Bias	40.10**	32.51**	45.95**	44.00**	42.19**	1.64	1.25
Positive Size Bias	35.48**	39.86**	38.37**	35.96**	38.10**	2.77**	1.61
Joint Test	76.36**	74.88**	85.94**	81.03**	82.22**	6.95	5.37
MAFE	2.178×10^{-5}	2.213×10^{-5}	1.693×10^{-4}	2.144×10^{-5}	2.180×10^{-5}	2.258×10^{-5}	1.858×10^{-5}
R^2	0.1978	0.2036	0.0818	0.1649	0.1709	0.2300	0.3052

Note: QML estimates are reported with robust standard errors in parentheses (knot coefficients for Spline-GARCH not reported). Also reported are $\ln L(\eta)$, the value of the maximized log-likelihood function, and the Akaike and Schwarz (or Bayesian) information criteria, respectively. The values of the Ljung-Box portmanteau statistic for up to K'th order serial dependence in the standardized residuals, $\hat{\varepsilon}_t/\hat{\sigma}_t$, and the absolute standardized residuals, $|\hat{\varepsilon}_t/\hat{\sigma}_t|$, are denoted Q_K and Q^A_K, respectively. Finally, we report the Engle & Ng (1993) sign/size bias tests, for which one and two asterisks denote rejection at the 5% and 1% level, respectively, and the mean absolute forecast error (MAFE) and squared correlation (R^2) for 200 one-day-ahead out-of-sample forecasts.

Table 2: GARCH-M models for CRSP value-weighted cum-dividend returns, 1926.1.2–2006.12.29

Parameter	GARCH-M	Spline-GARCH-M	FIGARCH-M	A-FIGARCH-M	EGARCH-M	FIEGARCH-M$_h$	FIEGARCH-M$_g$
μ	9.143×10^{-4}	1.213×10^{-3}	9.587×10^{-4}	9.657×10^{-4}	4.759×10^{-4}	4.727×10^{-4}	4.957×10^{-4}
	(7.040×10^{-5})	(9.279×10^{-5})	(7.080×10^{-5})	(6.924×10^{-5})	(1.088×10^{-4})	(5.649×10^{-5})	(5.354×10^{-5})
λ	-3.824	-1.736	-4.277	-4.289	-6.609×10^{-5}	-3.019×10^{-3}	-3.654×10^{-3}
	(0.8419)	(1.210)	(0.8779)	(0.8612)	(8.182×10^{-5})	(4.442×10^{-4})	(4.563×10^{-4})
ω	6.889×10^{-7}	7.455×10^{-7}	2.308×10^{-6}	7.318×10^{-7}	-8.995	-8.937	-8.990
	(1.166×10^{-7})	(4.601×10^{-7})	(4.415×10^{-7})	(9.038×10^{-8})	(0.1317)	(0.1609)	(0.1471)
δ	7.750×10^{-6}	4.763×10^{-6}	7.567×10^{-6}	6.852×10^{-6}	0.2330	0.2372	0.2365
	(1.463×10^{-6})	(2.805×10^{-6})	(1.433×10^{-6})	(1.409×10^{-6})	(0.03698)	(0.03931)	(0.03906)
θ	–	–	–	–	-0.1106	-0.1364	-0.1285
					(0.01229)	(0.01342)	(0.01263)
γ	–	–	–	–	0.2326	0.2231	0.2068
					(0.01761)	(0.01327)	(0.01482)
ϕ_1	0.9948	0.9802	0.2815	0.2750	0.9919	0.7842	0.7337
	(1.939×10^{-3})	(3.384×10^{-3})	(0.03087)	(0.03272)	(1.350×10^{-3})	(0.06595)	(0.08663)
ϕ_2	0.03960	0.03688	–	–	–	–	–
	(0.01628)	(0.01541)					
ψ_1	-0.9232	-0.9035	-0.5925	-0.5698	-0.4664	-0.6364	-0.4976
	(7.118×10^{-3})	(8.717×10^{-3})	(0.04148)	(0.04781)	(0.05282)	(0.09476)	(0.1327)
d	–		0.4213	0.4113	–	0.5676	0.5475
			(0.03544)	(0.04008)		(0.02308)	(0.02554)
δ_1	–	–	–	0.02104	–	–	–
				(8.308×10^{-3})			
δ_2	–	–	–	-0.03181	–	–	–
				(9.302×10^{-3})			
$\ln L(\eta)$	71,882.63	71,990.86	71,953.29	71,980.50	72,150.14	72,267.63	72,280.53
AIC	$-143,751.27$	$-143,949.71$	$-143,892.57$	$-143,942.99$	$-144,284.27$	$-144,517.26$	$-144,543.05$
SIC	$-143,695.43$	$-143,822.09$	$-143,836.74$	$-143,871.20$	$-144,220.46$	$-144,445.47$	$-144,471.26$
Q_{10}	268.84	380.78	290.60	300.83	252.16	127.94	102.13
Q_{100}	365.57	722.52	393.01	403.70	351.04	233.69	208.25
Q^A_{10}	20.65	31.35	12.26	13.05	42.67	45.04	34.83
Q^A_{100}	103.33	147.18	134.36	149.71	130.04	231.86	239.79
Sign Bias	60.71**	24.82**	63.21**	64.50**	7.05**	1.86	2.62**
Negative Size Bias	40.11**	20.69**	43.37**	41.29**	1.48	2.84×10^{-4}	0.58
Positive Size Bias	29.97**	0.12	29.31**	30.83**	2.64**	0.10	0.07
Joint Test	71.16**	33.49**	74.19**	74.59**	7.19	2.77	3.04
MAFE	2.176×10^{-5}	1.537×10^{-4}	2.145×10^{-5}	2.115×10^{-5}	2.253×10^{-5}	1.862×10^{-5}	1.884×10^{-5}
R^2	0.2037	0.0094	0.1697	0.1761	0.2312	0.3051	0.3122

Note: QML estimates are reported for models with in-mean terms, using the same definitions and layout as Table 1. M$_h$ applies h_t in the mean equation, M$_g$ applies $g(z_{t-1})$, the first four models apply σ_t^2, and EGARCH-M applies $\ln(\sigma_t^2) - \omega - \ln(1 + \delta N_t)$ in the mean equation.

Table 3: GARCH-M models with lagged returns for CRSP value-weighted cum-dividend returns, 1926.1.2–2006.12.29

Parameter	GARCH-M	Spline-GARCH-M	FIGARCH-M	A-FIGARCH-M	EGARCH-M	FIEGARCH-M$_h$	FIEGARCH-M$_g$
μ_0	8.283×10^{-4}	1.089×10^{-3}	8.744×10^{-4}	8.788×10^{-4}	4.382×10^{-4}	4.848×10^{-4}	4.882×10^{-4}
	(6.803×10^{-5})	(9.259×10^{-5})	(6.860×10^{-5})	(6.588×10^{-5})	(9.954×10^{-5})	(5.221×10^{-5})	(5.133×10^{-5})
μ_1	0.1119	0.1100	0.1141	0.1332	0.1003	0.1037	0.1036
	(7.764×10^{-3})	(7.591×10^{-3})	(7.668×10^{-3})	(7.993×10^{-3})	(5.911×10^{-3})	(8.863×10^{-3})	(9.798×10^{-3})
μ_2	−0.04510	−0.04598	−0.04545	−0.04492	−0.02117	−0.01657	−0.01717
	(7.502×10^{-3})	(7.441×10^{-3})	(7.498×10^{-3})	(7.482×10^{-3})	(8.653×10^{-3})	(7.997×10^{-3})	(0.01092)
μ_3	8.502×10^{-3}	5.002×10^{-3}	5.754×10^{-3}	8.995×10^{-3}	7.563×10^{-3}	-2.155×10^{-3}	-2.259×10^{-3}
	(7.593×10^{-3})	(7.478×10^{-3})	(7.633×10^{-3})	(7.569×10^{-3})	(6.188×10^{-3})	(7.697×10^{-3})	(8.741×10^{-3})
λ_1	−9.457	−4.860	−9.080	−6.712	-7.461×10^{-4}	-7.795×10^{-4}	-7.908×10^{-4}
	(4.471)	(3.887)	(4.361)	(4.508)	(4.575×10^{-4})	(4.449×10^{-4})	(4.642×10^{-4})
λ_2	7.698	2.855	7.472	7.126	1.321×10^{-3}	1.284×10^{-3}	1.073×10^{-3}
	(3.882)	(2.573)	(4.061)	(4.255)	(4.584×10^{-4})	(4.305×10^{-3})	(4.163×10^{-4})
λ_3	−1.645	−0.2449	−2.256	−4.391	-5.969×10^{-4}	-6.978×10^{-4}	-5.436×10^{-4}
	(4.262)	(1.469)	(4.283)	(4.268)	(4.221×10^{-4})	(3.416×10^{-4})	(3.396×10^{-4})
ω	6.773×10^{-7}	5.137×10^{-7}	2.341×10^{-6}	6.793×10^{-7}	−9.036	−8.956	−8.959
	(1.173×10^{-7})	(4.272×10^{-7})	(4.471×10^{-6})	(8.968×10^{-7})	(0.1266)	(0.1479)	(0.1476)
δ	7.653×10^{-6}	3.237×10^{-6}	7.428×10^{-6}	6.369×10^{-6}	0.2240	0.2191	0.2194
	(1.579×10^{-6})	(2.914×10^{-6})	(1.548×10^{-6})	(1.542×10^{-6})	(0.03741)	(0.03795)	(0.03793)
θ	–	–	–	–	−0.1122	−0.1189	−0.1187
					(0.01183)	(0.01347)	(0.01332)
γ	–	–	–	–	0.2225	0.2141	0.2141
					(0.01695)	(0.01647)	(0.01622)
ϕ_1	0.9950	0.9803	0.2670	0.2496	0.9915	0.7232	0.7226
	(1.938×10^{-3})	(3.747×10^{-3})	(0.03168)	(0.03487)	(1.382×10^{-3})	(0.08853)	(0.08785)
ϕ_2	0.03544	0.03048	–	–	–	–	–
	(0.01631)	(0.01519)					
ψ_1	−0.9227	−0.9024	−0.4192	−0.5346	−0.4388	−0.4803	−0.4791
	(7.611×10^{-3})	(9.598×10^{-3})	(0.03509)	(0.04963)	(0.05541)	(0.1372)	(0.1349)
d	–	–	0.5803	0.3937	–	0.5472	0.5470
			(0.04338)	(0.03591)		(0.02654)	(0.02642)
δ_1	–	–	–	0.02744	–	–	–
				(8.453×10^{-3})			
δ_2	–	–	–	−0.02825	–	–	–
				(9.182×10^{-3})			
$\ln L(\eta)$	72,008.65	72,116.28	72,083.13	72,136.59	72,269.34	72,353.62	72,353.64
AIC	−143,993.30	−144,190.55	−144,142.26	−144,245.17	−144,512.68	−144,679.24	−144,679.27
SIC	−143,897.58	−144,023.04	−144,046.54	−144,133.50	−144,408.99	−144,567.57	−144,567.60
Q_{10}	18.33	62.13	21.66	16.46	15.73	17.12	17.16
Q_{100}	112.62	318.59	121.28	117.03	112.48	121.53	121.56
Q_{10}^A	25.01	30.18	13.18	14.87	40.97	37.01	37.00
Q_{100}^A	110.00	133.73	135.12	146.09	128.57	199.97	199.88
Sign Bias	48.93**	38.27**	50.95**	49.99**	5.08**	4.08**	4.13**
Negative Size Bias	42.16**	69.43**	46.55**	44.06**	1.75	1.39	1.41
Positive Size Bias	27.46**	4.82**	27.83**	25.90**	0.99	0.68	0.69
Joint Test	64.17**	73.34**	68.09**	65.16**	5.14	4.16	4.20
MAFE	2.141×10^{-5}	3.268×10^{-5}	2.075×10^{-5}	2.683×10^{-5}	2.361×10^{-5}	1.861×10^{-5}	1.861×10^{-5}
R^2	0.1946	0.0035	0.1777	0.1759	0.1733	0.2982	0.2982

Note: QML estimates are reported for models with lagged returns and in-mean terms, using the same definitions and layout as Table 2. All models include three volatility-in-mean terms and three lagged returns.

Table 4: FIEGARCH-M models with lagged returns and bootstrap standard errors for CRSP value-weighted cum-dividend returns, 1926.1.2–2006.12.29

Parameter	FIEGARCH-M$_h$	FIEGARCH-M$_g$
μ_0	4.848×10^{-4}	4.882×10^{-4}
	(5.221×10^{-5})	(5.133×10^{-5})
	(7.296×10^{-5})	(4.854×10^{-5})
μ_1	0.1037	0.1036
	(8.863×10^{-3})	(9.798×10^{-3})
	(7.811×10^{-3})	(8.596×10^{-3})
μ_2	−0.01657	−0.01717
	(7.997×10^{-3})	(0.01092)
	(8.759×10^{-3})	(9.133×10^{-3})
μ_3	-2.155×10^{-3}	-2.259×10^{-3}
	(7.697×10^{-3})	(8.741×10^{-3})
	(8.066×10^{-3})	(8.402×10^{-3})
λ_1	-7.795×10^{-4}	-7.908×10^{-4}
	(4.449×10^{-4})	(4.642×10^{-4})
	(4.218×10^{-4})	(3.784×10^{-4})
λ_2	1.284×10^{-3}	1.073×10^{-3}
	(4.305×10^{-4})	(4.163×10^{-4})
	(4.703×10^{-4})	(3.984×10^{-4})
λ_3	-6.978×10^{-4}	-5.436×10^{-4}
	(3.416×10^{-4})	(3.396×10^{-4})
	(3.498×10^{-4})	(3.526×10^{-4})
ω	−8.956	−8.959
	(0.1479)	(0.1476)
	(0.1128)	(0.06511)
δ	0.2191	0.2194
	(0.03795)	(0.03793)
	(0.01642)	(3.823×10^{-3})
θ	−0.1189	−0.1187
	(0.01347)	(0.01332)
	(8.154×10^{-3})	(0.01029)
γ	0.2141	0.2141
	(0.01647)	(0.01622)
	(0.01425)	(0.01018)
ϕ_1	0.7232	0.7226
	(0.08853)	(0.08785)
	(0.09775)	(0.04792)
ψ_1	−0.4803	−0.4791
	(0.1372)	(0.1349)
	(0.06773)	(0.08151)
d	0.5472	0.5470
	(0.02654)	(0.02642)
	(0.02688)	(0.02117)

Note: QML estimates are reported with robust standard errors in the first parentheses, and wild bootstrap standard errors in the second parenthesis.

Table 5: FIEGARCH-M models for CRSP value-weighted cum-dividend returns, 1926.1.2–2006.12.29

Parameter	FIEGARCH-M$_h$	FIEGARCH-M$_g$	FIEGARCH
μ_0	4.812×10^{-4} (5.273×10^{-5})	5.024×10^{-4} (4.911×10^{-5})	4.804×10^{-4} (5.019×10^{-5})
μ_1	0.1019 (7.488×10^{-3})	0.1008 (3.542×10^{-3})	0.1134 (5.923×10^{-3})
μ_2	–	–	−0.03377 (4.079×10^{-3})
μ_3	–	–	7.074×10^{-3} (5.267×10^{-3})
λ_1	-8.357×10^{-4} (4.311×10^{-4})	-8.896×10^{-4} (3.796×10^{-4})	–
λ_2	1.676×10^{-3} (3.537×10^{-4})	1.456×10^{-3} (3.398×10^{-4})	–
λ_3	-7.505×10^{-4} (3.321×10^{-4})	–	–
ω	−8.972 (0.1473)	−8.989 (0.1449)	−8.945 (0.1487)
δ	0.2175 (0.03782)	0.2183 (0.03746)	0.2176 (0.03792)
θ	−0.1206 (0.01165)	−0.1208 (0.01271)	−0.1180 (0.01318)
γ	0.2116 (0.01342)	0.2115 (0.01476)	0.2153 (0.01633)
ϕ_1	0.7210 (0.08537)	0.7142 (0.09048)	0.7250 (0.08976)
ψ_1	−0.4739 (0.1242)	−0.4679 (0.1384)	−0.4826 (0.1381)
d	0.5466 (0.02631)	0.5482 (0.02616)	0.5446 (0.02687)
$\ln L(\eta)$	72, 351.71	72, 350.42	72, 345.84
AIC	−144, 679.42	−144, 678.85	−144, 669.69
SIC	−144, 583.70	−144, 591.11	−144, 581.94
Q_{10}	20.97	22.23	17.64
Q_{100}	126.14	127.59	121.75
Q^A_{10}	35.44	35.00	36.73
Q^A_{100}	199.60	199.53	198.42
Sign Bias	1.72	1.78	4.13^{**}
Negative Size Bias	1.15	1.12	1.50
Positive Size Bias	0.63	0.61	0.78
Joint Test	3.12	3.32	4.19
MAFE	1.864×10^{-5}	1.867×10^{-5}	1.851×10^{-5}
R^2	0.2993	0.2999	0.2996

Note: QML estimates are reported for the final models, using the same layout and definitions as Table 2.